

GRASS interfacing with DBMSs

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Abstract

The work here presented is related to one problem we had to face in building the geodetic GIS named GEOGRASS [3], [5].

This software package, already presented at the IAG International Symposium on Gravity, Geoid and Geodynamics 2000 (July 31 – August 4, 2000 Banff, Alberta Canada) allows to compute in a GIS environment the geoidal heights in points not belonging to the collocation grid solution. The user is only asked for the point in which he wants the geoid, the window size for the topographic geoidal component computation, the global geopotential model to be used and the system returns the punctual geoidal height.

The GIS selected for this aim was GRASS (<http://www.baylor.edu/~grass>; <http://www.geog.uni-hannover.de/grass>).[7]

GRASS, as other GIS, is not only a data analyser and viewer, but allows also to efficiently store data in its own hierarchical archive, organised in GISDBASE, LOCATIONS, MAPSETS and elements levels. The data we have to manage within GEOGRASS are raster (e.g. the DTM) and sites points data (e.g. the observed gravity anomalies).

With regard to the raster data, they are stored within Grass in a bidimensional matrix $N \times M$. The corresponding file contains N lines of M columns of integer values. These values, representing the attributes associated to each raster cell, are called categories. In the case of a Digital Terrain Model (DTM), for instance, each different eight may correspond to a different Grass category.

The information describing a raster data are distributed within more subdirectories in the MAPSET according to their contents

The site points data are stored in GRASS in ascii files within the sites_list directory. These files have an header containing:

- the name of the points contained in the file;
- the description of the attributes associate to points.

The data follow, each point is stored in a record, with the coordinates (east, north and eventually height and time) and the attributes:

east|north|[height|[time |...]|#category] [[@text %][floating point] ...]

The data we have to manage in geographically related problems are characterised at least by the two planar coordinates. In this case, if we would like to interface our GIS with an external DBMS, it is better to use relational data structures with spatial extensions and some particular indexing method instead of archiving them in a simple flat ascii file.

In our work we have focused on the Oracle 8 Spatial Data Cartridge and the PostgreSQL ([1],[2],[8]) DBMSs, as they have the previously mentioned characteristics and run both under Solaris and Linux operating systems.

New commands were implemented to displaying raster and site points data stored in the DBMS tables (d.ora.raster, d.ora.sites, d.pg.raster, d.pg.sites).

Some tests showed that, in case of the raster data, the time needed for the data retrieval, by using the DBMSs instead of GRASS archive, is higher; on the contrary, in case of sites the use of DBMS

makes the procedure faster. Moreover further tests proved that the use of a special built up quad-tree index directly in the GRASS site points archive is, in term of waiting time, the best solution. This result suggests the implementation of a new indexing method in PostgreSQL (the DBMS we have evaluated to be the most adequate and fast for our needs) more suitable for sites (e.g.: quadtree instead of R-tree).

Alternatively the proposal is, at least, to introduce this indexing method directly in the management of site points.

The archive access

The GRASS display and georeferenced data processing commands exploit their own routines to gain access directly to disks and to manage both the search and storage of data (Figure 1).

When another software tries to gain access to same data used by GRASS, this direct approach can generate problems.

These are the most critical situations to face:

- Data Redundancy: it is sometimes needed to store data in different formats; this is not only a wasting of resources, but can also lead to data inconsistency (due to unsynchronised updates)
- Concurrency: a software tries to gain access to files already opened by another application; this can lead to mutual switching off.
- Integrity: data rules must be defined for each software accessing data archives.
- Data Reliability: data access criteria must be defined for each software accessing data
- Performances: data processing softwares not always have their built-in, fast-search algorithms.

It is better to use an external DBMS¹ to data archives managing (Figure 2): doing so, GRASS will have only processing and display duties.

With a DBMS we get the following advantages [6]:

- Limited redundancy: storing format is managed by DBMS (tables)
- Concurrency reduced from whole file level to single record level.
- Integrity rules depending directly from data and not from software behaviour
- Reliability: usually built-in
- Performances: DBMS use indexing methods to store and access data faster.

To allow GRASS to use DBMS functionality (at least the basic ones), some new commands have to be written. The link between GRASS and the DBMS can be developed in two ways:

- Direct connection with the chosen DBMS, using its specific driver set (Figure 3).
- Generic DBMS connection using ODBC²: this allows a link that works with every DBMS supporting ODBC (almost everyone).

The use of ODBC leads to lower performances, because of the higher number of software layers, but allows to realise a single interface which works with the majority of DBMS; nowadays almost every well-developed DBMS comes with built-in ODBC driver ready to use³.

Moreover, this choice (shown in Figure 4) allows an easy change between different DBMSs, without rewriting the interface and without converting tables' formats.

On the other side, an ODBC interface could not exploit local characteristics of a particular DBMS at its best.

¹ Data Base Management System

² Open DataBase Connectivity

³ <http://www.jepstone.net/FreeODBC/>

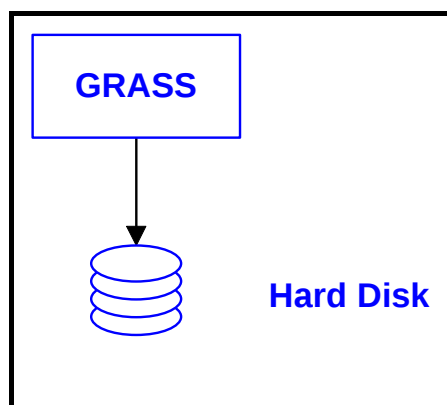


Figure 1 The direct access

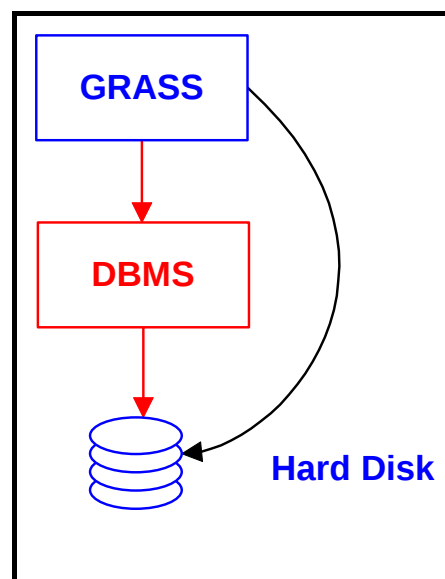


Figure 2 The access via DBMS

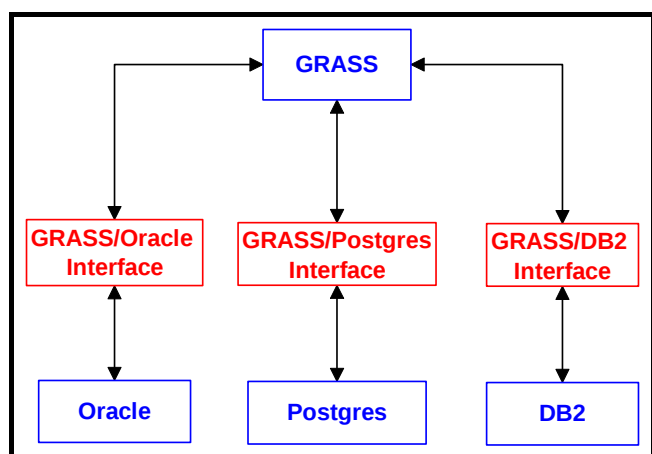


Figure 3 The direct interface

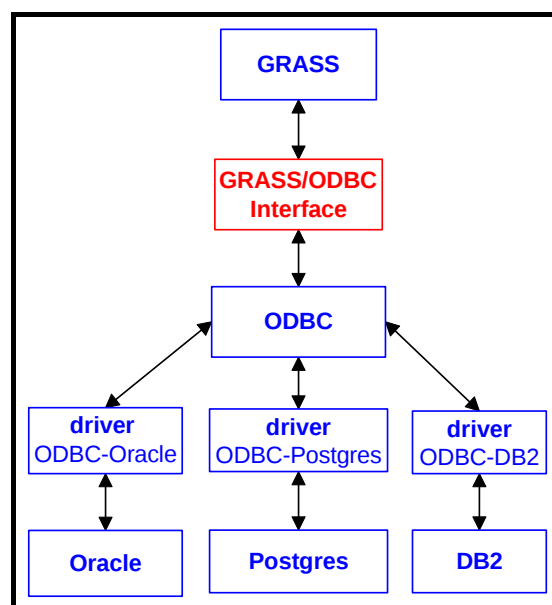


Figure 4 The ODBC interface

The archive management using DBMS

The simple use of a DBMS, to manage GRASS data archive, doesn't automatically raise access performances: we must remember that GRASS works with bidimensional data; to obtain a substantial improvement it is necessary that the DBMS adopts suitable methods to manage these spatial data [4]. A traditional relational DBMS (usually designed to manage monodimensional data) is not inevitably faster than GRASS.

PostgreSQL⁴ is a freeware DBMS that has the built in ability to define classes, types, functions, constants, triggers and integrity rules. These features allow to classify PostgreSQL as an "Object-

⁴ <http://www.postgresql.org/>

Relational DataBase". For spatial data, PostgreSQL provides a predefined set of types (point, line, box, ...), geometrical operators, and managing functions. It also allows two different indexing methods: B-tree (for monodimensional data) and R-Tree (for spatial data). Finally it comes with an ODBC driver.

The Shevlakov interface

Another GRASS - PostgreSQL interface was already written by Alex Shevlakov (<http://www.geog.uni-hannover.de/grass/projects/postgresqlgrass/>); its main purpose is to manage attributes of spatial data (and not spatial data themselves). The interface is organised as follows:

- geographical positioning data are still managed by GRASS.
- new data tables are created to store sets of attributes related to geometrical elements; these tables are managed by PostgreSQL.

With this interface we leave all monodimensional attributes managing to the DBMS: GRASS archives will contain only spatial information, while querying tables for every other information are in PostgreSQL.

By doing so, these data are easily accessed also by other programs with a simple and standard query on the database.

Worthy of note: only monodimensional data are managed by DBMS, so the interface doesn't exploit all the capabilities to manage multidimensional data.

Our interface

With the main purpose of using a DBMS to manage spatial data, we have written and tested some new commands for an interface GRASS - PostgreSQL that is able to exploit all the capabilities to store and access multidimensional elements in a faster way.

In particular (see the commands syntax in Figure 5):

- d.pg.rast: raster map is not any more stored in a binary file inside "cell" directory, but single values are saved inside a PostgreSQL table. When a user runs a visualisation command, GRASS generates a query on this table that performs an extraction of all points contained in current region's boundaries.
- d.pg.sites: it is similar to d.pg.rast, but the table contains sparse points coordinates and attributes.

Comparative tests have been performed between original GRASS commands (d.rast d.sites) and these two new commands; the tests evaluated retrieving time in many situations, involving different region size and position.

The results show that for raster map, GRASS alone is still the best choice, whereas for sites point the R-tree indexing method allows substantial time cut. (see tables 1 and 2)

```

Mapset <PERMANENT> in Location <modena>
GRASS 5.0beta6 > d.pg.sites -help

Usage:
d.pg.sites pgtable=name pgdatabase=name [color=name] [size=value]
[type=name]

Parameters:
  pgtable      Name of a postgres table with point data
  pgdatabase   Name of a postgres database containing previous table
  color        Sets the current color to that stated
               options: red,orange,yellow,green,blue,indigo,white,black,
               brown,magenta,aqua,gray,gray
               default: gray
  size         Size, in pixels, in which the icon is to be drawn
               options: 0-1000
               default: 5
  type         Specify the type of the icon
               options: x,diamond,box,+
               default: x

Mapset <PERMANENT> in Location <modena>
GRASS 5.0beta6 > d.pg.rast -help

Usage:
d.pg.rast pgtable=name pgdatabase=name

Parameters:
  pgtable      Name of a postgres table with raster data
  pgdatabase   Name of a postgres database containing previous table

Mapset <PERMANENT> in Location <modena>
GRASS 5.0beta6 >

```

Figure 5 The d.pg.sites and d.pg.rast commands syntax

Test	Rectangular region				Extraction time			n° points
	N	S	W	E	Postgres (sec)	GRASS (sec)	$\frac{T_{GRASS}}{T_{Postgres}}$	
A1	36.1167	34.5167	16.8083	18.4083	0.14	0.01	0.071429	368
A2	42.5167	40.9167	12.0083	13.6083	0.13	0.01	0.076923	368
A3	48.9167	47.3167	4.0833	5.6083	0.15	0.01	0.066667	368
B1	37.7167	34.5167	16.8083	20.0083	0.22	0.01	0.045455	1472
B2	44.1167	40.9167	12.0083	15.2083	0.22	0.01	0.045455	1472
B3	48.9167	45.7167	4.0833	7.2083	0.21	0.01	0.047619	1472
C1	48.9167	44.1167	5.6083	10.4083	0.35	0.01	0.028571	3312
C2	44.1167	39.3167	10.4083	15.2083	0.34	0.01	0.029412	3312
C3	39.3167	34.5167	15.2083	20.0083	0.34	0.01	0.029412	3312
D1	44.1167	37.7167	8.8083	15.2083	0.51	0.01	0.019608	5888
E1	44.1167	36.1167	8.8083	16.8083	0.72	0.01	0.013889	9200
F1	45.7167	36.1167	7.2083	16.8083	1.00	0.01	0.01	13248
G1	48.9167	34.0833	4.0833	20.9167	2.50	0.01	0.004	36134

Table 1: Performance results on raster data (Postgres versus GRASS)

Test °	Rectangular Region				Extraction time			n° points
	N	S	W	E	Postgres (sec)	GRASS (sec)	$\frac{T_{GRASS}}{T_{Postgres}}$	
A1	37.716	36.116	13.608	15.208	2.3	15.9	6.91	15767
A2	42.516	40.916	12.008	13.608	2.1	15.7	7.48	8914
A3	45.716	44.116	5.608	7.208	0.3	15.5	51.67	390
A4	45.716	44.116	10.408	12.008	4.7	15.8	3.36	24032
A5	42.516	40.916	13.608	15.208	4.4	16.1	3.66	29704
A6	40.916	39.316	8.809	10.408	0.4	14.9	37.25	1804
B1	39.316	36.116	13.608	16.908	3.2	15.5	4.84	25027
B2	44.116	40.916	12.008	15.208	6.9	16.4	2.38	58512
B3	47.316	44.116	5.608	8.808	3.8	15.3	4.03	16857
B4	40.916	37.716	7.208	10.408	0.8	15	18.75	3918
C1	40.916	36.116	12.008	16.808	7.2	16.4	2.28	63367
C2	47.316	42.516	5.608	10.408	5.3	15.9	3	40111
C3	47.316	42.516	10.408	15.208	8.5	16.6	1.96	72043
C4	42.516	37.716	7.208	12.008	0.8	14.9	18.63	5014
D1	47.316	40.916	5.608	12.008	9.9	17	1.72	81650
D2	42.516	36.116	12.008	18.408	13.6	18	1.32	127237
D3	42.516	36.116	5.608	12.008	0.7	15	21.43	5020
D4	47.316	40.916	10.408	16.908	14	18.2	1.3	125025
E1	47.316	39.316	7.208	15.208	17.4	18.9	1.09	161466
F1	47.316	37.716	7.208	16.808	22	20.5	0.93	213602
G1	48.916	34.083	4.083	20.916	34	21.6	0.648	244708

Table 2: Performance results on sites point data (Postgres versus GRASS)

The inner solution

The R-tree indexing method available in PostgreSQL is better than a simple sequential access to a flat ascii file, but it is not the best solution: this indexing method give high performances in case of vector data but are less suitable for sites points data.

We are now considering the hypothesis of modify the way in which GRASS stores and gains access to sites point: this can be an alternative to DBMS. We are planning to realise a new GRASS indexing method using a "point quadtree" algorithm.

This kind of indexing method creates a record for every element in the archive, which contains geographical information, attributes and four pointer fields, one for each quadrant (NW,NE,SW,SE); each of them points to another record in the tree, following these rules:

- 1- The first point is called root (R).
- 2- Every other point (N), starting from root is inserted, in one of the four branches (if empty) according to the following criterions:
 - if (M.lat > N.lat) and (M.lon > N.lon), N will be inserted in branch north-east (M->ne)
 - if (M.lat > N.lat) and (M.lon < N.lon), N will be inserted in branch north-west (M->nw)

if (M.lat < N.lat) and (M.lon > N.lon), N will be inserted in branch south-east (M->se)
if (M.lat < N.lat) and (M.lon < N.lon), N will be inserted in branch south-west (M->sw)

If the resulting branch is already filled by another node, this node becomes the new starting sub-point for the original record N.

Every time a new point is inserted in the tree, a region (or a sub region) is divided into four quadrants.

An example of the procedure is shown in (Figure 6):

Point 1: This is the first point to be inserted, so it must be taken as the root of the tree. It also divides the original region (see Figure 7) in four parts (A, B, C, D) using longitude 12°20' and latitude 44°50' as coordinates for the division.

Point	latitude	longitude
1	44°50'	12°20'
2	43°60'	13°60'
3	42°50'	11°30'
4	41°90'	16°10'

Figure 6 The point of the example

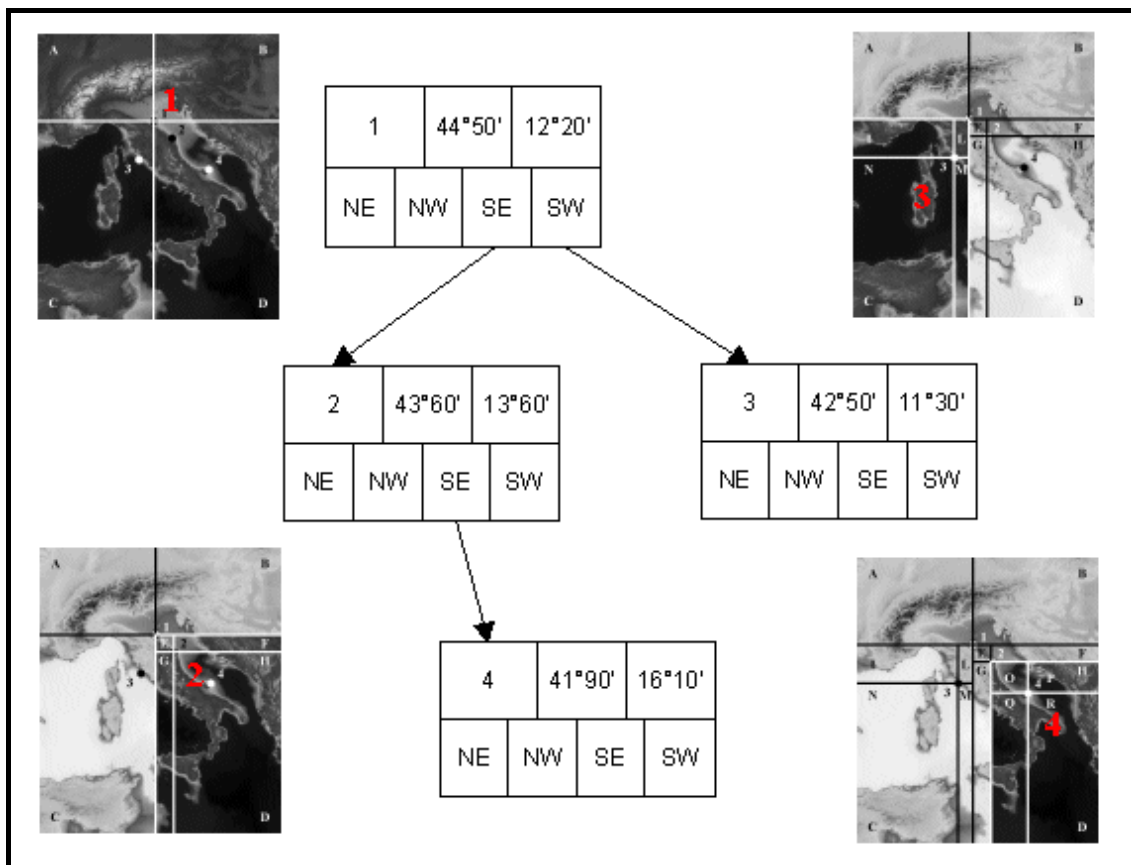


Figure 7 A point quadtree creation

Point 2: it is placed south-east of Point 1, so a link to point 2 must be placed in the SE pointer of Point 1. Point 2 divides region D in four parts (E, F, G, H)

Point 3: it is placed south-west of Point 1, the link will be created in the SW pointer of Point 1. Point 3 divides region C in four parts (I, L, M, N)

Point 4: it is located south-east of Point 1, but SE pointer of point one is already occupied by Point 2. Point 4 is placed south-east of Point 2 and the SE pointer of Point 2 is free: the link must be created here. Region H is divided in four parts (O, P, Q, R).

Conclusions

This work doesn't pretend to give a unique solution to the problem of managing GRASS' archive data, but it's more an analysis of all various possible methods. The best choice is strictly dependent from the particular context of utilisation.

If many softwares have to gain access to the same data for reading/writing operations, the best solution is the use of a DBMS to manage these data, to eliminate concurrency and integrity problems and to conform accessing methods. In any case for site points data the use of a DBMS with some spatial indexing method makes the retrieval of each single point (and then reduces the global processing time) faster. An alternative to this latter point, especially in case that only one user a time accesses the data, is the possibility to implement a suitable indexing method directly in GRASS. In this way we are now working.

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URUGEOIDE 2000 PROJECT A GRAVIMETRIC GEOID FOR URUGUAY

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ABSTRACT

The data, methodology and results of the UruGeoide 2000 project, aiming a determination of an accurate gravimetric geoidal model for Uruguay (latitude -30° to -35° and longitude $301,5^\circ$ to 307°), are presented on this paper.

Data types used were a high degree geopotential model, namely the EGM96 spherical harmonic coefficient set; a set of 15860 gravity stations and a 2 km x 2 km digital terrain model for the uruguayan terrestrial data, completed with bathymetric and GTOPO30 model data for the rest of the area.

The remove-restore technique (Sideris, 1997) was successfully applied, combining the three mentioned datasets. The external masses to the geoid was consider it with the second Helmert condensation method, taking in account the indirect effect. The gravimetric contribution was calculated using residual free-air anomalies in the spherical Stokes formula, evaluated with 1D-FFT as suggested by (Haagmans *et al.*, 1993).

The geoid model was referred both to WGS84 (G873) geodetic system and Montevideo 1948.0 vertical system and has been tested using a set of GPS/leveling geoid undulations, obtaining a 0.25 m and 2 ppm absolute and relative precision respectively. After a bias and tilt were estimated, a standard error of 0.13 m was found. Results are presented in form of graphics and contour maps.

1. INTRODUCTION

The GPS (Global Positioning System) is today one of the most surveying tools used on geodesy and topography. Its offers a simple method to get good precision for point position with low costs of execution. The tridimensional cartesian coordinates obtained for a station are easily transformed in geodetic (φ , λ , h) or planes ones (E , N , h), however the ellipsoidal altitude h , are physical meaningless, being refereed just to an arbitrary ellipsoid surface. The reduction of the ellipsoidal altitude to a orthometric one, referred to a specific geoid or zero local level, require of the accurate knowledge of the separation of the two surfaces, ellipsoid and geoid, namely geoidal undulation, N . On a global basis, the modern geopotential models of high degree, offer geoidal undulations with some ± 1 m error, but when a better precision is needed, its must be used gravimetric and topographic data of high resolution.

In Uruguay the SGM (Military Geographical Survey), as the official cartographic institution of the country, started the systematic use of the GPS observation techniques for mapping purposes in 1993. Since that time it was recognized the needed of an accurate transformation of the ellipsoidal altitudes to the local defined Montevideo Vertical Datum (1948.0). In 1994 a joint project SGM/USP (São Paulo University, Brazil), allows to calculate a geoidal model for Uruguay. The model was based on the geopotential model GEMT2 up to degree 36, combined with gravimetric anomalies. Terrain effects were not took in account it due to lack of a detailed DTM. Some tests of the model showed systematic errors of up to 2 m on the geoidal undulation calculated. New geopotential models (Lemoine *et al.*, 1998), as wee as gravimetric and topographic data (Subiza, 1998a) resulted in a new project, aiming a high-resolution geoidal model for Uruguay and named UruGeoide 2000. The main characteristics of the project are:

Its *objective* is calculate a high resolution geoidal model for Uruguay, located at latitude -30° to -35° and longitude 301.5° to 307° , with the main application being the transformation between ellipsoidal and orthometric altitudes.

The *calculation method* of the geoidal model is the gravimetric one, through the spherical Stokes formula, with rigorous kernel and evaluated via the one dimensional Fast Fourier Transform (1D-FFT).

The *main technique* used is the remove-restore technique (Sideris, 1997), combining a high degree of a geopotential model (EGM 96 up to 360 degree), gravimetric anomalies and a digital terrain model. In order to consider the external masses to the geoid, the second condensation method is applied; taking in account the indirect effect associated (Heiskanen e Moritz, 1985).

2. THEORETICAL CONCEPTS

The problem of obtain geoidal undulations using gravimetric anomalies was solved by Georg Stokes in 1849, through the well known formula, named after him:

$$N = \frac{R}{4\pi\gamma} \int \Delta g S(\psi) dS, \quad (1)$$

being N the geoidal undulation, R a mean terrestrial radius, γ the theoretical gravity value, Δg the gravity anomaly and $S(\psi)$, the so-called Stokes' function, depending only on the spherical distance between the calculated point and the anomaly being integrated. The integral is carried out onto the whole geoidal surface.

Integrals as (1), are called convolution integrals, being easily evaluated in the frequency domain via Fast Fourier Transform methods, allowing to process great amount of data, a must for regional geoids. In order to apply the Stokes' formula, same modifications has to be made. Thus the integral is replace by a discrete summation of discrete data, where the spatial density information is usually not better than $5' \times 5'$. Besides, the integral is apply over a restricted area and not over the whole geoidal surface. The remove-restore technique solve these problems, being described as follows:

In the remove step, are obtained the residual gravimetric anomalies using:

$$\Delta g^{res} = \Delta g_{AL} - \Delta g_{MG} - \Delta g_H. \quad (2)$$

being Δg_{AL} the Free-air gravimetric anomalies (corrected by the atmospheric effect) Δg_{MG} is the geopotential model contribution and Δg_H is the topographic contribution (direct effect), accounting for the Helmert's second condensation method. The formulae for the mentioned contribution are:

$$\Delta g_{MG} = \bar{g} \sum_{n=2}^{n_{max}} (n-1) \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda_p + \bar{S}_{nm} \sin m\lambda_p) \bar{P}_{nm} \sin \varphi_p \quad (3)$$

$$\Delta g_H = -c = \frac{1}{2} G \rho \int_E \frac{H^2 - H_P^2}{l^3} dx dy - H_P G \rho \int_E \frac{H - H_P}{l^3} dx dy \quad (4)$$

where $\bar{g}$ is a mean terrestrial gravity value; $\bar{C}$, $\bar{S}$ and $\bar{P}$ are the normalized coefficient of the geopotential model used and the normalized Legendre associated functions of first kind respectively; φ_P and λ_P , are the point geodetic coordinates; n and m are the geopotential model degree and order; c is the so-called *classical terrain correction*; G stands for the universal gravitational constant; ρ is the density of the terrestrial masses; H is the station orthometric altitude; l , is the distance between the calculation point and the integrated one and finally E is the restricted area of the integral.

In the second step the residual geoidal undulation are calculated, applying the Stokes formula in the following convolution form:

Considering that in order to calculate the undulation on a parallel φ_l , using data of another parallel φ_j , ψ changes only $\lambda_k - \lambda_i$, and Δg^{res} changes with λ_j , the Stokes' formula can be expressed as:

$$N_2(\varphi_l, \lambda_k) = \frac{R}{4\pi\gamma} \sum_{j=0}^{N-1} \left[\sum_{i=0}^{M-1} \Delta g^{res}(\varphi_j, \lambda_i) S(\varphi_l - \varphi_j, \lambda_k - \lambda_i) \cos \varphi_j \Delta \lambda \right] \Delta \varphi \quad (5)$$

with $\varphi_l = \varphi_1, \varphi_2, \dots, \varphi_n$. Using the addition theorem of the Fourier Transform, eq. (5) is evaluated by FFT as:

$$N_2(\varphi_l, \lambda_k) = \frac{R}{4\pi\gamma} F_1^{-1} \left\{ \sum_{j=0}^{N-1} F_1 [\Delta g^{res}(\varphi_j, \lambda_i) \cos \varphi_j] F_1 [S(\varphi_l, \varphi_j, \Delta \lambda)] \right\} \quad (6)$$

In (6), F_1 and F_1^{-1} , are the direct and inverse Fourier transform, given the geoidal undulations for all points located at parallel φ_l . These results are identical to the classical numerical integration.

In the restore step, the geopotential model contribution and the terrain indirect effect are added to the residual geoidal undulations, obtaining the final geoidal undulations according to the formula:

$$N = N_1 + N_2 + N_3, \quad (7)$$

N_1 is given by the geopotential model, in this case EGM96 up to degree and order of 360, calculated with the general formula:

$$N_1 = R \sum_{n=2}^{n_{max}} \sum_{m=0}^n (\bar{C}_{nm} \cos m \lambda_P + \bar{S}_{nm} \sin m \lambda_P) \bar{P}_{nm} \sin \varphi_P \quad (8)$$

In (7), N_2 was obtained with (6) and N_3 is the indirect effect of the Helmert method of condensation, calculated as:

$$N_3 = -\frac{\pi G \rho}{\gamma} H_P^2 - \frac{G \rho}{6\gamma} \int_E \frac{H^3 - H_P^3}{l^3} dx dy. \quad (9)$$

In order to evaluate the geoidal model, GPS observations over benchmarks were used. The geoidal undulation at that points (N_{GPS}), related both ellipsoidal (h) and orthometric altitudes (H), as:

$$N_{GPS} = h - H \quad (10)$$

If a geoidal undulation calculated from a geoidal model (N_{GEOI}), has the same geodetic reference system as the one obtained through (10), it is possible to compare both in an absolute form, as:

$$DN = N_{GPS} - N_{GEOI} = h - H - N_{GEOI}. \quad (11)$$

A better evaluation is made in a relative form, comparing the height differences at two stations:

$$\Delta DN = \Delta N_{GPS} - \Delta N_{GEOI} = (h_2 - h_1) - (H_2 - H_1) - (N_{GEOI}^2 - N_{GEOI}^1), \quad (12)$$

and relating the calculated value to the distance between station in order to get the relative error in one km.

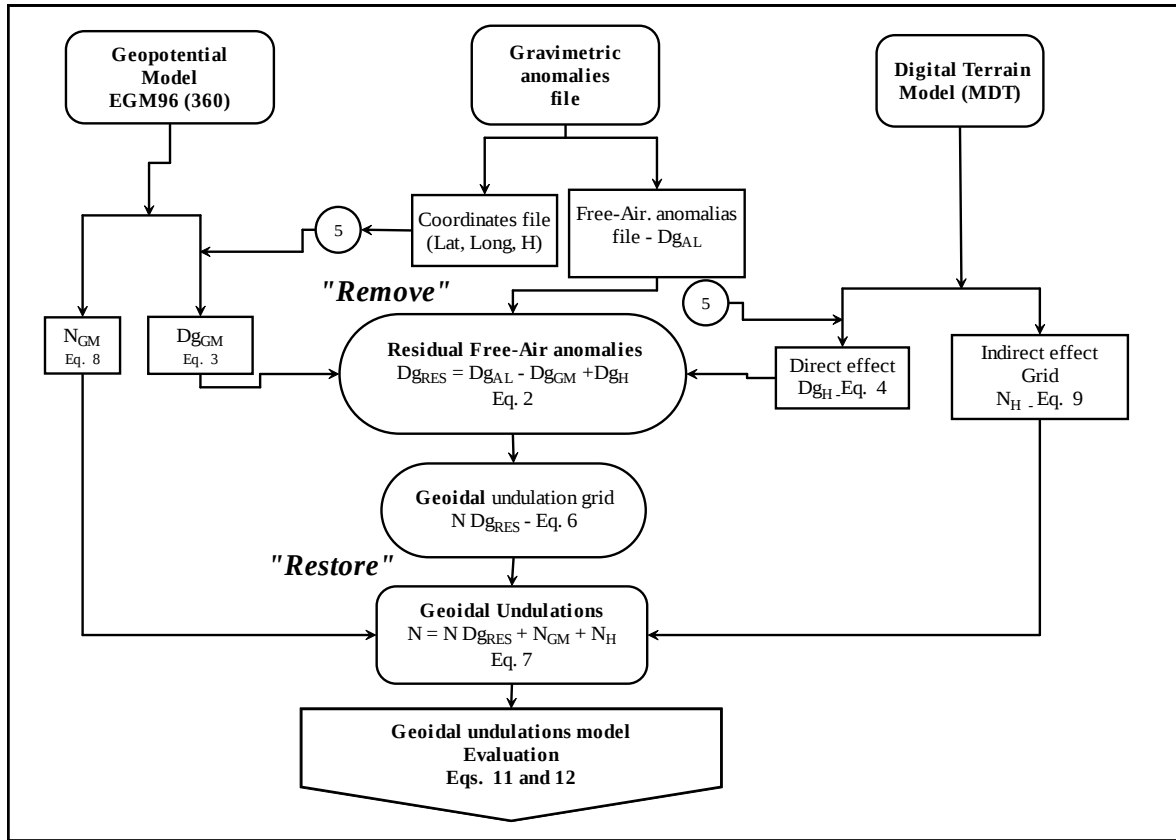


Figure 1. Procedure to calculate a gravimetric geoidal model via FFT and using the remove-restore technique.

3. DATASETS USED

The geodetic reference system adopted was the WGS84 (G873) (NIMA, 1999), thus the main purpose of the geoidal model i.e. the transformation between ellipsoidal altitudes and orthometric one, is well achieved. The new geodetic reference system adopted in Uruguay, based on SIRGAS 1995.4 is compatible to a cartographic level with WGS84. The gravimetric and topographic datasets were previously transformed from the old local reference system to the new one. The vertical reference system in Uruguay, was defined in 1948.0, having a systematic bias not well established yet of about +0,5 m, when compared to others regional/global vertical datum (Subiza, 1998b). The Fig. 2 shows the geographical data and type distribution used.

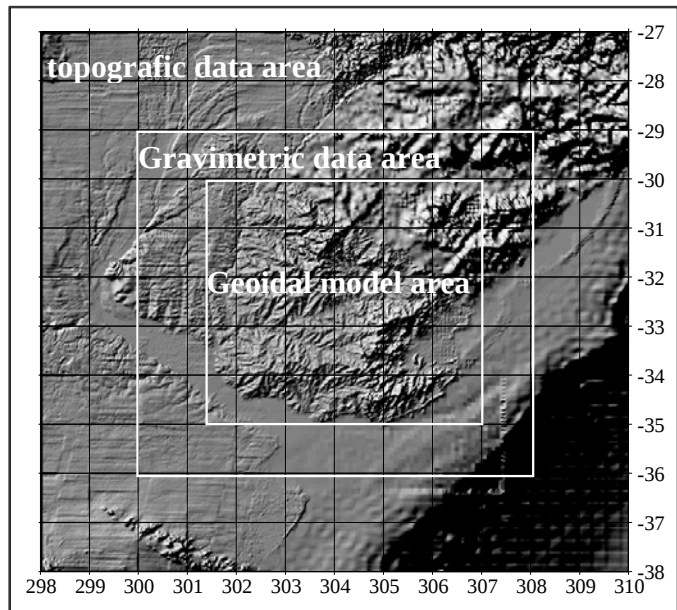


Figure 2. Geographical data and type distribution

The terrestrial gravity data density in Uruguay is better than 5' x 5', but at the remained area can reach 10' x 10' or worse. The gravimetric data were collected from the SGM databank, BGI (Bureau Gravimetrique Internationale), SCGGSA (South American Sub-Commission for the Geoid and Gravimetry) and IfE-PFA3 (Hanover University). The marine area was completed with the global marine Free-Air anomalies model GMGA9706 (Hwang *et al.*, 1997). A total of 15860 gravity stations were used, being 5873 marine and 9987 marine ones.

The Fig. 3 shows the geographical distribution of the gravity data and Table 1 shows the correspondent statistics.

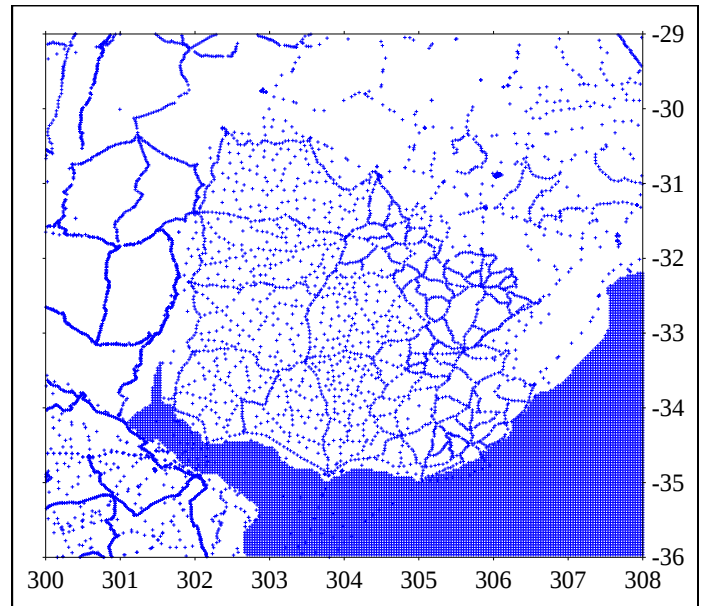


Figure 3. Free-Air gravity anomalies

Table 1. Statistic values of the Free-Air anomalies file.

15.860 stations	Maximum	Minimum	Mean	St. Deviation	Range
Latitude	-29°	-36°	34.97°	-	7°
Longitude	308°	300°	304,71°	-	8°
Altitude (m)	692,9	-37,5	35,46	75,46	730,4
Free-Air (mGal)	100,639	-35,788	13,317	16,246	136,427

The topographic data were compiled from the SGM (46014 terrestrial and marine altimetry datapoints, resolution of 2 km x 2 km), USP (41881 terrestrial and marine altimetry datapoints, resolution of 2.5' x 2.5'), from the global topographic model GTOPO30 (UGSC, 1997) (190351 datapoints, resolution of 1' x 1') and finally were added the altitudes from the gravity stations used. Some missing bathymetric values were detected at SE corner of the total area. Two grids with 1.5' and 3' regular resolution were calculated with the final topographic of 263376 datapoints. Table 2 and 3 shows the statistics from these files.

Table 2. Statistics values from final combination of topographic files

Source	Nr.	Maximum (m)	Maximum (m)	Mean (m)	St. Deviation	Resolution
GTOPO30	190351	732	1	68,15	58,34	1'
USP	21228	1340	-2459	219,96	359,85	2,5'
SGM	46014	425	-2300	104,13	85,03	2 km
Gravity st.	5783	629,9	-37,5	97,01	98,01	5'~10'
DTM	263376	1340	-2459	87,52	126,37	≈ 1,3'

Table 3. Statistics of the derived files

File	Maximum (m)	Minimum (m)	Mean (m)	St. Deviation	Resolution	Resolution (km)	Raw x Column
DTM15	1340,0	-2390,9	- 0,4	421	1,5'	≈ 2,8	441 x 480
DTM30	1340,6	-2258,8	+0,4	420	3'	≈ 5,5	221 x 240

4. RESULTS

In order to apply the remove-restore technique, the geopotential model EGM96 (up to 360 degree and order) and the topographic direct effect contributions for each gravity station observed were calculated.

Using eq. (2) the residual Free-Air gravimetry anomalies were obtained, being show on Fig. 4 and its statistics on Table 4.

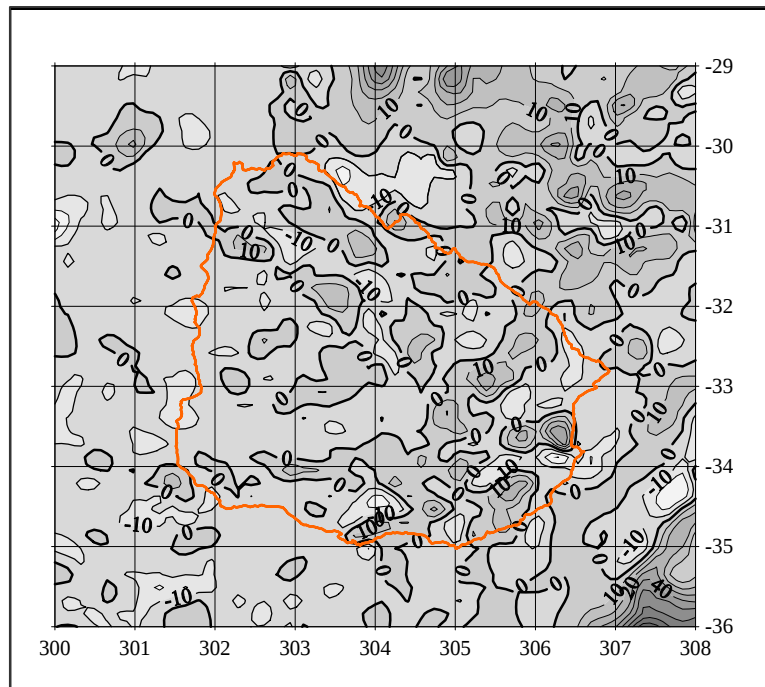


Figure 4. Residual Free-Air anomalies (in mGal)

Table 4. Residual Free-Air gravity anomalies.

15860 Stations	Latitude (°)	Longitude (°)	Altitude (m)	Free-Air anomaly	Terrain effect	EGM96 Free-air	Residual anomaly
Max.	-29	308	692,9	100,639	103,080	51,066	96,737
Min.	-36	300	-37,5	-35,788	0	-27,664	-59,139
Mean	-34,07	304,7	35,5	13,318	3,764	13,063	4,019
St. Deviation	-	-	75,7	16,247	9,483	13,901	12,577
Range	7	8	730,4	136,427	103,080	78,730	155,876

Using the Free-Air anomalies with Stokes' formula in the one-dimensional convolution form of eq. (6), the residuals geoidal undulations are obtained.

The minimum and maximum values of this component were 0.8 and 3 m respectively, being shown on Fig. 5

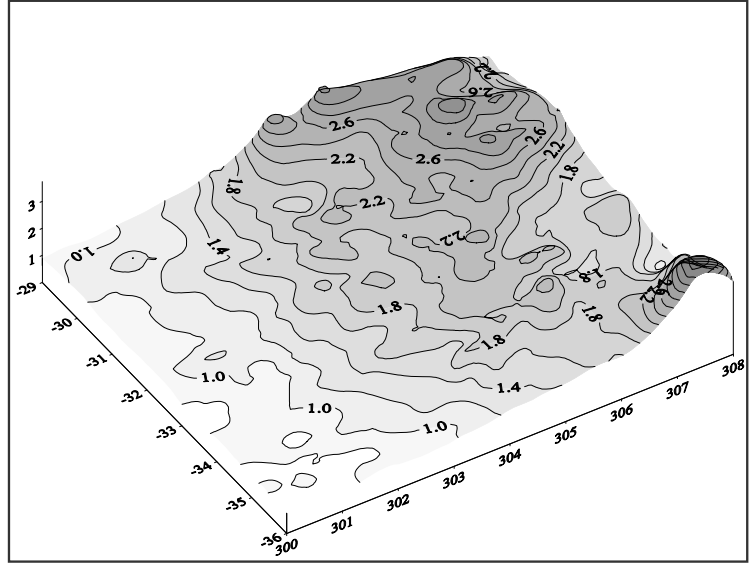


Figure 5. Residual geoidal undulations (in m)

Finally the reintroduction of the wavelengths corresponding to the geopotential model and the indirect effect of the Helmert second condensation method, allows getting the final geoidal undulations. The indirect effect values were sub centimetric ones and thus neglected.

As the process used was FFT, the results are obtained directly in gridded form, ready for practical use. Fig 6 and 7 show the geopotential model EGM96 (up to degree and order of 360) contribution and the final geoidal model.

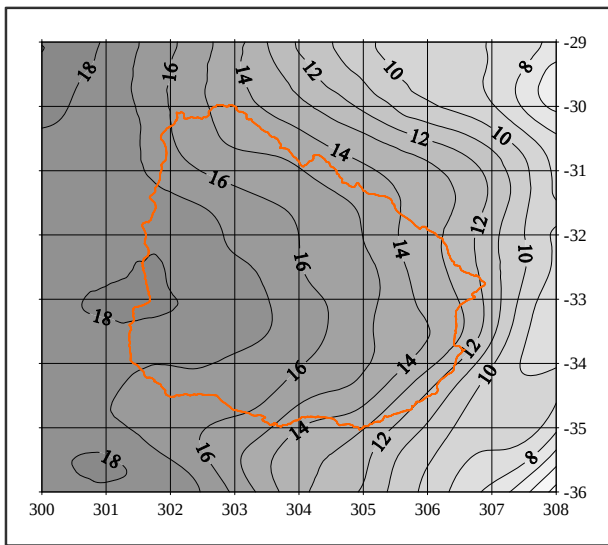


Figure 6. EGM96 (360) geoidal undulations

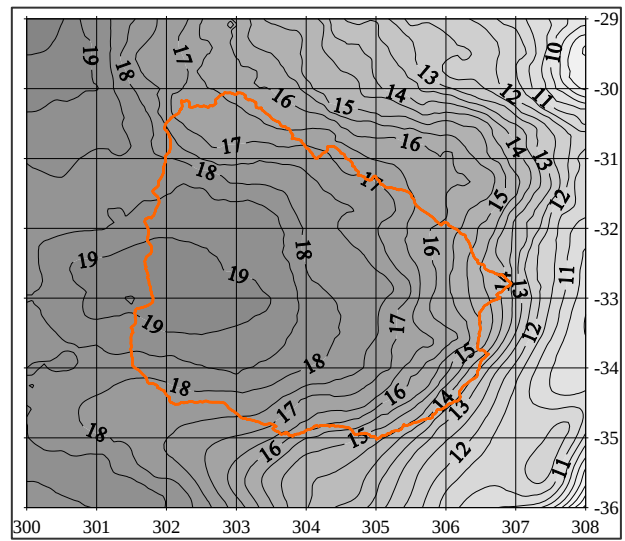


Figure 7. Final geoidal undulations

5. EVALUATION

A systematic bias between the geoid model and the vertical datum for Uruguay and Argentina, ranging from -1.05 to -1.95 m was found. This bias is caused mainly by different local vertical definitions, systematic geopotential coefficient errors of long wavelengths and error at the national leveling networks.

The following Table presents the results of the evaluation made after the systematic biases was removed, using two datasets of relative and absolute of GPS observations over bench marks in Uruguay and one dataset of absolute observations for the Argentinean area calculated. At Uruguay were used 24 relative and 22 absolute GPS stations, meanwhile in Argentina were used a total of 29 absolute ones. The values presented at the Table are for the relative data: mean difference between GPS geoidal undulation and geoidal model one in m, mean value in ppm and its standard deviation. For absolute data the values are maximum and minimum difference between GPS geoidal undulation and geoidal model one in m, range of the difference and standard deviation of the zero mean value.

		UruGeoide2000
Relative evaluation Uruguay		Mean Diff.
		0.26 m
		ppm
		1.99
		St. Deviation
		2.24
Absolute evaluation	Uruguay	máximo
		0.31 m
		mínimo
		-0.46 m
	Argentina	Range
		0.77 m
		St. Deviation
		0.25
		máximo
		0.21 m
		mínimo
		-0.14 m
		Variación
		0.35 m
		St. Deviation
		0.10

Table 5.

Table 5 shows that the UruGeoide 2000 has in Uruguay, a relative precision of 2 ppm, with an absolute error of 0.25 m at one sigma level of confidence. For the Argentinean dataset the results are better, probably because the GPS observations were made exclusively at the 1st. Order Leveling Network. The model at that region has an error of 0.1 m at one sigma level of confidence.

A further evaluation included not only a bias but also a tilt in order to account for the differences between the gravimetric geoid and GPS/levelling undulations. It was used 8 GPS/Levelling stations, covering the Uruguay area and the following transformation function (Denker *et al.*, 1997 and Lyscowicz & Forsberg, 1997):

$$N = N_{GEOD} + \cos \varphi \cos \lambda X + \cos \varphi \sin \lambda Y + \sin \varphi Z \quad (13)$$

where X, Y, and Z are the translation parameters between the implied GPS and the geoidal system, in a cartesian mode.

The two following tables 6 and 7 show the calculated parameters, the correspondent bias and tilt for the UruGeoide 2000 model and the results of a similar than Table 5 absolute evaluation. A clearly 50 % improvement could be seen in the range and in standard deviation of the difference (Table 7).

	Parameters	Bias&tilt
Uruguay	X= +0.2288 m	-1.952 m
	Y= +5.6881 m	N-S= +0.108"
	Z= -3.5652 m	E-W=+0.178"

Table 6

UruGeoide 2000		
Uruguay	maximum	0.09 m
	minimum	-0.28 m
	Range	0.37 m
	St. Deviation	0.131

Table 7

An interpolation program, using the *inverse distance* method within a cell (Franke, 1982) and these cartesian parameters mentioned, was implemented for practical purposes for the SGM's field GPS surveys.

6. CONCLUSIONS

The main purpose of this paper was the presentation of the UruGeoide 2000 project. The remove-restore technique was successfully applied, with main application through the FFT, an approximation not used before for Uruguay. In the calculation process was used a combination of the EGM96 geopotential model, gravimetric data and a 2 x 2 km resolution DTM.

As a subproduct of the research, the Uruguayan Gravimetric Database was transformed from a text form to a relational database with better possibilities for control and search information.

The UruGeoide 2000 model has a relative precision of 2 ppm and an absolute one of 0,25. Using the modeling parameters of Table 6, a 0,13 m standard deviation error could be expected. The results represent an improvement of one order, over the former model and geopotential models alone calculations.

In order to achieve future improvements for geoidal models in Uruguay, more attention must be paid to the DTM and areas lack of high-resolution gravimetric data. The CHAMP and GRACE satellite mission could be in great help in these matters. New methods/procedures as Fast collocation technique (FC) in geoidal calculations, will be also tested.

The geoidal model UruGeoide 2000 is available on demand at IGeS and SGM, Uruguay.

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Geoid Computations in Estonia

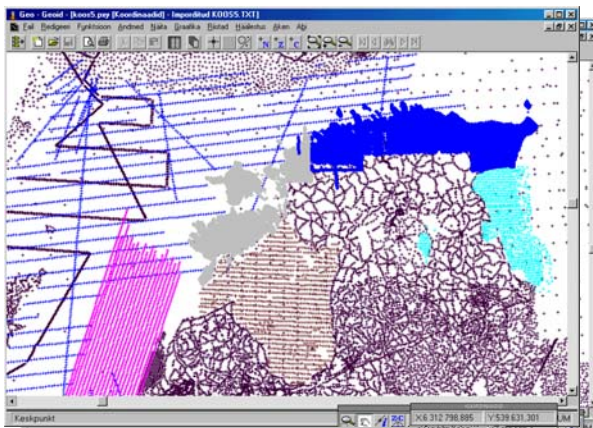
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Abstract

Calculations of the geoid surface in relation to the ellipsoid GRS-80 were launched in Estonia in the 1990's parallel to the development of the geoid for the Nordic countries. First, data obtained from measurements on 4,200 points in Estonia performed in the 1950's were added to the calculations. In 2000 the database for the geoid calculations was further implemented with gravimetric data on 85,000 points (West and North part of the Estonia) measured by the Estonian Center of Geology during the last 30 years. Further, data obtained from aerogravimetric measurements performed on the Baltic sea in 1999 and at the bottom of the Gulf of Riga in 1968 were also added to the calculations. This article analyzes the impact of the supplementary data on the geoid deformation in Estonia, particularly in the region of the West Estonian archipelago. The scarcity of data from these regions has been the primary obstacle to the preparation of the precise geoid. The gravimetric geoid is improved using data from the new high-precision GPS network as well as the levelling network.

The Gravity Data Used In Geoid Calculation

Until 2000 calculations of the gravimetric geoid were based on the approximately 4,200 gravity points measured by V. Maasik in 1949-1958 as well as 151 second-order or third-order points measured after the 1970s (Sildvee, 1997). The accuracy of V. Maasik's data was about 0.5 mgal and that of the network data about 0.05 mgal. This summer additional data was obtained from a number of large water areas, such as the Gulf of Riga, the three largest lakes in Estonia, the Baltic Sea and the Gulf of Finland. These additional data signalled a substantial reduction in some important "blank sheets", which allowed to start with the final computations of the accurate geoid. The new data are presented in Figure 2b. One problem that has still remained is the lack of gravimetric data from the eastern regions of Estonia.



(a)

(b)

Figure 2. (a) Gravimetric Data In 1999, (b) Gravimetric Data In 2000.

A Comparison Of The Aerogravimetric Measurements On The Baltic Sea With The Previous Measurements In The Gulf Of Riga

During the aerogravimetric measurements in 1999, data for the Baltic Sea and Finnish Gulf were obtained. During the measurements six points were measured on the Gulf of Riga also. These points were located near the gravimetric points

that were measured in 1968 at the bottom of the gulf. An overlap occurs near the southern part of the island of Saaremaa (see fig. 3). The accuracy of the previously measured points is estimated to be 0.4 mgal. If their values are interpolated onto the aerogravimetric points it appears that the differences remain within the range of -1.8 and $+0.5$ mgal. The previously measured points are computed into the IGSN71 system using the constant 14.0. This confirms the validity of the estimated precision of 2 mgal of the aerogravimetric measurements.

Table 3. Differences of the aerogravimetric and sea measurements.

Pno.	Difference in mgal		
60001	-1,8	60009	+0,5
60003	-1,0	60011	-0,5
60005	-0,6	60013	-1,3
60007	+0,3		

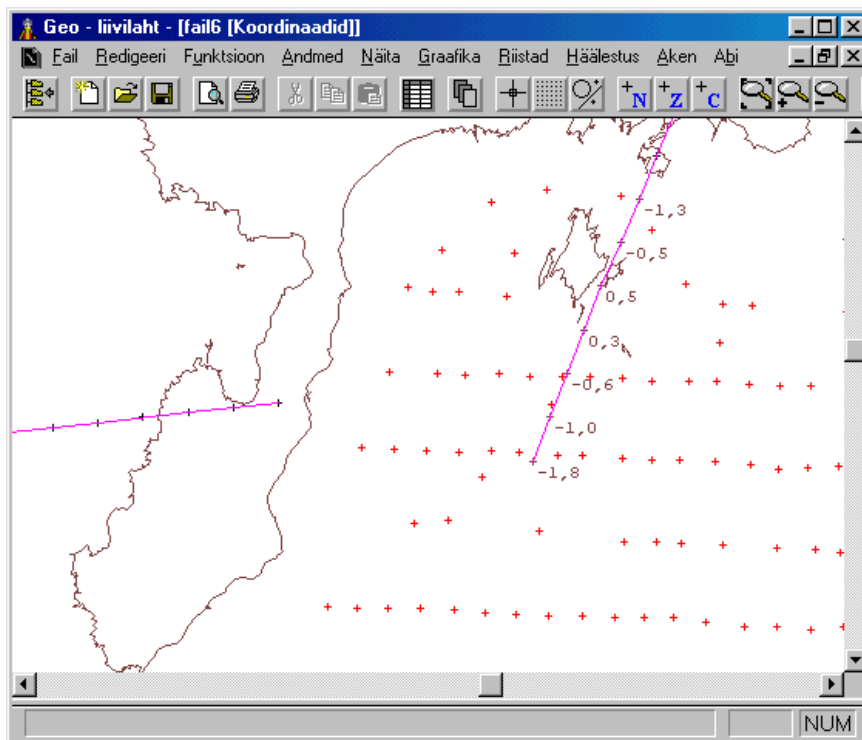


Figure 3. The continuous line corresponds to aerogravimetric points, single red points correspond to those measured in 1968.

Calculation Of The Gravimetric Geoid

The software package GRAVSOF (Tscherning, Forsberg and Knudsen, 1992) was used for gravimetric geoid computations. Point number, northing, easting, normal height, gravity anomaly and weights of the measured data served as initial data in the computation. It was not necessary to take into account the terrain correction in the Estonian area, since the preliminary test manifested its value to be less than the data accuracy. The highest point in Estonia is only 318 meters above the sea level, the mean height being less than 120 m.

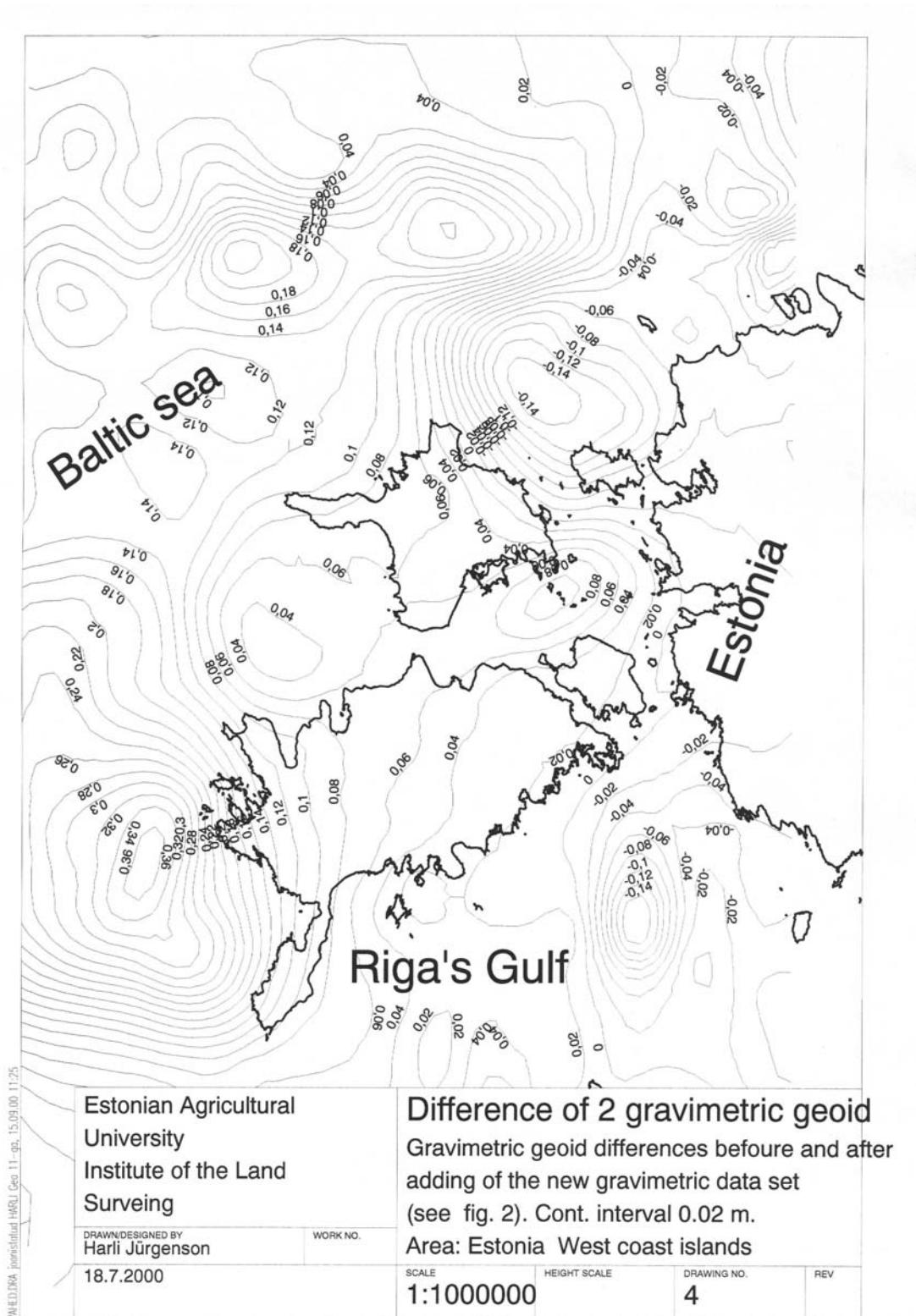
Four stages were used in the computations of the gravimetric geoid:

1. Interpolation of gravity anomalies from the global spherical-harmonious model EGM96 onto the points used in the geoid computations.
2. Subtraction of the gravity anomalies obtained from the EGM96 model from the actual gravity anomalies of the points included in the computations. The result is residual gravity anomalies.
3. Gridding these residuals at an interval of 0.025 degrees northing and 0.05 degrees easting. The removal of the mean value during the computation. Interpolation method: collocation.
4. Transformation of residual gravity anomalies into residual geoid heights using the Stokes formula (FFT).
5. Re-addition of the heights of the geoid model EGM96 to residual geoid heights.

Until 2000 a problem was posed by the scarcity of data from around the West Estonian archipelago. During this year new data from the Baltic Sea, the Gulf of Riga, the West Estonian archipelago, and North Estonia. Particularly problematic until then was the region of the West Estonian islands. Data from the Baltic Sea were nearly non-existent, those from the inland water bodies scarce. This resulted in a geoid deformation in these regions. Figure 4 presents the differences in the gravimetric geoid before and after the inclusion of new data (see also fig. 2). New models have been subtracted from the old model. The figure 4 shows that the differences were 2-10 cm, on the Gulf of Riga even 10-20 cm. The particularly high density (5 points per 1 km²) of gravimetric data on the islands and North Estonia provides a good opportunity for calculating the detailed geoid.

Such structure of the gravity geoid can provide high resolution and high relative accuracy in the difference of the calculated geoid heights.

Absolute accuracy (about 40 cm in Estonia) is poor at the moment due to long-wavelength errors in the adopted reference. These errors are corrected using GPS levelling points.



The Fitting Of The Gravity Geoid To GPS/Levelling Points

The ellipsoidal and normal heights of the 26 Estonian base network points were used for fitting the gravimetric geoid. The ellipsoid heights were measured during the measurements on the new base network in 1997 and the normal heights were measured mainly in 1998 using the first order levelling (Level DiNi 11). Also used were 5 points of the Latvian base network with a height precision similar to that of the Estonian base network. The data for the Tallinn and Ristna points were taken from the measurements performed within the framework of the “Baltic Sea Level” project in 1993 (Kakkuri, 1995). The locations of the points are given in Figure 5.

The following computations were performed for fitting the gravimetric geoid to GPS/levelling (using GRAVSOF):

1. The computation of geoid heights to the fitting points from the gravimetric geoid model. The interpolation method was bilinear interpolation.
2. The Subtraction of GPS/levelling geoid heights from the gravimetric geoid heights of the points. This results in differences between the gravimetric geoid (GG) and the Baltic Heights System (BHS).
3. The computation of the 0.025×0.05 grid for the Estonian territory and the gridding of geoid differences. Method: collocation using 11 nearest points.
4. The inclusion of geoid differences to the GG model. This results in a fitted geoid.
5. For checking purposes, the geoid heights of the fitting points are computed from the fitted geoid model.

Differences Between The Gravimetric And The Fitted Geoid

The gravimetric geoid computations were performed using 2 different data set:

- 1) Gravimetric data shown in Figure 2a
- 2) Including the new data presented in Figure 2b.

After the fitting in the first case the differences between the GG and the BHS were -50.2 at a maximum and -7.9 cm at a minimum. Thus the amplitude was 40 cm and the standard deviation 9.5 cm (Table 3).

In the second case the difference between the GG and the BHS was -45.7 cm at a maximum and -18.0 cm at a minimum. Thus the amplitude was 29 cm and the standard deviation 6.5 cm (Figure 5, Table 4).

Thus the inclusion of new data resulted in a much smaller difference between the gravimetric geoid and the Baltic Height System. The standard deviation improved by 32%. However, these calculations used data obtained from around Estonia with just a 300-kilometre radius, with no data from additional areas. Involving gravimetric data from further afield resulted in a further improve in the standard deviation by about 50 percent.

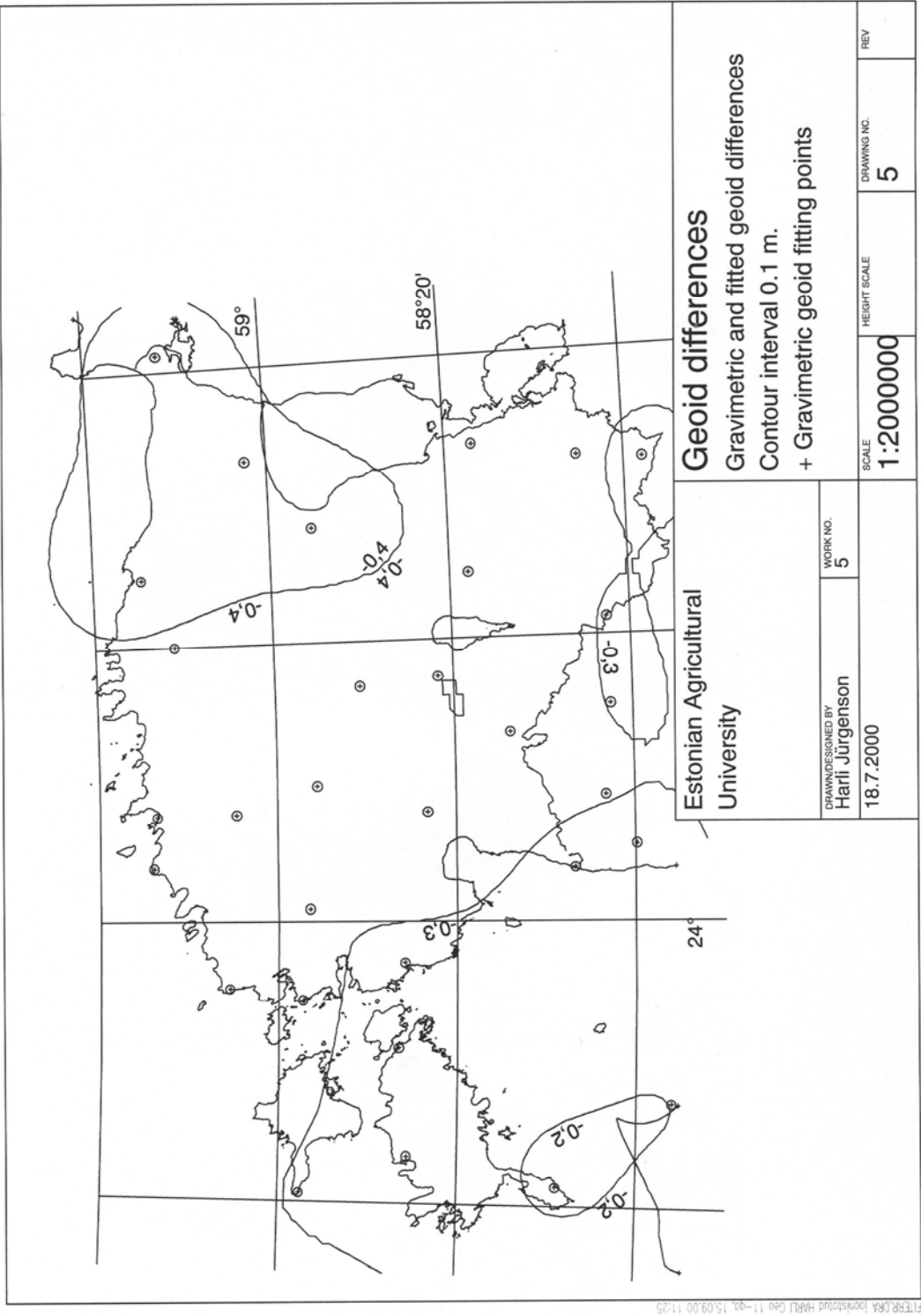


Table 3. Differences Between The Gravimetric And The Fitted Geoid Before The Inclusion Of New Data Sets.

```

--- G E O I P ---
- bilinear interpolation -
- subtraction of interpolated values from pointfile -
number of prediction points:      32
points predicted:      32, skipped points:      0
minimum distance to grid edges for predictions:      2.3 km
statistics:      mean  std.dev.  min  max  un
grid interpolation results:19.719  1.114 16.643 21.026  0
predicted values output  :-0.358  0.095 -0.502 -0.079  0

```

Table 4. Differences Between The Gravimetric And The Fitted Geoid After The Inclusion Of New Data Sets.

```

--- G E O I P ---
- bilinear interpolation -
- subtraction of interpolated values from pointfile -
number of prediction points:      32
statistics:      mean  std.dev.  min  max  un
grid interpolation results: 19.700  1.131 16.624 21.011  0
predicted values output  : -0.335  0.065 -0.457 -0.180  0
-----

```

The Testing Of The Fitted Geoid.

For checking the fitted geoid, the author used two sets of data not included in the fitting computations. The Estonian National Maritime measured ellipsoidal heights on approximately 90 points in 1998 using Ashtech Z-12 receivers (Fig. 6). The Estonian first-order base network points served as initial points. Points located near benchmarks were selected to minimise levelling. These 90 points are situated in the area between mainland Estonia and the islands of Saaremaa and Hiiumaa, where the determination of the geoid surface is the most problematic. From the fitted geoid model, geoid heights for these 90 independent points were computed and the results compared with the measured values. The differences were predominantly less than 5 cm (Fig. 6). Greater errors occurred in the case of some islands (particularly Vormsi), where the levelling network is not precise (taken from observations of water measuring posts). The standard deviation, including the data from the above mentioned area, is 0.04 m. This corroborates the ~5 cm accuracy of the fitted geoid model for the archipelago in West Estonia.

The second test was performed using geodetic heights of the second order network points. Normal heights were taken from the third-order or second-order levelling. Many points were from the south-east part of Estonia. The residuals were predominantly less than 5 cm. The residuals for selected points all over Estonia are given in Table 5. The standard deviation was 1.4 cm. Of course, the errors of the test points values themselves need to be taken into account here (their values may be several cm).

Conclusion And Further Development

It may be said that the accuracy of the fitted geoid is $\sim 0.5\text{???}$. For a further improvement of the geoid surface on the Estonian territory, the following additional tasks have been planned:

1. The levelling of additional base network GPS points, about 15 on the mainland and 10 on the West Estonian islands, or the determining of geodetic heights for precise stationary benchmarks.

2. The checking of the existing data sets of the gravimetric points and the removal of inaccurate data.

3. The request for obtaining gravimetric data from areas east of Estonia.

Since there are no high mountains on the territory of Estonia, the aim is to achieve a fitted geoid with an accuracy of 1-2 cm.

Tabel 5. Residuals from the test of the fitted

c geoid			
No	X m	Y m	Residuals, m
11	6496259	417377	0,055
24	6515027	468873	0,002
27	6478106	483913	0,042
35	6467480	510502	0,01
38	6575870	521859	0,026
54	6443047	559487	0,03
65	6558760	595368	-0,048
68	6593566	599811	0,009
78	6586677	630811	-0,012
94	6563895	657662	0,026
96	6540692	660519	0,015
115	6588328	718709	-0,006
22	6519384	600619	-0,032
72	6513621	603269	-0,017
73	6523803	600680	-0,014
749	6471880	656190	-0,01
751	6470088	657296	0,039
756	6474242	662460	0,012
3144	6503933	616046	0,002
2586	6503618	613547	0,018
2916	6504128	614378	0,02
8929	6505099	613925	0,028
9216	6502780	616625	0,007
9404	6505173	612025	0,022
9641	6501996	614808	0,01
Std.dev.			0.014 m

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A local geoid in Droning Maud Land - The NARE 96/97 Geoid

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Abstract

The main purpose of the present paper is to estimate the orthometric height of the Global Positioning System (GPS)-station at Troll, Dronning Maud Land, Antarctica. Located at approximately 1300 m above sea level Troll has been connected to the Geodetic Infrastructure for Antarctica (GIANT)-network, but its orthometric height remains undetermined. A local geoid has been determined based on gravity-, tidal- and GPS-observations and available gravity anomalies. Information about the ice-thickness has been included in the terrain model. Based on tidal measurements the mean sea level was determined in Jutulgryta, a rift zone in the Fimbulisen ice shelf. Several gravity- and GPS-stations were measured between the two points (Jutulgryta and Troll), a distance of approximately 100 km.

1 Introduction

The work described in this paper was mainly performed on the Norwegian Antarctic Research Expedition (NARE) 1996-97 (Barstad et al., 1997). The Norwegian station Troll, located in the mountain range of Dronning Maud Land, Antarctica, at approx. 72°S ; $2^{\circ} 30'\text{E}$ at an elevation of 1300 m. The main purpose of the work was to determine an orthometric height of the stations reference point. The station, located more than 200 km from the sea, is in an area mapped in the late 1950s using barometric reference heights. With the use of GPS precise ellipsoidal heights have become available, but a more precise orthometric height is still to be fixed.

Even if the coast (the ice shelf edge) is more than 200 km away, open water is accessible in Jutulgryta, a rift zone in the Fimbulisen ice shelf, about 100 km from Troll. In this location a tide gauge was deployed as a part of an oceanographic measurement in the 1991-92 season (Østerhus and Orheim, 1994), and the results showed that a mean sea level was accessible within a distance of approx. 100 km from the station Troll. Between Troll and Jutulgryta the ice sheet is gently sloping down from 1300 m to about 50 m elevation, with a steep ice cliff towards the rift zone. The area has several crevasses. With the help of helicopter transport, several gravity and GPS stations were measured between Troll and Jutulgryta. The stations were all located on glacier ice, except the Troll station on rock, and the Jutulgryta station on thin floating ice on the sea. The glacier in this area is quite slow moving, and as the work was performed within a few days, the glacier movement was not a source of problem.

The field work was done as one of several projects for a geodetic group of four, and the amount of work was limited, both because of the extent of the area making helicopter transport necessary, the changing weather conditions and the total amount of time available during the expedition. Even if it was desirable to carry out more observations, the results have been generated from the available field measurements.

2 Test area and data

The area of interest was located around the Troll station in Dronning Maud Land, Antarctica. The chosen geographical area, spanned by $71.0^{\circ} - 72.25^{\circ}\text{S}$, $0.25^{\circ}\text{W} - 3.25^{\circ}\text{E}$, encloses both the Troll station and Jutulgryta, with access to sea level.

The topography of the area was represented by a digital terrain model with grid-spacing 1 km, generated from the Antarctic Digital Database (ADD) (see <http://www.scar.org/>).

2.1 Gravity data

The available gravity points were mainly located along two tracks between Troll and Jutulgryta, with a few additional measurements around Troll. A La Coste - Romberg gravimeter, measuring relative values only, was used. However, due to logistical capacity no link to an absolute gravity

value was obtained.

Instead, the gravity data was linked to the Kort og Matrikelstyrelsen (KMS) marine free-air gravity data (Andersen and Knudsen, 1998) through the gravity point closest to the tide gauge in Jutulgryta, Δg_{jg} . By removing the normal potential (on the terrain), γ , and linking the observed gravity data, g_{obs} , to the KMS marine free-air gravity data, Δg_{kms} , a set of “free-air” anomalies was obtained. Mathematically,

$$\Delta g^* = g_{obs} - \gamma \quad (1)$$

$$\Delta g_c = \Delta g_{jg} - \Delta g_{kms} \quad (2)$$

$$\Delta g_{fa} = \Delta g^* - \Delta g_c, \quad (3)$$

where Δg_c is the correction level applied to all gravity points, Δg^* is the initial, first approximation, free-air anomaly, and Δg_{fa} is the final free-air anomaly.

The distribution of the used gravity points is shown in Fig. 1, with statistics listed in Table 1.

2.1.1 Bouguer anomalies

The Bouguer anomalies were obtained from the free-air anomalies, Δg_{fa} , by removing the effect of the topography.

$$\Delta g_{ba} = \Delta g_{fa} - 2\pi G \rho h + c_p, \quad (4)$$

where ρ is the density, h is the height of the topography at the calculation point, G is the constant of gravity, and c_p is the terrain correction. The Bouguer anomalies were used in the calculation of the *quasigeoid minus geoid* separation which in Sect. 3 was used to obtain the geoid. See Table 1 for statistics.

2.2 GPS-data

During the NARE 1996-97 the Troll station which is part of the GIANT-network was occupied with an Ashtech Z-XII GPS receiver as part of the SCAR GPS Epoch 97 Campaign. Data from this station were also used as reference data in the local geoid project.

A GPS network consisting of points between Troll and Jutulgryta (Fig. 1) was observed using Ashtech Z-XII GPS receivers. The GPS data were post-processed using the PRISM GPS software (Ashtech, 1994) and the network were adjusted using the FILLNET (Ashtech, 1994) network adjustment software. The adjusted coordinates of the stations are referred to ITRF97 and have a precision ranging from 0.003 m to 0.028 m horizontally and from 0.007 m to 0.060 m vertically.

2.3 Deflection of the vertical

In order to verify the trend of both existing (e.g. Earth Gravity Model (1996) (EGM96) Lemoine et al. (1996)) and the estimated geoid models, local values for deflections of the vertical were

estimated from reciprocal zenith angle measurements and relative GPS measurements.

Reciprocal zenith angle observations between pair of survey marks were carried out simultaneously every 10 minutes over a period of 2 hours using Wild T2 theodolites. The distance between the survey marks were approximately 5 km and halogen lights were used as targets.

The theodolites were then interchanged with geodetic GPS antennas and GPS observations were collected for a period of 2 hours using Ashtech Z-XII GPS receivers. The GPS observations from the pairs of survey marks as well as from the continuously running reference receiver at Troll were post-processed with the PRISM software to estimate GPS vectors between Troll and one of the survey marks as well as between the pairs of survey marks. The “in-house” utility DEFLECT was used to derive zenith angles relative to the ellipsoid from the GPS vectors, “refraction-free” zenith angles relative to the geoid from the theodolite observations and to output values for the deflection of the vertical in one of the survey marks in the direction of the accompanying survey mark. The precision of the estimated deflection of the vertical between pair of survey marks in terms of standard deviation is 1 arcsecond.

2.4 Tidal measurements

An attempt was made during the 1996-97 season to collect tidal data, and an Aanderaa WLR7 pressure tide gauge was deployed through a hole in the thin winter ice in Jutulgryta, and left on the bottom at 400 metres for 38 days. Unfortunately an error in the tape spooling destroyed the data set. The tidal reference in this paper have been based on the series collected by S. Østerhus on NARE 1991-92 (Østerhus and Orheim, 1994). A prediction of the tide have been made using the analysis software of M. Foreman (Foreman, 1977) with an harmonic analysis of the 1991 data, and a prediction for the actual 1997 period. Unfortunately the 1991 data set is only 27 days long, and hence a little too short to get a very good analysis. The precision of the predicted values are estimated to be better than 10 cm. The predicted sea levels have been connected to kinematic GPS height differences between a GPS receiver on the floating ice in Jutulgryta, and a GPS receiver on the (grounded) glacier ice. In this way a height above the calculated mean sea level of Jutulgryta can be found for the point on grounded ice. The kinematic GPS measurements were performed between Jutulgryta and points on grounded ice on three different days, and the resulting heights (above mean sea level) is estimated to be better than 5 cm.

2.5 Ice thickness data

As one of the sub-projects of NARE 1996-97 mission ice thickness measurements were obtained by radio echo sounding (Näslund, 1997). Only a single flight line was made in the area between Troll and Jutulgryta (see 2). A helicopter equipped with radio echo sounding instruments was used to generate the ice thickness profile. The flying height was 30 m. Along this profile the maximum ice thickness was more than 1200 m. The general slope of the ice thickness profile was assumed

to be valid for the entire test area. The precision of the ice thickness is approximately 10% of the measured thickness.

3 Computational algorithm

The method used (Residual Terrain Model (RTM)), in the quasigeoid calculation, utilize the remove-restore (R-R) technique. The main idea with this technique is to remove as much as possible of the gravity signal making it small and easy to grid. Usually, the effect due to the global geopotential model and the various terrain effects are removed and later restored.

The quasigeoid (or geoid by adding the $N - \zeta$ separation) is estimated using Stokes formula with gravity anomalies, Δg_{faye} , as input data. The gravity data, Δg_{fa} , is reduced to the sea level by the remove step of the R-R technique,

$$\Delta g_{red} = \Delta g_{fa} - \Delta g_{egm} - 2\pi G\rho(h - h_{ref}) + c_p, \quad (5)$$

where Δg_{fa} is the free-air anomalies, given as point values. Δg_{egm} is the reference gravity anomalies and $2\pi G\rho(h - h_{ref})$ is the RTM effect.

The classical terrain correction, c_p , is calculated at point $(x, y, z = h)_p$ by (Moritz, 1968)

$$c_p = G\rho \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{h_p}^h \left(\frac{z - h_p}{r^3} \right) dz dy dx, \quad (6)$$

where G is the constant of gravity, ρ is the density of the masses at the point of estimation, r is the slope distance and (x, y, z) is the point of integration with height $z = h$.

Next, the RTM effect is restored by

$$\Delta g_{faye} = \Delta g_{red}^{grid} + 2\pi G\rho(h - h_{ref})_{grid}, \quad (7)$$

which effectively yield gridded Faye anomalies ($\Delta g_{fa} + c_p$) relative to the EGM96. The statistics of the original and reduced gravity data are shown in Table 1.

The residual height anomalies (or the quasigeoid), ζ_{resi} , is calculated using the multi-band spherical Stokes FFT with the gravity data as input. The basic formula is (Forsberg and Sideris, 1993)

$$\zeta_{\Delta g} = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} \sum_{\Delta\phi} \sum_{\Delta\lambda} F^{-1}(F(\Delta g_{faye})F(S(\psi))), \quad (8)$$

where F and F^{-1} are the Fourier and the inverse Fourier transform, respectively. The gravity anomaly is given as Δg_{faye} , and $S(\psi)$ is the Stokes' function. Due to the limited number of gravity stations, indicating an approximate geoid, it is sufficient to use the Faye anomalies in the Molodensky formula (Eq. (8)).

As described in Forsberg and Sideris (1993) the multi-band spherical Stokes FFT method uses several reference latitudes in the computational area. The *exact* reduced height anomaly is found

at the central latitudes per band. Linear weighting of 2 overlapping bands is applied to determine the residual height anomaly between the reference latitudes.

By extending the Digital Terrain Model (DTM) with zeros in all directions by 50% (100% zero-padding) the effects of cyclic convolution in the FFT computation was avoided.

The height anomalies (or the quasigeoid) are obtained after applying the restore step,

$$\zeta = \zeta_{egm} + \zeta_{\Delta g} \quad (9)$$

where $\zeta_{\Delta g}$ is estimated from Eq. (8) and ζ_{egm} is the reference height anomaly extracted from the EGM96.

The resulting geoid, N , was obtained by adding the *geoid minus quasigeoid* separation.

4 Results

A local gravimetric geoid (NARE) was calculated based on the scarce distribution of gravity data, a standard density and an assumed reference gravity value. By varying the density of the topography and by changing the gravity reference value, several different additional geoids were created. The different geoids were used as a source for the estimation of the different std. dev. included in Eq. (11).

Since a fix of the measured gravity data to an absolute level was not achieved, they were mathematically connected to the KMS marine free-air gravity data at Jutulgryta. The KMS marine free-air gravity data was chosen instead of the EGM96, due to the assumption that the first represented a better fit to the correct gravity value. The two data sets differ by approximately 1 mgal at Jutulgryta. Due to the lack of an absolute reference level additional geoid estimates were calculated by changing the absolute reference level by ± 5 mgal. As seen from Fig. 5 the geoid was lowered by as much as 27.5 cm, due to a 5 mgal increase in the gravity anomaly level. The same is valid for a -5 mgal shift, however with opposite effect on the geoid. This imply a 5 cm change in the resulting geoid for every mgal the reference level at Jutulgryta is off.

As most of the bedrock around Troll is covered by as much as 1200 m of ice, assuming a standard density of 2.67 g/cm³ is probably incorrect. A new estimate of the density was based on the available ice thickness profile assuming a standard density of ice and bedrock of 0.92 g/cm³ and 2.67 g/cm³, respectively. With only one ice thickness profile available the estimated density is at most only approximative. The general slope of the profile is believed to be valid for the entire area of interest. The estimated density is given as

$$\rho = \frac{1}{n} \sum_{i=1}^n \left(\frac{\rho_{ice} h_{ice} + \rho_{rock} h_{rock}}{h_{ice} + h_{rock}} \right), \quad (10)$$

where h_{ice} is the ice thickness, h_{rock} is the height of the bedrock above sea level and n is the number of ice thickness measurements. An estimate based on the ice thickness profile, using Eq.

(10), gave a density of 1.29 g/cm³. As seen from Fig. 6 the difference in the geoid due to the change in density is approximately 10 - 20 cm.

In the Troll/Jutulgryta area only one estimate of the “true” geoid height was obtained. Combining the information from a tide gauge and a GPS receiver, located at Jutulgryta, gave the “true” geoid height (see Section 2.4). Based on one “true” geoid height only the difference between the real and estimated geoid (bias) may be removed, while a possible tilt of the geoid can not be determined. All geoids were linked to the “true” geoid height at Jutulgryta ($N = 13.38$ m).

The accuracy of the geoid is usually controlled by comparison to available GPS/levelling stations. However, in the actual area no such stations exists. An analysis of possible error sources connected to the estimation of the geoid/orthometric height give an impression of the accuracy of the geoid. From the error analysis and by assuming that the different error sources are independent of one another, the std. dev. of the geoid may be given as

$$\sigma = \sqrt{\sigma_{tg}^2 + \sigma_{kms}^2 + \sigma_{gps}^2 + \sigma_{\rho}^2 + \sigma_{remains}^2}, \quad (11)$$

where σ_{tg} is the std. dev. of the mean sea level measurement made by the tide gauge, σ_{kms} represents the uncertainty of the gravity anomaly’s reference level, σ_{gps} is the std. dev. of the measured ellipsoidic height, σ_{ρ} is the std. dev. due to the use of an erroneous density in the geoid calculation, and $\sigma_{remains}$ is the std. dev. of all other error sources assumed with the estimation of the geoid.

Based on the assumption that $\sigma_{tg} = \sigma_{remains} = 0.1$ m, $\sigma_{\rho} = 0.07$ m, $\sigma_{kms} = 0.05$ m and $\sigma_{gps} = 0.05$ m, Eq. (11) gave a std. dev. of 0.17 m. The values of σ_{kms} and σ_{ρ} were extracted from the generation of Fig. 5 and Fig. 6.

The deflection of the vertical between pair of survey marks were estimated based on zenith angle measurements (see Section 2.3). The results are listed in Table 3 together with estimated values of the deflection of the vertical based on EGM96 and the new NARE geoid. The coherence between the measured and the estimated values, in Table 3, are good, showing that the new NARE geoid fit the observations better than the EGM96. Figure 7 give the location of the survey marks.

A comparison of the new orthometric height, based on the NARE-geoid, with the height obtained from the EGM96 and OSU91a Rapp et al. (1991) was made. As seen from Table 4 and Fig. 4 there is a 2 m difference between the new geoid and the EGM96 and OSU91a. The large difference is due the lack of short wavelength information in both the EGM96 and OSU91a.

5 Conclusion and further work

Despite the limited amount of time, equipment, gravity measurements, etc, a new improved estimate of the orthometric height was obtained. As a fundamental basis for the estimate was the new NARE 96/97 geoid. Combining the new geoid and GPS measurements at the Troll station

an improved orthometric height was achieved. The new estimated orthometric height at Troll is 1296.97 m with a standard deviation of 0.17 m.

Further work is suggested in connection with the next NARE (2000/2001) mission. In relation to that mission the plans are to link the gravity measurements to an absolute gravity value (e.g. in South Africa), add several new ground gravity measurements, and cover a larger area around the Troll station with airborne gravity measurements.

If these suggestions do get funding a highly improved geoid together with a more accurate orthometric height at the Troll station will be the result.

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Tables

Table 1. Statistics of the available gravity points. All values in mgal

	Mean	SD	Min	Max
Original Δg_{fa}	13.066	51.112	-56.603	176.195
minus EGM96	37.502	40.256	-21.449	162.704
Faye anomalies	36.36	37.37	-20.87	142.25
Δg_{ba}	-94.559	42.485	-174.572	-22.165

Table 2. The orthometric height at three test points (indicated by $\diamond$ in Fig. 3) and the Troll station. The first column (Old) give the old height based on barometric measurements. All values in m

	Old	NARE	+5 mgal	-5 mgal	ρ_{new}
Vend	2227	2231.31	2231.39	2231.32	2231.28
Tern	2678	2687.66	2687.68	2687.66	2687.45
Rabb	1432	1437.64	1437.63	1437.68	1437.56
Troll	—	1297.00	1296.97	1297.05	1296.90

Table 3. The deflection of the vertical between pair of survey marks. All values in arcsec

	Measured	Estimated	EGM96
Klovningen - Troll	-9.1	-10.2	-6.3
Troll - Blåis	-24.5	-22.4	-11.3
Jutul N - Rabben	-12.2	-10.7	-7.7

Table 4. Comparing different orthometric heights from different sources. N is the geoid height at Troll while H is the orthometric height. All values in m

	EGM96	OSU91a	NARE 96/97
N	14.61	14.30	16.51
H	1298.90	1299.21	1297.00

Figure Captions

Fig. 1. A digital terrain model covering the test area; contour interval 100 m. The location of the 26 gravity data points are indicated by triangles and a square (the Troll station)

Fig. 2. The ice thickness profile

Fig. 3. The resulting geoid linked to the tide gauge in Jutulgryta; contour interval 0.5 m

Fig. 4. The NARE 96/97 geoid minus EGM; contour interval 0.25 m

Fig. 5. The change in geoid due to a five mgal change in gravity anomaly level; contour interval 0.025 m

Fig. 6. Difference in geoid when estimating the geoid using density equal to 1.29 g/cm^3 instead of the standard density 2.67 g/cm^3 ; contour interval 0.025 m

Fig. 7. The deflection of vertical were measured along line of sight between these survey marks. Part of satellite image map: Jutulsessen 1:100000, Norsk Polarinstitut 1992

Glacier terrain effects in geoid determination — the Iceland example

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Abstract

Glacier thickness measurements have been used in a study on glacial terrain effects on the (quasi)geoid. Iceland was chosen as a test area due to the existence of a detailed thickness database derived from radio echo sounding for the major ice caps.

Inclusion of ice thickness measurements in the geoid calculation by remove-restore techniques altered the computed gravimetric geoid by as much as 38 cm. A discrepancy of a few cm was apparent as far as 50 km from the glaciers. Two different geoid models, based on two terrain reduction methods (Residual Terrain Model (RTM) and Isostasy), were computed and compared to each other. Only the RTM method showed an improvement of results, compared to GPS levelling points, using ice thickness data. However, the largest changes in the geoid occurred across the glaciers, where no GPS levelling points were located. Standard deviations of approximately 12 cm were obtained comparing the gravimetric quasigeoids/geoids to the available GPS levelling points, reflecting the estimated errors of GPS levelling, so the GPS data did unfortunately not provide good control of the different methods.

As a further test gravimetric geoids were calculated based on either glacier gravity data (neglecting ice thicknesses), or using thickness data (but omitting gravity data located at the glaciers). The former geoid gave the best accuracy, implying that observed glacier gravity data give more information about the geoid than the use of ice thickness data alone.

Key words: Thickness of glacier · Terrain correction · Terrain effects · Geoid · Iceland

1 Introduction

During the last Ice Age large areas of the world were covered with ice or glaciers, estimated to have been 30% of the land surface. Today, glaciers cover 11% of the land surface containing about $30 \times 10^6 \text{ km}^3$ of ice.

Major glacier areas are located in the polar regions (Arctic, Antarctica, Greenland), the Himalayas, Northwestern Canada, Alaska and Patagonia.

Iceland was chosen as a test area to study the effect of large ice masses on the geoid. Almost 11% of the island is at present covered by glaciers (Björnsson, 1978, 1979). The glaciers range from small glaciers to extensive ice caps covering several hundred km^2 . Four of the ice caps in Iceland cover more than 500 km^2 , and for three of them detailed data of ice thickness are available (Björnsson, 1988).

The main purpose of this paper is to describe studies of different effects of ice masses on the geoid. In many areas of the world neither detailed ice thickness data are available, nor are gravity

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data located on the glaciers. If digital elevation models are used for geoid determination using terrain reductions in such areas, large systematic errors are possible, since rock density (2.67 g/cm^3) is usually assumed rather than the proper ice density (0.92 g/cm^3). In Iceland, however, data coverage is good, as both radar ice thickness data at three major glaciers and on-ice gravity observations are available.

Based on the knowledge of the thickness (and the density) of three glaciers the effect on the geoid has been estimated. The glacier density is assumed equal to the mean density of ice (0.92 g/cm^3), even though the uppermost uncompacted layers has less density.

To visualise the influence of the glaciers, gravimetric geoids have been calculated both with and without the glacier thickness, and also with and without glacier gravity data on the ice. Two different methods have been used to illustrate any dissimilarities between the methods or in the treatment of the terrain and ice thickness.

It was the purpose to compare the different geoid solutions to GPS-levelling data. However, no proper levelling network exists in Iceland, only indicative comparisons could be done, with no clear separations between methods.

Different routines in the GRAVSOFT-package (Tscherning et al., 1992) were applied in all calculations of this paper.

2 The thickness and topography data

Approximately 11% or 11200 km^2 of Iceland is covered by glaciers. The largest ice cap, Vatnajökull, covers an area of 8200 km^2 , while Hofsjökull, Myrdalsjökull and Langjökull cover areas of 925, 596 and 920 km^2 , respectively.

The sub-ice topography of the ice caps was mapped by radio echo soundings (Björnsson, 1988). Continuous echo soundings were obtained along sounding lines, typically arranged in a grid with a separation of 500 m to 1000 m between the lines. The position of the lines, logged by Loran-C and later GPS, is considered to be accurate to 30 m. The ice surface elevation was recorded by precision barometric altimetry at 50 m intervals along the sounding lines. The vertical depth of ice was computed from the echo soundings using general inversion techniques and the accuracy of the final estimates of ice depth is considered to be $\pm 15 \text{ m}$ or 2%, whichever is greater. The bedrock elevation was obtained as the difference between the ice surface altitude and the ice thickness.

Maps of the bedrock and surface topography were compiled using data from the sounding lines, existing geodetic maps of the areas surrounding the glaciers, and additional data from recent triangulation measurements. The outlines of the glaciers were obtained from Landsat images and aerial photographs. Digital maps were constructed by interpolation between the surface- and bed-elevation data on the sounding lines. The final digital map is a numerical matrix with a $200 \times 200 \text{ m}$ grid spacing. On Vatnajökull the data set for each map (ice thickness, surface and bed)

contains 6773 points, 995 on Hofsjökull and 563 on Myrdalsjökull. Statistics of the ice thickness measurements are given in Table 2.

For the purpose of geoid determination of the present paper, ice thickness data was computed on a 1 km resolution grid, and merged with land Digital Elevation Model (DEM) data from the GLOBE CD-ROM of global topography, provided by NGDC (Fig. 1). Table 1 give statistics of the DEM of Iceland and the DEM of the glaciers. The glaciers have a maximum thickness of approximately 900 m, while the maximum height of the terrain is 2005.0 m.

For Iceland's third largest glacier, Langjökull (located around $64.60^\circ N, 20.25^\circ W$, 25 km west of Hofsjökull), no thickness data were available, but several gravity measurements have been made on the glacier, as shown in Fig. 3.

3 Gravity anomalies

The (quasi)geoid was calculated based on 17341 gravity points (free-air anomalies, Δg_{fa}) within the geographical area spanned by $\phi = 62^\circ - 68^\circ N, \lambda = 11^\circ - 27^\circ W$. Gravity data was kindly provided by G. Thorabergsson, Orkustofnun (Iceland Energy Authority) and NIMA. The distribution of the gravity points was divided into two separate parts: points located at the three glaciers, and all other gravity data. There were a total of 96 points located at the glaciers and 17245 points located either on the ocean or the ice-free land. These were included in the database underlying the current joint Nordic geoid model NKG96 (Forsberg et al., 1996).

Gravity data was selected by rejecting older gravity points with assumed errors greater than 8 mgal. In Fig. 3 the distribution of the used gravity points is shown, with statistics listed in Table 3.

3.1 Bouguer anomalies

The Bouguer anomalies, was used as an intermediate quantity in the "Isostasy method", computing the total terrain effect due to topography and the isostatic compensation separately. The Bouguer anomalies were obtained from free-air anomalies, Δg_{fa} , by removing the effect of the topography. Mathematically, the Bouguer anomaly is approximated as a sum of Bouguer plates

$$\Delta g_{ba} = \Delta g_{fa} - 2\pi G \sum_{i=1}^n \rho_i h_i + c_p, \quad (1)$$

where $(\rho_i h_i)$ is a slice of the topography with density ρ_i and thickness h_i and n can be 1 or 2 (corresponding to ocean or rock only, or ice on top of rock). For simplicity the terrain correction c_p is only computed for the "top" layer (rock topography or ice, with density 2.67 g/cm^3 respectively 0.92 g/cm^3).

The simple Bouguer anomalies (Eq. (1) neglecting c_p) was additionally used to estimate the quasigeoid minus geoid ($\zeta - N$) separation on land.

4 GPS and levelling data

The Icelandic GPS network, ISNET, consists of 62 GPS levelling points evenly distributed across Iceland (see Fig. 4) with heights determined mainly by triangulation. The data was provided by Landmaelingar Islands. However, none of the GPS levelling points were located at a glacier. The triangulation height network is tied to local sea-level at several sites around the island, and therefore the accuracy of the GPS levelling data are hardly more than 1-2 dm relative to a true equipotential surface.

Table 4 shows that approximately 2/3 of the GPS points were located in the region 0-250 meters, with a maximum at 917 meters. In the test area the terrain has a maximum value of more than 2000 meters (Table 1).

The computed quasigeoids and geoids were compared to the 62 GPS points. The height anomaly (quasigeoidal undulation) ζ , was obtained from

$$\zeta = h - H^*, \quad (2)$$

where the ellipsoidal height h was available from GPS measurement, while the normal height, H^* , was taken from the triangulated height. Actually, due to no proper levelling network the levelled height are neither orthometric nor normal.

5 Computational algorithm

The quasigeoid/geoid covering Iceland was calculated with respect to two different methods, named "RTM" and "Isostasy" referring to different kinds of terrain reduction, cf. Forsberg (1984). These methods used a multi-band spherical Stokes FFT in the quasigeoid calculation (Forsberg og Sideris, 1993), with a remove-restore technique for the spherical harmonic reference field (EGM96, cf. Lemoine et al. (1996)) and the terrain. The two methods, described in Sect. 5.1 and 5.2, use the topography and ice data in different ways, and with different approximations.

5.1 "RTM/Helmert"-method

The gravimetric quasigeoid was calculated with the available free-air anomalies as a starting point, from which the terrain effects and the reference field, Earth Gravity Model (1996) (EGM96), were removed producing smooth (and easy to grid) gravity anomalies. The method used in practice grids RTM-reduced data, but restores the free-air (Faye) effects prior to Fast Fourier Transform (FFT). Therefore we use the composite name "RTM-Helmert".

The gravimetric terrain correction in planar coordinates for a calculation point $(x, y, z=h)_p$ is given as

$$c_p = G\rho \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{h_p}^h \left(\frac{z - h_p}{r^3} \right) dz dy dx, \quad (3)$$

where G is the constant of gravitation, ρ is the density of the masses at the point of estimation, r is the distance and (x, y, z) is the point of integration with height $z = h$. The terrain correction, c_p , was calculated from the irregularities of the topography, after the removal of a Bouguer plate through the calculation point. Because terrain corrections are to be used in combination with RTM or isostatic effects there was no need to consider spherical effects, and c_p was computed to a maximum cap size of 50 km only.

The RTM-anomalies were determined by the approximative formula (Forsberg, 1984)

$$\Delta g_{\text{RTM}} = \Delta g_{\text{fa}} - 2\pi G\rho(h - h_{\text{ref}}) + c_p, \quad (4)$$

where Δg_{fa} is the free-air anomaly and h_{ref} is a smooth average height surface selected here with a resolution of 20 km. The RTM anomalies refer to the surface of the topography but the effect of the short-wavelength topography has been removed. The reference surface is generated by low-pass filtering the DEM of the area. Gravity points located on the glaciers or the ice-free areas were estimated separately, with a density of either $\rho = 0.92 \text{ g/cm}^3$ or $\rho = 2.67 \text{ g/cm}^3$, respectively. This kind of reference surface might yield problems at the edge of glaciers, due to the assumed discontinuity in the assumed mass density (from $\rho = 2.67 \text{ g/cm}^3$ to 0.92 g/cm^3).

After the reference field (EGM96) was removed, the reduced gravity data, Δg_{red} ,

$$\Delta g_{\text{red}} = \Delta g_{\text{fa}} - \Delta g_{\text{egm}} - 2\pi G\rho(h - h_{\text{ref}}) + c_p, \quad (5)$$

was gridded, $\Delta g_{\text{red,grid}}$, by rapid collocation, (GRAVSOF program *geogrid*). The reference values, Δg_{egm} , in principle are computed at the surface of the topography.

Next, the RTM-effect was restored by

$$\Delta g_{\text{faye}} = \Delta g_{\text{red,grid}} + 2\pi G\rho(h - h_{\text{ref}})_{\text{grid}}, \quad (6)$$

effectively yielding gridded Faye anomalies relative to EGM96. The difference between the height, h , and the reference height, h_{ref} , was calculated with the glacier ice body condensed to density $\rho = 2.67 \text{ g/cm}^3$. The restoring of the residual topography (Eq. (6)) is performed to save one FFT operation. Doing a first-order computation of “restore” RTM terrain effect would be equivalent to taking the FFT (Stokes integration) of the right hand term of Eq. (6), cf. Forsberg (1985). By restoring Faye anomalies by Eq. (6) prior to Stokes integration the RTM method in fact turns into a close variant of the Helmert condensation method.

The residual quasigeoid height (height anomaly) may be calculated from the Molodensky formula

$$\zeta_r = \frac{R}{4\pi\gamma} \int \int \left(\Delta g_{\text{fa}} + g_1 \right) S(\psi) \cos \phi d\phi d\lambda, \quad (7)$$

where ϕ, λ are geographical coordinates, γ is the normal gravity at the surface, g_1 is the first term in the Molodensky expansion and $S(\psi)$ is the Stokes function given as

$$S(\psi) = \frac{1}{t} - 4 - 6t + 10t^2 - (3 - 6t^2)\ln(t + t^2), \quad (8)$$

where $t = \sin \frac{\psi}{2}$ and ψ is the spherical distance. Because the present computations serve to illustrate the magnitude of the glacial effects, we will approximate $\Delta g_{\text{fa}} + g_1$ with the Faye anomaly ($\Delta g_{\text{fa}} + c_p$).

In the *spfour*-routine of the GRAVSOF-T-package Molodensky's formula has been transformed into a convolution integral (Forsberg og Sideris, 1993)

$$\zeta_r = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} F^{-1}(F(\Delta g_{\text{faye}} \cos \phi) F(S(\psi))). \quad (9)$$

in ϕ and λ with kernel $S(\psi)$.

The implemented method calculates exact values along several reference latitudes in the area in question. The height anomaly at a latitude ϕ is obtained by interpolation of the height anomaly at two reference latitudes ϕ_i and ϕ_{i+1} . The formula is given as

$$\zeta_r = \frac{\phi - \phi_{i+1}}{\phi_i - \phi_{i+1}} \zeta_{r,i} + \frac{\phi_i - \phi}{\phi_i - \phi_{i+1}} \zeta_{r,i+1}. \quad (10)$$

After the spherical quasigeoid computation, the final quasigeoid is obtained by restoring the reference field

$$\zeta_{\text{rtm}} = \zeta_r + \zeta_{\text{egm}}. \quad (11)$$

5.2 Isostasy method

In the Isostasy method the isotastic compensation, Δg_{comp} , was calculated by assuming a Moho density contrast of 0.4 g/cm³ and a normal crust thickness of 32 km. The isostatic compensation was removed from the Bouguer anomalies, Δg_{ba} , which served as a starting point.

The reduced gravity anomalies, $\Delta g_{r,\text{iso}}$, can be written as,

$$\Delta g_{r,\text{iso}} = \Delta g_{\text{ba}} - \Delta g_{\text{comp}} - \Delta g_{\text{egm}} \quad (12)$$

where Δg_{egm} is the reference field (EGM96).

The residual quasigeoid, $\zeta_{r,\text{iso}}$, was calculated using the multi-band spherical Stokes FFT with gravity data, $\Delta g_{r,\text{iso}}$, as input,

$$\zeta_{r,\text{iso}} = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} F^{-1}(F(\Delta g_{r,\text{iso}} \cos \phi) F(S(\psi))). \quad (13)$$

After restoring the isostatic compensation, ζ_{comp} , and the reference field the quasigeoid was obtained by

$$\zeta_{\text{iso}} = \zeta_{r,\text{iso}} + \zeta_{\text{comp}} + \zeta_{\text{egm}}. \quad (14)$$

The effects of the isostatic compensating masses was computed by FFT, cf. Forsberg (1985). Both the RTM and the Isostasy method give in principle quasigeoid heights, and to obtain the classical geoid the first-order correction $N - \zeta = \frac{\Delta g_{\text{ba}} h}{\bar{\gamma}}$ should be applied, cf. Heiskanen og Moritz (1967). The comparison in the sequel were based on the quasigeoid. The GPS-levelling data was not sufficiently well defined to be belonging to either of the systems. Table 8 shows differences were not significant.

6 Results

In each of the two methods (RTM and Isostasy) the quasigeoid was calculated four times. First, by omitting the used ice thickness data (assuming all glaciers to be rock with density 2.67 g/cm^3), and secondly by correctly using the glacier thicknesses (assuming a density of 0.92 g/cm^3) in the quasigeoid calculation. Gravity data located on glaciers were included in the quasigeoid determination. The third and fourth alternative are the same as the first and second, but gravity data located on the glaciers have been omitted. These alternatives correspond to the examples often encountered in practice with regional or global (quasi)geoid calculations covering glacier areas. Such areas are most frequently void of gravity data, ice thickness data, or both.

To examine the change in the quasigeoid due to the inclusion of ice thicknesses, the results of the two first alternatives were compared. Figures 5 and 6 show the difference between quasigeoids computed *using* and *not using* the ice thickness measurements for the two methods, RTM and Isostasy, respectively. Statistics of the change in the quasigeoid are given in Table 5. In both methods, the effect on the quasigeoid due to the change in density, was more than 27 cm, with a maximum of 38 cm in the RTM method. As can be seen by comparing Figs. 5 or 6 to Fig. 2 the change is highly correlated with the shape of the ice caps. The deviation is naturally largest where the ice caps are thickest, but the quasigeoid is also influenced in a wider area around the glaciers. A 5 cm change in the quasigeoid is visible out to a few km from the glaciers in all models, while a 2.5 cm change stretches out to a maximum of approximately 50 km from the glacier margins.

Both terrain reduction methods are compared in Table 6, and the (raw) difference in quasigeoids between the RTM and the Isostasy method, assuming a glacier density of 0.92 g/cm^3 , are visualised in Fig. 7. As the figure illustrates the main differences occur around the glaciers and where the terrain is most irregular (north, north-west and east). It is also apparent that there is a major trend in the differences, probably due to the handling of different data biases in the FFT method (see Table 3). The difference in data biases between the two methods are probably due to an incorrect isostatic model. Iceland is located in a geological active area where the used isostatic model (Sect. 5.2) is known not to represent the real geophysical situation, since a substantial part of the isostatic compensation along the the mid-atlantic ridge is assumed to be associated with hot, low-density upper mantle material. In addition the way the residual gravity is modelled by FFT assumes no data bias to be present, even though the isostatic effects may have significant long-wavelength signals. Such signals are just not recovered by the used FFT methods. The deviation between the two methods is also slightly correlated to the height of the terrain as Fig. 7 indicates. This is due to the different approximations, used to estimate the terrain effect, in the two methods (see Sect. 5.1 and 5.2).

Statistics of the residual of the GPS levelling comparisons, of the four possible combinations of gravity and thickness data, are listed in Tables 7 and 8. The residuals have been fitted directly and after the removal of a 4-parameter “Helmert shift” trend surface (Forsberg, 1990), indicated

by “after fit” in the tables. The standard deviation of the quasigeoids are listed in Table 8, while Table 7 gives more detailed information about the residuals of the first two alternative combinations.

The result of Tables 7 and 8 show, in the RTM method, that the quasigeoid prediction accuracy was marginally improved due to the inclusion of glacier thickness measurements. The combination of gravity and thickness data also gave the best fit to the GPS levelling points. However, a small decrease was detected using the Isostasy method, and almost identical results (1-2 mm in difference) in three of the combinations of Table 8. It can be concluded that with the present accuracies and distribution of GPS levelling points, the effect of taking glaciers into account is insignificant in the Isostasy method. However, it should be borne in mind that the maximum local effect on top of the ice cap can be several dm, as shown in Fig. 6.

A few comments to Table 8 should be made. It appears that gravity data instead of thickness data are preferable, if no previous gravity or thickness data are available. This is quite natural, since gravity measurements on glaciers are strongly affected by the ice thickness, and may in fact often be used to determine ice thicknesses by inversion methods. This indicates that even sparse gravity data gives more information about the quasigeoid compared to dense glacier thickness measurements, at least as long as one is not concerned with the detailed quasigeoid over the glacier itself.

The changes in the quasigeoid due to the inclusion of ice thickness measurements, closely resembles the shape of the ice caps. In the present example only changes in the quasigeoid prediction accuracy are expected due to the remote location of the GPS levelling points compared to the ice caps. No GPS levelling point is located closer than approximately 10 km from any of the glaciers. Out of 62 GPS points, 19 points in the Isostasy method deviated by more than 10 mm, with a maximum of 48 mm and a standard deviation of 10 mm. However, the maximum change in the quasigeoid was 27.4 cm. This may explain the small differences between three of the four possible combinations with gravity data and thickness measurements (see Table 8). To draw any certain conclusions about the use of glacier thickness measurements, in the Isostasy method, additional GPS levelling points are needed in the immediate vicinity of the glaciers.

7 Conclusion

Available ice thickness measurements of three glaciers have been used in the calculation of the terrain correction, terrain effect and the (quasi)geoid. The inclusion of thickness data implied a small change in the (quasi)geoid. The largest discrepancy (> 27 cm) occurred close to the centre of the glaciers. However, a significant effect was computed as far as 50 km away from the glacier margins.

Only in the RTM method the inclusion of thickness measurements in the quasigeoid calculation

gave a significant improvement of the quasigeoid fit to the GPS levelling points. For the isostatic method, a slight decrease in the quasigeoid prediction accuracy was observed. This was somewhat surprising, but probably mainly reflects the inaccurate GPS control, but may also in part be due to the approximations used in our isostatic computations. To properly test the methods GPS levelling data should be used on the ice caps themselves, such data are however not available.

Given the expense of detailed radar echo sounding of glaciers, our examples show that a sparse coverage of gravity data on the glaciers itself may produce just as good (or bad) quasigeoid results as the utilization of detailed thickness models. Such data may be less costly to acquire, especially by airborne methods, (which ideally could be combined with radar ice sounding). The overall fit of the quasigeoid models, with or without using glacier thicknesses, was on the order of 12 cm. This is probably mainly due to the limited accuracy of the available GPS geoid data, reducing the usefulness of these data in discriminating between different models. The investigations of the present paper should therefore mainly be viewed as examples of order-of-magnitude examples.

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Tables

Table 1. Statistics of the used digital elevation model of Iceland, and the grid of glacier thickness

	Min	Max	Mean	SD
$\lambda(^{\circ})$	-26.0	-13.0		
$\phi(^{\circ})$	63.0	67.0		
Iceland (m)	0	2005.00	191.40	336.24
Ice thicknesses (m)	0	898.85	11.22	71.92

Table 2. Statistics of the thickness of the three glaciers. All values in metres

	Min	Max	Mean	SD
Vatnajökull	0.04	924.47	407.23	207.33
Myrdalsjökull	2.11	718.44	236.04	149.28
Hoffsjökull	0.10	757.10	217.44	126.91

Table 3. Statistics of the used gravity points. In Δg_{faye} and $\Delta g_{\text{r,iso}}$ both the effect of the gravity data on ice and the ice thickness data are included. All values in mgal

	Min	Max	Mean	SD
Δg_{fa}	-28.67	172.63	36.91	19.84
Δg_{faye}	-73.705	182.524	0.661	12.829
$\Delta g_{\text{r,iso}}$	-63.216	47.722	-1.711	12.326
Δg_{ba}	-67.46	74.55	21.19	27.51

Table 4. Statistics of the height of the GPS levelling points

Height (metres)				Number of points			
Min	Max	Mean	SD	0 - 250 m	250 - 500 m	500-750 m	750-1000 m
70.11	916.93	270.08	245.61	40	8	11	3

Table 5. The change in the quasigeoid due to the inclusion of the ice thickness data in the quasigeoid calculation. All values in metres

Method	Min	Max	Mean	SD
RTM	-0.380	0.225	-0.003	0.017
Isostasy	-0.274	0.178	-0.001	0.013

Table 6. Quasigeoid differences between the RTM and isostatic methods when using (density 0.92 g/cm^3) and not using (density 2.67 g/cm^3) the thickness of the glaciers in the quasigeoid calculation. All values in metres

Ice density	Method	Min	Max	Mean	SD
0.92 g/cm^3	R - I	-0.288	0.984	0.290	0.172
2.67 g/cm^3	R - I	-0.279	0.991	0.292	0.176

Table 7. The fit of the quasigeoids to GPS levelling points. Gravity data located at the glaciers are included in the (quasi)geoid prediction

Method	Ice density	Fit to quasigeoid				
		Min	Max	Mean	SD	SD
	(g/cm^3)	(m)	(m)	(m)	(m)	after fit (m)
RTM	0.92	-1.596	-0.968	-1.315	0.141	0.120
Isostasy	0.92	-1.469	-0.744	-1.130	0.131	0.121
RTM	2.67	-1.647	-0.972	-1.318	0.145	0.125
Isostasy	2.67	-1.466	-0.751	-1.138	0.128	0.119

Table 8. GPS comparison results when on-ice gravity data and glacier ice thickness data are combined in four possible variations. The fitted standard deviation of the quasigeoids and the geoids is listed. All values in metres

	Quasigeoid				Geoid
Gravity data on glacier used	Yes	No	Yes	No	Yes
Ice thickness data used	Yes	No	Yes	No	Yes
RTM	0.120	0.125	0.126	0.128	0.122
Isostasy	0.121	0.119	0.122	0.156	0.123

Figure Captions

Fig. 1. Digital terrain model of Iceland with the glaciers included; contour interval 250 m. The major ice caps include Vatnajökull (V), Hofsjökull (H), Myrdalsjökull (M) and Langjökull (L)

Fig. 2. The digital thickness model of the three major glaciers; contour interval 100 m

Fig. 3. The distribution of the used gravity points, with the boundary of the glaciers indicated

Fig. 4. The geographical distribution of the 62 GPS levelling points

Fig. 5. The change in the quasigeoid (RTM method) due to the inclusion of the glacier thickness in the calculation of the quasigeoid; contour interval 0.025 m

Fig. 6. The change in the quasigeoid (Isostasy method) due to the inclusion of the glacier thickness in the calculation of the quasigeoid; contour interval 0.025 m

Fig. 7. Difference between the RTM and the Isostasy method. Both glacier thickness measurements and gravity data located at the glaciers have been used in the quasigeoid calculation; contour interval 0.05 m

Fig. 8. The quasigeoid over Iceland calculated with the RTM method; contour interval 0.25 m

Quasi-geoid estimations in South America

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Abstract

A report on quasi-geoid computations in South America, based on the existing gravity data base collected in the framework of the South America Gravity Project (SAGP) is presented. Two estimation methods will be described. One of these is the standard “remove-restore” technique. The gridding of the residual gravity values (i.e. free air gravity anomalies minus the global geopotential model and the terrain effect) has been performed using, with some refinements, the GEOGRID program of the GRAVSOF package (Tscherning et al., 1994) and the residual quasi-geoid signal has been evaluated through the 1D FFT approach (Haagmans et al., 1993) which allows a fast and rigorous computation of this component.

The second approach is based on 1D FFT method applied on mean Helmert's gravity anomalies, having removed the low frequency components using EGM96 model up to degree 50.

1 Introduction

Geoid computations over large continental areas, aiming at refining the global geopotential model implied undulations, are nowadays one of the most important tasks in Geodesy.

This can be done in a fast and efficient way by means of FFT techniques (Sideris, 1994; Forsberg and Sideris, 1993) or fast collocation (Bottoni and Barzaghi, 1993) which lead to geoid estimates over large areas, so avoiding subareas computation and patching of subsolutions.

Furthermore, accurate global geopotential models (e.g EGM96) and, over some regions, detailed DTM allow an effective reduction of the gravity data, thus implying an optimal application of the “remove-restore” procedure.

Continental geoid solution have been computed in such a way in Europe, USA and Canada, leading to high precision estimates of the geoid undulation which, as it is well known, is needed in connection to GPS to derive orthometric heights.

The similar approach is presented here for quasi-geoid computation in South America. The main difference with the previously mentioned examples is the coverage and the quality of the gravity data.

In South America there are large unsurveyed areas; hence, large data gaps are present in the database, implying unreliable geoid solutions over those areas.

Furthermore, the existing data have been collected from many different sources over a large time span. So, different quality of the data and also possible data inconsistency are to be expected. Thus, a relevant effort must be done in collecting and in homogenizing the gravity data base in this particular area of the world. A decisive step forward in this direction has been accomplished in the last ten years with the SAGP project (Green and Fairhead, 1991). In the framework of this project, gravity data in South America have been collected to form a data base consisting at the moment of 291866 gravity points on land.

Argentina, Uruguay, Ecuador, Paraguay, Venezuela, Chile, Colombia, Brazil (with the Anglo-Brazilian Gravity Project) contributed in forming such valuable gravity data set in a initiative of SCGGSA (Sub-commission of the Geoid and Gravity in South America).

One quasi-geoid computation which is presented in this paper took also benefit from a new DTM that NIMA has prepared over South America. This has greatly improved the quality of the existing height data base since a detailed and consistent 30"×30" DTM was made available for this computation.

Using the collected gravity data, DTM models and the EGM96 global geopotential field, new estimates of the South America quasi-geoid have been computed based on a cooperation among NIMA, IGeS and EPUSP and they will be soon distributed, through the IGeS website, to the geodetic community.

These are new refined estimates (following preliminary geoid computations made at EPUSP; Blitzkow, 1999) which, we hope, will be used as a common reference field for local quasi-geoid computations/comparisons in South America.

2 The quasi-geoid computation procedures

The gravity data coverage is presented in fig. 1

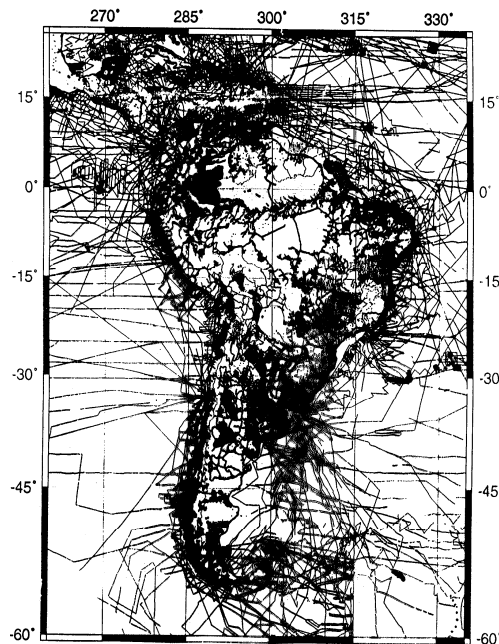


Fig. 1 Gravity data distribution

As one can see, relevant gaps are present in large areas, mainly in the Amazonian forest, where we probably computed biased geoid undulations.

- IGeS/NIMA solution

The remove step was based on the EGM96 geopotential model (Lemoine et al., 1998), that we used up to degree 360, and on a 30"×30" DTM which was assembled at NIMA. The gravity model component as well as the Residual Terrain Correction (RTC) gravity effect were computed pointwise at DTM derived elevations. Particularly, the RTC effect was evaluated using the TC software (GRAVSOF package; Tscherning et al., 1994). The reference DTM has been obtained by means of a moving average applied to the detailed one with a cap size of 30' and a sampling rate of 10' both in latitude and longitude.

RTC effect was calculated using two grids: the detailed one (30"×30") up to 22 km from the computation point and a coarser grid (5'×5') from 22 km to 220 km (the 5'×5' DTM was derived by averaging the 30"×30" DTM).

Point residual gravity data, were then gridded using the GEOGRID software (GRAVSOF package). For computing the quasi-geoid estimate, we selected a 5'x5' resolution grid covering the area $-60 \leq \varphi \leq 15$; $-85 \leq \lambda \leq -30$. Although it seems quite an ambitious target, the 5'x5' grid probably have a reasonable resolution at least in those areas where we have a good data coverage and where we can expect to have reliable estimates.

The gridding procedure that was adopted was based on the collocation option of GEOGRID and was improved using a local tuning of the covariance function parameters.

For each grid point, residual gravity values have been selected up to a distance of one degree and an empirical covariance has been estimated. On such an estimate, model covariance function parameters used in the prediction have been set up. Hence, each gridded value has been predicted using a covariance which reflects the local features of the residual gravity field.

The plot of the gridded residual gravity field Δg_r^G and the associated prediction error are plotted in fig.2 and fig. 3 respectively ; statistics of the Δg_r^G are listed in the table 1.

As one can see, the accuracy plot reflects only partially the plot of the coverage; the worst accuracies are reached along the Andes probably due to the combination of poor data coverage and roughness of the gravity field.

Table 1. Statistics of the gravity residuals Δg_r^G

#	average	σ	min	max
594000	0.46	13.58	-351.40	647.30

The residual quasi-geoid component ζ_r has been evaluated using 1D FFT technique (Haagmans et al., 1993), implemented in the FFTGEOID software by M. Sideris (Sideris, 1994): integration cap has been fixed to 1.5 degree.

The statistics of the 5'x5' ζ_r are presented in table 2 while the plot of the values is shown in fig. 4.

As expected, high frequency pattern in Δg_r^G (see fig. 2) are spread on a larger area in terms of ζ_r : a careful outliers rejection will be performed in the near future to avoid such effects, probably connected to outliers or inconsistency in the gravity data.

Table 2. Statistics of the quasi-geoid residuals ζ_r

#	average	σ	min	max
594000	0.09	0.76	-12.52	13.38

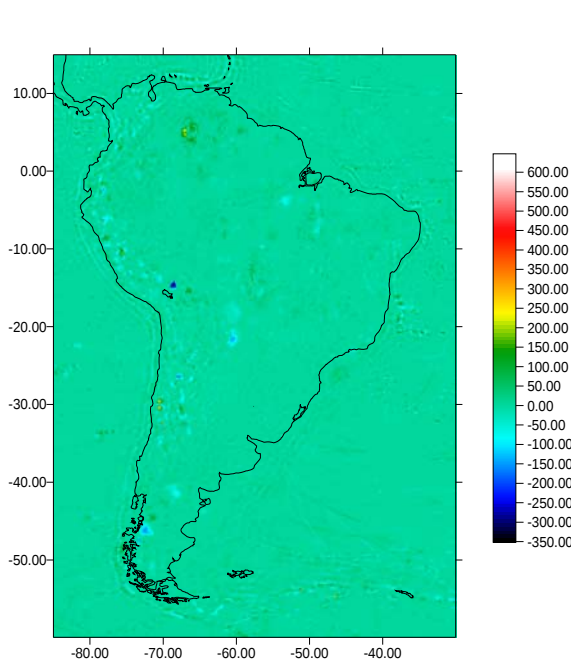


Fig. 2 Residual gravity data Δg_r^G (mGal)

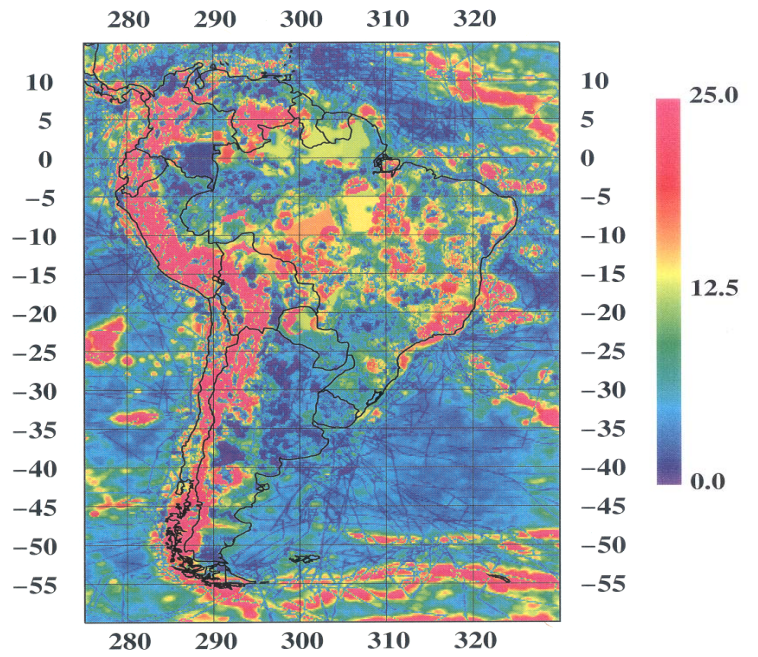


Fig. 3 Prediction error of the gridded residual gravity field data Δg_r^G (mGal)

The total quasi-geoid estimate is plotted in fig. 5

The values have been obtained through the "restore" step using the TC program for the RTC component and the EGM96 model up to degree 360 for the long wavelength part. Computation of ζ_{rtc} and ζ_M have been carried out on the 5'x5' prediction grid at DTM derived elevations. Heights for each grid knot have been estimated via bilinear interpolation on the 30"x30" high resolution DTM used in the RTC effect computation.

Statistics of the total quasi-geoid values are in tab. 3.

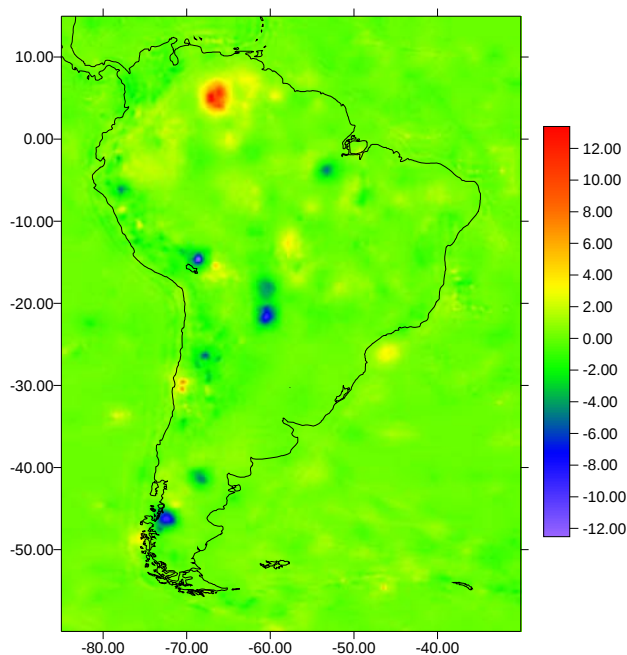


Fig. 4 Residual quasi-geoid ζ_r (m)

Table 3. Statistics of the quasi-geoid ζ

#	Average	σ	min	max
594000	1.90	16.86	-58.155	50.94

- EPUSP solution

A parallel computation has been also carried out at EPUSP. The point gravity data available at GETECH have been processed in order to derive 5' mean free air and Bouguer gravity anomalies. A grid of 10' mean values has been derived from 5'. A digital terrain model of 3' (DTM3) has been used to derive mean heights for the 10' grid; the terrain correction has been estimated using the DTM3 grid and then averaged to 10'. The quasi-geoid solution has been computed on the 10' grid using mean Helmert's gravity anomaly and 1D FFT with an integration cap of 4°. The long wavelength component has been removed and restored using EGM96 geopotential model up to degree and order 50. The quasi-geoid obtained following this procedure is shown in fig.6.

The quasi-geoid estimate by IGeS and NIMA has been interpolated on the EPUSP geoid grid to compare the two solutions. The two computations show differences even up to 17 m and the standard deviations value is about 2.8 m (tab. 4). As the database is quite the same, these high discrepancies are attributed to the two difference computation methodologies, that differ specially for the maximum degree of the global geopotential model EGM96 used, 50 and 360 respectively for the EPUSP and the IGeS/NIMA solution.

Table 4. Statistics of the differences between the two solution: quasi-geoid ζ by IGeS/NIMA minus quasi-geoid by EPUSP (m)

#	Average	σ	min	max
147392	0.343	2.785	-17.276	17.095

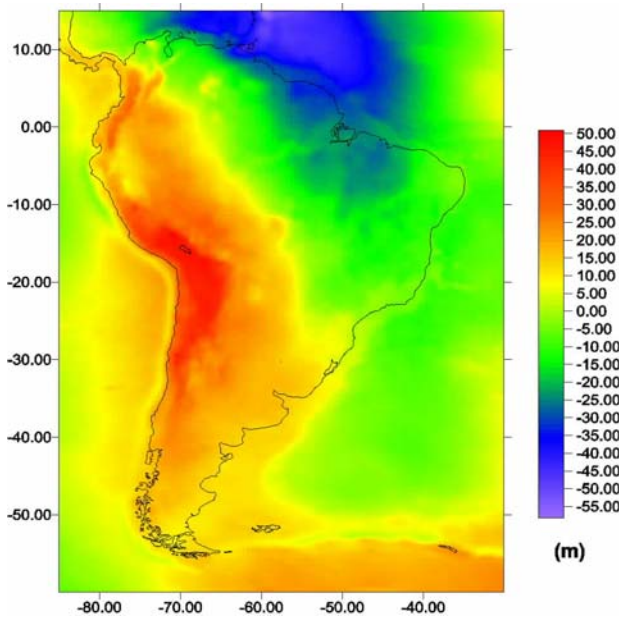


Fig. 5 Quasi-geoid undulation ζ (m) computed by IGeS and NIMA

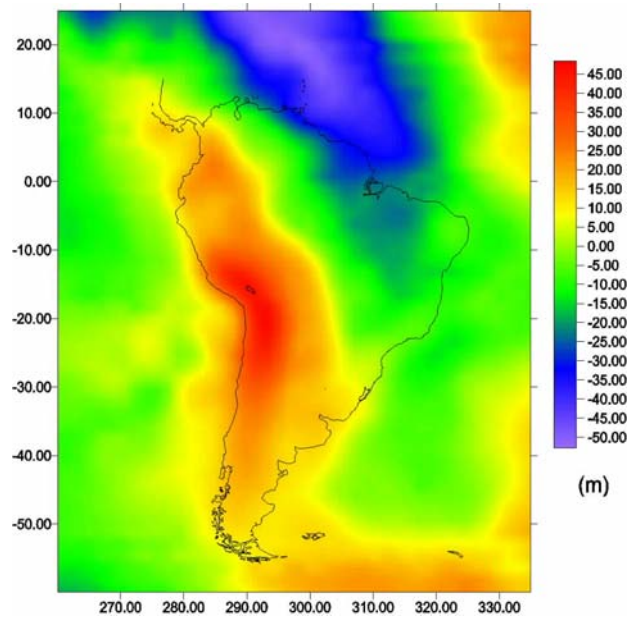


Fig. 6 Quasi-geoid undulation ζ (m) computed by EPUSP.

3 Comparisons with GPS/leveling geoid undulations

The solutions have been compared with 418 GPS/leveling geoid undulation values, collected for the SIRGAS project (SIRGAS Relatorio Final, 1997) and distributed as in fig.7. These values have been compared also to the geopotential model EGM96, to evaluate if the two solutions are able to improve the geoid knowledge with respect to the geopotential model EGM96.

The statistics in tab. 5 show that no significant improvements have been reached in the IGeS/NIMA and EPSUP solutions. Refined solutions have been compared with EGM96 to $n=360$ and $n=50$; these are the model degrees used in the IGeS/NIMA and EPSUP solutions respectively. A 5% Fischer test proved that the variances of the residuals with respect to the EGM96 model and to the detailed solution are equivalent in both cases. So, the new solutions don't improve with respect to the global geopotential model EGM96, probably because most of the gravity data used in these new solutions belong also to the gravity database used in the EGM96 model.

From the comparisons with GPS/leveling geoid undulations it only comes out that the IGeS/NIMA solution is more precise than the EPUSP estimate (tab.5 and fig. 8), as consequence of the greater degree used in the global potential model.

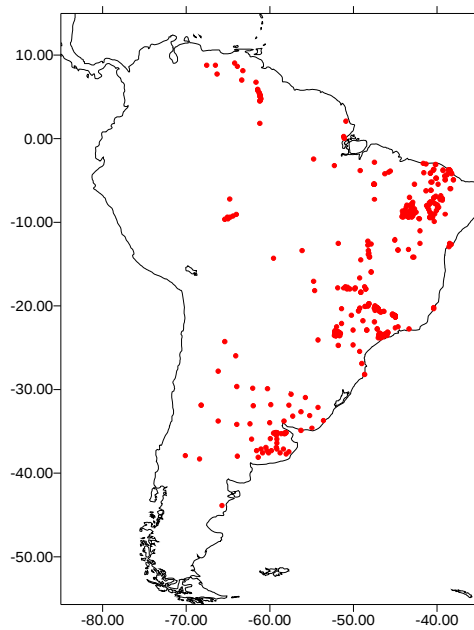


Fig. 7 GPS/leveling points.

Tab.5 Comparisons with GPS/leveling undulation values

	$\zeta_{\text{IGeS/NIMA}} - N_{\text{GPS/lev}}$	$N_{\text{EGM96 (360)}} - N_{\text{GPS/lev}}$	$\zeta_{\text{EPSUP}} - N_{\text{GPS/lev}}$	$N_{\text{EGM96 (50)}} - N_{\text{GPS/lev}}$
point	418	418	418	418
average	0.44	0.28	0.30	-0.01
sqm	0.86	0.94	1.22	1.33
min	-2.94	-3.89	-3.74	-4.05
max	3.90	3.67	3.50	5.50

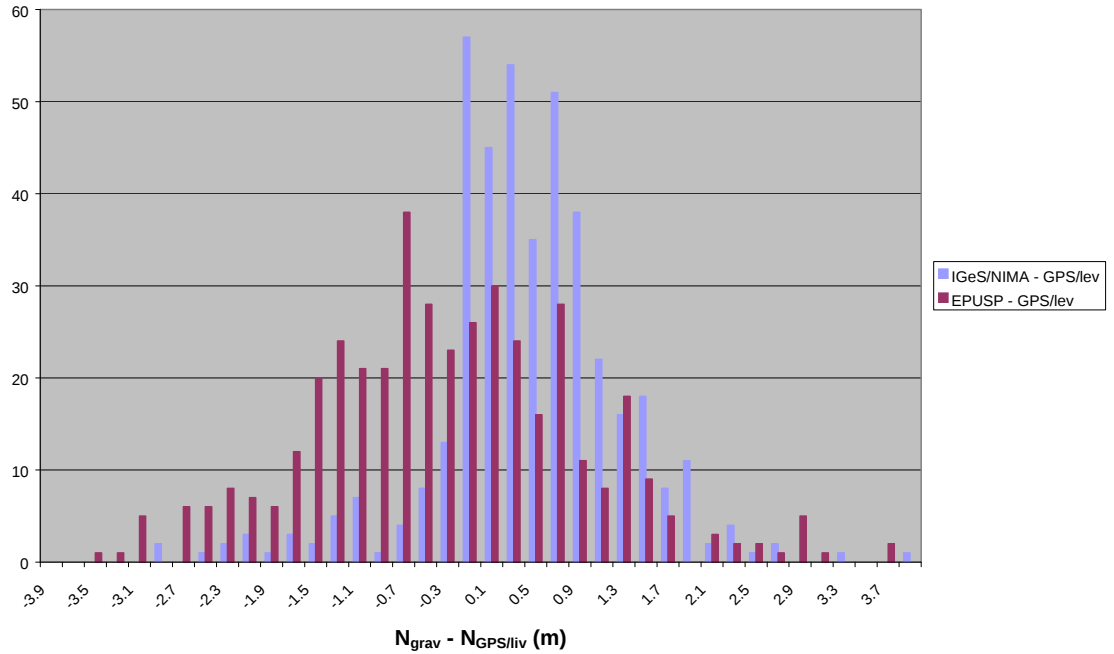


Fig. 8 Histogram of the differences with GPS/leveling geoid undulation.

Conclusions and future perspectives

These quasi-geoid estimates over the area covering South America have been computed using the existing gravity data bases, following different approaches.

These solutions must be considered a first step in continuous efforts for future computations both of global and local type, i.e. for computations on the whole South America continent and for estimates covering only limited areas.

Key points for improved estimates will be a new careful outliers rejection, based on collocation, and a better data coverage which will be achieved due to the remarkable ongoing efforts of the Sub-Commission for Gravity and Geoid in South America (SCGGSA).

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(*) Note by the editorial board

Note that the following paper has been considered as accepted by the reviewing process. Yet one reviewer has outlined a number of questions related to the paper which seem to the editorial board interesting and worth to be published. It seems obvious that to answer to these questions would require to the authors a significant piece of work. This might come in future; yet, given the character of this bulletin, we have deemed it useful to proceed in this way.

A letter on the paper

The paper deals with the computation of an astrogeodetic geoid by using geophysical information which in turn can be given in the form of an Airy-Heiskanen compensation model or in a form of seismically derived Moho. The results are then compared with GPS-leveling geoid undulations. It seems interesting to observe that the Airy-Heiskanen model, though less physically justified, in particular for a small area like Austria, is able to produce a smoothing of residual data and ultimately a collocation solution which seems to be more fitting than those derived from seismic Moho information. Is it possible that this is due to the fact that, though artificial, the Airy-Heiskanen reduction leaves a field of residual deflections which looks more stationary than the realistic one? In fact the Moho map of Figure 2 displays a very deterministic behaviour (a significantly non linear trend) in the area of Austria and in a way the same pattern seems to emerge from the error map of the corresponding geoid (Fig. 5 – Fig. 6). If this is the explanation, it has to be taken as warning, when using collocation, to be sure that the final residual field in the window where data are given has more or less to look as an oscillating de-trended field.

Astrogeodetic Geoid Determination Using Seismic Moho Information (*)

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Abstract

The depths of the Mohorovicic discontinuity (Moho depths) derived by seismic observations are available for this study. The computation of the terrain reduction is carried out using those pre-estimated seismic Moho depths. The TC-program (originally written by Forsberg, 1984) has been modified to compute the effect of the compensating masses on the disturbing potential (and hence on other gravitational quantities) using the Digital Moho Models. Both fine (11.25"×18.75") and coarse (90"×150") Digital Height Models are available for this investigation. Deflection components represent the gravitational field data for the current investigation. The geoid for Austria has been computed using the seismic Moho depths representing the compensating masses by the least-squares collocation technique. For the sake of comparison, another geoid for Austria has been computed using the Airy-Heiskanen isostatic model for generating the compensating masses. A wide comparison between both geoids is carried out. The results show that implementing the seismic Moho depths in the geoid computation dose not improve the geoid accuracy.

1. Introduction

The well-known remove-restore technique is frequently used in the modern geoid computation. According to that technique, the effect of the topography and its compensation has to be removed from the observed gravity field quantities (such as gravity anomalies and deflection components) and to be restored to the computed geoid undulations. The Airy-Heiskanen isostatic model is usually used in practice to generate the compensating masses, which can be defined as the distance from the normal crustal thickness to the depth of the Mohorovičić discontinuity (Moho depth). However, the Moho depths can be derived in practice from seismic observations. Hence,

the main aim of this investigation is to study the effect of implementing the seismic Moho depths in the geoid computation instead of the Moho depths generated by the isostatic models.

The basis for a numerical test is a program for computing the effect of the topography and its compensation (either using Airy-Heiskanen isostatic model or a seismic Moho model) on the gravity field quantities. One of the widely used programs to compute the effect of the topographic and isostatic masses on gravity field quantities by the Airy-Heiskanen isostatic model is the TC-program (originally written by Forsberg, 1984). As the TC-program does not include the possibility of working with variable Moho depths, it was modified to include the effect of seismic Moho depths. As the TC-program works with a fine grid for the neighbourhood of the computational point and a coarse grid for the outer zones, this concept had to be applied for the seismic Moho depths as well. Hence a set of fine and coarse digital Moho models was established.

The used data sets for the current investigation are outlined. The computation of the terrain reduction using both the seismic Moho depths and the Airy-Heiskanen isostatic model is carried out. Removing the reference field from the deflection components is performed using the EGM96 geopotential earth model. Two geoids are computed for Austria using the seismic Moho depths and the Airy-Heiskanen isostatic model by the least-squares collocation technique. The geoids are fitted to the available GPS stations. A wide comparison between the geoids computed in the current investigation is carried out.

2. Source Data

2.1. Deflection Data

The gravitational data set for this investigation is a set of deflection components ξ and η at 716 stations in Austria. Figure 1 shows the dense and good distribution of the deflection components of this data set.

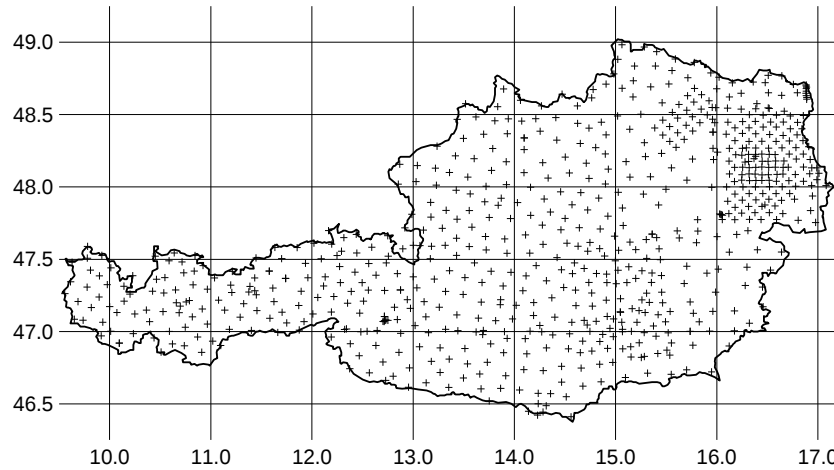


Figure 1: The distribution of the deflection data in Austria.

2.2. Digital Height Models

Two different Digital Height Models are available for this investigation. A coarse model of $90'' \times 150''$ resolution in the latitude and the longitude directions, respectively, and a fine model of $11.25'' \times 18.75''$ resolution in the latitude and the longitude directions, respectively. The fine DHM covers Austria. The coarse model covers the complete window $45^\circ \text{ N} \leq \phi \leq 50^\circ \text{ N}$; $7^\circ \text{ E} \leq \lambda \leq 20^\circ \text{ E}$.

2.3. Seismic Moho Data

The seismic Moho data set available for this investigation has been computed by Geiss (1987). This data set covers the complete window $35^\circ \text{ N} \leq \phi \leq 50^\circ \text{ N}$; $5^\circ \text{ W} \leq \lambda \leq 25^\circ \text{ E}$ having a $12' \times 20'$ resolution in the latitude and the longitude directions, respectively.

An interpolation process has been carried out using the Kriging interpolation technique with zero error variance to interpolate the Moho depths on a grid size of $90'' \times 150''$ in the latitude and the longitude directions, respectively. This resolution is the same as that of the coarse digital height model. Hence this Moho model represents the coarse digital seismic Moho model. It covers the window $40^\circ \text{ N} \leq \phi \leq 50^\circ \text{ N}$; $7^\circ \text{ E} \leq \lambda \leq 23^\circ \text{ E}$. Figure 2 shows the coarse digital seismic Moho model for the data window. It shows the deeper Moho depths under the Alps.

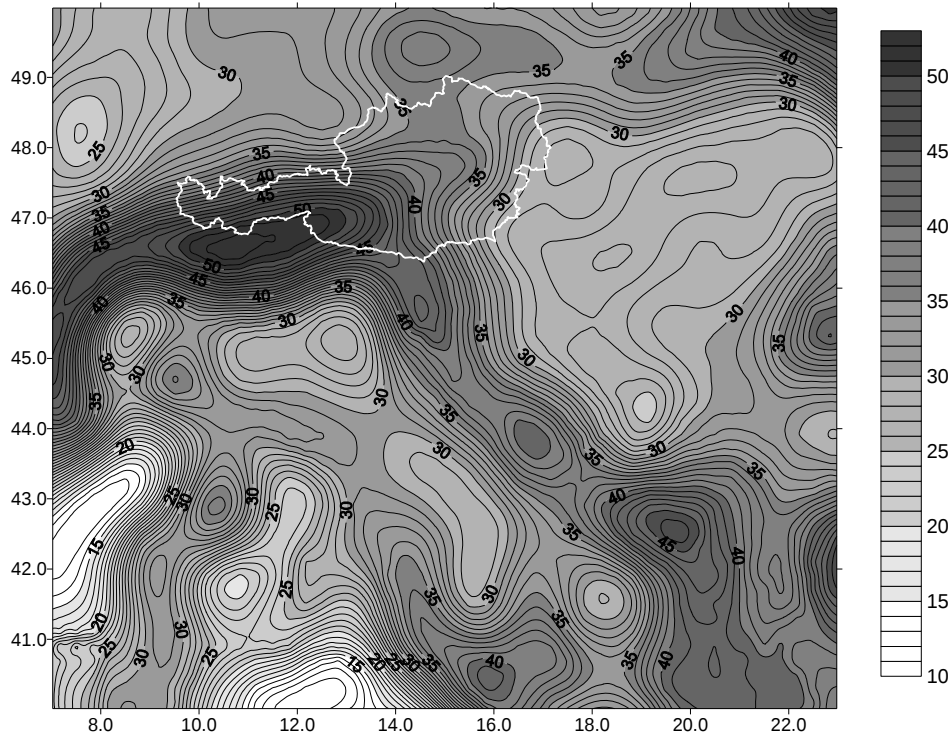


Figure 2: The coarse $90'' \times 150''$ digital seismic Moho model for the data window.
Contour interval: 1 km

An additional digital seismic Moho model has been established by interpolating the coarse one using the Kriging interpolation technique with zero error variance on a grid resolution of $11.25'' \times 18.75''$ in the latitude and the longitude directions, respectively. This resolution matches the resolution of the available fine digital height model. The fine digital seismic Moho model covers the window $42.25^\circ \text{ N} \leq \phi \leq 49.25^\circ \text{ N}$; $9.5^\circ \text{ E} \leq \lambda \leq 19.5^\circ \text{ E}$.

2.4. GPS Benchmarks

Figure 3 shows the distribution of about 40 available GPS benchmarks with known orthometric heights in Austria. It shows that most of the stations are located in the eastern part of Austria. Only few stations are located at the mountainous western part of Austria. Figure shows, more or less, a good distribution of the GPS benchmarks in Austria.

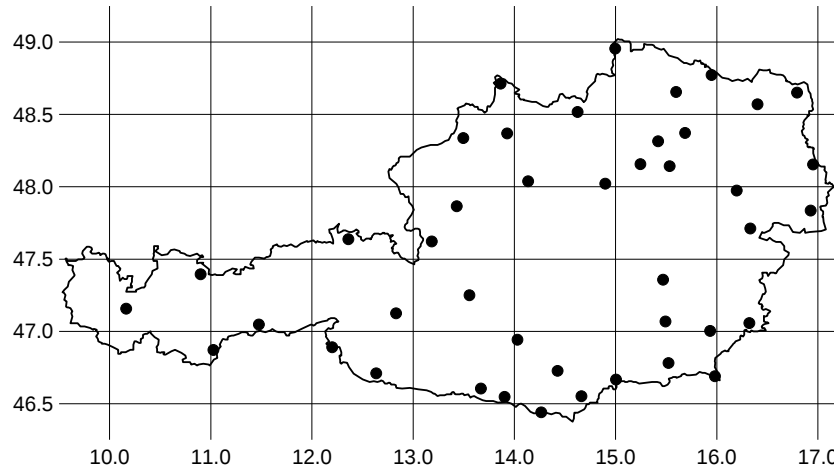


Figure 3: GPS benchmarks with known orthometric heights in Austria.

3. Data Reduction

The geoid computation in this investigation depends on the remove-restore technique. This implies that the effect of the reference field and the effect of the topography and its compensation should be removed from the observed deflection components and then restored to the computed geoid. Hence, the reduced deflections ξ and η can mathematically be expressed as follows:

$$\xi = \xi_{obs} - \xi_h - \xi_{GM} \quad , \quad (1)$$

$$\eta = \eta_{obs} - \eta_h - \eta_{GM} \quad , \quad (2)$$

where the subscript *obs* refers to the observed deflections, the subscript *h* refers to the effect of the topography and its compensation on the deflection components and the subscript *GM* refers

to the effect of the reference field on the deflection components. Alternatively, the computed geoid N can be written as:

$$N = N_{GM} + N_{def} + N_h , \quad (3)$$

where N_{GM} stands for the contribution of the global reference field, N_{def} refers to the contribution of the deflection components and N_h refers to the contribution of the topography and its compensation (the indirect effect).

The EGM96 global geopotential earth model complete to degree and order 360 has been used to compute the effect of the reference field on the deflection components ξ_{GM} and η_{GM} . The reader who may be interested in the software which can be used for such computations is kindly invited to refer to, e.g., (Rapp, 1982; Tscherning et al., 1983; Tscherning et al., 1992; Abd-Elmotaal, 1998).

The effect of the topography on the deflection components has been computed using the TC-program (Forsberg, 1984). For the effect of the compensating masses, two methods were applied. In the first method, the Airy-Heiskanen isostatic model has been used to generate the compensating masses. Then the effect of the Airy-Heiskanen compensating masses on the deflection components has been computed using the TC-program. In the second method, the seismic Moho depths were used to compute the compensating masses by the formula

$$t = T_{Moho} - T_o , \quad (4)$$

where t represents the depth of the compensating masses, T_{Moho} represents the Moho depth and T_o is the normal crustal thickness. Hence the TC-program has been modified to compute the effect of the seismic Moho depths (or rather, the seismic compensating masses t) on the deflection components.

The following parameters have been used to compute the effect of the topography and its compensation on the deflection components:

$$\rho = 2.67 \text{ g/cm}^3 , \quad (5)$$

$$T_o = 30 \text{ km} , \quad (6)$$

$$\Delta\rho = 0.3 \text{ g/cm}^3 , \quad (7)$$

where ρ stands for the constant density of topography and $\Delta\rho$ stands for the density contrast between the lower crust and the upper mantle. In the case of using the seismic Moho depths, an additional parameter set of using

$$\Delta\rho = 0.2 \text{ g/cm}^3 \quad (8)$$

has been used.

Table 1 shows the deflection components after each step of the reduction process. Here the subscript *Airy* refers to the effect of topography and its compensation when using the Airy-Heiskanen isostatic model to generate the compensating masses. The subscript *Moho* refers to the effect of topography and its compensation when using the seismic Moho depths to represent the compensating masses. The subscript 3003/3002 refers to the normal crustal thickness and the

density contrast (e.g., 3003 means $T_o = 30$ km; $\Delta\rho = 0.3$ g/cm³). In Table 1, the term $\delta\xi$ refers to the effect of the plumb line curvature on the ξ component, given by (Heiskanen and Moritz, 1967, p. 196)

$$\delta\xi = -0.17 h \sin 2\phi \quad (9)$$

where h is the height of the topography in km, ϕ is the geodetic latitude and $\delta\xi$ is in arcsecond.

Table 1: Statistics of the reduced deflection components after each reduction step (716 measured stations for both deflection components)

Deflection components	statistical parameters			
	minimum	maximum	Average	Standard deviation
	arcsecond	arcsecond	Arcsecond	arcsecond
ξ_{obs}	-14.22	24.74	3.67	6.33
$\xi_{obs} - \xi_{GM}$	-12.56	15.98	0.71	4.07
$\xi_{obs} - \xi_{GM} + \delta\xi - \xi_{Airy3003}$	-12.66	10.21	-1.61	4.68
$\xi_{obs} - \xi_{GM} + \delta\xi - \xi_{Moho3003}$	-10.36	13.15	-1.11	5.05
$\xi_{obs} - \xi_{GM} + \delta\xi - \xi_{Moho3002}$	-12.83	13.31	-1.91	5.72
η_{obs}	-15.52	17.73	2.82	5.56
$\eta_{obs} - \eta_{GM}$	-14.49	13.63	0.91	3.71
$\eta_{obs} - \eta_{GM} - \eta_{Airy3003}$	-9.49	8.93	-0.87	2.73
$\eta_{obs} - \eta_{GM} - \eta_{Moho3003}$	-10.05	7.78	0.06	2.54
$\eta_{obs} - \eta_{GM} - \eta_{Moho3002}$	-9.94	9.48	-1.06	2.82

4. Least-Squares Collocation Technique

The contribution of the deflection components on the geoid undulation N_{def} is computed in this investigation using the least-squares collocation technique. The normalized observation equation for the least-squares collocation technique can be written as (Moritz, 1980, p. 99)

$$l = t + n \quad , \quad (10)$$

where l denotes the measurements, n denotes the measuring errors (noise) and t denotes the signal part of the measurements, which is related to the earth's gravitational field. Equation (10) refers to the so-called *least-squares collocation without parameters*.

The estimated signals $\hat{s}$ are given by (ibid., p. 102)

$$\hat{s} = C_{st} (C_{tt} + C_{nn})^{-1} l \quad , \quad (11)$$

where C_{st} is the cross-covariance matrix between the measurements and the estimated signals, C_{tt} is the auto-covariance matrix of the measurements and C_{nn} is the covariance matrix of the noise. The error covariance matrix of the estimated signals E_{ss} is given by (ibid., p. 105)

$$E_{ss} = C_{ss} - C_{st} (C_{tt} + C_{nn})^{-1} C_{st}^T, \quad (12)$$

where C_{ss} is the auto-covariance matrix of the estimated signals.

It should be noted that the quantities t and s are related to the earth's gravitational field, which are linear functionals of the anomalous potential T . Hence the matrices C_{tt} , C_{ts} and C_{ss} can be obtained from the basic covariance function of the anomalous potential by covariance propagation (ibid., pp. 86–87). For practical applications, the covariance function of the gravity anomalies plays the role of the basic covariance function, from which all other covariance matrices can be derived.

5. Covariance Functions

The used covariance function model in this investigation is the well-known Tscherning-Rapp covariance function model. The global covariance function of the gravity anomalies $C_g(P, Q)$ is given by (Tscherning and Rapp, 1974, p. 29)

$$C_g(P, Q) = A \sum_{n=3}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos \psi), \quad (13)$$

where $P_n(\cos \psi)$ denotes the Legendre polynomial of degree n , ψ is the spherical distance between P and Q and A , B and s are the model parameters. A closed expression for (13) is available in (ibid., p. 45).

The Tscherning-Rapp local covariance function of gravity anomalies $C(P, Q)$ can be defined as

$$C(P, Q) = A \sum_{NN+1}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos \psi). \quad (14)$$

This expression may be written in the form (ibid., p. 62)

$$\begin{aligned} C(P, Q) &= A \sum_{n=3}^{\infty} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos \psi) - A \sum_{n=3}^{NN} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos \psi) \\ &= C_g(P, Q) - A \sum_{n=3}^{NN} \frac{n-1}{(n-2)(n+B)} s^{n+2} P_n(\cos \psi), \end{aligned} \quad (15)$$

where $C_g(P, Q)$ is given by (13) and its closed form.

Modelling the covariance function means in practice fitting the empirically determined covariance function (through its three essential parameters; the variance C_0 , the correlation length ξ and the variance of the horizontal gradient G_{oH}) to the covariance function model. Hence the four parameters A , B , NN and s are to be determined through this fitting procedure.

Table 2 shows the essential parameters of the empirical covariance function of the gravity anomalies for about 5000 gravity stations in Austria for both cases of reduction using Airy-

Heiskanen isostatic model and seismic Moho depths (using two different values of the density contrast). The values of the horizontal gradient variance G_{oH} have been roughly estimated. Table 2 shows that using different values of the density contrast has a significant effect on the reduced gravity anomalies for Austria in case of using the seismic Moho depths in the reduction process.

Table 2: Essential parameters of the empirical gravity anomaly covariance function

Reduction type	T_o	$\Delta\rho$	C_o	ξ	G_{oH}
	Km	g/cm ³	mgal ²	km	E ²
Airy-Heiskanen isostatic model	30	0.3	605.48	38.43	100
Seismic Moho depths	30	0.3	578.22	39.19	100
Seismic Moho depths	30	0.2	923.13	45.29	100

Table 3 shows the Tscherning-Rapp covariance function model parameters for the different cases of reduction using Airy-Heiskanen isostatic model and seismic Moho depths. A fixed value of

$$B = 24 \text{ exact}$$

has been chosen for all cases. This value matches the same value for the global gravity anomaly covariance function.

Table 3: Tscherning-Rapp covariance function model parameters

reduction type	T_o	$\Delta\rho$	s	A	NN
	km	g/cm ³	—	mgal ²	—
Airy-Heiskanen isostatic model	30	0.3	0.998659	630.18	89
Seismic Moho depths	30	0.3	0.998761	535.63	81
Seismic Moho depths	30	0.2	0.998229	1124.53	81

6. Geoid Computation and Comparison

The geoid for Austria has been computed using the remove-restore technique, eqs. (1), (2) and (3). The contribution of the deflection components on the geoid undulation N_{def} has been computed by the least-squares collocation technique, eq. (11). The contribution of the topography and its compensation N_h has been computed using the modified TC-program. The EGM96 global geopotential earth model has been used to compute the contribution of the reference field on the geoid undulations N_{GM} .

Figure 4 shows the absolute difference between the geoid computed by using the Airy-Heiskanen isostatic model $N_{Airy3003}$ and the geoid derived from the combination of the GPS and

levelling N_{GPS} for Austria. It shows that the difference ranges from 2.52 m to 3.94 m with an average of 2.94 m and a standard deviation of 0.33 m. The structure of the differences shows a long-wavelength behaviour, which may be considered more or less linear (especially at the western part).

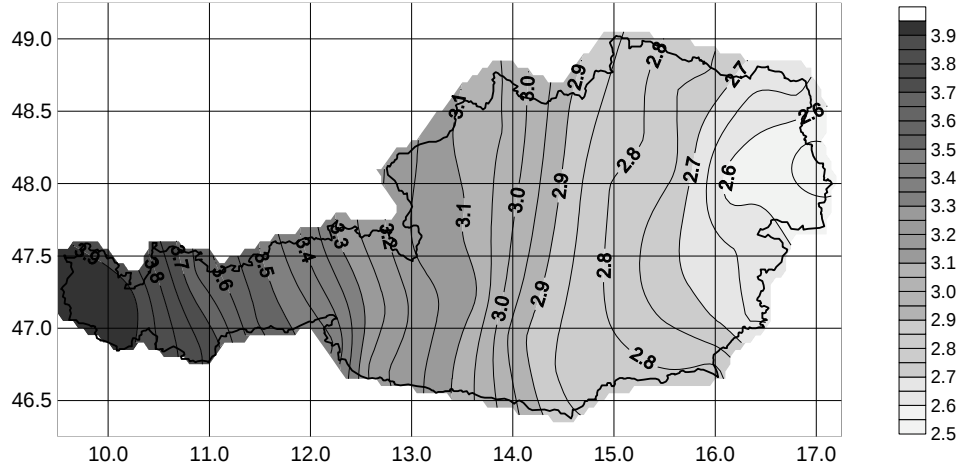


Figure 4: Absolute difference $N_{Airy3003} - N_{GPS}$ for Austria ($T_0 = 30$ km, $\Delta\rho = 0.3$ g/cm³). Contour interval: 5 cm.

Figure 5 shows the absolute difference between the geoid computed by using the seismic Moho depths $N_{Moho3003}$ and N_{GPS} for Austria. It shows that the difference ranges from 0.74 m to 4.58 m with an average of 2.94 m and a standard deviation of 0.91 m. The structure of the differences shows a more complicated structure than that in the case of the Airy-Heiskanen geoid. Figure shows a higher-order polynomial trend of the differences.

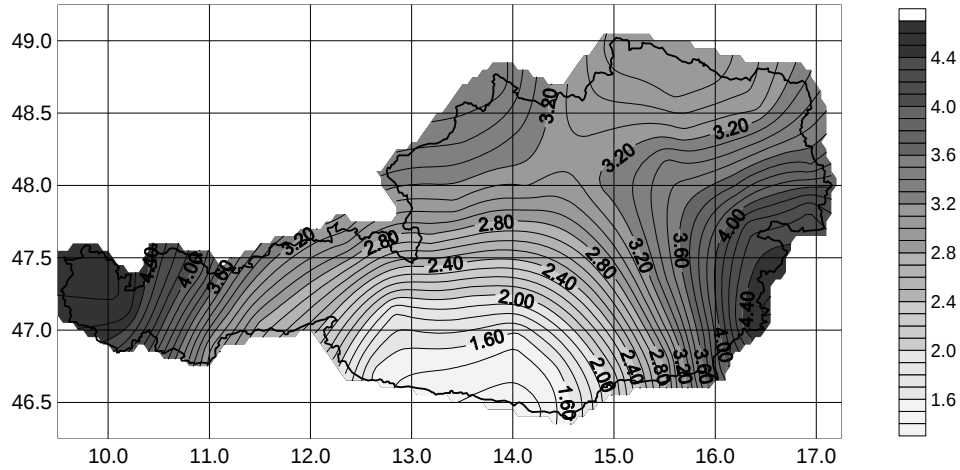


Figure 5: Absolute difference $N_{Moho3003} - N_{GPS}$ for Austria ($T_0 = 30$ km, $\Delta\rho = 0.3$ g/cm³). Contour interval: 10 cm.

Figure 6 shows the absolute difference between the geoid computed by using the seismic Moho depths $N_{Moho3002}$ and N_{GPS} for Austria. It shows that the difference ranges from 5.34 m to 9.36 m with an average of 6.46 m and a standard deviation of 0.89 m. Figure 6 shows a linear trend in the western part of Austria, and a higher-order polynomial trend in the eastern part of Austria. Hence, it shows that changing the value of the density contrast may improve the geoid fitting in case of using the seismic Moho depths in the reduction process (compare Figs. 5 and 6).

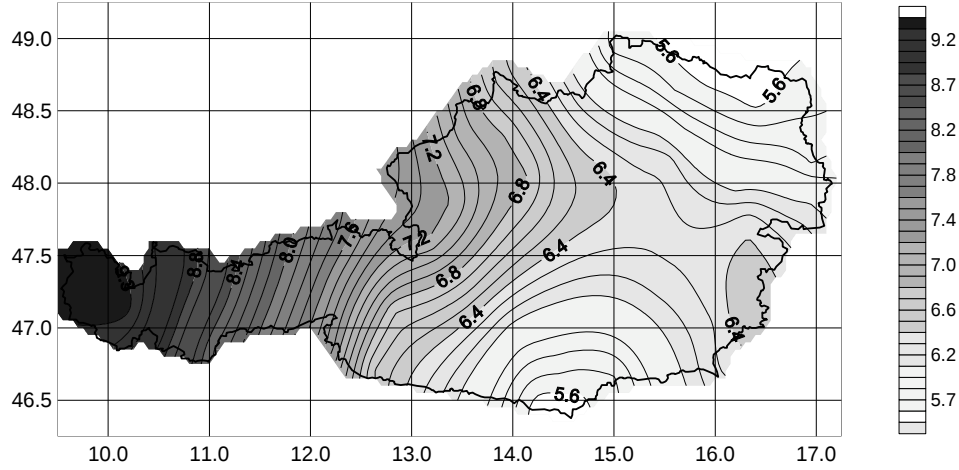


Figure 6: Absolute difference $N_{Moho3002} - N_{GPS}$ for Austria ($T_0 = 30$ km, $\Delta\rho = 0.2$ g/cm³). Contour interval: 10 cm.

The computed geoids $N_{Airy3003}$, $N_{Moho3003}$ and $N_{Moho3002}$ have been fitted to the GPS geoid N_{GPS} using about 40 GPS stations in Austria. A third degree polynomial has been removed from the absolute geoid difference in the case of the Airy geoid. A fourth degree polynomial has been removed from the absolute geoid difference in the case of both Moho geoids. The remaining differences represent the accuracy of the fitted geoid.

Table 4 lists the statistics of the remaining differences at the GPS benchmarks after removing the trend function. Table 4 shows that the standard deviation of the remaining differences in the case of the seismic Moho geoids is larger than that of the Airy-Heiskanen geoid. This indicates that implementing the seismic Moho depths does not improve the computed geoid accuracy. Table 4 also shows that changing the value of the density contrast improves the geoid accuracy in case of using the seismic Moho depths in the reduction process.

Table 4: Statistics of the remaining differences at the GPS benchmarks after removing the trend function. All values are in cm

differences in geoidal heights	statistical parameters		
	min.	max.	st. dev.
$N_{Airy3003} - N_{GPS}$	-7.1	8.0	3.6
$N_{Moho3003} - N_{GPS}$	-20.7	22.0	10.8
$N_{Moho3002} - N_{GPS}$	-17.0	16.6	8.0

It should be noted that we have assumed constant density contrast between the lower crust and the upper mantle for both cases, Airy-Heiskanen isostatic model and seismic Moho depths. In the case of Airy-Heiskanen it is obvious that the resulting Moho depths are computed by this constant density contrast and hence the reduction process using the same density contrast should give consistent results. In the case of the seismic Moho depths, the situation is completely different. The Moho depths are derived through seismic observation (observed Moho depths) in which no assumption concerning the density contrast has been made. Hence postulating a constant density contrast here falsifies the reality (in which the seismic Moho depths are referring to). Therefore, it is believed that assuming *variable* density contrast in the case of using seismic Moho depths in the geoid computation should yield better geoid accuracy.

Figure 7 shows the Airy fitted astrogeodetic geoid for Austria using $T_0 = 30$ km, $\Delta\rho = 0.3$ g/cm³. Figure 8 shows the Moho fitted astrogeodetic geoid for Austria using $T_0 = 30$ km, $\Delta\rho = 0.2$ g/cm³. Both figures have almost the same main features of the geoid. Slight differences are shown especially at the central part of Austria. The differences are in the order of only few cm.

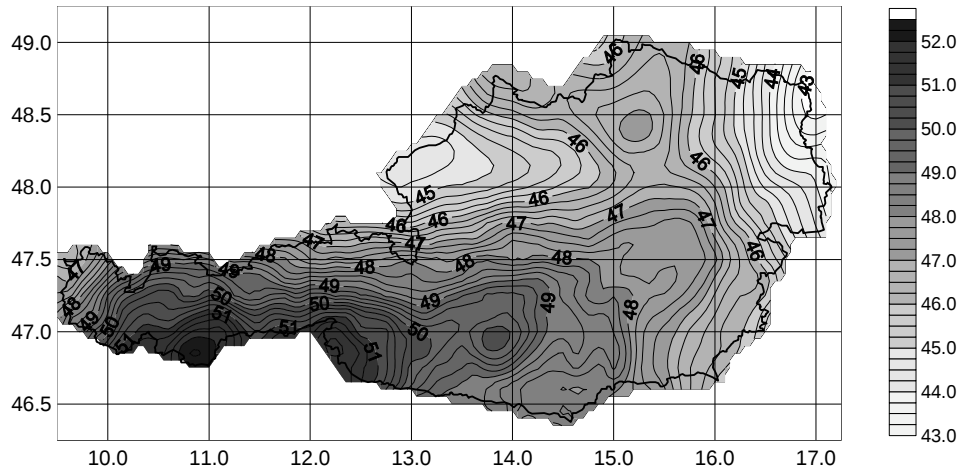


Figure 7: Airy fitted astrogeodetic geoid $N_{Airy3003}$ for Austria ($T_0 = 30$ km, $\Delta\rho = 0.3$ g/cm³). Contour interval: 25 cm.

7. Conclusion

The effect of implementing the seismic Moho depths in the geoid computation process has been investigated as the main aim of this paper. The geoid for Austria has been computed using the Airy-Heiskanen isostatic model and the seismic Moho depths. Using the seismic Moho depths in the reduction process with constant density contrast gives worse geoid accuracy. By changing the value of the density contrast, one may slightly improve the geoid accuracy in case of using the seismic Moho depths in the reduction process.

In this investigation, a constant density contrast between the lower crust and the upper mantle has been assumed. For the case of the Airy-Heiskanen isostatic model, this gives consistent results and good geoid accuracy. For the case of the seismic Moho depths, it is

believed that employing a variable density contrast between the lower crust and the upper mantle would give better geoid accuracy.

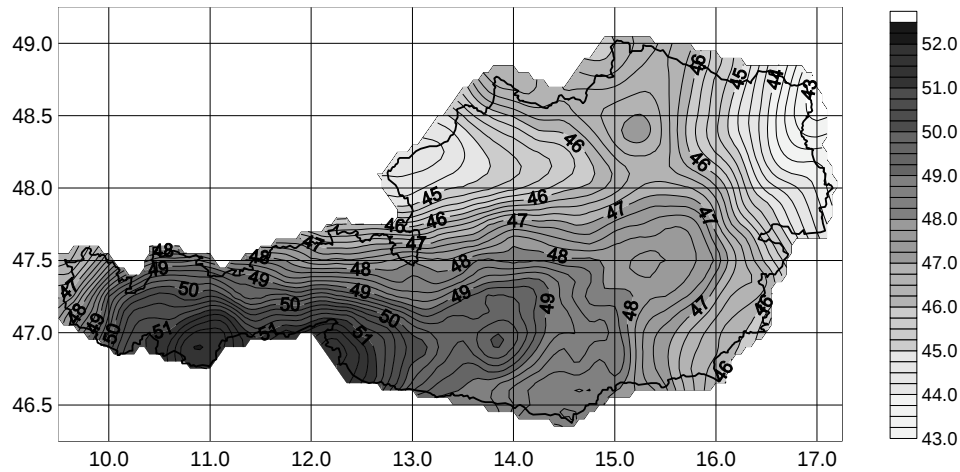


Figure 8: Moho fitted astrogeodetic geoid N_{Moho3002} for Austria ($T_0 = 30$ km, $\Delta\rho = 0.2$ g/cm³). Contour interval: 25 cm.

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Can we filter non-stationary noise from geodetic data with fast spectral (FFT) techniques ?

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Abstract

An important problem in the practical implementation of optimal spectral methods in gravity field modelling is the stationarity assumption for the input data noise and the underlying unknown signals. Such a restriction is required, according to the standard Wiener-Kolmogorov linear estimation theory for random fields, in order to obtain signal approximation algorithms of simple filtering (i.e. convolution) structure that can be evaluated very efficiently through fast Fourier transform numerical techniques. Often, the observation errors in the input data have significant spatial variations in their statistical behaviour, thus making the noise stationarity assumption unrealistic for many practical situations. Additionally, a stochastic interpretation of the true values of the various gravity field signals as random variables with similar statistical parameters is rather questionable, since they describe physical phenomena that are not random (probabilistic) and certainly do not have a uniform (stationary) behaviour over their domain. The aim of the paper is to present a spectral Wiener-type optimal noise filter that can be used in geodetic estimation problems regardless of the spatio-statistical properties of the underlying signals and the measurement errors. Some numerical examples have also been included to demonstrate the performance of the new filter under non-stationary uncorrelated noise at different sampling resolution levels, using a synthetic two-dimensional grid of noisy gravity anomaly data.

1. Introduction

The use of spectral methods in gravity field approximation problems has reached a considerable level of maturity over the past years, placing them among the standard tools of modern operational geodesy. Particularly the optimal combination and processing of regional data grids (e.g., gravity anomalies, altimetric data, digital elevation models, deflections of the vertical, etc.), for either land or marine geoid modelling, can be very efficiently implemented with frequency-domain techniques, resulting to what is commonly known in the geodetic literature as an input-output (I/O) linear estimation system. In SIDERIS [1996] a general description of such optimal spectral methods for solving physical geodesy estimation problems was given, which closely adheres to the mathematical formalism and terminology found in the textbooks by BENDAT AND PERSOL [1986, 1993]; see also SCHWARZ ET AL. [1990]. The comparison of the I/O systems theory with other linear approximation techniques traditionally used in geodesy (i.e. least-squares collocation) was discussed in detail by SANSONO AND SIDERIS [1997] within the unifying framework of the Wiener-Kolmogorov (W-K) optimal prediction theory for random fields. Many numerical studies in gravity field modelling have been performed over the last years using the versatility of optimal spectral methods, including: Wiener filtering of gravity anomaly data prior to gravimetric geoid computations [LI AND SIDERIS, 1994], optimal separation of the gravity anomaly signal from external noise (and other residual) effects for the identification of certain crustal geological features [PAWLOWSKI AND HANSEN,

1990], optimal combination of shipborne gravity and altimetric data for marine geoid modelling [LI, 1996; TZIAVOS ET AL., 1996, 1998], simultaneous optimal noise filtering of airborne gravity vector data [WU AND SIDERIS, 1995], and optimal estimation of the anomalous potential from airborne gradiometry data [VASSILIOU, 1986], among numerous other publications.

A crucial point in the practical implementation of a frequency-domain I/O estimation system for geoid computations has always been the assumption that the measurement noise in the various data sets follows a stationary probabilistic model. Such a hypothesis, in conjunction with: (i) an additional stationarity assumption for the underlying (true) gravity field signals, (ii) a mean square error (MSE) optimal criterion, and (iii) the fact that the unknown fields involved in geodetic problems are related through convolution operators^{*}, lead to simple signal estimation equations of filtering type which can be evaluated very efficiently using fast Fourier transform (FFT) numerical techniques. This provides a definite computational advantage over the equivalent space-domain algorithm of least-squares collocation, especially for large data grids with high sampling resolution. In the case of non-stationary noise and/or signals, on the other hand, the W-K optimal estimation theory can no longer be expressed in terms of straightforward linear filtering (i.e. convolution) operations, thus making the spectral algorithms of I/O systems theory quite complex and unsuitable for efficient evaluation via FFT routines; for more details, see SANSONO AND SIDERIS [1997].

The goal of this paper is to assemble a convolution-type algorithmic procedure (suitable for FFT-based implementation) that can be used in geodetic estimation problems regardless of the statistical properties of the underlying signals and the data noise. The true fields will not even be associated with any probabilistic behaviour at all, but they will be treated as arbitrary deterministic signals. Our analysis is going to be restricted for the simple case where an unknown deterministic field is observed under the masking of non-stationary random observation errors, and the desired output corresponds to an improved ('de-noised') linear interpolating model of the noisy input data. This will provide us with a modified Wiener-type filter that can be used either as an independent practical tool for geodetic data pre-processing, or as an integral component of a more general I/O linear estimation system (e.g., for optimal spectral geoid determination from gravity anomaly grids). An important point in our approach is that the sampling resolution of the input data will be explicitly taken into account within the optimization procedure, resulting in a *resolution-dependent* noise filter. The interesting interplay that exists between measurement noise and data resolution in linear signal estimation will thus become more apparent, since in the relevant geodetic literature dealing with spectral gravity field approximation problems the roles of these two important factors have not been clearly distinguished. A numerical example, using a synthetic two-dimensional gravity anomaly grid, has also been included at the end of the paper to demonstrate the performance of our optimal noise filter under non-stationary additive noise at different sampling resolution levels.

^{*} This is actually not a strict requirement in order to have estimation algorithms of convolution structure. However, it allows the efficient application of variance-covariance propagation in gravity field signals through simple filtering spectral relationships.

2. Notation and other preliminary issues

The main assumptions and the basic mathematical notation that will be used throughout the rest of this paper are presented in this section. We will follow a relatively simple approximation framework in a two-dimensional (2D) planar setting which, nevertheless, can fit nicely to many spatial geodetic estimation problems of local or regional scale. The unknown object of the estimation procedure will be modelled as a 2D deterministic signal $g(x, y)$ with compact spatial support over the real plane $\mathfrak{R}^2$. Its finite extent along the two orthogonal axes x and y is denoted by X and Y , respectively. No specific smoothing restrictions on the behaviour of the unknown field will be imposed, apart from the assumption that it posses a well-defined 2D Fourier transform, i.e.

$$G(\omega_x, \omega_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x, y) e^{-i(\omega_x x + \omega_y y)} dx dy \quad (1)$$

The two parameters ω_x and ω_y are the spatial circular frequencies along the x and y axes, respectively. A regular grid of true signal values will be denoted by $g(nh_x, mh_y)$, where the symbols h_x and h_y correspond to the orthogonal sampling intervals along the x and y directions. Without loss of generality, we can assume that the compact support of the unknown field is enclosed by the region $0 \leq x \leq X$ and $0 \leq y \leq Y$, and thus the integer sampling indices can practically be restricted within the finite range $0 \leq n \leq N-1$ and $0 \leq m \leq M-1$, where $X = (N-1)h_x$ and $Y = (M-1)h_y$. The 2D Fourier transform of such a noiseless signal grid will be denoted by $\overline{G}(\omega_x, \omega_y)$ and it is given by the summation formula [DUDGEON AND MERSEREAU, 1984]

$$\begin{aligned} \overline{G}(\omega_x, \omega_y) &= \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} g(nh_x, mh_y) e^{-i(nh_x \omega_x + mh_y \omega_y)} \\ &= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} g(nh_x, mh_y) e^{-i(nh_x \omega_x + mh_y \omega_y)} \\ &= \frac{1}{h_x h_y} \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} G(\omega_x + \frac{2\pi k}{h_x}, \omega_y + \frac{2\pi l}{h_y}) \end{aligned} \quad (2)$$

where the last part in the above equation expresses the aliasing effect on the Fourier transform of the original continuous signal. The overbar symbol will generally be used to indicate a periodic function, and the lower case letters n, m, k, l are always reserved to denote integer numbers. The input data obtained from the unknown field is given in a discrete gridded form according to the linear observation equation

$$d(nh_x, mh_y) = g(nh_x, mh_y) + v(nh_x, mh_y) \quad (3)$$

where $v(nh_x, mh_y)$ is a 2D random noise sequence that is generally assumed non-stationary. The associated stochastic model used to describe the behaviour of the measurement noise, in terms of second-order moment information, is defined by the following equations:

$$E\{v(nh_x, mh_y)\} = 0 \quad (4a)$$

$$E\{v^2(nh_x, mh_y)\} = \sigma_v^2(nh_x, mh_y) = \sigma_v[(nh_x, mh_y)(nh_x, mh_y)] \quad (4b)$$

$$E\{v(nh_x, mh_y)v(kh_x, lh_y)\} = \sigma_v[(nh_x, mh_y)(kh_x, lh_y)] \quad (4c)$$

where E is the probabilistic expectation operator. The symbol $\sigma_v^2(\cdot)$ denotes the noise variance at a specific data point on the real plane, whereas $\sigma_v[(\cdot)(\cdot)]$ corresponds to the noise covariance (CV) between two data points. We will also use the symbol $\bar{V}(\omega_x, \omega_y)$ for the 2D Fourier transform of the random observation errors, which is defined as

$$\begin{aligned} \bar{V}(\omega_x, \omega_y) &= \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} v(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \\ &= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} v(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \end{aligned} \quad (5)$$

Similarly, the 2D Fourier transform of the gridded data is given by the equation

$$\begin{aligned} \bar{D}(\omega_x, \omega_y) &= \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} d(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \\ &= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \\ &= \bar{G}(\omega_x, \omega_y) + \bar{V}(\omega_x, \omega_y) \end{aligned} \quad (6)$$

Note that the noise signal is zero outside the input data grid (NM points), since the unknown field $g(x, y)$ has been assumed to have finite spatial support and thus no measurements are performed outside this region.

3. Problem formulation

Two basic properties will be imposed a-priori in the estimation algorithm, namely linearity and translation-invariance. The reason for introducing the second property is to obtain a signal estimate $\hat{g}(x, y)$ that is independent of the reference system used to describe the position of the data points. Stated in a simplified way, if we change the origin of the 2D reference system xy on the real plane by arbitrary translations (without ‘moving’ the unknown field or the associated data grid), we want the new signal approximation to be just a translated version of the initial estimate that was obtained in the original reference system.

The justification of such a modelling choice relies basically on simple logic and mathematical intuition, and it is not affected by the spatio-statistical properties of the true signal and noise involved in the specific approximation problem. If one chooses to follow a non translation-invariant methodology (e.g., Tikhonov regularization in a Hilbert space with a non-homogeneous reproducing kernel), he should at least be able to explain physically the dependence of the output signal estimate on the origin of the coordinate system used to reference the unknown field and its discrete input data. Note that the translation-invariance condition has often been applied in the theoretical formulation of optimal estimation methods using errorless discrete data [SANSO, 1980; KOTSAKIS, 2000a], although its justification is not altered by the presence of noise in the input observations.

Based on the two assumptions of linearity and translation-invariance, the signal estimation formula will have the general convolution-type expression

$$\hat{g}(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d(nh_x, mh_y) \xi_h(x - nh_x, y - mh_y) \quad (7)$$

where $\xi_h(x, y)$ is a 2D filtering kernel that needs to be determined in some optimal sense. The subscript h is used to indicate that the estimation kernel will generally depend on the data resolution levels, h_x and h_y . The above equation can be illustrated through the linear I/O system shown in Figure 1.

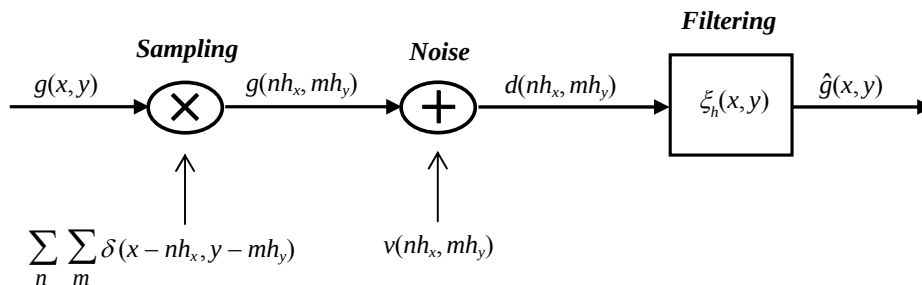


Figure 1. Linear and translation-invariant signal estimation from discrete noisy data.

4. Minimization of the noise-dependent signal estimation error

The output signal error produced by the filtering formula in Eq. (7) can generally be decomposed into two components

$$\begin{aligned} e(x, y) &= g(x, y) - \hat{g}(x, y) \\ &= e_h(x, y) + e_v(x, y) \end{aligned} \quad (8)$$

where $e_h(x, y)$ is the part of the total estimation error caused from the use of discrete data with finite sampling resolution (aliasing error), and $e_v(x, y)$ is the additional part due to the noise presence in the signal samples. In the absence of any noise from the discrete input data, the best we could do is to obtain just an *interpolated model* $\tilde{g}(x, y)$ for the unknown field that will depend on the true signal values at the given spatial resolution. It will be assumed that such a noiseless signal model is given in terms of a linear and translation-invariant formula, as follows:

$$\tilde{g}(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} g(nh_x, mh_y) \varphi_h(x - nh_x, y - mh_y) \quad (9)$$

where $\varphi_h(x, y)$ is some basic interpolating kernel that generally depends on the sampling intervals h_x and h_y . The noise-dependent estimation error will be measured with respect to such a linear interpolating model for the unknown field, i.e.

$$e_v(x, y) = \tilde{g}(x, y) - \hat{g}(x, y) \quad (10a)$$

whereas the (pure) aliasing error is

$$e_h(x, y) = g(x, y) - \tilde{g}(x, y) \quad (10b)$$

The specific form of the modelling kernel in Eq. (9) is irrelevant for the purpose of this paper. A very popular choice that covers many different linear interpolating schemes, including band-limited (Shannon) interpolation, spline-based interpolation and also more general wavelet approximation models, is based on the use of certain scaling functions $\varphi(x, y)$ which adapt to the data grid resolution through a dilation operation [UNSER, 2000], i.e.

$$\varphi_h(x, y) = \varphi\left(\frac{x}{h_x}, \frac{y}{h_y}\right) \quad (11)$$

The optimal determination of such interpolating scaling kernels, and their connection with the statistical collocation and wavelet theory, were discussed in KOTSAKIS [2000a, b]. The behaviour of the aliasing error term $e_h(x, y)$ for different choices of the scaling function $\varphi(x, y)$ and the data resolution level was also studied in KOTSAKIS [2000b] and KOTSAKIS AND SIDERIS [2000]. For the purpose of this paper, it is sufficient to consider $\varphi_h(x, y)$ in Eq. (9) as an arbitrarily chosen interpolating kernel with a well-defined Fourier transform $\Phi_h(\omega_x, \omega_y)$, which is used to obtain a continuous signal approximation in the absence of any noise from the discrete input data. In addition, it can be assumed that $\varphi_h(x, y)$ is such that: (i) the signal expansion in Eq. (9) is always stable, and (ii) the aliasing error component $e_h(x, y)$ vanishes as the data resolution increases; for more details, see KOTSAKIS [2000b] and BLU AND UNSER [1999].

The filtering kernel in Eq. (7) will be determined by minimizing the noise-dependent part of the total signal error. In this way, the signal estimation problem is essentially reduced to a problem of finding an optimal modification for the interpolating kernel $\varphi_h(x, y)$ that minimizes the effect of the propagated data noise in the final output field $\hat{g}(x, y)$. The optimization procedure will be carried out in the frequency domain using the familiar MSE criterion

$$P_{e_v}(\omega_x, \omega_y) = E\left\{\left|E_v(\omega_x, \omega_y)\right|^2\right\} = \text{minimum} \quad (12)$$

where $E_v(\omega_x, \omega_y)$ is the 2D Fourier transform of $e_v(x, y)$, and $P_{e_v}(\omega_x, \omega_y)$ is the (noise-dependent) mean error power spectrum of the estimated output signal. From Eqs. (7), (9) and (10a), we have that

$$\begin{aligned} E_v(\omega_x, \omega_y) = & \Phi_h(\omega_x, \omega_y) \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} g(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \\ & - \Xi_h(\omega_x, \omega_y) \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \end{aligned} \quad (13a)$$

where $\Xi_h(\omega_x, \omega_y)$ is the Fourier transform of the unknown filtering kernel $\xi_h(x, y)$. Using the shorthand notation according to Eqs. (2) and (6), the last equation takes the form

$$E_v(\omega_x, \omega_y) = \Phi_h(\omega_x, \omega_y) \bar{G}(\omega_x, \omega_y) - \Xi_h(\omega_x, \omega_y) \bar{G}(\omega_x, \omega_y) - \Xi_h(\omega_x, \omega_y) \bar{V}(\omega_x, \omega_y) \quad (13b)$$

By multiplying the above expression with its complex conjugate and taking the expected value, we can finally obtain the (noise-dependent) mean error power spectrum of the output signal, as follows:

$$\begin{aligned}
P_{e_v}(\omega_x, \omega_y) = & \Phi_h(\omega_x, \omega_y) \Phi_h^*(\omega_x, \omega_y) |\overline{G}(\omega_x, \omega_y)|^2 \\
& - \Phi_h(\omega_x, \omega_y) \Xi_h^*(\omega_x, \omega_y) |\overline{G}(\omega_x, \omega_y)|^2 \\
& - \Phi_h^*(\omega_x, \omega_y) \Xi_h(\omega_x, \omega_y) |\overline{G}(\omega_x, \omega_y)|^2 \\
& + \Xi_h(\omega_x, \omega_y) \Xi_h^*(\omega_x, \omega_y) |\overline{G}(\omega_x, \omega_y)|^2 \\
& + \Xi_h(\omega_x, \omega_y) \Xi_h^*(\omega_x, \omega_y) \overline{P}_v(\omega_x, \omega_y)
\end{aligned} \tag{14}$$

where * denotes complex conjugation, and the auxiliary term $\overline{P}_v(\omega_x, \omega_y)$ corresponds to the noise ‘power spectral density (PSD)’ function

$$\overline{P}_v(\omega_x, \omega_y) = E\left\{\overline{V}(\omega_x, \omega_y) \overline{V}^*(\omega_x, \omega_y)\right\} = E\left\{|\overline{V}(\omega_x, \omega_y)|^2\right\} \tag{15}$$

For the derivation of the result in Eq. (14) we have used the fact that $E\{\overline{V}(\omega_x, \omega_y)\} = 0$, in accordance with the zero-mean stochastic model introduced for the data noise in Eq. (4a). It can easily be verified that the optimal estimation filter will be finally given by the following formula:

$$\begin{aligned}
\Xi_h(\omega_x, \omega_y) &= \frac{|\overline{G}(\omega_x, \omega_y)|^2}{|\overline{G}(\omega_x, \omega_y)|^2 + \overline{P}_v(\omega_x, \omega_y)} \Phi_h(\omega_x, \omega_y) \\
&= \overline{W}(\omega_x, \omega_y) \Phi_h(\omega_x, \omega_y)
\end{aligned} \tag{16}$$

5. The separable structure of the optimal estimation filter

The final result in Eq. (16) indicates that the optimal estimation procedure can be decomposed into two individual steps which are connected in a linear cascading manner. The first step, expressed by the periodic filter component $\overline{W}(\omega_x, \omega_y)$, has the role of ‘de-noising’ the discrete input data using information about the average behaviour of the input noise and the unknown field at the given resolution level. The second filter component $\Phi_h(\omega_x, \omega_y)$, on the other hand, is solely used to obtain a continuous representation for the output signal based on an a-priori selected interpolating/modelling kernel $\varphi_h(x, y)$. These two basic steps of the optimal estimation procedure are illustrated in the linear I/O system of Figure 2.

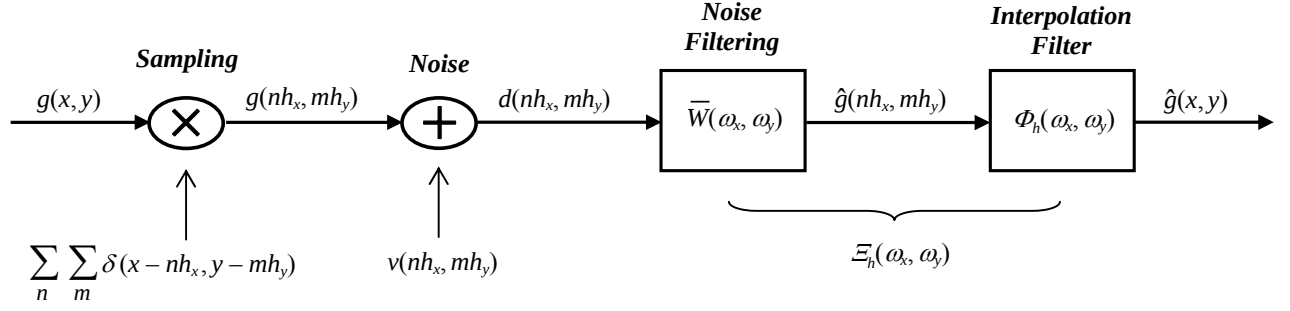


Figure 2. The cascading structure of the optimal linear estimation filter.

As it can be seen from the above figure, it is not really necessary to ‘modify’ the interpolating kernel $\phi_h(x, y)$ of the reference signal model in Eq. (9) when dealing with noisy input data. The optimization of the noise-dependent output error adds only an intermediate periodic filter that is applied to the original data grid, and it produces a new estimated signal sequence $\hat{g}(nh_x, mh_y)$ from which the effect of the random observational errors has been minimized in a certain translation-invariant MSE sense. We can then use this new sequence as input to the basic interpolating model of Eq. (9), in order to get a continuous (also linear and translation-invariant) approximation of the unknown field at the given resolution level. It should be noted that the interpolation filter in Figure 2 can be also optimized by following a separate MSE approach that takes into account only the noise-free error component $e_h(x, y)$, as it is described in KOTSAKIS [2000a, b].

The structure of the optimal noise filter in Eq. (16) is very similar to the classic Wiener estimation filter, since they are both defined in terms of a signal-to-noise ratio (SNR) expression. However, there do exist conceptual differences between the two filtering schemes because in our formulation: (i) the unknown field has been modelled as a deterministic (instead of stochastic) signal, and (ii) the additive data noise has not been restricted to being stationary. Therefore, it is important to clarify what is the exact meaning of the two frequency-domain terms that appear in the SNR expression of our noise filter $\bar{W}(\omega_x, \omega_y)$. From Eq. (16), we have that

$$\begin{aligned}
 \bar{W}(\omega_x, \omega_y) &= \frac{|\bar{G}(\omega_x, \omega_y)|^2}{|\bar{G}(\omega_x, \omega_y)|^2 + \bar{P}_v(\omega_x, \omega_y)} \\
 &= \frac{\frac{1}{NM} |\bar{G}(\omega_x, \omega_y)|^2}{\frac{1}{NM} |\bar{G}(\omega_x, \omega_y)|^2 + \frac{1}{NM} \bar{P}_v(\omega_x, \omega_y)} \\
 &= \frac{\bar{A}(\omega_x, \omega_y)}{\bar{A}(\omega_x, \omega_y) + \bar{B}(\omega_x, \omega_y)}
 \end{aligned} \tag{17}$$

where NM is the total number of points in the input data grid. The two auxiliary functions, $\bar{A}(\omega_x, \omega_y)$ and $\bar{B}(\omega_x, \omega_y)$, in the last equation correspond to the Fourier transforms of two associated sequences which have the CV-like expressions

$$a(nh_x, mh_y) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} g(kh_x, lh_y) g((k+n)h_x, (l+m)h_y) \quad (18a)$$

and

$$b(nh_x, mh_y) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v[(kh_x, lh_y) ((k+n)h_x, (l+m)h_y)] \quad (18b)$$

respectively. The first sequence in Eq. (18a) can easily be identified as the discrete spatial CV function of the true deterministic signal at the given data resolution level, and thus the term $\bar{A}(\omega_x, \omega_y)$ in Eq. (17) corresponds to the power spectrum of the true signal values $g(nh_x, mh_y)$. Note that $a(nh_x, mh_y)$ contains less spatio-statistical information than the continuous signal CV function, since it takes into account only the discrete values of the unknown field at a certain resolution.

The second sequence in Eq. (18b), on the other hand, does not exactly correspond to the discrete noise CV function and, as a result, the frequency-domain quantity $\bar{B}(\omega_x, \omega_y)$ in Eq. (17) should not generally be viewed as the PSD of the data noise. Such an interpretation is possible only in the special case where the input noise is stationary. Indeed, in such situation the noise covariance σ_v between two arbitrary data points with coordinates (kh_x, lh_y) and $((k+n)h_x, (l+m)h_y)$ becomes a function of their coordinate differences only, which are obviously equal to (nh_x, mh_y) . Therefore, σ_v can be taken outside of the summation operator in Eq. (18b), leaving the summation result equal to NM .

In the more general case of non-stationary noise, the sequence $b(nh_x, mh_y)$ can be interpreted as a ‘mean’ CV function of the random observation errors. Its value at the origin gives an average indication of the noise level at every point of the input data grid, i.e.

$$\begin{aligned} b(0, 0) &= \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v[(kh_x, lh_y) (kh_x, lh_y)] \\ &= \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v^2(kh_x, lh_y) \end{aligned} \quad (19)$$

whereas its values $b(nh_x, mh_y)$ at other points correspond to ‘averages’ of the noise covariance over pairs of data points with coordinate differences (nh_x, mh_y) . Note that both sequences in Eqs. (18a) and (18b) are always symmetric, and they take zero values outside the range $-(N-1) \leq n \leq (N-1)$ and $-(M-1) \leq m \leq (M-1)$ due to the finite spatial support of the input signal $g(x, y)$. Also, in practice the computation of the optimal noise filter $\bar{W}(\omega_x, \omega_y)$ can take place only at a discrete finite set of frequency values by using the FFTs of the two sequences given in Eqs. (18a) and (18b).

6. Additional remarks

A key point in our estimation procedure was the decomposition of the output signal error into an aliasing part $e_h(x, y)$ and a noise-dependent part $e_v(x, y)$. The advantage of such a partition is that it allows us to study and optimize individually the effects of the finite data resolution and the input noise on the final signal estimate $\hat{g}(x, y)$, using appropriate error measures and criteria for each case. It should be kept in mind that the error component $e_h(x, y)$ is a purely deterministic signal whose average behaviour (at a given data resolution level) can only be modelled in a spatio-statistical sense through the concept of different ‘sampling phases’ [KOTSAKIS, 2000b; KOTSAKIS AND SIDERIS, 2000], whereas the noise-dependent error term $e_v(x, y)$ is a random signal whose average behaviour is described probabilistically with the notion of different ‘experiment repetitions’ (expectation operator E).

An important aspect for geodetic applications is also the numerical evaluation of the optimal noise filter $\bar{W}(\omega_x, \omega_y)$ given in Eq. (17). Although the noise PSD-like term $\bar{P}_V(\omega_x, \omega_y)$ can always be determined from the known noise variances and covariances using FFT techniques, the signal ‘PSD’ term $|\bar{G}(\omega_x, \omega_y)|^2$ is generally unknown in practice. In order to overcome this difficulty, we can use the power spectrum of the available noisy data $d(nh_x, nh_y)$ to infer the behaviour of the signal ‘PSD’ function that is needed in the computation of the SNR-type noise filter. From Eq. (6), we can express the power spectrum of the data values as follows:

$$\begin{aligned} |\bar{D}(\omega_x, \omega_y)|^2 &= (\bar{G}(\omega_x, \omega_y) + \bar{V}(\omega_x, \omega_y)) (\bar{G}(\omega_x, \omega_y) + \bar{V}(\omega_x, \omega_y))^* \\ &= |\bar{G}(\omega_x, \omega_y)|^2 + \bar{G}(\omega_x, \omega_y) \bar{V}^*(\omega_x, \omega_y) + \bar{G}^*(\omega_x, \omega_y) \bar{V}(\omega_x, \omega_y) + |\bar{V}(\omega_x, \omega_y)|^2 \end{aligned} \quad (20a)$$

and by applying the expectation operator to the above formula, we finally get

$$\begin{aligned} E\left\{|\bar{D}(\omega_x, \omega_y)|^2\right\} &= |\bar{G}(\omega_x, \omega_y)|^2 + E\left\{|\bar{V}(\omega_x, \omega_y)|^2\right\} \\ &= |\bar{G}(\omega_x, \omega_y)|^2 + \bar{P}_V(\omega_x, \omega_y) \end{aligned} \quad (20b)$$

The unknown signal term $|\overline{G}(\omega_x, \omega_y)|^2$ can now be determined empirically through Eq. (20b) by taking the available realization $|\overline{D}(\omega_x, \omega_y)|^2$ of the data power spectrum as an estimate of its expected value.

Finally, let us briefly comment on the effect of the data sampling resolution on the noise-dependent output signal error. The use of the optimal estimation filter according to Eq. (16) leads to the following expression for the mean power spectrum of the signal error $e_v(x, y)$:

$$\begin{aligned} P_{e_v}(\omega_x, \omega_y) &= \frac{|\overline{G}(\omega_x, \omega_y)|^2}{|\overline{G}(\omega_x, \omega_y)|^2 + \overline{P}_v(\omega_x, \omega_y)} \overline{P}_v(\omega_x, \omega_y) |\Phi_h(\omega_x, \omega_y)|^2 \\ &= \overline{W}(\omega_x, \omega_y) \overline{P}_v(\omega_x, \omega_y) |\Phi_h(\omega_x, \omega_y)|^2 \end{aligned} \quad (21)$$

The above formula can easily be derived by substituting the optimal result of Eq. (16) into Eq. (14). Note that the noise-dependent error component $e_v(x, y)$ is not completely unaffected by the input data resolution, but it actually depends on it. In order to see more clearly this important implicit relationship, let us adopt a rather general model for the reference interpolating kernel $\varphi_h(x, y)$. In particular, we shall consider the case where the noiseless signal interpolating model has the scaling form

$$\varphi_h(x, y) = \varphi\left(\frac{x}{h_x}, \frac{y}{h_y}\right) \quad (22)$$

where $\varphi(x, y)$ is some basic scaling kernel (e.g., *sinc* function in the case of band-limited approximation). Taking into account the fundamental scaling property of the 2D Fourier transform [DUDGEON AND MERSEREAU, 1984], the frequency-domain expression in Eq. (21) can now be written as

$$P_{e_v}(\omega_x, \omega_y) = (h_x h_y)^2 \overline{W}(\omega_x, \omega_y) \overline{P}_v(\omega_x, \omega_y) |\Phi(h_x \omega_x, h_y \omega_y)|^2 \quad (23)$$

where $\Phi(\omega_x, \omega_y)$ is the Fourier transform of the scaling function $\varphi(x, y)$. Avoiding further mathematical details, it can be seen from the last equation that the noise-dependent estimation error will decrease as the resolution level of the input data increases (i.e. smaller sampling intervals h_x and h_y). Such a result is not surprising and it just confirms the (well-known from signal analysis theory) fact that oversampling leads to noise reduction in the final signal estimate (see, e.g., CVETKOVIĆ AND VETTERLI, 1998). An additional indication of this interesting behaviour is also given in the next section using simulated numerical data.

7. Numerical simulations

In this section we will test numerically the noise filtering component of the optimal estimation kernel that was derived in Sect. 4; see Eq. (16). Hence, we shall not implement the whole I/O linear estimation system shown in Figure 2, but we will restrict our attention only on its first ‘de-noising’ part that transforms the original input data into an improved filtered signal sequence. The second interpolatory step through the use of a reference modelling kernel $\varphi_h(x, y)$ will be omitted.

A two-dimensional deterministic signal $g(x, y)$, assumed to represent some local gravity anomaly field, was initially synthesized using a truncated Fourier series expansion with a record length of 200×200 km (see Figures 3 and 4). The continuous signal was sampled at various uniform resolution levels $h_x \times h_y$ to obtain noiseless gridded values $g(nh_x, mh_y)$. Four different sampling resolutions were selected, namely 0.5×0.5 , 1.0×1.0 , 2.5×2.5 and 5.0×5.0 km. The sample statistics of the true signal values are given in Table 1. All the signal grids at each resolution level were partitioned into four equal blocks-quadrants, labeled as northwest (NW), northeast (NE), southwest (SW) and southeast (SE). The simulated data noise, which is going to be added to the true signal values, will have a different stochastic behaviour in each of the four grid quadrants.

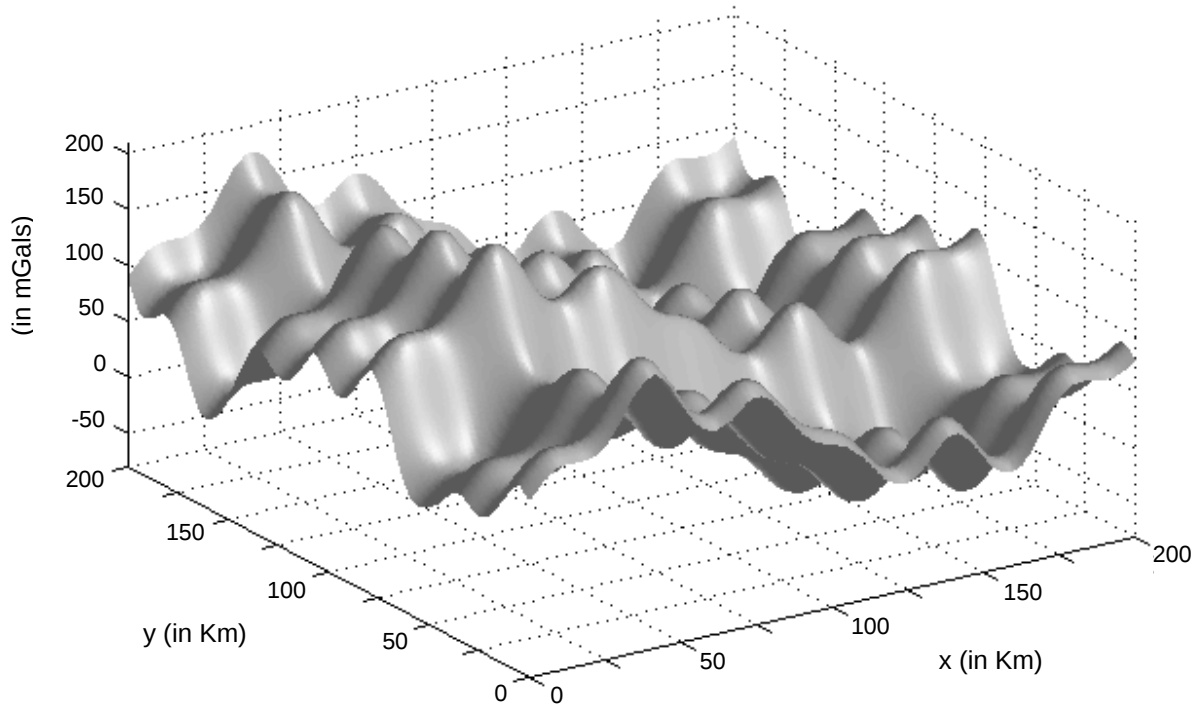


Figure 3. The original (simulated) gravity anomaly field shown as a 3D surface plot.

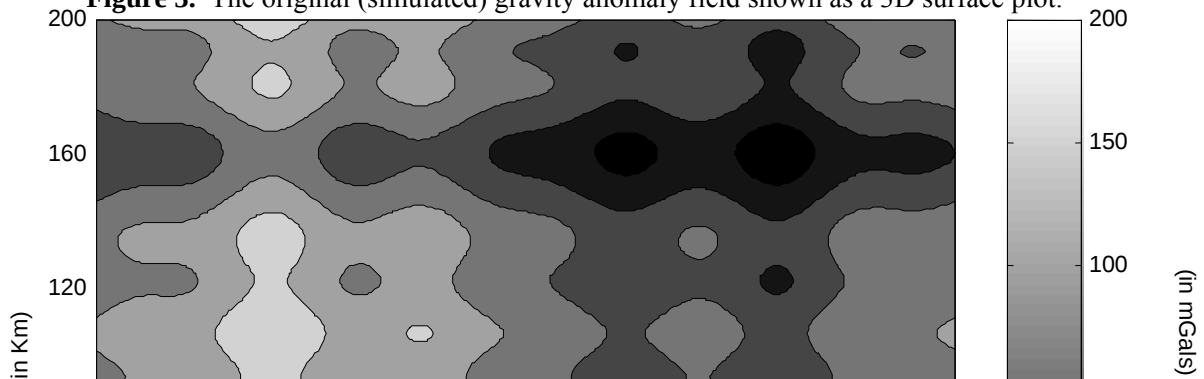


Figure 4. The simulated gravity anomaly field shown as a black and white image plot.

Table 1. Statistics of the true signal values at various sampling resolution levels (in mGals).

Data resolution (in Km)	0.5 × 0.5	1.0 × 1.0	2.5 × 2.5	5.0 × 5.0
Max	206.64	206.55	206.30	206.30
Mean	62.33	62.39	62.56	62.84
Min	-75.49	-75.36	-75.02	-74.50
Std	52.12	52.07	51.92	51.67
RMS	81.26	81.26	81.29	81.36

A zero-mean noise sequence $v(nh_x, mh_y)$ was added to the samples of the true signal in order to create the input data at every resolution level, according to the form $d(nh_x, mh_y) = g(nh_x, mh_y) + v(nh_x, mh_y)$. The noise values originated from a non-stationary and uncorrelated Gaussian stochastic process, using the routines for random number generation of the MATLAB™ software package. Note that the noise values were generated separately at each resolution, instead of simply decimating the noise sequence with the smaller sampling interval. The noise variance $\sigma_v^2(nh_x, mh_y)$ was constant within each quadrant (NW, NE, SW and SE) of the data grids, with its values set to 144 mGals², 9 mGals², 144 mGals² and 49 mGals², respectively (see Figure 5). The sample statistics of the total noise sequence $v(nh_x, mh_y)$ at every resolution level are given in Table 2, whereas the individual sample statistics of the noise values in the four different parts of the data grids are shown in Tables 3a and 3b.

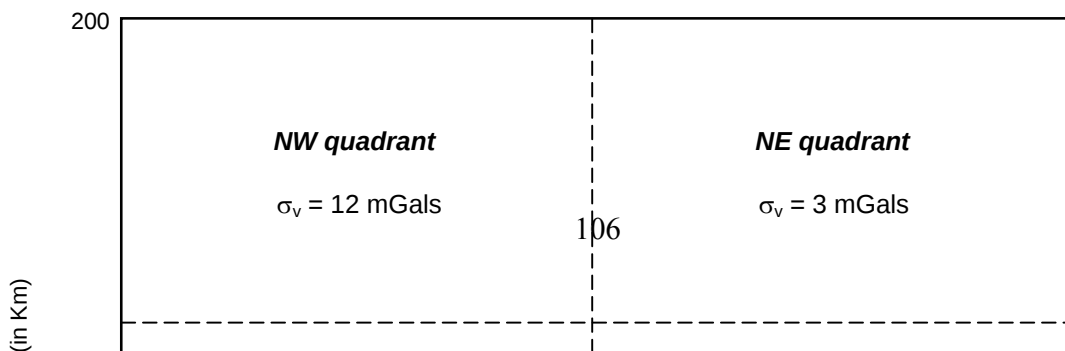


Figure 5. Partition of the input data grid into four equal quadrants. The standard deviation of the uncorrelated random noise that is added to the true signal samples varies spatially according to the simulating values shown in the figure.

Table 2. Statistics of the total additive noise at various sampling resolution levels (in mGals).

Data resolution (in Km)	0.5 × 0.5	1.0 × 1.0	2.5 × 2.5	5.0 × 5.0
Max	55.64	53.15	43.24	35.46
Mean	0.00	-0.01	0.05	-0.10
Min	-52.61	-52.71	-41.22	-38.84
Std	9.32	9.37	9.42	9.12
RMS	9.32	9.37	9.42	9.12

Table 3a. Statistics of the additive noise at different sampling resolutions in the four quadrants of the data grids (in mGals).

Data resolution (in Km)	0.5 × 0.5				1.0 × 1.0			
Grid quadrants	NW	SW	NE	SE	NW	SW	NE	SE
Max	49.61	55.64	12.47	27.37	53.15	43.04	11.45	25.88
Mean	-0.04	0.02	0.02	-0.01	0.03	0.01	0.02	-0.08
Min	-52.61	-46.34	-12.14	-27.14	-52.71	-47.87	-10.24	-28.58
Std	11.99	12.05	3.00	6.98	12.07	12.08	2.95	7.01
RMS	11.99	12.05	3.00	6.98	12.07	12.08	2.95	7.01

Table 3b. Statistics of the additive noise at different sampling resolution in the four quadrants of the data grids (in mGals).

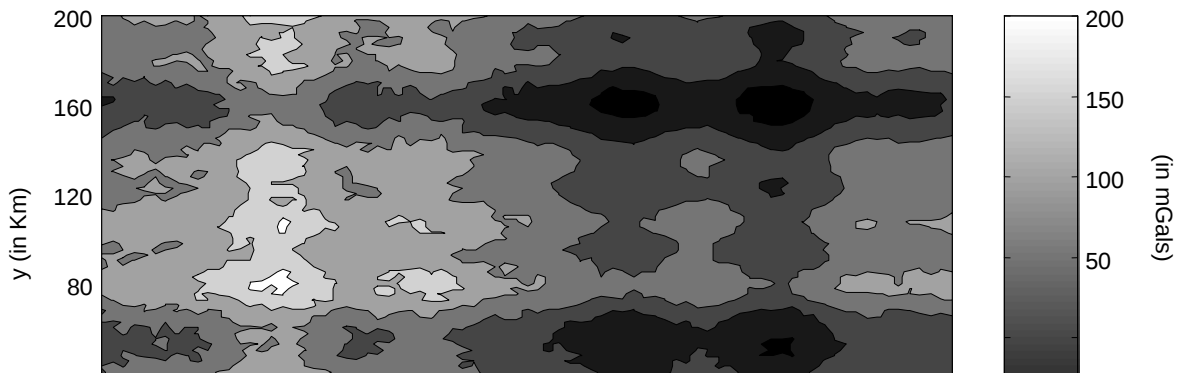
Data resolution (in Km)	2.5 × 2.5				5.0 × 5.0			
Grid quadrants	NW	SW	NE	SE	NW	SW	NE	SE
Max	41.39	43.24	10.97	23.92	35.46	32.49	9.57	18.04
Mean	0.45	-0.24	0.10	-0.10	-0.92	0.11	0.29	0.10
Min	-40.49	-41.22	-11.15	-22.53	-34.00	-38.84	-9.81	-19.02
Std	11.94	12.19	3.07	7.12	11.86	11.36	3.07	6.84
RMS	11.95	12.20	3.07	7.12	11.90	11.36	3.08	6.84

The optimal noise filter $\bar{W}(\omega_x, \omega_y)$ was computed through an FFT algorithm at each resolution level $h_x \times h_y$, according to the SNR expression given in Eq. (17). It was then multiplied by the FFT of the noisy gridded data $d(nh_x, mh_y)$, and the result was finally transformed back to the space domain as an estimated ('de-noised') signal sequence $\hat{g}(nh_x, mh_y)$. The original noisy data grids are plotted in Figure 6 for some selective sampling resolution levels, whereas the corresponding filtered signal values are shown in Figure 7. The differences between the true signal samples and the estimated signal values are also shown in Figure 8, and their statistics are given in Table 4 below.

It is interesting to observe that the output estimation error of the filtered signal values $\hat{g}(nh_x, mh_y)$ is decreasing, as the data sampling intervals h_x, h_y become smaller. This is evident from the comparison of the three graphs shown in Figure 8, as well as from the error RMS values given in Table 4, and it confirms our earlier theoretical remark at the end of Sect. 6. Note also that the parts of the input data grid with the highest noise level (NW and SW quadrants) show larger estimation errors after the data filtering than the other two grid quadrants, as it should be intuitively expected (see also the values given in Tables 5a and 5b).

Table 4. Statistics of the differences between the true and the filtered signal values at various sampling resolution levels (in mGals).

Data resolution (in Km)	0.5 × 0.5	1.0 × 1.0	2.5 × 2.5	5.0 × 5.0
Max	3.09	3.51	5.53	6.26
Mean	0.00	0.01	-0.05	0.10
Min	-3.25	-4.00	-4.86	-8.01
Std	0.66	0.91	1.43	2.26
RMS	0.66	0.91	1.43	2.26



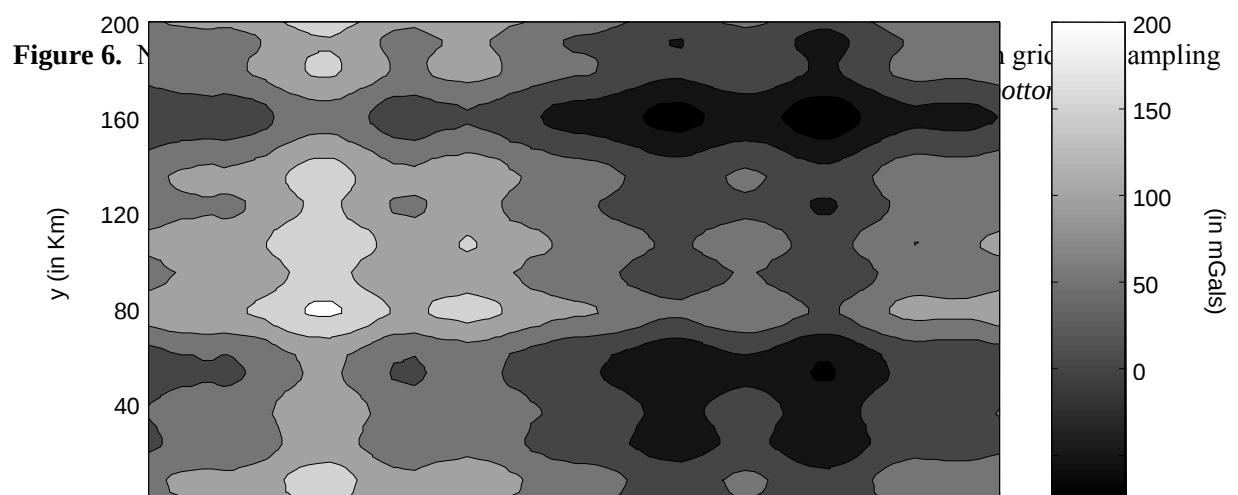
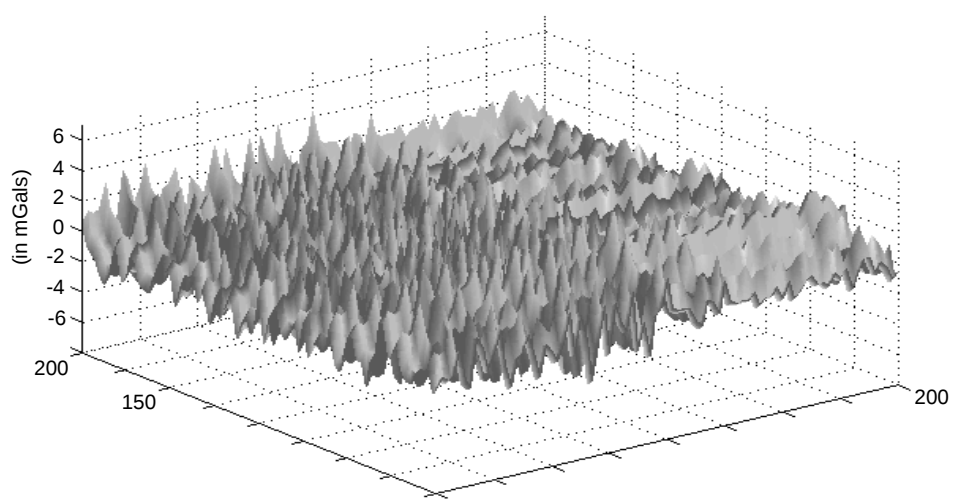


Figure 1



uniform
tom).

Figure 8. Differences between the true and the estimated (filtered) gravity anomaly values, at various data resolution levels. The uniform gridding sampling intervals are 2.5×2.5 km (*top*), 1.0×1.0 km (*middle*) and 0.5×0.5 km (*bottom*).

Table 5a. Statistics of the differences between the true and the filtered signal values at various sampling resolution levels in the four quadrants of the data grids (in mGals).

Data resolution (in Km)	0.5×0.5				1.0×1.0			
Grid quadrants	NW	SW	NE	SE	NW	SW	NE	SE
Max	2.75	3.09	1.97	2.30	3.51	3.38	2.63	2.50
Mean	0.02	0.00	0.00	-0.02	-0.05	0.01	0.00	0.06
Min	-2.89	-3.25	-2.08	-2.44	-3.59	-4.00	-2.35	-2.75
Std	0.72	0.78	0.52	0.60	1.06	1.04	0.76	0.74
RMS	0.72	0.78	0.52	0.60	1.06	1.04	0.76	0.74

Table 5b. Statistics of the differences between the true and the filtered signal values at various sampling resolution levels in the four quadrants of the data grids (in mGals).

Data resolution (in Km)	2.5×2.5				5.0×5.0			
Grid quadrants	NW	SW	NE	SE	NW	SW	NE	SE
Max	4.27	5.53	3.17	4.43	5.40	6.26	3.83	4.69
Mean	-0.33	0.12	-0.23	0.23	0.62	0.18	0.03	-0.41
Min	-4.86	-4.62	-2.93	-2.70	-8.01	-7.36	-5.30	-4.65
Std	1.57	1.71	1.02	1.22	2.47	2.60	1.76	1.93
RMS	1.60	1.71	1.04	1.24	2.55	2.61	1.76	1.97

8. Conclusions

A modification of the classic Wiener filtering method has been presented, which allows us to work with deterministic (instead of stochastic) signals that are masked by additive non-stationary noise at different sampling resolution levels. This provides a very useful estimation tool for many geodetic problems related to optimal spectral gravity field modelling. It has been shown that non-stationary noise filtering of geodetic data using fast spectral (i.e. FFT) techniques is possible, if we are willing to incorporate a simple translation-invariance condition into our signal approximation framework. Note that the traditional W-K linear prediction theory cannot lead to filtering (convolution) operations when the data noise is non-stationary. In such cases the estimation algorithm is reduced to a Fredholm equation of the first kind (Wiener-Hopf equation), whose solution determines the best linear (but *not* translation-invariant) signal estimate in a probabilistic MSE sense. In our approach, on the other hand, the a-priori imposed condition of translation-invariance allows us to treat both stationary and non-stationary noise cases within a unified linear filtering setting, which can be very efficiently implemented in practice via FFT numerical techniques.

Many important problems related to the work presented herein remain and require further research. Future work should include the extension of the planar spectral filtering algorithms for non-Euclidean domains of interest in geodesy, such as the sphere or the ellipsoid. Additional modifications are also needed in

order to handle signal estimation applications that involve more than one type of input data (multiple-input/single-output systems), and not just the single-input/single-output case that was studied here. Nevertheless, the presented methodology can be proven a useful tool in many existing geodetic problems of regional or local scale, such as the optimal spectral geoid determination from noisy gridded gravity data or the FFT computation of various terrain-dependent gravity field quantities (e.g., indirect effect, terrain correction, isostatic potential, etc.) from noisy digital elevation models.

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Problems and new concepts in local geoid solutions

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Abstract The local modeling of the gravity field has always been a theoretical challenge for geodesy, particularly at wavelengths comparable with the data window. Several solutions have been proposed all of which are known to leave long wavelength errors in the geoid. The current belief on the origin of this error is criticized. A new proposal, namely the use of the Slepian basis to overcome this difficulty, is illustrated in general terms.

1 Introduction: a bunch of problems

We consider the classical problem of computing a gravimetric geoid and we identify a number of critical points like:

- a) what can we do if the global model is not a good approximation of local data and in particular if the residual field has long wavelength features in the area where we have the data?
- b) how good is the formula for R.T.C. specially due to the systematic behaviour of gravity close to prism edges?
- c) how can we construct more flexible models of covariance functions to be able to interpolate more accurately the empirical data?

In particular how can we avoid using an error covariance function for the low degrees, which according to the discussion on point a) is not necessarily a right concept?

The International Geoid Service is interested in developing research on these points and, hopefully, in producing new software based on non-traditional solutions.

Here we address the first problem, i.e. to discuss the role of the global model in the geoid computation.

2 The role of the global model and its "local errors"

Typically it is believed that the global model should subtract to the global gravity field the long wavelength part; but the long wavelength components can come from very far away the area where we want to compute the geoid and they do not necessarily reflect the long wavelength behaviour of the local gravity field.

Example (in 1D) The discrete Fourier transform of the function $f(t)$ (with $0 \leq t \leq 2\pi$)

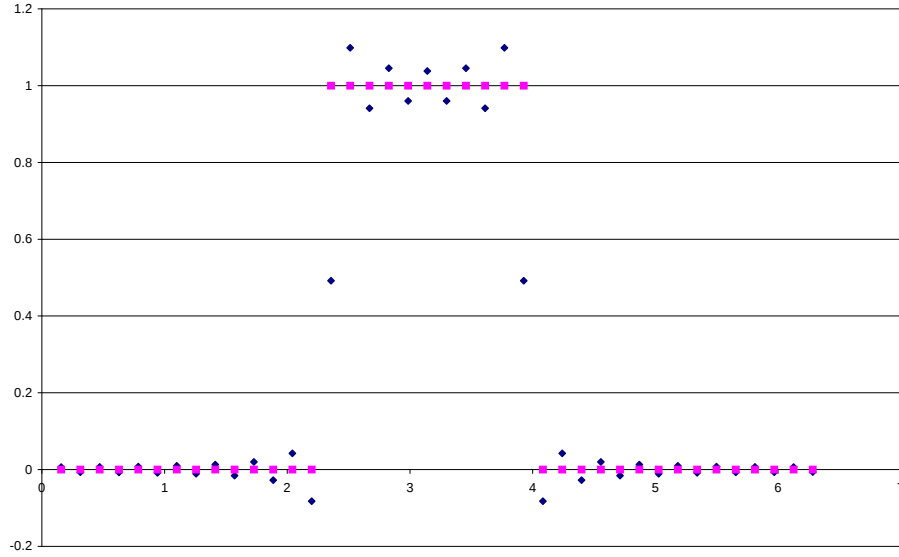


Figure 1: The functions $f(t)$ and $\hat{f}(t)$.

$$f(t) = \begin{cases} 1 & \frac{3}{4}\pi \leq t \leq \frac{5}{4}\pi \\ 0 & \text{otherwise} \end{cases}$$

is

$$\hat{f}(t) = \frac{1}{4} + \sum_{n=1}^N \frac{2}{\pi} \frac{1}{n} (-1)^n \sin n \frac{\pi}{4} \cos nt$$

This has a very slow convergence and out of $\left[\frac{3}{4}\pi, \frac{5}{4}\pi\right]$ goes to zero more by interference rather than by pointwise convergence.

So if we take $\hat{f}(t)$ as a global model for $f(t)$ we get a residual

$$f_r(t) = f(t) - \hat{f}(t)$$

which has a clear long wavelength signature.

It is for this reason that the research on the concept of "local" (or windowed) Fourier transform has started, ultimately leading to the concept of wavelets.

In geodesy this concept has been taken up in the form of taylorred potential coefficients $\tilde{T}_{lm}(P_0)$ which are in reality functions of the "area" where we compute them, centered at the point P_0 .

Basically what we do is just to adapt the coefficients to a local data set, e.g.

$$\Delta g(P) \sim \sum_{l,m=n}^N \frac{l-1}{R} T_{lm} Y_{lm}(P)$$

$$P \in C(P_0, \underline{\psi}) \quad \text{cap with center } P_0 \text{ and radius } \underline{\psi}$$

This can be done in 3 ways:

a) either by least squares determining T_{lm} as parameters from

$$\sum_i \left[\Delta g(P_i) - \sum \frac{l-1}{R} T_{lm} Y_{lm}(P_i) \right]^2 = Min$$

b) or via orthogonality relations with data on C only

$$\tilde{T}_{lm} = \frac{1}{4\pi} \frac{R}{l-1} \int_C \Delta g Y_{lm}(P) d\sigma.$$

c) or by minimizing the quadratic form

$$\int_C \left| \Delta g - \sum \frac{l-1}{R} T_{lm} Y_{lm} \right|^2 d\sigma = Min$$

and then discretizing the solution.

Approaches a) and c) give non well conditioned problems because Y_{lm} are by far non orthogonal on C ; the approach b) corresponds to taking the coefficients of the function T as equal to those of the function $T\chi_C$, i.e.

$$\tilde{T}_{lm} = [\Delta g(P)\chi_C(P)]_{lm}$$

with

$$\chi_C(P) = \begin{cases} 1 & P \in C \\ 0 & P \notin C \end{cases}$$

(This is more properly corresponding to the "local" Fourier transform concept). In all cases the basis functions used $\{Y_{lm}(P)\}$, although linearly independent, are widely redundant in the sense that their Graham matrix

$$\begin{aligned}
\Gamma_{lmjk} &= \frac{1}{4\pi} \int_C Y_{lm} Y_{jk} d\sigma = \\
&= \frac{\delta_{mk}}{2} (1 + \delta_{m0}) \int_0^{\bar{\psi}} \bar{P}_{lm}(\psi) \bar{P}_{jm}(\psi) \sin \psi d\psi \quad e, j \leq N
\end{aligned}$$

is more and more badly conditioned as N increases.

Consequently we cannot expect the $\tilde{T}_{lm}$ to be really near to the true coefficients; therefore when we transform our function with some non local operator, like when we compute the geoid from Δg , we may expect a significant error to be committed.

We believe that two proposals are on the table to overcome this problem: one aims at the concept of wavelets on the sphere, the other one uses the idea of Slepian bases for spherical caps.

The wavelets concept, which is rather new for the sphere, is being currently developed by the University of Kaiserslauten (W. Freeden et al.); this concept is very nice but presents some technical complications due to the very nature of wavelets which prevents them from being on the same time symmetric orthogonal and with finite support.

Here we present the other new idea, namely the Slepian approach, which seems to join a basic theoretical simplicity with a clear numerical efficiency.

3 The Slepian concept for the sphere

Assuming one has local data in an area, which can be approximately considered as a spherical cap C , and one would like to make the analysis and synthesis and even some functional transformation of the field of measurements (e.g. computing a geoid from Δg) one is faced with the following requirements:

- a) to have a basis $\{S_{lm}\}$ possibly locally orthogonal

$$\frac{1}{4\pi} \int_C S_{lm}(\sigma) S_{jk}(\sigma) d\sigma = D_{lm} \delta_{lj} \delta_{mk} \quad ;$$

if the data are well distributed on C this condition guarantees that the normal matrix used to find the coefficients of a function f with respect to $\{S_{lm}\}$ is almost diagonal and generally well conditioned;

- b) that this basis is related in a simple way to spherical harmonics $\{Y_{lm}\}$ so that the forward and backward transformations could be efficiently computed and harmonic calculations, e.g. the upward or downward continuation, could be easily performed.

The Slepian concept gives a positive answer to these questions with the additional advantage that the "eigenvalues" D_{lm} become a measure of the importance of S_{lm} on

C ; in fact the quantities

$$D_{lm} = \frac{1}{4\pi} \int_C S_{lm}^2 d\sigma \leq 1 \quad ,$$

when S_{lm} are normalized on the whole sphere, tell us of the "information" carried by S_{lm} on C which is great for D_{lm} close to 1 and negligible for small values of D_{lm} .

The analytical development of the computation of $\{S_{lm}\}$ is basically elementary.

Having observed that, if we imagine the cap C placed at the pole, due to its cylindrical symmetry we expect in any way a dependence of S_{lm} on the longitude λ through the trigonometric functions $\cos m\lambda$, $\sin m\lambda$, we can reduce our attention to subspaces of $L^2(\sigma)$ with fixed order m ; moreover if we want to treat a signal which by definition has a threshold N_{max} , we can reduce our attention to subspaces

$$L^m(\sigma) = \text{Span} \{Y_{lm}(\sigma); m \leq l \leq N_{max}\}$$

If we order Y_{lm} in a vector $\underline{Y}_m$, with $N_{max} - m + 1$ components, we can consider the matrix

$$\Gamma^m = \langle \underline{Y}_m, \underline{Y}_m^+ \rangle_{L^2(C)} = \left\{ \frac{1}{4\pi} \int_C Y_{lm} Y_{jm} d\sigma \right\}$$

If we take its spectral decomposition

$$\Gamma^m = U^m D^m U^{m+}$$

we immediately realize that the vector function

$$\underline{S}_m = (U^m)^+ \underline{Y}_m$$

is indeed such that

$$\langle \underline{S}_m, \underline{S}_m^+ \rangle_{L^2(C)} = D^m = \{D_l^m, m \leq l \leq N_{max}\} \quad ;$$

this is precisely the Slepian basis.

Let us remark that in addition $\{S_{lm}\}$ turns to be orthogonal on the whole sphere as

$$\begin{aligned} \langle \underline{S}_m, \underline{S}_m^+ \rangle_{L^2(\sigma)} &= U^{m+} \langle \underline{Y}_m, \underline{Y}_m^+ \rangle_{L^2(\sigma)} U^m = \\ &= U^{m+} U^m = I \end{aligned}$$

Since U^m is an orthogonal matrix.

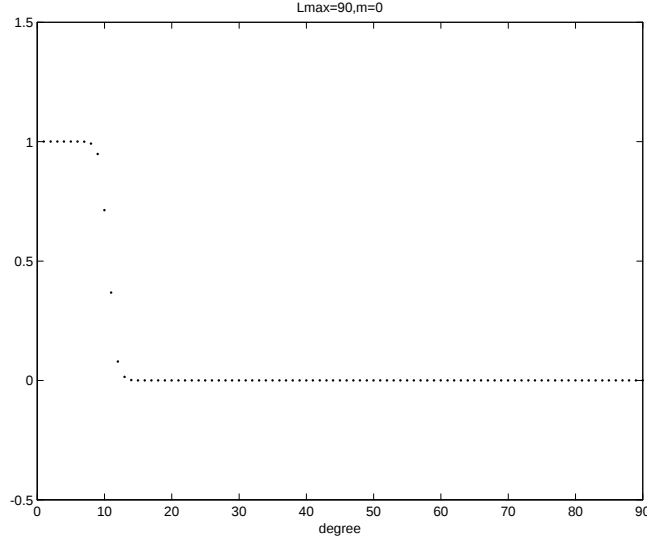


Figure 2: Typical behaviour of D_{l0} as function of the degree l .

In Figure 2 a typical plot of D_{lm} against l is shown. For each m there are a number of functions of the $\{S_{lm}\}$ system which really don't carry any information on C and those functions are precisely the ones which create the instability in performing the local analysis; a good idea seems to be to ignore them completely in making the analysis and synthesis of our signal.

Numerical examples show that this concept is fully acceptable.

Let us conclude this note by showing as an example how one can perform the upward harmonic continuation of the basis $\{\underline{S}_{lm}\}$.

Noting that the harmonic continuation of $\underline{Y}_m$ from a radius R to a radius $r > R$ is performed through the diagonal operator

$$A\underline{Y}_m = \left\{ \left(\frac{R}{r} \right)^{l+1} Y_{lm} \right\} = A^m \underline{Y}_m$$

$$A^m = \text{diag} \left\{ \left(\frac{R}{r} \right)^{l+1} \right\}$$

we can write

$$\begin{aligned} A\underline{S}_m &= U^{m+} A\underline{Y}_m = U^{m+} A^m Y_m = \\ &= U^{m+} A^m U^m U^{m+} Y_m = [U^{m+} A^m U^m] \underline{S}_m \end{aligned}$$

which is the sought formula.

As expected the upward continuation operator is not any more diagonal on the basis $\underline{S}_m$, so for its practical implementation it is convenient to go back to $\underline{Y}_m$ (always skipping the pathological S_{lm}), to apply A^m and then to return to $\underline{S}_m$. Now the word is to simulations and numerical work.

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The Height Datum Problem: the Italian Case

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1 Foreword

(Fernando Sansò)

The following three small contributions all together constitute a report of a working group set up by IGeS in Italy with the purpose of clarifying the state of the art of the height datum problem as far as the Italian case is concerned. As such the paper does not want to provide a new theoretical insight to solve the problem, but rather to clarify the problem itself, i.e. how to transform a “local” relative height datum into a global one, the data needed to perform this transformation and a plan for its realistic numerical solution. In particular we want to thank the Istituto Geografico Militare for providing a detailed knowledge of the procedures by which the classical local height datum has been established and all the members of the Working Group, namely D. Baldi, R. Barzaghi, B.Benciolini, A. Coticchia, M. Fermi, F. Sacerdote, F. Sansò. We present this report in the International Bulletin of IGeS because we believe that it could serve as a stimulus for other national groups to repeat the same experience.

(Note: the References are common to three contributions and placed at the end of the report.)

2 Symbols

- W potential of gravity
- U normal potential
- W_0 value of W on the geoid
- U_0 value of U on the reference ellipsoid
- $\vec{g}$ gravity vector
- g magnitude of the gravity vector, or simply gravity
- $\vec{n} = -\frac{\vec{g}}{g}$ vertical unit vector
- $\bar{g}$ mean value of gravity along the plumb line, between a point and the geoid
- $\vec{\gamma}$ normal gravity vector
- γ magnitude of the normal gravity vector, or simply normal gravity
- $\vec{\nu} = -\frac{\vec{\gamma}}{\gamma}$ unit vector associated to normal gravity
- $\bar{\gamma}$ mean value of γ between a point and the reference ellipsoid
- H orthometric height
- H^* normal height
- H_D dynamic height
- γ_{rif} conventional reference value of normal gravity
- ζ height anomaly
- N geoid undulation
- $\vec{\xi}$ vertical deflection
- $\vec{r}_{AB}$ displacement vector between points A and B
- l height increment (or difference) measured by spirit levelling; (l_i measurement between points A_i and A_{i+1})
- $\vec{ds}$ line differential
- h ellipsoidal height
- T disturbing potential
- C geopotential number

3 Developments in height datum definition

(F. Sacerdote)

3.1 Introduction

The global height datum definition with an accuracy of a few centimeters is still an open problem in geodesy. Relevant improvements in geoid computation and in the determination of high-accuracy global reference systems with world-wide permanent GPS network have been obtained, as well as in the knowledge of ocean circulation and sea surface topography. Significant theoretical results on the role of continental height datum differences in the formulation and solution of geodetic boundary-value

problems have been achieved, and numerical methods for the simultaneous processing of heterogeneous kinds of data have been proposed. Yet, it is still necessary to go through a number of steps to reach the final result: some aspects must be more deeply investigated and, most of all, procedures to collect, validate and process large amount of data must be more rigorously defined. Furthermore, variations in time are very significant, and their interpretation is relevant for the understanding of important geophysical phenomena, such as ocean level growth, land subsidence or uplift. Therefore, when dealing with historical data, it is essential to know the epoch of their measurement, and possibly to have time series available to model the evolution. Unfortunately these conditions do not always occur, particularly for levelling lines, that in some countries are not sufficiently updated, and for tide gauge data, that cannot be easily recovered in suitable form for comparisons. The present paper tries to illustrate the state of the art and the main problems that must still be faced, particularly in relation to the Italian and the European situation.

The Italian height datum is officially defined by the high-precision levelling network, established by the Istituto Geografico Militare (IGM). Its extension is about 13000 km. An enlargement with new levelling lines about 5500 km long is scheduled (Figure 1)([11]). The reference benchmark is the Genova tide gauge. Geometric levelling campaigns have been carried out for several decades since about 1950, and the whole network has been adjusted along closed lines; furthermore, Italy contributed to UELN (United European Levelling Network) providing adjusted geopotential numbers at nodal points, obtained from gravity measurements with 3-5 km spacing. The a-posteriori standard deviation of the unit weight referred to 1 km of levelling for the Italian levelling network is 1.78 kgal*mm, about at the same level as for most other countries ([30]); it must be pointed out, however, that in many regions, owing to the long time elapsed from measurement epoch, relevant crustal deformations and subsidence certainly occurred, as illustrated in more detail in the sequel; hence at present official height data are not always completely reliable, and the measurements of a number of levelling lines ought to be shortly repeated. A short illustration of the vertical crustal deformations in Italy, with reference to some of the main geophysical investigations on the topic, is presented in Section 4.

In the last decade the use of GPS for height determination has acquired growing relevance. It is well-known that from the cartesian coordinates of a point it is possible to obtain its height over an ellipsoid with given geometric parameters a , e^2 , centered at the origin of the cartesian axes; for GPS the geocentric reference ellipsoid for WGS84 reference system, with $a = 6378137\text{m}$, $e^2 = 6.694379990 * 10^{-3}$, is adopted. A number of vertices of IGM95, the Italian official GPS network established by IGM ([35]), are altimetrically connected with the high-precision levelling network, so that in those points geoid undulation can be computed with the classical formula

$$N = h - H \quad (1)$$

Yet, geoid undulation, being influenced by mass distribution and consequently by terrain conformation, cannot be interpolated by simple numerical devices (for example low-degree polynomials) to determine orthometric height from GPS measurements at arbitrary points with an acceptable accuracy level. In ([7]) data for some test areas are analysed, and it is shown that interpolated geoid undulations lead to unacceptable discrepancies between estimated and measured orthometric heights. On the other hand, even using gravimetrically computed geoid undulations, namely ITALGEO95 model ([5]), residual discrepancies are obtained, due to the fact that the reference ellipsoid



Figure 1: The altimetric densification project in Italy

used for gravimetric geoid computations, essentially defined by global geopotential models, does not exactly coincide with WGS84 ellipsoid ([6]). The results of some campaigns carried out in the United States ([27]; [26]; [17]) show that, at least for regions with relatively small extension (maximum distances less than 200km), equation (1), with H obtained by levelling, h by GPS measurements and N by gravimetric geoid computation, is satisfied at an accuracy level of 2-3cm.

3.2 Progress in European Height Datum Definition

It has been already recalled that Italy participates in UELN activities. The goal of UELN is to establish a unified high-precision vertical datum for Europe. It started in 1973 and since then new global adjustments were carried out in 1986 and in 1994, with weights related to the a-posteriori standard deviations of individual network blocks. Since 1995 the global network has been repeatedly enlarged with the introduction of new data sets, some of which replaced old network blocks (Germany, Austria, Netherlands, Denmark), some else covering regions from East Europe and Balkans previously missing. As reference point the Amsterdam tide gauge was chosen. Network density is highly non-uniform, as the new networks of some Central Europe countries recently introduced are much denser than the previous ones (Figure 2); as a consequence, errors

are very small in the continental part of Europe (at 1cm level) and grow up to about 5cm at the ends of southern peninsulas and Scandinavia.

In last years, with the establishment of the global GPS network of the International GPS Service (IGS), and of a specific European network (EUREF) including about 90 stations ([10]), most of which are part of IGS, the goal of defining a high-precision reference system and monitoring its time changes with GPS permanent stations has been achieved. Solutions are produced weekly combining the adjustments of a number of local networks computed by regional analysis centres ([9]). Therefore the problem of connecting the results produced by EUREF concerning heights with UELN data has been tackled. To this purpose the EUVN (European Vertical Network) project has been started. A GPS campaign has been carried out during a week in May 1997 on a network of about 200 stations spread all over Europe, including EUREF sites (with a number of IGS stations), UELN nodal points and tide gauges (Figure 3); levelling connection lines between UELN points, tide gauges and GPS points are established ([19], [20]). One of the main goals is the knowledge of sea level variations along European coasts, due both to water level change related to global climate and to vertical crustal displacements recorded by GPS measurements. The results of the GPS campaign have been published ([21]), whereas connections with levelling lines and tide gauges are still in progress ([19], [20]).

3.3 Some Basic Theoretical Remarks

As already remarked, from the knowledge of ellipsoidal height from GPS measurements and of orthometric height from levelling at the same points geoid undulation can be obtained using formula (1). As both GPS and levelling provide centimeter accuracy, the same accuracy is achieved for geoid undulation at measurement points. On the other hand, in order to obtain orthometric heights with GPS measurements only, certainly less costly, it is necessary to use geoid undulations, that cannot be interpolated with acceptable accuracy from levelled points, so that they must be computed independently, using gravimetric data; only recently computation methods that ensure the required accuracy (at the level of few centimeters) have been elaborated. Nowadays dedicated research centers can manage these computing procedures to determine geoid undulation at arbitrary points in areas covered with suitably dense gravity data and terrain models. The International Geoid Service has been established as IAG service with the purpose of collecting, validating and distributing data and software for geoid computation. In Italy the geoid model ITALGEO95 has been produced ([5]); its updating ITALGEO99 has been prepared and will be soon published. Yet, it must be pointed out that the use of formula (1) is not straightforward and requires caution for a number of reasons:

- first, H is orthometric height, whose determination requires information on mass distribution in surface layers (see Section 5); most of the European countries do not adopt it as official altimetric coordinate, and prefer normal height, that requires only gravity information along levelling lines. On the other hand, in order to compute geoid undulations with gravimetric methods, gravity anomalies must be reduced onto the geoid surface, that requires internal mass distribution information too. If, on the contrary, gravity anomalies are “free-air” reduced, and the disturbing potential T obtained by Stokes formula or collocation is analytically continued to the Earth’s surface height, applying Bruns formula height anomaly, $\zeta = T/\gamma$, is obtained. The surface whose ellipsoidal height

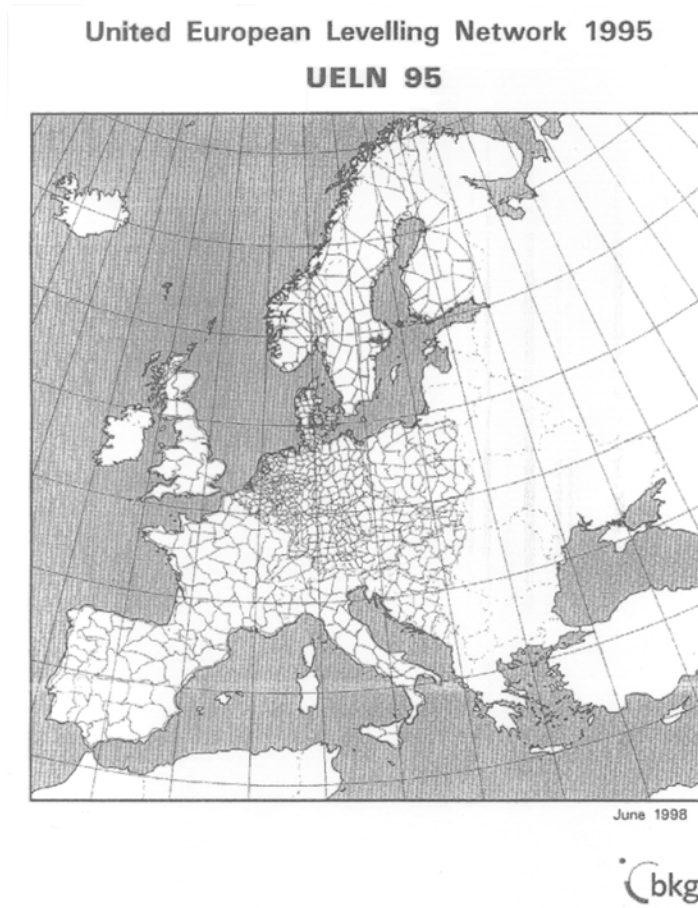


Figure 2: United European Levelling Network 1995 (from BKG - EUREF Publication No. 7/II)

is ζ is called quasi-geoid and has no physical meaning; yet, it is important to remark that the quantity

$$H^* = h - \zeta \quad (2)$$

obtained subtracting height anomaly from ellipsoidal height is exactly normal height. Therefore equation (1) can be profitably replaced by (2) in practical situations. Indeed, the procedures adopted in ITALGEO95 lead to the computation of a quasi-geoid;

- second, the geoid (or quasi-geoid) gravimetrically computed is affected by small reference system differences with respect to the geoid obtained from GPS measurements and levelling. Indeed, the main contribution to the gravimetric geoid comes from a geopotential global model, computed from continental gravity

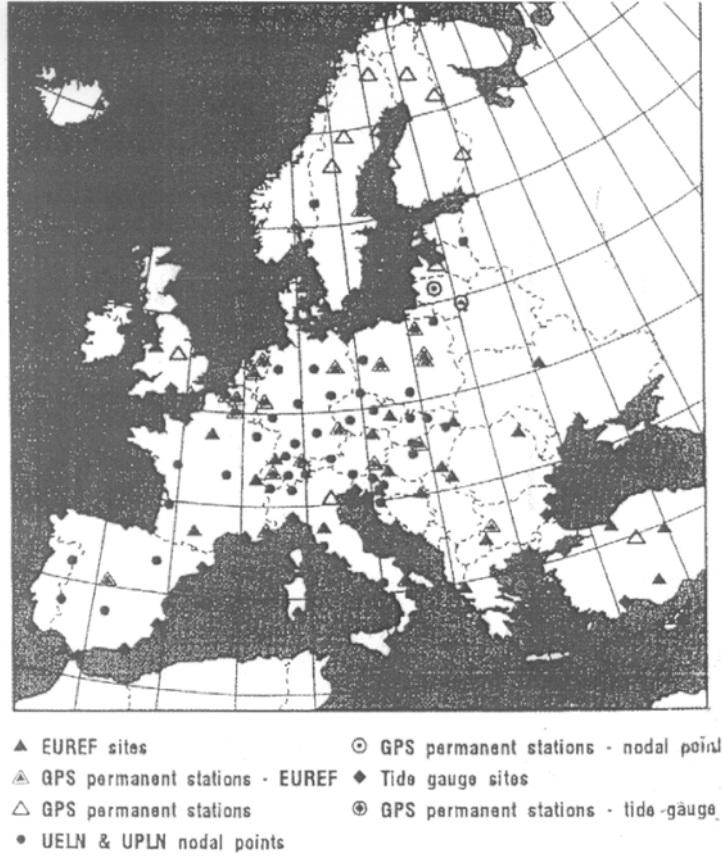


Figure 3: Distribution of EUVN points (from BKG - EUREF publication No. 7/II)

anomalies and satellite altimetry data over the oceans; the latter in last years have been mainly provided by ERS and Topex-Poseidon missions. Although these satellites are equipped with GPS on-board and their orbits are nominally expressed in WGS84 reference system, yet discrepancies up to a few meters can occur between the reference ellipsoid for the geoid model obtained via Bruns' formula from the disturbing potential coefficients computed from a global adjustment and the reference ellipsoid for local GPS networks. On the other hand, local corrections to the geoid global model, computed by collocation or Stokes formula from local gravity anomalies, are weakly influenced by the reference system, implicitly fixed by the geodetic coordinates ϕ , λ of measurement points; indeed, even a displacement of a few hundreds of meters has a negligible effect on gravity anomalies.

In principle these reference system discrepancies can be determined by estimating transformation parameters by least squares from the residuals of the equation $h - \zeta - H^* = 0$ at points along levelling lines, where h , ζ and H^* (or h , N and H) can be obtained with independent methods. In small regions the complete 7-parameter transformation can be replaced by a linear interpolation with respect to geodetic or cartographic coordinates. It must be remarked that what is actually interpolated is not geoid undulation itself, but discrepancies between geoid undulations computed with different procedures, due only to reference system differences. A detailed illustration of numerical problems arising from the simultaneous elaboration of height data of different nature and origin is for example contained in ([12]).

3.4 Boundary-Value Problem Approach

The geoid, whose separation from the reference ellipsoid is introduced in formula (1), is defined as an equipotential surface of the geopotential $W(P)$. If it is computed via Bruns' formula, $N = T/\gamma$, where $T(P) = W(P) - U(P)$, $U(P)$ is the normal potential and P is on the geoid, then the potential value on the geoid is defined by $W(P) = U_0$, where U_0 is the value of the normal potential on the level ellipsoid. Similarly, using formula (2), if ζ is defined via Bruns' formula, the value of the normal potential on the telluroid, i.e. the surface whose separation from the physical surface of the Earth is ζ , is equal to the geopotential at the point on the Earth's surface along the same normal to the ellipsoid. The problem is that orthometric and normal heights, introduced in (1) and (2), are measured via levelling, so that they require the physical definition of a reference benchmark, $\bar{P}$, whereas, on the other hand, the absolute value of the geopotential cannot be determined with high precision. Therefore it is necessary to assume a small discrepancy between the geopotential value at the reference benchmark and U_0 , that is uniquely defined by some choice of the parameters a, e^2, GM, ω : $W(\bar{P}) = U_0 + \delta W$. What can be measured at any point connected to $\bar{P}$ by a levelling line, is the geopotential number, $C(P) = W(\bar{P}) - W(P)$ and consequently

$$\widetilde{W}(P) = U_0 - C(P) = W(P) - \delta W \quad (3)$$

can be determined. If the geoid is defined as the level surface through $\bar{P}$, a modified Bruns' formula must be applied, $N = (T - \delta W)/\gamma$; a similar formula holds for ζ if the telluroid is defined by $U(Q) = \widetilde{W}(P)$, where P is on the Earth's surface, and Q and P lie on the same ellipsoid normal. Yet, $\bar{P}$ can be chosen as reference point only for a region whose points can be connected to it by levelling; in principle for each continent, and even for each island, a different reference benchmark must be chosen, with a different geopotential value. Therefore δW is constant only on individual continental areas, and the definition of geoid given above is ambiguous. Alternatively, the geoid can be defined as the equipotential surface at level U_0 , and in formulas (1) and (2) an additional quantity δh , different for each continental area, must be introduced to account for the height difference between the geoid and the reference benchmark.

The disturbing potential T can be determined from Stokes formula:

$$T(P) = \frac{R}{4\pi} \int_{\Sigma} \Delta g(Q) S(\psi_{PQ}) d\sigma_Q \quad (4)$$

where Σ is the unit sphere, S is the Stokes function, ψ_{PQ} is the spherical

distance between P and Q . Here the points P and Q are on the geoid and gravity anomalies are free-air reduced using some estimate of their radial derivatives. Indeed, it turns out that gravity anomalies are very weakly correlated with height, and their radial derivative is very small ([5]).

It is well-known that the function $T(P)$ obtained from Stokes formula does not contain 0- and 1-degree spherical harmonics.

Using Stokes' formula, it is possible to determine, for each continental region, the quantity δW . Indeed, at a point P on the Earth's surface

- the ellipsoidal height $h(P)$ can be measured with GPS;
- the geopotential number $C(P) = W(P_0) - W(P)$ with respect to a reference point P_0 (benchmark) can be determined by levelling and gravity measurements;
- U_0 , the normal potential on the level ellipsoid, is computed with suitable formulas as a function of the ellipsoid parameters. Consequently, $\widetilde{W}(P)$ can be determined from (3);
- the quantity $\widetilde{H}(P)$, implicitly defined by the equation

$$U(\phi(P), \lambda(P), \widetilde{H}(P)) = \widetilde{W}(P) \quad (5)$$

can be in principle computed, if the geocentric GPS position of P is known. $\widetilde{H}(P)$ can be assumed as normal height, but is not exactly equal to H^* defined in (2), as $\widetilde{W}(P)$ differs from $W(P)$.

- for the "height anomaly" $\zeta = h(P) - \widetilde{H}(P)$ the modified Bruns formula holds: $\zeta(P) = (T(P) - \delta W)/\gamma(P)$;
- as remarked above, if T is computed using Stokes formula from an arbitrary set of gravity anomaly data, it does not contain 0- and 1-degree harmonic components, as it must be, if the mass inside the ellipsoid is equal to the earth's mass and the ellipsoid is centered in the Earth's centre of mass;
- If gravity anomaly $\widetilde{\Delta g}$ is obtained subtracting γ at ellipsoidal height $\widetilde{H}(P)$ from measured gravity on the physical surface of the Earth, its relation to the disturbing potential is (in spherical approximation)

$$\widetilde{\Delta g} = -\frac{\partial T}{\partial r} - \frac{2}{r}(T - \delta W) \quad (6)$$

i.e., it contains the additive term $(2/r)\delta W$ with respect to the usual expression from which Stokes' formula is obtained. If Stokes' integral is taken over the whole sphere, the contribution of this term vanishes;

Practically, from the gravity anomaly data the global model and the terrain correction contributions are removed, and the Stokes formula is applied to the residuals, which are localized in a small region, so that the integral is extended to this region. This restriction is acceptable, provided that the residuals contain only high-degree components; therefore, if, owing to the presence of the discrepancy δW , a component constant in the region, $(2/R)\delta W$ is contained in the data, it must be cancelled (all the formulation leading to Stokes formula assumes spherical approximation);

- let $\widetilde{\Delta g}$ be the data (obtained by measuring $g(P)$ and subtracting γ at ellipsoidal height $\widetilde{H}(P)$); $\widetilde{\Delta g}_R = \widetilde{\Delta g} - \Delta g_{\text{mod}} - \Delta g_{\text{RTC}}$; $\widetilde{\Delta g}_R = \Delta g_R + (2/R)\delta W$, where Δg_R are the reduced data to which localized Stokes formula can be applied. Therefore, the disturbing potential T to which the modified Bruns formula must be applied is $T = \widetilde{T} - (R/4\pi)(2/R)\delta W \int_{\Sigma_C} S(\psi)d\sigma$, where Σ_C is a cap where measured gravity data are given, $\widetilde{T}$ is the result of the application of Stokes formula to $\widetilde{\Delta g}_R$ and the addition of the model and terrain correction contributions. Finally

$$\zeta = \frac{T - \delta W}{\gamma} = \frac{\widetilde{T}}{\gamma} - \frac{\delta W}{\gamma} \left(1 + \frac{1}{2\pi} \int_{\Sigma_C} S(\psi)d\sigma\right) \quad (7)$$

More generally let $T = \mathcal{S}(\Delta g)$ express any procedure to obtain the disturbing potential from gravity anomalies (for example, collocation from data available over a certain area). Consequently, $\widetilde{T} = \mathcal{S}(\widetilde{\Delta g} = \mathcal{S}(\Delta g) + (2/R)\delta W \mathcal{S}(1))$, where $\mathcal{S}(1)$ expresses the result of applying collocation (over the same region) to data all equal to 1. Then

$$\zeta = \frac{\mathcal{S}(\widetilde{\Delta g})}{\gamma} - \frac{\delta W}{\gamma} \left(1 + \frac{2}{R\gamma} \mathcal{S}(1)\right) \quad (8)$$

Introducing this expression for ζ into $h - H^* - \zeta = 0$ an equation in δW is obtained, from which an estimate of δW can be obtained, if one point is given for which ellipsoidal height and geopotential number are given and the gravimetric geoid can be computed from gravity anomalies on a surrounding area. If this situation occurs for a number of points in a region with the same reference benchmark, the estimate of δW can be obtained by least squares.

4 Vertical crustal deformations in Italy

(P. Baldi)

Ground uplift and subsidence of the Earth's crust in a tectonically active region is due essentially to processes caused by horizontal compressional or extensional phenomena, and isostatic forces induced by temporary gravitational instabilities.

The geodynamical processes which regulate the tectonic activity in the Italian region are dominated by the collision of African plate with Euroasia which causes a wide variety of deformation patterns; in the past even high intense strain phenomena have been recognised, but also at present a complex space-time distribution of compressional and extensional events are active.

A reconstruction of the recent strain phenomena has been obtained on the basis of neotectonical study ([1]); starting from the middle Pliocene, the areas interested by continuous ground uplift were identified (Alps, Apennine chain, Adriatic slope of the southern Apennines), while generalised earth's crust subsidence displacements has interested to Po basin, the Adriatic and Tyrrhenian edges and the northern and central part of the Italian Peninsula, and less extended areas like the Neapolitan region and the straits of Messina. The amount of these differential displacements can be evaluated in the range of some mm per year.

Recent studies on the focal mechanism of major earthquakes, measurements of stress in situ ([23]) analytical or finite element modelling ([13]), have tentatively reconstructed the stress field and the consequent deformation pattern of some area involved in the geodynamical processes of the Italian region; for example, in the Calabrian Arc the mean uplift is evaluated to be lower than 1 mm/yr.

Nowadays the modern geodetic techniques, (GPS, V.L.B.I., S.L.R.) can be successfully used to detect the crustal deformations, even if the low velocity of vertical movements needs observations over a long time span. At the moment a quantitative evaluation of recent vertical deformations is given by the comparison of spirit levelling measurements performed by the Geographic Institute of Italian Army (IGM) in the periods 1877-1903 and 1950-1956, in northern and central Italy. A subsidence of about 450 mm has been measured in the Po delta, while the Sempione area and the Monferrato region are involved in an uplift process which gave deformations of about +175 and +250 mm respectively. An inversion of the displacement field is present approximately along the Genova-Bolzano line. Also repeated levelling campaigns in the Swiss Alps have provided the amount of uplift of the Alpine chain (about 1.5 mm/yr), which may be correlated both to the strong isostatic gravity anomalies and to the compressional stress regime active in the area. Moreover, more detailed information on vertical deformations are available for regions involved in seismic, volcanic, and strong anthropic activity. Quantitative observations of ground movements of the Neapolitan volcanic area started at the early 19th century, and recorded a sinking of the ground in the Pozzuoli harbour at an average rate of 15 mm/yr until 1968, when an inversion of the trend produced significant ground uplift ([8]). Episodic strong deformations have been observed during and after the major earthquakes occurred in the Italian peninsula (e.g. [22]; [36]; [3]; [24]; [18]); in this case the area involved is limited to few hundred Km² around the epicenter. Of major extension is the subsidence process active in the Po plain and along the North Adriatic coast; this phenomena, already measured by means of levelling campaign fifty years ago by IGMI, and considered part of the natural development of an alluvial plain, has been more recently accelerated as the result of anthropic factors such as underground exploitation (gas and water); the rate of subsidence is evaluated of the order of 1 cm/yr in the next future ([14]).

5 Height coordinates

(B. Benciolini)

5.1 Summary

This section describes some different kinds of height coordinates and their relations with the observable quantities and with the gravity field. The last paragraph contains an attempt of categorization based on the geometric and physical meaning of the coordinates.

5.2 An ancient and modern problem

The problem of the choice of a vertical datum and of height coordinates is both ancient and modern. It is treated with some extent already by ([28]). The book contains a clear distinction between ellipsoidal height and height above a level surface. The Author clearly points out that the leveled increments are non-holonomic and mentions some

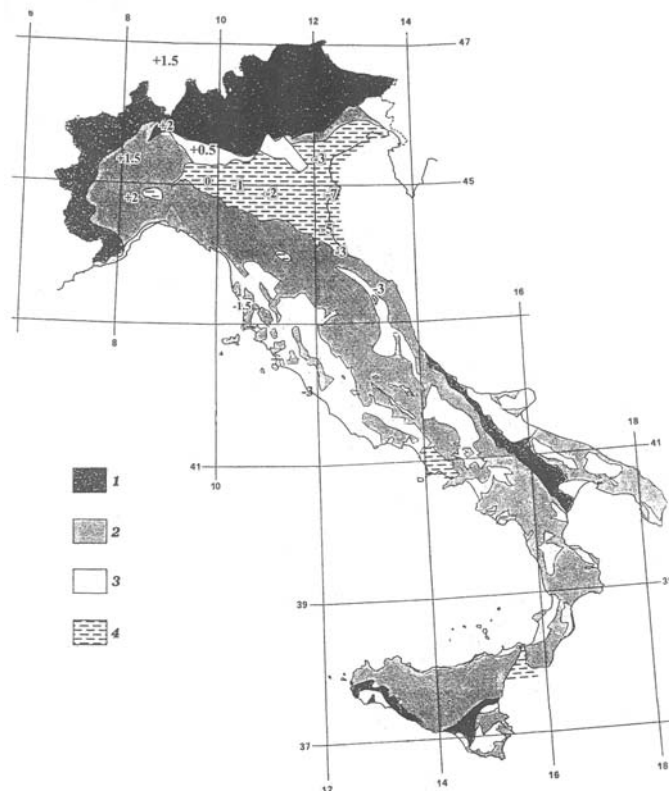


Figure 4: Distribution of Vertical Movements in Middle Pliocene.// Numbers represent velocities (mm/yr) from repeated IGM levelling campaigns (1897-1957) ([2], [31])

modern problems such as the definitin of a global vertical datum and the need of the determination of the geoid.

5.3 Definitions of height coordinates

5.3.1 Ellipsoidal height

The ellipsoidal height of a point P is the distance of the point from the reference ellipsoid measured along the normal. It is designated by h . The most relevant aspect of h is just its purely geometrical definition.

5.3.2 Orthometric height

The orthometric height of P is its distance from the geoid measured along the plumb line. It is designated by H .

(The geoid is an equipotential surface chosen conventionally. In practice it is fixed by selecting a point as the origin of the heights. This procedure implicitly fixes a particular value of W_0 .) The orthometric height has an evident geometrical meaning but it does not give a fully geometrical information about the position of the point because the shape of the geoid is in principle unknown. It also does not give a real physical information, since the surfaces of constant H are not equipotential surfaces. The discussion on “holonomy of heights” created some conceptual problems clarified by ([32]).

5.3.3 Dynamic height

The dynamic height is nothing else than the value of the potential W subtracted from a reference value W_0 and divided by a purely conventional constant value of normal gravity. Again W_0 is implicitly determined by the choice of the origin. The definition is therefore:

$$H_D = -(W(P) - W_0)/\gamma_{rif}.$$

The difference $(W(P) - W_0)$ is named geopotential number. The most important property of the dynamic height is its physical meaning; it tells us *the direction the water flows*.

5.3.4 Normal height

Consider the point Q on the plumb line passing through P and such that $U(Q) = W(P)$. The normal height of P is, by definition, the distance of Q from the ellipsoid and is denoted by H^* .

As P varies on the physical surface of the earth Q describe a surface named *telluroid*. A geometrical representation of H^* as the distance of P from a reference surface leads to the definition of the quasigeoid. We first define the *height anomaly* ζ as the distance of P from Q . The *quasigeoid* is the surface with a displacement from the ellipsoid equal to the height anomaly of a point on the earth surface. Therefore H^* is the height of P above the quasigeoid.

The normal height is related to the geopotential number by the relation:

$$H^* = -(W(P) - W_0)/\bar{\gamma}$$

where $\bar{\gamma}$ is the mean value of γ between Q and its projection to the ellipsoid.

5.4 Relations between height coordinates

5.4.1 H , H^* , H_D

It is clear that:

$$\begin{aligned} H &= H_D \frac{\gamma_{rif}}{\bar{g}} \\ H &= H^* \frac{\bar{\gamma}}{\bar{g}} \\ H^* &= H_D \frac{\gamma_{rif}}{\bar{\gamma}} \end{aligned}$$

where $\bar{g}$ is the mean value of g between P and its projection onto the geoid. The evaluation of the conversion factor of the first two relations requires some geophysical hypothesis for the computation of $\bar{g}$.

5.4.2 H and H^* versus h

The relations are formally simple and they look quite similar:

$$H = h - N$$

$$H^* = h - \zeta$$

but it must be noted that the computation of the geoid is quite more difficult than the computation of the quasigeoid. (The backward continuation of the field would require the knowledge of the density of the topographic masses, and it is unstable also in the free air.)

5.5 Height coordinates and geodetic observables

5.5.1 Spirit levelling

The observable of the spirit levelling, l , is related to the gravity field and to the relative position of points by:

$$l = \vec{r}_{AB} \cdot \vec{n}$$

where A and B are the points occupied by the rods and $\vec{n}$ is the vertical unit vector at a point in the middle; the points directly connected are close to each other, so that the variation of $\vec{n}$ can be neglected. (This is for the single measurement, not for a chain of measurements along an arbitrary path). The measured l can be considered the discrete version of the differential

$$\delta l = \vec{n} \cdot \vec{ds}$$

which is not an exact one. In fact the integral of δl (or the sum of a chain of l) along a closed loop does not vanish and the same integral (or the same summation) between two distinct points is not independent of the path. This fact is due to the lack of parallelism of the equipotential surfaces. We denote g_i the mean value of g between points A_i and A_{i+1} . From the well known differential relations:

$$dW = \vec{g} \cdot \vec{ds} = -g \vec{n} \cdot \vec{ds} = -g \delta l.$$

we obtain the integral relation

$$W(A_N) - W(A_1) = - \int_{A_1}^{A_N} g \delta l$$

and its discrete form:

$$W(A_N) - W(A_1) = - \sum_{i=1}^{N-1} g_i l_i$$

that tell us that the measure of the spirit levelling must be combined with measures of gravity to compute potential differences. The gravity g acts as integrating factor of the non exact differential form δl .

The potential difference can be converted into a difference of dynamic or normal or orthometric heights. In practice it is convenient to compute the height difference as the sum of the measured quantities and a small "correction".

The dynamic height results:

$$H_D(A_N) - H_D(A_1) = \sum_{i=1}^{N-1} l_i + \sum_{i=1}^{N-1} \frac{g_i - \gamma_{rif}}{\gamma_{rif}} l_i$$

and the normal height is :

$$\begin{aligned} H^*(A_N) - H^*(A_1) &= \\ &= \sum_{i=1}^{N-1} l_i + \sum_{i=1}^{N-1} \frac{g_i - \gamma_{rif}}{\gamma_{rif}} l_i + H^*(A_N) \frac{\bar{\gamma}(A_N) - \gamma_{rif}}{\gamma_{rif}} - H^*(A_1) \frac{\bar{\gamma}(A_1) - \gamma_{rif}}{\gamma_{rif}} \end{aligned}$$

and finally the orthometric height is:

$$\begin{aligned} H(A_N) - H(A_1) &= \\ &= \sum_{i=1}^{N-1} l_i + \sum_{i=1}^{N-1} \frac{g_i - \gamma_{rif}}{\gamma_{rif}} l_i + H(A_N) \frac{\bar{g}(A_N) - \gamma_{rif}}{\gamma_{rif}} - H(A_1) \frac{\bar{g}(A_1) - \gamma_{rif}}{\gamma_{rif}} \end{aligned}$$

The terms added to the sum of the measured increments are called dynamic correction, normal correction and orthometric correction respectively. The value of the orthometric correction for a levelling line of about 25 Km with north-south orientation can reach some millimetres in a hilly terrain and some centimetres in a mountainous region (see [34] and [16]) and Heiskanen and Moritz (1967)). The normal correction has a similar value, while the dynamic correction can be much larger (-2.7 meters for a levelling line near the equator with a height difference along the line of 1000 meters, (see [16])). It is once more clear that only the computation of the orthometric height requires a geophysical hypothesis.

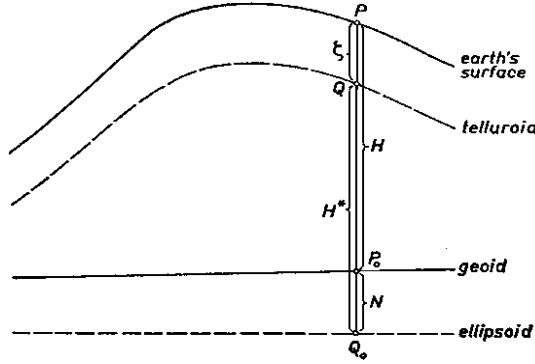


Figure 5: The geoid lies at a depth H below the earth's surface (from [16])

5.5.2 Zenith distances

The observation equation of the zenith angle is

$$\theta_{AB} = \arccos \frac{\vec{r}_{AB} \cdot \vec{n}}{(\vec{r}_{AB} \cdot \vec{r}_{AB})^{1/2}}.$$

For precision determinations it is convenient to combine two reciprocal simultaneous measurements because their difference is almost free of the effect of refraction. The corresponding observation equation is:

$$\tan \frac{\theta_{BA} - \theta_{AB} + \alpha}{2} \tan \frac{\psi}{2} = \frac{h_B - h_A}{2R + h_B + h_A}$$

where α is the mean between the two values of the vertical deflection in A and B (component in the direction of $\vec{r}_{AB}$), R is the radius of the osculating sphere and ψ is the spherical distance between A and B . With some approximation (e.g.: $\tan \frac{\psi}{2} = \frac{\psi}{2}$) we can obtain

$$h_B - h_A = s(1 + \frac{h_m}{R}) \tan \frac{\theta_{BA} - \theta_{AB} + \alpha}{2}$$

where s is the distance between the points projected on the reference sphere and h_m is the mean of the two heights (this quantity must be known, but it is not needed with high accuracy). If α is neglected the same expression gives a difference of the orthometric height (with some further approximation).

5.5.3 GPS survey

A geodetic survey with GPS is based on measurements that are purely geometrical, i.e. insensitive to the gravity. (Orbits are precomputed.) Therefore the result can be expressed as latitude, longitude and ellipsoidal height h . The need of a connection between classical networks and GPS networks, and the need of the production of physically significant heights from modern techniques, demands the computation of a precise (quasi)geoid.

5.6 Synopsis

coordinates	geometrical meaning	physical meaning	computable in Molodensky's theory
h	YES	NO	-
H	yes	NO	NO
H^*	indirect	NO	YES
H_D	NO	YES	YES

The *geometrical meaning* is intended as a simple and evident one : only h and H are the distances of P from a reference surface with a simple definition. On the other hand only the dynamic height H_D has a precise *physical meaning*.

Computable in Molodensky's theory means that it does not require to know the density of the topographic masses.

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II Section

"Communications and News"

During the GGG 2000 Symposium, in Banff (July 31 - August 4, 2000), an assembly has held on the item "A proposal for a new IAG-Service; Gravity Field and Figure of the Earth Service (GFFS)".

This idea has been discussed among already existing Services like BGI, IGeS, ICET and it was decided to bring it to the public, so that possible new proposals from other groups/institutions could be collected.

Here is the report of the meeting.

GFFS Meeting in Banff at GGG 2000 Symposium

The meeting was called by F. Sansò (IGeS President) and J. Barriot (BGI Director) as well as members of ICET and NIMA were present.

In addition 29 scientists have participated to the meeting.

The report on the proposal by F. Sansò has touched on the following points:

1) Area of action of GFFS

- Collection, validation and dissemination of gravity data.
- Collection, validation and dissemination of global gravity models (satellite only, integrated models,...).
- Collection, validation and dissemination of computed geoids.
- Collection, validation and dissemination of DTM's.
- Participation to projects, e.g. global models determination, geoid calculations for large areas, etc.
- Applied research and software implementation in the above areas.
- Calculation and dissemination of gravity tidal effects, local tidal model, etc.
- Calculation and dissemination of marine gravity from altimetry.
- Realization of schools/tutorials e.g. geoid schools, gravimetry tutorials, etc.
- Issue bulletin(s), e.g. a unified bulletin for BGI-IGeS.

2) Data

- Global models (harmonic potential coefficients)
- Gravity data (land and marine measurements)
- Geoid data.
- Digital terrain model data
- Marine gravity derived from altimetry.
- Tidal gravity data.

3) Software

- Global models manipulators.
- Validation and preprocessing of geographically distributed data
- Terrain corrections.
- Geoid estimators (Stokes, collocation,...).
- Cross over adjustment and gravity estimator.
- Software for earth tides computation

4) Services

- Data validation, archiving and dissemination.
- Support to the community and national agencies for gravity, geoid etc. calculations.
- Software validation and dissemination.
- Training in observation techniques and gravity field calculations.
- Information through bulletin(s) and web sites.
- Participation to important international projects.

5) Structure

- Advisory Board:
Directors of Centers/Bureaux.
Delegates of relevant IAG Commissions or Services.
Members elected among fellows.
- Centers/Bureaux:
Based on their own support, governing bodies elected according to internal rules; they can participate to other Services/Institutions.
- Fellows:
Individual members wishing to help the Service to provide necessary power to projects and connection to national agencies.

6) State of the art

At present we have two existing Centers, namely BGI and IGeS, that have declared their intention to join GFFS; ICET is still in a decisional phase. As it is obvious in case ICET preferred another solution, all items concerning gravity tides would be cancelled from this project.

In addition NIMA has offered to set up a Center of GFFS; specialized in collection, validation, dissemination of DTM's and indeed cooperating with other Centers on general Service activities.

7) Schedule

The actions that will come after this meeting are summarized as follows:

- 2000 Sept.-Dec.: issue of a call for proposals and discussion with institutions offering their cooperation.
- 2001 Jan.-Feb.: finalize the terms of reference and provisory list of fellows, approved by Commission XIII (e-mail vote).
- 2001 March: nominations for the GFFS advisory board (e-mail vote).
- 2001 April: discussion at the IAG E.C. Meeting; in case of approval.
- 1001 September: announcement of the new GFFS in Budapest and start of the Service.

8) Conclusions and call for proposals

After a good discussion among attendees the following conclusions have been reached

- a general acceptance of the proposal, understanding its utility for the scientific community and IAG;
- the timeliness of the proposal in view of the IAG restructuring and an invitation to go on in any event along the line presented using the actual IAG bylaws;
- the necessity to open the project to a more general discussion issuing a call for proposals to individuals as well as to institutions ;

- to announce, already at the beginning 2001 the unification of IGeS Bulletin and Bulletin d'information du BGI.

Based on the above conclusions we open in the IGeS web a new page where any member of the geodetic scientific community at large interested in the items of GFFS, is invited to contribute with opinions, proposals for specific actions, nomination of possible fellows that could also guarantee the necessary link between the new GFFS and national Institutes and Agencies.

In addition we invite any Institution willing to contribute to GFFS with a general support and/or with the project of installing a new GFFS Cneter/Bureau, to send their proposals to F. Sansò at DIIAR - Politecnico di Milano, P.za L. da Vinci 32, 20133 MILAN (Italy) - Ph: +39-02 23996504 /7518 - FAX +39 - 02 23996530, e-mail: fsanso@ipmtf4.topo.polimi.it before 2000 November 30.

The Unified Height Reference System for the Americas (SIRGAS 2000)

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Abstract

A network of 185 stations all over the North, Central and South American continent was observed in a continuous ten days GPS campaign in May 2000 in order to establish a unified height reference system for the Americas and to connect the individual national height systems. In conjunction with the gravimetrically corrected leveled heights and a precise geoid computation, the ellipsoidal heights determined by GPS will form the unified continental vertical reference frame as a basis for the national vertical datums. By this means it will be feasible to apply the GPS leveling procedure for routine users' height determinations.

Motivation

The official heights in almost all the countries of the world refer to the mean sea level at a selected tide gauge averaged over a certain time interval. They are classically transferred from the tide gauge to the benchmarks inside the country by spirit leveling, which is corrected for the effects of gravity anomalies in order to refer them to an equipotential surface of the Earth's gravity field, the geoid. The different tide gauges which define the height systems of different countries, however, do not necessarily coincide with the geoid, because the mean sea level reflects the sea surface topography (caused by oceanic currents, temperature, salinity etc.) with deviations up to two meters from the geoid. The height systems of different countries may therefore deviate from each other by several meters, i.e., points at the borders between two countries have different heights in this order of magnitude in both national reference systems. This is in particular true for the American countries, where some height systems refer to tide gauges at the Atlantic and others to those at the Pacific Ocean. A discrepancy between official heights in the order of meters is not acceptable for many applications, e.g., engineering (tunnels, bridges, dams etc.), precise navigation, water supply (channels, tubes etc.).

At present, and even more in future, heights are more and more determined by global satellite positioning systems such as the NAVSTAR GPS. The heights provided by these techniques are heights above a global ellipsoid. The transformation to user's heights (above the geoid) may be done by subtracting the geoid heights. In any case, however, they refer to a global reference surface (ellipsoid or geoid) and are not directly compatible with the individual countries' height systems. Therefore, a unification of all existing height systems is indispensable.

Objectives

The present project is to establish a unified height reference system for North, Central and South America, and to derive the transformation parameters (height offset) for all existing national height systems. For this purpose, all the tide gauges defining a national height system have to be connected by precise, simultaneous GPS observations, providing their heights above the ellipsoid, and by a precise geoid computation in order to determine the mean sea level offset.

In order to control the geographic variability of mean sea level, other tide gauges should be included in such a GPS campaign. The connection of the classical national height systems all over the countries to the unified height system is improved by including other benchmarks inside the continent. It is stabilized in the continental scale by observing points at the borders between countries with known heights in both national systems.

A principal requirement for the future height determination by GPS is a precise geoid computation for the entire continent. For this purpose, all available gravity anomalies over the entire continent have to be collected in a one center and to be processed in a common procedure.

The International Association of Geodesy (IAG) assists these activities by its Scientific Commissions and Scientific Representatives in the project to fulfil both objectives, the GPS height determination and the geoid computation.

History

The activities for a unified geocentric reference system for South America (Sistema de Referencia Geocéntrico para América del Sur, SIRGAS) started in 1993 with a workshop at Asunción, Paraguay, sponsored by the International Association of Geodesy (IAG), the Panamerican Institute of Geography and History (PAIGH), and the US National Imagery and Mapping Agency (NIMA, at that time DMA). It was decided to install a unified continental reference frame (SIRGAS Working Group I) and to connect it with all the horizontal datums of national control networks in order to derive transformation parameters for all the countries (SIRGAS Working Group II).

The SIRGAS continental reference frame was established by a GPS observation campaign over 10 days in May/June 1995. A total of 58 stations were observed simultaneously in ten South American countries, and three-dimensional coordinates were computed in the International Terrestrial Reference Frame (ITRF). The processing was done in two centers at Deutsches Geodaetisches Forschungsinstitut (DGFI), Muenchen, Germany, and at NIMA, USA. The final solution is a combination of both evaluations and was presented officially at the Scientific Assembly of the IAG at Rio de Janeiro, Brazil, in 1997.

The connection of the national horizontal datums to the SIRGAS continental frame was done individually by each country. Dedicated GPS campaigns were performed to observe national networks as a densification of the SIRGAS reference frame. A typical national network (first order densification) includes about 60 stations.

In 1998, during a SIRGAS Workshop in Santiago, Chile, it was decided to install a unified vertical reference system (SIRGAS Working Group III) besides the three-dimensional one, which mainly serves as a unification of the horizontal datums. This new activity includes three principal tasks:

- Definition of the type of heights,
- definition and realization of the height reference surface,
- realization of the vertical reference frame and connection with national height systems.

As a consequence of discussions in the PAIGH, the North and Central American countries joined

the SIRGAS project in the following years. The project name was accordingly changed into "Sistema de Referencia Geocéntrico para las Américas" (SIRGAS).

Definition of the type of heights

There are principally two types of heights, geometrically defined and physically defined ones. Among the geometric heights we have in particular the ellipsoidal heights (perpendicular above the ellipsoid) and the leveled heights (sum of differences of readings of spirit leveling). Ellipsoidal heights are obtained from GPS observations and will be provided routinely in the future. Leveled heights are not corrected for anomalies of the gravity field and depend therefore on the trajectory of leveling, i.e., they are not unique and unequivocal. Therefore they are not applicable for precise surveys.

Physical heights refer to the Earth's gravity field and are distinguished by the way of reduction of gravity anomalies. Orthometric heights refer to the geoid but their computation requires the knowledge of the mass distribution inside the Earth and the associated gravity field. As this information is not available they cannot be computed rigorously, but only as an approximation by applying hypotheses on the mass distribution. Normal heights start from the normal Earth's gravity field and can be computed in an objective way. The reference surface of normal heights, however, is not an equipotential surface. Dynamic heights are derived by simply dividing the geopotential numbers of the Earth's gravity field by a constant. They are the only ones that describe an equipotential surface by equal height values. However, they deviate considerably from leveled heights, i.e., a gravity correction is required even for short leveling lines.

After long discussions in SIRGAS Working Group III there was a resolution to adopt officially two types of heights, ellipsoidal ones and another one based on geopotential numbers of the Earth's gravity field. Among the latter ones it was recommended to adopt normal heights because they can easily and unequivocally be computed without hypotheses and they have the same practical use as the other ones which have to be computed in a more complicated way.

Definition and realization of the height reference surface

The reference surface corresponding to the ellipsoidal heights is the reference ellipsoid. It was already decided in SIRGAS Working Group I (Asunción 1993) to adopt the ellipsoid of the Geodetic Reference System 1980 (GRS80) which is practically identical with the World Geodetic System 1984 (WGS84). It is used in almost all global geodetic computations, e.g., the IAG services for positioning

The reference surface corresponding to the normal heights is the quasi-geoid. This is the surface which normally is computed when using gravity anomalies without detailed consideration of the mass distribution inside the Earth. The realization of the quasi-geoid has to be done by a sophisticated common processing of all available gravity anomalies over the entire continent. It was decided to cooperate for this purpose with the Subcommittee for South America of the IAG Gravity and Geoid Commission. All American countries are encouraged to provide the gravity data to this Subcommittee and to participate in the common processing.

Realization of the vertical reference frame and connection with national height systems

The reference frame of the unified height system for the Americas is realized by a set of stations

with both types of adopted heights, ellipsoidal and normal heights. In order to allow a direct connection with the classical national height systems, the reference stations should be included in the leveling networks. It was decided to include three types of stations in the reference frame:

- SIRGAS 1995 reference stations for connection with the geocentric reference frame,
- tide gauges which define classical national height systems and other tide gauges for sea level control,
- other stations inside the countries with leveled heights and gravity observations, in particular those at the borders between two countries with known heights in both national systems.

A simultaneous GPS observation campaign was performed from May 10 to 19, 2000 (ten days continuously) in order to determine the ellipsoidal heights of the vertical reference frame. A summary of the stations and their attribution to the different types (SIRGAS 95, tide gauges, additional new sites) is given in table 1, the geographical distribution is shown in figure 1. A total of 185 sites in 20 countries was observed. The data processing is presently done by DGFI, Germany, and by Instituto Brasileiro de Geografia e Estatística (IBGE), Rio de Janeiro, Brazil.

Tab. 1: SIRGAS Vertical Reference Frame: Type of stations per country

Country	SIRGAS 1995	New Sites	Tide Gauges	Total
Argentina	10	7	3	20
Bolivia	6	3	-	9
Brasil	11	4	6	21
Canada	-	10	3	13
Caribbean	-	2	2	4
Chile	7	9	5	21
Colombia	5	1	2	8
Ecuador	3	3	1	7
Fr. Guayana	1	-	-	1
Guatemala	-	3	1	4
Guayana	-	2	-	2
Honduras	-	1	-	1
Mexico	-	13	2	15
Nicaragua	-	2	-	2
Paraguay	1	-	-	1
Peru	4	3	3	10
Trinidad/Tobago	-	2	-	2
Uruguay	3	2	3	8
U.S.A.	-	12	12	24
Venezuela	5	3	3	11
Antarctica	1	-	-	1
All	57	82	46	185

Two different procedures will be followed to determine the normal heights. Firstly, all the leveled heights in all the countries have to be reduced by normal gravimetric corrections. For this

purpose, missing gravity surveys along the leveling lines have to be performed. This will take some time and requires enormous financial resources. Secondly, a quasi-geoid computation will be done for the entire North, Central and South American continent. The principal requirement is the collection of all available gravity data which has already been started by the IAG Gravity and Geoid Commission. Subtracting the heights of the quasi-geoid (height anomalies) from the ellipsoidal heights determined by GPS will provide the normal heights and give a control of the leveled heights reduced by normal gravimetric correction (see above).

Maintenance of the reference frame

In the same way as all the geodetic networks, the SIRGAS vertical reference frame has to be maintained by regular control observations and by monitoring the time evolution of coordinates (heights). Following the establishment of the SIRGAS geocentric reference frame in 1995, a number of stations was equipped with permanently observing GPS receivers. All these stations are routinely processed by the Regional Network Associate Analysis Center (RNAAC) of the International GPS Service (IGS) at DGFI, Germany, in weekly intervals. They are included in the combined global IGS solutions and provide the station coordinate changes (velocities) in a global scale. This is very important to maintain the consistency between the terrestrial reference frame (SIRGAS) and the satellite orbit reference frame (ITRF at observation epoch). When performing precise geodetic positioning, the terrestrial coordinates have to be transformed by the velocities to the observation epoch. The geographical distribution of GPS stations processed by the RNAAC for SIRGAS is shown in figure 2.

The time series of station coordinates computed by the RNAAC and the IGS combination centers provides also information on recent crustal movements (plate motions and regional deformations). They may be used for geophysical interpretation and deformation models.

Conclusion

The GPS observation of the vertical reference frame for the Americas (SIRGAS 2000) provides the basis for a unified height reference system for. Considerable efforts have to be undertaken to complete its installation. Urgent requirements are the normal gravimetric corrections of all the leveled heights in all countries and the delivery of all gravimetric data for a common computation of the quasi-geoid. The International Association of Geodesy and several associated institutions offer their collaboration and assistance in this great task.

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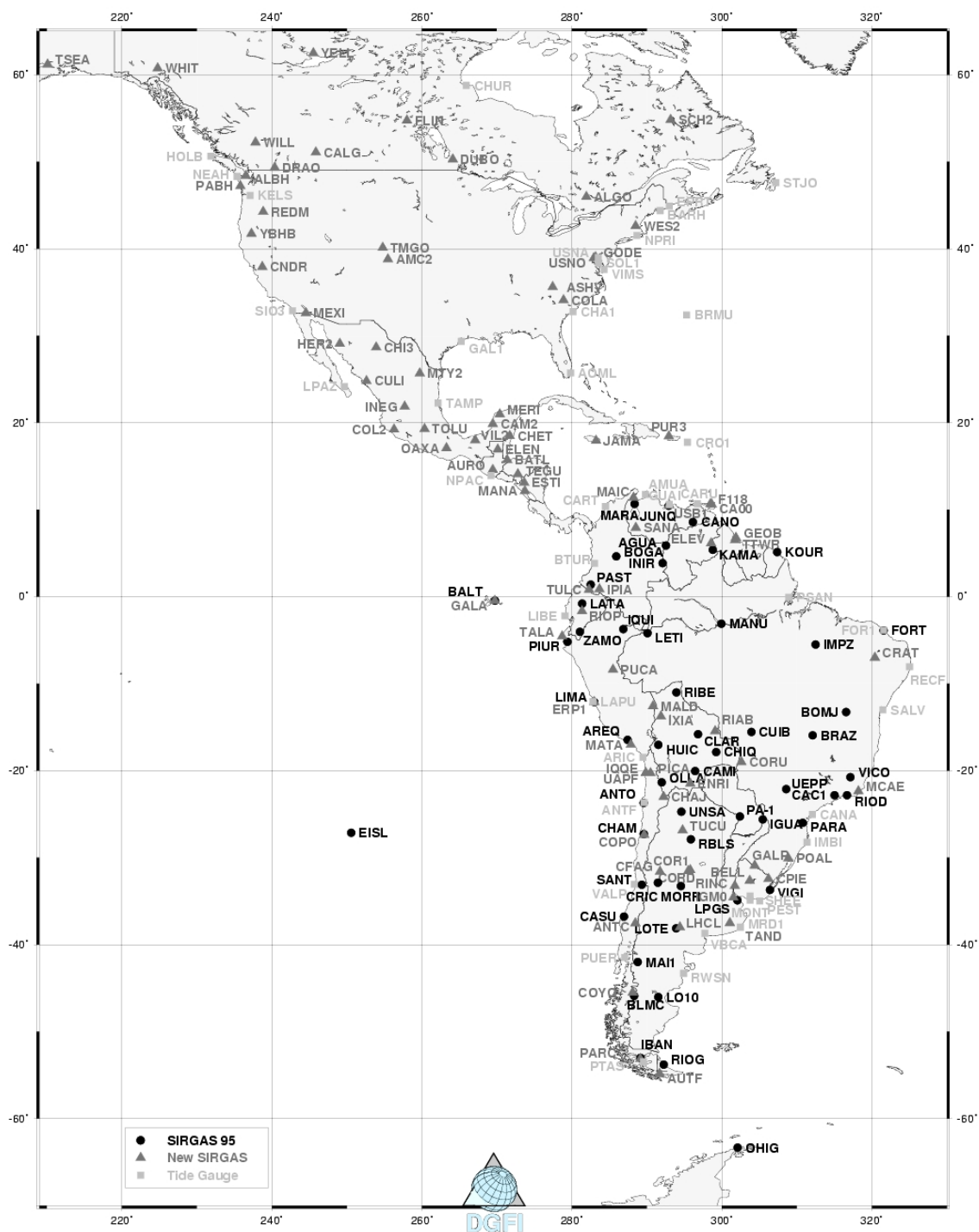


Fig.1: SIRGAS 2000 GPS Stations

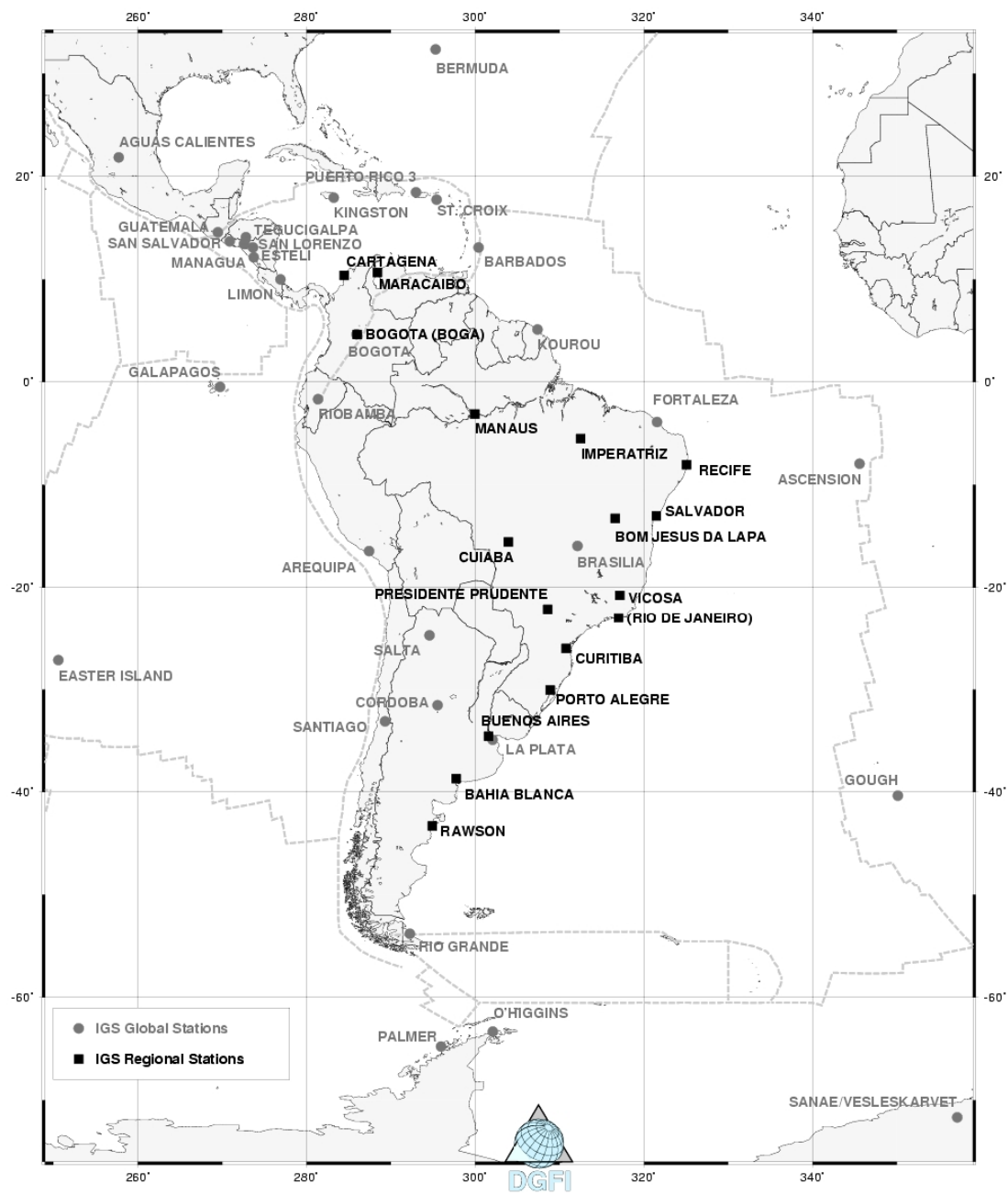


Fig.2: Stations of the IGS Regional Network Associate Analysis Center for SIRGAS

AFRICAN GEOID PROJECT

(A Project of the IAG Committee for Developing Countries, in co-operation with the International Geoid Service)

Rationale:

Precise geoid models have become increasingly important for many geodetic and surveying applications. In particular, the geoid (quasi-geoid) is needed to transform GPS-derived ellipsoidal heights to the local height datum (orthometric heights or normal heights). Precise geoid models have been developed in many parts of the world, in particular in North America and Europe. Recently, a highly successful project to determine the geoid in South America has been carried out.

The purpose of this project is to carry out a similar determination of the geoid in Africa. A major part of the project will be to collate and merge gravity anomaly data sets for Africa. Because of the paucity of these data and their poor distribution, the geoid that will result will not be very precise, but it should still be a substantial improvement over the global EGM96 model. An equally important part of the project will be to develop geoid computation expertise within Africa.

The goals of the project are best achieved by setting up a working group of African geodesists, under the aegis of the Committee for Developing Countries of the IAG and with the support of the IGeS.

Working Group Members:

Charles Merry (South Africa) - Chairman
Hussein Abd-Elmotaal (Egypt)
Benahmed Daho (Algeria)
Hassan Fashir (Sudan)
Saburi John (Tanzania)
Peter Nsombo (Zambia)
Francis Podmore (Zimbabwe)

Working Group Objectives:

- Identifying and acquiring data sets - gravity anomalies, DEM's, GPS/levelling
- Training of African geodesists in geoid computation
- Merging and validation of gravity data sets, producing 5' gridded and mean Δg
- Computation of African geoid, and evaluation using GPS/levelling data

Working Group Tasks:

1. Contact BGI, NIMA, GETECH regarding the use of their data (chairman)
2. Seek gravity, DEM, GPS/levelling data in their regions (all members)
3. Obtain KMS altimetry data, other altimetry data (chairman)
4. Obtain GLOBE DEM and evaluate it (chairman, others?)
5. Merge DEM data sets, if needed (chairman, others?)
6. Collaborate with GETECH and merge data sets to produce single 5' gridded and mean Δg data set for distribution to working group (chairman)
7. Canvas African geodesists re need for geoid school (all members)
8. If need exists, collaborate with IGeS re geoid school in Africa, possibly in 2002, possibly associated with a geodetic conference in Africa (chairman)
9. Computation and evaluation of regional and/or national geoids in Africa (interested members)
10. Computation and evaluation of African geoid (IGeS, interested members)

Time Frame:

The tasks outlined above will take several years to complete, and it is impossible to set fixed deadlines. As a rough guideline, we should expect to accomplish the following:

- Gravity and DEM data acquisition by June 2002
- Merging of data sets and production of 5' gridded and mean Δg by September 2002
- Geoid school – late 2002 (exact timing dependent upon the existence of a geodetic conference in Africa)
- Geoid computation and evaluation, and final report – by late 2003

D Blitskow
C L Merry

January 2001

International School for "THE DETERMINATION AND USE OF THE GEOID"



**Organized by The Survey Group
Faculty of Engineering, Ain Shams University
CAIRO, Egypt - 19-24 January, 2002**

Due to organizing difficulty the Geoid School in Egypt is shifted one year, to January 2002.

The Program is the same.

Program School :

1st day:

1000 – 1700	Participants check-in into respective Accommodation
1400 – 2200	Course Registration

2nd day:

Dr. Fernando Sanso (Lect. 1, 2)
Dr. Christian Tscherning (Lect. 3, 4)

0800	Course Registration
0830	Opening Ceremony
0930	Tea Break
1000	Course commences
	Lecture 1 – General Theory for geoid computation
	Lecture 2 – The remove restores concept
1300 Lunch	
1400	Lecture 3 – Collocation theory
1530	Tea Break
1545	Lecture 4 – Local datum estimation
1930	Dinner by Local Organizing Committee

3rd day:

Dr. Riccardo Barzaghi (Lect. 5, Ex. 1)
Dr. Christian Tscherning (Ex. 2)

0830	Lecture 5 – Theory and application of global gravity models
1000	Tea Break
1015	Lecture 5 (Continues)
1200	Lunch
1400	Exercises 1 - Global models
1530 Tea Break	
1545	Exercises 2 - Collocation
1745	Day Ends

4th day:	Dr. Christian Tscherning (Ex. 1) Dr. A.H.W Kearsley
0830	Exercises 1 - Collocation
1000	Tea Break
1015	Lecture 6 – Practical geoid evaluation by Ring Integration
1200	Lunch
1400	Exercises 2 - Ring integration
1530	Tea Break
1545	Exercises 3 - Ring integration and height datum
1700	Day Ends
5th day:	Dr. Rene Forsberg
830	Lecture 7 – The theory of terrain correction for gravity anomalies and potential
1000	Tea Break
1015	Lecture 7 (Continues)
1200	Lunch
1430	Exercises
1530	Tea Break
1545	Exercises (Continue)
1700	Day Ends
6th day:	Dr. Michael Sideris
0830	Lecture 8 – The FFT Approach to the Geoid Determination
1000	Tea Break
1015	Lecture 8 (Continues)
1200	Lunch
1400	Exercises
1600	Closing Ceremony
1745	Course Ends

IMPORTANT: Please live your name to the LOC (Naser El-Sheimy: naser@geomatics.ucalgary.ca) if you an still interested to participate in the school. You will get information.