

A survey of gravity field modeling applications of the Input–Output System Theory (IOST)

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Abstract

A review of the theoretical background of the Input–Output System Theory (IOST) is presented. The simplest case of a single input–single output system as well as its generalization to the multiple input–multiple output system are analyzed. Special emphasis is given to the output accuracy estimates through the construction of the output error Power Spectral Density (PSD). Some remarks on the data preprocessing are also given, and different cases of input noise generation are itemized. Finally, the presentation of various applications related to gravity field modeling are analyzed and some conclusions on the optimum use of the method are drawn.

1 Introduction

The use of spectral techniques in gravity field modeling has been extremely popular over the past two decades. The algebraic simplicity of the convolution integrals in the frequency domain led to the development of various applications in the area of gravity field approximation. Spectral methods have been used for the terrain correction convolution integrals evaluation, e.g., Sideris (1984), Li (1993); in geoid determination, e.g., Sideris and Li (1993), Forsberg and Sideris (1993), Strang van Hees (1990), Haagmans et al. (1993), Tziavos (1993), Tziavos (1995); in computations with the Vening-Meinesz formulas, e.g., Kearsley et al. (1985), Liu et al. (1997), Tziavos and Andritsanos (1998); and in Molodensky's problem solution, e.g., Sideris (1987). The use of spectral techniques in physical geodesy is summarized in Schwarz et al. (1990).

In all the above applications, homogeneous noiseless data were used. This assumption was recently overcome by the heterogeneous data combination and the appropriate error propagation presented by Sideris (1996) and Sansò and Sideris (1997). The Input–Output System Theory (IOST) is based on the analysis given by Bendat and Piersol (1986) and the proper adaptation in gravity field studies is discussed in Sideris (1996). Simulation studies based on IOST were presented in Li (1996); Tziavos et al. (1997); Tziavos et al. (1996); Tziavos et al. (1996); Tziavos et al. (1998). The application of the method to airborne gravimetry data processing was presented by Wu and Sideris (1995), Wu (1996) and Wu and Sideris (1997). An analysis of the basic similarities and differences between IOST and Least-Squares Collocation (LSC) was presented by Sansò and Sideris (1997). A generalization of the IOST to the Multiple Input–Multiple Output System Theory (MIMOST) was carried out by Andritsanos and Tziavos (1999), where some simulation studies were performed in the Labrador Sea. Finally, a real data application to the Quasi-stationary Sea Surface Topography (QSST) estimation, using pre-determined error covariances by a multiple track analysis of the recent altimetry missions, was discussed in Andritsanos et al. (1999).

In the present paper, we review the theoretical background of the IOST, present some gravity field modeling schemes based on this analysis, discuss some critical points on the data properties, the PSD computations and the input noise treatment, and present some numerical examples of the method. Some recommendations on the improvement of the method are given in the final part of the paper.

2 Theoretical background

2.1 Single input-single output system

A single input-single output system with noise is presented in Fig. 1, where Y_o is the spectrum of the input observation y_o , Y is the spectrum of the input signal y , M is the spectrum of the input noise m , X is the spectrum of the output signal x and E is the spectrum of the output error e . The frequency response function $H_{x_o y_o}$ is the connecting operator between the output and the input. The relationship between the input and the output is given in the

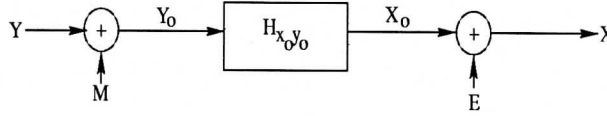


Figure 1: Single input / single output system with noise

spectral domain as:

$$X_o = H_{x_o y_o} Y_o \Rightarrow \quad (1)$$

$$X - E = H_{x_o y_o} (Y + M) \quad (2)$$

The determination of the optimum frequency response function is carried out by minimizing the Power Spectral Density (PSD) of the output error. The procedure is described in Bendat and Piersol (1986):

$$E = X - H_{x_o y_o} (Y + M) \Rightarrow \quad (3)$$

$$EE^* = [X - H_{x_o y_o} (Y + M)][X - H_{x_o y_o} (Y + M)]^* \Rightarrow \quad (4)$$

$$P_{ee} = P_{xx} - H_{x_o y_o}^* P_{xy_o} - H_{x_o y_o} P_{y_o x} + H_{x_o y_o} P_{y_o y_o} H_{x_o y_o}^* \quad (5)$$

In the above equations, P represents the PSD of the quantity noted by the respective subscript. The expectation operator needed to arrive at eq. (4) from eq. (5) is omitted in all PSD computation formulas for the simplicity of the presentation. The minimization of P_{ee} is done by the nullification of the partial derivative with respect to the complex conjugate of the frequency response function:

$$\frac{\partial P_{ee}}{\partial H_{x_o y_o}^*} = 0 \Rightarrow \quad (6)$$

$$-P_{xy_o} + P_{y_o y_o} \hat{H}_{x_o y_o} \Rightarrow \hat{H}_{x_o y_o} = \frac{P_{xy_o}}{P_{y_o y_o}} \quad (7)$$

After the determination of the optimum frequency response function the estimation of the output $\hat{X}_o$ is given by:

$$\hat{X}_o = \hat{H}_{x_o y_o} Y_o. \quad (8)$$

The information related to the estimation accuracy of the output is derived using the optimum transfer function and the known function H_{xy} which connects theoretically the input

and the output signals. The following series of equations is used:

$$\hat{E} = X - \hat{X}_o \Rightarrow \hat{E}\hat{E}^* = XX^* - X\hat{X}_o^* - \hat{X}_oX^* + \hat{X}_o\hat{X}_o^* \Rightarrow \quad (9)$$

$$P_{\hat{E}\hat{E}} = H_{xy}H_{xy}^*P_{yy} - H_{xy}\hat{H}_{x_o y_o}^*P_{y_o y_o} - \hat{H}_{x_o y_o}H_{xy}^*P_{y_o y_o} + \hat{H}_{x_o y_o}\hat{H}_{x_o y_o}^*P_{y_o y_o} \quad (10)$$

The derivation of $P_{y_o y}$ or $P_{y_y y_o}$ is simple under the assumption of uncorrelated input signals with input noises as follows:

$$P_{y_o y} = (Y + M)Y^* = YY^* + MY^* = P_{yy}. \quad (11)$$

Additionally, the input-output PSD can be derived using the known function H_{xy} as follows:

$$P_{xy_o} = XY_o^* = X(Y^* + M^*) = XY^* + XM^* = H_{xy}YY^* + H_{xy}YM^* = H_{xy}(P_{y_o y_o} - P_{mm}). \quad (12)$$

Under the above mentioned assumption, the final equations for the estimation of the output and the computation of the output error PSD are:

$$\hat{X}_o = \frac{H_{xy}(P_{y_o y_o} - P_{mm})}{P_{y_o y_o}}Y_o \quad (13)$$

$$P_{\hat{E}\hat{E}} = H_{xy}P_{yy}(H_{xy}^* - \hat{H}_{x_o y_o}^*) - \hat{H}_{x_o y_o}P_{y_o y_o}(H_{xy}^* - \hat{H}_{x_o y_o}^*) + \hat{H}_{x_o y_o}P_{mm}H_{xy}^* \Rightarrow \quad (14)$$

$$P_{\hat{E}\hat{E}} = [H_{xy}(P_{y_o y_o} - P_{mm}) - \hat{H}_{x_o y_o}P_{y_o y_o}](H_{xy}^* - \hat{H}_{x_o y_o}^*) + \hat{H}_{x_o y_o}P_{mm}H_{xy}^*. \quad (15)$$

The computation of $P_{y_o y_o}$ can be done directly from the observed data following the periodogram approach (Marple, 1987). As it is seen from the above equations, the input noise PSD P_{mm} must be known. This is the major difficulty in the application of the method. In practice, only the variances of the measurements are known. A more detailed discussion of this problem is presented in Section 2.4.

2.2 Multiple input-multiple output system

The extension of the single input-single output system to the multiple input-multiple output system is depicted in Fig. 2, where y_{io} are the input observations, y_i are the pure input signals, m_i are the input noises, x_j are the unknown output signals and e_j are the output noises. The total number of input data i may be equal or different from the number of output signals j . The interaction between every input and output is also presented in this figure. In this case, matrix notation is applied for the system description. According to

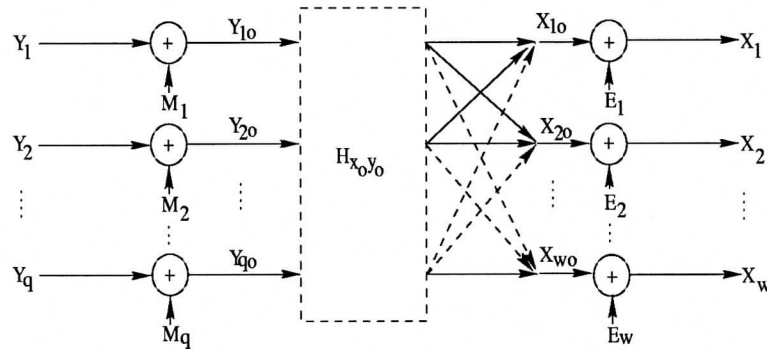


Figure 2: Multiple input-multiple output system with noise

Andritsanos and Tziavos (1999) and Andritsanos et al. (1999), the input and output vectors are:

$$\mathbf{Y}_o = \begin{bmatrix} Y_1 + M_1 \\ Y_2 + M_2 \\ \vdots \\ Y_q + M_q \end{bmatrix} \quad \mathbf{X}_o = \begin{bmatrix} X_1 - E_1 \\ X_2 - E_2 \\ \vdots \\ X_w - E_w \end{bmatrix}, \quad (16)$$

where capital letters stand for the spectra of the respective quantities. The transfer function matrix is of the form:

$$\mathbf{H}_{x_o y_o} = \begin{bmatrix} H_{x_{1o} y_{1o}} & H_{x_{1o} y_{2o}} & \cdots & H_{x_{1o} y_{qo}} \\ H_{x_{2o} y_{1o}} & H_{x_{2o} y_{2o}} & \cdots & H_{x_{2o} y_{qo}} \\ \vdots & \vdots & \ddots & \vdots \\ H_{x_{wo} y_{1o}} & H_{x_{wo} y_{2o}} & \cdots & H_{x_{wo} y_{qo}} \end{bmatrix}, \quad (17)$$

which clearly shows the dependence of each output on all inputs. Following the same procedure as previously the output error PSD matrix is (Andritsanos et al., 1999):

$$\begin{aligned} \mathbf{E}\mathbf{E}^{*T} &= \mathbf{X}\mathbf{X}^{*T} - \mathbf{X}(\mathbf{Y}^{*T} + \mathbf{M}^{*T})\mathbf{H}_{x_o y_o}^{*T} - \mathbf{H}_{x_o y_o}(\mathbf{Y} + \mathbf{M})\mathbf{X}^{*T} \\ &\quad + \mathbf{H}_{x_o y_o}(\mathbf{Y} + \mathbf{M})(\mathbf{Y}^{*T} + \mathbf{M}^{*T})\mathbf{H}_{x_o y_o}^{*T} \Rightarrow \end{aligned} \quad (18)$$

$$\mathbf{P}_{ee} = \mathbf{P}_{xx} - \mathbf{P}_{xy}\mathbf{H}_{x_o y_o}^{*T} - \mathbf{H}_{x_o y_o}\mathbf{P}_{y_o x} + \mathbf{H}_{x_o y_o}\mathbf{P}_{y_o y_o}\mathbf{H}_{x_o y_o}^{*T} \quad (19)$$

and the optimum transfer function matrix is derived by the minimization of the error PSD matrix with the assumption of uncorrelated input signals with noises:

$$\hat{\mathbf{H}}_{x_o y_o} = \mathbf{P}_{xy}\mathbf{P}_{y_o y_o}^{-1} = \mathbf{H}_{xy}(\mathbf{P}_{y_o y_o} - \mathbf{P}_{mm})\mathbf{P}_{y_o y_o}^{-1}. \quad (20)$$

Of course, one cannot minimize a matrix like $\mathbf{P}_{ee}$ in general, yet the specific result is the same as minimizing $\text{tr}(\mathbf{P}_{ee})$ and in any way it provides a true minimum of the family of matrices (19). The output vector and the output error PSD matrix are as follows:

$$\hat{\mathbf{X}}_o = \hat{\mathbf{H}}_{x_o y_o} \mathbf{Y}_o = \mathbf{P}_{xy}\mathbf{P}_{y_o y_o}^{-1} \mathbf{Y}_o = \mathbf{H}_{xy}(\mathbf{P}_{y_o y_o} - \mathbf{P}_{mm})\mathbf{P}_{y_o y_o}^{-1} \mathbf{Y}_o \quad (21)$$

$$\mathbf{P}_{\hat{\mathbf{e}}\hat{\mathbf{e}}} = [\mathbf{H}_{xy}(\mathbf{P}_{y_o y_o} - \mathbf{P}_{mm}) - \hat{\mathbf{H}}_{x_o y_o}\mathbf{P}_{y_o y_o}] (\mathbf{H}_{xy}^{*T} - \hat{\mathbf{H}}_{x_o y_o}^{*T}) + \hat{\mathbf{H}}_{x_o y_o}\mathbf{P}_{mm}\mathbf{H}_{xy}^{*T} \quad (22)$$

In gravity field modeling applications, both the input-output PSD $\mathbf{P}_{xy}$ and the output signal are unknown. In this specific case, the evaluation of the input-output PSD matrix is possible only if the input noise PSD matrix is known. Then $\mathbf{P}_{xy}$ can be computed by:

$$\mathbf{P}_{xy} = \mathbf{H}_{xy}\mathbf{P}_{yy} = \mathbf{H}_{xy}(\mathbf{P}_{y_o y_o} - \mathbf{P}_{mm}), \quad (23)$$

where $\mathbf{H}_{xy}$ is the transfer function matrix which connects the pure input and output signals.

The similarities between MIMOS theory and LSC were presented by Sansò and Sideris (1997) and can be observed by a direct comparison of the solution equation presented here and the final solution equation presented by Moritz (1980).

2.3 Data preprocessing

A careful preprocessing of the data is needed for the efficient application of the IOST to gravity field modeling applications. The spectral characteristics of the method impose the observation preprocessing. The common problems of the discrete Fourier transform are present. Data reductions and terrain corrections must be applied in order to minimize aliasing effects and the contribution of a global geopotential model must be subtracted for spectral leakage minimization. Zero padding must be applied prior to the transform

procedure for the minimization of circular convolution effects. For an accurate PSD computation, the mean value of the data set must be subtracted and restored as DC value to the PSD computation results. Bendat and Piersol (1986) proposed an additional trend removal. Nevertheless, a trend should be removed only if it is physically expected or clearly apparent in the data.

2.4 Input noise treatment

As it is observed from (21), the solution can be derived only if the input noise PSD P_{mm} is known. This is not the case in practice, since only the variances of the measurements are known and not the errors themselves. Therefore, a direct computation of the input noise PSD following the periodogram approach is not feasible. Different PSD estimation techniques were analyzed by Marple (1987), Kay (1987) and Andritsanos and Tziavos (1999), including classical non-parametric and modern parametric methods. The weakness of direct input noise PSD estimation is the major difficulty in the application of the method in gravity field modeling. Nevertheless, some procedures are proposed here in order to overcome this disadvantage.

1. *Noise simulation*: Assumptions for the input errors are introduced into the IOST. A noise field is simulated using a random number generator. Normal as well as uniform distribution can be chosen and the variance of the field can be selected from some a-priori information on the observation accuracy. The input error PSD is then computed by the periodogram method using FFT (Marple, 1987).
2. *PSD models*: Instead of the assumption made for the errors themselves, one can accept some models for the input noise PSD directly. White noise PSD models (constant value noise PSD) as well as colored noise models (non-constant value noise PSD) can be used.
3. *Estimation of input error PSD - Altimetry case*: The repeated track configuration of the recent altimetry missions provide a PSD estimation procedure. The original idea is due to Sailor (1994). Analytical equations and the procedure of the input noise and signal PSD estimation are given in Andritsanos et al. (1999), where a QSST model is computed using the repeat track information.

2.5 Advantages and drawbacks of the method

One of the main advantages of the IOST is that the spectral characteristics of the data are used. This spectral technique leads to a fast and efficient handling of large amounts of data. With this new spectral method, heterogeneous data can be combined and their error can be propagated to the results. This is possible through a proper modification of the transfer function. The frequency impulse response function which connects the input with the output data can be estimated by the minimization of the noise-to-signal ratio. With this procedure, input noises are filtered out and error estimates for the predicted results are provided through the output error PSD estimation. Another advantage is the use of smaller matrix dimensions in comparison to space domain techniques (Sideris, 1996). The technique is based on matrix division (frequency-by-frequency division) in the spectral domain rather than complicated matrix inversion in the spatial domain.

A major drawback of the method is the requirement of a-priori known input error PSD. This can be overcome by some simulated noise field or PSD models as well as the proper consideration of the repeated configuration in altimetry cases. Some assumptions for the signal and the noise are also needed. These assumptions are based on the data stationarity and the uncorrelateness between the input signal and noise. The simply algebraic equations described in the previous sections become complicated integral equations when the

stationarity assumption is eliminated. This problem is discussed in detail in Sansò and Sideris (1997).

3 Applications to gravity field modeling

3.1 Single input–single output system (Stokes application)

The application of IOST to geoid approximation is depicted in Fig. 3. The single input–

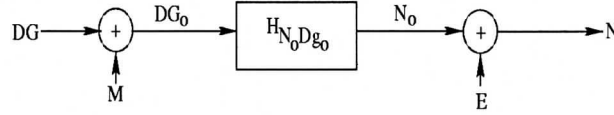


Figure 3: IOST geoid determination application

single output system can be solved using the derivations of Section 2.1. The input observations are gravity anomaly data and the output are geoid heights. The required input error PSD is usually approximated by a stationary model. The gravity anomalies are referred to a global geopotential model and corrected for terrain effects. The relationships between the input–output signal and noise are:

$$\Delta g_o = \Delta g + m \quad (24)$$

$$N_o = N - e \quad (25)$$

The optimal frequency impulse response function can be estimated by:

$$\hat{H}_{N_o \Delta g_o} = \frac{P_{N \Delta g}}{P_{\Delta g_o \Delta g_o}}. \quad (26)$$

The input–output PSD $P_{N \Delta g}$ can be computed in this case using the well known theoretical connection between pure input and output signals. Let $H_{N \Delta g}$ be the Stokes's operator in the frequency domain. The following equation holds:

$$P_{N \Delta g} = N \Delta G^* = H_{N \Delta g} \Delta G \Delta G^* = H_{N \Delta g} P_{\Delta g \Delta g} = H_{N \Delta g} (P_{\Delta g_o \Delta g_o} - P_{mm}). \quad (27)$$

The output geoid estimate can be computed by:

$$\hat{N}_o = \hat{H}_{N_o \Delta g_o} \Delta G_o = \frac{H_{N \Delta g} (P_{\Delta g_o \Delta g_o} - P_{mm})}{P_{\Delta g_o \Delta g_o}} \Delta G_o \quad (28)$$

The output geoid error PSD is given by:

$$P_{\hat{e}\hat{e}} = \left[H_{N \Delta g} (P_{\Delta g_o \Delta g_o} - P_{mm}) - \hat{H}_{N_o \Delta g_o} P_{\Delta g_o \Delta g_o} \right] (H_{N \Delta g}^* - \hat{H}_{N_o \Delta g_o}^*) + \hat{H}_{N_o \Delta g_o} P_{mm} H_{N \Delta g}^* \quad (29)$$

The application of this model was first discussed in Sideris (1996).

3.2 Multiple input–multiple output system (Data filtering)

The two input–two output system of Fig. 4 represents a filtering procedure of gravity and geoid observations. Noisy input data are introduced into the system and the result is the estimate of the output vector using the optimum transfer matrix. In this case, vector and

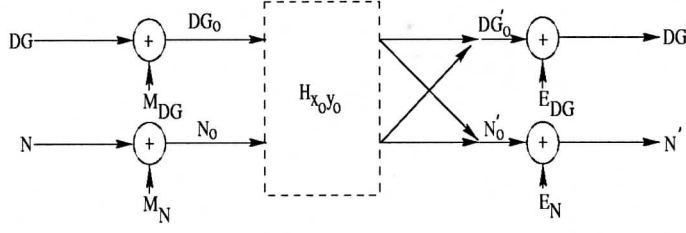


Figure 4: IOST data filtering application

matrix notations must be used in order to combine the two input signals and consider their interrelations. The output and input vector are:

$$\mathbf{x}_o = \begin{bmatrix} \Delta G'_o \\ N'_o \end{bmatrix} \quad \mathbf{y}_o = \begin{bmatrix} \Delta G_o \\ N_o \end{bmatrix}, \quad (30)$$

where primed quantities denote the output data. The optimum frequency impulse response function is:

$$\hat{\mathbf{H}}_{\mathbf{x}_o \mathbf{y}_o} = \mathbf{P}_{\mathbf{xy}} \mathbf{P}_{\mathbf{y}_o \mathbf{y}_o}^{-1}, \quad (31)$$

where, in this specific case,

$$\begin{aligned} \mathbf{P}_{\mathbf{xy}} &= \begin{bmatrix} P_{\Delta g' \Delta g} & P_{\Delta g' N} \\ P_{N' \Delta g} & P_{N' N} \end{bmatrix} = \begin{bmatrix} P_{\Delta g \Delta g} & H_{N' \Delta g} P_{\Delta g \Delta g} \\ H_{N' \Delta g} P_{\Delta g \Delta g} & P_{NN} \end{bmatrix} = \\ &= \begin{bmatrix} (P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}) & H_{N' \Delta g} (P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}) \\ H_{N' \Delta g} (P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}) & (P_{N_o N_o} - P_{m_N m_N}) \end{bmatrix} \end{aligned} \quad (32)$$

The inverse of the observation auto-PSD is given directly by:

$$\mathbf{P}_{\mathbf{y}_o \mathbf{y}_o}^{-1} = \frac{1}{\Delta} \begin{bmatrix} P_{N_o N_o} & -P_{\Delta g_o N_o} \\ -P_{N_o \Delta g_o} & P_{\Delta g_o \Delta g_o} \end{bmatrix}, \quad (33)$$

where $\Delta = P_{\Delta g_o \Delta g_o} P_{N_o N_o} - P_{N_o \Delta g_o} P_{\Delta g_o N_o}$. The optimum transfer function follows:

$$\begin{aligned} \hat{\mathbf{H}}_{\mathbf{x}_o \mathbf{y}_o} &= \begin{bmatrix} \hat{H}_{\Delta g'_o \Delta g_o} & \hat{H}_{\Delta g'_o N_o} \\ \hat{H}_{N'_o \Delta g_o} & \hat{H}_{N'_o N_o} \end{bmatrix} = \frac{P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}}{\Delta} \begin{bmatrix} 1 & H_{N' \Delta g} \\ H_{N' \Delta g} & \frac{P_{N_o N_o} - P_{m_N m_N}}{P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}} \end{bmatrix} \times \\ &\quad \begin{bmatrix} P_{N_o N_o} & -P_{\Delta g_o N_o} \\ -P_{N_o \Delta g_o} & P_{\Delta g_o \Delta g_o} \end{bmatrix}. \end{aligned} \quad (34)$$

The final solution and the output error PSD are given in matrix notation:

$$\begin{aligned} \begin{bmatrix} \Delta G'_o \\ N'_o \end{bmatrix} &= \frac{P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}}{\Delta} \begin{bmatrix} 1 & H_{N' \Delta g} \\ H_{N' \Delta g} & \frac{P_{N_o N_o} - P_{m_N m_N}}{P_{\Delta g_o \Delta g_o} - P_{m_{\Delta g} m_{\Delta g}}} \end{bmatrix} \times \\ &\quad \begin{bmatrix} P_{N_o N_o} & -P_{\Delta g_o N_o} \\ -P_{N_o \Delta g_o} & P_{\Delta g_o \Delta g_o} \end{bmatrix} \begin{bmatrix} \Delta G_o \\ N_o \end{bmatrix} \end{aligned} \quad (35)$$

$$\begin{aligned} \mathbf{P}_{\hat{\mathbf{e}} \hat{\mathbf{e}}} &= \begin{bmatrix} P_{\hat{e}_{\Delta g} \hat{e}_{\Delta g}} & P_{\hat{e}_{\Delta g} \hat{e}_N} \\ P_{\hat{e}_N \hat{e}_{\Delta g}} & P_{\hat{e}_N \hat{e}_N} \end{bmatrix} = \\ &\quad \left\{ \begin{bmatrix} 1 & H_{\Delta g' N} \\ H_{N' \Delta g} & 1 \end{bmatrix} \left(\begin{bmatrix} P_{\Delta g_o \Delta g_o} & P_{\Delta g_o N_o} \\ P_{N_o \Delta g_o} & P_{N_o N_o} \end{bmatrix} - \begin{bmatrix} P_{m_{\Delta g} m_{\Delta g}} & 0 \\ 0 & P_{m_N m_N} \end{bmatrix} \right) - \right. \end{aligned}$$

$$\begin{aligned}
& \begin{bmatrix} \hat{H}_{\Delta g_o \Delta g_o} & \hat{H}_{\Delta g_o' N_o} \\ \hat{H}_{N_o' \Delta g_o} & \hat{H}_{N_o' N_o} \end{bmatrix} \begin{bmatrix} P_{\Delta g_o \Delta g_o} & P_{\Delta g_o N_o} \\ P_{N_o \Delta g_o} & P_{N_o N_o} \end{bmatrix} \Big\} \times \\
& \left(\begin{bmatrix} 1 & H_{N' \Delta g}^* \\ H_{\Delta g' N}^* & 1 \end{bmatrix} - \begin{bmatrix} \hat{H}_{\Delta g_o' \Delta g_o} & \hat{H}_{N_o' \Delta g} \\ \hat{H}_{\Delta g_o' N_o} & \hat{H}_{N_o' N_o} \end{bmatrix} \right) + \\
& \begin{bmatrix} \hat{H}_{\Delta g_o \Delta g_o} & \hat{H}_{\Delta g_o' N_o} \\ \hat{H}_{N_o' \Delta g_o} & \hat{H}_{N_o' N_o} \end{bmatrix} \begin{bmatrix} P_{m_{\Delta g} m_{\Delta g}} & 0 \\ 0 & P_{m_N m_N} \end{bmatrix} \begin{bmatrix} H_{\Delta g' \Delta g}^* & H_{N' \Delta g}^* \\ H_{\Delta g' N}^* & H_{N' N}^* \end{bmatrix} \quad (36)
\end{aligned}$$

In the above equations, $H_{N' \Delta g}$ is Stokes's operator and $H_{\Delta g' N}$ is the inverse Stokes operator in the frequency domain. Although in the space domain the relation between the direct and the inverse Stokes operators is quite complicated, in the spectral domain this relationship is very simple, i.e., $H_{N' \Delta g} = 1/H_{\Delta g' N}$.

3.3 Single input-multiple output system (Vening-Meinesz application)

Another interesting application of the IOST is the estimation of the meridian and prime vertical components of the deflection of the vertical using observed gravity data and the equations of Vening-Meinesz. The one input-two output system is presented in Fig. 5. In

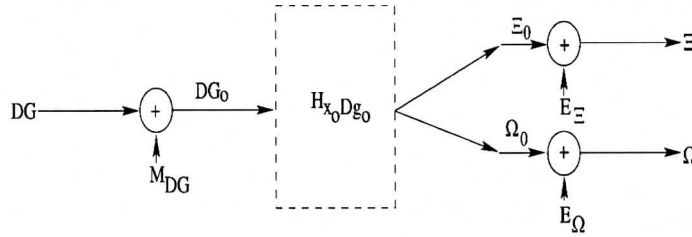


Figure 5: IOST deflection of the vertical estimation application

this case also the input noise PSD can be approximated using stationary models and the a-priori information of the measurement variances. The output and input vector in this application take the form:

$$\mathbf{x}_o = \begin{bmatrix} \Xi_o \\ \Omega_o \end{bmatrix} \quad \mathbf{y}_o = \begin{bmatrix} \Delta G_o \end{bmatrix}, \quad (37)$$

where Ξ_o is the Fourier transform of the output meridian component and Ω_o is the Fourier transform of the output prime vertical component. We used Ω for the prime vertical component notation instead of the classical H (and η) in order to avoid any confusion with the transfer function notation.

The optimum transfer function is:

$$\hat{\mathbf{H}}_{\mathbf{x}_o \mathbf{y}_o} = \mathbf{P}_{\mathbf{xy}} \mathbf{P}_{\mathbf{y}_o \mathbf{y}_o}^{-1} = \frac{P_{\Delta g \Delta g}}{P_{\Delta g_o \Delta g_o}} \begin{bmatrix} H_{\xi \Delta g} \\ H_{\omega \Delta g} \end{bmatrix} \quad (38)$$

and the final solution is given in matrix notation as follows:

$$\hat{\mathbf{x}}_o = \hat{\mathbf{H}}_{\mathbf{x}_o \mathbf{y}_o} \mathbf{y}_o = \frac{(P_{\Delta g_o \Delta g_o} - P_{mm}) \Delta G_o}{P_{\Delta g_o \Delta g_o}} \begin{bmatrix} H_{\xi \Delta g} \\ H_{\omega \Delta g} \end{bmatrix}, \quad (39)$$

where $H_{\xi \Delta g}$ and $H_{\omega \Delta g}$ are the frequency response functions which connect the pure input and output signal, i.e., the Vening-Meinesz kernels in the frequency domain. Finally, the

output error PSD $P_{\hat{e}\hat{e}}$ can be written as:

$$\begin{aligned}
\mathbf{P}_{\hat{e}\hat{e}} &= \begin{bmatrix} P_{\hat{e}_\xi \hat{e}_\xi} & P_{\hat{e}_\xi \hat{e}_\omega} \\ P_{\hat{e}_\omega \hat{e}_\xi} & P_{\hat{e}_\omega \hat{e}_\omega} \end{bmatrix} = \\
& (P_{\Delta g_o \Delta g_o} - P_{mm}) \begin{bmatrix} H_{\xi \Delta g} \\ H_{\omega \Delta g} \end{bmatrix} (1-1) \times \\
& \left(\begin{bmatrix} H_{\xi \Delta g}^* & H_{\omega \Delta g}^* \end{bmatrix} - \begin{bmatrix} H_{\xi \Delta g} & H_{\omega \Delta g} \end{bmatrix} \frac{P_{\Delta g_o \Delta g_o} - P_{mm}}{P_{\Delta g_o \Delta g_o}} \right) + \\
& \frac{P_{mm}(P_{\Delta g_o \Delta g_o} - P_{mm})}{P_{\Delta g_o \Delta g_o}} \begin{bmatrix} H_{\xi \Delta g}^2 & H_{\xi \Delta g} H_{\omega \Delta g}^* \\ H_{\omega \Delta g} H_{\xi \Delta g}^* & H_{\omega \Delta g}^2 \end{bmatrix} \Rightarrow \\
P_{\hat{e}\hat{e}} &= \frac{P_{mm}(P_{\Delta g_o \Delta g_o} - P_{mm})}{P_{\Delta g_o \Delta g_o}} \begin{bmatrix} H_{\xi \Delta g}^2 & H_{\xi \Delta g} H_{\omega \Delta g}^* \\ H_{\omega \Delta g} H_{\xi \Delta g}^* & H_{\omega \Delta g}^2 \end{bmatrix}. \quad (40)
\end{aligned}$$

3.4 Airborne gravimetry application

The application of MIMOS theory in airborne gravity measurements post-processing was presented by Wu and Sideris (1995), Wu (1996) and Wu and Sideris (1997). Simulated data were used in these applications and the noise filtering from the measured components of the disturbing potential was investigated. A sequential multiple input-single output system was used because of the complicated form of the generalized algorithm (Wu and Sideris, 1995). The post-processing of the measured components of the disturbing potential is presented in Fig. 6. In practice, surveyed areas are usually relatively small. The

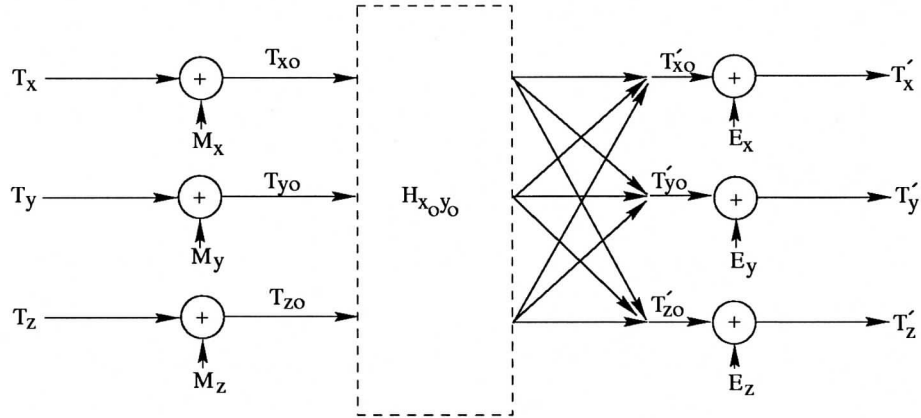


Figure 6: MIMOS in airborne gravimetry application

surface of the Earth can be approximated by a tangent plane without causing significant errors. According to this assumption, T_x , T_y and T_z are the three components of the gravity disturbance vector, defined as:

$$T_x = \partial T / \partial x \quad T_y = \partial T / \partial y \quad T_z = \partial T / \partial z. \quad (41)$$

The output and input vectors are:

$$\mathbf{X}_o = \begin{bmatrix} T_{x_o} \\ T_{y_o} \\ T_{z_o} \end{bmatrix} \quad \mathbf{Y}_o = \begin{bmatrix} T'_{x_o} \\ T'_{y_o} \\ T'_{z_o} \end{bmatrix} \quad (42)$$

According to Wu and Sideris (1995) and Wu (1996), the estimation of the input-output PSD can be based on the Fourier transform of the gravity disturbing potential P_{TT} :

$$\mathbf{P}_{xy} = 4\pi P_{TT} \begin{bmatrix} u^2 & uv & -juq \\ uv & v^2 & -jvq \\ -juq & -jvq & q^2 \end{bmatrix}, \quad (43)$$

where u and v are the frequencies corresponding to x and y , j is the imaginary unit ($j = \sqrt{-1}$) and $q = \sqrt{v^2 + u^2}$. A special treatment is needed for the noise estimation. A first choice, as mentioned in section 2.4, is the simulation of some random noise fields. This is not the case in airborne gravimetry applications since the estimation accuracy of the horizontal components is much less than the accuracy of the vertical components (Wei and Schwarz, 1994). The noise in the vertical component and the noise in the horizontal components must be generated separately considering the accuracy difference. The main difference is the color noise behaviour in the low frequency part and, therefore, some noise can be generated according the following PSD models (Wu and Sideris, 1995):

$$P_{vv} = \frac{A}{f^2} + W \quad (44)$$

$$P_{hh} = \frac{B}{f^4} + W, \quad (45)$$

where P_{vv} is the noise PSD of the vertical component, P_{hh} is the noise PSD of the horizontal components, f is the frequency, A and B are color noise constants, and W is white noise constant. An alternative method for the error PSD estimation is the use of the along-track 1D noise PSD of each component (which it is known) for the 2D error PSD approximation (Wu and Sideris, 1997).

The final solution and the output error PSD are obtained by (21).

3.5 Sea Surface Topography application

A MIMOS can be used for a Sea Surface Topography (SST) estimation and the filtering of the input data. Marine gravity anomalies and Sea Surface Heights (SSHs) from various satellite altimetry missions can be combined. Let t be the SST and s_i the SSH from n different satellite missions. The output and input vectors follow:

$$\mathbf{X}_o = \begin{bmatrix} T_o \\ S'_{1o} \\ \vdots \\ S'_{2o} \\ \Delta G'_o \end{bmatrix} \quad \mathbf{Y}_o = \begin{bmatrix} S_{1o} \\ \vdots \\ S_{2o} \\ \Delta g_o \end{bmatrix} \quad (46)$$

A MIMOS system of the present scheme is depicted in Fig. 7. The input output PSD has the form:

$$\mathbf{P}_{xy} = \begin{bmatrix} P_{ts_1} & \cdots & P_{ts_n} & P_{t\Delta g} \\ P_{s'_1s_1} & \cdots & P_{s'_1s_n} & P_{s'_1\Delta g} \\ \vdots & \ddots & \vdots & \vdots \\ P_{s'_ns_1} & \cdots & P_{s'_ns_n} & P_{s'_n\Delta g} \\ P_{\Delta g's_1} & \cdots & P_{\Delta g's_n} & P_{\Delta g'\Delta g} \end{bmatrix}. \quad (47)$$

The input / output PSD components can be computed following Andritsanos and Tziavos (1999) and Andritsanos et al. (1999):

$$P_{ts_i} = TS_i^* = (\bar{S} - N)S_i^* = \frac{S_1 + \cdots + S_n}{n} S_i^* - H_{N\Delta g} \Delta GS_i^* =$$

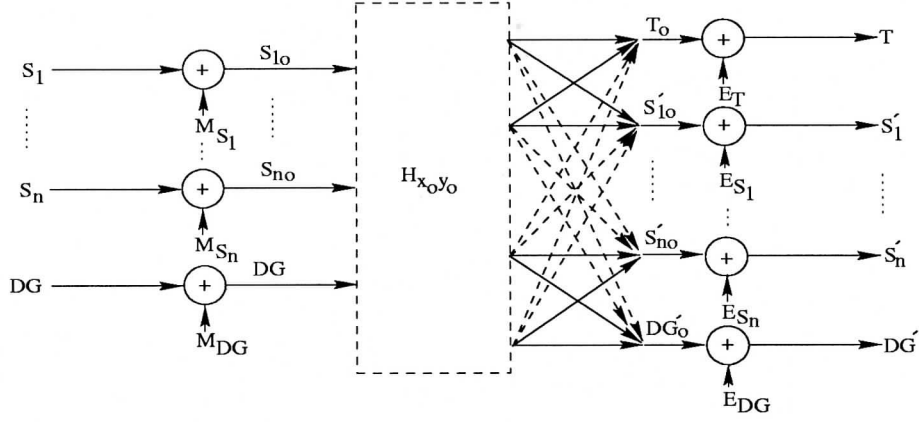


Figure 7: MIMOST Sea Surface Topography estimation application

$$= \frac{P_{s_1 s_i} + \dots + P_{s_n s_i}}{n} - H_{N\Delta g} P_{\Delta g s_i} \quad (48)$$

If the assumption of uncorrelated input signal with noise is extended to the MIMOS case then the following equation holds:

$$P_{\Delta g s_i} = P_{\Delta g o s_{io}} \quad (49)$$

The input /output PSD for the SSHs can be computed using the formula:

$$P_{s'_i s'_j} = \begin{cases} i = j & P_{s'_i s'_i} = P_{s_i s_i} = P_{s_{io} s_{io}} - P_{m_{s_i} m_{s_i}} \\ i \neq j & P_{s'_i s'_j} = P_{s_i s_j} = P_{s_{io} s_{jo}} \end{cases} \quad (50)$$

In this case the input noise PSD can be simulated as previously or can be computed using the repeated information of the ERM missions as described in Andritsanos et al. (1999). The input auto-PSD can be calculated directly from the observations and has the form:

$$P_{y_o y_o} = \begin{bmatrix} P_{s_{1o} s_{1o}} & \dots & P_{s_{1o} s_{no}} & P_{s_{1o} \Delta g_o} \\ \vdots & \ddots & \vdots & \vdots \\ P_{s_{no} s_{1o}} & \dots & P_{s_{no} s_{no}} & P_{s_{no} \Delta g_o} \\ P_{\Delta g_o s_{1o}} & \dots & P_{\Delta g_o s_{no}} & P_{\Delta g_o \Delta g_o} \end{bmatrix}. \quad (51)$$

The final solution is given by (21). The estimation of the output error PSD is not possible: the transfer function which connects the pure signal of SSH and gravity anomaly is not known theoretically. The only information that one can get is from some external comparisons with oceanographic estimations of SST. Such comparisons can be done using the spherical harmonic expansion of Levitus SST (Engelis, 1985) and the one of Parallel Ocean Circulation Model (POCM-4B) presented by Pavlis et al. (1998).

4 Numerical results

The numerical tests presented in this paper are the results of various previous papers. The description of these results in a common review paper is done for an integrated presentation of the IOST applications.

4.1 Simulation tests

In these tests simulated data and noise were used as input to the system. These tests show the effectiveness of the method in the input noise filtering and the low sensitivity of the method to the applied error model.

4.1.1 Data filtering

A first numerical comparison between results derived by the IOST and LSC were presented by Li (1996). Gravity anomaly and geoid fields were simulated and some noise models based on both Gaussian and uniform distribution were generated. The simulated observations were created by the combination of input signal and noise fields. A first test was done considering the effect of the input noise level to the output results, when these were compared to the pure input signal. As it is observed in Fig. 5-6 and 5-7 of Li (1996), the IOST method has the smallest sensitivity to input noise as compared to LSC and Least Squares Adjustment in the Frequency Domain (LSAFD). This fact is also valid in both the cases of Gaussian and uniform distribution of the noise field.

A second test in Li (1996) was performed using real data from ERS1-GM and shipborne measurements in the Central Mediterranean. The noise fields were generated either using a-priori information of the covariance in the area (correlogram - indirect method (Marple, 1987)) or using simulated noise with known standard deviation -sd- (periodogram - direct method (Marple, 1987)). The numerical results show a better performance when the direct method is used.

The same conclusions were drawn by Tziavos et al. (1996), where non-isotropic a-priori information was used in the test area of Central Mediterranean. The comparisons between the results obtained by IOST and LSC show an agreement of the order of 4-5 cm sd in geoid height estimates and 1.43 mGal sd in gravity anomaly prediction. The non-isotropic noise information provided always better results than the use of the pre-computed covariance function and the correlogram method.

Some other comparisons using Fast Collocation method (Bottoni and Barzaghi, 1993) and the one-dimensional spherical FFT approach introduced by Haagmans et al. (1993) were presented by Tziavos et al. (1996). In all the tests, the IOST method performed better than the other methods in effectively by filtering out the input noise.

The recovery of gravity field quantities using IOST was presented by Tziavos et al. (1997) employing non-isotropic information. The area under consideration was the off Newfoundland sea area at the east coast of Canada. ERS1-GM adjusted data were used in combination with shipborne gravity anomalies. The recovery of geoid signal was tested against SSH derived from Topex/Poseidon (T/P) altimetry mission and the agreement was at the level of 4 cm sd after a bias and tilt fit, while the recovery using one-dimensional FFT was of the order of 8 cm sd.

Results from the combination of satellite altimetry data and shipborne gravity anomalies were presented by Tziavos et al. (1998) and Tziavos et al. (1998). Andritsanos and Tziavos (1999) presented the results of the application of the IOST method at the Labrador Sea. The output noise in the simulation case *"is 2-3 times less in magnitude than the input noise in gravity signals and 5-10 times less in the geoid signals. This means that most of the input noise is filtered out and only a small portion of it is propagated into the output results. This fact shows that the MIMOS method sensitivity to input noise is quite small."* In all the above mentioned studies, the SST was ignored and the SSH observations were treated as geoid data.

4.1.2 Airborne gravimetry

The application of the IOST to the airborne gravimetry was presented by Wu and Sideris (1995), Wu (1996) and Wu and Sideris (1997). Both simulated gravity field and noise were

used. The noise in the vertical component and the noise in the horizontal components were generated separately considering the accuracy difference. The effect of different input combinations between the components of the gravity disturbance vector was studied. The authors concluded that *"when the input signal noise is considered, better results for every component can be obtained by combining more components."* They also examined the effect of the flight height in the downward continuation procedure. They show *"the increase rate of the noise in vertical component with the increase rate of the noise in horizontal components when the height increases."* Additionally, they concluded that *"the downward continuation error increases when larger data grid interval is used. But this result depends on the roughness of the gravity field"*. They also investigated the difference between 1D and 2D filtering concluding that *"two dimensional filtering is more effective in reducing the measurement noise than one dimensional filtering"*. They finally compared MIMOST with the frequency domain least-squares adjustment method (FDLSA). Using noisy data, *"the MIMOST yields better results than the FDLSA method"*.

4.2 Real data application

4.2.1 Quasi stationary SST estimation

The application of the MIMOS theory to the Quasi-stationary Sea Surface Topography (QSST) and the filtering of the input observations was discussed in Andritsanos et al. (1999). Input SSH observations from ERS1 phase C and G, ERS1-GM, ERS2 phase A and T/P were combined with shipborne gravity anomalies and the QSST was estimated for a six-year period (1993-1998). The repeated information of the altimetry missions can be used for the estimation of input error and signal PSD. The procedure is based on a successive difference and summation between each respective cycle of the satellite. In this case, a multiple sample configuration can lead to a PSD estimation. More details on the error and signal PSD computations can be found in Andritsanos et al. (1999). An analogous treatment of the gravity anomaly error PSD can be done, if some information of the cruise dates is available (Behrend, 1999). Some comparisons of the predicted QSST based on MIMOS theory and the global harmonic solution of Parallel Ocean Circulation Model 4B (POCM-4B) (Pavlis et al., 1998) have shown an agreement of 7.3 cm sd between the two solutions after a bias and tilt fit.

4.3 SST estimation in the Central Mediterranean

The application of the MIMOST method in the SST estimation in the Central Mediterranean will be presented. The area is bounded by $33^{\circ}.083333 \leq \phi \leq 38^{\circ}$ and $16^{\circ} \leq \lambda \leq 20^{\circ}.916666$. Gridded ERS1-GM SSHs and marine gravity data were used in a 2 inputs-3 outputs system. The grid interval was 5'. The altimetry data were corrected from atmospheric, orbit and geophysical corrections as described in AVISO - Corrected Sea Surface Heights (1997). The contribution of the global geopotential model EGM96 (Lemoine et al., 1998) were subtracted from both input fields. Simulated noise fields were generated using a Gaussian random number generator with 5 cm standard deviation (sd) for the SSH noise and 5 mGal sd for the marine gravity noise. The input data sets as well as the respective noise field are presented in Tab. 1. The output SST as well as the differences between the input data and the output marine gravity anomalies and ERS1-GM SSHs are presented in Tab. 2. Considering Tab. 2, the remarks made on the sensitivity of the method in the input noise can be justified. The input noise sd in the case of marine gravity anomaly was 5 mGal, while the differences between the input and the output field had only a 1.351 mGal sd. The same can be said in the case of SSHs: the input noise sd was 5 cm, while the differences had a 1.4 cm sd. The output SST estimation is depicted in Fig. 8. A low-pass filter was applied to the SST output grid. The cut-off wavelength was chosen at 100 km. In this way, the high frequency

Table 1: Input data and noise field for Central Mediterranean

	max	min	mean	rms	sd
Marine gravity (mGal)	116.736	-105.079	0.122	14.045	14.044
ERS1-GM SSHs (m)	1.358	-1.810	-0.012	0.247	0.247
Gravity noise (mGal)	16.249	-19.500	0.018	4.940	4.940
SSH noise (m)	0.179	-0.181	0.000	0.050	0.050

Table 2: SST estimation results and difference statistics between input-output data

	max	min	mean	rms	sd
SST (m)	0.500	-0.578	-0.040	0.134	0.128
Gr. anom. differences (mGal)	11.548	-9.950	0.023	1.351	1.351
SSH differences (m)	0.108	-0.081	0.000	0.014	0.014

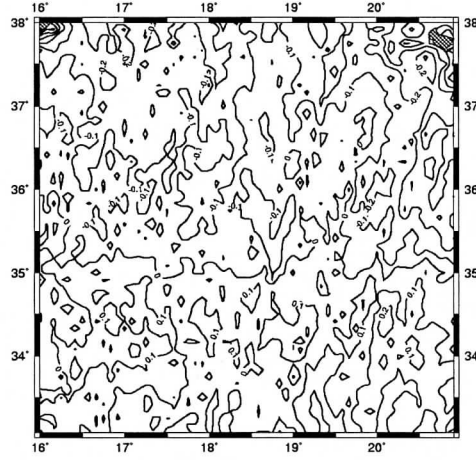


Figure 8: SST estimation in the Central Mediterranean by MIMOST (Contour Interval: 10cm)

part of the estimation is subtracted. The high frequency information may be attributed to the datum uncertainty in marine gravity data, as well as the erroneous SSH data near the coastlines. The filtered field is depicted in Fig. 9.

5 Summary - Conclusions

The applications of the IOST to gravity field modeling were presented in this paper. The theoretical presentation of the single input-single output system and its extension to a matrix-form MIMOST were analyzed. This method can provide us with an estimation of the prediction accuracy, by approximating the output error PSD matrix. An accuracy estimate can now be computed in a frequency domain technique and the major disadvantage of error propagation weakness in spectral domain methods can be overcome. Additionally, the optimum combination of heterogeneous data can be achieved by the consideration of their spectral properties and the estimation of the optimum transfer matrix, which connects the inputs with the outputs.

The major drawback in the application of the method is the fact that the input noise

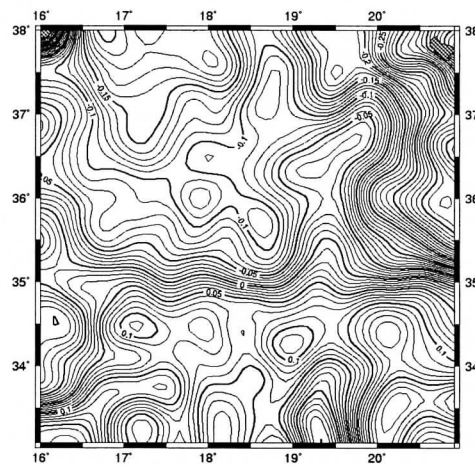


Figure 9: Filtered SST estimation in the Central Mediterranean by MIMOST (Contour Interval: 1cm)

PSD must be known. The problem can be overcome by a noise field simulation or the use of PSD models. Additionally, the repeat track information of the altimetry missions can be used to approximate of the input noise PSD. This repeated information does not take into account the oceanic variability, which is considered as part of the noise (Andritsanos et al., 1999). Nevertheless, the choice of the noise model is not so critical. The sensitivity of the IOST method to the input noise is the smallest than any other approximation method. This fact is related to the proper modification of the frequency response function by the minimization of the noise-to-signal ratio.

This method can be used in areas with a vast number of observations, where all other approximation methods are very time consuming. The optimization of the data PSD estimation will improve the results of the method. The use of modern spectral estimation techniques (Marple, 1987) will contribute in this direction, as well as the optimization of some gridding procedures. The advent of the recent gravity field dedicated satellite missions will provide a great number of observations all over the world and the application of IOST in other areas of interest will be feasible.

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The IGeS Cross Over Adjustment With a New Software for the SST in the Mediterranean Sea

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Abstract

The goal of our work is to determine the Stationary Sea Surface Topography (SST) of the Mediterranean Sea combining the different radar altimetry data nowadays available. This implies to manage observations with different distributions in time and space and tracks with different inclinations. In addition the choice of a closed basin make us to work with short and discontinuous arcs. In the following paper after a description of the procedure built up to get the stationary SST from the fully corrected Sea Surface Height (SSH), the main characteristics of the Cross Over Adjustment Software implemented to fulfill the specific needs of our data processing will be discussed. Finally the first results obtained will be shown.

1 A short description of the data processing

The methodology to get the Stationary SST of the Mediterranean Sea is strictly connected with the different resolution in time and space of the altimetry data.

1.1 Track wise data processing

We start to treat the repeated data, that are observations of the same ground track after a fixed time period (e.g. 35 days for ERS1 and ERS2 or 10 days for Topex/Poseidon), which we will refer to as the track wise data processing. Figure 1 shows the ground tracks pattern of the repeated

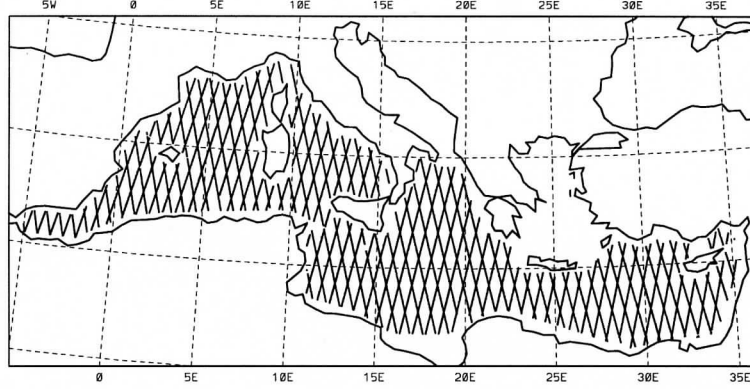


Figure 1: Ground tracks pattern of the repeated observations

observations in the Mediterranean sea. Figure 2 shows 11 cycles of observations along the same ground track.

The *SSH* values (reduced to *SST* by subtracting the Geoid undulation) along the same track, observed in different periods of the year, contain that part of the *SST* which is constant during the year, namely correlated in the different cycles, together with the time varying part, which is uncorrelated. A 1D collocation filtering is therefore able to separate the stationary/correlated part of the signal from the time dependent part. This filtering procedure is performed after estimating and removing a linear trend in each repeated track [1].

1.2 Area wise data processing

At this point the area wise data processing starts. The filtered tracks, all belonging to the stationary surface we look for, should have the same value in the cross over points. The discrepancies are due to the residual radial orbit errors and the residual tidal errors. These errors, in the case of short arcs like in the Mediterranean, can be modeled as linear trends [7] for each arc and are estimated by a cross over adjustment, more deeply described in the next section. Note that the track wise trend removal, performed before the filtering procedure, is geometrically too weak to consistently model the mentioned errors [2].

The stationary signal, area wise corrected by the orbit errors constitutes what we will refer to as the stationary reference network profiling the stationary *SST*.

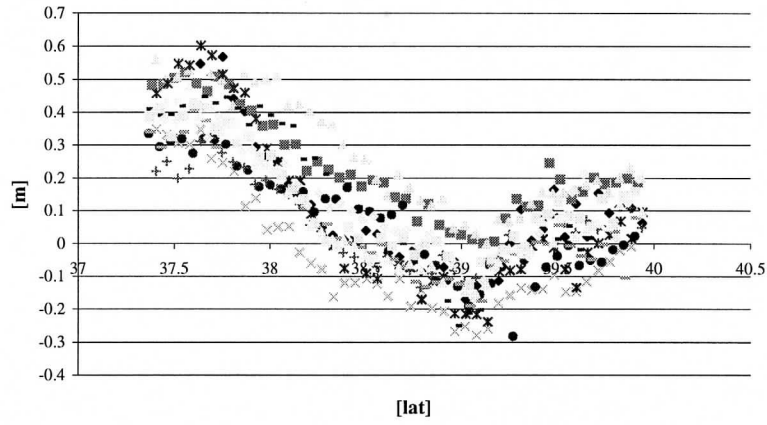


Figure 2: 11 cycles of observations along the same ground track

This net is however too loose in space to get a good interpolation of the surface itself. Yet, if one consider the data coming from the geodetic phase of the satellite missions, the observation distribution in space is strongly densified. Figure 3 shows the geodetic mission observations pattern.

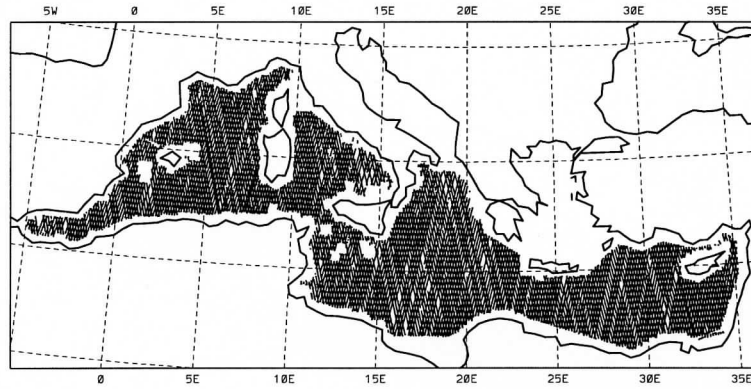


Figure 3: Geodetic mission observations pattern

On the other hand these data are not repeated during one year so that the track wise collocation filtering is not possible. In order to get the stationary *SST* from these data, we first remove the residual orbit and tidal errors by means of a cross over adjustment in which the repeated tracks are used as reference frame, and therefore perform a 2D collocation, using a covariance function estimated on the reference frame tracks only, i.d. on the stationary signal we look for.

2 The IGeS new software for Cross Over Adjustment

The Cross Over Adjustment is the crucial connection point between the track wise and area wise determination of the stationary *SST*. It is needed to form the reference network of the track wise filtered *SST* and to globally attach the geodetic tracks to this reference frame before performing the 2D collocation.

The IGeS software intends to satisfy the specific needs resulting in the analysis of the Mediterranean Sea data using the described methodology. They can be summarized as follows:

- analysis of irregularly distributed data ;
- analysis of short arcs only (short if compared to oceanic tracks);
- merging of different satellite mission data and of different phases within the same mission.

In addition we implemented a rigorous general solution (master arcs or minimum norm) of the rank deficiency problem and the possibility to fix the parameters of a whole set of tracks.

Let us start to summarize the data processing flow in the simplest case of one mission only. As it is well known the first step in the cross over adjustment is to determine the cross over positions, in which the differences (the observations) of the *SST* are to be computed. The main steps of the research algorithm [6] are the following:

- partition of the whole basin in rectangular sub areas small enough to make admissible plane approximation of the reference ellipsoid;
- determination of an approximate position of the cross over in the plane system as the intersection point of the straight line located by the extreme points of the tracks;
- testing the existence of the cross over, that is to find four points belonging to the intersecting tracks which surround the cross over position, and computation of the final cross over position from these points. A bilinear interpolation is used to determine the cross over difference.

Once the observations are computed two further problems arise. The first is the choice of the coordinate system to which we refer the bias and tilt parameters of the cross over adjustment linear model. The second is the rank deficiency problem solution. As for the coordinates system we fix two tracks which cross each other in a barycentric position. The observation point positions would be ideally defined as the spherical distance from the origin in the direction of the two reference arcs and are denoted as ψ_1 and ψ_2 , for the ascending and descending arcs respectively. Nevertheless the equation $\psi_i = \text{const}$, due to the non parallelism of the altimetric tracks, does not describe one track, or other way round the points belonging to the same track do not have in common one of their two coordinates. We therefore introduced a system of pseudo-coordinates ψ_i^* by averaging the ψ_1 coordinate of all the points belonging to an ascending arc and the ψ_2 coordinate of all the points belonging to a descending one. The cross over positions are therefore defined from the equations of the two intersecting arcs. Note that these coordinates strictly depends on the number and position of points considered for each arc [4].

Once the coordinates system is defined, at the observation point the cross over difference can be expressed as the difference between the residual radial orbit error and tidal error of the two intersecting tracks, modeled as a function linear in ψ_i^* . In every cross over difference a bias and a tilt parameter for each track are therefore involved. This model, in the case of a well connected network, has a rank deficiency of four. A network is defined well connected if it is obtained by intersecting two couples of non parallel arcs in four points, and by adding at each step an arc which has at least two intersections with the previous network. Before solving the rank deficiency problem an algorithm to split the whole network into well connected sub networks was needed. This was developed and implemented into our software. Once reduced the whole set of tracks in well connected sub nets, one can perform the least square adjustment solving the rank deficiency problem within each data subset. We implement the following solutions. The first one consists in solving the rank deficiency by fixing the four parameters of two parallel arcs. This is the so called master arc solution. The second one is called minimum norm. We use the master arc estimated parameters as a particular solution of the linear model. The general solution is obtained by adding to it the null space, which is a surface bilinear in ψ_1^* and ψ_2^* , that we explicitly express in our coordinates system. The minimum norm is that particular solution for which a weighted norm of the parameters is minimum.

As for the stochastic model, we neglect the correlation among observations and give the same variance to each of them.

Let us now consider the case of data coming from different phases of the same mission or from different missions. The following situations are possible: the data have the same track inclination, the data have a different track inclination. In the first case we only introduce the possibility to modify the stochastic model weighting in a different way the observations coming from tracks of the same missions and those coming from different missions. What we call a fixed frame solution is also possible. It consists in fixing the parameters of a whole set of tracks, for instance the repeated tracks already filtered and area wise adjusted by a previous cross over adjustment, automatically solving the rank deficiency problem. In the second situation the cross over search algorithm is to be slightly modified: ascending (or descending) tracks from different missions can cross together. In addition the minimum norm solution is to be modified taking into account that the rank deficiency is now reduced to one, i.e. the null space is a plane.

3 Our experiment

3.1 The data set

Our first experiment on the whole Mediterranean area was based on the following data set ¹

- ERS1.c: multidisciplinary phase c of ERS1 mission (35 days repeat cycle). We considered 11 cycles from the end of October 1992 to the middle of October 1993. The Mediterranean area is covered by 122 repeated passes.
- ERS2.a: multidisciplinary phase a of ERS2 mission (35 days repeat cycle). We considered

¹The data were delivered by dr. Remko Scharroo (DEOS, Delft)

12 cycles from the middle of October 1995 to the end of October 1996. The whole area is covered by 124 repeated passes.

- ERS1.e: geodetic phase e of ERS1 mission (168 days repeat cycle). We considered two cycles from April 1994 to October 1994. The area is covered by 567 passes

Note that the passes or tracks have all the same inclination.

3.2 Main steps of data processing

The first step was the track wise filtering of the SST on repeated tracks, namely on ERS1.c and ERS2.a data, via a 1D collocation approach.

The observed SSH were modeled as follows:

$$SSH = SST^{(0)} + SST^{(t)} + N + e_r \quad (1)$$

where $SST^{(0)}$ is the stationary SST and $SST^{(t)}$ is the time dependent SST , N is the geoid undulation and e_r the residual radial orbit error and the tidal error. For each pass the Geomed [3] updated geoid ² N was removed, then the residual radial orbit error and residual tides e_r was estimated by a linear regression on the data reduced to an along track abscissa. The pass by pass reduced data were therefore put together to estimate the empirical covariance function of the stationary signal. Each empirical function was interpolated with a finite definite positive model, a cubic spline. The 1D collocation filtering concluded this first step by separating the $SST^{(0)}$ from the $SST^{(t)}$. Figure 4 shows one of the empirical covariance function estimated from a block of repeated tracks and the interpolating model. Figure 5 shows the result of the filtering procedure.

The next step was to estimate by means of a cross over adjustment the residual radial orbit error and the tidal error in the filtered tracks. We performed the minimum norm adjustment of the ERS1.c and ERS2.a data together as the tracks have the same inclinations, giving to all the observations the same precision. Two connected networks were founded so that for the moment we have not a unique solution for the stationary SST but two separate solutions which differ for a bilinear surface. The resulting statistics on the data treated are reported in table 1 and table2.

The final step consisted in the geodetic mission data processing. It started with the estimate and removal of the linear trend by means of a cross over adjustment. A fixed frame solution was performed. The data already filtered and adjusted of the ERS1.c and ERS2.a missions were processed together with the ERS1.e data, but fixing their bias and tilt parameters to zero (by means of pseudo-observation equations) and weighting the cross over difference between tracks of the fixed

²The geomed geoid was updated by estimating and removing a bilinear surface belonging to the null space of the cross over adjustment model which interpolate the difference between Geomed and EGM96

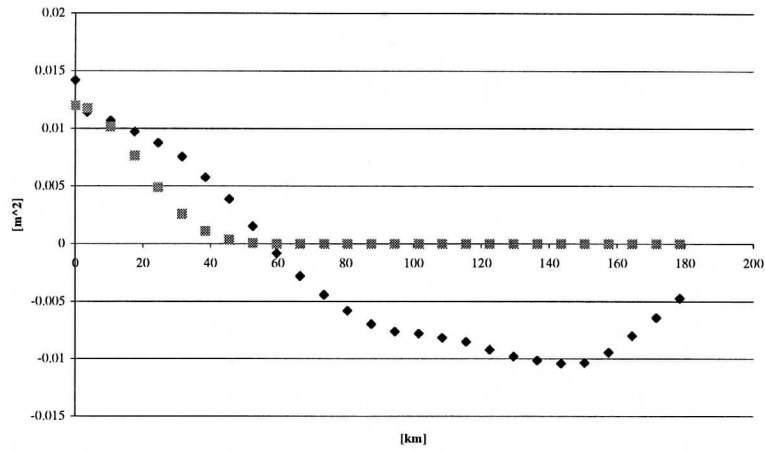


Figure 4: Empirical covariance function estimated from a block of repeated tracks and the interpolating model

before cross-over adjustment		after cross-over adjustment	
mean	stdv	mean	stdv
-0.011m	0.148m	0.000m	0.017m

Table 1: Western network of repeated tracks. Statistics of the cross over differences n=394

network and the geodetic ones more than the differences between geodetic tracks themselves. The latter in fact contains the non stationary SST which make the cross over differences much noisy. The statistics of the resulting eastern and western geodetic networks are reported in table 3 and table 4.

The result of a minimum norm solution of the geodetic mission only cross over adjustment are also shown in table 5 for comparison. It resulted a main well connected network only. It was split into two sub net to be referred to the eastern and western repeated tracks frames. Note that the internal consistency between the geodetic mission data only is higher than that resulting from the fixed frame solution. Nevertheless by introducing the reference frame networks we refer all our tracks to a much more significant stationary system.

before cross-over adjustment		after cross-over adjustment	
mean	stdv	mean	stdv
0.009m	0.130m	0.000m	0.011m

Table 2: Eastern network of repeated tracks. Statistics of the cross over differences n=116

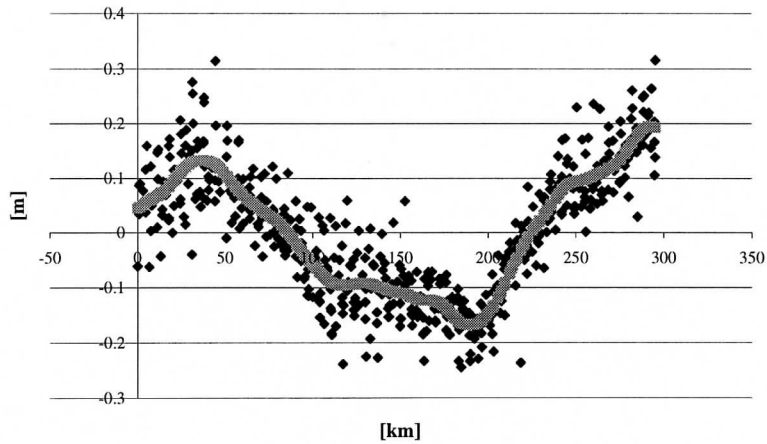


Figure 5: The result of the filtering procedure

before cross-over adjustment		after cross-over adjustment	
mean	stdv	mean	stdv
0.005m	0.6338m	-0.004m	0.123m

Table 3: Western network of repeated and geodetic tracks. Statistics of the cross over differences $n=4287$

Figure 6 and 7 show the eastern and western SST obtained by interpolating the data resulting from the fixed frame cross over adjustment by means of inverse distance squared moving average.

4 Future work

The data processing is not finished yet. The estimate of the 2D covariance function on the reference frame network is to be performed together with the 2D final collocation filtering of the whole set of data. Before doing that we must process the TOPEX/POSEIDON data, whose track inclinations are slightly different from the ERS1 and ERS2 data. We expect that the different mission data

before cross-over adjustment		after cross-over adjustment	
mean	stdv	mean	stdv
0.069m	1.059m	-0.002m	0.082m

Table 4: Eastern network of repeated and geodetic tracks. Statistics of the cross over differences $n=1427$

before cross-over adjustment		after cross-over adjustment	
mean	stdv	mean	stdv
-0.021m	0.130m	0.000m	0.050m

Table 5: Network of geodetic tracks only. Statistics of the cross over differences n=2943

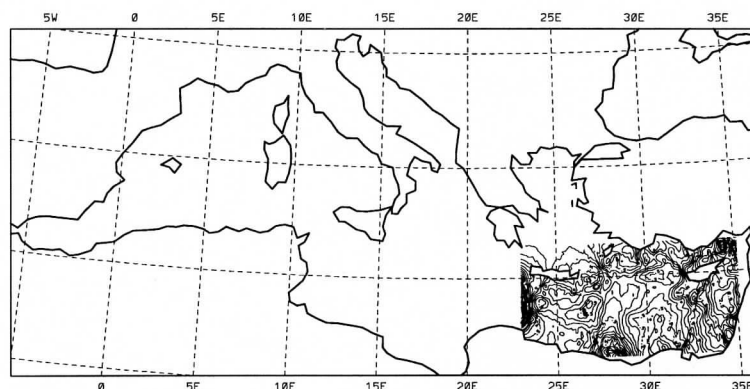


Figure 6: Eastern *SST*

merging will result in a unique well connected network. In addition it will reduce the rank deficiency to 1, of course as a counterpart we expect the rms of residuals to increase. The 2D data filtering of a so large amount of data will make us to perform a moving average collocation solution which will be deeply discussed in future.

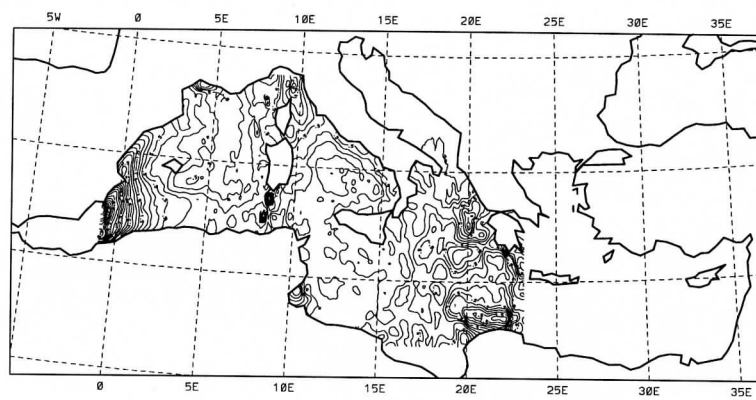


Figure 7: Western *SST*

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IMPROVEMENT OF THE REGIONAL GRAVITY MODEL IN CHINA AND ITS VICINITY SEA BY COMBINING THE SATELLITE ALTIMETER AND GRAVITY DATA

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Abstract

This paper develops the tailored method to improve a regional potential model using observed gravity anomalies in the continents and geoid undulations on the sea from satellite altimetry, and obtains an improved local solution to the so-called altimetry-gravimetry problem. By using the stack of four years (1992-1996) Topex/Poseidon and ERS-1 satellite altimeter data and the gravity anomalies (30'x30') in mainland China, we derive the geoid undulations of China and its adjacent sea. On the basis of the tailored method, we achieve a mixed regional geopotential model (IGG_LS_GU) to degree and order 360 for China and its adjacent sea. The gravity anomalies computed from the new model can achieve a RMS of ± 9 mGal in mainland China, and have a RMS of ± 12 mGal on the sea. The geoid undulations from the model have a RMS of ± 0.70 m compared with height anomalies derived from GPS/Leveling in mainland China, and have a RMS of ± 0.13 m compared with the geoid undulations derived from satellite altimetry on the China sea and its vicinity.

1. Introduction

From an accurate determination of the altimeter satellite orbit, the geoid undulations on the ocean are obtained over ocean with an accuracy of centimeters and with a great abundance of details. It is possible to solve numerically the mixed Boundary Value Problem (B.V.P.), the one of altimetry-gravimetry that is defined from gravity anomalies determined by gravimeter measurements mostly on continents and from geoid undulations on oceans from the satellite altimetry. A lot of authors described the theory in the altimetry-gravimetry boundary problem (Holota 1980, Sanso 1983, Sanso and Stock 1985, Sjöberg 1982, Svensson 1985). Most of them had no analytical expression to use the geoid undulations directly. However, when these undulations were transformed into gravity anomalies, the mixed B.V.P. becomes a single B.V.P.. In an analogous manner, the global geopotential model must be used, such as OSU91 (Rapp and Pavlis, 1990, Rapp et al, 1991) and EGM96 (Lemoine, et al, 1996). Mainville (1986) described a "Least-Squares method" (i.e. minimizing the integral using orthonormal base functions), which is different from a "Least-Squares adjustment" and can solve the altimetry-gravimetry problem directly, and provided numerical tests. The Least-Squares method needs a tremendous number of computations. The "tailored method" as the single B.V.P. has also been utilized to compute a regional geopotential model (Weber and Zomorrodian, 1988, Basic, 1989, Hsu and Lu, 1995, Lu et al, 1999). This method has been proven useful in some areas, even though the method has some problems in creation and application of tailored models (Kearsley and Forsberg, 1990, Lu, et al, 1999).

We intend to improve the geopotential model tailored to observed gravity anomalies in the continents and geoid undulation on the sea from satellite altimetry, and obtain an improved local solution to the so-called altimetry-gravimetry problem.

2. The tailored method

In the idea behind the tailored method, there exist discrepancies, called regional residual gravity field, between the observed regional gravity field and the reference field that obtained from global potential model. The main reasons are as follows. First, the initial global model (for example OSU91A1F), which is in a global-average sense, is not very accurate at short wavelength. Second, the data set used in the solution of model is incomplete, and the omission errors of model can not be avoided. Therefore, we can use higher resolution and accurate local gravity data to improve a global model that can fit to local gravity field well.

To some extent, the tailored method is a 'remove-restore' method. Firstly, regional residual gravity anomalies are obtained by removing reference gravity anomalies from observed gravity anomalies, which are the errors of reference model. Secondly, the corrections of new coefficients are calculated from residual gravity anomalies. Finally, a complete model is generated from reference model and corrections of coefficients.

In order to compute the corrections of coefficients, we develop following formulas based on the solutions by Rapp et al. (1991) in terms of ellipsoidal coordinate.

For gravity anomaly, we have:

$$\Delta \bar{C}_{nm}(\Delta g) = \frac{1}{4\pi\gamma} \sum_{t=1}^T r_t^E \sum_{k=0}^s \frac{L_{nmk}}{\bar{S}_{n-2k,|m|}(\frac{b}{E})} \frac{\bar{IP}_{n-2k,|m|}}{(n-2k-1)q_{n-2k}^t} \delta \Delta \bar{g}_t \begin{cases} IC_t & \text{if } m \geq 0 \\ IS_t & \text{if } m < 0 \end{cases} \quad (1)$$

For geoid undulation, we have:

$$\Delta \bar{C}_{nm}(N) = \frac{1}{4\pi} \sum_{t=1}^T \left(\frac{r_t^E}{a}\right)^2 \sum_{k=0}^s \frac{L_{nmk}}{\bar{S}_{n-2k,|m|}(\frac{b}{E})} \frac{\bar{IP}_{n-2k,|m|}}{q_{n-2k}^t} \delta N_{ij} \begin{cases} IC_t & \text{if } m \geq 0 \\ IS_t & \text{if } m < 0 \end{cases} \quad (2)$$

where

$$\bar{IP}_{n,|m|} = \int_{\delta_i}^{\delta_{i+1}} \bar{P}_{n,|m|}(\cos \delta) \sin \delta d\delta; \quad IC_j = \int_{\lambda_j}^{\lambda_{j+1}} \cos \lambda d\lambda; \quad IS_j = \int_{\lambda_j}^{\lambda_{j+1}} \sin \lambda d\lambda$$

Here, $\Delta \bar{C}_{nm}$ is the corrections of geopotential coefficients caused by regional residual field, $\delta \Delta g$ is residual gravity anomaly, N is residual geoid undulation, L_{nmk} is defined by Gleason (1988), $\bar{S}_{nm}$ is related to the associated Legendre function of the second kind (Gleason, (1988)), q_n is smoothing factor, s is the greatest integer which is less than or equal to $1/2(n-|m|)$, and T is the total amount blocks in local area. The new coefficients of geopotential model become

$$\bar{C}_{nm}^{new} = \bar{C}_{nm}^0 + \Delta \bar{C}_{nm} W(n) \quad \text{for } n, m = 2 - 360 \quad (3)$$

where $\bar{C}_{nm}^0$ are geopotential coefficients of reference model, the weight function $W(n)$ is as follows (T.Basic 1989),

$$W(n) = \begin{cases} 0 & n < N_{min} \\ (n - N_{min}) / (N_{max} - N_{min}) & N_{min} \leq n \leq N_{max} \\ 1 & N_{max} < n \end{cases} \quad (4)$$

where N_{min} is the lowest degree that should be kept in reference model and N_{max} is the starting degree that should be fully corrected.

3. The solution to the altimetry-gravimetry problem based on tailored method

The altimetry-gravimetry problem gives that,

$$\left. \begin{aligned} \Delta T &\equiv 0 & T \in \Omega \\ \frac{\partial T}{\partial \gamma} + \frac{2T}{\gamma} &= -\Delta g & T \in \Sigma_L \\ T &= \gamma N & T \in \Sigma_S \\ T &= O\left(\frac{1}{r}\right) & r \rightarrow \infty \end{aligned} \right\} \quad (5)$$

It is very complex to solve this problem. We try to use the tailored method to solve the altimetry-gravimetry problem.

Assuming as follows, there are gravity anomalies over land and geoid undulations over the oceans. First, gravity data over the land are used to compute the corrections of coefficients derived $\bar{C}_{nm}^L$.

$$\bar{C}_{nm}^L = \bar{C}_{nm}^o + W_L(n) \Delta \bar{C}_{nm}^L(\Delta g_L) \quad (6)$$

Then, geoid undulation data over the sea are used to modify the above corrections, and a mixed model is derived

$$\bar{C}_{nm}^{LS} = \bar{C}_{nm}^L + W_S(n) \Delta \bar{C}_{nm}^S(\Delta N_S) \quad (7)$$

where, $W_L(n)$ and $W_S(n)$ are computed from formula (4), $\Delta \bar{C}_{nm}^L(\Delta g_L)$ and $\Delta \bar{C}_{nm}^S(\Delta N_S)$ from the formula (1) and (2), respectively.

4. Computation of the regional geopotential model to degree 360

The area of interest is a $45^\circ \times 65^\circ$ (in latitude and longitude) region in China and its vicinity. The data used in the analysis include the mean free air gravity anomalies in mainland China, altimetry geoid undulations on the China Sea and its vicinity. As an initial approximate model, the global geopotential model OSU91A1F has been used to degree and order 360.

Totally 3360 terrestrial mean free air gravity anomalies on a $30' \times 30'$ grid in mainland China were used. The area is $20^\circ \sim 45^\circ \text{N}$ and $72^\circ \sim 125^\circ \text{E}$, named area 'L'.

For altimeter data, geoid undulations were obtained by subtracting SST model (Nerem et al., 1994) from the mean sea surface height (SSH), and the OSU91A1F geopotential model was used as reference of geoid undulation, on the China Sea and its vicinity. The area is $0^\circ \sim 45^\circ \text{N}$ and $105^\circ \sim 135^\circ \text{E}$, named area 'S'.

We use ERS-1 and Topex/Poseidon stacked tracks during four years (1992-1996) over the China Sea and its vicinity. During 13 cycles (6-18) of ERS-1 altimeter data, whose period is 35 days, there are totally 377008 valid observation data. During 148 cycles (2-1490) of Topex/Poseidon altimeter data, whose period is 10 days, there are totally 1399845 valid observation data. Figure 1 shows satellite altimeter tracks from ERS-1 and Topex/Poseidon over the area of interest. After stacking, there are 103 ERS-1 co-linear tracks, which contain totally 7739 mean sea-surface heights, and 31 T/P co-linear tracks, which contain totally 2461 mean sea-surface heights.

Moreover, using the inverse Stokes formula by FFT (GRAVSOF by R.Forsberg), we converted the geoid undulation into gravity anomalies on a $30' \times 30'$ grid on the China Sea and its vicinity. The calculated root mean square (RMS) is about ± 3.0 mGal. The comparisons between altimetry gravity anomalies and ship anomalies are summarized in Table 1, where 'Total' means the total area of interest; 'East' means the East region of interest: $27^\circ \sim 33^\circ \text{N}$ and $123^\circ \sim 128^\circ \text{E}$; 'South' means the South region of interest: $10^\circ \sim 18^\circ \text{N}$ and $110^\circ \sim 118^\circ \text{E}$; 'Pacific' means the west Pacific region: $9^\circ \sim 25^\circ \text{N}$ and $129^\circ \sim 134^\circ \text{E}$ (Wang, H.Y. et al., 1997).

When the above data are available for the combination solution, we have computed four test models. The flow chart of modeling shows in Figure 2. The first test model IGG_LAND_G was computed from the terrestrial gravity anomalies (Δg_L) in mainland China, which is in area

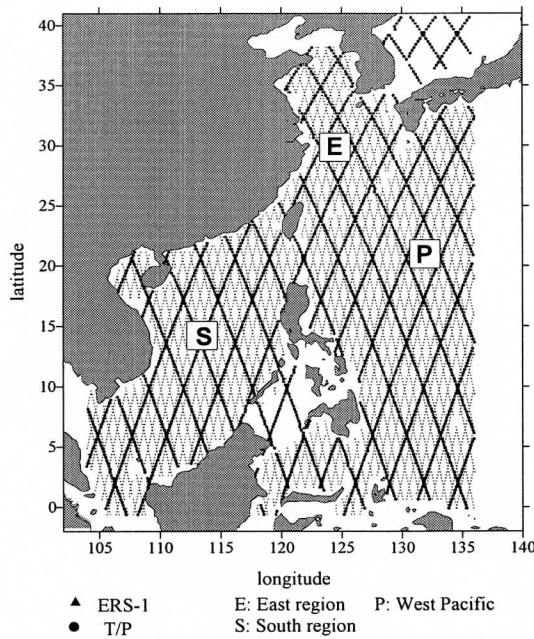


Fig.1 Satellite altimeter tracks from ERS-1 and Topex/Poseidon over the area of interest

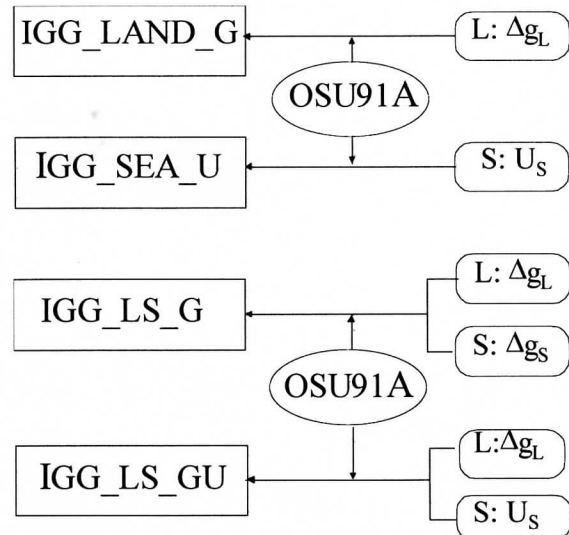


Fig.2 The flow charts of modeling

L. The second test model IGG_SEA_U was computed from the altimetry geoid undulations (U_s) on the China Sea and its vicinity, which is in area S. The third test model IGG_LS_G was computed from combining gravity anomalies (Δg_L and Δg_S) in the both of above area, i.e. area L and area S. The anomalies are derived from terrestrial and converted altimeter data, respectively. The last test model IGG_LS_GU was computed from gravity anomalies (Δg_L) over mainland China and geoid undulations (U_s) over the sea, which is in the same area as third.

Table 1 Comparisons of altimetry gravity anomalies and ship anomalies

	$\Delta g_{ALT} - \Delta g_{Ship}$			
	Total	East	South	Pacific
mean	-1.8	-1.3	-3.9	0.9
RMS	15.9	6.1	12.1	5.6

5. Comparisons of results from models

The geopotential models have been compared with observed gravity anomalies and geoid undulations in the research area. The results are summarized in Table 2 and Table 3. The gravity anomalies computed from the mixed model IGG_LS_GU could achieve a root mean square error (RMS) of ± 8.7 mGal in mainland China, and a RMS of ± 11.6 mGal on China Sea and its vicinity. The geoid undulations from the mixed model IGG_LS_GU have a RMS of ± 0.70 m compared with height anomalies derived from GPS/Leveling in mainland China, and have a RMS of ± 0.13 m compared with the geoid undulations derived from satellite altimetry on the China sea and its vicinity. In both of gravity anomalies and geoid undulations, the mixed geopotential model IGG_LS_GU can achieve higher accuracy compared with other models in each area of interest.

Table 2 Comparisons of gravity anomalies and that from models (mGal)

model area	OSU91A		IGG_LAND_G		IGG_SEA_U		IGG_LS_G		IGG_LS_GU	
	L	S	L	S	L	S	L	S	L	S
num.	5457	3293	5457	—	—	3293	5457	3293	5457	3293
mean	3.0	0.5	2.9	—	—	-1.3	2.9	-1.1	2.9	-1.4
RMS	23.7	19.2	7.8	—	—	11.8	8.9	5.4	8.7	11.6
min.dif	-120	-154	-42	—	—	-104	-81	-36	-54	-101
max dif	170	187	49	—	—	105	52	32	52	100

Table 3 Comparisons of geoid undulations and that from models (cm)

model area	OSU91A		IGG_LAND_G		IGG_SEA_U		IGG_LS_G		IGG_LS_GU	
	L	S	L	S	L	S	L	S	L	S
	GPS/L	Altm.	GPS/L	Altm.	GPS/L	Altm.	GPS/L	Altm.	GPS/L	Altm.
num.	73	3293	73	3293	73	3293	73	3293	73	3293
mean	-0.26	0.38	-0.16	0.27	-0.27	0.02	-0.14	-0.01	-0.15	0.02
RMS	1.54	0.60	0.71	0.58	1.54	0.13	0.72	0.36	0.70	0.13
min	-5.63	2.25	-1.82	-1.97	-5.62	-0.87	-1.81	-1.64	-1.83	-0.81
max	2.65	2.90	1.47	3.26	2.66	0.88	1.43	2.48	1.47	0.74

6. Conclusions

In fact, errors will occur in any method while conversing altimeter data into gravity anomalies, and have impacts on modeling. To minimize these effects, it is important to directly use altimeter data (i.e. altimetry geoid undulation) in the modeling. We developed a modeling method combining gravity data and altimetry geoid data by means of the tailored method. Based on this method, we computed the mixed regional geopotential model IGG_LS_GU to degree and order 360. The new mixed model IGG_LS_GU can fit the gravity field in China and its vicinity sea well.

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The Best Fitting High-Order Geopotential Model for the Medium to Long Wavelength Component of the Peninsular Malaysia Geoid

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ABSTRACT

The medium to long wavelength geoid signal (N_L) is the main component of the geoid height N , and this is evaluated from the coefficients of a spherical harmonic expansion, up to a certain degree and order. These coefficients are the end product of a complex process by which several data sources, including terrestrial gravity data (providing the fine detailed structure of the gravity field), observations of artificial earth satellites and satellite altimetry (information on the predominantly longwavelength characteristics of the gravity field over the ocean), are all combined in a mathematical solution to obtain the best approximation of the earth's gravitational field.

The accuracy of the gravimetric geoid in a region (contributing information on the short wavelength components of the geoid) depends on how well the global geopotential model that is used as the "reference model" in the gravimetric solution, matches the general features of the regional gravity field. In the paper the author reports on the performance of the geopotential models EGM96 and OSU91A, to determine which one of these models is more appropriate as the reference model for the gravimetric geoid solution in the Peninsular Malaysia region.

1. INTRODUCTION

Earlier investigations on the performance of "best fitting", high order, geopotential models to the Peninsular Malaysia local geoid parameters were reported in Ahmad & Kearsley (1993, 1994). Several models were tested to see how well the local gravity field could be recovered and the Ohio State University 91A (OSU91A) model was found to give the "best fit", implying that it best emulates the low frequency (long wavelength) components of the geoidal signals in this region.

In October 1996 a new global geopotential model was released known as the Earth Geopotential Model of 1996 (EGM96), representing the product of a joint project between the U.S. National Imagery and Mapping Agency (NIMA -- formerly the Defense Mapping Agency), NASA's Goddard Space Flight Center (GSFC), and the Ohio State University (OSU).

This paper compares the performance of EGM96 to OSU91A model (until 1996 considered the best geopotential model) in their ability to model the medium to long wavelength components of the Peninsular Malaysia geoid model.

2. THE ROLE OF GEOPOTENTIAL MODELS

The impact of the Global Positioning System (GPS) technology on the establishment and maintenance of geodetic control surveys has been immense. In the past geodesists relied upon line-of-sight instrumentation (theodolites and, since the mid-1950s, electronic distance measurement equipment) to propagate coordinates from the datum or origin station. With GPS, ground station intervisibility is no longer required, and it is possible to survey much longer baselines with unprecedented accuracy. Where as in the past different instrumentation and survey techniques were used to determine horizontal and vertical coordinates, resulting in separate horizontal control and vertical benchmark networks, GPS can provide three-dimensional coordinate information.

However, the height information obtained from GPS is intrinsically different to those historically obtained with geodetic levelling. GPS coordinates can be mathematically transformed to give so-called ellipsoidal height h . This is the height above (or below) a simple biaxial ellipsoid model of the earth. On the other hand, geodetic levelling gives rise to orthometric height H , often referred to carelessly as height above mean sea level. These are the heights typically found on topographic maps, stamped on markers, or stored in innumerable digital data sets. To relate these two height systems one requires the quantity geoid height N , so that: $h = H + N$.

The medium to long wavelength geoid signal (N_L) is the main component of N , and this value is easily generated from a global geopotential model, defined in terms of a set of spherical harmonic coefficients up to a certain degree and order (representing the shortest wavelength of the gravity field that can be modelled). The spherical harmonic model is the end product of a complex process by which several data sources, including terrestrial gravity data (providing the fine detailed structure of the gravity field), observations of artificial earth satellites and satellite altimetry (information on the predominantly longwavelength characteristics of the oceanic gravity field), are all combined in a mathematical solution to obtain the best approximation of the earth's gravitational field.

2.1 GENERAL DESCRIPTION OF EGM96

EGM96 is a geopotential model of the earth consisting of a set of spherical harmonic coefficients complete to degree and order 360. It is a composite solution (the data that was used are described below), consisting of: (i) a combination solution to degree and order 70; (ii) a block diagonal solution from degree 71 to 359; and (iii) the quadrature solution at degree 360.

New surface gravity data from many different regions of the globe, including data newly released from the NIMA archives, were used in the solutions. Major terrestrial gravity acquisitions by NIMA since 1990 include the airborne gravity surveys over Greenland and parts of the Arctic and the Antarctic (surveyed by the U.S. Naval Research Laboratory), and cooperative gravity collection projects (several of which were undertaken in partnership with the University of Leeds). These data gathering efforts have improved the data holdings over many of the world's land areas that formerly were very sparsely sampled, including Africa, Canada, parts of South America and Africa, South East Asia, Eastern Europe, and the former Soviet Union. There have also been major efforts to improve NIMA's existing 30' mean gravity anomaly database through contributions from various countries in Asia.

NIMA has also computed and made available 30'x30' mean altimeter-derived gravity anomalies from the GEOSAT Geodetic Mission. EGM96 also included altimeter-derived anomalies from ERS-1 provided by Kort and Matrikelstyrelsen (KMS -- the National Survey and Cadastre, Denmark) over portions of the Arctic, and the Antarctic, including the altimeter-derived anomalies over the Weddell Sea (Schoene, 1996).

Other data that contributed to the EGM96 model are direct satellite altimetry observations from the TOPEX/POSEIDON, ERS-1 and GEOSAT altimeter missions. Satellite tracking to over 20 satellites using SLR, GPS, DORIS, the Tracking and Data Relay Satellite System (TDRSS), and TRANET are also included. In addition, a new topographic database was assembled from all available sources in order to reduce the raw surface gravity observations, and to prepare the 30'x30' mean gravity anomaly values. This database was also used to calculate the isostatic "fill-in" values in the approximately 3% of the globe where surface gravity data was not available.

This new geopotential model will not only be used as a geodetic reference to complement the WGS84 (and other earth-centred, earth-fixed) reference frame, but also the state-of-the-art geopotential model for precise orbit, oceanographic, and geophysical investigations. See Lemoine et. al, 1996 and 1998.

2.2 GENERAL DESCRIPTION OF OSU91A

The data used in developing the earlier OSU91A model included altimeter data, the GEM-T2 potential coefficients (with its error covariance matrix) and surface gravity data represented by 0.5°x0.5° mean gravity anomaly values. The model exhibited an orbit rms fit of ± 34 cm for the GEOSAT satellite altimeter-derived geoid undulations (or height of sea level above the reference ellipsoid). The GEOSAT altimeter data that was used was based on the orbits computed using the GEM-T2 geopotential model and associated tracking station coordinates.

The fundamental terrestrial 1°x1° mean gravity anomaly data used for this solution, designated in October 1991 as the OSU91 model, had been steadily improved over the years with the incorporation of 30' terrestrial data gathered until 1989 from 38 different sources (see Rapp et al., 1990; 1991, for more details).

2.3 SOME COMMENTS

The large amount of artificial earth satellite tracking data, an oceanographic coverage of altimetrically-derived free-air gravity anomalies and the application of state-of-the-art space techniques to derive the spherical harmonic coefficients and tracking station coordinates would imply that the EGM96 model should be superior to the OSU91A model. More importantly, the wider geographic extent and greater density of terrestrial gravity data used in EGM96 is a major improvement over the surface gravity data sets available for OSU91A. However, the terrestrial gravity data from the Peninsular Malaysia region used for both model solutions is generally the same. Therefore, the differences between these two high order geopotential models over this region will be significant only as far as the results of modelling the

mean regional gravity signals. On the other hand, the variations (or rms) in the residual gravity is not expected to be significant since there has not been any upgrade in the quantity of terrestrial gravity data from this region.

3. DATA AVAILABILITY

The updated terrestrial gravity database of this region, collated mainly from the Department of Surveying and Mapping Malaysia (DSMM) and the Geological Survey Malaysia (GSA) from 1993 to the end of 1996 is used as the control data, totalling 7248 values (Ahmad & Kearsley, 1996).

This combination of both randomly distributed and gridded (at an approximate density of one measurement for every 25 km square area -- covering only 65% of the country) surface gravity data was first reduced to free air anomalies (see Equation 3.1), and then values from each respective models subtracted to obtain two sets of the residual gravity values (see Equation 3.2):

$$\Delta g_{FA} = g_{obs} - \gamma - \left(\frac{dg}{dH} H \right) \quad (3.1)$$

$$\Delta g' = \Delta g_{FA} - \Delta g_L \quad (3.2)$$

where

Δg_{FA} = free-air anomaly (Torge, 1980, p. 161)

g_{obs} = observed (terrestrial) gravity

γ = Normal gravity (GRS80) (Ibid, 1980)

$\left(\frac{dg}{dH} H \right)$ = free-air correction (Ibid, 1980)

$\Delta g'$ = residual gravity anomaly

Δg_L = gravity anomaly from a geopotential model

Neither marine gravity observations nor altimeter-derived free-air anomalies have been used in these tests. The region of computation is defined by the latitude and longitude bands: $1^\circ < \phi < 7^\circ$ and $99.5^\circ < \lambda < 104.5^\circ$, with the residual gravity anomalies divided into 61 $0.5^\circ \times 0.5^\circ$ blocks (see Figure 1).

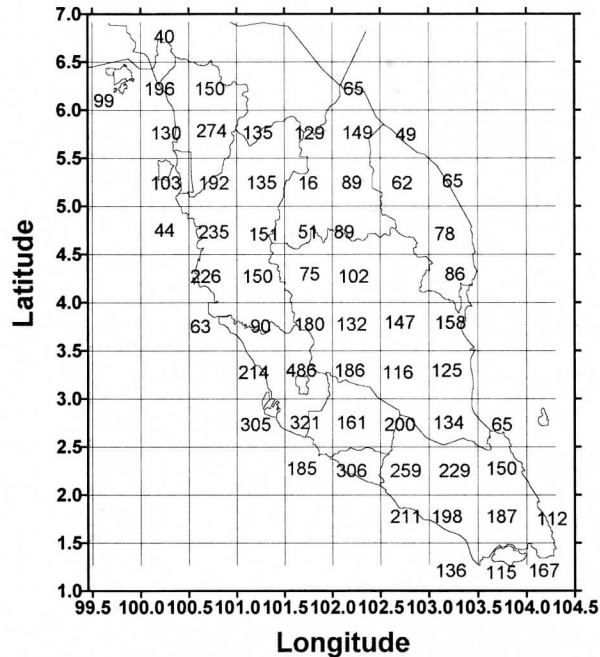


Figure 1: The Sample Size of Terrestrial Gravity Data for each $0.5^{\circ} \times 0.5^{\circ}$ Block used in the Statistical Analyses

4. TESTING PROCEDURES

A description of the analysis techniques and statistical tests are given in Ahmad & Kearsley (1993), including the flow chart of the procedures and a detailed description of the software used.

5. TEST RESULTS

Results of the statistical tests on the residual gravity anomalies from both models are discussed in Section 5.1 for the mean and Section 5.2 for the rms values, including the differences in both the mean and rms residuals for all the $0.5^{\circ} \times 0.5^{\circ}$ blocks.

5.1 MEAN RESIDUALS

The mean of the residuals based on the EGM96 reference model ranges between -61 to 11mGal, with an average of -11.5mGal. In case of the OSU91A model a wider range is obtained, between -74 to 14mGal with an average of -12mGal. The difference in the mean values of the residuals is about 0.5 mGal. As can be seen from Table 1 and Figure 2, both models show a strong negative bias (more than 75%) which indicates that the long wavelength features from these geopotential models have “underestimated” gravity field features over the Peninsular Malaysia region. This is to be expected because the amount of terrestrial gravity used in the geopotential model solutions, was not only about

15% (or less) of the current terrestrial gravity database (see Section 3), but are mainly concentrated in certain parts of this region.

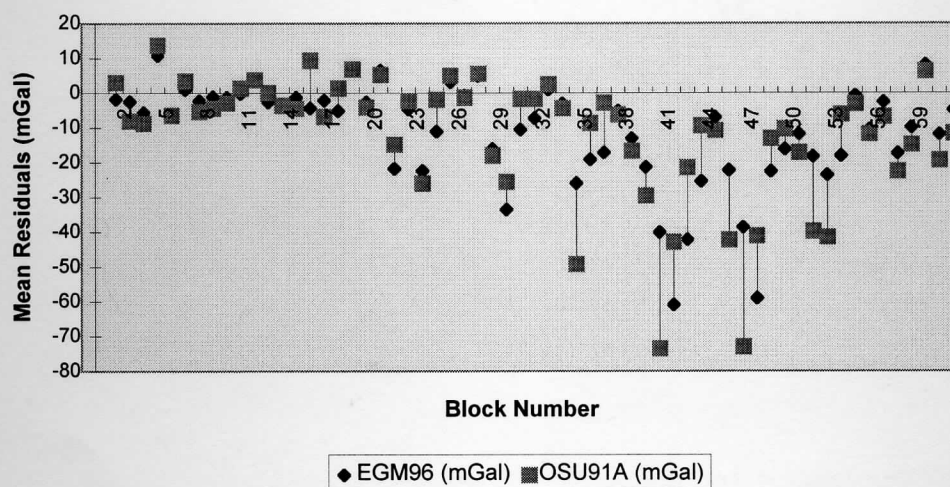


Figure 2: Mean Residuals from EGM96 and OSU91A, and their Differences.

The trend of the negative biases is strongly demonstrated in Figure 2, where most of the $0.5^\circ \times 0.5^\circ$ blocks are negative for both models. The bars in this figure graphically illustrate the magnitude of the differences in the mean residuals between both models, for each $0.5^\circ \times 0.5^\circ$ block. In general, where the differences are greatest, the mean residuals from EGM96 *always* have the smaller bias; for instance, the mean residual from EGM96 is -40.2mGal in Block 40 cf. -73.6mGal from OSU91A (hence, a difference of 33.4mGal, which translates to about 1.77 metres in height) and -38.7mGal in Block 46 from EGM96 cf. -73.1mGal for OSU91A (giving a difference of 34.4mGal, which is about 1.83 metres in height). These height differences are certainly significant when considering the millimetre accuracy sought in the conversion of GPS-derived ellipsoidal heights to the orthometric height system, although the geoid height values generally improve (depending on the quality of the surface gravity) with the inclusion of the short wavelength signals (N_s). Note also that the statistical average between these two models are only about 0.6mGal (about 32mm difference in geoid height), which is the general trend for most of the region. Since the mean residuals for most of the blocks are generally better for EGM96, certainly this high order geopotential model is *more* suited for modelling the medium to long wavelength components of the geoid in this region.

The areas of large bias over the mainland (see Figure 3) are mainly located in the mountainous regions, which include the Titiwangsa (which runs from the Thai border to south east of Kuala Lumpur, rising to about more than 2000 meter in elevation) and Tahan Ranges (the highest peak on Peninsular Malaysia, about 2187 meter). The main reason for these biases is because no surface gravity is available (see Figure 4 below), primarily due to the inaccessibility of these areas (mountainous ranges and dense rainforest). The mean gravity anomalies from the EGM96 model for the Titiwangsa and Tahan Ranges, is in the high 30s to -61mGal, while OSU91A performed even poorer, ranging from -40 to -70mGal. The improvement shown by the EGM96 model is very likely because a new topographic database has been used in the preparation of the $30' \times 30'$ gravity anomaly values, and the calculation of the isostatic fill-in values for the 3% of the globe where surface gravity data was not available (see Section 2.1).

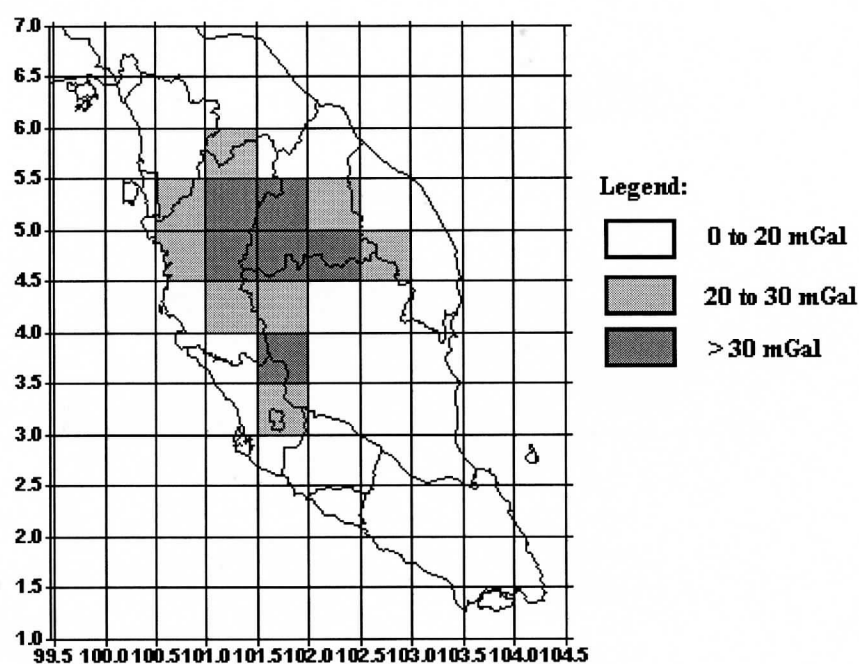


Figure 3: Mean Residuals from EGM96

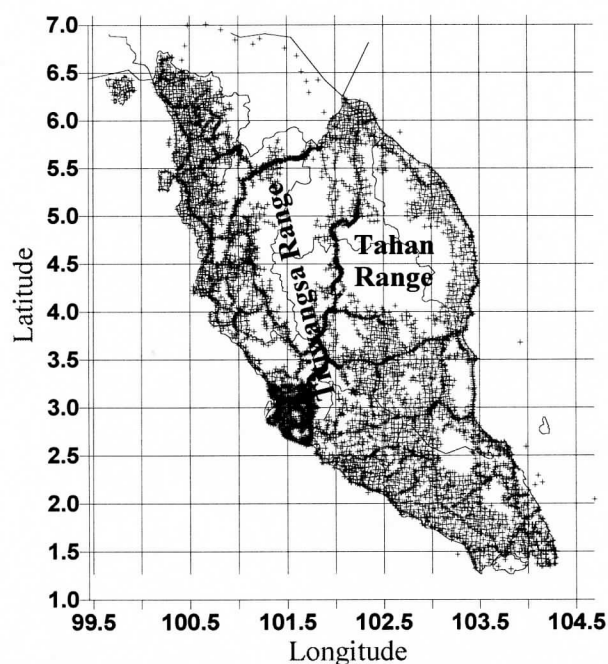


Figure 4: Distribution of Terrestrial Gravity Database for the Peninsular Malaysia Region.

5.2 RMS RESIDUALS

A statistical analysis of the rms residuals shows that the results are generally similar for about 90% of the $0.5^\circ \times 0.5^\circ$ blocks for both models (see Table 1, Columns 7, 8 and 9; and Figures 5). EGM96, however, performs slightly better with an average rms fit of $\pm 11.7\text{mGal}$ compared to $\pm 12.2\text{mGal}$ for

OSU91A. This is clearly see in Figure 5, where in the case of most of the blocks the rms residuals are generally smaller for the EGM96 model.

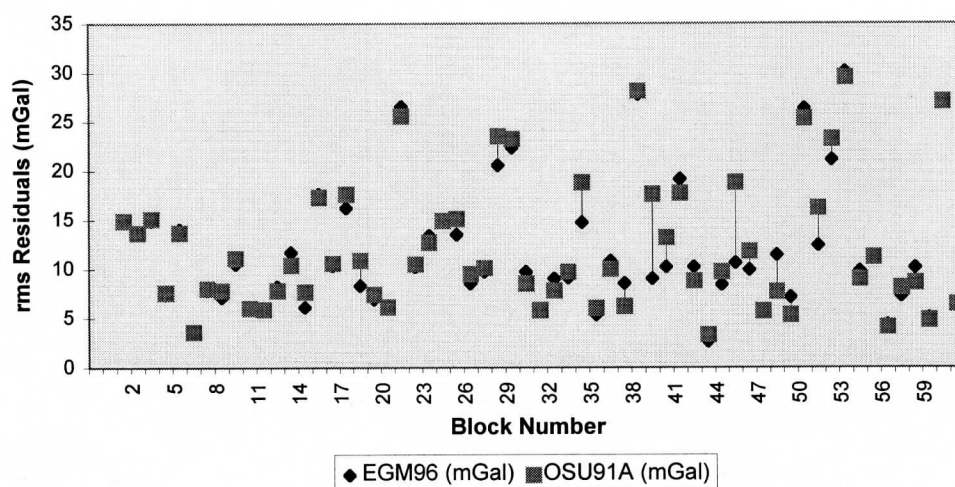


Figure 5: rms Residuals from EGM96 and OSU91A, and their Differences.

The fact that about 90% of these blocks give similar results is hardly surprising. The same set of terrestrial gravity has been used in the solutions of the potential coefficients for both models. In other words, the “sensitivity” or “roughness” of the terrestrial gravity is, overall, the same, and when tested at a resolution of $0.5^\circ \times 0.5^\circ$ (55km), the rms values are not expected to vary very much. The areas of largest bias (Figure 6) are predominantly over some part of the mountain ranges, coastal and swampy areas. However, since 10% of all blocks indicate the superiority of the potential coefficients of EGM96, this is further evidence that this high order geopotential model is better for modelling the long wavelength components of the geoid in Peninsular Malaysia.

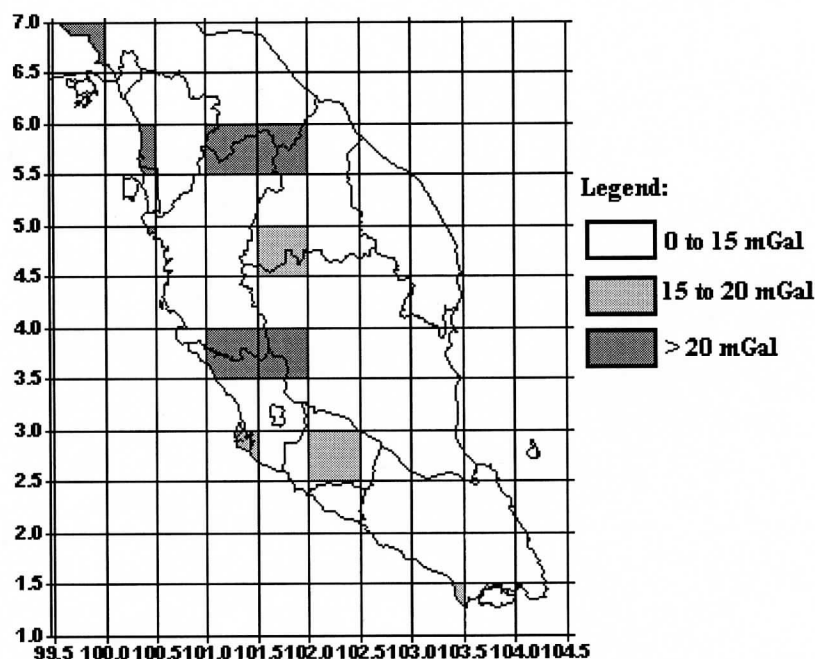


Figure 6: rms Residuals from EGM96

6. N_L COMPARISON

N_L values are generated for some 93 GPS first-order network from EGM96 and OSU91 A (Table 1, Columns 1 and 2, respectively), which are compared geometrically using the model equation $h - H - N = 0$ (Table 1, Columns 3 and 4). The orthometric height system (H) of this GPS network is of the accuracy of third-order levelling, and the ellipsoidal heights (h) are quoted to have an internal precision of $\pm (8 \text{ mm} + 1.2 \text{ ppm})$ (DSMM, 1993). The values of N_L from EGM96 ranges from -5.6 to 10 meter, with an average of 4.4 ± 3.21 meter, a smaller standard deviation compared to OSU91A's 4.18 ± 3.46 meter. EGM96 excels again in the statistical results of the geometric comparisons carried out on both models, not only with a much smaller range (-1.02 to 0.22 meter) compared to OSU91A (-1.02 to 0.82 meter), but also has a better standard deviations of the mean value (± 0.25 meter from EGM96 cf. ± 0.33 meter from OSU91A).

	1	2	3	4
	N_L (EGM96)	N_L (OSU91A)	Δ (EGM96)	Δ (OSU91A)
min	-5.622	-6.013	-1.020	-1.023
max	9.962	10.111	0.217	0.820
mean	4.431	4.185	-0.228	0.019
sdev	3.208	3.461	0.254	0.331

Table 1 : Statistical Analyses in the Geometric Comparisons of the N_L values from EGM96 and OSU91A

7. CONCLUSION

From the analysis of the mean and rms of the residual gravity anomalies, the EGM96 model has proven to be superior to OSU91A for modelling the medium to low frequency signals of the geoid (which contribute to the bulk of the geoid height) for the Peninsular Malaysia region. In Section 5.1, the mean residuals between the model and terrestrial gravity ranges from -61mGal (about 3.2m in height, and situated in the mountainous region) to 11mGal, with an average of -11.5mGal (about 0.6m in height), and more than 75% have a strong negative bias. This is an indication that the medium to long wavelength geoid features have been underestimated, which on the average could be up to 0.6m. This, of course, could be improved if a better coverage of terrestrial gravity data from this region were to be used in the calculation of the coefficients of spherical harmonic of the geopotential model(s). Indeed, the mountainous regions, and areas where there is lack of gravity (including swamps and the coastal plains), have the larger mean and rms values. However, on the whole, a large part of this region has both small mean bias and rms values, indicating that EGM96 can model the geoid undulations better than OSU91A. In much of the region tested, EGM96 is expected to model the relative geoid to better than 5ppm.

8. ACKNOWLEDGEMENTS

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Exploring The Detailed Structure Of The Local Earth's Gravity Field Using Fractal And Fourier Power Spectrum Techniques

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Abstract

As one of the world's oldest continents, Australia has experienced a complicated geological history and thus has a distinctive landscape and an extensively weathered topography. As such, the gravity field of Australia behaves differently to that in other countries. This paper investigates the features of the Australian gravity field. The methods used are a simple statistical comparison, Fourier power spectrum analysis and the Hurst fractal technique. Three test gravity and height profiles, which represent extreme features of the Australian gravity field (ie the Hamersley Ranges, Central Australia and The Snowy Mountains), have been selected. It is shown that the gravity field of Australia is not always correlated with the terrain and the topography often contains longer wavelength features than the gravity anomalies. It is demonstrated that the simple statistical analysis and Fourier power spectral methods are the most informative tools for measuring the smoothness of the gravity field. It is revealed that none of the free-air, Bouguer or topographic-isostatic gravity anomalies is consistently the smoothest type in Australia. Sometimes, the Bouguer anomaly is more variable than the free-air anomaly and thus should not necessarily be used for gravity field gridding in Australia.

1. Introduction

With the advent of the Navstar Global Positioning System (GPS), there is a pressing demand for a precise geoid model with which to transform GPS-derived ellipsoidal heights to orthometric heights. This has triggered a large amount of geoid-related research over the past decade. Generally, large amounts of data are involved in the evaluation of the Earth's gravity field, particularly in geoid determination and terrain reduction. For the interpretation of geophysical data, it is often convenient if the quantity to be analysed is decomposed as a sum of components, a process known as spectral analysis. The Fast Fourier Transformation (FFT) and Fast Hartely Transform (FHT) techniques (Brigham, 1988; Bracewell, 1986a, 1986b) have proven to be a very powerful tool for the efficient evaluation of gravity field convolution integrals (eg. Tziavos, 1996; Zhang *et al*, 2000) and present a very attractive alternative to the classical, time consuming numerical methods.

However, the FFT/FHT approaches normally require that gridded gravity data are utilised (Li *et al.*, 1995; Schwarz *et al*, 1990). Therefore, the irregularly spaced gravity observations must be re-sampled onto a regular grid over the area of interest. The conventional approach to such gravity gridding is to smooth the gravity field using reductions - the remove stage, interpolate these smoothed quantities onto the desired grid, then add back the appropriate gravity reduction - the

restore stage (Forsberg, 1984). In many instances, the Bouguer anomalies are used for data smooth purposes prior to gravimetrical geoid computation (Bian and Zhang, 1991). However, if the reduced gravity data are not smooth, interpolation errors will result, which will then propagate into the gravimetric geoid (Hipkin, 1988).

This research investigates the special features of the Australian gravity field in terms of relative roughness of various gravity anomalies and its spectral relationships with corresponding topography. The data used are various gravity anomalies and spot heights. The methods used are fractal geometry and Fourier spectrum analysis, in addition to conventional statistics and graphical visualisation. Fourier spectral analysis and the fractal geometry methods provide a new paradigm for understanding detailed structure and features of the Earth's gravity field.

Studies show that the gravity field in Australia is relatively complicated and does not adhere to the traditional axioms. Not one of the free-air, Bouguer or (Airy-Heiskanen) topographic-isostatic gravity anomalies is consistently the smoothest in continental Australia. This suggests that in order to grid the gravity field prior to geoid determination, some hybrid approach should be used, where the smoothest gravity anomaly type is used in different areas. Obviously, this has implications for interpolation, gridding and subsequent gravimetric geoid determination in this part of the world.

2. Data sets and investigation procedures

Gravity data over the Australian continent were supplied by the Australian Geological Survey Organisation (AGSO). This includes 526,091 land and 111,396 marine gravity observations. These data have been reformatted and validated using the procedures described in Featherstone *et al.* (1996) and Zhang (1997). A digital file of approximately six million spot heights was supplied by the Australian Surveying and Land Information Group. These spot heights have been combined with the gravity observation elevations, then gridded at 1km by 1km to produce the digital terrain model (DTM). Then, various gravity anomalies are calculated using the AGSO gravity databank and the 1km by 1km DTM.

Due to the variable features of the gravity field (Zhang, 1997), three profiles were selected (cf Table 1), which exhibit different topographic and gravity features. Profile 1 (P_1) is located in the Hammersley Ranges of Western Australia, Profile 2 (P_2) in central Australia and Profile 3 (P_3) in the Snowy Mountains. Statistics over the whole continent and three specific test profiles which

exhibit different topographic and gravity features are used for comparisons. The power spectral and fractal techniques are used to analyse the terrain, free-air anomalies, Bouguer and topographic-isostatic gravity anomalies across the three profiles.

Table 1 Location of the three test profiles

Profiles	P ₁	P ₂	P ₃
Latitude	25°S	26°S	36°S
Longitude	116°~120°E	126°~132°E	145°~149°E

Four different techniques are used to determine which type of gravity anomaly is the smoothest in Australia. These are:

1. Statistics of the various gravity anomalies across different profiles
2. Direct visual comparisons of the profiles
3. Power spectrum comparison at the three profiles
4. Hurst fractal dimension comparisons

Both the power spectral and the fractal analyses are well documented in the reference (cf: Barton and Pointe, 1995; Farge et al, 1993; Brace well, 1986a). However, the fractal analysis method is a relatively new technique and offers a new paradigm for understanding detailed structure and features of the Earth's gravity field. The fractal characterisation technique was initially applied to time-based phenomena, but can also be used for elevation-like profile analysis (Russ, 1994; Mandelbrot and Benoit, 1982; Barnsley, 1988). The best known applications of fractals are to surfaces and profiles (Barton and Pointe, 1995). The fractal dimension of a surface or a profile can be used to quantify its smoothness through its fractal dimension (*ibid.*). The larger the fractal dimension, the rougher the surface or profile. There are many different approaches to estimate fractal dimensions (Russ, 1994; Barnsley, 1988). The underlying characteristic of a fractal set is the self-similarity of the scales in the sense that there are large and small scales which maintain some relation between them (Farge *et al.*, 1993). Therefore, the fractal dimension can be a useful tool to measure the roughness of a profile or a surface in terms of classifying the texture of various gravity anomaly surfaces.

Hurst, or rescaled range analysis of fractals, was initially performed on time-based historical data (Hurst *et al.*, 1965). The basic idea to construct the Hurst fractal is to use a log-log plot of the maximum differences in a given window versus the size of the window (range). The size of this

window is progressively increased while the maximum difference is determined accordingly. The slope fitted to the log-log plot of the maximum differences and corresponding window size using linear regression is the slope value ($\hat{S}$). The slope value thus defined is used to determine the Hurst dimension (D).

Assuming the gravity anomalies (Δg) in a limited area to follow self-similarity, the log maximum differences of the gravity anomalies ($\log \Delta g$) and log differences ($\log L$) for a given window size (L) is related by:

$$S_i = \frac{\log_{10}(\Delta g_i)}{\log_{10}(L_i)} \quad (1)$$

and for a profile, the Hurst fractal dimension is:

$$D = 2 - \hat{S} \quad (2)$$

where S_i is the slope of the log-log graph corresponding to the i^{th} window; $\hat{S}$ is the regression slope of the log-log plot with an RMS regression error σ ; D is the Hurst fractal dimension for a profile (or a surface) and Δg_i is the maximum differences of the gravity anomalies within the i^{th} window size (L_i).

3. Test results and analysis

3.1 Statistics

The first indication of the relative roughness of the Bouguer anomalies in Australia can be seen by simply comparing the statistical properties of each gravity anomaly type across the whole Australian continent.

Table 2. The statistics of land gravity anomalies over Australia (units in mGal)

	MAX	MIN	MEAN	RMS	STD
Free-air	172.89	-121.10	0.40	25.31	25.10
Bouguer	82.56	-164.00	-24.79	41.27	33.00

Generally, the refined Bouguer anomaly surface should be smoother than the free-air anomaly surface. However, the STD of the Bouguer anomalies is higher than that of free-air anomalies over the whole continent. This indicates that the Bouguer anomalies are more variable, and thus more rough, so would be expected to introduce larger interpolation errors than if free-air anomalies alone are used during the gridding process. This implies that the application of a constant

topographical density model can not effectively remove the irregularities of the gravity field of the topography in Australia, and thus it is concluded that a complicated geological structure exists.

The Australian free-air anomaly shows a preponderance of negative features in the southwest and more positive features in the north and east flanks. It is found that a largely negative Bouguer anomaly field is a prominent feature of the Australian gravity (cf. Zhang, 1997). This finding is of importance for subsequent gravity field gridding and precise geoid determination.

Topographic-isostatic (TI) gravity anomalies were computed along these profiles and were also compared statistically to determine whether these are less variable than the Bouguer and free-air anomalies. Table 3 shows the statistics of the free-air, Bouguer and topographic-isostatic gravity anomalies for each test profile.

Table 3. The statistics of gravity anomalies over the three test profiles

Statistics (mGal)	Profile P ₁			Profile P ₂			Profile P ₃		
	FA	BG	TI	FA	BG	TI	FA	BG	TI
Max-min	61.3	74.6	51.2	128.0	219.5	137.4	161.0	71.7	53.6
Mean	8.8	-40.2	4.8	-31.4	-48.4	-37.0	2.7	-20.7	1.8
Rms	27.3	64.6	25.7	99.0	48.2	52.1	24.8	25.6	12.7
Std	15.8	9.8	12.9	15.4	71.5	15.2	23.5	18.6	10.2

By comparing the standard deviations values in each of the three study areas, there is no indication that any gravity anomaly type is consistently the smoothest. For example, in the Hammersley Ranges (P₁), the Bouguer anomalies are the least variable, whereas in the Snowy Mountains (P₃), the topographic-isostatic gravity anomalies are the least variable. However, this does not provide conclusive evidence in its own right as to the roughness of these gravity anomaly types. Therefore, spectral and fractal analysis are also used to support this hypothesis.

3.2 Profiles

The profiles of the free-air, Bouguer and topographic-isostatic gravity anomalies and terrain height are plotted for profiles P₁, P₂ and P₃ in Figures 1~3 respectively. Through a simple visual inspection of the profiles in Figure 2, the Bouguer anomalies are not necessarily smoother than the free-air anomalies. This is because the topography is of longer wavelength than the gravity anomalies. This observation suggests that, in some regions of Australia, the Bouguer anomalies are not necessarily the optimum gravity anomalies to use for interpolation.

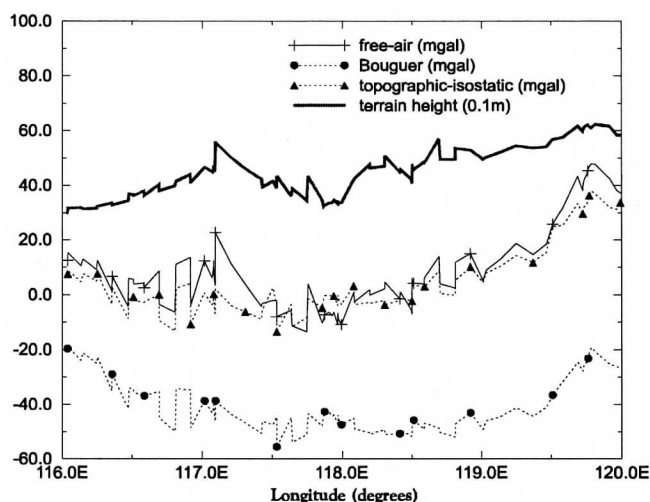


Figure 1 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P₁

In Figure 1, the profiles of free-air, and terrain height are very similar in shape and they thus have a strong correlation. The free-air anomaly profile is rougher than other two kinds of anomaly profiles in this area. The Bouguer anomaly profile is the smoothest.

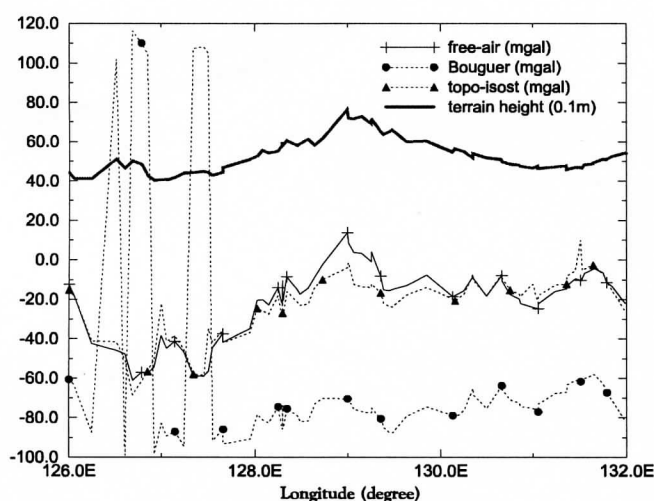


Figure 2 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P₂

For the profile P₂, it is revealed that Bouguer anomalies are relatively rough and some steep gradients exist at the intervals [126.0°E~128.0°E]. The free-air and topographic-isostatic gravity anomalies have a slightly negative correlation with terrain heights. This implies that a complicated geological structure exists in this region and $\rho=2,670\text{kgm}^{-3}$ is not representative of the true topographic density. In the intervals [128.0°E~130.0°E] topographic-isostatic anomaly has a slight correlation with terrain height. The topographic-isostatic gravity anomaly profile is the smoothest profile in this area, whereas the free-air gravity anomaly profile is slightly rougher than the topographic-isostatic profile.

The interesting fact is that the terrain profile, in contrast to the gravity anomaly profiles, is relatively smooth which means that the topography is of a longer wavelength in nature than the gravity. Therefore, it is concluded that terrain heights do not have strong correlation with free-air gravity anomalies along this profile at high (or even medium) frequencies, suggesting that there exists a large density anomaly below this region. From this point of view, the topographical density information is very important for the terrain reduction in the application of the remove-restore techniques, and free-air gravity anomaly reduction, where the gradient of the gravity cannot be represented by the normal gravity gradient, can cause errors in the free-air reduction. In addition, the free-air anomalies are correlated with terrain height, but not as strongly as in profiles P_1 and P_2 .

This finding can be confirmed by Anfiloff (1982), who used eighteen elevation and gravity profiles across Australia to study crust and tectonic processes using sparse gravity observations (~260,000). Anfiloff (*ibid.*) concluded that the gradient of the Bouguer anomalies along profile 133°E is very steep. Unfortunately, he was not able to give more analysis on several other profiles where similar features can be observed (ie. profiles 22°S, 24°S, 26°S, 29°S, 133°E and 144°E).

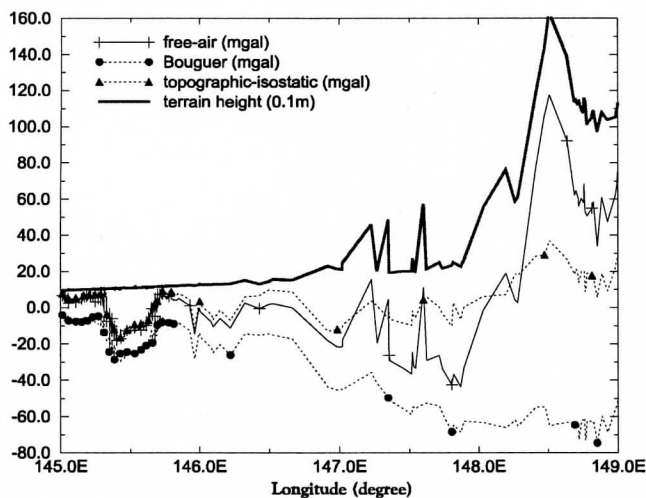


Figure 3 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_3

As seen from Figure 3, the free-air anomalies are highly correlated with the terrain heights in the interval [146.5°E~149°E]. However, all the three types of gravity anomalies are not correlated with the terrain heights in the interval [145°E~146.5°E]. This is most probably due to the relatively flat terrain and complicated geological structure along this interval. If a usual terrain reduction is applied to the gravity observations in this region using terrain heights only, the residual gravity anomaly will become rougher rather than smoother. Terrain and free-air

anomalies are very rough in this area. The Bouguer anomaly is smoother than free-air anomaly. The topographic-isostatic anomaly profile is the smoothest.

This study shows none of the gravity anomaly types are consistently smoothest in the three test profiles.

3.3 Power spectrum analysis

To analyse the spectral characteristics of the gravity anomaly profiles, and thus the gravity field, the Fourier power spectral analysis method (Brigham, 1988; Bracewell, 1986a) has been implemented. This method is very useful to analyse the detailed spectral structure of the gravity field in terms of power distribution versus wavelength. The power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies on profiles P₁, P₂ and P₃ are shown on Figures 4~6 respectively.

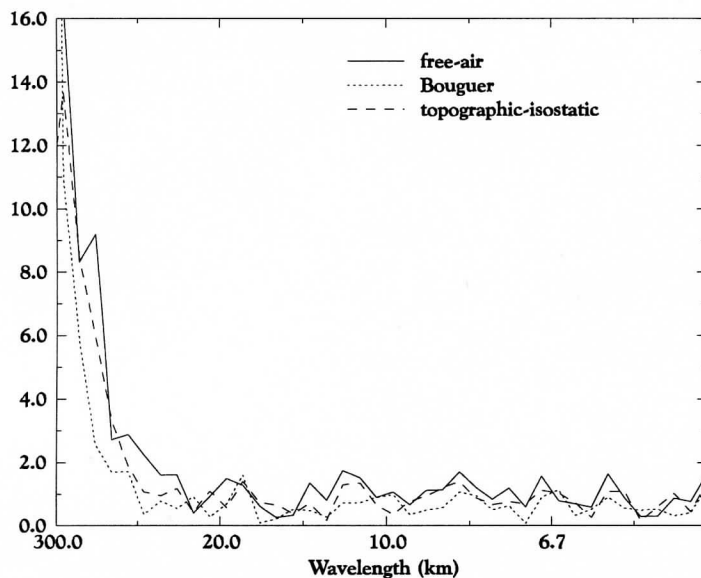


Figure 4 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies on profiles P₁

In Figure 4, all the three gravity profiles (ie. Bouguer, free-air and topographic-isostatic gravity anomalies) contain very similar power spectra in the short wavelength components (5km~20km). However, the free-air profile is the roughest and Bouguer anomaly profile is slightly smoother than that of topographic-isostatic gravity anomaly profile.

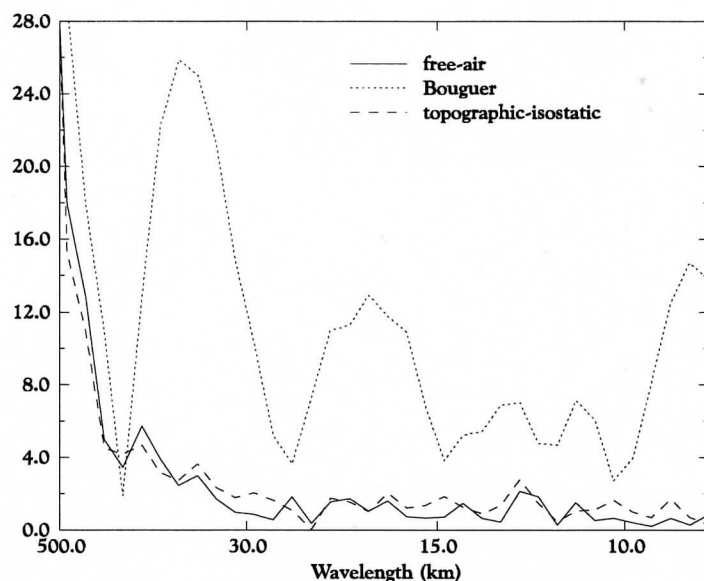


Figure 5 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at profile P_2

In Figure 5, it is clearly seen that Bouguer anomaly is much rougher than those of free-air and topographic-isostatic anomaly profiles for the short and medium frequency parts (10km~300km). The roughness of the free-air profile is very similar to that of the topographic-isostatic anomaly. It is hard to tell which is smoother directly from Figure 5. The topographic-isostatic anomaly, however, is slightly smoother than free-air anomaly can be observed if Figure 5 is rescaled. This conclusion complies with the previous statistical analysis.

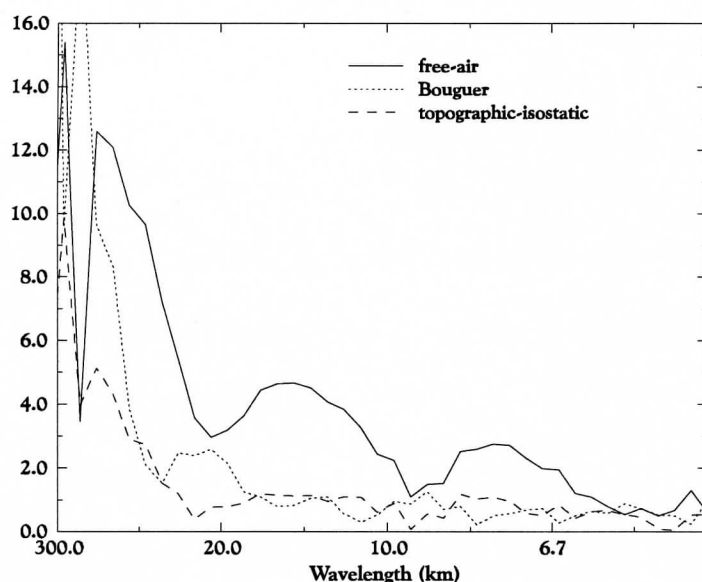


Figure 6 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P_3

From profile P_3 , it is concluded that the free-air anomaly is very rough in the short wavelength range ($< \sim 50$ km). The Bouguer and topographic-isostatic profiles are relatively

smooth in this wavelength band. However, the topographic- isostatic anomaly profile is slightly smoother than that of Bouguer anomaly.

3.4 Hurst Fractal Dimension

The fractal dimension of a surface can be used to quantify its relative smoothness. The Hurst fractal dimension (Russ, 1994) uses the logarithm of the difference in quantities versus the logarithm of their direct separation. The gradient of this line can give an estimate of the relative smoothness of quantities. This approach has been applied to gravity and terrain profiles across the Australian continent and the results from two typical profiles at P_1 , P_2 and P_3 are summarised in Table 4. One advantage using the Hurst fractal dimension method is that the gravity anomaly and terrain height profiles can be inter-comparable, otherwise not possible for other techniques.

Table 4. The Hurst fractal dimension at P_1 , P_2 and P_3

Profiles	P_1	P_2	P_3
free-air	1.9907 ± 0.0026	1.9985 ± 0.0001	1.9517 ± 0.0071
Bouguer	1.9920 ± 0.0020	1.9999 ± 0.0000	1.9616 ± 0.0047
TI	1.9923 ± 0.0022	1.9985 ± 0.0001	1.9517 ± 0.0071
terrain	1.9773 ± 0.0028	1.9527 ± 0.0071	1.9165 ± 0.0075

In Table 6, the Hurst fractal dimension indicates that the free-air gravity anomalies are the smoothest at both P_1 and P_2 . For these, and most other profiles in Australia, the free-air anomalies appear smoother (low fractal dimension) than the Bouguer anomalies (higher fractal dimension). This implies that interpolation errors could be decreased by using free-air anomalies alone, which also reduce the computational requirements when gridding gravity data prior to geoid determination.

Note that the terrain is smoother than the free-air anomalies, when they are expected be closely correlated, and thus equally rough. This feature suggests that the rougher terrain information is effectively contaminating the Bouguer anomalies and making them rougher instead of smoother. The gravity anomaly is more variable than the terrain which indicates that the terrain in Australia is of long wavelength in nature.

4. Reasons for the Complicated Australian Gravity Field

Australia is one of the world's oldest continents, which has a complicated geological history and a relatively smooth landscape. The highest mountain in Australia is 2228m (Mt Kosciuszko) and the lowest point is -15m (Lake Eyre). Correspondingly, the gravity field of Australia behaves quite differently to that in other countries. The gravity field is variable and not always correlated with the terrain. This explains why the Bouguer and terrain corrections will not necessarily smooth the gravity field.

According to this study, it is evident that none of the three gravity anomaly types tested are necessarily the smoothest over the Australian continent. It is most likely that the highly variable gravity field in Australia is due to the complicated geological structure of the continent in conjunction with the topography, which has been highly weathered and is thus relatively smooth. Despite this, the use of terrain data does not necessarily smooth the gravity field and appears to make it more rough. Therefore, the refinement of the Australian gravity field, especially for gravity field gridding prior to geoid determination is a delicate task.

5. Concluding Remarks

The gravity field of Australia has been found to be unusual. The gravity is not strongly correlated with the terrain but more so with the geology in some regions. It is also revealed that the Australian gravity field is highly variable, which is likely due to the complicated geological structure of the continent in conjunction with the relatively smooth topography. None of the gravity anomalies are consistently the smoothest over Australia. The Bouguer and topographical-isostatic anomalies are not necessarily smoother than free-air gravity anomalies in Australia. The roughness of the three kinds of gravity anomalies behaves quite differently in the three test areas. More importantly, this finding may also be pertinent to other parts of the world. Therefore the relative roughness of different gravity anomalies should be assessed before these data are gridded.

The power spectral analysis gives results consistent with the statistical comparisons. However, the power spectral analysis can give more detailed information for specific frequencies in the gravity field. The Hurst fractal dimension method can only approximately indicate the relative roughness of the surfaces. Due to regression errors in the estimation of the fractal dimension, the simple 2-D Hurst analysis is not always informative.

Of the three methods used for quantifying the smoothness of the various gravity anomalies, direct statistical comparison and Fourier power spectral method are the most informative. The statistical analysis method is simple, intuitive and easily implemented. The power spectral method also performs very well to scrutinise the detailed spectral distribution. The Hurst fractal analysis can also give relative roughness information approximately. However, when the smoothness of the profiles/surfaces is close, due to regression errors, the Hurst method does not perform very well. Therefore, the simple statistical comparison and Fourier power spectral analysis methods are recommended for the analysis of the gravity anomalies.

Given the complex nature of the Australian gravity field, 3-D density model is required for further refinement of the Australian gravity field where possible.

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The Sudanese Gravimetric Geoid 1998: Stokesian Approach

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Abstract

A new, high-resolution and high-precision detailed geoid has been computed for the whole of Sudan, ranging from 4° N to 24° N in latitude and 22° E to 38° E in longitude. The EGM96 geopotential model truncated to degree and order 70 was combined with surface gravity data and modified Stokes's kernel to generate the geoid file. The surface data consists of $30' \times 30'$ and $10' \times 10'$ mean gravity anomalies. Comparison of the gravimetric geoid with the Doppler/ levelling derived geoidal heights of 98 stations all over Sudan shows that the absolute agreement with respect to the Doppler/ levelling datum is generally better than 1.43 m RMS. Other geoids, such as OSU91A and EGM96 complete to degree and order 360 were also included in the comparison, showing that the new gravimetric geoid achieves the best agreement with the Doppler/ levelling data.

1. Introduction

The determination of the geoid has been one of the prime objectives of geodesy. The knowledge of the geoid with respect to some reference ellipsoid, either on a global or local scale is valuable to geodesy, surveying and geophysics for a number of purposes such as the reduction of measured distances to a reference surface and the processing of satellite observations. The geoid represents the datum to which height differences and the gravity potentials are referred. The geoid heights are essential for verification of global datums and transformations of local datums to the world datum. Also the combination of an accurate geoid model with GPS co-ordinates, plays a dominant role in achieving high accuracy levelling results. Spirit levelling is tedious, time consuming and costly in conventional surveying exercise. If the geoid undulation (N) of station is known, and the ellipsoidal height (h) is determined from GPS observations, then the orthometric height (H) of the station can be determined directly: ($N=h-H$). The knowledge of the geoid is also essential in geophysical explorations (reconnaissance survey), in control surveys, in large scale mapping, in engineering surveys and in height control.

The lack of gravity measurement over much of the earth surface is still the major problem in gravimetric geodesy. Many well known algorithms do exist to compute geoid undulations from surface gravity data, namely, Stokes's integration, Fast-Fourier Transform (FFT) and Least Squares Collocation (LSC). The combination of spherical

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harmonics potential coefficient set with terrestrial gravity data in order to reduce the latter requirement to a localised region for geoid height computation has been carried out by many researchers (e.g Molodenskii et al,1960;Paul,1973;Rapp,1981; Jekeli,1981 ; Sjoberg,1985;Vanicek et al,1986).In this investigation, a modified spheroidal Stokes's kernel is used in the geoid computation instead of the conventional ellipsoidal Stokes's kernel. It is found that spheroidal function tapers off more rapidly than the ellipsoidal function for increasing spherical distances. Thus we can expect that a truncation of the spheroidal (modified) integration at a certain spherical distance leads to smaller truncation errors compared to the truncation of the ellipsoidal (original) Stokes's integration.

2. Mathematical Models

As is well known, the geoidal height N , at any point of geographical co-ordinates (ϕ, λ) can be computed from a surface integral derived by Stokes in 1949 as:

$$N(\phi, \lambda) = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\psi) d\sigma \quad (1)$$

where:

- R : mean radius of the earth;
- γ : mean value of gravity over the earth;
- Δg : free air gravity anomaly;
- $S(\psi)$: original Stokes's Kernel
- $d\sigma$: surface element

In this paper a procedure to obtain the geoid over the whole Sudanese subcontinent is based on the combination of spherical harmonics potential coefficient set with terrestrial gravity data. The final formulae to compute gravimetric geoid heights were given in Fashir (1991c) as follows:

$$N = N^S + N^C + \delta N \quad (2)$$

$$N^S = R \sum_{n=2}^L \sum_{m=0}^n [\bar{C}_{nm}^* \cos m\lambda + \bar{S}_{nm}^* \sin m\lambda] \bar{P}_{nm}(\sin \phi) \quad (3)$$

$$N^C = \frac{R}{4\pi\gamma} \iint_{\sigma_c} S_L^m(\psi) \Delta g^L d\sigma \quad (4)$$

$$\delta N = \frac{R}{2\gamma} \sum_{n=2}^L \Delta g_n Q_{nm}^L(\psi_0) \quad (5)$$

with:

- N^S : long wavelength component
- N^C : short wavelength component
- δN : is the Truncation error

with:

$$\Delta g^L = \Delta g - \Delta g^S \quad (6)$$

$$\Delta g^S = \gamma \sum_{n=2}^L (n-1) \sum_{m=0}^n [\bar{C}_{nm}^* \cos m\lambda + \bar{S}_{nm}^* \sin m\lambda] \bar{P}_{nm}(\sin \phi) \quad (7)$$

$$S_L^m(\psi) = S(\psi) - \sum_{n=2}^L \frac{2n+1}{n-1} P_n(\cos \psi) - \sum_{k=0}^L \frac{2k+1}{2} T_k(\psi_0) P_k(\cos \psi) \quad (8)$$

$$S(\psi) = \operatorname{cosec} \frac{\psi}{2} - 6 \sin \frac{\psi}{2} + 1 - 5 \cos \psi - 3 \cos \psi \ln \left(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} \right) \quad (9)$$

The coefficients $T_k(\psi_0)$ are solved for from the system of linear equations:

$$\sum_{k=0}^L \frac{2k+1}{2} e_{nk}(\psi_0) T_k(\psi_0) = Q_n^L(\psi_0) \quad (10)$$

with:

$$e_{nk}(\psi_0) = \int_{\psi=\psi_0}^{\pi} P_n(\cos \psi) P_k(\cos \psi) \sin \psi d\psi \quad (11)$$

$$Q_n^L(\psi_0) = Q_n(\psi_0) - \sum_{k=2}^L \frac{2k+1}{k-1} e_{nk}(\psi_0) \quad (12)$$

$$Q_{nm}^L(\psi_0) = Q_n^L(\psi_0) - \sum_{k=0}^L \frac{2k+1}{2} T_k(\psi_0) \quad (13)$$

$$Q_n(\psi_0) = \int_{\psi=\psi_0}^{\pi} S(\psi) P_n(\cos \psi) \sin \psi d\psi \quad (14)$$

where;

$S_L^m(\psi)$: the modified Stokes's kernel

$Q_{nm}^L(\psi_0)$: truncation coefficient associated with the modified Stokes's kernel

L : the degree of the spheroid

$\bar{C}_{n,m}^*, \bar{S}_{n,m}^*$	: are the fully normalised potential coefficients
$\bar{\phi}$	: geocentric latitude
λ	: geocentric longitude
$\bar{P}_{nm}(\sin \bar{\phi})$	: associated Legendre function
Δg^L	: residual gravity anomaly
$Q_n(\cos \psi)$	: original Molodenskii truncation coefficients
$P_n(\cos \psi)$	: Legendre polynomial of degree n
$Q_n^L(\cos \psi)$	: spheroidal Molodenskii type truncation coefficients
$T_k(\psi)$	: modifying truncation coefficients
σ_c	: spherical cap from the computation point

3. Data Used In The Computations

3.1 The 30' x 30' (NIMA) Terrestrial Mean Free-Air Gravity Anomaly Data

These data originate from the National Imagery and Mapping Agency (NIMA), the former Defence Mapping Agency (DMA). This data set is termed the 30' mean terrestrial gravity anomalies from surface and airborne data (Lemoine, et al, 1997). In this study only data from the region 8° S to 36° N, and from 10° E to 50° E are used. The original data set contains 5945 gravity anomaly records in that region. Each record contains 9 pieces of information. These information are latitude, longitude, mean elevation, number of points used to estimate gravity anomaly, mean free-air gravity anomaly, standard deviation of gravity anomaly, code of method of evaluation, code of source and code of type. Also the anomalies are referred to WGS-84 horizontal datum. The anomalies are corrected for the effect of atmosphere. The orthometric heights used for the compilation of gravity anomaly database were derived from JGP95E global topographic database.

3.2 30' x 30' (NIMA) Altimetric Mean Free-Air Gravity Anomaly Data

This data set originates from NIMA. This data set is termed the 30' mean gravity anomalies derived from satellite altimeter data (Lemoine, et al, 1997). In this study only

data from the region 8° S to 36° N, and from 10° E to 50° E are used. The original data set contains 1068 gravity anomaly records in that region. Each record contains 9 pieces of information identical to that of the 30' x 30' (NIMA) terrestrial mean free-air gravity anomaly data. Anomalies are referred to WGS-84 horizontal datum and corrected for atmospheric effect. The orthometric heights used for the compilation of gravity anomaly database were derived from JGP95E global topographic database.

3.3 30' x 30' (OSU) Terrestrial Mean Anomalies

These data set originate from Ohio State University. This data set is termed the 30' x 30' terrestrial mean free-air anomaly database (Kim and Rapp, 1990). The original data set contains 4967 gravity anomaly records in the area of interest 8° S to 36° N and 10° E to 50° E. Each records contains 9 pieces of information identical to those of the 30' x 30' terrestrial mean anomaly data. Gravity anomalies are referred to WGS-84 datum. These anomalies were corrected for atmospheric effect.

3.4 The 10' x10' Mean Gravity Anomaly Data

These data set originate from the Geological Research Authority of Sudan (GRAS). In this study only data from the region 2° N to 26° N and from 20° E to 40° E are used. The original data set contains 15210 gravity anomaly records in that region. Each record contains five pieces of information, the latitude, the longitude, the gravity anomaly, the standard deviation of gravity anomaly and the elevation. The anomalies are referred to WGS-84 horizontal datum. Also anomalies are corrected for the effect of atmosphere.

3.5 The Quasi- Geoid to Geoid Coefficients

These data set were provided in terms of correction coefficients (Rapp,1997a). The corrections are used to convert height anomaly (ζ) into geoid height (N). The magnitude of (N- ζ) reached -3.4 m in the Himalaya Mountains with smaller but still significant magnitudes in other mountainous regions (ibid.,p.282). The conversion of the height anomaly to a geoid undulation is described in Heiskanen and Moritz (1967,p. 237) as:

$$N(\phi, \lambda) \cong \zeta(\phi, \lambda, r) + \frac{\Delta g_B}{\bar{\gamma}} H \quad (15)$$

where;

Δg_B : Bouguer anomaly

$\bar{\gamma}$: average value of normal gravity

H : orthometric height

Equation (15) was developed further by Rapp(1997a) to read:

$$N(\phi, \lambda) = \zeta(\phi, \lambda, r) + C_1(\phi, \lambda) + C_2(\phi, \lambda) \quad (16)$$

with;

$$C_1(\phi, \lambda) = \frac{d\zeta}{dr} H + \frac{d\zeta}{dr} \frac{dh}{dh} H \quad (17)$$

$$C_2(\phi, \lambda) = \frac{\Delta g_B}{\gamma} H \quad (18)$$

The C_1 and C_2 terms were expanded in spherical harmonics using OSU91A model and the 30' x 30' topographic data. Finally the sum of C_1 and C_2 of equation (16) is computed. As it seems from equations (17) and (18), the C_1 terms represents a height anomaly gradient term, that computes the difference between a height anomaly like value on the station. The C_2 term represents the difference between the geoid undulation and height anomaly, this term depends on the Bouguer anomaly and the elevation of the station.

3.6 EGM96 Geopotential Model Coefficients

The NASA Goddard Space Flight Centre, NIMA and Ohio State University (OSU) have collaborated to produce EGM96, an improved degree 360 spherical harmonic model representing the earth gravitational potential. This model was developed using terrestrial, airborne and altimetry data together with satellite tracking data from more than 20 satellites, including new data from GPS and TDRSS, as well as altimeter data from TOPEX, GEOSAT and ERS-1. The final model, EGM 96, is a composite solution, It consists of a combination solution to degree and order 70, a block diagonal solution from degree 71 to 359, and a quadrature solution at degree 360 (Lemoine, et al, 1997). The truncated EGM96 (70,70) coefficient set has been chosen out of the variety of potential field models published so far, as probably the most recent solution available at the time of our computations. The truncated EGM96 model is complete to degree and order 70. This implies that the smallest gravity field features represented in the truncated model have a spatial extent of 2.6° spherical distance or 288 km.

4. Computation of The Gravimetric Geoid

4.1 Selection of optimal integration Cap Size (Ψ_0)

Before the final computations of the geoid heights, we had to decide upon the optimal integration spherical cap beyond which gravity contribution is negligible. To decide upon the selection of Ψ_0 , the gravimetric geoid was computed for the whole of the Sudanese subcontinent ($4^\circ \leq \phi \leq 24^\circ$, $22^\circ \leq \lambda \leq 38^\circ$), on a grid of 4° interval. The integration cap size was varied from 2° to 9° and the residual geoid heights at the above mentioned grid points were computed from Stokes's integration using modified Stokes's

kernel. The results were shown in table.1. From Table.1, it is evident that geoid contribution beyond cap size of 6° was almost constant. That is to say, geoid gain beyond cap size of 6° was of the order of 1cm. Hence a cap size of 6° was chosen as an optimal integration cap size for our computations.

degr

φ	λ	Capsize 2°	Capsize 3°	Capsize 4°	Capsize 5°	Capsize 6°	Capsize 7°	Capsize 8°	Capsize 9°
4	22	0.88	0.87	0.86	0.87	0.86	0.85	0.85	0.85
4	26	-.15	-.29	-.39	-.46	-.52	-.53	-.53	-.53
4	30	0.14	0.14	0.13	0.13	0.13	0.14	0.14	0.14
4	34	3.01	3.17	3.28	3.35	3.43	3.44	3.44	3.44
4	38	2.11	2.22	2.29	2.35	2.38	2.39	2.39	2.39
8	22	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
8	26	1.95	2.04	2.07	2.07	2.06	2.05	2.05	2.05
8	30	-0.48	-0.54	-0.59	-0.64	-0.68	-0.67	-0.67	-0.67
8	34	-0.47	-0.39	-0.34	-0.31	-0.28	-0.29	-0.29	-0.29
8	38	-1.41	-1.55	-1.64	-1.71	-1.75	-1.74	-1.74	-1.74
12	22	0.73	0.76	0.77	0.78	0.79	0.78	0.78	0.78
12	26	0.15	0.15	0.14	0.14	0.13	0.13	0.13	0.13
12	30	0.98	1.00	0.99	0.98	0.97	0.96	0.96	0.96
12	34	-0.11	-0.13	-0.12	-0.11	-0.11	-0.11	-0.11	-0.11
12	38	1.96	1.94	1.94	1.95	1.96	1.96	1.96	1.96
16	22	0.14	0.18	0.20	0.21	0.22	0.21	0.21	0.21
16	26	-0.43	-0.45	-0.47	-0.48	-0.49	-0.50	-0.50	-0.50
16	30	-0.72	-0.79	-0.84	-0.87	-0.90	-0.91	-0.91	-0.91
16	34	0.49	0.52	0.54	0.54	0.53	0.53	0.53	0.53
16	38	2.46	2.57	2.65	2.71	2.75	2.74	2.74	2.74
20	22	0.70	0.67	0.69	0.71	0.72	0.72	0.72	0.72
20	26	-1.09	-1.23	-1.33	-1.41	-1.47	-1.48	-1.48	-1.48
20	30	-1.30	-1.46	-1.58	-1.67	-1.75	-1.76	-1.76	-1.76
20	34	-1.47	-1.59	-1.66	-1.72	-1.78	-1.77	-1.77	-1.77
20	38	-1.50	-1.59	-1.66	-1.71	-1.75	-1.74	-1.74	-1.74
24	22	1.71	1.85	1.92	1.97	2.00	2.01	2.01	2.01
24	26	-0.17	-0.29	-0.38	-0.44	-0.48	-0.47	-0.47	-0.47
24	30	1.12	1.11	1.05	1.00	0.96	0.95	0.95	0.95
24	34	-0.10	-0.03	-0.04	-0.11	-0.18	-0.19	-0.19	-0.19
24	38	-0.03	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05

Table 1. Residual geoid contribution from different spherical capsizes in metres.

4.2 The Long wavelength Component of The Geoid (N^S)

A major step forward in geodesy and geophysics is the rapid progress in the determination of the shape of the earth and its gravity field from the satellite orbit analysis to the global solutions expanded in the spherical harmonics up to degree 360. Better knowledge of the geopotential models has created dramatic advances in point positioning, in the study of the earth's kinematics and tectonics, in understanding the earth's rheology and interior and in the study of global oceanic process with space-borne instrumentations. These high degree expansions can be used to evaluate quantities such as geoid undulations, height anomalies, gravity anomalies, gravity disturbances and deflections of the vertical.

In this investigation, The long wavelength geoid component (N^S) was computed using equation (3) from the truncated EGM96 potential coefficient set complete to degree and order 70 and the parameters of the WGS-84 ellipsoid. Also together with these model coefficients are the associated topographic correction coefficients developed in terms of spherical harmonics. Hence at first the program computes height anomalies (ζ) and uses the corrections to obtain the geoid.

The geoid heights were computed on a grid defined by 4° N to 24° N and 22° E to 38° E. The grid interval is 10'. A total of 11737 grid points were considered in the computation. The contoured geoid on WGS-84 ellipsoid was shown in Fig 2. The contour interval is 2 m. From Fig 2. The geoid slopes from a high of about 13 m in north Sudan to a low of about -13 m in South Sudan. In general there is a north southerly down slope of undulations.

4.3 The Residual (Short & Medium) Wavelength Component of The Geoid (N^C).

The residual geoid was recovered from the numerical integration of the modified Stokes's function using surface gravity data. Stokes's integral of equation (1) is certainly one of the most famous formula in physical geodesy. This formula requires integration of the gravity anomaly over whole earth. The lack of gravity coverage has led to a natural approximation, whereby the integration is limited to spherical cap (truncation of the integral). In this investigation the residual geoid was computed from the numerical evaluation of the spheroidal Stokes's integration using modified Stokes's function. This function differs from the ellipsoidal Stokes's function of equation (1) by the fact that it lacks the low frequency part and it tapers off more rapidly. The original Stokes's kernel $S(\psi)$, the spheroidal Stokes's kernel of degree 70 $S_{70}(\psi)$ and the modified spheroidal Stokes's kernel $S_{70}^m(\psi)$ were shown in Fig 1.

The spherical integration cap $\psi \leq \psi_0$ is approximated by one-degree square of ellipsoid co-ordinates ϕ, λ . This approximation technique is well known in literature (eg. Kearsley

et al,1985). The complete integration area, delineated by bold lines in Fig 3 is divided into three main zones:

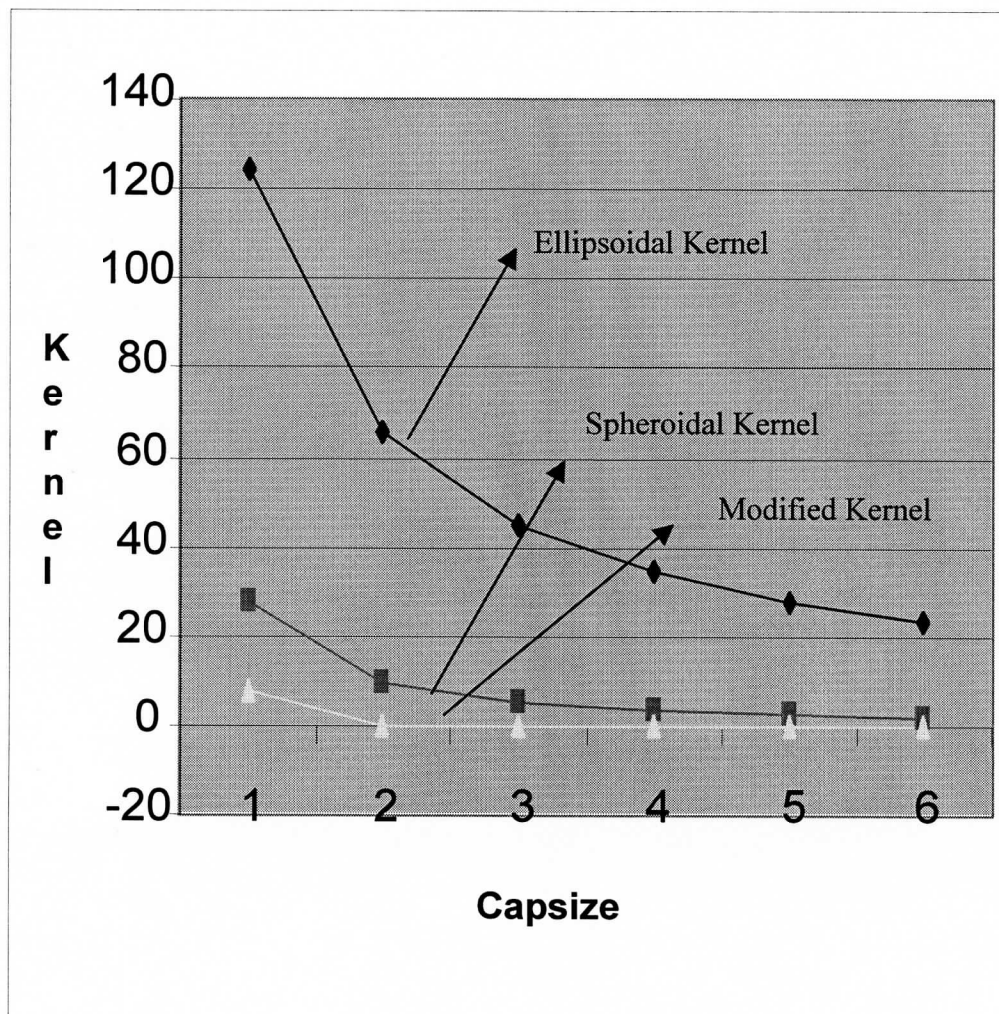


Fig.1 The contribution of the residual geoid for different capsizes in metres

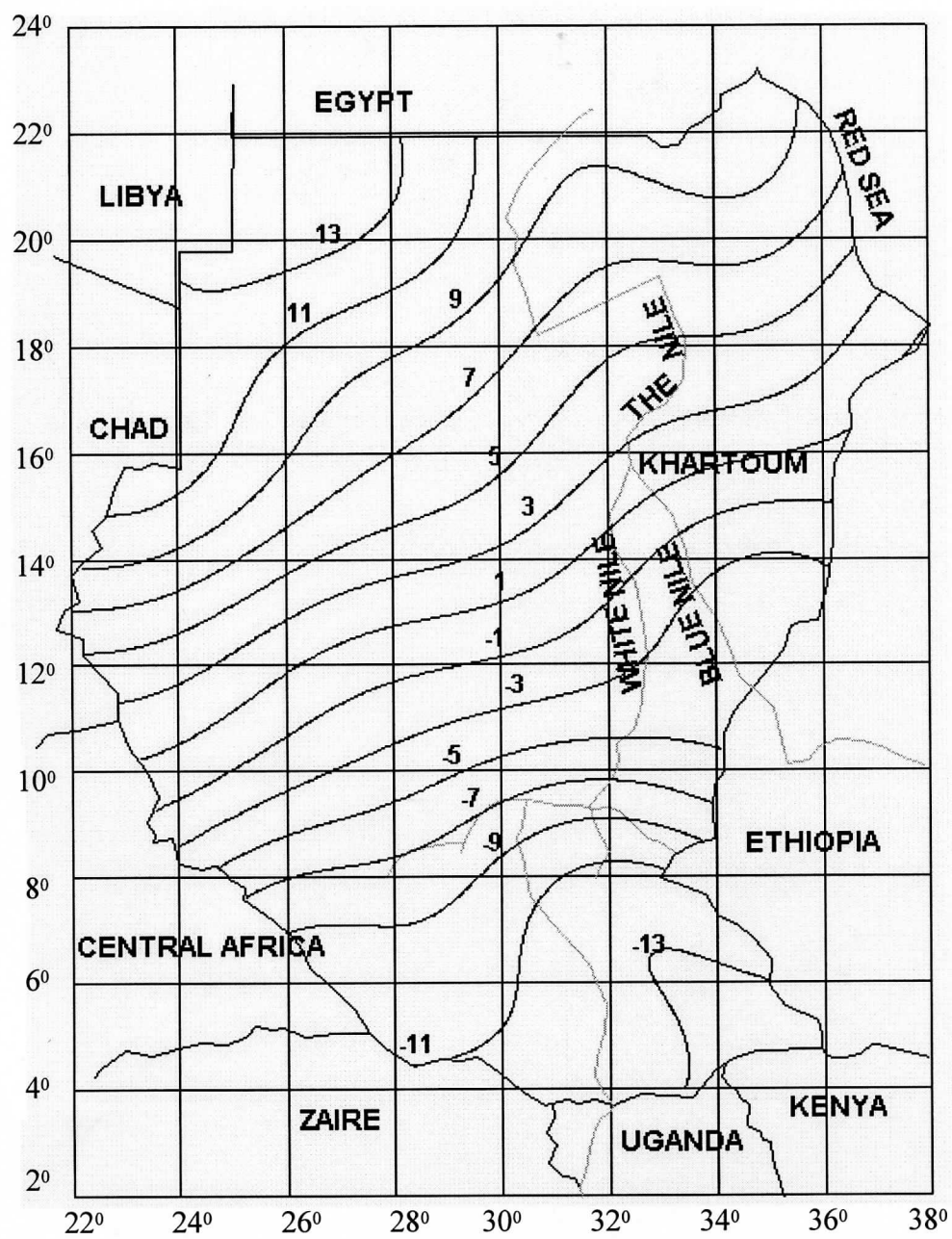


Fig 2 The long wavelength geoid component, ($N_{\max} = 70$). Contour interval 2 m.

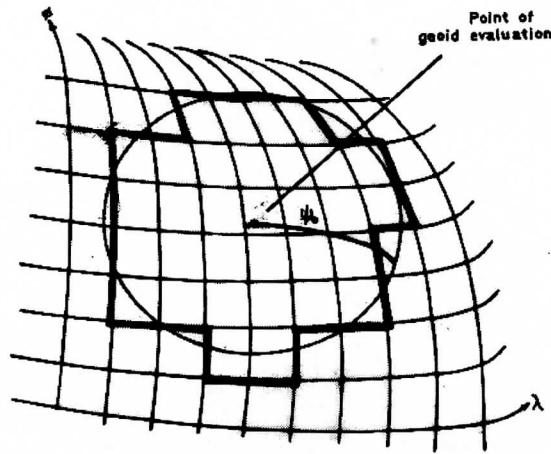


Fig .3 Approximation of spherical cap (ψ_0)

- **The innermost zone**

This covers the immediate neighbourhood of the point of geoid evaluation. The size of the innermost zone is $20' \times 20'$. The boundaries of the innermost zone coincide with the grid division of the $10' \times 10'$ gravity anomaly data file. The geoid contribution in this zone is computed from:

$$N_1 = \frac{R}{4\pi\gamma} \int_{x_1}^{x_2} \int_{y_1}^{y_2} S_L^m(\psi) \Delta g^L(x, y) dx dy \quad (19)$$

with;

$$\psi = \sqrt{(x^2 + y^2)} \quad (20)$$

$$\Delta g^L(x, y) = c_0 + c_1 x + c_2 y$$

$$S_L^m(\psi) = \frac{2}{\psi} - 3 \ln\left(\frac{\psi}{2}\right) + M(\psi_0)$$

$$M(\psi_0) = - \left[.4 + \sum_{n=2}^L \frac{2n+1}{n-1} + \sum_{k=0}^L \frac{2k+1}{2} T_k(\psi_0) \right] \quad (21)$$

At every time the four ($10' \times 10'$) gravity anomaly data around the computation point were considered in the integration, due to the fact that no point gravity anomalies were available in this investigation.

• The Inner Zone

This zone extends over an area of $2^0 \times 2^0$ minus the innermost zone. The boundaries of the innerzone coincide with the grid division of the $30' \times 30'$ gravity anomaly data file. In this zone the geoid is computed using the one hundred and fourty ($10' \times 10'$) gravity anomaly data in the inner zone. This is carried out as :

$$N_2 = \frac{a^2 R}{4\pi\gamma} \sum_{i=1}^n S_L^m(\psi_i) \cos\phi_i \Delta g_i^L \quad (22)$$

where;

n : is the number of cells in the inner zone

a : is the cell size ($10'$) in radians.

• The Outer Zone

This is the area between the boundaries of the inner zone and the integration area (6^0 spherical distance from the computation point). Also the geoid contribution in this zone is recovered from:

$$N_3 = \frac{b^2 R}{4\pi\gamma} \sum_{i=1}^k S_L^m(\psi_i) \cos\phi_i \Delta g_i^L \quad (23)$$

with,

k : is the number of cells in the outer zone

b : is the cell size ($30'$) in radians.

The subdivision of the integration area into three zones is a matter of convenience to allow for different integration techniques and different data input in the integration. The summation of the integration results in the three zones resulted in the residual geoid. This residual geoid is contoured and shown in Fig 4. The contour interval is 1 m. From Fig 4 it is evident that the residual geoid attains a maximum of 5 m. Finally, the total gravimetric geoid, the summation of the residual geoid, the long wavelength geoid and the truncation error of (Eq.5) is contoured in Fig.5. From fig.5 The shape of the geoid with respect to WGS-84 ellipsoid remains the same as the geoid shown in Fashir (1991c), but there are substantial differences in geoid values for some identical points, ranging between 3-5 m. This is mainly due to two reasons: our better knowledge of the gravity field in Sudan since our last attempt to compute the gravimetric geoid (additional $10' \times 10'$ mean anomalies were obtained, besides $1^0 \times 1^0$ mean anomalies were replaced by $30' \times 30'$ mean anomalies). Also the correction coefficients used to convert height anomalies to their equivalent values of geoid heights may have profound effect on points with rather

higher elevations (Rapp,1997a). There is a general north southerly down slope of the geoid. The geoid slopes from a high of about 13 m in north Sudan to a low of -13 m in Southern Sudan.

4.4 Point Testing Against Doppler Derived Undulations

As far as the geoid undulations are concerned, any two solutions from different techniques can be compared after reducing the two solutions to a common datum. Due to the fact that GPS data co-located with Benchmarks were not available in Sudan, The data used in this investigation consisted of 98 Doppler stations associated with levelling. The Doppler data was carried out by point positioning using precise ephemeris. The Doppler data were given on the world wide absolute Figure NWL-9D. The geoid heights at the 98 stations were computed on NWL-9D and transformed to WGS-84 Datum (Fashir,1991a) Three sets of geoid undulations were computed at the 98 Doppler stations, using the gravimetric geoid model, the EGM96 model and the OSU91A model. The distribution of the Doppler stations were shown in Figure 6. The differences between these geoids and the Doppler heights have then been analysed statistically, regarding the 98 values as a statistical sample. The results were shown in Table 2. Since we considered N_D as an external standard of comparison then the RMS differences provide a measure of the accuracy of the gravimetric, the OSU91A and the EGM96 geoids.

Statistics	$N_G - N_D$	$N(\text{EGM96}) - N_D$	$N(\text{OSU91A}) - N_D$
Minimum difference (m)	-1.78	-3.51	-1.89
Maximum difference (m)	3.99	4.44	5.01
Mean difference (m)	1.57	0.96	1.51
RMS difference (m)	1.43	1.67	1.48

Table 2. Comparison of Doppler / levelling Data To Different Geoid Solutions

All Doppler stations have been connected to the first order levelling network in Sudan. In general the vertical datum inconsistency may reach up to 2 m (Rapp,1992) due to the fact that mean sea level at different locations does not lie on the same equipotential surface. Hence the expected accuracy of the residual geoid (ΔN), is computed from:

$$\Delta N = N_G - N_D \quad (24)$$

$$= N^S + N^C + \delta N - h + H \quad (25)$$

where;

- N_G : gravimetric geoid
- N^S : long wavelength component
- N^C : short wave length component
- δN : The truncation error
- h : ellipsoidal height

H : orthometric height

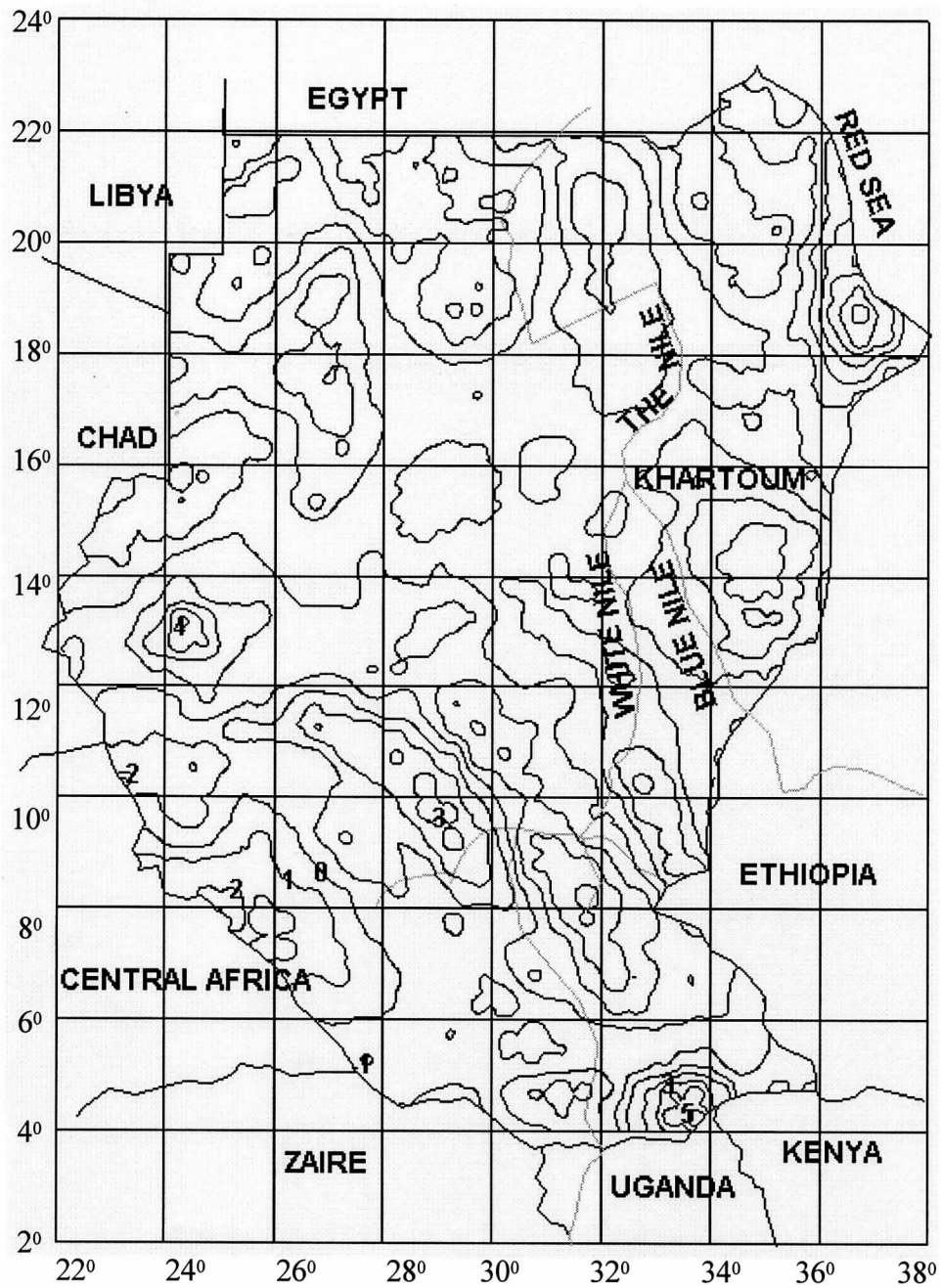


Fig 4 The residual gravimetric geoid component. Contour interval 1 m

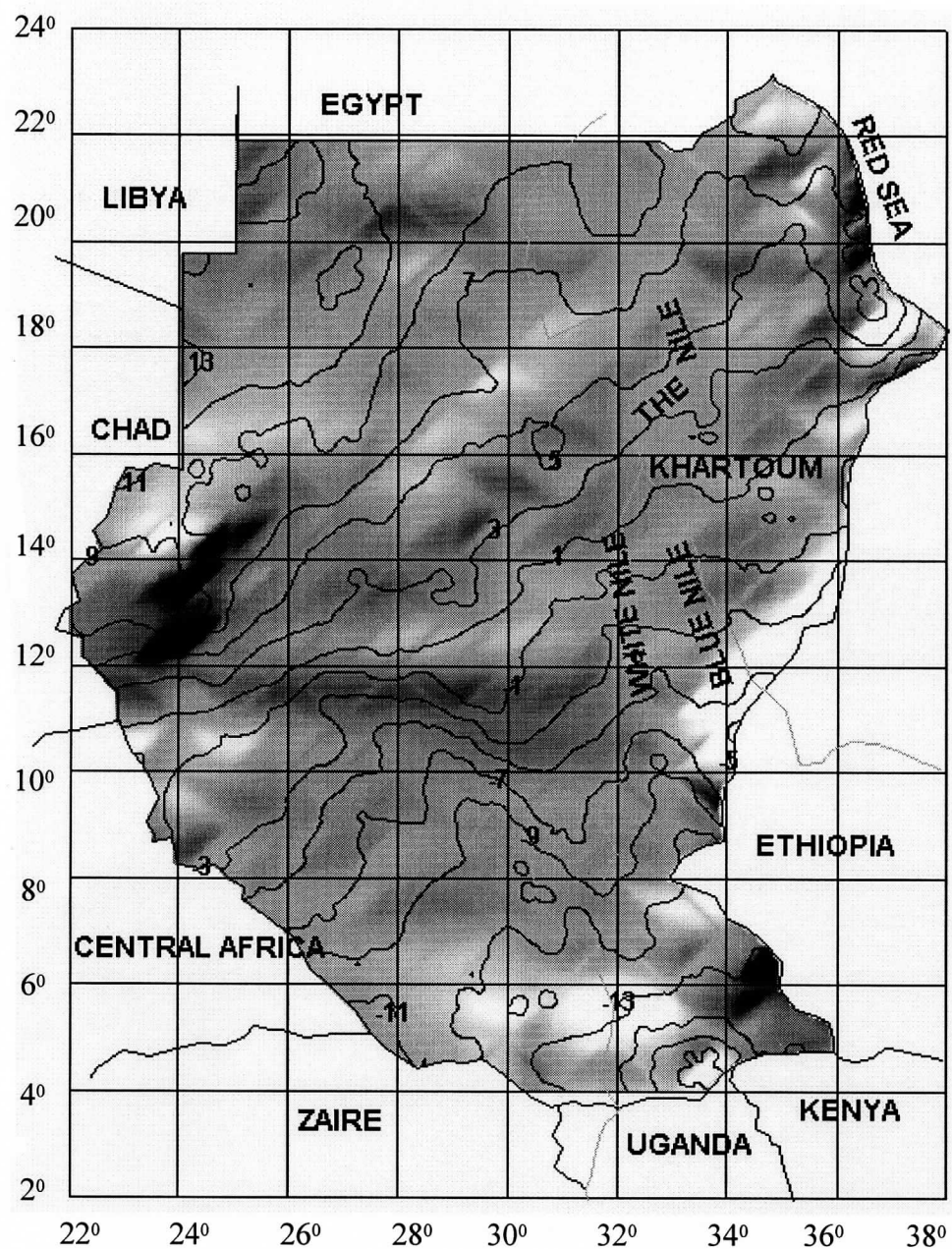


Fig 5 The total gravimetric geoid. Contour interval 2 m.

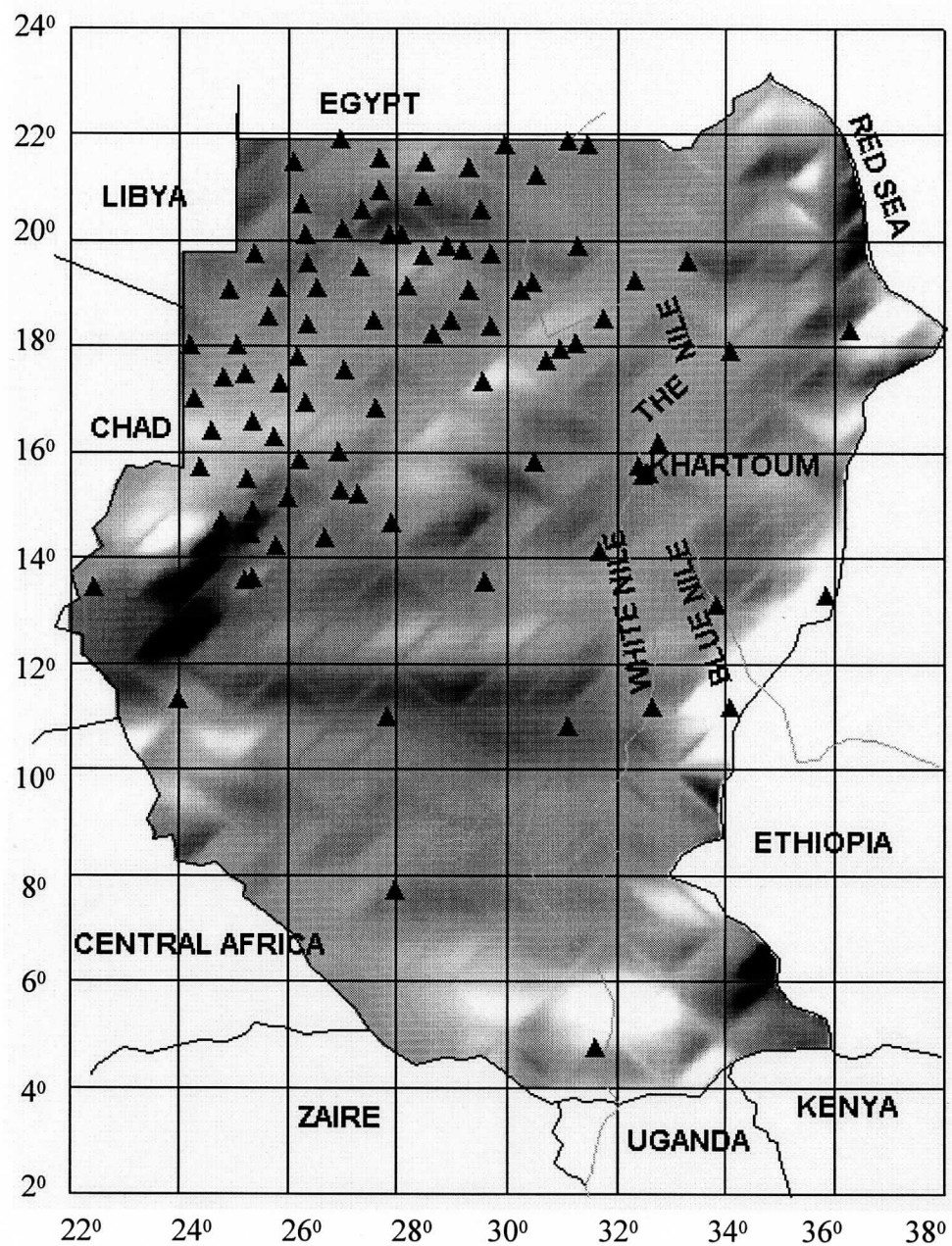


Fig.6 Distribution of Doppler Data in Sudan

Hence applying the law of error propagation to (Eq.25) and assuming the parameters are statistically independent yields:

$$\sigma_{\Delta N}^2 = \sigma_{N^S}^2 + \sigma_{N^C}^2 + \sigma_h^2 + \sigma_H^2 \quad (26)$$

Assuming $\sigma_h = 0.3\text{m}$, $\sigma_N^S = 0.7\text{ m}$, $\sigma_H = 1.1\text{ m}$ and $\sigma_N^C = 0.6\text{ m}$. The law of error propagation yields $\sigma_{\Delta N} = 1.43\text{m}$. From Table 2 , we can conclude that the accuracy of gravimetric geoid is in well agreement with expected accuracy of (ΔN). This precise geoid could be used for all regional purposes such as datum connections, adoption of a unified vertical datum, engineering works, national and regional planning maps for the country, as well as to contribute needed geophysical data to the other branches of earth sciences.

5. conclusions

The outcome of this investigation is a detailed gravimetric geoid of Sudan. The geoid is recovered using 30' x 30' and 10' x 10' mean surface gravity anomaly data combined with truncated EGM96 (70, 70) spherical harmonics potential coefficient set. To compute the gravimetric geoid in Sudan, the modified Stokes's kernel is being used instead of the original ellipsoidal Stokes's kernel, to reduce truncation errors as the former tapers off more rapidly than the latter (the influence of distant gravity anomalies on local geoid heights is reduced). The reduction is proportional to the degree L of the satellite model being used to recover the long wavelength component of the geoid.

To recover the long wavelength contribution of the gravimetric geoid, the EGM96 (70,70) model coefficient set complete to degree and order 70, has been chosen out of the variety of potential field models published so far, as probably the most recent solution available at the time of our computations. This implies that the smallest gravity field features represented in EGM96 (70,70) model have spatial extent of 2.6° spherical distance or 288 km. The short wavelength component of the geoid is recovered by means of mean gravity anomalies after subtracting the corresponding EGM96 (70,70) model values from gravity anomalies. The optimal integration cap size in this case was found to be 6° from the computation point.

The total gravimetric geoid was obtained from the summation of the long wavelength, geoid component, the short wavelength geoid components, and the truncation error of (Eq.5). This solution was referred to WGS-84 datum. The comparison of Doppler

geoid heights with the corresponding gravimetric values showed a reasonable agreement with RMS of 1.43 m.

Finally, to meet the 1cm geoid which has been the goal of geodesists and geophysicists, we suggest it is very important to densify the regional gravity data in Sudan. The suggestion can not be met unless a mutual co-operation between geodetic communities in Sudan, oil-exploring companies in Sudan and the bonafide potential users of the gravity data in the country is established. Hence, if a determination of the geoid to an accuracy of the order of 1cm is attempted, the effect of the atmosphere, the topography and the ellipticity of the reference surface on the gravity as well as the indirect effect on the computed geoid must be taken into account.

To verify our gravimetric solution, an independent geoid should be derived from Global Positioning System (GPS) network established throughout the subcontinent with its station located at known benchmark heights.

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THE NEW GRAVIMETRIC GEOID IN ALGERIA

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Abstract

A new gravimetric determination of the geoid over Algeria has been made using the following data types: a) a global geopotential model, namely the OSU91A spherical harmonic coefficients set (Rapp and Al., 1991), complete to degree and order 360, b) a set of 12183 validated point free air gravity anomalies covering the region, and c) a two elevation grids; 1 km x 1 km digital terrain model for the north of Algeria and the ETOPO5 for the rest of the area.

The method used for the computations is the remove-restore technique in connection with the Fast Collocation. The input data were gridded gravity anomalies. The terrain correction was applied to the data and the corresponding indirect effect was taken into account.

The final result is a gravimetric quasi-geoid of Algeria on a 5'x 5' grid in the area with limits $20^{\circ} \leq \varphi \leq 37^{\circ}$, $-7^{\circ} \leq \lambda \leq 10^{\circ}$. The gravimetrically derived quasi-geoid has been adapted to 20 geoidal undulations provided from GPS and levelling observations. The discrepancies between them amount to about ± 11 cm especially for the north part of the country.

Key words: Quasi-Geoid, Geopotential model, Fast Collocation, RTM, Remove-Restore.

1. Introduction

Geoid determination is a major problem of Physical Geodesy. Today the whole international geodetic and geophysical community is interested in this task and a great number of international workshops and symposiums on this topic are held. The precise geoid model knowledge have various applications, notably in geodesy for the adequate exploitation of the measurements derived from GPS, and for the establishment of the altimetric networks, in oceanography for the determination of the topography of the ocean surface, and in geophysics for mining and oil prospecting.

The first determination of a preliminary geoid in a small zone in the north of Algeria was made in 1997 (Benahmed daho, 1997), using de least squares collocation (LSC) method and the Gravsoft software.

Now, a new geoid has been computed for the whole of Algeria between the limits $20^{\circ} \leq \varphi \leq 37^{\circ}$ for latitudes and $-7^{\circ} \leq \lambda \leq 10^{\circ}$ for longitudes, in a grid with mesh of 5'x 5'.

The method used to compute the quasi-geoid is the Remove-Restore and the Fast collocation technique. The terrain effect has been computed with the available DTM using the TC program of the Gravsoft software. The OSU91A model has been used as a reference field in order to remove and restore the long wavelength components of the gravity and the geoid respectively.

In order to test the quality and the accuracy of the quasi-geoid calculated, the results of this computation are compared to GPS/Levelling data collected from the TYRGEONET project and the local GPS/Levelling survey.

2. Data Description

2.1 Gravity Anomalies

Gravity anomalies used in this work, and composed of 12183 validated land data covering the territory of Algeria, were supplied by the B.G.I. These measurements with an initial precision of 5 mGals were expressed in the Geodetic Reference System GRS67. All gravity measurements were transformed from the GRS67 to GRS80 system.

Moreover, we applied an atmospheric correction recommended by the International Association of Geodesy (IAG, 1971) in order to eliminate the influence of the atmospheric masses. Figure 1 shows the geographical distribution of the available gravity data.

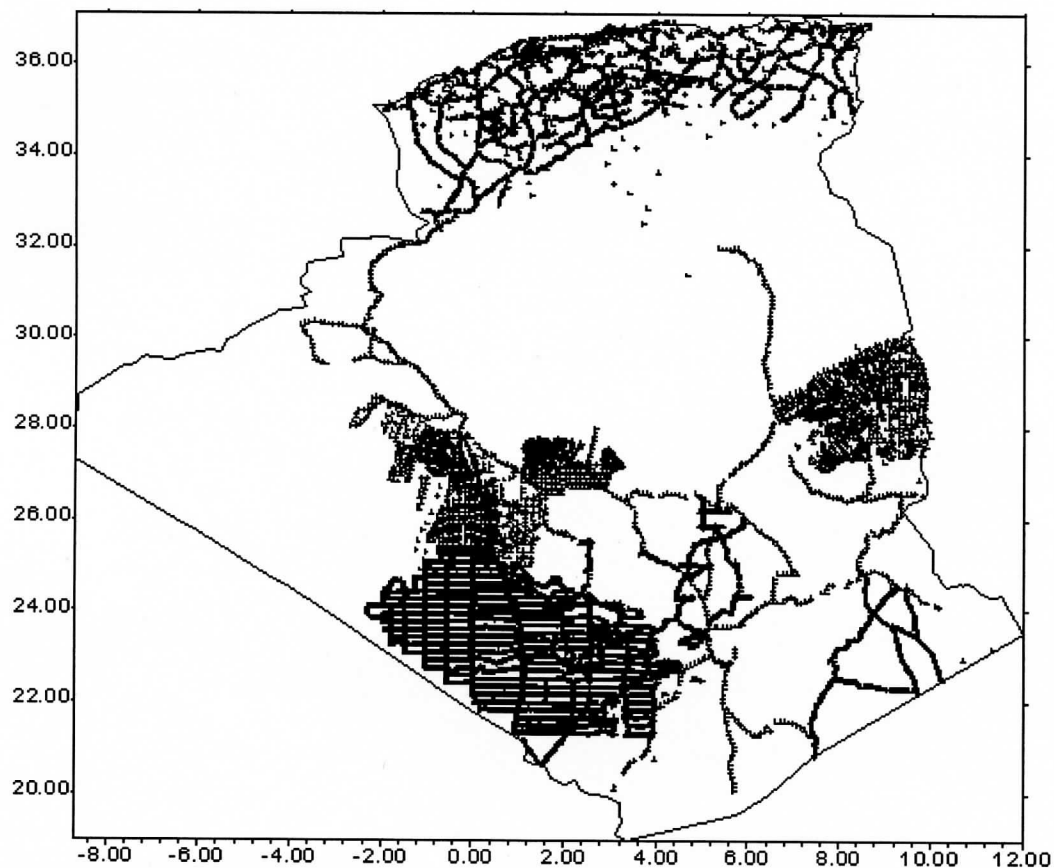


Figure 1. Geographical distribution of gravity measurements.

2.2 Geopotential model

For this work, the high-degree global geopotential model OSU91A (Rapp & al., 1991), complete to degree and order 360, was adopted as a reference in order to eliminate the long wavelengths of the gravity field. Gravity anomalies and geoid undulation can be computed in a spherical approximation from a geopotential coefficient set by:

$$\Delta g = \frac{GM}{r^2} \sum_{n=2}^{N_{\max}} (n-1) \sum_{m=0}^n [\bar{C}_{nm} \cdot \cos m\lambda + \bar{S}_{nm} \cdot \sin m\lambda] \bar{P}_{nm}(\cos \theta) \quad (1)$$

$$N = \frac{GM}{r\gamma} \sum_{n=2}^{N_{\max}} \sum_{m=0}^n [\bar{C}_{nm} \cdot \cos m\lambda + \bar{S}_{nm} \cdot \sin m\lambda] \bar{P}_{nm}(\cos \theta) \quad (2)$$

where θ , λ are the geocentric colatitude and longitude of the point where Δg and N will be determined; $\bar{C}_{nm}$, $\bar{S}_{nm}$ are the fully normalised spherical geopotential coefficients of the anomalous potential; $\bar{P}_{nm}$ are the fully normalised associated Legendre functions; $N_{\max}$ is the maximum degree of the geopotential model.

2.3 Topographic data

The computation of the terrain effects on the quasi-geoid required a detailed DTM model. For this purpose, two elevation grids of different resolution were used:

- 10 km DTM covering the area $15^\circ \leq \varphi \leq 40^\circ$, $-10^\circ \leq \lambda \leq 15^\circ$ subtracted from ETOPO5.
- 1 km DTM covering $33^\circ \leq \varphi \leq 38^\circ$, $-3^\circ \leq \lambda \leq 9^\circ$ derived from GLOBE database.

3. Quasi-Geoid computation

Fast collocation and the Remove-Restore procedure were used to compute the quasi-geoid estimate. In order to smooth the gravity field, the gravity data must be reduced of the geopotential model and terrain effect. The spherical harmonic coefficient set OSU91A has been used to remove the long wavelength component of gravity data. The computation of the effects of the topography according to the RTM reduction modelling method is based on two elevation grids; 1km x 1km digital terrain model for the north of Algeria and a global topographic model ETOPO5 of 5' x 5' for the rest of the area which were used up to a distance of 200 km. The reference surface of 15' x 15' needed for the RTM reduction is formed from the 5' x 5' blocks. This grid is smoothed further by taking moving averages over the 3 x 3 adjacent blocks. Table 1 shows the statistics in mGals of the gravity reductions for the gravity data.

Anomalies	mean	std. dev.	min.	max.
Δg_{obs}	4.89	26.84	- 82.59	136.20
$\Delta g_{\text{obs}} - \Delta g_{\text{OSU91A}}$	0.12	12.31	-67.75	123.46
$\Delta g_r = \Delta g_{\text{obs}} - \Delta g_{\text{OSU91A}} - \Delta g_{\text{RTM}}$	3.09	13.00	-34.17	110.38

Table 1. Statistics of gravity reductions.

The gridding of Δg_r to produce a 5'x 5' grid of residual gravity anomalies was done with the GEOGRID program using a fast quadrant-search collocation algorithm. The prediction of the grid gravity value was computed from the 5 closest points in each quadrant around the prediction point, using a second-order Markov model covariance function fitted to local data (correlation length $x_{1/2}=25$ km), and taking into account individual variance of the gravity data. The maximum value of the estimated formal error, which is defined by this covariance function and by the assigned error on the gravity anomalies (5 mGals in our case) is 13 mGals on the area where there is no data, and 4 mGals along the gravimetric survey, while the r.m.s. is 10.9 mGals.

However, for estimation of the residual quasi-geoid (ζ_r) via fast collocation using the FASTCOL program, an analytic expression of the covariance function is more convenient. For this purpose, we adopted a classical analytic covariance function for the disturbing potential proposed by Tscherning & Rapp (1974), which is expressed by a sum of series of Legendre polynomials (Moritz, 1980) as:

$$K(\psi) = \alpha \left[\frac{GM}{R_E} \right]^2 \cdot \sum_{l=2}^N \varepsilon_l \left[\frac{R_E^2}{rr'} \right]^{l+1} P_l(\cos\psi) + \sum_{i=N+1}^{+\infty} \frac{A}{(i-1)(i-2)(i+4)} \left[\frac{R_B^2}{rr'} \right]^{i+1} P_i(\cos\psi) \quad (3)$$

where ε_l : error degree variances of the reference field,
 N : the maximum degree of the reference field coefficients (here 360),
 α : scale factor of errors associated with the reference field,
 R_B : radius of a Bjerhammar sphere,
 r and r' : radial distances of the points,
 R_E : mean earth radius,
 A : scale factor of the degree variances.

The actual use of this model as a local covariance function requires the estimation of three parameters: the radius of the sphere of Bjerhammar (R_B) and two scale factors α and A .

The results of the adjustment of the empirical covariance function on the Tscherning & Rapp model were obtained by the program COVFIT8 (Knudsen, 1987). Figure 2 shows the empirical and analytic covariance functions of the residual anomalies.

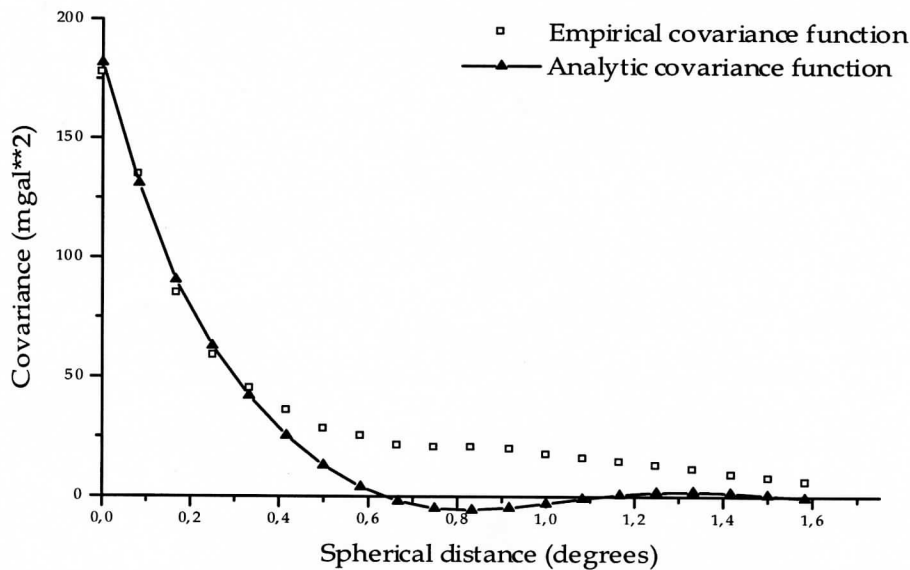


Figure 2. Covariance functions of residual gravity anomalies

The final quasi-geoid was obtained by adding the model and the residual terrain effect on the 5' x 5' residual quasi-geoid grid. Figure 3 shows a map of the quasi-geoid solution in Algeria contoured with 1 m interval.

With the gravity anomalies variances on the grid and by means of the error propagation, the maximum value of the geoid undulation accuracy estimated by the FFTGEIOD program is about 13 cm in the area where gravity data is clearly missing.

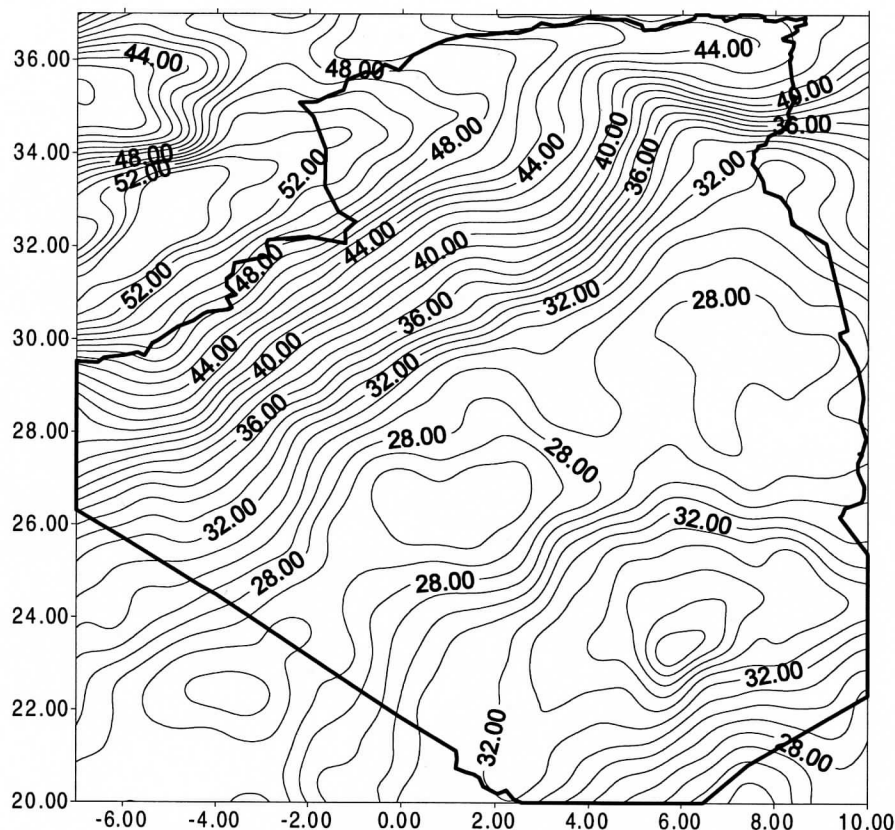


Figure 3. Quasi-geoid solution in Algeria. (in meters)

4. Comparison with GPS/Levelling data

Many GPS campaigns have been carried out in the past years in Algeria. Furthermore, in the framework of the TYRGEONET project, two sites located in the North of Algeria have been determined in the WGS84 system, which have been used later for the densification and improvement of accuracy of the local geodetic network.

All these GPS stations are connected to the national height system through spirit levelling so that an estimation of N in these points is possible.

To minimise the long wavelength errors, the systematic datum differences between the gravimetric geoid and the GPS/levelling data were removed by the following four-parameter transformation equation:

$$\Delta N = x_0 + x_1 \cos(\varphi) \cdot \cos(\lambda) + x_2 \cos(\varphi) \cdot \sin(\lambda) + x_3 \sin(\varphi) \quad (4)$$

where x_0 is the shift parameter between the vertical datum implied by the GPS/levelling data and the gravimetric datum, and x_1 , x_2 and x_3 are the three translation parameters in x , y , z axes.

The comparisons were made on 20 benchmarks; all situated in the north of Algeria whose 15 points are close to the station of Arzew (see figure 4). The statistics of the differences before and after fitting out the systematic biases and tilts using a four-parameter transformation are summarised in table 2.

These results show the existence of the systematic biases between the two kinds of geoid representations with a mean value of -1.74 m and standard deviation of ± 0.73 m. The use of the four-parameter transformation fit eliminates the possible tilt of the gravimetric geoid as well, yielding a geoid fit with ± 11 cm. The remaining differences, mainly due to the effect of the high frequency errors in the reduced Faye gravity anomalies, the errors in the levelling and in GPS derived ellipsoidal heights, and the geopotential coefficient errors, can be further reduced by improving the data accuracy and, probably, by using a smaller grid size for the gravity anomalies and heights.

	Mean	Min.	Max.	Stand. dev.
before fitting	-1.740	-2.290	0.730	0.730
after fitting	0.000	-0.062	0.387	0.105

Table 2. Comparison of GPS/Levelling results to gravimetric quasi-geoidal heights (in meters).

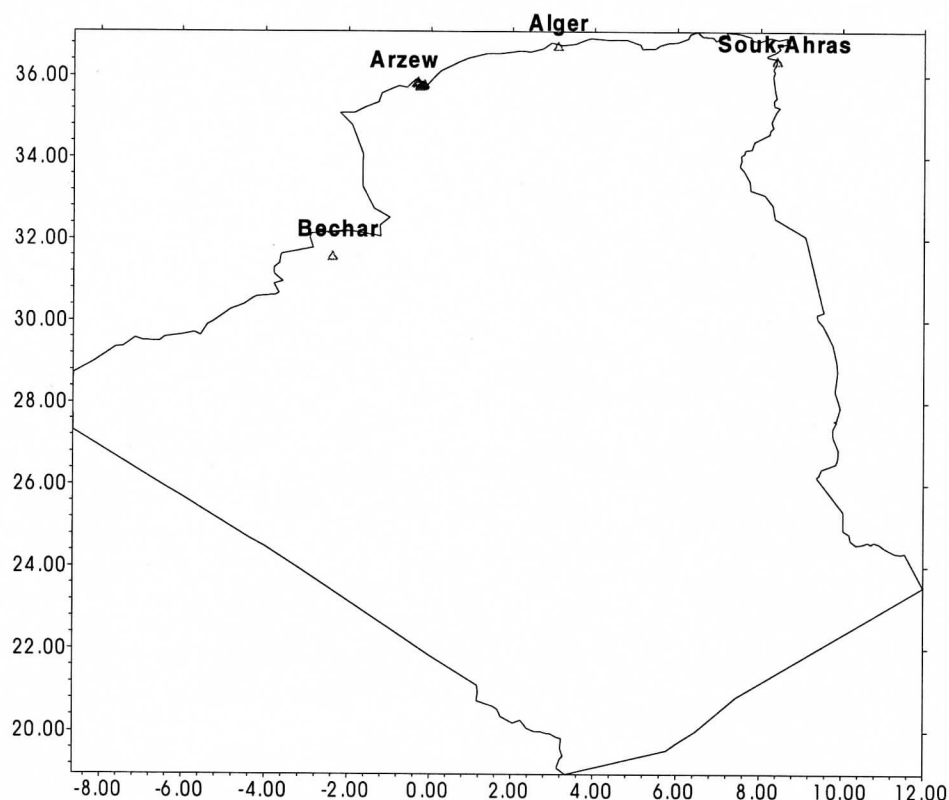


Figure 4. GPS points in the north of Algeria.

Conclusion

The new Algerian quasi-geoid was computed via fast Collocation using the Remove-Restore procedure. The comparisons of the quasi-geoid with GPS/Levelling data in 20 benchmarks provide, after fitting, an RMS differences about ± 11 cm for the north part of Algeria over distances of 3 to 1000 km.

With the precision of the available gravity data, and its distribution, the geoid in Algeria was computed with a maximal error of 13 cm in the area where gravity data is clearly missing.

This work constitutes the first test of calculation of the gravimetric geoid over Algeria via the Fast Collocation method. The results obtained were satisfactory, so in the near future the new solutions will be proposed. These will include the maximum of gravimetric, topographic and geodetic informations in order to reach a centimetric accuracy on all the country.

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GEOPOTENTIAL MODELS COMPARISON IN ALGERIA

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Abstract

The choice of the best geopotential model to reduce geodetic data is one of the critical steps in computing the geoid. Several studies have shown that the geopotential models tailored to regional or local gravity data are best suited for high precision geoid computations. In this paper, an analysis was carried out to define the geopotential model, which fits best the gravity field in Algeria.

Global geopotential solutions, which are used in the comparison, are GPM2, OSU81, OSU91A and EGM96. The test of the fitting of these high order geopotential models to the gravity field in Algeria is based on the gravity data supplied by the B.G.I. The comparisons were made at all gravity points by calculation of the residual gravity anomalies (i. e. observed data minus model).

The result obtained for the geopotential models comparison shows that the OSU91A model fits best the observed values in Algeria.

Key words: Geoid, Geopotential model, Disturbing potential, Spherical harmonic.

1. Introduction

The gravitational potential of the Earth can be represented outside the masses of the Earth by a set of spherical harmonic coefficients $\bar{C}$ and $\bar{S}$, called a geopotential model. Over the last 40 years, continuing improvements and refinements to the basic theory have been paralleled by the availability of more accurate and complete data and by improvements in the computational resources available for numerical modelling studies. These advances have led to solutions that are complete to degree and order of 360 with about 55 km resolution.

Accurate geopotential models are required for many applications in solid Earth sciences, as e.g. to study the structure of the Earth, to compute the orbit of a satellite, and to compute the geoid.

Furthermore, the geopotential models allow also the computation of any gravity field parameter outside the Earth's masses. The currently most accurate method for the determination of the geoid, the remove-restore method, is based on the approximation of the disturbing potential by a high degree geopotential model.

2. Geopotential model

The spherical harmonic representation of the Earth's gravity field W is (Moritz, 1980) :

$$W = \frac{GM}{r} \sum_{n=0}^{+\infty} \sum_{m=0}^n \left(\frac{a_e}{r} \right)^n [\bar{C}_{nm} \cdot F_{nm} + \bar{S}_{nm} \cdot G_{nm}] + \Phi \quad (1)$$

where $F_{nm} = \bar{P}_{nm}(\cos \theta) \cos m\lambda$
 $G_{nm} = \bar{P}_{nm}(\cos \theta) \sin m\lambda$

(θ, λ) are the geocentric colatitude and longitude of the point where W will be determined, GM is the gravitational constant, a_e is the semi major axis of the reference ellipsoid, $\bar{C}_{nm}, \bar{S}_{nm}$ are the fully normalised spherical geopotential coefficients of the Earth's gravitational field, $\bar{P}_{nm}$ are the fully normalised associated Legendre functions of the first kind and Φ is the potential of the centrifugal force.

The potential of a rotational reference ellipsoid is represented by the expansion :

$$U = \frac{GM}{r} \left[1 - \sum_{n=1}^{+\infty} \bar{J}_{2n} \left[\frac{a_e}{r} \right]^{2n} P_{2n}(\cos \theta) \right] + \Phi \quad (2)$$

where M' is the mass of the reference ellipsoid.

The representation of the disturbing potential (difference between the gravity potential W and the normal potential U) is given by:

$$T = \frac{GM}{r} \sum_{n=2}^{+\infty} \left(\frac{a_e}{r} \right)^n \sum_{m=0}^n [\Delta \bar{C}_{nm} \cdot F_{nm} + \Delta \bar{S}_{nm} \cdot G_{nm}] \quad (3)$$

where ΔC_{nm} and ΔS_{nm} are the differences between the coefficients of the gravitational and ellipsoid potential (the difference $M-M'$ is assumed to be small) and M' can be replaced by M .

Taking into account the boundary condition of the physical geodesy we get the following expansions:

for gravity anomalies

$$\Delta g_M = \frac{GM}{r^2} \sum_{n=2}^{+\infty} (n-1) \left(\frac{a_e}{r} \right)^n \sum_{m=0}^n [\Delta \bar{C}_{nm} \cdot F_{nm} + \Delta \bar{S}_{nm} \cdot G_{nm}] \quad (4)$$

and, with the Bruns equation, for geoid undulation

$$N = \frac{T}{\gamma} = \frac{GM}{r\gamma} \sum_{n=2}^{+\infty} \left(\frac{a_e}{r} \right)^n \sum_{m=0}^n [\Delta \bar{C}_{nm} \cdot F_{nm} + \Delta \bar{S}_{nm} \cdot G_{nm}] \quad (5)$$

where γ is the normal gravity at $P(r, \theta, \lambda)$. In practical works the index n runs up to the maximum degree of the model.

3. Gravity Anomalies

Gravity anomalies used in this work, and composed of 12472 land data covering the territory of Algeria, were supplied by the B.G.I. These measurements with an initial precision of 5 mGals were expressed in the Geodetic Reference System GRS67. All gravity measurements were transformed from the GRS67 to GRS80 system.

Moreover, we applied an atmospheric correction recommended by the International Association of Geodesy (IAG, 1971) in order to eliminate the influence of the atmospheric

masses. The statistics of observed data are given in table 1. Figure 1 shows the geographical distribution of the available gravity data.

	mean	st. dev.	minimum	Maximum	range
Latitude (degrees)	26.99	4.72	19.56	37.07	17.51
Longitude (degrees)	2.67	3.13	-8.43	11.32	19.75
Height (m)	499.18	268.82	1.00	2538.00	2537.00
Free -Air (mGal)	5.25	27.43	-82.59	165.37	247.96

Table 1. Statistics of BGI data in Algeria

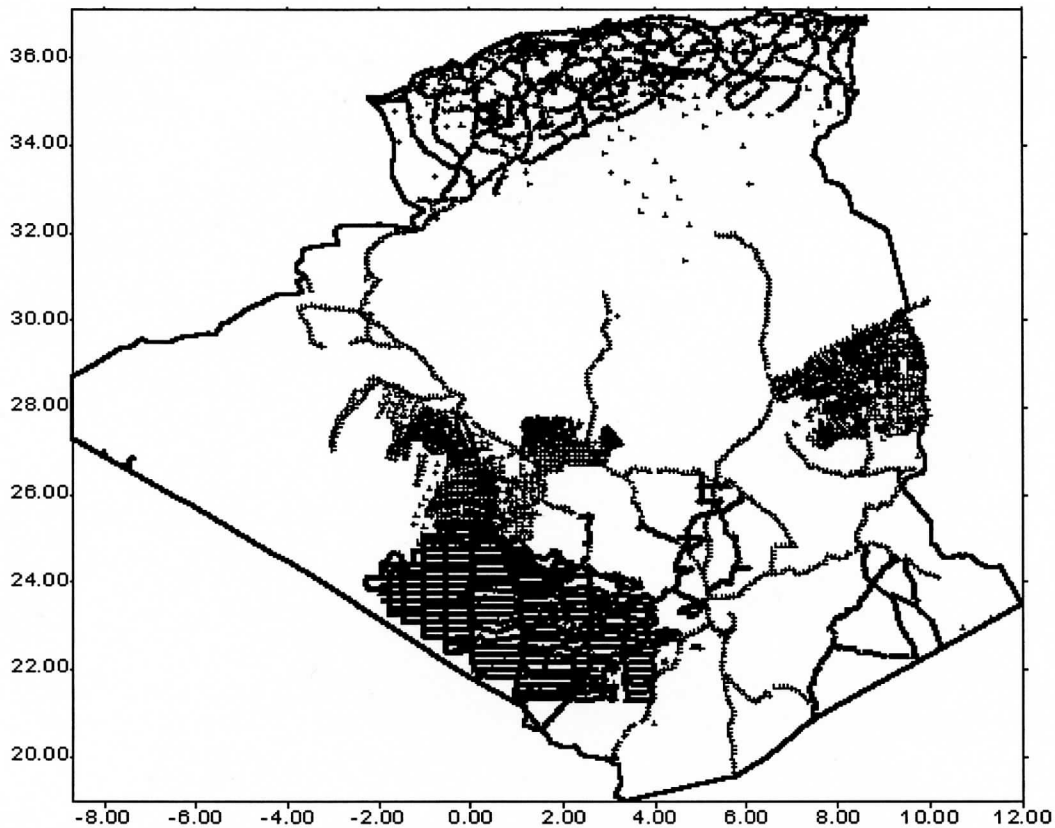


Figure 1. Geographical distribution of gravity measurements

4. Geopotential models used in the computation

The sets of potential coefficients considered are: GPM2, OSU81, OSU91A and EGM96. The OSU models were supplied by R. Rapp; the OSU81 model is complete up to degree and order of $n = 180$, while the OSU91A is complete up to $n = 360$. The GPM2 model computed by Wenzel and the EGM96 model by Lemoine et al. are respectively complete up to $n = 200$ and $n = 360$.

In the tests and for each geopotential model, the gravity anomalies Δg_M were calculated at the nodes of a 10-minute grid covering Algeria using the GEOCOL program (Tscherning,

1994). The value Δg_M was then obtained at the locations of the observation points using bicubic interpolation.

This approach also makes use of the fast recursive relations for the associated Legendre functions along parallels. The results produced using this technique differ by a few microgals from a point by point solution. However, the computation time is reduced by almost two orders of magnitude.

5. Results

In order to determine the fit of the geopotential model to the gravity data, the model value Δg_M was subtracted from the observed data value at the observation location, and the statistics of the difference calculated for all data points. The statistical parameters considered in this work are the mean, the standard deviation and the smooth covariance function necessary for all estimation by the collocation method.

The table 2 shows the statistics, in mGals, of reduced data.

Geopotential models	min.	max.	mean	std. dev.
GPM2	-82.05	138.37	2.79	24.81
OSU81	-72.26	115.19	1.55	18.17
OSU91A	-97.04	125.56	0.30	13.16
EGM96	-100.96	112.03	-2.17	13.54

Table 2. Statistics of the difference between the BGI gravity data and the geopotential models.

The following figures show the empirical and analytical covariance functions of the reduced gravity data for each geopotential model obtained by using EMPCOV (Tscherning, 1994) and COVFIT (Knudsen, 1987) programs. In our case, we have adopted a classical analytic covariance function for the disturbing potential proposed by Tscherning & Rapp (1974), which is expressed by a sum of series of Legendre polynomials (Moritz, 1980) as:

$$K(\psi) = K_G(\psi) + K_L(\psi) \quad (6)$$

with:

$$K_G(\psi) = \alpha \left[\frac{GM}{R_E} \right]^2 \sum_{l=2}^N \varepsilon_l \left[\frac{R_E^2}{rr'} \right]^{l+1} P_l(\cos \psi)$$

$$\text{and } K_L(\psi) = \sum_{i=N+1}^{+\infty} \frac{A}{(i-1)(i-2)(i+4)} \left[\frac{R_B^2}{rr'} \right]^{i+1} P_i(\cos \psi)$$

where ε_l : error degree variances of the reference field,

N : maximum degree of the reference field coefficients,

α : scale factor of errors associated with the reference field,

R_B : radius of a Bjerhammar sphere,

r and r' : radial distances of the points,

R_E : mean Earth radius,

A : scale factor of the degree variances.

The actual use of this model as a local covariance function requires the estimation of three parameters: the radius of the Bjerhammar sphere (R_B) and two scale factors (α) and A .

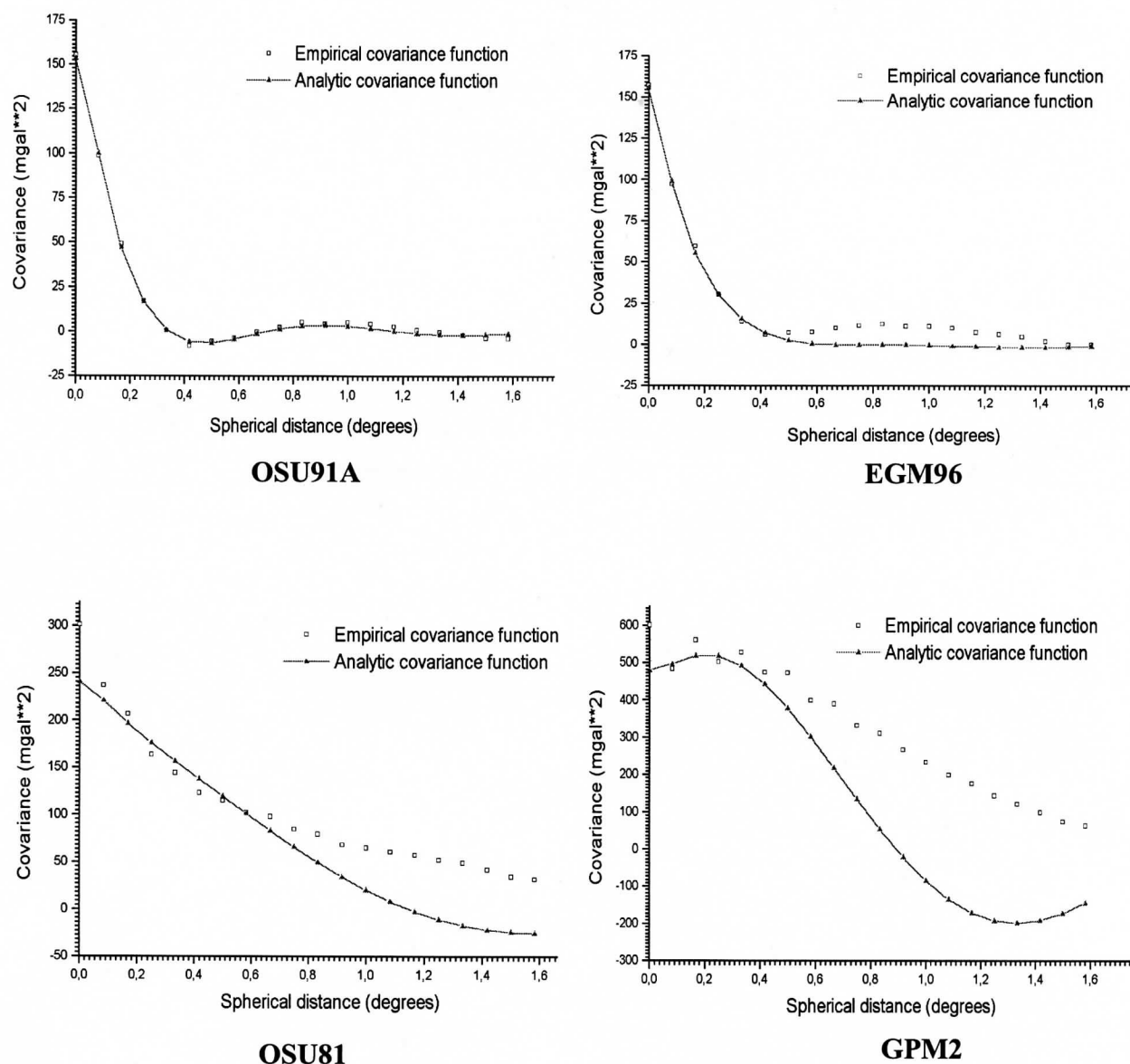


Figure 2. The empirical and the model covariance functions of the reduced gravity data for each geopotential model.

Comparing the results of Table 2 and taking into account the agreement between the analytical and empirical covariance functions, it comes out that the OSU91A model fits best the observed values.

Conclusion

The result of this test comparison shows that the OSU91A model describes the gravity field most closely in Algeria. According to the data used, the OSU91A model is able to recover gravity anomalies over 43% of the Algerian territory to within 5 mGals and over 71% of the country to within 10 mGals.

The perspective of the establishment of an absolute gravity network on all the country and the integration of the new local gravity measurements, will certainly improve the choice of the optimal geopotential model on the area.

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- Wenzel G.; 1999: Global models of the gravity field of high and ultra-high resolution. Lecture notes for the international school for the determination and use of the geoid, Milan, February 15-19, 1999.

The effect of high precision bathymetric model on geoid computation

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ABSTRACT

The effect of a new detailed regional terrain model on geoid computation has been studied. A new detailed bathymetric model around Azores Islands has been constructed based on new ocean depth information obtained from echo soundings during ship surveys. These bathymetric data were merged with high precision height data on land producing a new terrain model (AZDTM98) with 1'x1' resolution on sea and 20''x20'' on land. Global digital terrain model JGP95E was compared with AZDTM98 model. Sea and land gravity data covering the area was merged with satellite derived gravity anomalies for geoid computation. Least squares collocation was used for geoid estimation according to the remove-restore technique using EGM96 geopotential model and RTM method. Two geoid solutions were obtained from AZDTM98 and JGP95E models and compared with 134 GPS stations on land and with ERS-1 and ERS-2 altimetric data on sea. From this comparison it was found, on sea, a slight improvement for AZDTM98 model after the removal of the trend surface, reaching the level of 5 cm in the difference between the two models. The new detailed bathymetric/altimetric data did not improve the geoid solution as it was expected.

1. INTRODUCTION

Topographic effects on geoid computation have long been the subject of several investigations (Arabelos, 1998; Dahl and Forsberg, 1998). On land areas, topographic effects on gravity anomalies reduction and on geoid computation are very well quantified with results that follows the expected (Sevilla, 1995; Tziavos et al., 1996). However, on oceanic areas, bathymetric models and its terrain effects on geoid computation has given some disappointing results (Catalao and Sevilla, 1998; Dahl and Forsberg, 1998), which are believed to be due to gross errors on bathymetric models.

Azores archipelago and surrounding area (Fig. 1) have been the subject of systematic bathymetric survey, with swath bathymetric systems, in the aim of international programs on ridge research (FARA and RIDGE projects). A complete description of these data may be found in Lourenço et al. (1999). In Fig. 2 it is shown all bathymetric data acquire during these projects and also bathymetric data obtained from several cruises complemented with Sandwell and Smith (1997) bathymetric data in areas with poor coverage.

With this high precision bathymetric data (depth accuracy estimated to be 0.1% of water depth) complemented with altimetric data on the islands a new high precision digital terrain model was constructed, AZDTM98, with 1'x1' resolution on sea and 20'' x 20'' on land.

The main purpose of this study was to evaluate the influence of a new detailed bathymetric/altimetric model on the gravity field modelling, in particular, on geoid estimation. In order to assess the influence of this bathymetric data, two geoid solutions were compared considering AZDTM98 model (Fig. 3) and JGP95E model (Fig. 4). Geoid was calculated by means of collocation and Stokes Fast Fourier Transform using remove-restore method for geopotential model EGM96 (Lemoine et al., 1997) and terrain effects.

2. GRAVITY DATA

The original data bank, used in previous studies, was the result of a compilation from BGI, NGDC and DMA data banks. Most of the data were acquired from USA, United Kingdom and France Institutions in the period from 1970 to 1990. The gravity data was validated by crossover error adjustment fixing the longest track from United Kingdom.

This data bank was recently improved with two gravimetric campaigns held in 1998 in the aim of PDIC/C/Mar project (Fernandes et al., 1998) and MARFLUX project (Luis et al., 1999). A new crossover adjustment was done considering PDIC/C/Mar tracks as error free. After adjustment the standard deviation of the crossover error drops to 2.61 mGal (from an initial value of 7.21 mGal). In order to full fill remaining areas with poor coverage satellite derived gravity anomalies from Andersen and Knudsen (1998), named A&K on the sequel, were merged with observed gravity anomalies. The merging process was done in three steps:

- a) first, the residuals between A&K anomalies and observed anomalies were calculated on a grid of $3.75' \times 3.75'$ by means of collocation with a second order Markov covariance model with a 20 km correlation distance;
- b) Second, the correction surface obtained previously was applied to the A&K anomalies. The values of the correction surface were set to zero were the distance of these points to the nearest track (observed anomaly) were greater than 30 km.
- c) Finally a sample of the observed anomalies were selected with one observation in each cell of a grid with spacing of $3.75' \times 3.75'$. On land a finer resolution of $1' \times 1'$ was selected due to the finer resolution of land gravimetric data. The empty cells were fulfilled with the A&K gravity anomalies, corrected for the observed anomalies. The plot of this data (Fig. 5) shows a complete cover of the study area. In Fig. 6 it is shown the contour lines of the gravity anomalies designated by AZGRA98.

3. GRAVIMETRIC GEOID DETERMINATION

The geoid was calculated using Least Squares Collocation (Moritz, 1980) combined with remove-restore method for EGM96 spherical harmonic reference field and the terrain effects determined from the residual terrain model, RTM, (Forsberg, 1984). For each terrain model (AZDTM98 and JGP95E) the residual terrain model and its effects on gravity data were computed and removed from the free air gravity anomalies. EGM96 reference field was also removed yielding two different data sets of residual anomalies, one for each terrain model.

	Mean	Std	Min.	Max.
Free air	47.72	45.53	-91.21	263.71
Free air-EGM96	9.13	34.49	-102.98	211.19
Free air-EGM96-RTM _{AZDTM98}	1.65	12.51	-61.60	61.79
Free air-EGM96-RTM _{JGP95}	0.65	18.29	-116.15	81.27

Table 1. Statistics of the gravity anomalies (Unit is mGal).

The statistical parameters of the residual anomalies (6737 data values on land and on sea), whose only difference reside on the DTM used on RTM reduction, are presented in Table 1. For both terrain models the gravity smoothing reaches a level of more than 50%, with AZDTM98 model revealing a better adjustment to the reality reducing 6 mGal on the Std of the residual anomalies when compared with JGP95E model.

The residual geoid undulation was computed using LSC and Stokes FFT having as input the residual gravity anomalies. The aforementioned procedure, in principle, yields the quasi-geoid (ζ). On land areas (Azores islands) the quasi-geoid was transformed in the geoid by computing the expression (Heiskanen and Moritz, 1967):

$$N = \frac{\Delta g_B}{\gamma} H + \zeta$$

from the Bouguer anomalies (Δg_B). The maximum difference between ζ and N is 14 cm in S. Miguel island. The ellipsoidal height was available from GPS measurements in Pico and S. Miguel islands in 134 data points. The residuals between these data points, or ERS1 (Fernandes et al., 1998) altimetric data on sea, and the gravimetric geoid were detrended both on land and on sea by a four parameter fit surface. The results of this comparison between the gravimetric geoid and SSH obtained from ERS1 and ERS2 and GPS height data are presented in Table 2, with the values "fit" meaning the residuals after the removal of the fit surface. On sea, the results obtained from the two models are almost identical in terms of mean and Std values, but with a slight improvement for AZDTM98 model was found after the removal of the trend surface, reaching the level of 5 cm in the difference between the two models. On land, an opposite effect was seen with considerable differences between the two models on the original solution, decreasing for the fitted solution.

	Original				Fit
	Mean	Std	Min.	Max.	Std
AZDTM98 Model					
Geoid _{Col} - SSH	-0.09	0.16	-0.47	0.64	0.13
Geoid _{FFT} - SSH	-0.33	0.20	-0.87	0.56	0.20
Geoid _{Col} -GPS (S.Miguel)	-1.01	0.29	-1.38	-0.30	0.07
Geoid _{FFT} -GPS (S.Miguel)	-1.14	0.26	-1.47	-0.49	0.08
Geoid _{Col} -GPS (Pico)	-1.41	0.20	-1.90	-1.03	0.14
Geoid _{FFT} -GPS (Pico)	-1.38	0.19	-1.90	-0.85	0.14

	Original				Fit
	Mean	Std	Min.	Max.	Std
JGP95 Model					
Geoid _{Col} - SSH	0.08	0.18	-0.34	0.80	0.18
Geoid _{Col} -GPS (S.Miguel)	-0.36	0.37	-0.86	0.41	0.09
Geoid _{Col} -GPS (Pico)	-0.50	0.32	-1.16	0.24	0.18

Table 2. Statistics of the differences between geoid and SSH from ERS-1, on sea, and GPS heights on land. Units are m.

4. CONCLUSIONS

A new bathymetric model around Azores Islands has been constructed based on new ocean depth information obtained from echo soundings during ship surveys. These bathymetric data were merged with high precision height data on land and with other ship surveys producing a new terrain model (AZDTM98). The gravity data used on geoid computation was improved with two recent oceanic gravimetric campaigns held in 1998, which were included in our gravimetric data bank after a new crossover adjustment. Satellite derived gravity anomalies were also included in the final data bank, in areas with poor data coverage, through a process that "forces" these anomalies to be in accordance with the ship surveys.

The smoothing of the gravity anomalies was considerably improved using the new model, the std decreased from 18.29 mGal for JGP95E model to 12.51 mGal for AZDTM98 model. This smoothing effect is also seen in the correlation distance, which increase from 7' in JGP95E model to 12' in AZDTM98 model.

Considering the comparison between the two geoid solutions, the results obtained, on sea, are almost identical in terms of mean and Std values. A slight improvement for AZDTM98 model was found after the removal of the trend surface, reaching the level of 5 cm in the difference between the two models. On land, an opposite effect was seen with considerable differences between the two models on the original solution, decreasing for the fitted solution. The new detailed bathymetric/altimetric data did not improve the geoid solution as it was expected.

As a future work we plan to include the mean sea level data from coastal station in order to get a better control between the islands and sea interface. We plan also to analyse the effect of density variations on terrain corrections accordingly with some geophysical density models available on the islands and on the sea.

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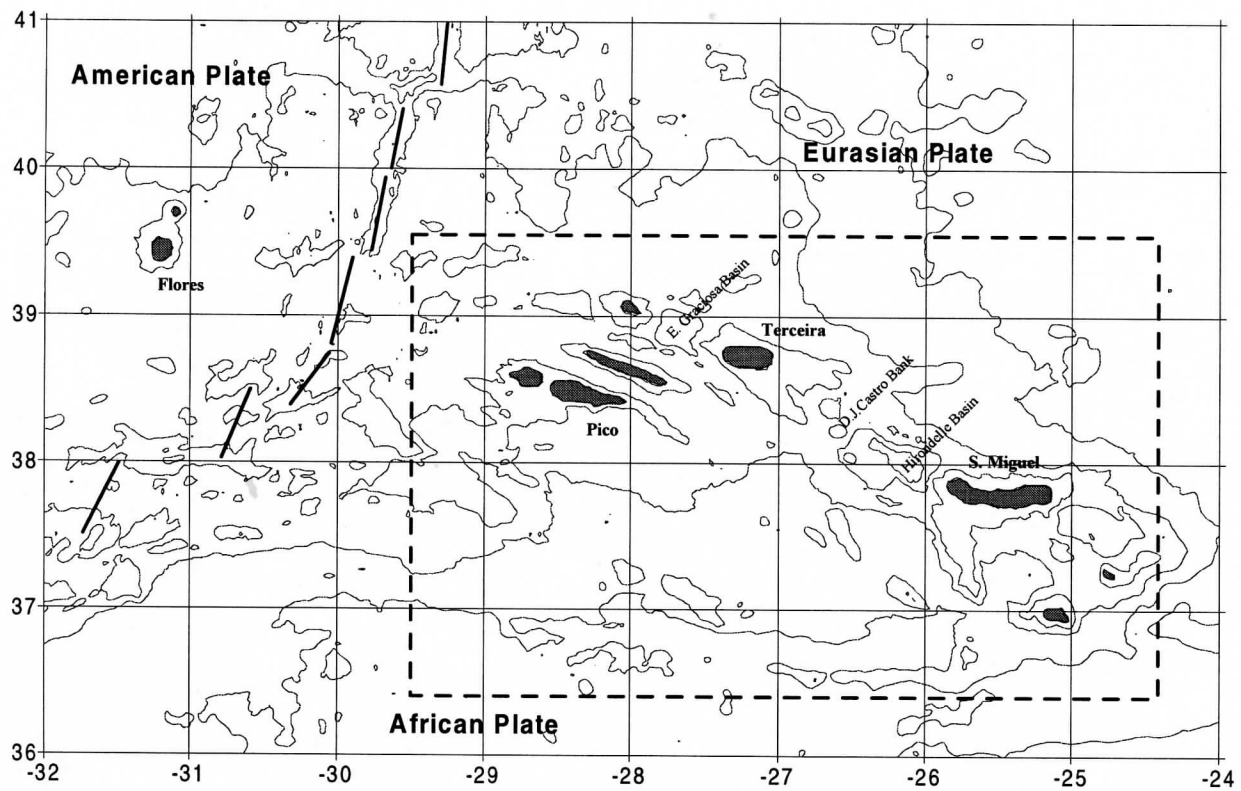


Fig. 1 -Azores Archipelago and surrounding area showing the location of the main tectonic features. The dash line indicate the study area.

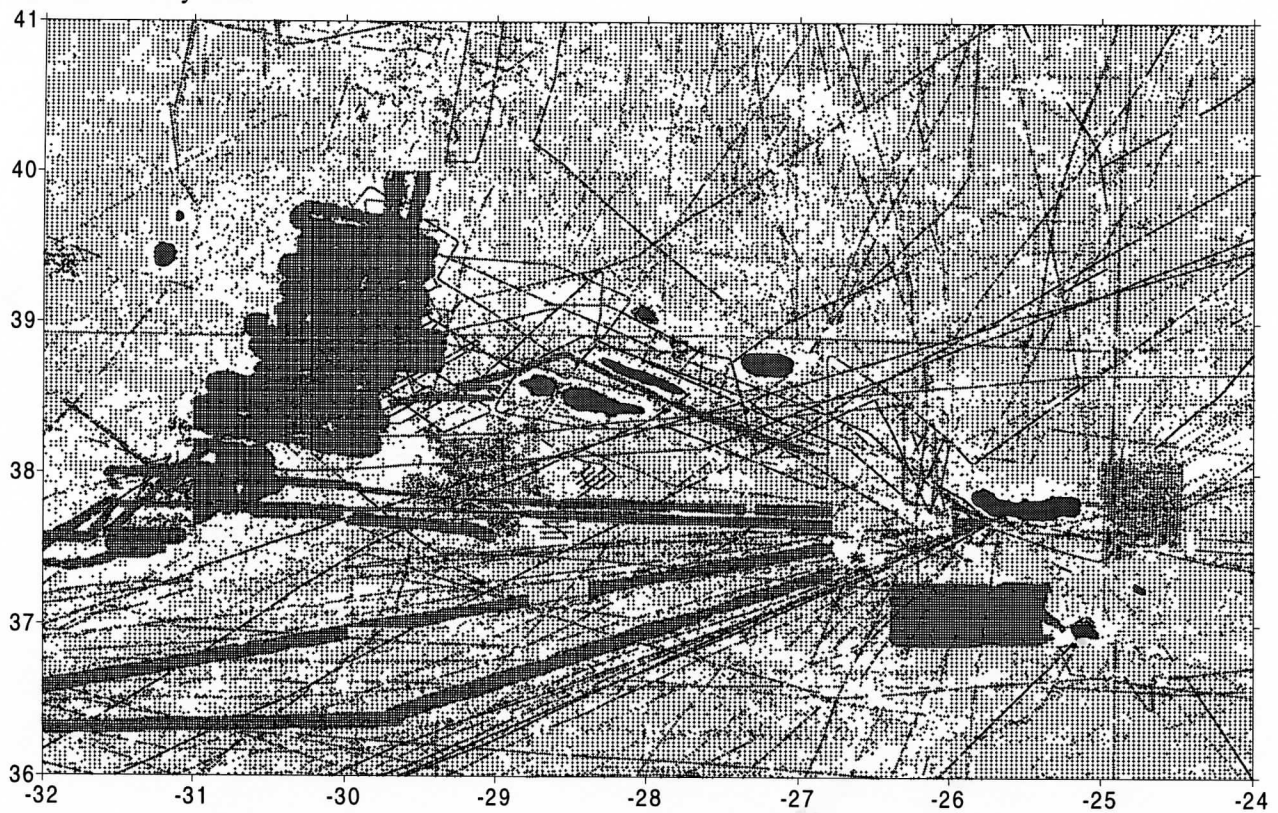


Fig. 2 - Distribution of points used in the construction of AZDTM98 digital terrain model. Dense coverage areas are the result of swath surveys.

AZDTM98 Model

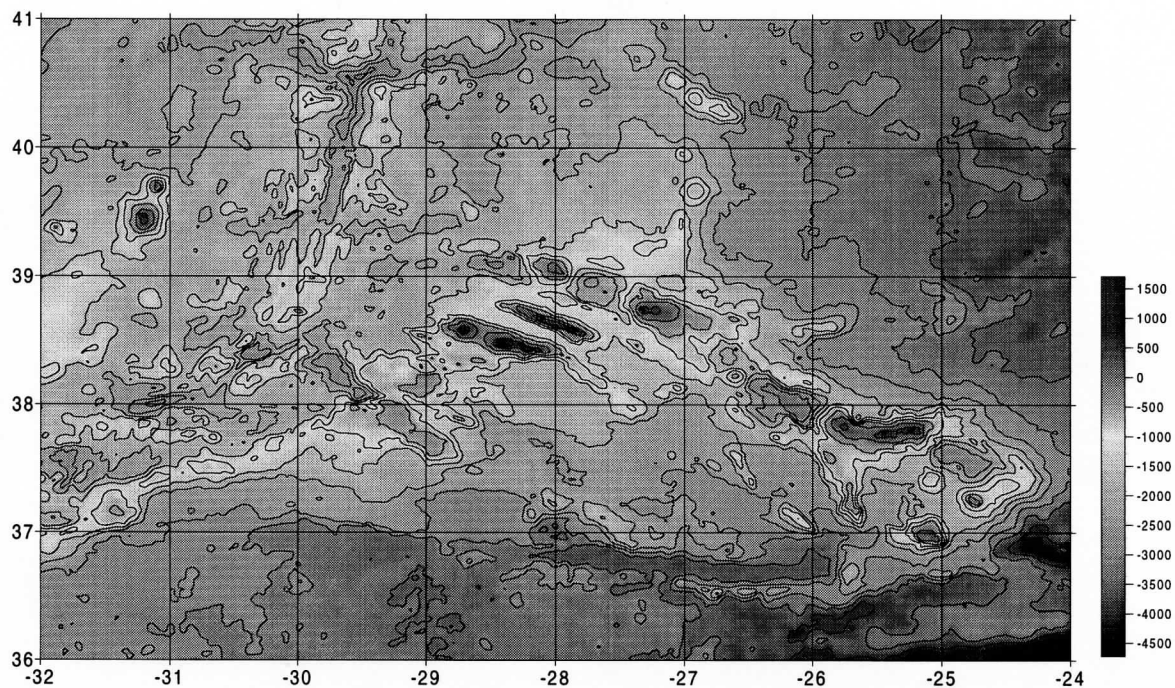


Fig 3 - AZDTM98 bathymetric/altimetric map contoured every 500 m.

JGP95E Model

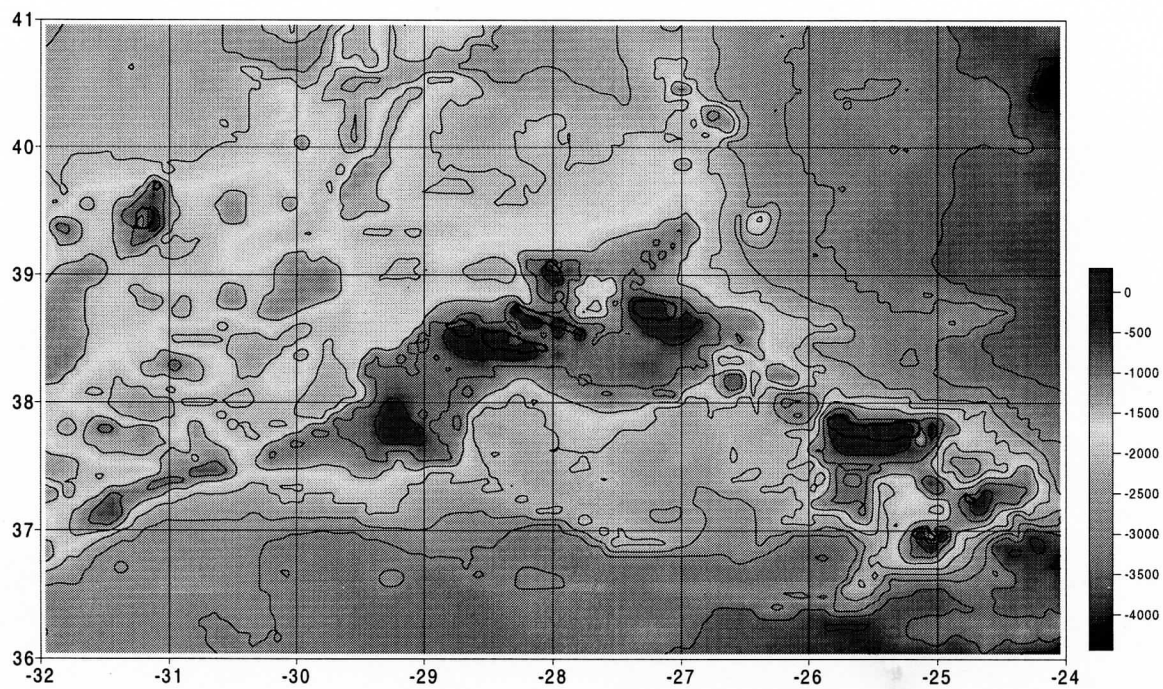


Fig. 4 - JGP95E bathymetric/altimetric map contoured every 500 m.

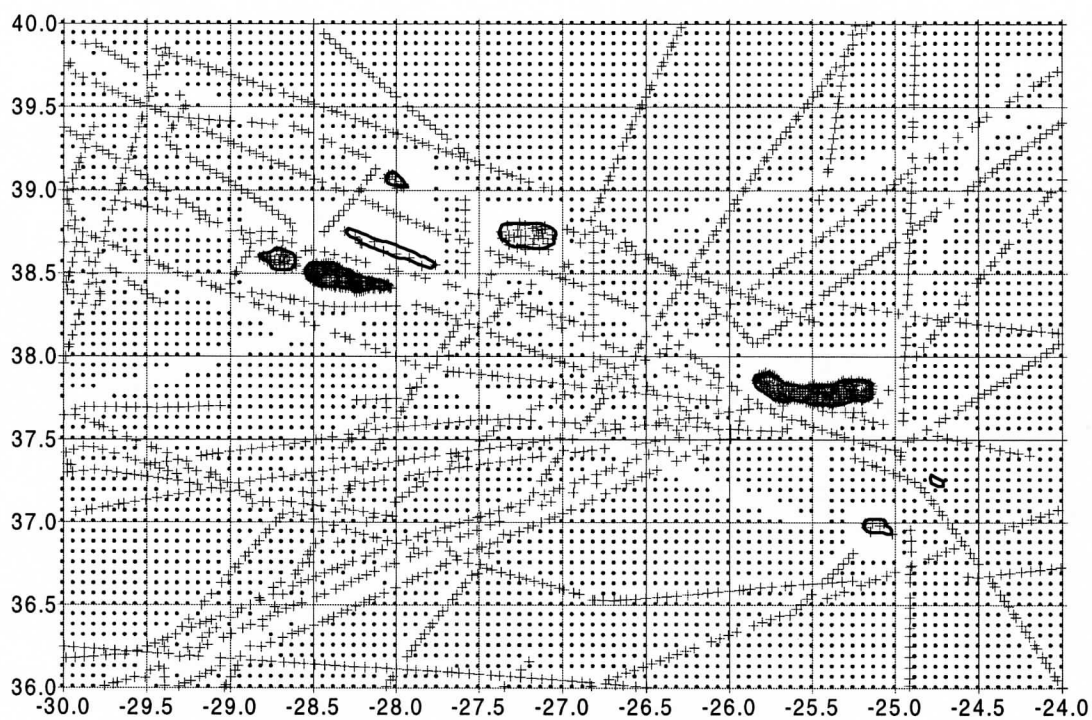


Fig. 5 - Final gravity data distribution including observed gravity anomalies on sea and on land (+) and derived satellite gravity anomalies in areas with absent coverage (•).

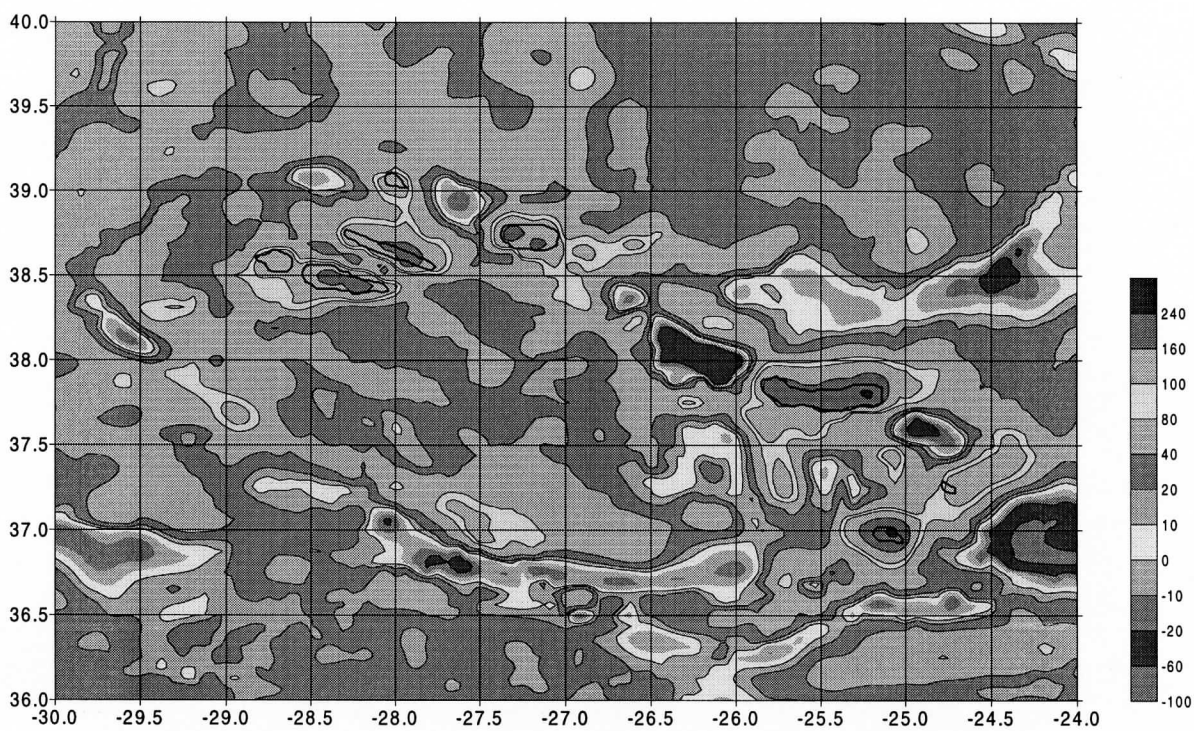


Fig. 6 -Free air gravity anomaly map (contour interval is 20 mGal). There are a clear correlation between this map and AZDTM98 digital terrain model which reflects the effects of the short-wavelength features of the topography/bathymetry on the free air anomalies. The lowest values correlate basins and with Gloria's fault in the south and the highest values correlate with the archipelago islands.

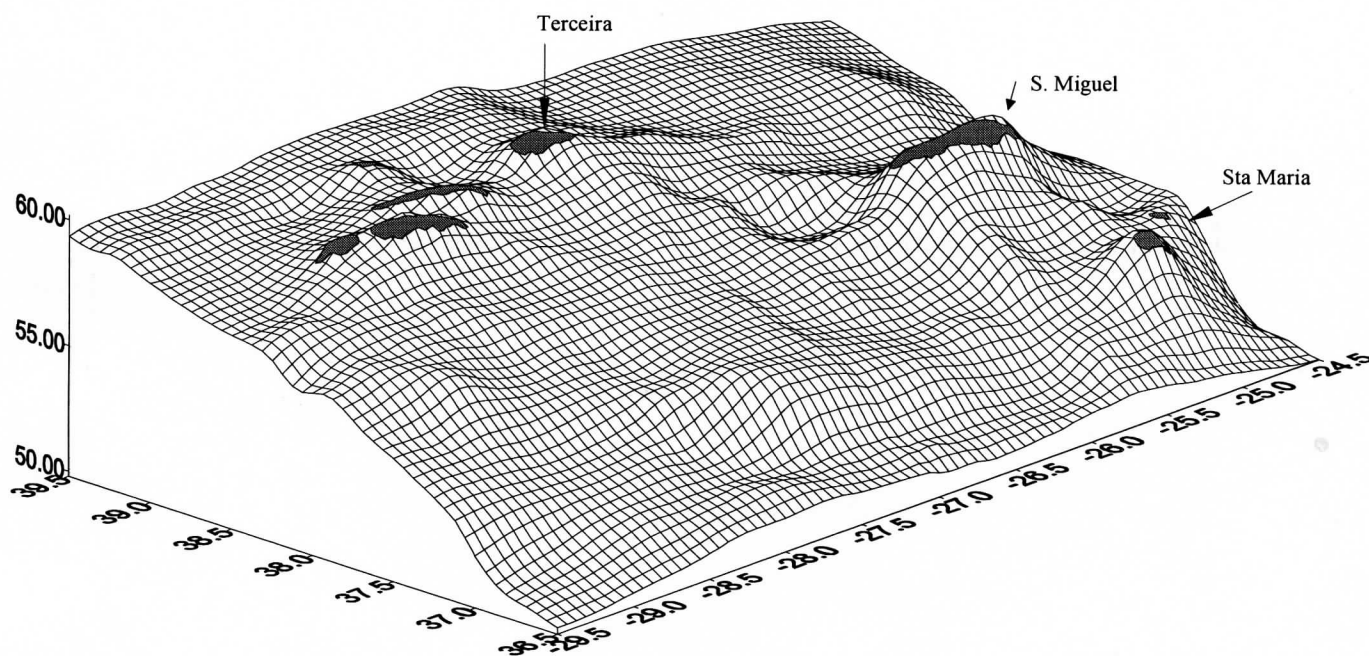


Fig. 7 - Gravimetric geoid model around Azores archipelago. Geoid height is in meter.

GPS/levelling Traverses for Testing a Gravimetric Geoid Solution in Eastern Andalusia (Spain)

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SUMMARY

To test the accuracy of a gravimetric geoid computed in a test area located in the South of Spain two GPS traverses have been established with benchmarks belonging to the first order Spanish levelling network in the east of Andalusia. The GPS constellation was tracked for 3-4 hour sessions over baselines ranging from 3 to 85 km. The equipment used consisted of three Leica SR399 and one Leica 9500 dual frequency carrier phase GPS receivers. Bernese 4.0 with precise ephemerides was used to process the GPS observations and NETGPS software for the adjustment of the network. The quasigeoid model computed by fast collocation method has been transformed to geoid model in order to be compared with GPS/levelling geoid undulations. The standard deviation of the discrepancies after a fit of tilted plane amounts to ± 5 cm.

1. INTRODUCTION

Most of the Iberian Peninsula is occupied by the Meseta, a large plateau that is almost completely surrounded by mountain range. A region of Andalusia has been chosen to test the accuracy of our gravimetric geoid. This geoid solution has been computed from a validated gravity data set covering the area $36^{\circ}.7 \leq \varphi \leq 38^{\circ}.42$, $-4^{\circ}.33 \leq \lambda \leq -1^{\circ}.2$ and a Digital Terrain Model of land area $36^{\circ}.12 \leq \varphi \leq 38^{\circ}.82$, $5^{\circ}.30 \leq \lambda \leq -0^{\circ}.67$ having a grid mesh of 100 m. Morelli bathymetry on sea with grid spacing $5' \times 7'.5$ has been merged.

The method used to compute the quasigeoid is the fast collocation with the remove-restore technique. The terrain effect has been computed with the DTM and Morelli bathymetry: The EGM96 geopotential model has been used as a reference field in order to remove and restore the long wavelength components of gravity and geoid respectively. The final result is a gravimetric geoid on a $3' \times 3'$ grid in the area with limits $36^{\circ}.7 \leq \varphi \leq 38^{\circ}.42$, $-4^{\circ}.33 \leq \lambda \leq -1^{\circ}.2$.

The accuracy of this geoid solution has been tested by using GPS/levelling geoid undulations in benchmarks belonging to the first order levelling network. After elimination of the possible tilt of the gravimetric geoid by means a similarity transformation the standard deviation of the discrepancies between gravimetric geoid and GPS/levelling values amounts to ± 5 cm.

In this paper a summary of the work done is presented. In section 2 the data used are shown. In section 3 the quasigeoid computation is explained and in section 4 the results of the comparison between the geoid model and GPS/levelling undulations are presented.

2. DATA USED

GPS SURVEYS. A GPS survey was performed in March 1998. In this survey two traverses containing 26 GPS control points, 22 of which are benchmarks of the first order levelling network, and the others belong to the second order levelling network. For

geodetic establishment the GPS constellation was tracked for 3-4 hours sessions over baseline lengths averaging less than 21km, 38 baselines were observed during 25 observation sessions. The equipment used throughout the survey consisted of three Leica SR399 and one Leica 9500 GPS double frequency carrier phase GPS receivers. Bernese 4.0 (Rothacher et al., 1996) with precise ephemerides was used to process the GPS observations and NETGPS (Crespi, 1996) software for the adjustment of the GPS traverses. A statistical summary of the GPS processing can be seen in Table 1.

GRAVIMETRY DATA. A set of 3277 free-air gravity anomalies covering the area between limits $36^{\circ}.7 \leq \varphi \leq 38^{\circ}.42$, $-4^{\circ}.33 \leq \lambda \leq -1^{\circ}.2$ has been used for this work.

SPHERICAL HARMONIC COEFFICIENTS. The high-degree global geopotential model EGM96 has been employed. This model is considered complete up to degree and order 360 (Lemoine et al., 1997).

TOPOGRAPHY DATA. The Digital Terrain Model is provided by Servicio Geográfico del Ejército. This data base contains 3001 x 2001 heights on a regular UTM grid of 100 x 100 spacing. Morelli bathymetry on sea with grid spacing 5' x 7'.5 has been used. Both of them were merged together to form a unique homogeneous height data base over the area $36^{\circ}.12 \leq \varphi \leq 38^{\circ}.82$, $-5^{\circ}.30 \leq \lambda \leq -0^{\circ}.67$ with 100 x 100 resolution.

Table 1. Minimum constrained adjustment global parameters.

sessions	equations	Unknown	Redundancy	σ_o^2 ($\sigma_o=1$)	χ_{ex}^2	χ_{th}^2 ($\alpha=5\%$)	rms x (m)	rms y (m)	rms z (m)	S_{max} (m)	S_{med} (m)	CI_h (m)
25	186	78	108	1.11	132.57	133.26	0.025	0.006	0.019	0.029	0.015	0.061

χ_{ex}^2 : experimental χ^2 with "redundancy" degrees of freedom

χ_{th}^2 : theoretical χ^2 95% with "redundancy" degrees of freedom at the 95% confidence level

S_{max} : semimajor axis of the error ellipse at the 95% confidence level

S_{med} : average error ellipse semiaxis at the 95% confidence level

CI_h : height confidence interval at the 95% confidence level

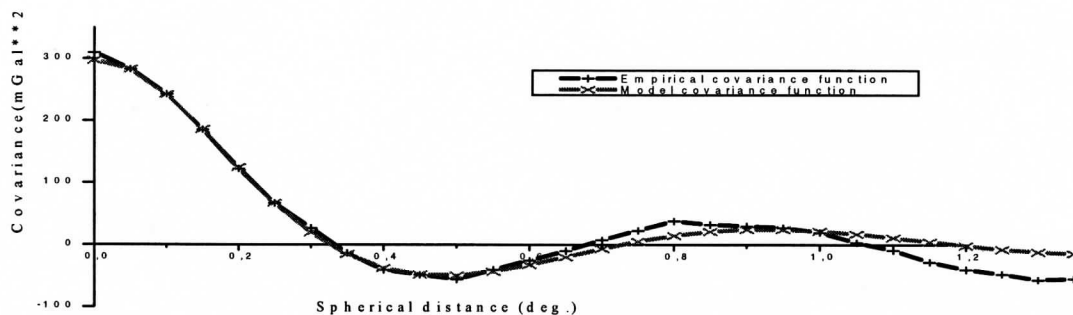


Figure 1. The synthetic and empirical covariance functions

3. QUASIGEOID COMPUTATION IN THE TEST AREA

Fast collocation and the remove-restore procedure were used to compute the quasigeoid estimate (Barzaghi et al., 1996). In order to obtain a smoothed gravity field as input for the collocation procedure gravity data must be reduced for the geopotential model and terrain effect. The geopotential model EGM96 has been used to remove the long wavelength

component of the gravity data. The residual terrain effect was computed to a 10' x 10' reference grid. Only residual topography distance of 38 km was taken into account. To compute the effect of topography TC program was employed (Tscherning et al., 1992). Finally, a set of 3277 residual gravity anomalies was available. In order to detect outliers in this data set a comparison of the difference between the observed and the predicted values has been done. 1.4% outliers has been detected and removed from the data.

The gridding of Δg_r to produce a 3' x 3' grid of residual anomalies has been done with GEOGRID program of the GRAVSOFT package (Tscherning, 1992). To estimate the residual quasigeoid via fast collocation the empirical and model covariance functions of the gravity anomalies are needed.

The covariance functions used in the prediction of residual quasigeoid anomaly via fast collocation are plotted in Figure 1. For the analytical approximation of the empirical covariance functions and for computation of height anomalies COVFIT and FASTCOLB programs have been used.

The last step to get the final product is the restoring of the model and the residual terrain effect on the 3' x 3' residual quasigeoid grid. The quasigeoid model has been transformed to geoid model (Heiskanen and Moritz, 1967), (Figure 2). The numerical results of the estimation procedure are summarised in Table 2.

Table 2. Statistical summary of remove-restore procedure

	Δg_{fa} (mGal)	$\Delta g_{fa} - \Delta g_m$ (mGal)	Δg_r (mGal)	Δg_r (mGal)	ζ_r (m)	ζ (m)	N (m)
n° of points	3277	3277	3230	2331	2331	2331	2331
mean	2.25	22.03	-7.2	-0.43	0.05	50.88	50.85
st. dev.	33.83	24.10	21.33	17.61	0.36	1.47	1.41
max.	177.65	67.70	72.20	63.68	1.50	54.45	54.43
min.	-80.45	-52.82	-75.20	-67.50	-0.96	46.73	46.70

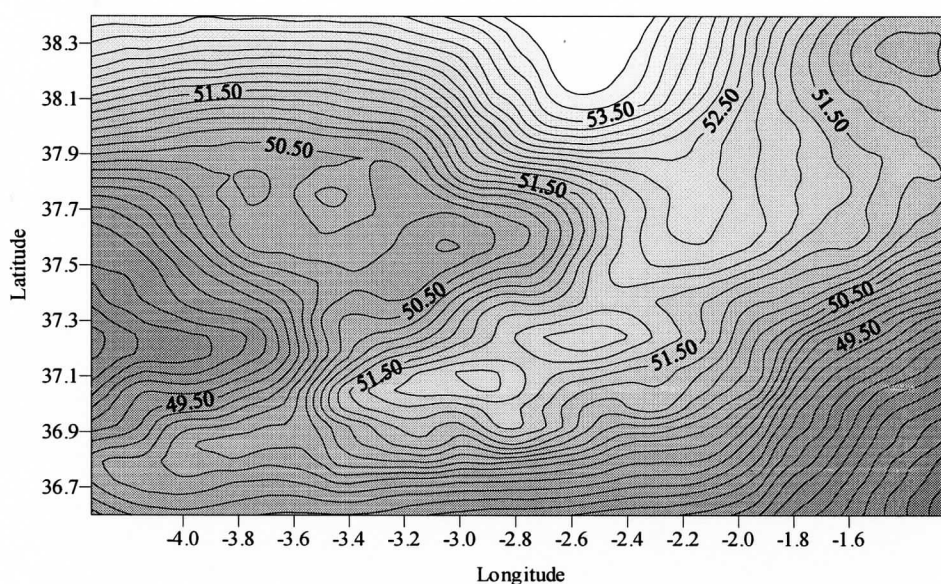


Figure 2. Gravimetric geoid solution

4. COMPARISON WITH GPS/LEVELLING

Geoid undulations from fast collocation have been interpolated to the 22 benchmarks belonging to the first order levelling network. To minimise the long wavelength error the systematic datum difference between the gravimetric geoid and the GPS/levelling values was removed by a four parameter transformation. The use of the datum shift eliminates the possible tilt of the gravimetric geoid as well yielding a fit only ± 5 cm. This fact proves a good fit between the geoid of the test area and GPS/levelling data has been reached.

5. CONCLUSIONS

This work has presented a study of the accuracy of a gravimetric geoid in a test area in Northeast Andalusia (Spain) by comparing with GPS/levelling data.

The standard deviations of the discrepancies between the gravimetric geoid solution and GPS/levelling geoid undulations amounts to ± 5 cm after fitting a plane. Therefore a good fit between them has been reached.

The results of this work show the fast collocation technique and the remove-restore procedure has been used successfully for the determination of a geoid in a test area. Thus we intend to apply the same method to compute a geoid over whole of Andalusia in a near future.

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II Section
"Communications and News"

REPORT ON ACTIVITIES OF IAG COMMISSION X : SUB COMMISSION FOR SOUTH EAST ASIA AND THE PACIFIC

Background

At the time of the Canberra IUGG conference in 1979, an IAG sub commission on South East Asia and the Pacific was formed with Tony Bomford as chairman. Activities primarily focused on the establishment of a network of Doppler points across the region and a preliminary geoid as the basis for regional Geodesy. On the retirement of Tony Bomford, in the early 1980s, Con Veenstra and Jacob Rais became co-chairs of this sub-commission. The last meeting of the IAG Commission X South East Asia and the Pacific was held in Fiji in 1986. But activities lapsed with the retirement of Con Veenstra in 1987 and the sub-commission became defunct.

In 1995 under the banner of the United Nations Regional Cartographic Conference, a regional Permanent Committee was formed with membership from National agencies as the permanent Committee on GIS Infrastructure for Asia and the Pacific (PCGIAP).

Current Activities

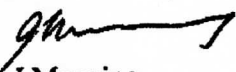
Subsequently a PCGIAP Regional Geodesy Networks Working Group was established in 1996 to develop a regional datum on which to build a unified geodetic framework for GIS applications. Chaired by Indonesia this working group has actively undertaken regional geodetic campaigns, aimed at establishing a common framework base for spatial data across the region. Composition of the working group is strong being comprised of representatives from National surveying and mapping agencies.

A regional workshop was held by the group in Canberra in July 1998 to finalise results from the first ,multi- technique, field campaign of October 1997. At that meeting it was realised that the work of the PCGIAP, in establishing regional geodetic networks, was also fulfilling the aims of Commission X. A regional sub commission was proposed and accepted by Claude Boucher as chairman of Commission X co chaired by JY Chen and J Manning. It is desirable that the project based activity of the PCGIAP WG and the activity of Commission X is in unison.

A second regional field campaign was arranged in November 1998, to coincide with Geodeyssea 98, and Vietnam subsequently hosted a workshop in July 1999 to present results from that campaign. A sub commission report on activities of the PCGIAP Working Group was provided to the Birmingham IUGG meeting. Since that time a status report on activities of the working group was provided to the PCGIAP executive meeting in Melbourne in October 1999 (attached).

Next workshop

The third regional geodesy workshop of the group is planned for August 2000 in Mongolia to present results of the 1999 field campaign, which was undertaken in November 1999. It is hoped to get IAG endorsement of this workshop and an official representative of Commission X or of Section 1 to attend.



J. Manning
Chairman

Sub commission on South East Asia and the Pacific
10 February 2000

REPORT FOR WORKING GROUP 1 : REGIONAL GEODETIC NETWORK

Key activities undertaken since 5th PCGIAP Meeting in Beijing:

1. Asia Pacific Regional Geodetic project (APRGP98)

A second field campaign (APRGP98) was undertaken in November 1998 concurrent with the Geodyssea98 campaign. There was greater participation in this campaign than APRGP97 but there are still significant areas to be infilled when countries gain access to GPS resources. Seventeen nations were able to participate and provided GPS observations from some 87 sites in addition to the existing IGS sites. The GPS data (except from four sites in India) was collated by AUSLIG and distributed on CD ROMs to all countries for processing, analysis and presentation of results.

A VLBI campaign was again arranged by China (Shanghai Observatory) through APSG cooperation. SLR was arranged through WPLTN, with DORIS observations made available through CNES. The APRGP98 project was assessed as being very successful with:

- increased participation by member countries;
- increased number of stations in the campaign; and
- excellent results produced from GPS, SLR and DORIS observations.

2. Regional Geodetic Network Workshop

The General Department of Land Administration, Vietnam hosted a Regional Geodetic Network Workshop in Ho Chi Minh City on 12th & 13th July 1999 on behalf of PCGIAP. This was a very successful event with representatives from thirteen member countries attending. The workshop welcomed and encouraged the participation of Mongolia and Nepal in future campaigns. Both these countries requested assistance in applying GPS/GIS technology in their countries and member countries were encouraged to respond and assist wherever possible.

Results from the Asia Pacific Regional Geodetic Project 1998 (APRGP98) campaign were presented thus concluding the core component of the project. This will form the basis for transformation of individual country geodetic networks to the ITRF reference frame. The success of technology transfer was obvious in some nation's GPS analysis, presentations and technical expertise.

3. Workshop Outcomes Summary

Completion of APRGP97 Campaign Computations:

- SINEX files to AUSLIG by end of July 1999.
- Australia, China (State Bureau of Surveying and Mapping), China (Shanghai Observatory), Japan, Indonesia and Malaysia,
- Combined GPS solutions.
- Combined all techniques solution.

- Complete all computation for APRGP97 before APRGP99 commences.

Publication of results

Following combination of results an official set of coordinates for APRGP97 and APRGP98 will be prepared for distribution to member countries.

1999 regional campaign

A sub committee was established to plan the APRGP99 observation campaign, committee members are Australia (coordinator), Fiji, Iran, Japan, and Vietnam. Committee to address the following criteria:

- Focus on the "Empty Zones" – Russia, China, India, etc and the "Pacific Group" for densification.
- Committee members to encourage and coordinate participation in APRGP99 by PCGIAP members in sub regions:
- Fiji – Pacific Islands sub-region members.
- Iran – West Asia sub-region members.
- Japan – permanent trackers in Pacific Islands sub-region.
- Vietnam – Indo-China sub-region members.
- Focus on site selection for connection to a universal height datum – GPS observations at tide gauge benchmarks and leveled benchmarks
- APSG5 and APSG6 VLBI campaign – 1st November and 8th November 1999 being coordinated by Shanghai Observatory.
- GPS campaign from 0hr, Sunday 31 October 1999 to 0hr Wednesday 10th November 1999.
- SLR and DORIS data for the whole month of October will be required. WPLTN to coordinate the SLR campaign.

Permanent Network of Base Stations

Establish a list of all continuous GPS receivers in the region together with contact agencies and data availability e.g. Pacific, Nepal, etc.

- Fiji to look at situation in Pacific sub-group,
- Nepal to investigate US and Italian campaigns in Nepal (and Absolute Gravity),
- Japan to advise of location of their receivers in Pacific (including proposed sites) with aim of making data available during APRGP99.
- Make information available on PCGIAP web site

GPS Observations Specifications

A sub committee was established comprising Australia, China (State Bureau of Surveying and Mapping), Japan and Malaysia to develop standards for the 1999 campaign including:

- Antenna descriptions.
- Antenna heights.
- Station IDs.
- RINEX headers conformity.
- Data transfer medium – ftp, CD-ROM, Zip, Floppy – in preferred order.
- Specifications for RINEX conversion software.

Classification of networks

Indonesia to provide a position paper on Accuracy Classification and Standards of coordinates. Interpretation of "Zero Order", "First Order" and "Second Order" in the context of the APRGP results.

Development of Transformation parameters.

Member countries to consider the precision datum transformation proposals of Iran and Australia for implementation after achieving an adequate densification of ITRF points within individual geodetic networks. APRGP99 planning committee to also consider transformation requirements in site selection process.

Absolute Gravity Network

A subcommittee was established comprising Australia, Indonesia, Iran, Japan, Malaysia and Vietnam to develop a proposal for a regional Absolute Gravity Observation Campaign to connect relative gravity networks to gravity reference frames

Regional Geoid

A sub committee was established comprising Iran and Vietnam to study the terrestrial (relative) gravity data input data into the EGM96 – identifying weaknesses and developing a work plan for further observational requirements leading to an improved regional Geoid.

Regional Geoid School

The Workshop supported the arrangements for an International Geoid School to be held in Malaysia in 2000, and strongly encourages member countries to attend. It also encourages the planning and hosting of Tropical Schools of Geodesy with specific reference to a school on GPS.

Electronic communications

- The need for enhanced communication between WG1 members was identified.
 - AUSLIG to further develop the Working Group web site with diagrams and meta data, and
 - Establish an email List Server with mailing addresses for all WG1 members and APRGP campaign participants.

4. Asia Pacific Regional Geodetic project (APRGP99)

At the workshop plans were developed for the next regional observation campaign in November 1999, with a results workshop to be subsequently hosted by Mongolia in Ulaanbaatar in July/August 2000. The aim of this campaign (APRGP99) is to expand the number of key geodetic sites occupied in individual countries and where possible to undertake GPS connections to sea level at established tide gauges. Additionally it will also incorporate data from all permanent GPS, VLBI, SLR and DORIS to further densify the regional ITRF. The intention is to begin to expand activities into the Pacific nations. The field campaign will include:

- GPS observations 1-10 November
- DORIS whole month November
- VLBI 1st and 4th November
- SLR whole month of November with support from ILRS and WPLTN

Currently eleven nations have responded positively to provide GPS observations and an additional 4 or 6 countries are expected to participate. As the Pacific nations are sorely under-resourced in GPS equipment it will be difficult for them to participate fully this time. However extensive research has been undertaken to get access to existing GPS base stations in the Pacific region (established for scientific purposes) and these will be included in the regional data set to give reference sites which can be tied to the local geodetic networks in the future. A further focused campaign will be considered in year 2000.

5. PCGIAP WG1 Meta data Web Site

AUSLIG is expanding the Working Group 1 web page to be a meta data site for geodetic information mainly for permanent sites, such as

- VLBI
- GPS base stations
- SLR
- DORIS
- Absolute Gravity geodetic

6. Absolute Gravity Project

At the workshop in Vietnam, it was proposed to consider the need to upgrade the regional Geoid in relation to vertical reference datums and the use of GPS for heighting. A questionnaire has been circulated and responses are being received, gathering information on individual countries gravity networks, and ties to global reference gravity networks.

The intention is to prepare a plan for a strengthening of the regional gravity network by tying countries together by the use of absolute gravity at key sites. Any project implementation would need support in the provision of FG5 equipment and expertise.

7. General

- An email listserver has been set up and several newsletters circulated electronically.
- Malaysia proposes to hold a Geoid school in Jodoh in February and **PCGIAP endorsement of this initiative is recommended**
- Korea is establishing a network of GPS base stations
- Malaysia has established a network of GPS base stations
- Linkages with other organisations has continued to benefit Working Group 1 activities
- **Vietnam should be officially thanked by PCGIAP for hosting the Regional Geodetic workshop and their generous hospitality and magnificent venue.**

7. Workplan Summaries

The work plans for the next 12 months as approved at 5th PCGIAP are:

Project 1 – Regional Precise Geodetic Network - Coordinator Australia

Workplan Summary

Action	Responsibility	Target Date	Progress
1. Finalise APRGP98 collection and distribution of GPS data	Australia,	30 April 1999	Completed
2. Individual countries process and present results for combined solution	Participating members,	12/13 th July 1999	Completed
3. Arrange Technical Workshop for presentation of results from APRGP98	Vietnam	12/13 th July 1999	Completed
4. Prepare strategies to maintain precise reference framework and further develop the regional network of permanent sites	Australia,	12/13 th July 1999	Workshop discussions
5. Publish proceedings of Technical workshop and enhance directory of permanent network sites, and details of access to data, on WWW	Australia, Vietnam	September 1999	Publication target now November
6. Monitor development of GLONASS and present strategy for using GLONASS global base station observations for Working Group consideration	Russia	November 1999	New strategy required following IGEX workshop at ION meeting September 1999
7. Investigate regional capability for ongoing VLBI for WG1 discussion and submission to the Permanent Committee	Japan	12/13 th July 1999	China has arranged ongoing annual VLBI campaigns through APSG (next 1&4th November 1999)

Project 2 – Linkages from Local to Regional Horizontal Geodetic Datum Co-ordinator - Australia

Workplan summary

Action	Responsibility	Target	Progress
1. Develop Observation campaign strategy and workplan for APRGP99	Australia	August 1999	Draft prepared for discussion
2. Arrange epoch campaign to link with international projects in the region	Australia,	November 1999	In hand
3. Undertake processing of APRGP98 observation campaign	Australia, in parallel with other countries	Sequentially from April 1999	Initial processing completed. Integration still required
4. Prepare papers on strategy for computation of transformation parameters	Australia, China, Iran	12/13 July 1999	Papers prepared for further consideration when needed
5. Individual countries readjust geodetic network to regional points	Individual countries	UNRCCAP 2000	ongoing
6. Individual countries publish results, post to WWW	Individual countries	UNRCCAP2000	ongoing

Project 3 Regional Vertical Datum Coordinators: Japan and Australia

Workplan summary

Action	Responsibility	Target	Progress
1. Prepare discussion papers on aspects of vertical datum for consideration as possible approaches for establishing a regional vertical datum	Japan, Australia, China	12/13 th July 1999	Workshop discussion lead to consideration of Absolute gravity program for Geoid
2. Represent the WG on IAG Special Study Groups on sea level. Prepare progress reports to WG	Vietnam, Australia	Ongoing	Commission X sub commission on SE Asia reformed. Report presented to IAG July 1999

CHECKLIST SUMMARY OF ACTIONS ARISING FROM VIETNAM WORKSHOP

Action	Responsibility	Target date	Progress
Completion of ARGP97			
GPS SINEX files to AUSLIG	China, SBSM (Weiping Wuhan)	End July 1999	Action required
	China, Shanghai		Action required
	Japan, Shigeru		Action required
	Indonesia, Cecep		Action required
	Malaysia, Samad		Received
Combined GPS solution	AUSLIG	One month later	Now due December
Completion of ARGP98	China, SBSM	30th August 1999	Received
SINEX files to AUSLIG	China, Shanghai		
	Japan, Shigeru		
	Indonesia Cecep		
	Malaysia, Samad		Received
Combined GPS solution for 98	AUSLIG	One month later	December
Total combined GPS solution	AUSLIG	30 th Sept 1999	March 2000
Publication	Vietnam, AUSLIG	September	Due November
Planning ARGP99 committee	Auslig, Fiji, Iran, Japan, Vietnam	August 1999	--
Arrange VLBI campaigns	China Shanghai	Campaign 1, 4 November	Arranged
Arrange SLR participation with WPLTN	AUSLIG	August 1999	Arranged
Arrange DORIS participation with CNES	AUSLIG	August 1999	Arranged
Register of Base Stations			
List all continuous receiver	Fiji -Pacific	August 1999	Nil to date
	Asia Iran Nepal		Nil to date
Post information on Website	AUSLIG	September 1999	
GPS Observation specifications	Australia, SBSM, Japan, Malaysia		In hand
Classification of network accuracies	Cecep	September 1999	Nil to date
Absolute Gravity strategy	Australia, Iran, Japan, Malaysia, Vietnam	September 1999	Now in hand
Regional Geoid			
Examine gravity coverage of region in EGM96, identify weaknesses in data	Vietnam, Iran	November 1999	Nil to date
Arrange Geoid school	Malaysia	March 2000	Arrangements in place
Communications			
Develop web site with details of all permanent sites	Australia	September 1999	Commenced
Establish list server for WG1	Australia	August 1999	Initiated

The Fourth International School on "THE DETERMINATION AND USE OF THE GEOID"

Johor Bahru, Malaysia 21- 25 February 2000

Introduction

In the effort of continuing and spreading the technical culture for the determination and use of the geoid, the International Association of Geodesy (IAG), through its International Geoid Service (IGeS), has organised the IGeS Geoid School in Johor Bahru, Malaysia on 21 – 25 February 2000.

Department of Survey and Mapping Malaysia (DSMM) was initially invited to be the local organiser in the late September 1998. The date for the school was changed twice due to some organisation and budgetary problems.

The general purpose of this school was to prepare new graduate students, young scientists or employees of national agencies and services or industry staff, to use and, when necessary, to compute gravimetric geoids for the many scientific and technical applications in geodesy, primarily in transforming ellipsoidal (GPS) heights into orthometric heights.

The school was structured in a way to be self-contained for any participant at graduate level with basic knowledge of geodesy.

The courses included:

Monday: A general introduction to physical geodesy, geoid computation and collocation theory by Fernando Sansò

Introduction and explanation on the role and present activities of BGI by Jean Pierre Barriot . In conjunction, a South East Asia Geodetic Committee Meeting was held in the afternoon to discuss various issues related to IAG.

Tuesday: The computation and use of global models of high and ultra-high degree, by Riccardo Barzaghi

Wednesday: Optimisation of Stokes' Integral in Geoid Computations by A.H.W. Kearsley

Thursday: The terrain and residual terrain correction,

by Rene Forsberg

Friday: Fast Fourier techniques to perform the computation of the main formulas in physical geodesy,
by Micheal Sideris

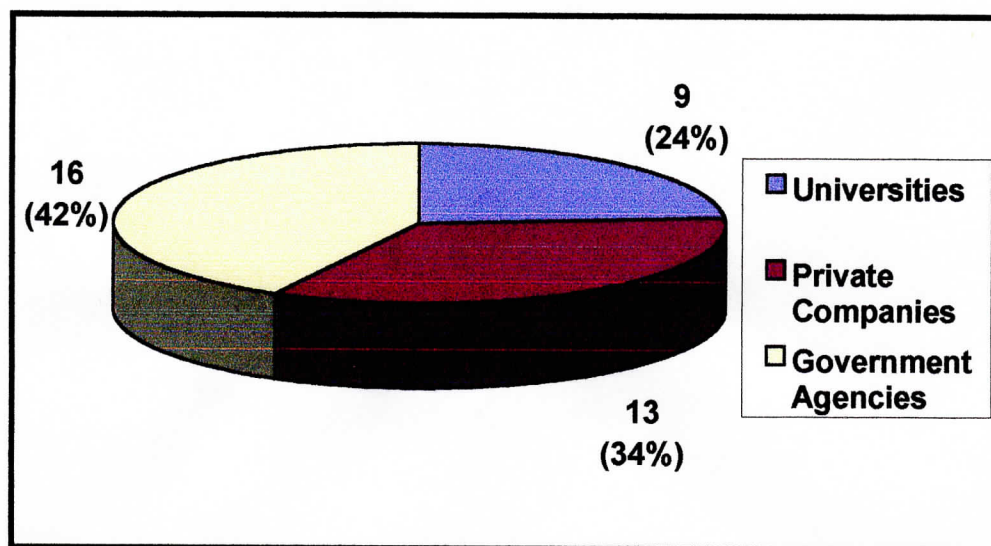
Participants

We were contacted by 65 people from the first announcement to the end, and finally we've got a final number of 41 students who came from Australia, Chile, China, Ethiopia, France, Hong Kong, Indonesia, Malaysia, Nepal, Singapore, Sweden, Republic of Maldives and USA. successfully attended the school. Old and new Lecture Notes have been provided and IGeS software as well as numerical exercises have been made available to the students. All participants were provided with software material in a CD which was distributed to each participant under a contract condition that it would not be used for commercial purposes.

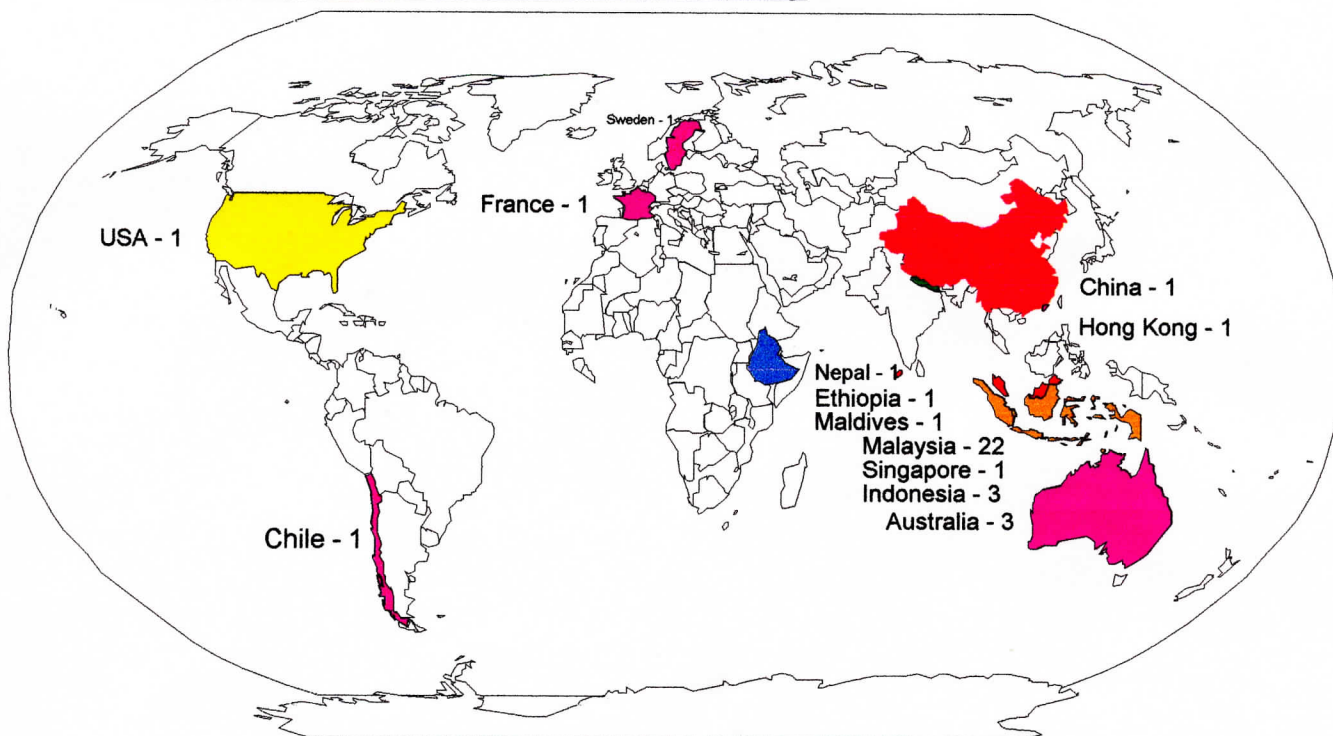


Lecturers, participants and organiser to the school

Main activity of the participants



Distribution of the students by country



Opening Dinner

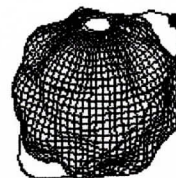
The opening dinner was given by the Local Organising Committee for the participants and the lecturers. It was held at the Poolside Restaurant of the Puteri Pan Pacific Hotel with a buffet Barbeque menu. A cultural show was presented to allow the participants, especially from the overseas to acquainted with the local traditions. In fact some were left with quite an unusual memories.

Transportation

Since the school was conducted at the hotel where all participants stayed, transportation was not necessary.

Suggestions

- The school should cater for various level of users so as not to compromise the standard and also would allow the beginners to follow much easily.
- A bus tour of the city should be included in the seminar to allow the participants to get to know of the local attractions and cultures.
- The duration of the school should be extended so as to allow for a more detailed lectures and comprehensive practicals on various topics.
- A topic on GPS heighting should be included on future course.
- More tutors should be provided in the practicals on software usage



South America Geoid 2000

Workshop

Local: Polytechnic School – University of São Paulo – Dept. of Eng. Transportation

Date: May 17 – 19, 2000

Organization: IGeS (International Geoid Service)

SCGGSA (Sub-Commission for Gravity and Geoid in South America)

Coordinator: Denizar Blitzkow

EPUSP – PTR

Caixa Postal 61548

05424-970 São Paulo – SP – Brasil

dblitzko@usp.br

Phone: (55) (11) 818-5501

Fax: (55) (11) 818-5716

Program:

- May 17**
- 9:00 – 9:30 Welcome Reception
 - 9:30 – 11:00 Activities of the Geoid in the World and IGeS (F. Sansó)
 - 11:00 – 12:00 Activities of Gravity and Geoid in South America (D. Blitzkow)
 - 12:00 – 13:30 Lunch time
 - 13:30 – 14:30 The geoid and oceanography (O. Andersen - IAPSO)
 - 14:30 – 15:30 TOPEX/POSEIDON data processing (C. A. de Sampaio França - IO)
 - 15:30 – 16:00 Coffee break
 - 16:00 – 17:00 NIMA: Data availability and validation (S. Kenyon)
- May 18**
- 9:00 – 12:00 Data availability in South America:
 - Argentina – (15 min)
 - Brazil – (15 min)
 - Chile – (15 min)
 - Colombia – (15 min)
 - Uruguay – (15 min)
 - Venezuela - (15 min)
 - Peru - (15 min)
 - Other possible countries
 - 12:00 – 13:30 Lunch time
 - 13:30 – 17:00 Geoid computation procedures (R. Barzaghi & D. Blitzkow)
- May 19**
- 9:00 – 12:00 Geoid computation procedures (R. Barzaghi & D. Blitzkow)
 - 12:00 – 13:30 Lunch time
 - 13:30 – 15:00 Report on local geoid computation procedures (R. Barzaghi)
 - 13:30 – 17:00 Resolution improvement on the geoid (R. Barzaghi & D. Blitzkow)

IGeS Working Group:

Preparation of Standard Procedures for Global Gravity Field Validation

Background:

Within the next years a number of dedicated gravity field missions from space will be launched into orbit. These missions are:

- CHAMP: high-low satellite-to-satellite tracking system using GPS; accelerometer for measuring non-gravitational forces; launch 4-2000; mission duration 5 years
- GRACE: low-low satellite-to-satellite tracking system using microwaves; GPS on-board receiver; Accelerometers for measuring non-gravitational forces on both satellites; launch 6-2001; mission duration 5 years
- GOCE: satellite gradiometry mission; GPS/GLONASS receiver on-board; drag forces control by differential measurements; launch 7-2004; mission duration 20 months

Goal of all of these missions is the provision of the static and time variable gravity field from long wavelengths to high resolutions with unprecedented accuracy compared to the state of the art global gravity field models. The validation of these gravity fields requires better test procedures and better test data sets as currently available. From experiences with the validation of gravity field models it became clear that for an independent quality assessment a standardized approach has to be developed. For example: Orbital fits for testing the long wavelengths highly depend on the standards used for orbit computation and gravity field determination. Using different standards imply higher residuals for the tracking data and a worse result for the tested gravity field. Or, usage or not of GPS derived geoid heights, which are flagged as possible bad quality data, can provide opposite results when two gravity field models are compared. These two examples show that it is necessary to define evaluated standard comparison data sets as well as standard procedures to perform the independent quality assessment of a global gravity field model.

For the definition of standard procedures and data sets, previous work performed by international groups like the EGM96 validation group, chaired by M. Sideris and the IAG special study group 3.165 (1995-1999), chaired by N. Pavlis should be continued. Especially the EGM96 gravity field validation group performed a wide range of tests, which partly could become standard procedures for gravity field validation.

Objective:

Objective of the working group is the definition of standard test procedures for global gravity field evaluation. This includes gathering and provision of standard test data sets as well as recommendations how to use these data sets. For this, existing experiences and data sets from BGI, from the EGM96 validation group, from global gravity field modelling groups and from a number of other national institutions and universities should be gathered.

Standard test data sets should be made available ideally on a Web-site of one of the existing IGGC services (BGI, IGeS). As a first step recommendations for standard test procedures using these data sets should be given along with the data sets. As a second step also Software to perform these standard test procedures should be made available on this Web-site. As a final step an option could be that a complete standardized test sequence for gravity field validation is operational at this service and that results are provided together with the gravity field model (to be discussed).

Activities:

The activities of the working group could be organized by the following sub-activities:

- Definition of types of test data sets, which can be used for gravity field validation.
- Find an institution, which is willing to archive and distribute gravity field test data sets (ideally an existing IGGC service).
- Gather as much of such test data sets worldwide and make them available to research community via the archiving institution.
- Convince as many data owners as possible to release gravity field test data sets to the archiving institution for further distribution. If original data are restricted, derived gravity field related products could be very helpful (e.g. gravity anomaly block mean values derived from restricted single point measurements).
- Discussion and definition of procedures how to use the available test data sets correctly for gravity field validation.
- Provision of Software modules via the service center, which can be used for gravity field validation (partly Software which is available at IGeS can be used).
- Discussion if and how a standardized procedure for satellite orbit tests can be realized (problem of different standards in orbit determination software).
- Discussion if a standard test procedure comprising various individual tests can be implemented at a service center, which distributes available gravity field solutions and the quality parameters.

As pointed out in the list of activities above, besides collecting test data sets mainly discussions on test procedures and the incorporation of existing IGGC services are the main activities. Discussions should not be endless, but should result within a reasonable time in recommendations how to test global gravity field models (which have to be written down somewhere). In parallel, also discussions about the incorporation of a "global gravity field service center" into existing IGGC services have to be initiated. It has to be discussed and specified to what extent existing services are willing and can support this task. Ideally, a service providing global gravity field test data sets together with gravity field models can

be the starting point. Later on the service can be upgraded step by step to provide Software and to perform gravity field quality tests on-site. From these ideas a very rough schedule for the period 2000-2003 can be drawn:

2000-2003	collection of test data sets
2000-2001	discussion on standard test procedures
IAG Assembly 2001	recommendations on test data sets and procedures (report)
2000-2001	discussion on inclusion of test data sets in existing IGGC service
2001-2003	discussion how to extend this service for Software and on-site gravity field validation
IUGG Assembly 2003	working group results recommendations

Proposed Group Members:

The group should be formed from representatives of both IGGC services, from representatives of global gravity field modellers and from independent scientists working in the field of gravity field modelling (mainly from the EGM96 validation group). The current proposal from my knowledge is:

Abd-Elomotaal H.; Minia Univ.; Egypt
 Albertella A.; Politecnico Milano; Italy
 Barriot J.-P.; CNES; France; BGI Director
 Bettadpur S.; UTCSR; USA
 Chen J.; Nat. Bureau of Surv.; China
 Featherstone W.; Curtin Univ.; Australia
 Forsberg R.; KMS; Denmark
 Gruber Th.; GFZ Potsdam; Germany; Chairman
 Kenyon S.; NIMA; USA
 Lemoine F.G.; NASA/GSFC; USA
 Lemoine J.M.; GRGS; France
 Lu Y.; Academy of Sciences; China
 Medvedev P.; Academy of Sciences; Russia
 Milbert D.; NGS; USA
 Pavlis N.; Raytheon ITSS Corp.; USA
 Ries J.; UTCSR; USA
 Schwintzer P.; GFZ Potsdam; Germany
 Sideris M.; Univ. Calgary; Canada; EGM96 Validation Group Chairman
 Tziavos I.N.; Univ. Thessaloniki; Greece
 Veronneau M.; Geodetic Survey; Canada

Thomas Gruber, GFZ Potsdam

IAG/IAPSO Symposium

Sea level measured by satellite altimetry (Topex/Poseidon, ERS-2 and the future Jason and Envisat) and the global gravity field as will be measured by the GRACE and GOCE satellites, are each of direct use in both oceanographic and geodetic studies. Geodesists use models of the continental and marine geoid to establish a unique reference to different national height systems. Oceanographers use the difference between an altimetric sea surface and the marine geoid to estimate total surface currents. Strong geoid and oceanographic signals occur along bathymetric gradients such as continental slopes, where separating the two is more difficult.

Satellite gravity missions, in addition to improving the geoid, hold the promise of measuring time changes in ocean bottom pressure, which is particularly important in regions of the world ocean very poorly covered by other oceanographic measurements. Thus a satellite gravity measurement whose primary purpose is to improve the geoid, may shed further light on crucial dynamic balances in the ocean.

In this Symposium we have therefore a unique convergence between studies in Geodesy and Ocean Dynamics. A gravity measurement is planned that will directly translate into an oceanographic variable, and ocean dynamic observations and models can improve gravity models. This Symposium will focus on these interactions, and will focus on fostering a closer scientific cooperation between IAG and IAPSO.