

Foreword

Dear friends,
we are about to build definitely the new International Gravity Field Service (IGFS) of which you will find attached the Bylaws
According to the new concept in particular BGI and IGeS will come closer in a number of actions including the publication of a new Bulletin.
So this is the last number of the Bulletin of IGeS alone.
The Bulletin died; long life to the new Bulletin.

Fernando Sansò

Evaluation of Geoid Models and Validation of Geoid and GPS/Leveling Undulations in Canada

Vergos G.S. and M.G. Sideris

Department of Geomatics Engineering, University of Calgary, 2500 University Drive N.W., Calgary AB, T2N 1N4, Canada, gsvergos@ucalgary.ca

ABSTRACT

The purpose of this paper is to evaluate available geoid models in the region of Canada and validate the differences between geoid and GPS/Leveling undulations. The geoid heights come from six different sources; three from global geopotential solutions, namely OSU91A, EGM96, and GPM98b; and three more from national gravimetric models for the area under study, namely GSD95, GARR98, and GSD2000. The GPS-derived ellipsoidal heights and the leveling-derived orthometric ones come from the Geodetic Survey Division's *GPS on BM's* database. To minimize the differences between geoid and GPS/Leveling undulations we tested a four- and a five-parameter transformation model performing tests in both a national and a regional scale. Results show that the differences between the available geoid models range between $\pm 65\text{cm}$ and $\pm 111\text{cm}$ in terms of the standard deviation when the ones between geoid and GPS/Leveling undulations vary between $\pm 10\text{cm}$ and $\pm 98\text{cm}$ after the bias and tilt fit. The overall best agreement ($\pm 10\text{cm}$) is achieved when GSD2000 geoid heights, the 1998 readjusted orthometric heights and the 5-parameter transformation model are used.

KEYWORDS

Geoid heights, GPS ellipsoidal heights, Leveling orthometric heights, transformation model.

1 Introduction

The determination of orthometric heights by traditional techniques, such as spirit leveling, is known to be a difficult task. This is especially evident in big countries like Canada where the establishment of a leveling network covering all parts of the country would be impractical from the financial point of view [Sideris et al. 1992]. Moreover leveling over areas with rough terrain, like the Rocky Mountains in western Canada, is very strenuous and time consuming. On the other hand the combined use of GPS and geoid heights presents an alternative potential to the classic geometric leveling. Differential GPS measurements are known for their high-accuracy and geoid models are becoming more and more accurate in the aim of achieving the 1cm geoid.

These facts make the study of Geoid and GPS/Leveling differences necessary for both practical surveying and scientific applications. To this extend many studies have been performed in different places all over the world and as examples we mention [Forsberg and Madsen 1990], [Fotopoulos et al. 1999b], [Kearsley et al. 1993], and [Mainville et al. 1992] among others. In this paper we want to investigate whether the combination of geoid undulations and GPS derived geometric heights, gives sufficiently accurate results in order to stand in for precise leveling.

The area under study extends from $41^{\circ} \leq \phi \leq 73^{\circ}$ and $215^{\circ} \leq \lambda \leq 315^{\circ}$ covering the entire territory of Canada, parts of northern USA and ocean regions in the west and east coast. For the geoid undulations we used six different models, namely three global geopotential and three national gravimetric geoid solutions for Canada. The first geopotential model was OSU91A complete to degree and order 360 and developed in Ohio State University [Rapp et al. 1991]. The second was the join NASA GSFC and NIMA geopotential model EGM96 complete to degree and order 360 [Lemoine et al. 1998]. The final one used was the new ultra-high-degree, complete to degree and order 1800, global geopotential model [Wenzel 1999]. The three other geoid models represent national gravimetric geoid solutions for Canada. GSD95 is the official geoid of the country and was computed, using OSU91A as a reference field, in 1995 by the Geodetic Survey Division (GSD) of Canada [Véronneau 1997]. GARR98 is a gravimetric geoid solution for Canada and parts of the US created, using EGM96 as a reference field, in the Department of Geomatics Engineering of the University of Calgary [Fotopoulos et al. 1999a]. Finally, GSD2000 is the newly computed gravimetric geoid for Canada, which is at

the final stages of its development by the GSD of Canada [Véronneau 2001]. In the next paragraph we present the comparisons between all these models in a national scale.

The GPS/Leveling data refer to 1587 benchmarks (BMs) across Canada, which are part of the first-order Canadian leveling network. On these points orthometric heights in the Canadian Vertical Geodetic Datum 1928 (CVGD28) and GPS derived ellipsoidal heights are available and will be denoted from now on as $H28$ and h respectively [Canon 1928]. We also had available orthometric heights on 1443 benchmarks from the readjustment of the Canadian leveling network performed by GSD in 1995, which will be denoted as $H98$ from now on. A brief outline of the available data sets is provided in the subsequent sections.

To assess the agreement between the geoid undulations and the GPS/Leveling ones, we computed the differences for all the available geoid models and for both the 1928 and the 1998 orthometric heights. The various tests were performed for all of Canada and then for three large regions. These three regions represent; western Canada, between $215^\circ \leq \lambda \leq 250^\circ$; central Canada, between $250^\circ \leq \lambda \leq 265^\circ$; and eastern Canada, between $265^\circ \leq \lambda \leq 315^\circ$. In order to minimize these deviations we used a four- and a five- parameters model, which absorb most of the datum inconsistencies between the available height datasets [Heiskanen and Moritz 1967].

2 Comparisons of geoid models

Global geoid models represent the long wavelength part of the gravity field and are obtained from global geopotential solutions which are given as a set of spherical harmonic coefficients [Sideris et al. 1992]. Different datasets are used to determine these coefficients ranging from satellite observations, which give the so-called satellite-only solutions, to models which incorporate satellite altimetry and surface gravity data in the previous ones, thus usually containing more coefficients. To compute a geoid undulation value N^{GM} in spherical approximation and from such a set of spherical harmonic coefficients we use the following formula [Heiskanen and Moritz 1967]:

$$N^{GM} = R \sum_{n=2}^{n_{max}} \sum_{m=0}^n \left(\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda \right) \overline{P}_{nm}(\sin \phi) \quad (1)$$

where n_{max} is the maximum degree of expansion, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalized coefficients of the disturbing potential, $\overline{P}_{nm}(\sin \phi)$ are the fully normalized associated Legendre functions, R is the mean radius of the earth, and ϕ and λ are the geodetic latitude and longitude.

Gravimetric solutions on the other hand give a better estimate of the geoid heights by taking into account local gravity measurements. In this way they represent the medium and high frequencies of the gravity field spectrum, thus giving a more detailed picture of local mass distributions and irregularities. For such a geoid computation the well-known remove-compute-restore method is used with the trend nowadays pointing to the use of spectral methods. Spectral methods like the FFT provide an efficient way of handling large datasets in shorter time periods comparing to the classical Least Squares Collocation (LSC). This is significant since modern satellite methods like satellite altimetry offer a vast amount of data. Following this method, and to derive residual geoid undulations from residual, free-air in most cases, gravity anomalies, we can apply the 1-D FFT spherical Stokes convolution using the following formula [Haagmans et al. 1993]:

$$N^{gr} = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} F_1^{-1} \left\{ \sum_{\phi=\phi_1}^{\phi_n} F_1 \{S\} F_1 \{\Delta g \cos \phi\} \right\} \quad (2)$$

where N^{gr} is the gravimetric geoid height, R is the mean earth radius, γ is the normal gravity, Δg denotes gravity anomaly, and $S(\psi)$ is Stokes' function.

The geoid models available for the present study incorporate one of the aforementioned methods for the determination of geoid undulations. Their statistics, for the area under study, are presented in Table 1 and in Figure 1 we depict the GPM98b geoid heights and GSD2000 geoid model. All five models do not have the same resolution and the three gravimetric solutions are computed on different points. In order to compare them, we used GSD95 points and its resolution of $5' \times 5'$ as a reference. Thus, GARR98 and GSD2000 geoid heights were interpolated on GSD95 points. GSD2000 has a resolution of $2' \times 2'$ and covers the entire Canada, and most of the USA and Greenland. A bilinear type of interpolation, available in GEOIP a program from the GRAVSOFIT software package, was used [Tscherning et al. 1994]. For the comparisons between the geopotential models and the gravimetric solutions we computed the contribution of each one of the

geopotential models to geoid heights on the GARR98, GSD95, and GSD2000 points. The statistics of the differences between the available geoid models are presented in Table 2. For better visualization we depict in Figures 2 and 3 some of these differences.

From Table 2 and Figures 2 and 3 we can see that the overall best agreement is achieved between GSD2000 and EGM96 and is at the $\pm 37\text{cm}$ in terms of the standard deviation of the differences. This is something expected since EGM96 was used as a reference field for the development of GSD2000, thus their difference represents the high frequency part of the gravity field signal in Canada. The greatest differences are present when comparing OSU91A with the other models and they range between $\pm 90\text{cm}$ to $\pm 111\text{cm}$. The latter corresponds to the differences between OSU91A and GPM98b and can be mainly attributed to the additional, more recent and accurate gravity data used in GPM98b.

Table 1. Geoid undulations in the area under study. Unit: [m]

	max	min	mean	rms	std
N^{OSU91A}	39.440	-48.983	-16.541	24.624	± 18.320
N^{EGM96}	36.349	-49.104	-16.545	24.545	± 18.130
N^{GPM98b}	48.869	-49.534	-15.557	25.391	± 20.068
N^{GSD95}	43.875	-48.995	-15.450	25.300	± 20.034
N^{GARR98}	48.462	-47.661	-14.550	24.947	± 20.297
$N^{GSD2000}$	49.005	-49.585	-15.823	25.574	± 20.092

Table 2. Geoid height differences in Canada. Unit: [m]

	max	min	mean	rms	std
$N^{OSU91A} - N^{GPM98b}$	12.213	-6.503	0.025	1.111	± 1.111
$N^{EGM96} - N^{OSU91A}$	6.941	-10.570	-0.041	0.904	± 0.903
$N^{EGM96} - N^{GPM98b}$	5.532	-2.919	-0.016	0.664	± 0.664
$N^{GSD95} - N^{OSU91A}$	10.475	-10.466	0.096	0.970	± 0.965
$N^{GSD95} - N^{EGM96}$	6.025	-5.553	0.120	0.663	± 0.653
$N^{GSD95} - N^{GPM98b}$	6.266	-6.006	0.107	0.895	± 0.888
$N^{GSD95} - N^{GARR98}$	-1.237	5.306	-0.900	1.099	± 0.631
$N^{GARR98} - N^{OSU91A}$	11.158	-10.230	0.945	1.384	± 1.011
$N^{GARR98} - N^{EGM96}$	7.477	-1.976	1.020	1.266	± 0.751
$N^{GARR98} - N^{GPM98b}$	7.845	-2.733	1.007	1.422	± 1.004
$N^{GSD2000} - N^{OSU91A}$	11.208	-10.769	-0.291	1.036	± 0.994
$N^{GSD2000} - N^{EGM96}$	3.238	-5.624	0.250	0.446	± 0.369
$N^{GSD2000} - N^{GPM98}$	6.228	-6.308	-0.373	0.638	± 0.518
$N^{GSD2000} - N^{GSD95}$	6.727	-3.567	-0.266	0.797	± 0.751
$N^{GSD2000} - N^{GARR98}$	7.751	-1.974	1.272	1.430	± 0.653

These big differences are especially evident in Western Canada, near the Rocky Mountains, and in the Northeastern parts of our area near Greenland. In these areas the discrepancies reach the 10.5m for the comparisons of OSU91A with EGM96 and GSD95, 11.2m with GARR98 and GSD2000, and 12.2m with GPM98b. The main reasons for these great differences can be found in the strong effect of the additional gravity data used and to the roughed and difficult to model terrain of the Rockies. Some big differences between EGM96 and OSU91A are also present in Hudson Bay, the St. Lawrence Gulf, and the Labrador Sea, which are mainly due to the additional and more accurate altimetry data used in EGM96 (see Figure 2(a)). The overall best agreement for the discrepancies between all models is found in Central Canada, ranging between 50cm – 1m. The main reason for these small values can be viewed in the absence of rough topography.

The differences between GSD95 and GARR98 have a standard deviation of $\pm 63\text{cm}$ with a value range of 6.5m. These deviations can be explained if we take into account that GSD95 uses OSU91A as a reference field when GARR98 uses EGM96. For the comparison between GSD95 and EGM96 the standard deviation of the differences is at the $\pm 65\text{cm}$, representing the high frequency information included in GSD95 with the local gravity anomalies used versus the long wavelength nature of EGM96.

In Figure 3(b) we depict the differences between GSD2000 and GSD95, which have a standard deviation of $\pm 75\text{cm}$. These differences are correlated with the topography across the Rockies and reach their greatest value of 6.7m in Greenland. This is so because of the use of additional gravity data in the development of EGM96 versus OSU91A. Additionally some significant values of the order of 50-70cm can be seen in the ocean areas,

which are mainly attributed to the multi-satellite altimetry data used in GSD2000, where Geodetic Mission data were incorporated as well. From Figures 2(b) and 3(a), where the discrepancies between GPM98b and EGM96 and GSD95, are depicted respectively, we can see a very interesting pattern. The structure of this pattern changes right across the border of Canada and the USA and it can be seen that there is a direct correlation with the borderline. We can attribute this to the use of new gravity data for USA when the same as in EGM96 data were used for Canada. The pattern of the differences that is seen in USA agrees very well with the case in the oceans where more recent and more accurate altimetry data were used.

3 Comparisons of geoid and GPS/Leveling undulations

The basic relationship between geoid, ellipsoidal, and orthometric heights is very simple, given by the following formula:

$$H = h - N \quad (3)$$

Because of the simplicity of Eq. 3, the combined use of geoidal undulations and GPS-derived ellipsoidal heights is so appealing in comparison to the use of orthometric heights derived from spirit leveling. For our study we used 1587 benchmarks, which are part of the first-order leveling network of Canada.

On these points orthometric heights in the CGVD28 vertical datum ($H28$) are available. They were determined in 1928 and they belong to the official height system of Canada. The heights were determined with a differential adjustment, tied to the Atlantic Mean Sea Level of 1928 as computed in the Halifax, Yarmouth, and Father Point tide gauge stations and an elevation agreed upon for Rouses Point [Canon 1928]. For the 1928 set of orthometric heights, approximate normal gravity values were used instead of true gravity measurements. The second set of orthometric heights ($H98$) comes from the adjustment performed by GSD in October 1995. This was an attempt to improve CGVD28 because of various systematic distortions occurred during the time period since 1928. Factors creating these distortions can be viewed in terms of mean sea level rise and post-glacial rebound (Fotopoulos et al. 1999a). The $H98$ set corresponds to Helmert like orthometric heights, which were computed using actual surface gravity measurements. A minimal constrained type of adjustment, of the entire leveling network of Canada, was performed, by retaining fixed a single point at the tide-gauge station in Rimouski, Quebec. This set of orthometric heights is characterized by GSD as a scientific database and is not to be used in practical applications. 1443 benchmarks with $H98$ heights are available and the lack of data comparing to $H28$ is located in the island of Newfoundland and British Columbia (see Figure 4). The known GPS-derived ellipsoidal heights (h^{GPS}) were determined during GPS campaigns conducted by GSD between 1987 and 1996, with observation periods ranging between 4 to 48 hours. They refer to ITRF96 reference frame (reference epoch 1997.0) and the GRS80 ellipsoid.

For our comparisons we used all available geoid models and both sets of orthometric heights. We first interpolated geoid undulations from the geoid models on the GPS benchmarks where orthometric and ellipsoidal heights were available, forming five N^{GM} datasets. To do so we used a bilinear type of interpolation implemented in GEOIP, a program from GRAVSOFIT. In a next step, using Eq. 3, we determined on the same points the GPS/Leveling undulations N^{GPS} . In this way three different datasets were formed. 1587 points with geoid heights using $H28$ set, 1443 points using $H98$ and 1443 points using $H28$ but only on the points where information for the $H98$ heights was available. The statistics of the interpolated geoid model undulations, the $H28$ and $H98$ orthometric heights and the geometric heights are presented in Table 3. Then for each one of the GPS/Leveling geoid height datasets we determined the differences with the previously interpolated N^{GM} on BMs.

The computed differences reflect datum inconsistencies between the available height data, long-wavelength geoid errors, and GPS and leveling errors included in the ellipsoidal and orthometric heights. In order to minimize these deviations we used a four- and a five-parameter transformation models. The four-parameter model is the most commonly used in such adjustments and is given by the following formula [Heiskanen and Moritz 1967, Sideris 1992]:

$$N_i^{GPS} - N_i^{GM} = h_i^{GPS} - H_i^k - N_i^{GM} = b_3 + b_0 \cos \phi_i \cos \lambda_i + b_1 \cos \phi_i \sin \lambda_i + b_2 \sin \phi_i + v_i \quad (4)$$

where the parameters b_0 , b_1 , b_2 and b_3 are calculated by a least squares technique, (i) denotes the benchmarks, (k) represents either $H28$ or $H98$, v_i are the residuals to be minimized, N^{GPS} and N^{GM} are the GPS/Leveling and geoid undulations, and ϕ and λ are the geodetic latitude and longitude respectively. The five-parameter model is

similar to Eq. 4 with one additional parameter added. Following Heiskanen and Moritz [1967] it is given by the following formula:

$$N_i^{GPS} - N_i^{GM} = h_i^{GPS} - H_i^k - N_i^{GM} = b_4 + b_0 \cos \phi_i \cos \lambda_i + b_1 \cos \phi_i \sin \lambda_i + b_2 \sin \phi_i + b_3 \sin^2 \phi_i + v_i \quad (5)$$

where now the parameters b_0 , b_1 , b_2 , b_3 and b_4 are determined minimizing the residuals v_i using a least squares technique. The other parameters are the same as in Eq. 4. An explicit derivation and geometrical interpretation of the two models is given in Heiskanen and Moritz [1967, Section 5.9]. We use the two different models in order to assess the possible improvement that the additional parameter offers in Eq. 5. As we mentioned before the residuals v_i , which are to be minimized, contain a wide spectrum of different in nature errors resulting from different in accuracy methods. With Eqs. 4 and 5 equal weight is given in all observations during the least squares adjustment. More rigorous models can be found in Kotsakis [2001], where an extended seven-parameter similarity transformation model is used with equal or slightly better results than that of Eq. 5, and Kotsakis [1999].

The statistics of the computed differences between the available datasets can be found in Tables 4 to 6. The differences between geoid and GPS/Leveling undulations, when using H28 orthometric heights on all 1587 benchmarks, are presented in Table 4, when using H98 on the 1443 points they refer to in Table 5, and when using H28 data but on the same points as H98, in Table 6. The differences after the bias and tilt fit using the four- and the five-parameter models are presented in parenthesis with bold letters and with italics respectively. From Table 4 and for the 1928-orthometric heights, we can see that the best agreement, at the ± 22 cm level, before the bias and tilt fit is offered by GSD2000 geoid when for GARR98 and GSD95 it is at the ± 28 cm and ± 32 cm respectively. This can be mainly attributed to the more accurate long-wavelength information contained in EGM96 in comparison to OSU91A, which is the reference field used for the development of GSD95. The ± 105 cm of difference when using geoid heights from GPM98b is due to the absence of new data for Canada in the development of this model. On the other hand new data were used for USA and the surrounding oceans influencing in this way the harmonics over Canada as well. In a study conducted by Vergos et al. [2001] in the Atlantic part of Canada, GPM98b gives better agreement with TOPEX/POSEIDON Sea Surface Heights (SSHs), comparing to EGM96, when used for marine geoid modeling.

Table 3. Geoid, orthometric, and ellipsoidal heights on benchmarks. Unit: [m]

	max	min	mean	rms	std
N^{OSU91A}	12.910	-46.617	-20.492	23.740	± 11.442
N^{EGM96}	11.940	-47.854	-20.555	23.566	± 11.527
N^{GPM98b}	13.014	-47.873	-20.511	23.488	± 11.446
N^{GSD95}	12.215	-47.680	-20.538	23.535	± 11.493
N^{GARR98}	12.786	-46.652	-19.843	22.916	± 11.462
$N^{GSD2000}$	11.554	-48.341	-20.855	23.820	± 11.509
H28	2010.086	1.101	436.141	562.179	± 354.720
H98	2011.220	2.104	472.072	588.913	± 352.089
<i>h^{GPS}</i>	1999.086	-42.465	414.982	548.242	± 358.273

This is further evidence that the lack of new data in the continental part of Canada is the main factor causing this high disagreement between GPM98b and GPS/Leveling geoid heights. For OSU91A and EGM96 the differences are at the ± 70 cm and ± 38 cm before the fit indicating the more accurate long-wavelength information included in the latter. After the bias and tilt fit, using the 4-parameter model, the improvement is at the 4cm for OSU91A, the 5cm for EGM96, the 18cm for GPM98b, the 9cm for GSD95 and GARR98, and the 2cm for GSD2000. The overall best agreement after the fit is achieved by GARR98 being at the ± 19 cm. When using the five-parameter model for the minimization of the residuals the improvement, comparing to the four-parameter one, is not great. The differences improve by 5mm for OSU91A, by 2cm for GPM98b and by 2cm for the two GSD models. The improvement for both EGM96 and GARR98 is negligible. These results indicate that the four-parameter model is adequate enough for the minimization of the differences in this kind of studies.

Table 4. Comparisons of various geoid models with 1928 orthometric heights before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	5.099 (5.203) (5.290)	-4.953 (-4.712) (-4.756)	-0.668 (0.000) (0.000)	0.963 (0.658) (0.653)	± 0.694 (± 0.658) (± 0.653)
$N^{GPS} - N^{OSU91A}$ (after 2rms)	1.787 (2.337) (2.427)	-1.905 (-1.385) (-1.459)	-0.700 (0.000) (0.000)	0.861 (0.474) (0.459)	± 0.501 (± 0.474) (± 0.459)
$N^{GPS} - N^{EGM96}$	1.262 (1.531) (1.477)	-1.468 (-1.055) (-1.113)	-0.605 (0.000) (0.000)	0.717 (0.332) (0.323)	± 0.384 (± 0.332) (± 0.323)
$N^{GPS} - N^{GPM98b}$	4.389 (4.280) (4.179)	-2.652 (-2.088) (-2.263)	-0.649 (0.000) (0.000)	1.241 (0.889) (0.868)	± 1.058 (± 0.889) (± 0.868)
$N^{GPS} - N^{GPM98b}$ (after 2rms)	2.365 (2.662) (2.515)	-2.373 (-1.813) (-1.965)	-0.694 (0.000) (0.000)	1.188 (0.811) (0.789)	± 0.965 (± 0.811) (± 0.789)
$N^{GPS} - N^{GSD95}$	0.921 (1.680) (1.601)	-1.338 (-0.613) (-0.618)	-0.623 (0.000) (0.000)	0.701 (0.231) (0.209)	± 0.322 (± 0.231) (± 0.209)
$N^{GPS} - N^{GARR98}$	-0.361 (1.278) (1.278)	-2.470 (-0.843) (-0.843)	-1.317 (0.000) (0.000)	1.346 (0.190) (0.189)	± 0.280 (± 0.190) (± 0.189)
$N^{GPS} - N^{GSD2000}$	1.204 (1.630) (1.552)	-1.012 (-0.620) (-0.375)	-0.350 (0.000) (0.000)	0.376 (0.200) (0.176)	± 0.220 (± 0.200) (± 0.176)

Table 5. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	4.087 (5.191) (5.210)	-5.923 (-4.719) (-4.759)	-1.112 (0.000) (0.000)	1.305 (0.668) (0.656)	± 0.682 (± 0.668) (± 0.656)
$N^{GPS} - N^{EGM96}$	0.359 (1.582) (1.574)	-2.574 (-1.306) (-1.425)	-1.053 (0.000) (0.000)	1.113 (0.302) (0.302)	± 0.360 (± 0.302) (± 0.302)
$N^{GPS} - N^{GPM98b}$	3.419 (4.075) (3.909)	-3.497 (-2.089) (-1.904)	-1.093 (0.000) (0.000)	1.454 (0.838) (0.815)	± 0.960 (± 0.838) (± 0.815)
$N^{GPS} - N^{GSD95}$	0.273 (0.616) (0.515)	-2.093 (-0.669) (-0.760)	-1.090 (0.000) (0.000)	1.176 (0.136) (0.129)	± 0.442 (± 0.136) (± 0.129)
$N^{GPS} - N^{GARR98}$	-1.198 (0.461) (0.431)	-2.702 (-1.057) (-0.999)	-1.742 (0.000) (0.000)	1.752 (0.143) (0.137)	± 0.205 (± 0.143) (± 0.137)
$N^{GPS} - N^{GSD2000}$	0.076 (0.470) (0.403)	-1.402 (-0.530) (-0.448)	-0.744 (0.000) (0.000)	0.786 (0.106) (0.101)	± 0.254 (± 0.106) (± 0.101)

Table 6. Comparisons of various geoid models with 1928 orthometric heights (on the H98 points) before and after the bias and tilt fit. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	5.099 (5.146) (5.192)	-4.953 (-4.716) (-4.816)	-0.681 (0.000) (0.000)	0.978 (0.672) (0.662)	± 0.702 (± 0.672) (± 0.662)
$N^{GPS} - N^{EGM96}$	1.262 (1.582) (1.534)	-1.468 (-1.004) (-1.075)	-0.622 (0.000) (0.000)	0.719 (0.326) (0.314)	± 0.360 (± 0.326) (± 0.314)
$N^{GPS} - N^{GPM98b}$	4.389 (4.078) (3.852)	-0.661 (-2.123) (-1.900)	-0.661 (0.000) (0.000)	1.252 (0.879) (0.838)	± 1.063 (± 0.879) (± 0.838)
$N^{GPS} - N^{GSD95}$	0.584 (0.951) (0.694)	-1.338 (-0.637) (-0.730)	-0.658 (0.000) (0.000)	0.721 (0.223) (0.193)	± 0.294 (± 0.223) (± 0.193)
$N^{GPS} - N^{GARR98}$	-0.690 (0.622) (0.552)	-2.470 (-0.812) (-0.856)	-1.310 (0.000) (0.000)	1.341 (0.176) (0.174)	± 0.285 (± 0.176) (± 0.174)
$N^{GPS} - N^{GSD2000}$	0.404 (0.621) (0.587)	-1.012 (-0.814) (-0.417)	-0.313 (0.000) (0.000)	0.382 (0.198) (0.169)	± 0.221 (± 0.198) (± 0.169)

From Table 5, and for the readjusted 1998-orthometric heights, GARR98 offers again the best agreement, at the ± 21 cm, before the fit, comparing to ± 25 cm and ± 44 cm for GSD2000 and GSD95. But, after the fit GARR98 and GSD95 reduce both to almost ± 14 cm and ± 13 cm, when GSD2000 gives the overall best agreement at the ± 10.6 cm and ± 10 cm level, for the four- and the five-parameter models respectively. This would suggest that even when using the superior EGM96 to model the long-wavelength part of the geoid, the

available resolution and accuracy of the gravity data is not sufficient enough to bring the agreement with GPS/Leveling at the sub-decimeter level. Nevertheless, the achieved $\pm 10\text{cm}$ for GSD2000 geoid heights provide promising results for the usual, low accuracy, surveying applications. When using the four-parameter model the fit improvement is almost at the 3cm level for OSUA91A, the 6cm for EGM96 and GARR98, the 14cm for GSD2000 and GPM98b, and the 31cm for GSD95. Again the use of the five-parameter model fails to give significant improvement comparing to its four-parameter counterpart, since their differences range from a few mm to 3cm. Nevertheless the overall best fit is seen when using GSD2000, the H98 orthometric heights, and the five-parameter model.

Comparing Tables 5 and 6, where the statistics of the differences when using the H98 and the H28 on the same points are presented respectively, we can see that in all comparisons the 1998-adjusted heights give better agreement with almost all geoid models. The improved fit ranges between 1cm and 4cm indicating the superiority of the new readjusted orthometric heights. As expected the H98 heights fail to give better results comparing to the H28 ones when computing their differences with the oldest geoid model, OSU91A. This offers an additional proof that OSU91A fails to model the low-frequency part of the geoid, at least as accurately as EGM96 does. Of course this outcome holds for the specific area under study and the specific data sets and methods used. In remote areas, where no new gravity data were added during the last 15 years, this conclusion may not hold.

Comparing the results of this study with the ones from Kotsakis et al. [2001], where an extended seven-parameter similarity transformation model was used, we can conclude that the differences are not significant. When using EGM96 and the five-parameter model we achieved an agreement of $\pm 31.4\text{cm}$, for the reduced 1928 orthometric heights, and $\pm 30.6\text{cm}$ for the 1998 ones respectively. For the same datasets the extended model resulted in $\pm 29.4\text{cm}$ and $\pm 30.6\text{cm}$ fit between the height data. Substituting EGM96 with GSD95 the five-parameter model gave $\pm 19.3\text{cm}$ fit for the reduced H28 dataset and $\pm 12.9\text{cm}$ for H98, when the extended one resulted in an agreement of $\pm 18.4\text{cm}$ and $\pm 12.7\text{cm}$ respectively. From these results we may conclude that a five-parameter transformation model is capable of minimizing the residuals enough so that an extended version is not necessary. Of course we must not neglect the fact that a few mm improvement will be important if we are after the cm-level fit between our height data. But since the present level of agreement is at the $\pm 10\text{cm}$, for the best case, such a constraint is not considered to be necessary.

As a final remark in this nationwide investigation, we present in Figure 5 West – East profiles of the residuals after the five-parameter model fit when using H98 orthometric heights and GSD95, GSD2000, OSU91A, EGM96, and GPM98b geoid heights. From (a), (b), (c), and (d) we can see that the greatest differences are located in the Western part of Canada where the Rocky Mountains are situated. This is something expected, since both geoid modeling and spirit leveling are less accurate in areas with rough topography comparing to flat areas like Central Canada. Additionally some big values appear in Eastern Canada and this, for GSD95 and GARR98, can be mainly attributed to the low-quality shipborne gravity data used for geoid modeling near the island of Newfoundland and the St. Lawrence gulf. From Figure 5(a) the improved residuals for the case of GSD2000, comparing to GSD95, are evident, since almost all large deviations are eliminated and the differences look constantly small across Canada. For OSU91A the lack of new and more accurate altimetry data adds to residuals at the $\pm 60\text{cm}$ for the same area. The improvement in modeling the low frequency part of the geoid that EGM96 offers, comparing to OSU91A, is evident from Figure 5(c). We can see that the very big differences in British Columbia, at the $\pm 350\text{cm}$ for OSU91A, are reduced to $\pm 80\text{cm}$ for EGM96. The same improvement can be seen in the Rocky Mountains where once again EGM96 outperforms OSU91A. The absence of new gravity data for Canada in the development of GPM98b is evident in Figure 5 (d), so that the differences are great across Canada and not in the Western part only. The new data used in USA and the oceans and the lack of new information over Canada clearly trigger the main part of the differences between the model and GPS/Leveling undulations. As a concluding remark we mention the obvious superiority of gravimetric geoid solutions in contrast to global geopotential models. It is important to mention at this point that even with gravimetric geoid models the best level of agreement is at the $\pm 10\text{cm}$ level, which is far from the cm-level goal. New, more accurate, and higher resolution gravity data are needed if cm-level geoid and GPS/Leveling residuals are to be obtained.

We will proceed to the regional comparisons where three sub-areas were selected. These three regions represent: western Canada, between $215^\circ \leq \lambda \leq 250^\circ$; central Canada, between $250^\circ \leq \lambda \leq 265^\circ$; and eastern Canada, between $265^\circ \leq \lambda \leq 315^\circ$. The total number of GPS benchmarks is 737, 172, and 534 for western, central, and eastern Canada respectively. For each one of these cases we readjusted the geoid and GPS/Leveling undulations using all available geoid models and orthometric height datasets.

The results obtained from these tests are far too many to be depicted individually, thus we present only the comparisons when using H98 orthometric heights, since these give the overall best agreement. These results are

presented in Tables 7, 8, and 9 for western, central, and eastern Canada respectively where we can see that the largest residuals are found in western Canada. This is expected, since the topography in this area is rough thus making both geoid modeling and traditional leveling less accurate than in plain regions.

For all regions GSD2000 gives the overall best agreement after the bias and tilt fit for both models used. It is interesting to see that the agreement between GSD2000 and GPS/Leveling undulations in western and eastern Canada is approximately at the $\pm 9.5\text{cm}$ when for the central region reaches the $\pm 6\text{cm}$. The latter level of agreement provides substantial evidence that the current method is appropriate for orthometric height determination over large regions with smooth terrain, like central Canada. Of course even for these cases it cannot reach the millimeter level requirements of a first-order leveling network. Comparing the statistics of the three geopotential solutions for the three regional cases, we can see that they provide a relatively good agreement for central Canada at the $\pm 12\text{cm}$ for OSU91A, the $\pm 11\text{cm}$ for EGM96, and the $\pm 24\text{cm}$ for GPM98b. But the statistics degrade for the eastern part and get worst in the western one, reaching the $\pm 85\text{cm}$, $\pm 36\text{cm}$, and $\pm 91\text{cm}$ for the aforementioned models respectively. This is an indication of the fact that global geopotential solutions fail to model the high-frequency part of the gravity field spectrum, in areas with varying topography like the Rocky Mountains, caused by the distribution of the masses. Thus, they are to be avoided when GPS and geoid heights are to be used for orthometric height determination.

Table 7. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Western Canada. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	4.087 (4.636) (3.800)	-5.923 (-5.369) (-4.759)	-1.297 (0.000) (0.000)	1.324 (0.851) (0.793)	± 0.878 (± 0.851) (± 0.793)
$N^{GPS} - N^{EGM96}$	0.359 (1.557) (1.525)	-2.574 (-1.286) (-1.273)	-1.236 (0.000) (0.000)	1.263 (0.359) (0.358)	± 0.369 (± 0.359) (± 0.358)
$N^{GPS} - N^{GPM98b}$	3.419 (3.642) (4.223)	-3.497 (-1.755) (-1.857)	-1.144 (0.000) (0.000)	1.459 (0.915) (0.830)	± 1.230 (± 0.915) (± 0.830)
$N^{GPS} - N^{GSD95}$	-0.888 (0.425) (0.395)	-2.093 (-0.614) (-0.602)	-1.405 (0.000) (0.000)	1.416 (0.120) (0.119)	± 0.148 (± 0.120) (± 0.119)
$N^{GPS} - N^{GARR98}$	-1.441 (0.402) (0.354)	-2.434 (-0.543) (-0.524)	-1.833 (0.000) (0.000)	1.845 (0.115) (0.111)	± 0.119 (± 0.115) (± 0.111)
$N^{GPS} - N^{GSD2000}$	-0.458 (0.388) (0.391)	-1.402 (-0.295) (-0.308)	-0.941 (0.000) (0.000)	0.957 (0.099) (0.097)	± 0.139 (± 0.099) (± 0.097)

Table 8. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Central Canada. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	-0.611 (0.377) (0.371)	-1.428 (-0.447) (-0.417)	-1.043 (0.000) (0.000)	1.051 (0.125) (0.125)	± 0.152 (± 0.125) (± 0.125)
$N^{GPS} - N^{EGM96}$	-0.551 (0.423) (0.422)	-1.582 (-0.531) (-0.524)	-0.980 (0.000) (0.000)	1.011 (0.116) (0.116)	± 0.142 (± 0.116) (± 0.116)
$N^{GPS} - N^{GPM98b}$	-0.120 (0.596) (0.621)	-2.643 (-0.788) (-0.777)	-1.270 (0.000) (0.000)	1.372 (0.243) (0.242)	± 1.230 (± 0.243) (± 0.242)
$N^{GPS} - N^{GSD95}$	-1.071 (0.217) (0.241)	-1.701 (-0.255) (-0.206)	-1.302 (0.000) (0.000)	1.304 (0.070) (0.065)	± 0.095 (± 0.070) (± 0.065)
$N^{GPS} - N^{GARR98}$	-1.479 (0.337) (0.336)	-2.272 (-0.233) (-0.168)	-1.841 (0.000) (0.000)	1.845 (0.096) (0.087)	± 0.116 (± 0.096) (± 0.087)
$N^{GPS} - N^{GSD2000}$	-0.527 (0.222) (0.243)	-0.994 (-0.235) (-0.170)	-0.726 (0.000) (0.000)	0.731 (0.065) (0.061)	± 0.074 (± 0.065) (± 0.061)

Table 9. Comparisons of various geoid models with 1998 orthometric heights before and after the bias and tilt fit in Eastern Canada. Unit: [m]

	max	min	mean	rms	std
$N^{GPS} - N^{OSU91A}$	-0.070 (0.791) (0.808)	-1.589 (-0.646) (-0.704)	-0.880 (0.000) (0.000)	1.051 (0.271) (0.262)	± 0.291 (± 0.271) (± 0.262)
$N^{GPS} - N^{EGM96}$	0.132 (0.898) (0.838)	-1.700 (-0.822) (-0.741)	-0.826 (0.000) (0.000)	0.851 (0.234) (0.227)	± 0.240 (± 0.234) (± 0.227)
$N^{GPS} - N^{GPM98b}$	0.955	-2.266	-0.965	1.251	± 0.534

	(1.819) (1.686)	(-1.469) (-1.411)	(0.000) (0.000)	(0.509) (0.486)	(±0.509) (±0.486)
$N^{GPS} - N^{GSD95}$	0.273 (0.456) (0.452)	-1.341 (-0.832) (-0.796)	-0.586 (0.000) (0.000)	0.728 (0.130) (0.128)	±0.302 (±0.130) (±0.128)
$N^{GPS} - N^{GARR98}$	-1.198 (0.466) (0.435)	-2.702 (-0.870) (-0.804)	-1.583 (0.000) (0.000)	1.737 (0.141) (0.137)	±0.225 (±0.141) (±0.137)
$N^{GPS} - N^{GSD2000}$	0.076 (0.327) (0.329)	-1.030 (-0.422) (-0.466)	-0.479 (0.000) (0.000)	0.491 (0.096) (0.094)	±0.147 (±0.096) (±0.094)

4 Conclusions

A detailed study on the possibility to substitute classic spirit leveling with GPS/geoid derived orthometric heights was presented. We used; three gravimetric geoid solutions and three global geopotential models to derive geoid undulations; two different sets of orthometric heights, one from the original 1928 adjustment of the entire Canadian leveling network and another from the 1998 readjustment when real gravity measurements were used; and finally GPS derived ellipsoidal heights on BMs. For the minimization of the residuals a four- and a five-parameter transformation models were used, successfully managing to absorb the datum inconsistencies between our height data and GPS, leveling, and long-wavelength geoid, errors.

The comparisons between the available geoid models indicate that the main problems exist in Western Canada mainly due to the roughed topography and the lack of sufficient gravity measurements. The superiority of EGM96 over OSU91A to model the long-wavelength part of the geoid is evident, when the comparisons of GPM98b indicate that the model may not be adequate enough for use in the continental part of Canada due to the lack of new data in this region during its development. For marine regions though, the advantages of the new altimetry data incorporated in GPM98b make it to give better results than EGM96 when ocean geoid modeling is concerned [Vergos et al. 2001].

The overall best agreement, ± 10 cm, between geoid and GPS/Leveling undulations was achieved when we used height information from GSD95, the 1998-readjustment of the Canadian leveling network, and the five-parameter similarity transformation model. The use of the five-parameter model slightly improves the residuals comparing to the four-parameter one, with their differences ranging between 1mm to 3cm. Orthometric heights from the 1998-readjustment provide better results than the 1928 ones at the 1cm to 4cm level. Furthermore the results for Central Canada reached the ± 6 cm level for GSD2000 indicating the advantages of this latest gravimetric geoid solution and the better agreement achieved in areas with smooth topography in contrast to those with more varying one.

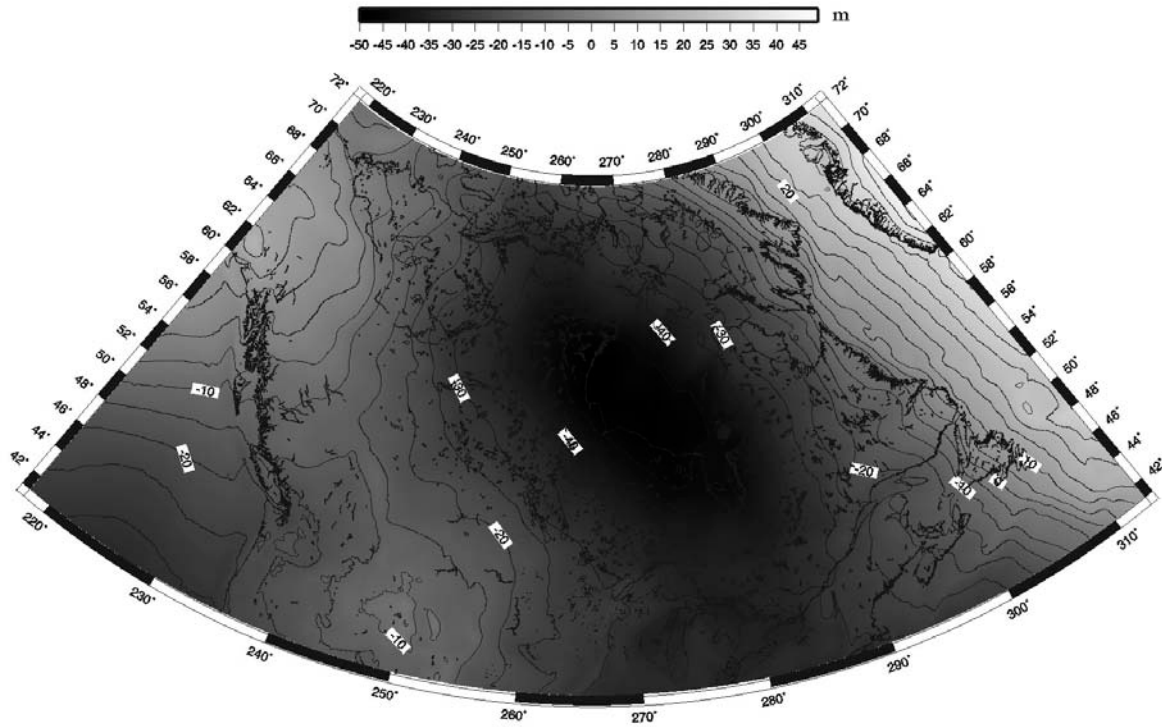
When low-order leveling surveys are conducted the combined use of geoid and GPS ellipsoidal heights offers a great alternative to traditional methods, providing accuracies at the ± 10 cm and ± 6 cm level for national and regional scales respectively. On the other hand, when first-order networks are concerned, meaning that the accuracies wanted are at the sub-cm-level, the presented method is not a feasible alternative to traditional leveling. To achieve a sub-cm-level accuracy with the use of geoid and GPS orthometric heights, more accurate and higher resolution gravity data should be obtained. Overall, more sophisticated methods, which will take into account the a-priori accuracies of the different height data, need to be employed and further investigated.

Acknowledgements

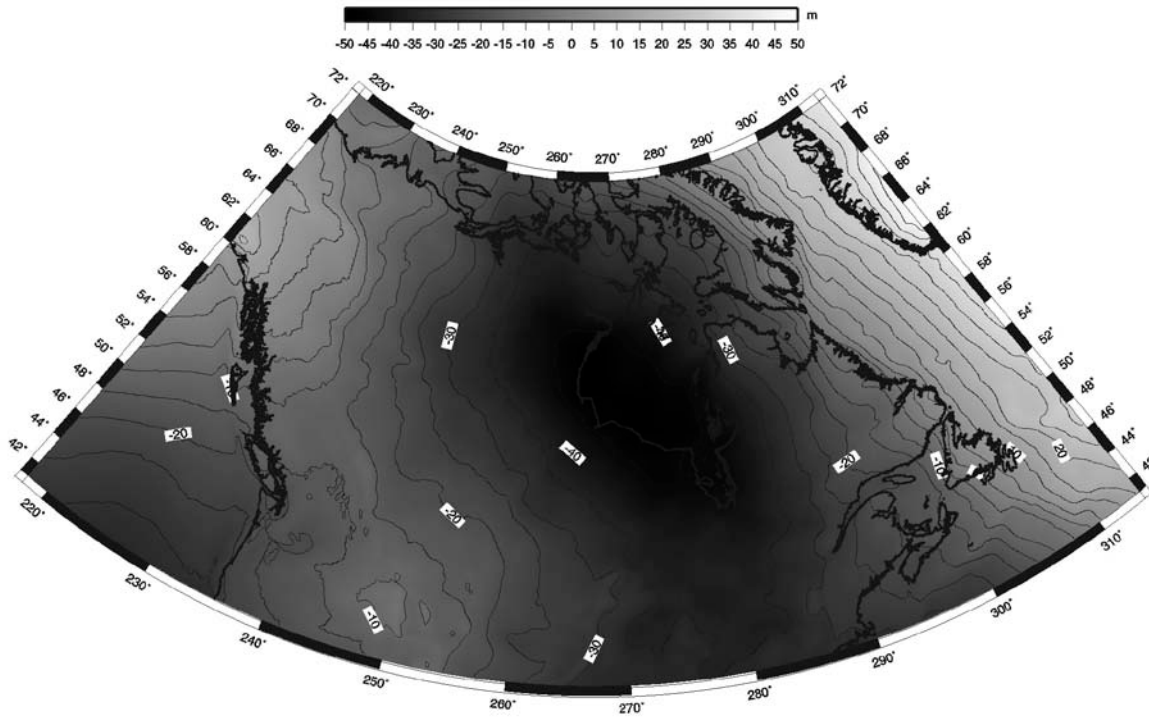
The first of the authors received financial support by the GEOIDE Network of Centers of Excellence. GPS and leveling data were provided by Geodetic Survey Division of Geomatics Canada. All this support is gratefully acknowledged. We extensively used the Generic Mapping Tools Version 3.1 [Wessel and Smith, 1998] in displaying our results.

References

- Canon, J.B, M.A. (1928). Adjustment of the Precise Level Net of Canada 1928. *Geodetic Survey of Canada*, Publication No. 28, F.A. Acland, Ottawa.
- Forsberg, R. and F. Madsen. (1990). High Precision Geoid Heights for GPS Leveling. *Proceedings of the 2nd International Symposium on Precise Positioning with the Global Positioning System*, Sept. 2-7, Ottawa, Canada, pp. 1060-1074.
- Fotopoulos, G., C. Kotsakis, and M.G. Sideris. (1999a). A New Canadian Geoid Model in Support of Leveling by GPS. *Geomatica*, Vol. 53, pp.53-62.
- Fotopoulos, G., C. Kotsakis, and M.G. Sideris. (1999b). Evaluation of Geoid Models and Their Use in Combined GPS/Leveling/Geoid Height Network Adjustment. *Technical Reports of the Department of Geodesy and Geoinformatics*, Universität Stuttgart, Report Nr. 1999.4.
- Haagmans, R., E. de Min and M. van Gelderen. (1993). *Fast evaluation of convolution integrals on the sphere using 1D FFT, and a comparison with existing methods for Stokes' integral*. Manuscr. Geod. 18, pp. 227-241.
- Heiskanen, W.A. and H. Moritz. (1967). *Physical Geodesy*. Institute of Physical Geodesy, Technical University of Graz, Austria, W.H. Freeman.
- Kearsley, A.H.W., Z Ahmad, and A. Chan. (1993). National Height Datums, Leveling, GPS Heights and Geoids. *Aust. J. Geod. Photogram. Surv.*, No. 59, pp. 53-88.
- Kotsakis, C. and M.G. Sideris. (1999). On the adjustment of combined GPS/leveling/geoid networks. *J. of Geodesy*, Vol. 73, pp.412-421.
- Kotsakis, C., G. Fotopoulos, and M.G. Sideris. (2001). *Optimal Fitting of Gravimetric Geoid Undulations to GPS/Leveling Data Using an Extended Similarity Transformation Model*. A paper presented at the Annual Scientific Meeting of the Canadian Geophysical Union (CGU), Ottawa, May 14 – 17.
- Lemoine, F.G., D.E. Smith, R. Smith, L. Kunz, E.C. Pavlis, N.K. Pavlis, S.M. Klosko, D.S. Chinn, M.H. Torrence, R.G. Williamson, C.M. Cox, K.E. Rachlin, Y.M. Wang, S.C. Kenyon, R. Salmen, R. Trimmer, R.H. Rapp, and R.S. Nerem. (1998). The development of the join NASA GSFC and NIMA geopotential model EGM96. *NASA Technical Paper TP-1998-206861*, Goddard Space Flight Center.
- Mainville, A., R. Forsberg, and M.G. Sideris. (1992). Global Positioning System Testing of Geoids Computed from Geopotential Model and Local Gravity Data: A Case Study. *J. Geophys. Res.*, Vol. 97, No. B7, pp. 11,137-11,147.
- Rapp, R.H., Y.M. Wang, and N.K. Pavlis. (1991). The Ohio State 1991 Geopotential and Sea Surface Topography Harmonic Coefficient Models. *Report No. 410*, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.
- Sideris, M.G., A. Mainville, and R. Forsberg. (1992). Geoid Testing Using GPS and Leveling (or GPS Testing Using Leveling and the Geoid?). *Aust. J. Geod. Photogram. Surv.*, No. 57, pp. 62-77.
- Tscherning, C.C., P. Knudsen, and R. Forsberg. (1994). Description of the GRAVSOFT package. *Geophysical Institute of Copenhagen*, Technical Report, 4th Ed.
- Vergos, G.S., R.S. Grebenitcharsky, and M.G. Sideris. (2001). *Combination of Multi-Satellite Altimetry and Shipborne Gravity Data for Geoid Determination in a Coastal Region of Eastern Canada*. A paper presented at the XXVI General Assembly of the European Geophysical Union (EGS), Nice, France, March 26 – 30, (To appear in the proceedings).
- Véronneau, M. (1997) The GSD95 geoid model for Canada. *Proceedings of the IAG International Symposium in Gravity, Geodesy and Marine Geodesy*, Spet. 30 – Oct. 5, Tokyo Japan, 1996, Springer Verlag, Vol. 117, pp.573-580.
- Véronneau, M. (2001) The GSD2000 geoid model for Canada. *Personal communication*.
- Wenzel, H.G. (1999). *Global models of the gravity field of high and ultra-high resolution*. In Lecture Notes of IAG's Geoid School, Milano, Italy.
- Wessel, P., and W.H.F. Smith. (1998). *New improved version of Generic Mapping Tools released*. EOS Trans. Amer. Geophys. U., vol. 79 (47), pp. 579.

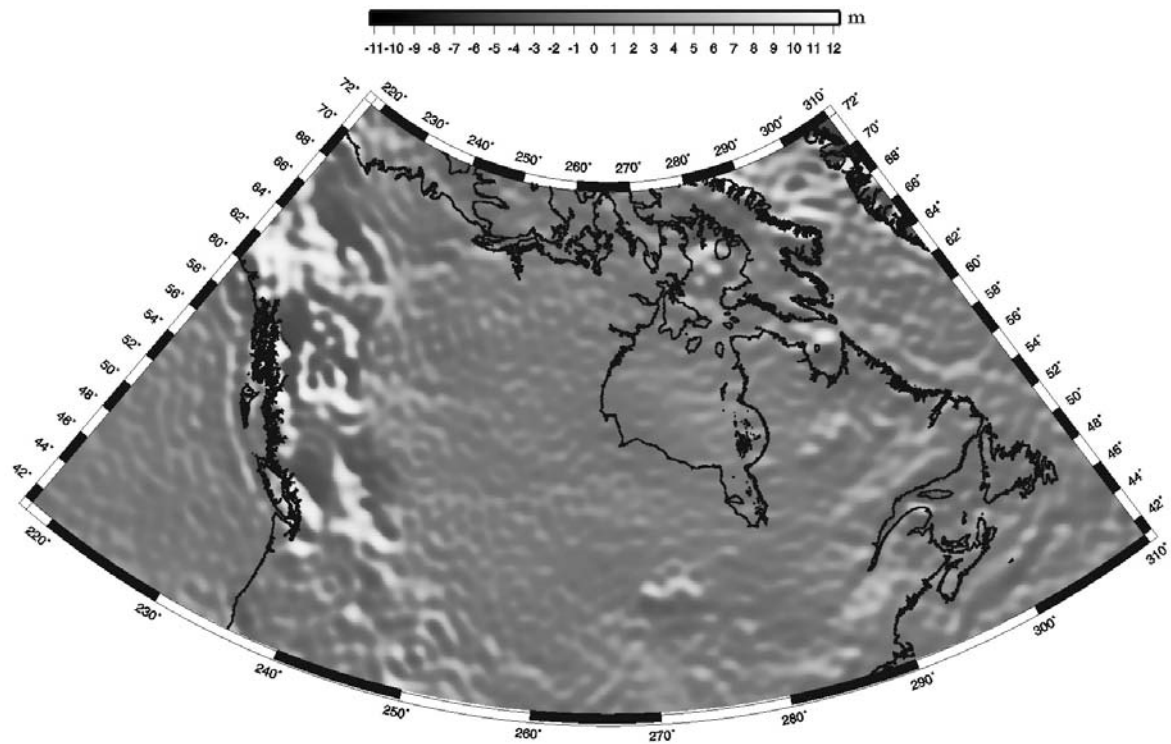


(a)

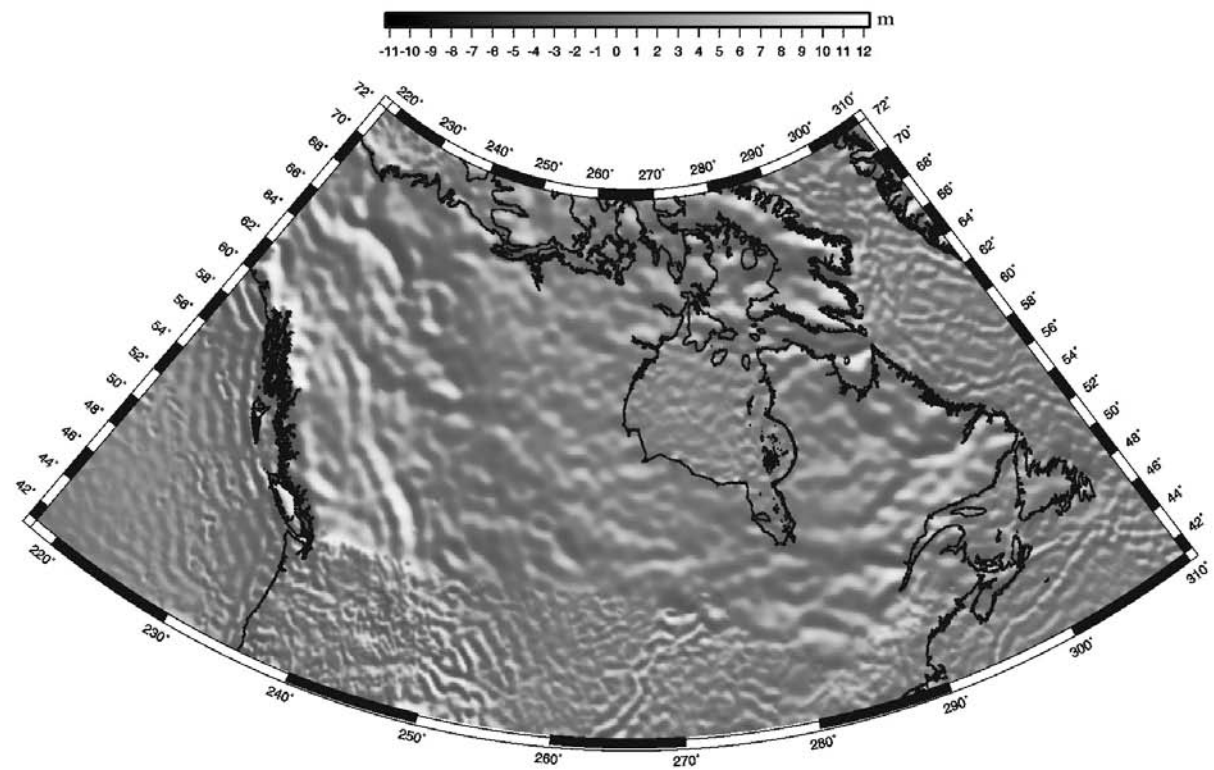


(b)

Figure 1: Geoid undulations from GPM98b geopotential model (a) and GSD2000 (b) (contour interval 5m).

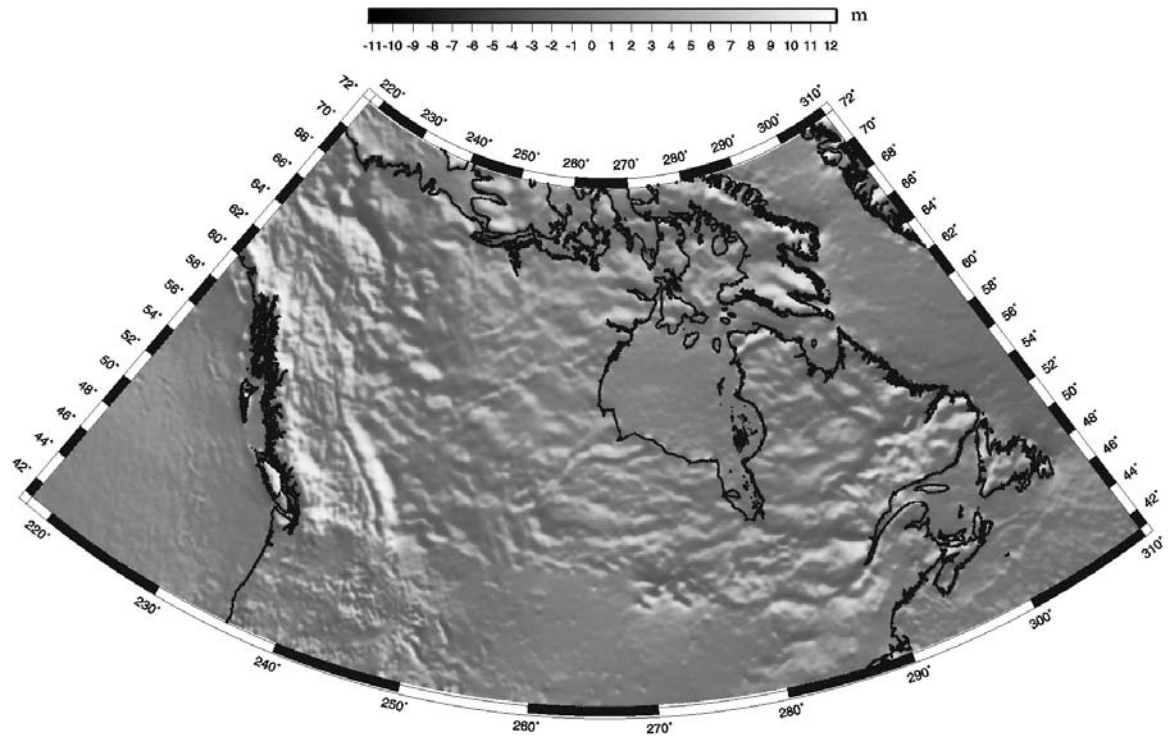


(a)

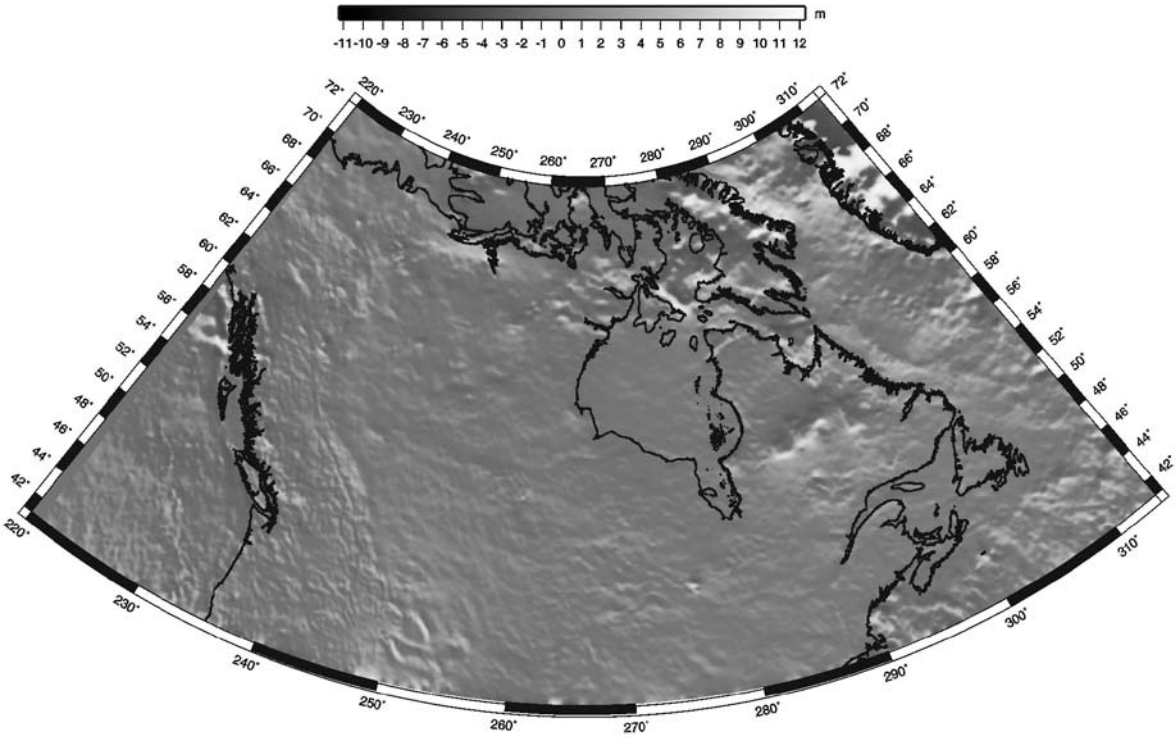


(b)

Figure 2: Geoid height differences between EGM96 and OSU91A (a) and EGM96 and GPM98b (b).



(a)

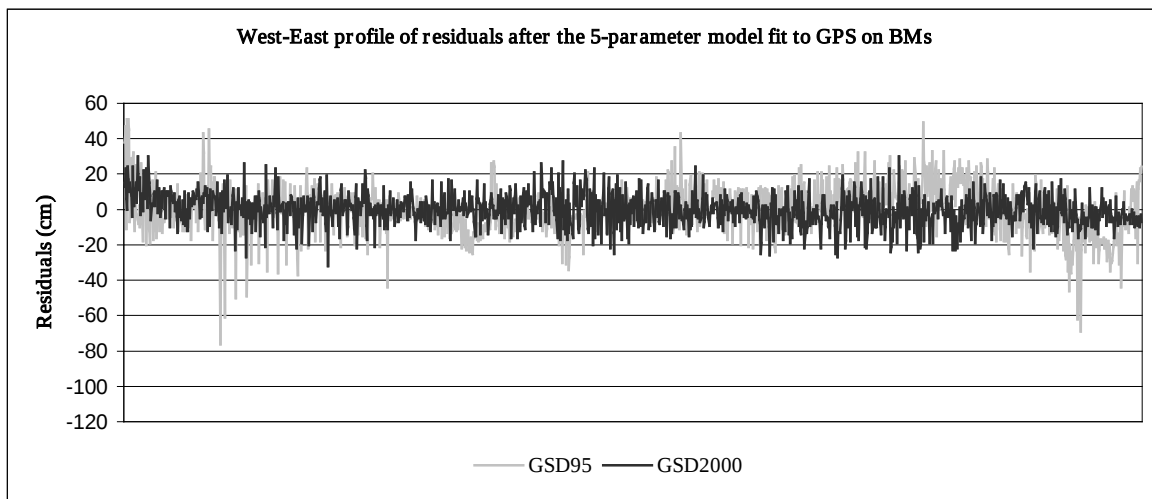


(b)

Figure 3: Geoid height differences between GSD95 and GPM98b (a) and GSD2000 and GSD95 (b).

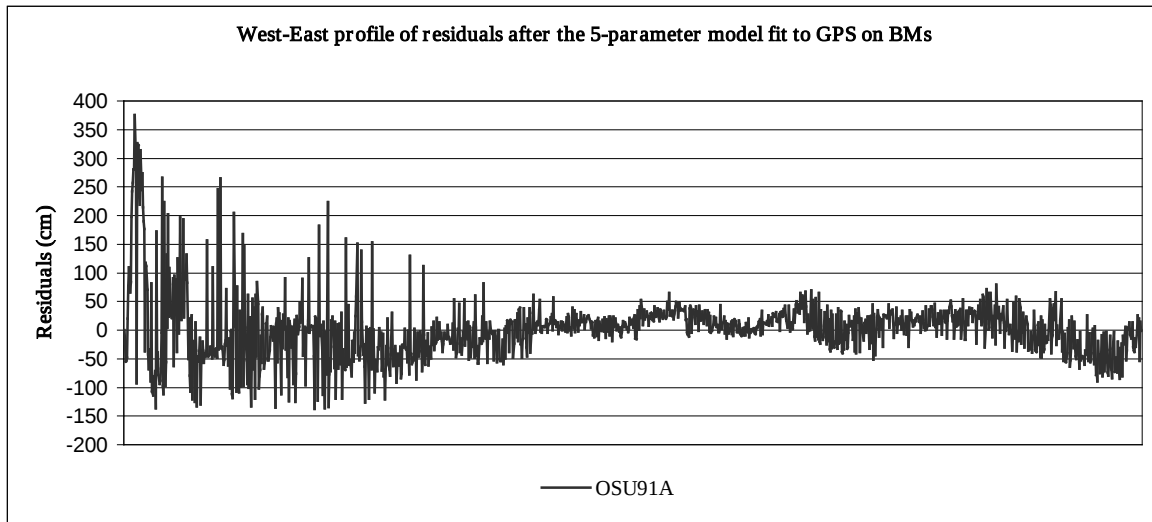


Fig. 4: Distribution of GPS benchmarks in Canada.

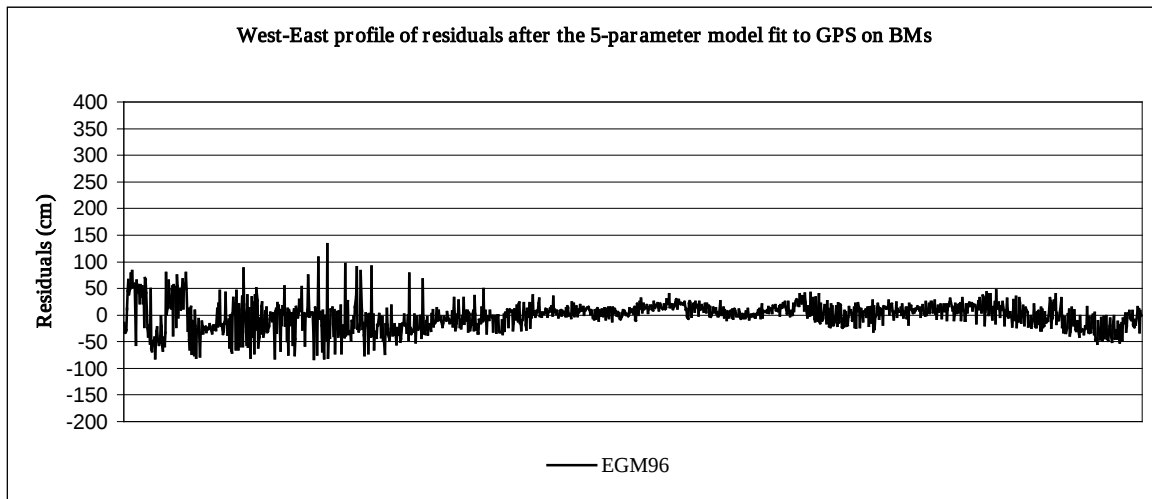


(a)

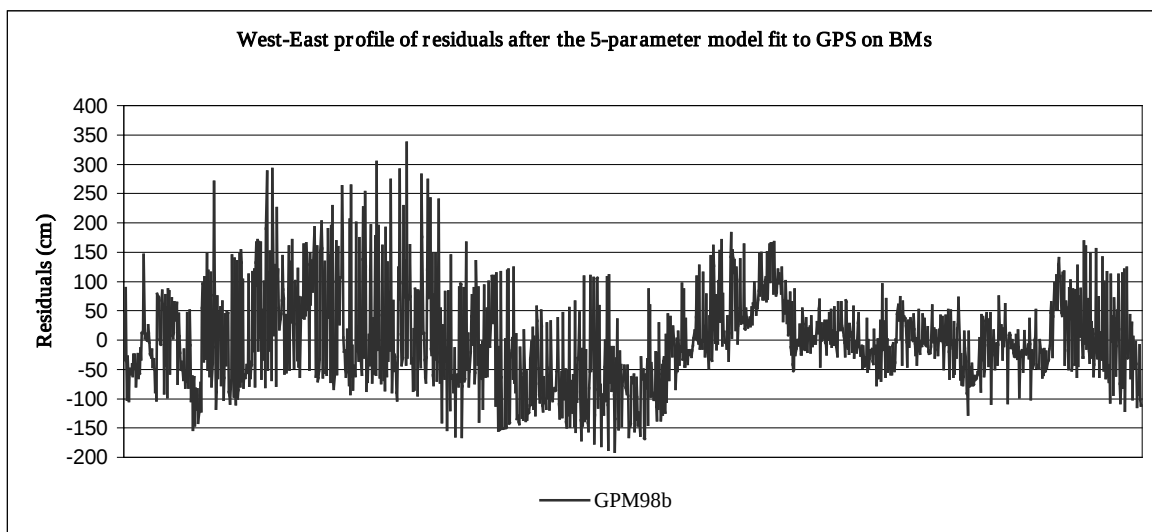
Figure 5: West – East profiles of residuals after bias and tilt fit with the five-parameter model for GSD95 and GSD2000 (a), OSU91A (b), EGM96 (c), and GPM98b (d).



(b)



(c)



(d)

Figure 5: (continued)

AIRBORNE GRAVITY SURVEY OF GREENLAND'S CONTINENTAL SHELF

A.V. Olesen, R. Forsberg and K. Keller

KMS, National Survey and Cadastre, Rentemestervej 8, DK-2400 Copenhagen, Denmark

ABSTRACT.

During four field campaigns in 1998, 1999, 2000 and 2001 National Survey and Cadastre - Denmark (KMS) has done airborne gravity measurements along the coast of Greenland, from 59 to 85 degrees north and 75 degrees west to 0 degree, covering the continental shelf. The aim has been to bridge the gap between the existing terrestrial data and gravity derived from satellite altimetry in the open ocean. In the northern part with permanent ice cover almost no gravity information existed before these measurements.

The gravity system consisted of a LaCoste & Romberg S gravimeter updated to airborne use, multiple GPS receivers, a laser altimeter and a low cost IMU unit. The system was installed in a Twin-Otter aircraft equipped with autopilot and additional fuel tank.

Flight conditions were generally good and a high accuracy has been obtained in the processing of the data. Crossover analysis and independent comparisons to shipborne and ice surface gravity measurements indicate a noise level around 2 mGal at 6 km resolution. Our results show that with careful processing the need for crossover adjustment of airborne gravity measurements can be eliminated.

The paper outlines the survey, processing methods, evaluation of results and the impact of the new data on gravimetric geoid models in the coastal region. Our measurements represent a contribution to the "Arctic Gravity Project", an international effort to compile a grid of all available and releasable gravity data for the Arctic region, in support a.o. of the upcoming satellite gravity field missions.

1. INTRODUCTION

The US Naval Research Laboratory pioneered large-scale airborne gravity surveys a decade ago, see Brozena (1991). The development in the quality of airborne gravimetry in recent years is closely linked to the better performance of the Global Positioning System. Today airborne gravimetry is a truly operational tool for gravity mapping and geoid determination, see Forsberg et al. (1996).

Airborne gravimetry is a fast, viable and economic method for gravity mapping. Some of the biggest advantages are the uniform and seamless coverage of land and sea, and the ability to cover remote and otherwise inaccessible areas. Also the bias free behaviour of

airborne gravity data obtained by spring type gravimeters is an important point for geodetic applications, see Olesen et al (2000).

2. BACKGROUND OF THE AIRBORNE GRAVITY SURVEYS

A number of airborne gravimetry campaigns have been done on a regular basis since 1998, covering the coastal region of Greenland, Svalbard and vicinity, and the Baltic Sea. All operations have been done with the same airplane, a DHC-6 Twin-Otter, a general-purpose charter airplane of Greenlandair, Nuuk. The measurements have all been low and slow, mostly flown at 500 ft above sea level at 130 knots airspeed, giving high-resolution

measurements with essentially no need for analytical downward continuation.

The measurements around Greenland were performed during four field campaigns in 1998, 1999, 2000 and 2001. The purpose of the surveys has been to complement the existing on-shore gravity coverage, mainly established by KMS by helicopter-based conventional gravimetry in the years 1991-97. The airborne surveys cover a relatively narrow band along the coast of Greenland, see Figure 1. The surveys were dedicated to provide gravity data bridging across the inner continental shelf areas, where data from satellite altimetry are less reliable than in the open sea. In North and North-East Greenland wider shelf areas have been surveyed (Forsberg et al., 1999); these are regions north of the coverage of the ERS radar altimetry, or areas covered by near-permanent sea-ice. The

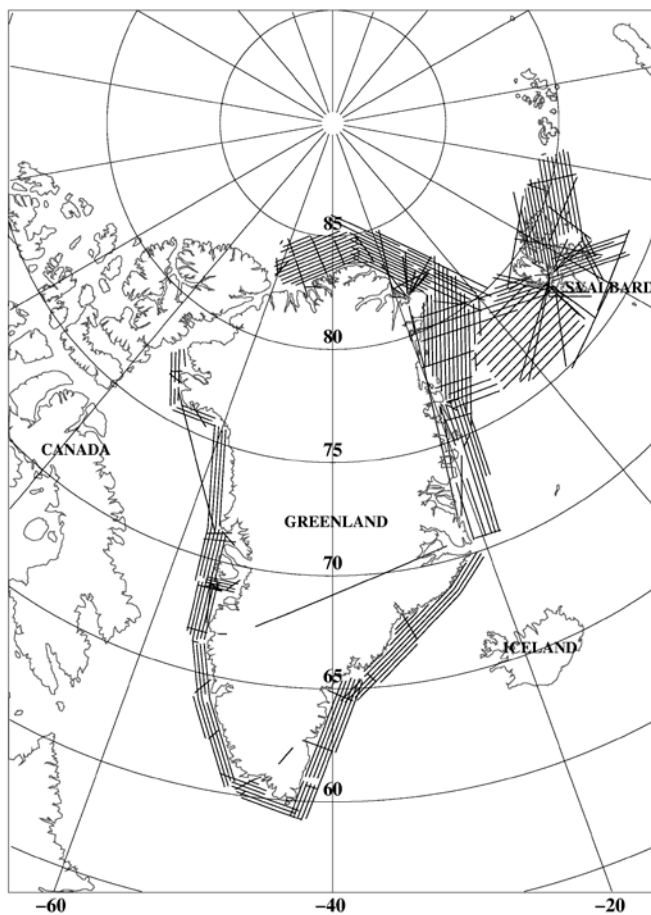


Fig. 1. Ground tracks for the Greenland surveys. Additional tracks around Svalbard flown in cooperation with Statens Kartverk, Univ. of Bergen and a Norwegian oil company are also shown.

airborne gravity surveys contribute to an ongoing effort to map the gravity and geoid of the entire Arctic region ("Arctic Gravity Project", cf. www.nima.mil/gandg/agp). The US National Imagery and Mapping Agency, NIMA, provided economic support for the Greenland surveys.

Use of the free space in the aircraft, allowed us to make joint side-by-side flight test in 1998 with a strapdown inertial gravity system from University of Calgary/Intermap (Glennie et al., 2000).

The overall dimensions of the survey are 3000 km from south to north and a total of 50,000 line kilometres of filtered data. The total flown distance is somewhat larger, approximately 65,000 kilometres. In most areas the line spacing is 10 nautical miles.

3. THE KMS AEROGRAVIMETRY SETUP

The KMS airborne gravimetry system is based on a ZLS-modified LaCoste and Romberg gravimeter (S-99, owned by University of Bergen), where the basic corrections for non-

gravitational and Eötvös effects is provided by kinematic long-range carrier-phase GPS positioning. The S-99 gravimeter has proven as an extremely stable and reliable field instrument, with typical bias drifts less than 1 mGal per week, thus making this type of system especially useful for geoid determination in previously unsurveyed areas.

The KMS gravimeter assembly is heritage from the AGMASCO project (Forsberg et al, 1996). The aircraft power converter and system control are in the KMS system mounted in a single rack, together with GPS units and a data logger. Numerous GPS receivers are usually flown (Ashtech, Trimble and Javad makes), fed by beam-splitters on two antennas (forward and aft). A custom-made inertial measurement unit, consisting of 3 Litef fibre-optics gyros and 3 Schaewitz accelerometers,



Fig. 2. Internal view of aircraft installation looking forward in cabin. Ferry tank to the left, LCR gravimeter to the right, and electronics rack in background.

provide aircraft attitude. The rack system and IMU have been custom-built by a small Danish engineering company (Greenwood Engineering). Since 2000 a medium grade integrated INS/GPS system (Honeywell H764G) has been flown as well, with the aim to provide improved attitude as well as gravity recovery redundancy. A single person operates the system in-flight, and a field crew of 2 to 3 geodesists makes for small and efficient survey operations.

For ocean ice sheet applications an Optech laser altimeter is used to map surface heights, whenever conditions permit (low fog is frequent in the arctic). Over the ocean the laser may be used as an independent source of vertical acceleration estimates, with a fit to GPS accelerations after filtering typically better than 1 mGal r.m.s. (Olesen et al., 1997).

4. GPS PROCESSING

To obtain acceleration estimates suitable for airborne gravimetry the relative position accuracy must be at the centimeter level, whereas the requirements for absolute positioning are more relaxed, and is determined by the free air gradient (0.3086 mGal/m). 0.5 m will be satisfactory for most applications. Normally this is obtained routinely with careful use of commercial GPS processing software for baselines up to several hundred kilometers.

Occasionally the GPS receiver may lose lock on the GPS carrier wave, e.g. due to ionospheric scintillation, multipath or satellite constellation changes and artificial jumps can be introduced in the position estimates. It is of crucial importance to identify such jumps and remove the effects on the resulting gravity. Due to the dynamics of the flight it is not possible to identify the jumps directly in the raw position estimates, but fortunately analysis of the high frequency part of the unfiltered gravity signal has proven to be a valuable help in the search for cycle-slip induced position jumps. The INS unit provides a similar possibility for finding GPS errors.

During 1999 and especially 2000 and 2001 the sunspot activity was quite high causing periods with unsettled ionosphere. Now and then we have had problems tracking the GPS signal, especially the L2 frequency, but it has been a minor problem causing only a few percent data loss.

5. NON-LINEAR GRAVITY SENSOR RESPONSE

Optimal modeling of the sensor response is of crucial importance in airborne gravimetry, due to the extremely high noise level. The gravimeter response to vertical accelerations was analyzed, with special attention on possible non-linearity. Figure 3 below shows the observed calibration factor for the gravimeter as a function of beam velocity and position.

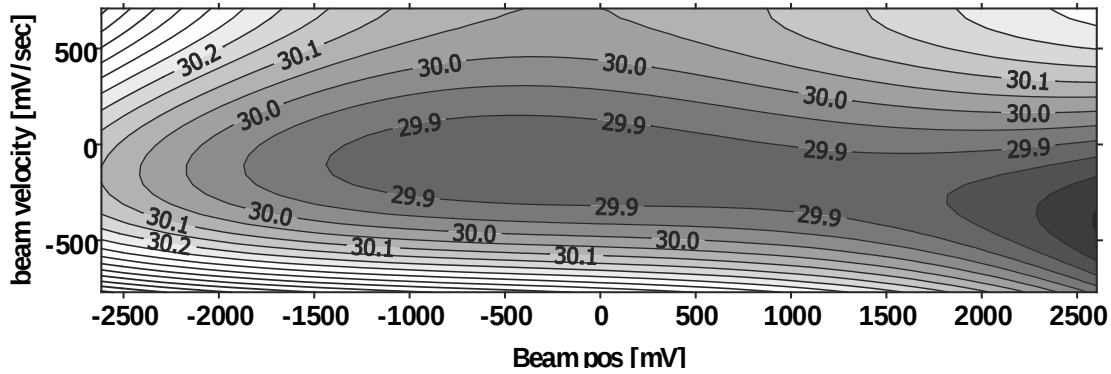


Fig. 3. Observed non-linearity of beam response

This so-called k-factor relates beam velocity with unbalanced forces acting on the beam and is normally assumed to be a constant, see Valiant (1991).

The introduction of a non-constant k-factor improves the modeling of the sensor response compared to a constant k-factor, and thereby reduces the noise signal left in the unfiltered gravimeter output. It is especially important to reduce the noise induced by the aircraft phugoid motion, since the spectral content of this noise overlaps with the gravity signal. An improved sensor modeling allows a shorter final low-pass filter to be applied and thereby enhances the resolution of the system.

Calibration factors for the platform horizontal accelerometers have been determined by a FFT technique, based on the frequency dependent behaviour of the platform, see LaCoste (1967) and Olesen et al (1997).

6. GRAVITY DATA REDUCTION

A proper synchronization of the involved data streams is obviously important. Both the gravimeter and GPS have a high frequency signal in common, the vertical acceleration of the aircraft. This signal is used to synchronize the data by cross correlation, see Olesen et al (1997).

The gravity sensor is mounted on a 2-axis gimbaled platform. The platform is kept horizontal by torque motors. A feedback loop with two horizontal accelerometers and two gyros gives the input signal to the torque motors. The gyros control the short-term behaviour of the platform, while the horizontal accelerometers control the long-term level. In the absence of horizontal accelerations the platform is driven so that the accelerometer outputs are zero. That is the condition for the platform to be orthogonal to the gravity vector and gyro drift is automatically compensated. Details of the operation principle of the LCR gravimeter can be found in Valiant (1991).

The LaCoste & Romberg meter uses a combination of two internal measurements - spring tension and beam velocity - to obtain the relative gravity variations. The basic gravimeter observation equation for relative gravity y is of the form

$$y = sT + kB' + C \quad (1)$$

where T is spring tension, s the scale factor, B' the velocity of the heavily damped beam and the factor k the beam velocity/acceleration scale. A beam-type gravimeter like the S-meter is sensitive to horizontal accelerations even when the platform is leveled, and a cross-coupling correction C is computed in real time by the gravimeter control computer.

Free-air gravity anomalies at aircraft level are (omitting second order terms) obtained by

$$\Delta g = y - y_0 - h'' + \delta g_{eotvos} + \delta g_{tilt} + g_0 - \gamma_0 - (h - N) \frac{\partial \gamma}{\partial h} \quad (2)$$

where h'' is the GPS vertical acceleration, δg_{eotvos} the Eötvös correction (computed by the formulas of Harlan, 1968), y_0 the base reading, g_0 the apron gravity value, γ_0 normal gravity, h the GPS ellipsoidal height and N the geoid undulation (EGM96 used throughout). The platform off-level correction δg_{tilt} is expressed as

$$\delta g_{tilt} = \sqrt{y^2 + A_x^2 + A_y^2 - a_x^2 - a_y^2} - y \quad (3)$$

where a and A denotes horizontal kinematic aircraft accelerations and horizontal specific forces measured by the platform accelerometers, respectively.

7. FILTERING

Low pass filtering plays a fundamental role in airborne gravity processing. The objective of the filtering is both to account for the difference in inherent filtering from the data acquisition, and to remove the high frequency part of the gravity signal. The gravimeter data acquisition system uses a 1 sec. boxcar filter on 200 Hz data, whereas the inherent filtering of the accelerations derived from GPS positions depends on the GPS processing software, and the algorithm applied for differentiation. This difference in filtering has no impact on the linear terms in the processing (the vertical GPS accelerations and the output from the vertical gravimeter sensor) due to the heavy final filtering, but may have a big influence on the nonlinear terms (mainly squared horizontal accelerations derived from GPS and the platform horizontal accelerometers). A short initial filtering, approximately 3 seconds, seems to mask this difference in inherent filtering. An improper initial filtering

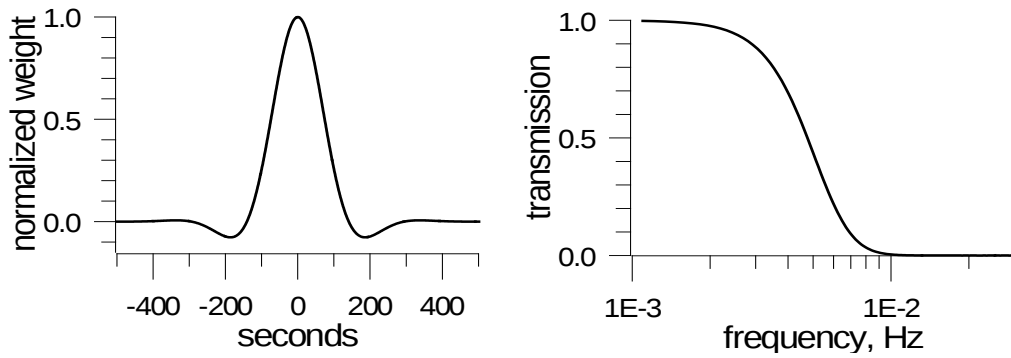


Fig. 4. Normalized convolution filter (left) and transfer function (right)

may introduce biases into the resulting gravity anomalies and thereby degrade the usefulness of the data set for geodetic purposes.

Air turbulence causes accelerations that often reach hundred thousand mGal, while we are looking for gravity anomalies of magnitude a few mGal. The low signal to noise ratio mainly affects the short wavelengths, therefore low pass filtering allows recovery of a useful signal. The filter design ultimately boils down to a trade-off between resolution and measurement accuracy.

The final filter applied to the actual gravity data is a multistage Butterworth filter with a half power point at 200 seconds, see figure 4. With an aircraft speed of 60 m/sec this filter gives a 12 km full Fourier wavelength resolution. It is common practice to use the half wavelength as a definition of resolution; in this case the resolution is 6 km.

8. DATA VALIDATION

The resulting free air anomalies for North-East Greenland shelf area are shown in Figure 5 as a contour plot. It illustrates the roughness of the gravity field around Greenland. The final processed data set comprised 479 crossovers and an analysis of the differences in the crossing points is shown in Table 1. The RMS difference of 2.8 mGal indicates a noise

Unit: mGal	no of crossings	RMS difference	max. difference
1998, 1999, 2000 and 2001	479	2.8	9.4
1998 subset	84	2.6	6.0
1999 subset	77	2.8	8.4
2000 subset	97	2.7	8.9
2001 subset	64	2.4	6.8

Table 1. Crossover statistics

level around 2 mGal (2.8 divided by square root 2), when the noise is assumed uncorrelated from track to track. It should be emphasized that no bias or tilt adjustment has been applied to the track data to obtain these numbers, as experience has shown that such an adjustment may actually degrade the data (in spite of improvements in RMS crossover statistics, cf. Forsberg et al., 1999). Any crossover adjustment will by nature distribute point errors at crossing points into along-track corrections, and thus provide a way for short-period errors to leak into longer wavelengths. For proper processed aerogravity data obtained with a nearly drift-free gravimeter, there is no physical justification to apply such a crossover adjustment. The crossover results for the 1998, 1999, 2000 and 2001 subsets of the data

show the same constant noise level for the four years, which indicates that 2 mGal noise level can be anticipated also for future work under similar not very favorable conditions, that is: long baselines for DGPS, unsettled ionosphere and a pretty rough gravity field.

For independent quality control the gravity values at existing surface data points were predicted from the airborne data, using a weighted-mean algorithm. Comparisons were only made for data points within 2 km from the airborne tracks. No downward continuation algorithms were applied to the airborne data due to the low flight altitude. Table 2 shows comparisons to Canadian gravity data collected on the sea ice in Lincoln Sea and to recent high quality marine data distributed along most of the east and west coast. The standard deviation for both comparisons is in a nice agreement with the error estimate from the cross over analysis, taking

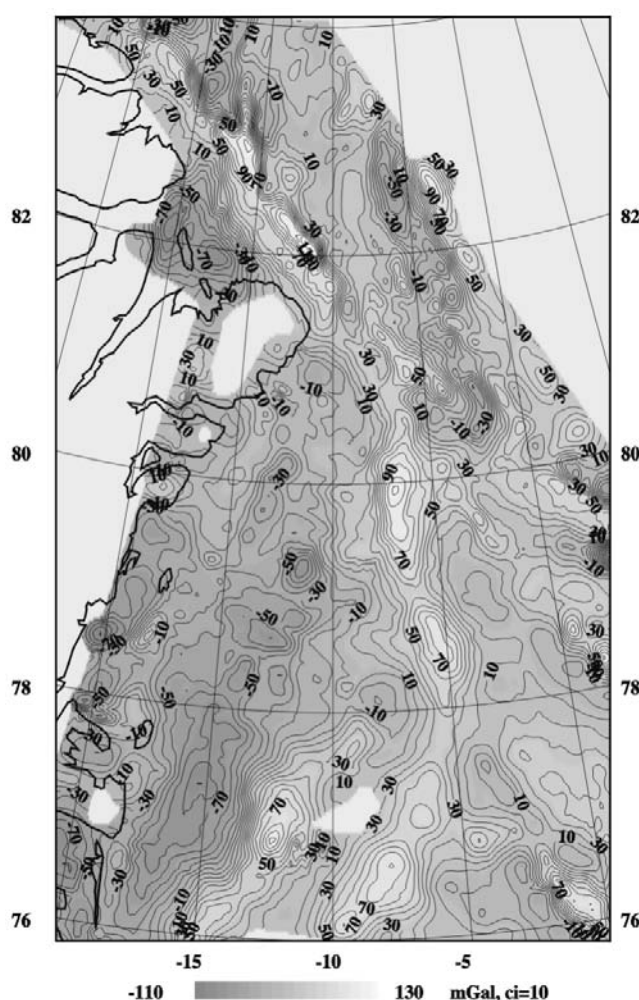


Figure 5. Free air anomalies of NE-Greenland shelf area, illustrating the roughness of the field. 10 mGal contour interval.

into account that the surface data as well as the prediction algorithm contribute to the noise. Also the table shows that the airborne data has little or no bias error. The data are therefore very suitable for geoid determination, where biases have an especially large impact, due to the inherent low-pass filtering nature of geoid determination.

Unit: mGal	max. dist.	no of points	max. diff.	mean diff.	stdv
KANUMAS marine data	2 km	2417	15.1	0.1	2.9
	1 km	1178	13.7	0.1	2.7
Canadian ice data	2 km	27	2.6	0.6	1.4
	1 km	12	2.2	0.4	1.3

Table 2. Comparison to surface data within 2 km from airborne tracks

9. IMPACT ON GEOID MODELLING

The geoid is determined from the airborne data by conventional methods of physical geodesy. With gravity errors at 2 mGal, typical relative geoid errors are at the 20-30 cm level for 18 kilometers (10 nautical miles) track spacing, as based on collocation error studies (Forsberg et al., 2000). To illustrate the impact of the new data on geoid models, a new geoid incorporating the airborne data was compared with the previous best geoid, computed by exact the same methods.

A traditional remove-restore technique was used for the computations, with the geoid signal constructed by subdividing it into three parts

$$N = N_1 + N_2 + N_3 \quad (4)$$

where the first part comes from a spherical harmonic expansion complete to degree and order 360 (EGM96), the second part from the topography, and the third part from residual gravity (i.e., gravity anomalies minus the global field model and gravimetric terrain effects). Terrain effects have been removed in a consistent "RTM" terrain data reduction scheme using rectangular prisms as basic integration element. Residual gravity anomalies are converted into residual height anomalies by spherical FFT (Strang van Hees, 1990). The method in principle evaluates Stokes' integral

$$N_3 = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_3 S(\psi) d\psi \quad (5)$$

through a series of band wise two-dimensional FFT operations of form

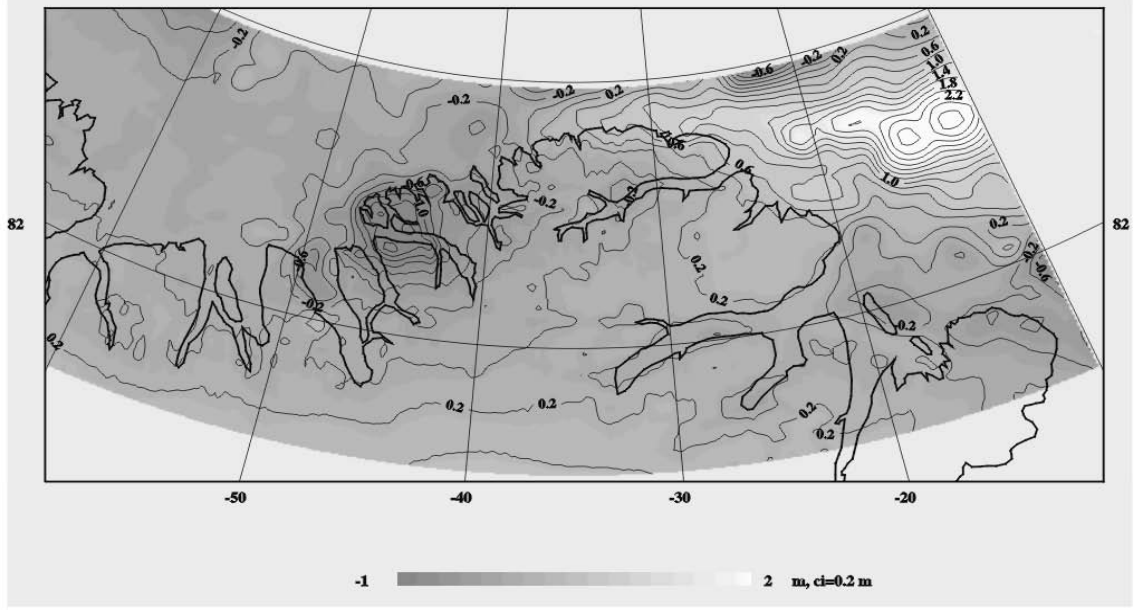


Fig. 6. Geoid differences in North Greenland

$$N_3 = S_{ref}(\Delta\varphi, \Delta\lambda) * [\Delta g_3(\varphi, \lambda) \sin \varphi] = F^{-1}[F(S_{ref})F(\Delta g \sin \varphi)] \quad (6)$$

where S_{ref} is a modified Stokes' kernel function, and $*$ and F the two-dimensional convolution and Fourier transform, respectively, for details see Forsberg and Siderius (1993). In the actual implementation of the method, the data are gridded by least-square collocation, and a zero-padding is used to limit the periodicity errors of the FFT.

Figure 6 shows the difference between the geoid models in the northern part of Greenland, where little data were available before the airborne measurements, causing the old geoid to be essentially identical to EGM96. Some major differences up to 2 meters are seen, illustrating the need for new data in this area to improve global models (e.g. EGM96).

9. CONCLUSION

The KMS airborne gravity system based on a LaCoste and Romberg S gravimeter and a Twin-Otter has proven to be a reliable tool for regional gravity mapping even under difficult logistic and operational conditions. It showed 2 mGal accuracy at 6 km resolution, and it should be noticed that this performance is obtained in an area with quite steep gradients in the gravity field. This performance was obtained repeatedly during four consecutive years with periods of unsettled ionosphere.

A careful calibration of the gravimeter on in-flight data, together with modeling of non-linearity in the sensor response resulted in a data quality in terms of accuracy and resolution

that is suitable for geodetic use. Especially the bias free properties of the resulting gravity data is an important point for geodetic applications, e.g. for geoid determination.

ACKNOWLEDGEMENT. US National Imagery and Mapping Agency, NIMA provided financial support for the airborne gravity measurements. Also thanks to Arne Gidskehaug, University of Bergen, Norway, for cooperation with the field measurements in Greenland.

REFERENCES

- Brozena, J. M.: The Greenland Aerogeophysics Project: Airborne gravity, topographic and magnetic mapping of an entire continent. In: *From Mars to Greenland: Charting Gravity With Space and Airborne Instruments*. IAG symposium series 110, pp. 203-214, Springer Verlag, 1991.
- Forsberg, R. and M. G. Sideris: Geoid computations by the multi-band spherical FFT approach. *Manuscripta geodaetica*, 18, 82-90, 1993.
- Forsberg, R., K. Hehl, U. Meyer, A. Gidskehaug, L. Bastos: Development of an airborne geoid mapping system for coastal oceanography (AGMASCO). *Proc. Int. Symp. of Gravity, Geoid and Marine Geodesy (GraGeoMar96)*, Tokyo, 1996.
- Forsberg, R., A. V. Olesen and K. Keller: Airborne Gravity Survey of the North Greenland Shelf 1998. Kort - og Matrikelstyrelsen Technical Report no. 10, 1999.
- Forsberg, R., A. V. Olesen, K. Keller: Airborne gravity survey of the North Greenland continental shelf. To appear in IAG proceedings volume of Gravity, Geoid and Geodynamics conference, Banff, 2000.
- Glennie, C L, K. P. Schwarz, A. Bruton, R. Forsberg, A. Olesen, K. Keller: A Comparison of Stable Platform and Strapdown Airborne Gravity. *Journal of Geodesy*, 74, 2000.
- Harlan, R. B.: Eotvos corrections for airborne gravimetry, *J. Geophys. Res.*, 3, 4675-4679, 1968.
- LaCoste, L. B. J.: Measurement of Gravity at Sea and in the Air. *Reviews of Geophysics*, 5, 477-526, 1967.
- Olesen A. V., R Forsberg, A. Gidskehaug: Airborne gravimetry using the LaCoste & Romberg gravimeter – an error analysis. In: M.E. Cannon and G. Lachapelle (eds.). *Proc. Int. Symp. on Kinematic Systems in Geodesy, Geomatics and Navigation*, Banff, Canada, June 3-6, 1997, Publ.Univ. of Calgary, pp. 613-618, 1997.
- Olesen, A. V., R. Forsberg, K. Keller, A. Gidskehaug: Airborne Gravity Survey of Lincoln Sea and Wandel Sea, North Greenland. *Chemistry and Physics of the Earth*, Vol 25 A, pp 25-29, 2000.
- Strang van Hees, G.: Stokes formula using fast Fourier techniques. *Manuscripta geodaetica*, vol. 15, pp. 235-239, 1990.
- Valliant, H.: The LaCoste & Romberg air/sea gravimeter: an overview. In: *CRC Handbook of Geophysical Exploration at Sea*, Boca Raton Press, 1991.

Determination of the Achievable Accuracy of Relative GPS/Geoid Levelling in Northern Canada¹

G. Fotopoulos, C. Kotsakis, and M.G. Sideris

Department of Geomatics Engineering, University of Calgary
2500 University Drive N.W., Calgary, Alberta, Canada, T2N 1N4
Tel: +1(403)2204984, Fax: +1(403)2841980, Email: gfotopou@ucalgary.ca

Abstract

It is well known that traditional spirit levelling, as a method for precise vertical positioning, suffers from a number of practical limitations caused by terrain roughness, harsh environmental conditions and restricted line-of-sight. In Canada, this is most evident when we look at the spatial distribution of vertical control stations, since the remote northern parts of the country are very poorly surveyed. A method that has been proven to be a useful and efficient alternative for vertical positioning in such environments is GPS-based levelling. A major advantage of GPS observations is that they are not affected (as much) by the practical limitations of spirit levelling. The achievable accuracy of this method, however, is still under question mainly because of datum inconsistencies and systematic errors inherent in the data. In this paper, a number of investigations are conducted to estimate the achievable accuracy of orthometric height determination in the northwestern parts of Canada, using GPS and geoid information in conjunction with various auxiliary parametric models (corrector surfaces) for describing datum offsets and systematic distortions. Specifically, the covariance (CV) matrix of the estimated parameters in the corrector surface model, and the combined relative accuracy of GPS and geoid data, are used to infer the accuracy of the orthometric height differences of newly established baselines in remote northern parts of Canada. A large test network consisting of the GPS benchmarks in western Canada is used for the computation of the covariance matrix of the estimated parameters in the corrector surface models, through a combined least-squares adjustment of GPS, levelling and geoid data. The results provide valuable insight into the role of the accuracy for the parameters in the corrector surface model for precise vertical positioning via GPS/geoid levelling.

1 Introduction

The purpose of this paper is to investigate the achievable accuracy of GPS/geoid levelling in northern Canada. There are several reasons for focusing our studies in northern Canada, including the numerous practical limitations that are involved in establishing vertical control and obtaining orthometric heights via differential spirit levelling, in such remote and largely unsurveyed territories. In an effort to obtain orthometric heights with respect to an established vertical datum, the concept of GPS-based levelling has been applied, which is based on the combination of three height types geometrically related by the following equation (Heiskanen and Moritz, 1967):

$$H_i = h_i - N_i \quad (1)$$

where h_i is the ellipsoidal height computed from GPS measurements, N_i is the geoidal undulation obtained from a gravimetric geoid model, and H_i refers to the Helmert orthometric height. In practice, the theoretical relationship given in Eq. (1) is not fulfilled, due to datum inconsistencies, data noise, and systematic biases inherent among the three height types. The major part of these discrepancies is usually

¹ This paper was previously presented at the IAG 2001 Scientific Assembly in Budapest, Hungary, Sept. 2-7.

attributed to the systematic effects and datum inconsistencies, which can be described by a *corrector surface* model such that

$$H_i = h_i - N_i - \mathbf{a}_i^T \mathbf{x} \quad (2)$$

where the bilinear term $\mathbf{a}_i^T \mathbf{x}$ describes the corrector surface.

The general idea described herein is to use the accuracy information of the different height types that is available from existing vertical control in densely surveyed areas (such as southwestern Canada) and propagate that information for determining the accuracy of the orthometric height difference for a newly established baseline in an area with no (or very limited) vertical control, such as northern Canada. The discussion begins with an overview of the formulations required for determining the accuracy of GPS/geoid levelling. This is followed by a description of how we simulate the accuracy for the different height data types. Finally, a number of numerical tests are performed, which provide interesting insight into the role of the corrector surface model accuracy for GPS/geoid levelling. Based on these results some conclusions are drawn and a brief mention for future work on this topic is provided.

2 GPS/Geoid Levelling Accuracy

For the purposes of this paper, we will focus on the relative model, where height differences are taken into account. Given the theoretical relationship among the three types of height data and the incorporation of an appropriate corrector surface model as shown in Eq. (2), the orthometric height difference ΔH_{kl} for a **new** baseline (k, l) as obtained from relative GPS/geoid levelling is given by

$$\Delta H_{kl} = \Delta h_{kl} - \Delta N_{kl} - (\mathbf{a}_l^T - \mathbf{a}_k^T) \hat{\mathbf{x}} \quad (3)$$

where $\hat{\mathbf{x}}$ is a vector containing the estimated parameters of the corrector surface model obtained through a combined adjustment of GPS/levelling/geoid data in a common control network, Δh_{kl} is the observed GPS height difference of the points (k, l), ΔN_{kl} is the geoid undulation difference, and $\mathbf{a}_l$, $\mathbf{a}_k$ correspond to the vectors of known coefficients of the selected parametric model (see below).

As it was mentioned previously, we are interested in the achievable *accuracy* of the orthometric height difference. By simply applying variance-covariance propagation to Eq. (3), the accuracy of GPS/geoid levelling for a new baseline can be obtained according to the following formula:

$$\sigma_{\Delta H_{kl}}^2 = \sigma_{\Delta h_{kl}}^2 + \sigma_{\Delta N_{kl}}^2 + (\mathbf{a}_l^T - \mathbf{a}_k^T) \mathbf{C}_{\hat{\mathbf{x}}} (\mathbf{a}_l - \mathbf{a}_k) \quad (4)$$

where $\sigma_{\Delta h_{kl}}^2$ and $\sigma_{\Delta N_{kl}}^2$ represent the relative accuracy of the ellipsoidal and geoidal heights of the baseline, and $\mathbf{C}_{\hat{\mathbf{x}}}$ is the a-posteriori CV matrix of the estimated parameters in the corrector surface model. This CV matrix is obtained from a multi-data adjustment of relative GPS/geoid/levelling heights. Details of such combined adjustment problems are given in Kotsakis and Sideris (1999). The final form of $\mathbf{C}_{\hat{\mathbf{x}}}$ is given by

$$\mathbf{C}_{\hat{\mathbf{x}}} = \left[\mathbf{A}^T (\mathbf{C}_{\Delta h} + \mathbf{C}_{\Delta H} + \mathbf{C}_{\Delta N})^{-1} \mathbf{A} \right]^{-1} \quad (5)$$

where, $\mathbf{C}_{\Delta h}$, $\mathbf{C}_{\Delta H}$, and $\mathbf{C}_{\Delta N}$ denote the CV matrices for the relative GPS ellipsoidal heights, orthometric height differences and relative geoid heights in the control network, respectively. The design matrix $\mathbf{A}$ corresponds to the pre-selected parametric model. Its type varies in form and complexity depending on a number of factors. In this paper, three models have been tested ranging from a simple 3-parameter trigonometric model (its pointwise formulation is given in Heiskanen and Moritz, 1967, sec. 5-9) to a more complicated 6-parameter differential similarity transformation model (see Kotsakis et al., 2001). The general form of the corrector surface model for the relative case can be represented by

$$f_{ij} = (\mathbf{a}_j^T - \mathbf{a}_i^T) \mathbf{x} = \mathbf{a}_{ij}^T \mathbf{x} \quad (6)$$

where $\mathbf{a}_j, \mathbf{a}_i$ are vectors of known coefficients that depend on the horizontal location of the points (i, j) and $\mathbf{x}$ is a vector of unknown parameters. The three parametric models tested in this paper are

$$\begin{aligned} (a) \text{ 3-parameter model} \quad \mathbf{a}_{ij} &= \begin{bmatrix} \cos \varphi_j \cos \lambda_j - \cos \varphi_i \cos \lambda_i \\ \cos \varphi_j \sin \lambda_j - \cos \varphi_i \sin \lambda_i \\ \sin \varphi_j - \sin \varphi_i \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ (b) \text{ 4-parameter model} \quad \mathbf{a}_{ij} &= \begin{bmatrix} \cos \varphi_j \cos \lambda_j - \cos \varphi_i \cos \lambda_i \\ \cos \varphi_j \sin \lambda_j - \cos \varphi_i \sin \lambda_i \\ \sin \varphi_j - \sin \varphi_i \\ \sin^2 \varphi_j - \sin^2 \varphi_i \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \\ (c) \text{ 6-parameter model} \quad \mathbf{a}_{ij} &= \begin{bmatrix} \cos \varphi_j \cos \lambda_j - \cos \varphi_i \cos \lambda_i \\ \cos \varphi_j \sin \lambda_j - \cos \varphi_i \sin \lambda_i \\ \sin \varphi_j - \sin \varphi_i \\ \frac{\sin \varphi_j \cos \varphi_j \cos \lambda_j}{W_j} - \frac{\sin \varphi_i \cos \varphi_i \cos \lambda_i}{W_i} \\ \frac{\sin \varphi_j \cos \varphi_j \sin \lambda_j}{W_j} - \frac{\sin \varphi_i \cos \varphi_i \sin \lambda_i}{W_i} \\ \frac{\sin^2 \varphi_j}{W_j} - \frac{\sin^2 \varphi_i}{W_i} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \end{aligned}$$

where, φ and λ are the horizontal geodetic coordinates of the network or baseline points, e is the eccentricity of a common datum ellipsoid and $W_{(\cdot)} = \sqrt{1 - e^2 \sin^2 \varphi_{(\cdot)}}$. In the following three sub-sections, a description of how to compute the relative accuracy of the three height types, required as input for Eq. (5), is given.

2.1 Accuracy of the Levelling Data

The full covariance matrix for the orthometric height differences $C_{\Delta H}$ in the multi-data adjustment of the control network, was computed the following formula:

$$C_{\Delta H} = A_{net} C_H A_{net}^T \quad (7)$$

where A_{net} is a design-type matrix (composed of -1 , 1 and 0) corresponding to the baseline configuration of the multi-data network adjustment (see Fig. 2), and C_H is the covariance matrix for the absolute orthometric heights at all points in the test network. The latter quantity was simulated through a separate minimally constrained least-squares adjustment of the levelling part of the control network, as follows:

$$C_H = \left(A_{lev}^T P_{lev} A_{lev} \right)^{-1} \quad (8)$$

where A_{lev} is a design matrix (composed of -1 , 1 and 0) corresponding to the baseline configuration of the levelling network adjustment (see Fig. 1), and P_{lev} is a diagonal weight matrix that takes into account the measuring accuracy of each levelling baseline in the test network. In this case, three different orders of accuracy were used for assigning the a-priori values $\sigma_{\Delta H}$ for each levelling baseline in the weight matrix P_{lev} , namely $0.7mm\sqrt{d(km)}$, $1.3mm\sqrt{d(km)}$ and $2mm\sqrt{d(km)}$, referring to first, second and third order respectively. National standards for the accuracy of vertical control vary depending on the country. In our case, the U.S. standards were implemented, as they were readily available (NGS, 1994). For the case of levelling, larger baselines ($d > 80km$) usually constitute part of a national levelling campaign and adhere to first order levelling standards, followed by denser regional levelling campaigns ($30km < d \leq 80km$) of second order, and finally local levelling lines ($d \leq 30km$) which are of third order accuracy.

2.2 Accuracy of the GPS Data

A similar procedure to the one described in the previous section was followed in order to obtain the relative accuracy of the ellipsoidal heights $C_{\Delta h}$ in the control test network used for the multi-data adjustment (see Eq. (5)). The final formulation for $C_{\Delta h}$ is given by

$$C_{\Delta h} = A_{net} C_h A_{net}^T \quad (9)$$

where A_{net} is the same design matrix that was used in Eq. (7) and it corresponds to the baseline configuration of the multi-data network adjustment, and C_h is the covariance matrix for the ellipsoidal heights at all points of the test network. The latter was obtained through a separate minimally constrained least-squares adjustment for the GPS part of the control network, according to the formula:

$$C_h = \left(A_{GPS}^T P_{GPS} A_{GPS} \right)^{-1} \quad (10)$$

where A_{GPS} is a design matrix (composed only of -1 , 1 and 0) corresponding to the baseline configuration of the GPS network adjustment (see Fig. 1). The stronger overall geometry used in the

GPS-based network as compared to a levelling network is evident from Fig. 1, as the GPS network is composed of all of the baselines in the levelling network as well as additional baselines (dashed lines). The weaker geometrical configuration of the levelling network is due to the stringent line-of-sight restrictions of spirit levelling (Ollikainen, 1997). Here we see the advantages of using GPS from a network point of view, especially in rough terrain areas such as western Canada, where traditional spirit levelling is a difficult and laborious task, to say the least. Finally, $\mathbf{P}_{GPS}$ is a diagonal weight matrix that takes into account the measuring accuracy of the vertical component for each GPS baseline in the test network. In our analysis, ten different orders for the relative ellipsoidal height accuracy are used as defined by the U.S. standards (NGS, 1994). The a-priori values $\sigma_{\Delta h}$ in the weight matrix $\mathbf{P}_{GPS}$ were assigned based on the length of each GPS baseline in the test network, such that the accuracy of the baseline degrades as the baseline length increases (due to the spatial decorrelation of GPS errors, see Fotopoulos et al., 2001).

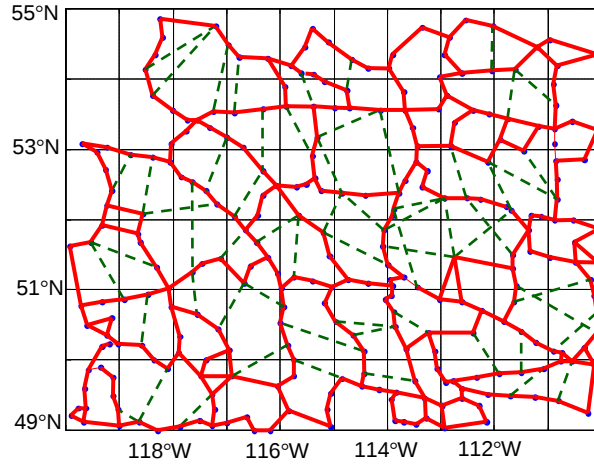


Fig. 1 Levelling (solid lines) and GPS (solid and dashed lines) network configurations

2.3 Accuracy of the Geoid Heights

Following the separate simulative adjustments of the levelling and GPS parts of the control network, full CV matrices $\mathbf{C}_{\Delta h}$ and $\mathbf{C}_{\Delta H}$ for the adjusted height differences were obtained. As stated previously, these CV matrices were used as input to a final integrated multi-data adjustment in order to obtain $\mathbf{C}_{\hat{x}}$ (see Eq. (5)). The input accuracy for the geoid undulation differences $\mathbf{C}_{\Delta N}$ in the multi-data test network was approximated in a slightly different manner. A $1^\circ \times 1^\circ$ world-wide grid of the commission errors of the global geopotential model, EGM96 (Lemoine et al., 1998), as computed from the accuracy of the spherical harmonic coefficients up to degree and order 70 was used to interpolate the σ_N of each benchmark in the control test network. For our test network area (southwestern Canada), the $1^\circ \times 1^\circ$ grid resolution of the geoidal undulation errors was sufficient as the gravity coverage employed for this area in the computation of EGM96 was reasonably dense and homogeneously spaced (*ibid.*). It should be noted that, although a higher degree of expansion (i.e. 180 or 360) may theoretically recover higher frequency information (thus reducing the aliasing error), the noise is also increased as the number of coefficients increases. By using this global geoid error model and bilinearly interpolating the grid of commission

errors for the points in the test network, a diagonal error CV matrix of the absolute geoid height errors $\mathbf{C}_N$ was obtained. The computation of the relative geoid height accuracy $\mathbf{C}_{\Delta N}$, required as input into Eq. (5), was obtained by propagating the absolute height accuracy as follows:

$$\mathbf{C}_{\Delta N} = \mathbf{A}_{net} \mathbf{C}_N \mathbf{A}_{net}^T \quad (11)$$

where $\mathbf{A}_{net}$ is the same design matrix that was used in Eqs. (7) and (9), corresponding to the baseline configuration of the multi-data network adjustment. This results in a fully populated form of $\mathbf{C}_{\Delta N}$.

3 Description of the Numerical Tests

Although this study is based on the *accuracy* information of the different height data types and therefore does not make use of actual height data values, the tests were designed to mirror realistic conditions. The test network area used for the multi-data adjustment contains a subset of the GPS benchmarks in the southwestern part of Canada spanning parts of British Columbia and Alberta (275 points in total, see boxed network in Fig. 2). This area has a fairly good distribution of GPS benchmarks covering $49^\circ \leq \phi \leq 55^\circ$ and $-120^\circ \leq \lambda \leq -110^\circ$.

Several test scenarios were conducted to determine the achievable accuracy of orthometric height differences via GPS/geoid levelling by varying the basic simulation parameters, such as:

- new baseline length
- location of new baseline with respect to original test network
- type of parametric model (see Sec. 2)
- form of the CV matrices $\mathbf{C}_{\Delta h}$, $\mathbf{C}_{\Delta H}$, and $\mathbf{C}_{\Delta N}$ required as input into Eq. (5)

It should be noted that sometimes, the estimation of the fully populated CV matrices may not be feasible or computationally efficient (due to the required inversion). In these cases, approximations of the CV matrices may be made in a diagonal form where only the variances of the baselines are included and the covariances between baselines are set to zero. In the following section, some of the key findings from the test scenarios mentioned above will be discussed.

4 Analysis of Results

Numerous tests were conducted by varying the new baseline length from a minimum of 10 km to a maximum of 100 km. As expected, it was found that the longer the length of the new baseline, the poorer the achievable ΔH accuracy from GPS/geoid levelling. This is partly due to the spatial decorrelation of GPS errors. The contribution of the corrector surface model error also increases as the baseline length increases due to its dependence with the horizontal locations of the points.

Overall, it was found that the major error contributor of the three height types to the final result was $\mathbf{C}_{\Delta N}$. As only a global geoid model was used to obtain the aforementioned CV matrix, the accuracy was significantly poorer than those selected for precise levelling and GPS heights. The error CV matrix for $\mathbf{C}_{\Delta N}$ may also be approximated by employing the internal precision of a local/regional geoid solution, which is supported with very dense local gravity, height and density data. In such cases, the variances and

covariances of the geoid height differences are usually smaller than those obtained from a global geoid error model. However, such a reliable model may not always be readily available, especially in remote areas.

By varying the type of corrector surface model used in the adjustment, the achievable accuracy also changed. For instance, for a 50 km baseline located in the north ($\varphi = 60^\circ\text{N}$, $\lambda = 116^\circ\text{W}$) with input accuracies of $0.6 \cdot \sqrt{d(\text{km})}$ and 23 cm for GPS and geoid height differences, respectively, the resultant relative orthometric height accuracy was 48.3 cm with the 3-parameter model and 52.4 cm with the 6-parameter model. Similarly, for a 100 km baseline the achievable accuracy degraded by ~ 13 cm with the 6-parameter model as compared to the 3 or even 4-parameter models. It is evident from these results that the more parameters in the model, the more amplified its error contribution is (see Table 1). This is interesting, as we are not evaluating the performance/fitting of different parametric models (no actual height data used), rather we focus on how the random errors flow through our model and affect the accuracy of the final value.

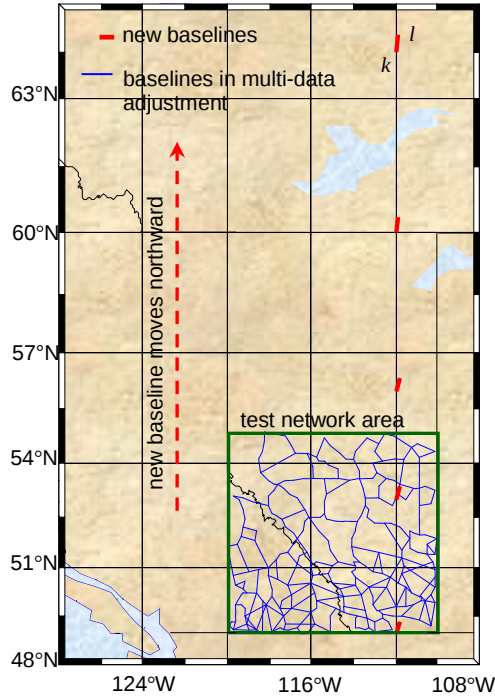


Fig. 2 Test network coverage area and locations of newly established baselines in the north

Table 1 provides a summary of some of the numerical results. The first column refers to the approximate latitude of a 40 km baseline ($\lambda \sim 112^\circ\text{W}$). The latitude varies as the newly established baseline is moved northward with respect to the test network area (see Fig. 2). The column labeled 3^{rd} term refers to the accuracy contribution of the corrector surface model (3^{rd} term in Eq. (4)). There are three main groups of results, where *Full CVs* refers to fully populated covariance matrices for the three height types, *Diag. CVs* refers to diagonal covariance matrices for the three height types, and *Full & Diag.* refers to fully populated CV matrices for levelling and GPS heights and a diagonal covariance matrix for the geoid heights. All results are based on input accuracies of $\sigma_{\Delta N} = 15$ cm and $\sigma_{\Delta h} = 0.15 \cdot \sqrt{d(\text{km})}$ cm for Eq. (4).

Table 1. Results of Baselines Moving Northward from the Test Network Area (all values in cm)

<i>3 - Parameter Corrector Surface Model</i>						
φ	Full CVs		Diag. CVs		Full & Diag.	
	$\sigma_{\Delta h}$	3 rd term	$\sigma_{\Delta h}$	3 rd term	$\sigma_{\Delta h}$	3 rd term
49°	17.8	0.1	18.0	2.7	18.0	2.7
53°	17.8	0.6	18.0	3.6	18.1	3.6
56°	17.8	1.1	18.8	6.0	18.8	6.0
60°	17.8	1.8	20.2	9.6	20.2	9.6
64°	17.9	2.5	22.2	13.3	22.2	13.3
<i>6 - Parameter Corrector Surface Model</i>						
49°	17.8	1.4	18.3	4.6	18.3	4.6
53°	17.8	0.8	18.3	4.5	18.3	4.5
56°	18.5	5.1	24.4	16.8	24.4	16.8
60°	24.2	16.4	54.5	51.6	54.6	51.6
64°	36.9	32.4	102.2	100.6	102.3	100.8

The results show that the achievable $\sigma_{\Delta h}$ for the new baseline was worse as it moved farther north from the control test network. This was mainly due to the increased error contribution of the corrector surface model parameters. Perhaps the most interesting result was the difference between using fully populated and (approximate) diagonal CV matrices for Δh , ΔH , and ΔN as input into Eq. (5). In studies where the newly established baseline was located *within* the test network coverage area, it was found that there was no difference between using fully populated versus (more approximate) diagonal CV matrices. However, in cases where the baseline is moved farther north (away from) the test network area, differences up to several tens of centimeters resulted. This result is quite significant as it indicates that approximate versions of the CV matrices should *not* be used, for new baselines situated *away from* the test network, in order to take advantage of the highest accuracy that GPS-based levelling provides.

5 Conclusions and Future Work

A method for evaluating the accuracy of relative GPS/geoid levelling was presented. Specifically, the achievable accuracy for tests in northern Canada was evaluated. The overall accuracy was affected by the relatively poor accuracy of the global geoid model (as compared to GPS and levelling accuracies). Of significance for future studies was the increased error contribution of the corrector surface based on the number of model parameters. Also, as the baseline was moved farther north, the significance of using fully populated versus diagonal CV matrices for the three height types became evident. In these cases, the achievable accuracy for ΔH varied as much as several tens of centimeters.

Acknowledgements

The authors would like to thank N. Pavlis, F. Lemoine, and E.J.O. Schrama, for their help with estimating the geoid height data accuracy. The financial support provided by NSERC and GEOIDE is also gratefully acknowledged.

References

- Fotopoulos G, Kotsakis C and Sideris MG (2001) How accurately can we determine orthometric height differences from GPS and geoid data? Submitted to the Journal of Surveying Engineering (May 2001).
- Heiskanen WA and Moritz H (1967) Physical Geodesy. WH Freeman, San Francisco.
- Kotsakis C and Sideris MG (1999) On the adjustment of combined GPS/levelling/geoid networks. Journal of Geodesy, Vol. 73, No. 8, pp. 412-421.
- Kotsakis C, Fotopoulos G and Sideris MG (2001) Optimal Fitting of Gravimetric Geoid Undulations to GPS/Levelling Data Using an Extended Similarity transformation Model. Presented at the Annual Scientific Meeting of the Canadian Geophysical Union, Ottawa, Canada, May 14-17.
- Lemoine FG, et al. (1998) The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96.
- National Geodetic Survey (1994) Input Formats and Specifications of the National Geodetic Survey Data Base: Volume II, Vertical Control Data. Systems Development Division, National Geodetic Survey (NGS), NOAA, Silver Spring, MD.
- Ollikainen M (1997) Determination of Orthometric Heights Using GPS Levelling. Publication of the Finnish Geodetic Institute, No. 123.

Aliasing Effects in Terrain Correction Computation Using Constant and Lateral Density Variation

S. Bajracharya, C. Kotsakis, M.G. Sideris

Department of Geomatics Engineering

The University of Calgary, 2500 University Drive N.W., Calgary, Alberta, T2N 1N4, Canada

E-mail: bajrachs@ucalgary.ca; fax: 403-284-1980

Abstract

Terrain correction (TC) needs to be treated in a precise and rigorous way in every gravity reduction technique within the context of precise geoid determination, especially for mountainous areas. Since the small grid spacing in the modern digital terrain models (DTMs) can represent the local features of rugged terrain very precisely, such high-resolution DTMs should be used, if available, in the numerical computation of TC according to the Newtonian attraction integral formula. Two areas within Canada are selected to study the effects of using different DTM grid spacing on TC computations. The DTM resolution levels used for this test are 15", 30", 45", 1' and 2'. Firstly, the computations are applied using constant crust density (2.67 g/cm^3) and the algorithm for the mass-prism (MP) topographic model, which is evaluated by fast Fourier transform technique in planar approximation. Secondly, we use lateral crust density variations in the TC computation for the MP model through the incorporation of the available digital density models (DDMs) with 30, 1' and 2' grid resolution. The comparison of the results using different DTM and DDM grid resolutions is carried out. Finally, the terrain effect (direct, indirect and total) on the geoid is studied using constant crust density with different DTM resolution levels and the direct effect on geoid is studied using lateral density variation with DTM and DDM of different grid resolutions.

Keywords. Terrain correction, geoid undulation, digital terrain model, digital density model

1 Introduction

The TC is a key auxiliary quantity in gravity reductions, which are used in solving the geodetic boundary value problem of physical geodesy and in geophysics. It contains the high frequency part of the gravity signal representing the irregular part of the topography, which deviates from the Bouguer plate. Helmert's second method of condensation is mostly used in practice as the mass reduction technique in the classical solution of the geodetic boundary value problem. Faye anomaly (or Helmert anomaly), which consists of free-air anomaly plus TC, represents the boundary values in the Helmert Stokes approach since TC alone is the difference between the attraction of the topography and the attraction of the condensed topography in planar approximation; see Moritz (1968), Wichiencharoen (1982) and Sideris (1990). In Molodensky's problem, which is regarded as modern boundary value problem, TC can replace the g_1 term under the assumption that the gravity anomalies are linearly dependent on the heights (Moritz, 1980).

Various computational approaches have been developed based on the conventional methods, which usually evaluate the TC integral using a model of rectangular prisms with flat tops (Nagy, 1966) or even with inclined tops (Blais and Ferland, 1984). TC computation based on these formulas is very time-consuming, but rigorous. Recently, Biagi et al. (2001) have given a new formulation for residual terrain correction (RTC) and Strykowski et al. (2001) have introduced a polynomial model for TC computation. The TC computation can be performed very fast in the frequency domain by means of FFT, having the TC convolution integral expanded in the form of Taylor series; see for example, Sideris (1984), Forsberg (1984), Tziavos et al. (1988), Harison and Dickinson (1989), Sideris (1990), Li and Sideris (1994), Li et al. (2000).

There are different resolutions of DTM available these days throughout the world. TC computation using FFT technique is one of the most efficient tools to handle the large amounts of height data efficiently. The convergence condition, that the distance between computation and running point should be larger than the difference between their heights, can be regarded as a major problem in the application of FFT to the series expansion of the TC integral, especially in rugged areas. Divergence of the series is observed with densely sampled height data in rough terrain; for example see Martinec et al. (1996) and Tziavos et al. (1996). A combination method, based on the evaluation of the numerical integration method in the intermediate zone around the computation point and the use of FFT in the rest of area, has been used to tackle the convergence problem by Tsoulis (1998) and Tziavos et al. (1998).

The knowledge of actual crust density is required in each gravity reduction method (including TC) in order to effectively remove all the masses above the geoid. Constant density is often used in practice instead of actual crust density because of lack of actual bedrock density information. However, two-dimensional DDMs are becoming available these days in some countries though a three-dimensional model is required to represent a real topographical density distribution. These density models should be incorporated in the TC computation. This has been studied by Tziavos et al. (1996), Huang et al. (2000), and Tziavos and Featherstone (2000). The effect on RTC of lateral density variation and approximations made on density modelling has been shown by Biagi et al. (2001).

This paper mainly focuses on investigating the importance of using actual density information and the use of various grid resolutions of DTM for TC computation in flat and rough areas, within the context of precise geoid determination. In this paper, the term aliasing represents the loss of detail information as terrain corrections are evaluated from a high-resolution DTM to a coarse one. In other words, the results from the densest DTM are taken as “control values” and the differences between these results and the results obtained by using sparser DTMs are considered to be the aliasing effects. It also studies terrain effects on geoid undulation for Helmert’s second method of condensation in planar approximation. Numerical tests are carried out in two areas in Canada, one in the most rugged areas of Canadian Rockies and another one in a modest area of Saskatchewan (Canada).

2 Computational formulas

The TC integral at a point (i,j) is given by (Heiskanen and Moritz, 1967)

$$c(i,j) = G \iint_E \int_{h_{ij}}^{h_p} \frac{\rho(x,y,z)(h_{ij} - z)}{r^3(x_i - x, y_j - y, h_{ij} - z)} dx dy dz, \quad (1)$$

where G is Newton’s gravitational constant, $\rho(x, y, z)$ is the topographical density at the running point, h_{ij} and h_p are the computation and running points respectively, and E denotes the integration area.

$r(x, y, z)$ is the distance kernel defined as

$$r(x,y,z) = (x^2 + y^2 + z^2)^{1/2}. \quad (2)$$

Equation (1) can be written for a gridded digital topographic model as (Li and Sideris, 1993)

$$c(i,j) = G \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \int_{y_m - \Delta y/2}^{y_m + \Delta y/2} \int_{h_{ij}}^{h_{nm}} \frac{\rho(x,y,z)(h_{ij} - z)}{r^3(x_i - x, y_j - y, h_{ij} - z)} dx dy dz \quad (3)$$

Integrating equation (1) with respect to z gives

$$c(i,j) = G \iint_E \left(\frac{\rho}{l} \left[1 - \left[1 + \left(\frac{\Delta h}{l} \right)^2 \right]^{-1/2} \right] \right) dx dy \quad (4)$$

where $l^2 = (x_i - x)^2 + (y_i - y)^2$ and $\Delta h = h_{ij} - h_p$

The term $\left[1 + \left(\frac{\Delta h}{l}\right)^2\right]^{-1/2}$, with the condition $\left(\frac{\Delta h}{l}\right)^2 \leq 1$, can be expanded into a series

$$\left[1 + \left(\frac{\Delta h}{l}\right)^2\right]^{-1/2} = 1 - \frac{1}{2}\left(\frac{\Delta h}{l}\right)^2 + \frac{13}{24}\left(\frac{\Delta h}{l}\right)^4 - \frac{13.5}{2.4.6}\left[\frac{\Delta h}{l}\right]^6 + \dots \quad (5)$$

Inserting equation (5) into equation (4) gives a principal formula for the FFT evaluation of the TC integral. Keeping only up to two terms in binomial series expansion, the following formula is obtained:

$$c(i,j) = G\rho \iint_E \left\{ \rho \left[\frac{\Delta h^2}{2l^3} - \frac{3\Delta h^4}{8l^5} \right] \right\} dx dy \quad (6)$$

2.1 Mass prism and mass line topographic models

A prism with a mean height of the topography represents the height within each cell in MP topographic representation. The mass of the prism is concentrated along its vertical axis representing topography as a line in mass line (ML) model. The unified two dimensional convolution formulas for equation (6) using MP and ML algorithms can be evaluated by means of FFT as (Li et al., 2000).

$$\begin{aligned} T_S^0 &= d_{00}F^{-1}\{H^0K_{S\alpha}^0\} - d_{01}F^{-1}\{H^0K_{S\beta}^0\} - d_{02}F^{-1}\{H^1K_{S\beta}^0\} \\ T_S^1 &= d_{10}F^{-1}\{H^0K_{S\alpha}^1\} - d_{11}F^{-1}\{H^0K_{S\beta}^1\} - d_{12}F^{-1}\{H^1K_{S\beta}^1\} - d_{13}F^{-1}\{H^2K_{S\beta}^1\} - d_{14}F^{-1}\{H^3K_{S\beta}^1\} \\ T_S^2 &= d_{20}F^{-1}\{H^0K_{S\alpha}^2\} - d_{21}F^{-1}\{H^0K_{S\beta}^2\} - d_{22}F^{-1}\{H^1K_{S\beta}^2\} - d_{23}F^{-1}\{H^2K_{S\beta}^2\} - d_{24}F^{-1}\{H^3K_{S\beta}^2\} \\ &\quad - d_{25}F^{-1}\{H^4K_{S\beta}^2\} - d_{26}F^{-1}\{H^5K_{S\beta}^2\} \end{aligned} \quad (7)$$

where $H^i = F\{\rho h^i\}$

$$K_{sw}^i = F\{K_s^i(x,y,w)\}, \quad s=x, y, w=\alpha, \beta, i = 0, 1, 2 \quad (8)$$

The coefficients d_{ij} and the analytical formulas for kernel functions used in above equations are given in details by Li et.al. (2000). α and β are parameters used to speed up the convergence of the series and the optimal value for this parameter is given by one-half of the standard deviation of the heights. The TC formula using DDM and MP algorithm is also given by Tziavos et. al. (1996) provided that the grid size of DTM is same as DDM.

2.2 Terrain effect on geoid undulation

The total geoid undulation in remove-restore technique can be expressed as

$$N = N_{GM} + N_{\Delta g} + N_{ind} \quad (9)$$

where N_{GM} represents the low frequency component of the geoid obtained from geopotential model, $N_{\Delta g}$ represents the medium frequency component of the geoid obtained from Stokes' formula and N_{ind} is the indirect effect on geoid, which depends on the gravity reduction method used. The Stokes' formula for the determination of medium wavelength part of the geoid for Helmert's second method of condensation in planar approximation can be formulated as

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g + c)S(\psi)d\sigma = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta gS(\psi)d\sigma + \frac{R}{4\pi\gamma} \iint_{\sigma} cS(\psi)d\sigma \quad (10)$$

where $(\Delta g+c)$ represents Faye anomalies and c is terrain correction, the negative value of which represents the difference between the attraction of the topography computed on the surface of the topography and the attraction due to the condensed masses computed on the geoid. The second term in equation (10) is direct terrain effect on geoid undulation. Indirect effect on gravity is not included in above formula. The indirect effect for this condensation scheme can be formulated in planar approximation as (Wichiencharoen, 1982)

$$N_{ind} = -\frac{\pi G \rho}{\gamma} h_{(i,j)}^2 - \frac{G \rho}{6\gamma} \iint \frac{h^3 - h_{(i,j)}^3}{r_0^3} dx dy \quad (11)$$

where r_0 is the planar distance between computation and running point. The total terrain effect on geoid undulation for this reduction scheme can be expressed as

$$N_{Total} = \frac{R}{4\pi\gamma} \iint_{\sigma} cS(\psi) d\sigma - \frac{\pi G \rho}{\gamma} h_{(i,j)}^2 - \frac{G \rho}{6\gamma} \iint \frac{h^3 - h_{(i,j)}^3}{r_0^3} dx dy \quad (12)$$

3 Numerical tests

Two tests areas, one in the Canadian Rockies bounded by latitude between 49°N and 54°N and longitude between 124° W and 114°W, and the other in Saskatchewan, bounded by latitude 49°N and 54°N and longitude between 110° W and 100° are selected for this numerical investigation. The statistics of the DTMs are presented in the table 1 for both test areas for different grid resolutions. The original grid resolution available for this test is 3' while 15", 30", 45", 1' and 2' grid files are produced by selecting the point height values from the 3' grid for the corresponding grid levels. The resolution of original DDM available for these test areas is 30 while 1' and 2' grid resolutions of DDM are produced from the 30 DDM picking up the point density values for the corresponding grid levels. Figure 1 represents the topography model of Canadian Rockies. Figure 2 shows large contrasts in topographic density of Canadian Rockies with maximum and minimum values of 2.98 (gm/cm³) and 2.63 (gm/cm³) respectively, whereas Saskatchewan has smooth geological structure of the topography with constant density of 2.56 (gm/cm³) except in some areas in the southern part.

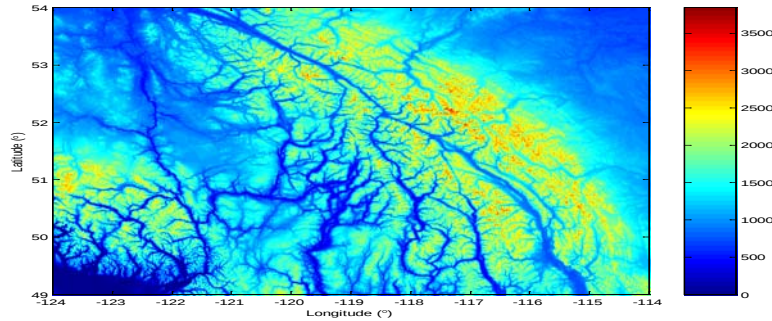


Fig.1 The topography in the Canadian Rockies

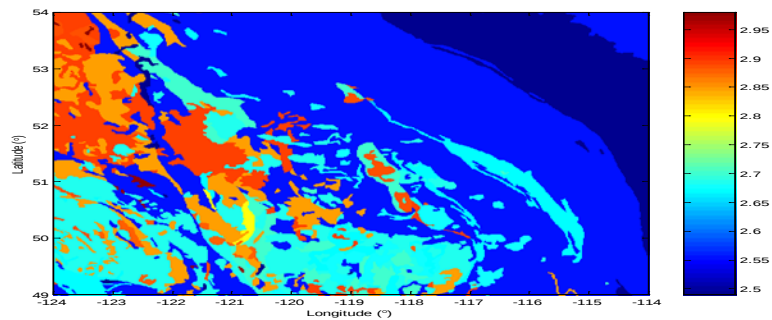
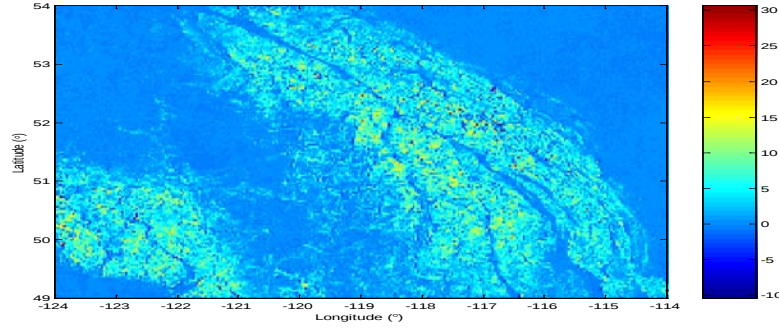


Fig. 2 The density model of Canadian Rockies

Table 1. Statistical characteristics of DTMs (m)

Test Area		Canadian Rockies					Saskatchewan				
Grid Resolution	Max	Min	Mean	RMS	STD	Max	Min	Mean	RMS	STD	
15"×15"	3840	0	1355	1460	543	1385	244	581	603	159	
30"×30"	3785	0	1355	1460	543	1381	244	581	603	159	
45"×45"	3656	0	1354	1459	543	1380	244	581	603	159	
1'×1'	3429	0	1354	1459	543	1379	244	581	603	159	
2'×2'	3275	0	1353	1458	544	1379	244	581	603	159	

**Fig. 3** Difference in TC using 15'' and 2' grid resolution (mGal).

3.1 Aliasing effects on TC with constant density

The TC computation is carried out using MP model for different grid resolution of DTM in both test areas for up to third term in Taylor series expansion. The kernel function is computed over the whole area and 100% zero padding is performed around the matrices of heights and around the distance kernel in order to remove circular convolution effects. The third term in Canadian Rockies shows divergence of the series giving out unrealistic results whereas that term in Saskatchewan does not change the final result of TC showing the convergence of the series right after second term and the results are presented just up to second term in this paper. Results on different tests are presented just for Canadian Rockies in this paper. Table 2 summarizes the statistics of TC results using different grid resolution of DTM. Figure 3 shows the difference in TC using DTM grid resolution between 15'' and 2', which shows the correlation of difference in TC using different DTM resolutions with topography. TC varies from 109 mGal to 42.7 mGal in maximum value and 9.9 mGal to 7.3 mGal in RMS in Canadian Rockies, while there is no considerable difference in the statistics of Saskatchewan using DTMs from 15'' grid resolution level up to 2' grid level. Maximum and RMS values of TC decrease from 2.6 mGal and 0.1 mGal to 1.5 mGal and 0.1 mGal respectively for Saskatchewan.

Table 2. Terrain correction in Canadian Rockies (mGal) (C1-first term, C2-second term)

Grid resolution	Terms	Max	Min	Mean	RMS	STD
15"×15"	C1	142.00	0.05	6.73	9.57	6.80
	C1+C2	108.76	0.05	7.06	9.89	6.93
30"×30"	C1	121.23	0.05	6.75	9.64	6.88
	C1+C2	100.06	0.05	6.96	9.77	6.86
45"×45"	C1	109.92	0.05	6.65	9.54	6.84
	C1+C2	83.38	0.05	6.71	9.44	6.64
1'×1'	C1	82.62	0.05	6.39	9.22	6.63
	C1+C2	59.83	0.05	6.36	8.98	6.34
2'×2'	C1	52.85	0.05	5.20	7.59	5.53
	C1+C2	42.73	0.05	5.09	7.31	5.24

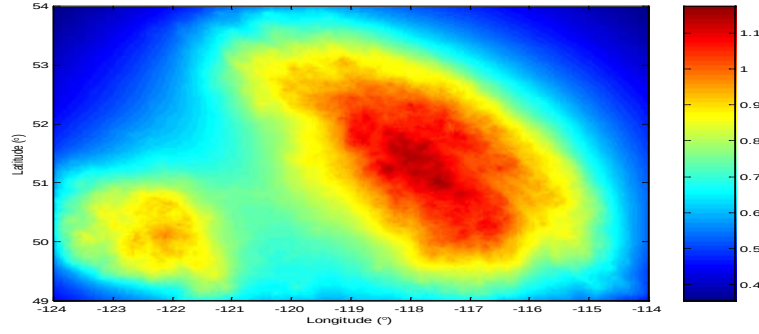


Fig. 4 Difference in total terrain effect on geoid undulation using between 15'' and 2' grid resolutions (m)

3.2 Aliasing effects on geoid with constant density

The direct effect on geoid undulation is computed from Stokes' formula with the rigorous spherical kernel by the one-dimensional fast Fourier transform algorithm. Both terms, a regular and an irregular part, are computed for indirect effects on geoid. Total terrain effects on geoid undulation vary from 3.777 m to 2.663 m in maximum value and 2.497 m to 1.728 m in RMS in Canadian Rockies, whereas there is just a

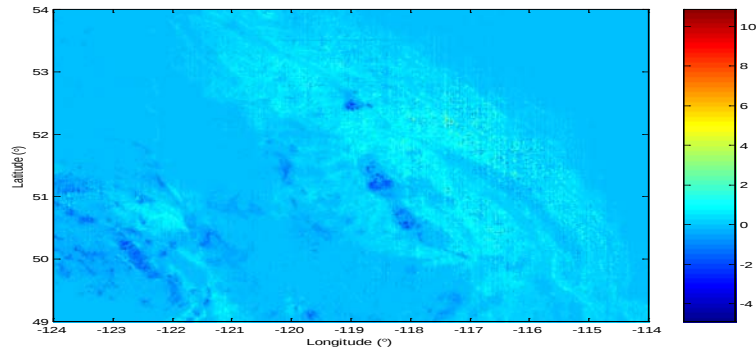


Fig. 5 Difference in TC using constant and variable density (mGal)

Table 3. Terrain effect on geoid undulation (m) (E-effect, D-direct effect, I-indirect effect, T-total terrain effect)

Grid Resolution	E	Max	Min	Mean	RMS	STD
15''×15''	D	3.866	1.207	2.536	2.614	0.634
	I	0.005	-0.470	-0.121	0.144	0.078
	T	3.777	1.188	2.418	2.497	0.623
30''×30''	D	3.779	1.179	2.482	2.556	0.612
	I	0.003	-0.470	-0.121	0.144	0.078
	T	3.691	1.160	2.362	2.439	0.609
45''×45''	D	3.619	1.128	2.376	2.448	0.588
	I	0.002	-0.463	-0.121	0.144	0.078
	T	3.535	1.104	2.257	2.324	0.554
1'×1'	D	3.410	1.066	2.247	2.314	0.556
	I	0.001	-0.468	-0.121	0.144	0.079
	T	3.337	1.042	2.127	2.191	0.524
2'×2'	D	2.729	0.852	1.795	1.850	0.445
	I	0.000	-0.457	-0.121	0.145	0.080
	T	2.663	0.827	1.677	1.728	0.416

change of 3-4mm in maximum value and RMS in Saskatchewan. The statistics of indirect effect are practically constant using different grid resolution levels for both test areas. Table 3 shows the direct, indirect and total terrain effect on geoid undulation for different DTM grid resolutions. Figure 4 shows the difference in total terrain effect on geoid undulation between using 1'5 and 2' grid resolution in Canadian Rockies. The maximum difference is at the top of the mountains.

3.3 Aliasing effects on TC using variable density

The TC is computed using DDM in both of the test areas using MP algorithm. The grid spacing of DTM and DDM used for this test are 30' and 2'. Table 4 shows the statistics of difference in TC using constant density and gridded actual density information. Their effect on geoid undulation is given in table 5. There is a difference of 10.9 mGal to 1.8 mGal in maximum value with RMS from 0.4 mGal to 0.2 mGal using constant and actual density information for the grid spacing of DTM and DDM from 30'' up to 2' arc minute in Canadian Rockies. There is no considerable difference in TC and their effects on geoid undulation using constant and actual density information in Saskatchewan. There is a maximum effect of 10 cm with an RMS of 4.6 cm on geoid undulation using constant and lateral density variation in Canadian Rockies. Figure 5 shows the difference in TC using constant and lateral density variation for 30' resolution while figure 6 shows its effect on geoid undulation. Figure 7 and figure 8 show the change in RMS value in TC and direct effect of TC on geoid undulation using constant and variable density.

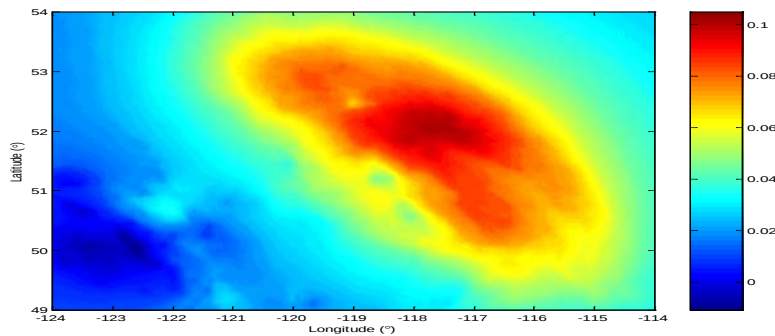


Fig. 6 Difference of Effects of TC on geoid undulation using constant and variable density (m)

Fig. 7 RMS value of TC using constant and variable density

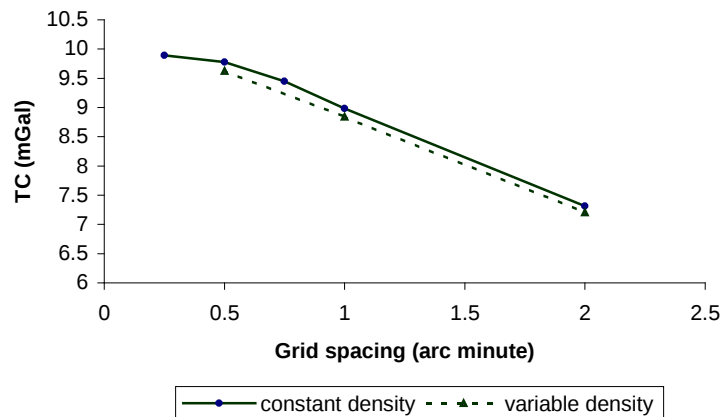


Fig. 8 RMS value of direct effect on geoid undulation using constant and variable density (m)

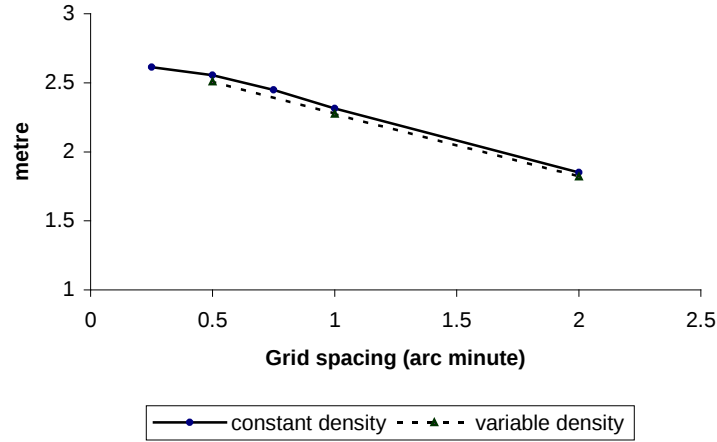


Table 4. The difference in TC using constant and variable density (mGal)

Grid Spacing	Max	Min	Mean	RMS	STD
30"×30"	10.87	-4.91	0.13	0.38	0.36
1'×1'	3.26	-2.94	0.11	0.29	0.27
2'×2'	1.77	-2.46	0.08	0.23	0.22

Table 5. Difference of Effects of TC on geoid undulation using constant and variable density (m)

Grid spacing	Max	Min	Mean	RMS	STD
30"×30"	0.105	-0.011	0.046	0.052	0.024
1'×1'	0.085	-0.012	0.038	0.043	0.021
2'×2'	0.068	-0.010	0.029	0.034	0.016

4 Conclusion

Fine DTM resolution should be used in precise geoid determination in rugged areas. Our results show that the difference in total terrain effect on geoid undulation using from 30 grid spacing of DTM up to 2' arc minute can vary from 3.777 m to 2.663 m in maximum value and 2.497 m to 1.728 m in RMS in Canadian Rockies. These values can even go higher if 3 grid resolution of DTM is used. The effect of second term of the TC Taylor series on geoid undulation is 16 cm in maximum value with RMS of 11 cm in Canadian Rockies. The use of high resolution DTM is not critical in the computation of terrain effect in non-mountainous regions. Divergence of the series was observed in the third term of Taylor series expansion. More studies regarding convergence problem should be carried out using FFT and high-resolution DTMs.

The actual crust density information, if available, should be used in precise geoid determination in high mountains. Our results show that the difference in TC effect on geoid undulation using constant density and horizontal density variation can go up to 10 cm in maximum value with an RMS of 5 cm. It would be wise to use at least the mean density of the area, if available, for terrain correction computation in mountains if DDM is not available. The knowledge of actual density information is not crucial in the TC computation of non-mountainous areas.

The difference in TC using different grid resolution of DTM and constant and lateral density variation is correlated with the topography and the maximum difference is observed in high mountains.

Acknowledgements: The authors are thankful to Dr. I.N. Tziavos from university of Thessaloniki, Greece for kindly providing software used in this paper. This research has been supported by grants from the GEOIDE NCE and the Natural Sciences and Engineering Research Council of Canada.

References

- Blais JAR and Ferland R (1984) Optimization in gravimetric terrain corrections. Canadian Journal of Earth Sciences. Vol.21, No. 5, pp.505-515.
- Biagi L, De Stefano R, Sanso F, Sciarretta C (1999) RTC and density variations in the estimation of local geoids. Bolletino Di Geofisica Teorica ed Applicata. Vol.40, N.3-4, pp589-595.
- Biagi L, Sanso F (2001) TcLight: a New technique for fast RTC computation. IAG Symposia, Vol. 123 Sideris (ed.), Gravity, Geoid, and Geodynamics 2000, Springer – Verlag Berlin Heidelberg 2001, pp61-66.
- Forsberg R (1984) A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modeling. Report No. 355, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio, April, 1984a
- Harrison JC, Dickinson M (1989) Fourier transform methods in local gravity field modeling. Bulletin Geodetique, Vol. 63, pp. 149-166
- Heiskanen WA, Moritz H (1967) Physical geodesy. W. H. Freeman and Company. San Fransisco.
- Huang J, Vanicek P, Pagiatakis SD, Brink W (2000) Effect of topographical density on geoid in the Canadian Rocky Mountains. Journal of geodesy, 74, 805-815.
- Li YC, Sideris MG (1994) Improved gravimetric terrain corrections. Geophys. J. Int., 119, 740-752
- Li YC, Sideris MG, Schwarz KP (2000) Unified terrain correction formulas for vector gravity measurements. PINSA, 66, A, No. 5, Sept-2000, pp.521-535
- Moritz H (1980) Advanced Physical Geodesy. Herbert Wichmann Verlag, Karlsruhe, Abacuss press, Tunbridge Wells Kent.
- Moritz H (1968) On the use of the terrain correction in solving Molodensky's problem. Rept. 108, Department of Geodetic Science and Surveying, The Ohio State University, Ohio, USA.
- Martinez Z, Vanicek P, Mainville A, Veronneau M (1996) Evaluation of topographical effects in precise geoid computation from densely sampled heights, Journal of geodesy, 70, 746-754.
- Nagy D (1966) The prism method for terrain corrections using digital computers. Pure Applied Geophysics. Vol.63, pp 31-39.
- Sideris MG (1984) Computation of gravimetric terrain corrections using fast Fourier transform techniques, UCSE Report #20007, Department of Surveying Engineering, The University of Calgary, Calgary, Alberta, Canada.
- Sideris MG (1990) Rigorous Gravimetric terrain modeling using Molodensky's operator. Manuscripta Geodaetica 15 : 97-106.

- Strykowski G, Boschetti F, Horowitz FG (2001) A fast spatial domain technique for terrain corrections in gravity field modelling. IAG Symposia, Vol. 123 Sideris (ed.), Gravity, Geoid, and Geodynamics 2000, Springer – Verlag Berlin Heidelberg 2001, pp67-72.
- Tsoulis D (1998) A combination method for computing terrain corrections. Phys.Chem.Earth, Vol.23, No. 1, pp.53-58.
- Tziavos IN, Featherstone WE (2000) First results of using digital density data in gravimetric geoids computation in Australia. IAG Symposia, Vol.123 Sideris (ed.), GGG2000, ©Springer Verlag Berlin Heidelberg (2001) pp 335-340
- Tziavos IN, Sideris MG, Sunkel H (1996) The effect of surface density variation on terrain modeling – A case study in Austria. Proceedings, EGS Society General Assembly. The Hague, The Netherlands, May, 1996. Report of the Finnish Geodetic Institute.
- Tziavos IN, Andritsanos VD (1998) Recent advances in terrain correction computations. Second continental workshop on the geoids in Europe. Budapest, Hungary, March, 1998. Report of Finish Geodetic Institute.
- Wichiencharoen C (1982) The indirect effects on the computation of geoids undulations. OSU Rept. 336, Department of Geodetic Science and Surveying, The Ohio State University, Ohio, USA.

Developing and testing the ellipsoidal gravity model manipulator ELGRAM

G.Sona

DIIAR, Sez. Rilevamento

Politecnico di Milano, Facolta' di Como, P.zza Gerbetto 6, Como, Italy

Abstract

The commonly used representation of potential as a truncated series of spherical harmonics leads to global models that consist of set of coefficients c_{nm} , s_{nm} (OSU91, EGM96), that can be successively used by a spherical harmonics manipulator to compute values of the potential and its functionals (geoid undulation, gravity anomaly, deflection of the vertical) at any point of given coordinates (ϕ, λ, h) .

The approximations involved both in the computations of the coefficients, from satellite and terrestrial measurements, and in their use in the synthesis of potential functionals make desirable the development of different computational techniques.

In view of the actual requirements of more and more precise potential representation it is useful to develop a manipulator working with series of ellipsoidal harmonic functions.

To this aim, appropriate equations relating geoid undulation, gravity anomaly, deflections of the vertical to anomalous potential T expanded in ellipsoidal series have been deduced, and a source code for their computation have been written and tested, comparing results with 'classical' spherical synthesis.

In the paper the main steps of analytical computations are shown, the structure and use of the new software is illustrated and some results of the comparisons are reported.

1 Ellipsoidal harmonic expansions

For many geodetic computations the anomalous gravitational potential of the earth is usually expanded in series of spherical harmonic functions:

$$T = \sum_{n=0}^{\infty} \sum_{m=0}^n v_{nm}^s \left(\frac{R}{r} \right)^{n+1} Y_{nm}(\theta, \lambda) \quad (1)$$

and usually represented as a truncated series, up to $n = N_{max}$

It is however possible to expand T using ellipsoidal harmonic functions (Heiskanen, Moritz (1990))

$$T = \sum_{n=0}^{N_{max}} \sum_{m=0}^n v_{nm}^e \frac{Q_{nm} \left(i \frac{u}{E} \right)}{Q_{nm} \left(i \frac{b}{E} \right)} Y_{nm}(\theta_e, \lambda) \quad (2)$$

The presence of Legendre function of second kind $Q_{nm} \left(i \frac{u}{E} \right)$ has been an obstacle up to now for the computation of such a series, the main difficulties in using truncated series of ellipsoidal harmonics coming from :

- The dependence on both order n and degree m of the Q_{nm} , that causes longer computational time for the terms containing the ellipsoidal "height" u (compared with spherical series (1) that

can be computed separating the angular part Y_{nm} from the simple radial terms $\left(\frac{R}{r}\right)^{n+1}$, independent of m).

- The computation of the Q_{nm} functions cannot be done using the simple recursive relations that hold identically for Q_{nm} and for P_{nm} , because in this case the recursive relations show an unstable behavior due to their imaginary argument (Sona (1995)).

The first problem cannot be avoided, being intrinsic in the ellipsoidal representation, (we can only simplify computations with some interpolations), but with the more and more powerful processors now available it is today a minor problem.

The second and more important obstacle can be overcome computing the Q_{nm} with hypergeometric functions, as suggested in Thong and Grafarend (1989) or in Sona (1995).

The difficulties arising in such a direct computation of Q_{nm} functions however (for each n, m and each value of 'height' u) are better reduced by introducing the 'boundary layer approach', that means by putting :

$$1 < \frac{u}{b} < 1 + 10^{-3} \quad (h < 6700 \text{ m}),$$

approximation that holds for most of the earth surface. This leads to the direct approximation of the terms

$$\bar{Q}_{nm}(u) = \frac{Q_{nm}\left(i \frac{u}{E}\right)}{Q_{nm}\left(i \frac{b}{E}\right)}$$

in (2) as

$$\bar{Q}_{nm}(u) = \left(\frac{b}{u}\right)^{n+1} \left(\frac{u}{b}\right)^{e^{2\frac{(n+1)(n+2)+m^2}{2n+1}}} \quad (3)$$

This expression has been proved to be very close to $\bar{Q}_{nm}$, up to a very high order and degree ($n < 1.5 \cdot 10^5$) and in the same time does not display any computational irregularity (singularity, instability) (Sona (1995)).

The 'limit layer' approximation is therefore a good solution to simplify and speed up the computation of $\bar{Q}_{nm}$ making thus sufficiently manageable a computer program working directly in ellipsoidal harmonics.

2 Spherical and ellipsoidal model coefficients

In order to develop a computer program that uses ellipsoidal harmonic truncated series to synthesize anomalous potential T and its functionals, it is necessary to transform the relationship exploited by the worldwide used classical 'spherical' software (for instance f477 or f388 by R.Rapp, (1982)) into the corresponding ellipsoidal ones, and, of course, it is necessary to use an ellipsoidal global model, that is, a set of ellipsoidal coefficients $v_{n'm}^e$ (eq.2).

This is not available, at the moment, and all the global geopotential models available to IGeS consist in the well known sets of spherical harmonic coefficients (e.g. OSU91A, EGM96, by Rapp, or Wenzel, (1999))

So the only way to have an ellipsoidal global model, with the purpose of checking results obtained with a new ellipsoidal model manipulator, (comparing values of ζ , Δg , ξ , η at several point ϕ_i , λ_i , h_i) is to translate a spherical model into the corresponding ellipsoidal one, that means, to transform the set of spherical coefficients v_{nm}^s , into the corresponding set of ellipsoidal coefficients $v_{n'm}^e$, through the well known exact relations existing between the two sets (Jekeli (1988)). Therefore we created a fictitious set of ellipsoidal coefficients by transforming the coefficients of the spherical global model EGM96 (Nmax=360, Lemoine et al.1998) ; the values of $v_{n'm}^e$ can be computed as linear combination of v_{nm}^s , with $n = n'$, $n' - 2$, $n' - 4$, $n' - 2k, \dots$:

$$v_{n'm}^e = S_{nm} \left(\frac{b}{E} \right) \sum_{k=0}^W \lambda_{knm} v_{n-2k,m}^s \quad (4)$$

where

$$\begin{aligned} \lambda_{knm} &= \frac{(2n-2k)! n!}{(2n)! k! (n-k)!} \sqrt{\frac{(2n-4k+1)(n-m)!(n+m)!}{(2n+1)(n-2k-m)!(n-2k+m)!}} \left(\frac{E}{R} \right)^2 \\ \bar{S}_{nm} \left(\frac{b}{E} \right) &= \frac{\left(\frac{R}{E} \right)^{n+1} (2n)!}{2^n n!} \sqrt{\frac{\varepsilon_m (2n+1)}{(n-m)!(n+m)!}} Q_{nm} \left(i \frac{b}{E} \right) = \\ &= \left(\left(i \frac{b}{E} \right)^2 - 1 \right) \left(i \frac{R}{b} \right)^{n+1} \left(i \frac{E}{b} \right)^m F \left(\frac{n+m+2}{2}, \frac{n+m+1}{2}; n + \frac{3}{2}; - \left(\frac{E}{b} \right)^2 \right) \end{aligned}$$

and F is the hypergeometric Gauss function (Jekeli (1988), Petrovskaya (2000)).

As it can be seen in eq. (4) each spherical coefficient v_{nm}^s is involved in the computation of all $v_{n'm}^e$ of the same order m and $n' \geq n$, $n'=n, n+2, n+4, \dots, n+2k$. Therefore a global model, described by spherical coefficients up to order and degree 360, yields through (4) ellipsoidal coefficients that are significantly different from zero also for $n' > 360$.

3 Ellipsoidal expansion of geodetic functionals of T

For the purpose to make the ellipsoidal computations comparable with the spherical one (up to spherical degree max Nmax=360), the (fictitious) ellipsoidal model EGM96ell has been built up to degree $N_{\max}^e=400$ (of course, coefficients with $m > 360$ are missing in this model).

Starting from the usual relations :

$$\zeta = \frac{T}{\gamma} \quad (5)$$

$$\Delta g = \frac{\gamma}{\gamma} \cdot \nabla T + \left(\frac{\partial \gamma}{\partial h} \right) \frac{T}{\gamma} \quad (6)$$

$$\varepsilon = - \frac{dN}{ds} = - \frac{1}{\gamma} \frac{dT}{ds} \quad (7)$$

expansion of main geodetic functionals in ellipsoidal harmonics can be deduced, in order to compute from a set of ellipsoidal coefficients the anomalous potential T and its functionals height anomaly ζ , gravity anomaly Δg and deflections of the vertical ξ and η .

Being simply related with the anomalous potential T , the height anomalies ζ can be easily computed inserting the coefficients c_{nm}^e , s_{nm}^e , in (2):

$$T = \sum_{n=0}^{N_{max}} \sum_{m=0}^n \bar{Q}_{nm}(u) P_{nm}(\theta_e) \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \dots (8)$$

On the contrary, for some terms in equations (6) and (7) more analytical computations are needed. In (6), the projection of the gradient of T on the normal gravity direction in ellipsoidal coordinates can be computed as:

$$\frac{\gamma}{\gamma} \cdot \nabla T = \frac{\gamma_u}{\gamma} \frac{\partial T}{\partial s_u} + \frac{\gamma_\beta}{\gamma} \frac{\partial T}{\partial s_\beta} \quad (9)$$

where γ_u and γ_β can be derived from the closed formula of normal potential U (Heiskanen and Moritz (1995))

$$\gamma_u = \frac{1}{w} \left[\omega^2 u \cos^2 \beta - \frac{kM}{u^2 + E^2} + \frac{\omega^2 a^2}{2q(b)} \left(\sin^2 \beta - \frac{1}{3} \right) \frac{dq(u)}{du} \right]$$

$$\gamma_\beta = \frac{\omega^2 \cos \beta \sin \beta}{w \sqrt{u^2 + E^2}} \left[a^2 \frac{q(u)}{q(b)} - (u^2 + E^2) \right]$$

where $w = \sqrt{\frac{u^2 + E^2 \sin^2 \beta}{u^2 + E^2}}$ and $q(u) = \frac{1}{2} \left[\left(1 + 3 \frac{u^2}{E^2} \right) \operatorname{artg} \frac{E}{u} - 3 \frac{u}{E} \right]$.

The ellipsoidal derivatives of the anomalous potential T :

$$\frac{\partial T}{\partial s_u} = \frac{1}{w} \frac{\partial T}{\partial u} \quad (10)$$

$$\frac{\partial T}{\partial s_\beta} = \frac{1}{w \sqrt{u^2 + E^2}} \frac{\partial T}{\partial \beta} = \frac{-1}{w \sqrt{u^2 + E^2}} \frac{\partial T}{\partial \theta_e} \quad (11)$$

($\theta_e = \pi/2 - \beta$) can be derived by writing the anomalous potential T with the expansion (8) and remembering that the line element in ellipsoidal coordinates is given by

$$ds^2 = w^2 du^2 + w^2 (u^2 + E^2) d\beta^2 + (u^2 + E^2) \cos^2 \beta d\lambda^2 \quad (12)$$

Inserting the approximate expression (3) for the $\bar{Q}_{nm}$ one gets

$$\frac{\partial T}{\partial u} = \sum_{n=0}^{N_{max}} \sum_{m=0}^n \frac{K_{nm}}{u} \bar{Q}_{nm}(u) P_{nm}(\theta_e) \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \quad (13)$$

with

$$K_{nm} = e^2 \frac{(n+1)(n+2) + m^2}{2n+1} - (n+1),$$

and

$$\frac{\partial T}{\partial \theta_e} = \sum_{n=0}^{N_{max}} \sum_{m=0}^n \bar{Q}_{nm}(u) \frac{\partial P_{nm}(\theta_e)}{\partial \theta_e} \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \quad (14)$$

As verified by testing one step at a time, the different approximations used in the two manipulators yield Δg values which cannot be compared because the gradient is taken along significantly different directions (while height anomalies results very close in values to the spherical ones). An additional check has therefore been planned, by comparing the values of $|\nabla T|^2$ in spherical and ellipsoidal coordinates, as this quantity is coordinate independent, thus testing whether the partial derivatives of T are computed properly or not:

$$|\nabla T|^2 = \left(\frac{\partial T}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial T}{\partial \theta} \right)^2 + \left(\frac{1}{r \sin \theta} \frac{\partial T}{\partial \lambda} \right)^2$$

$$|\nabla T|^2 = \left(\frac{1}{w} \frac{\partial T}{\partial u} \right)^2 + \left(\frac{1}{w \sqrt{u^2 + E^2}} \frac{\partial T}{\partial \beta} \right)^2 + \left(\frac{1}{\sqrt{u^2 + E^2} \cos \beta} \frac{\partial T}{\partial \lambda} \right)^2$$

All the terms needed to compute $|\nabla T|$ are already present in f477 as part of computation of ζ , Δg , ξ , η , therefore only few changes in that source code have been added.

The horizontal derivatives of T in ellipsoidal coordinates are also needed, to compute the two components of vertical deflection $\underline{\varepsilon}$:

$$\eta = -\frac{dN}{ds_\lambda} = -\frac{1}{\gamma} \frac{dT}{ds_\lambda} \quad \xi = -\frac{dN}{ds_\varphi} = -\frac{1}{\gamma} \frac{dT}{ds_\varphi}$$

The η component is easy to compute and to compare with corresponding spherical value, because both coordinate systems use the same definition for λ .

From eq. (12) we get the simple relation :

$$\frac{dT}{ds_\lambda} = \frac{1}{\sqrt{u^2 + E^2} \cos \beta} \frac{dT}{d\lambda}$$

On the contrary, ξ component needs more analytical computations, in fact we can write

$$\frac{dT}{ds_\varphi} = \frac{1}{f(u, \beta)} \frac{dT}{d\varphi}$$

where we have to compute $\frac{dT}{d\varphi}$ as

$$\frac{dT}{d\varphi} = \frac{\partial T}{\partial u} \frac{\partial u}{\partial \varphi} + \frac{\partial T}{\partial \beta} \frac{d\beta}{d\varphi} ,$$

and $f(u, \beta)$ is deduced from relation (12) , with $d\lambda = 0$ and written as :

$$ds_\varphi^2 = \frac{w^2 du^2 + w^2(u^2 + E^2)d\beta^2}{d\varphi^2} \cdot d\varphi^2$$

that gives :

$$ds_\varphi = w \sqrt{\left(\frac{du}{d\varphi}\right)^2 + (u^2 + E^2)\left(\frac{d\beta}{d\varphi}\right)^2} \cdot d\varphi .$$

The derivatives of T with respect to u and β expanded in ellipsoidal harmonic series have already been computed for Δg determination (eq (13) and (14)).

Derivatives of u and β with respect to ϕ have to be computed from the relations used to transform geodetic into ellipsoidal coordinates :

$$\begin{aligned} (N + h) \cos \phi &= \sqrt{u^2 + E^2} \cos \beta \\ \left(\frac{b^2}{a^2} N + h\right) \sin \phi &= u \cdot \sin \beta \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{du}{d\varphi} &= (M + h) \frac{\sqrt{u^2 + E^2}}{u^2 + E^2 \sin^2 \beta} \left[\sqrt{u^2 + E^2} \sin \beta \cos \varphi - u \sin \varphi \cos \beta \right] \\ \frac{d\beta}{d\varphi} &= (M + h) \frac{1}{u^2 + E^2 \sin^2 \beta} \left[\sqrt{u^2 + E^2} \sin \beta \sin \varphi + u \cos \varphi \cos \beta \right] \end{aligned}$$

(M and N are the principal radii of curvature)

Finally, calling

$$\begin{aligned}
S_{(0)} &= \sum_{n=0}^{N_{max}} \sum_{m=0}^n \bar{Q}_{nm}(u) P_{nm}(\theta_e) \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \\
S_{(1)} &= \sum_{n=0}^{N_{max}} \sum_{m=0}^n \frac{K_{nm}}{u} \bar{Q}_{nm}(u) P_{nm}(\theta_e) \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \quad (16) \\
S_{(2)} &= \sum_{n=0}^{N_{max}} \sum_{m=0}^n \bar{Q}_{nm}(u) \frac{\partial P_{nm}(\theta_e)}{\partial \theta_e} \left(c_{nm}^e \cos m\lambda + s_{nm}^e \sin m\lambda \right) \\
S_{(3)} &= \sum_{n=0}^{N_{max}} \sum_{m=0}^n \bar{Q}_{nm}(u) P_{nm}(\theta_e) \left(c_{nm}^e \sin m\lambda + s_{nm}^e \cos m\lambda \right)
\end{aligned}$$

ζ , Δg , ξ , η can be written as

$$\begin{aligned}
\zeta &= \frac{1}{\gamma} S_{(0)} \\
\Delta g &= \frac{1}{\gamma} \left[F_1 \cdot S_{(1)} + F_2 \cdot S_{(2)} + F_3 \cdot S_{(0)} \right] \\
\xi &= -\frac{1}{\gamma} F_4 \left[F_5 S_{(1)} + F_6 S_{(2)} \right] \quad (17) \\
\eta &= -\frac{1}{\gamma} F_7 S_{(3)}
\end{aligned}$$

These relations (similar to those used in spherical manipulators) link values of ζ and Δg to ellipsoidal coefficients c_{nm}^e , s_{nm}^e .

4 ELGRAM : an ellipsoidal harmonic manipulator

The program developed up to now computes ζ , Δg and ξ, η at sparse points (ϕ_i, λ_i, h_i) or on a regular (ϕ, λ) grid with constant height h , from a set of ellipsoidal coefficients and it is detailed in the following.

The main computational steps are:

1) geodetic coordinates (ϕ_i, λ_i, h_i) are transformed into the ellipsoidal coordinates $(\beta_i, \lambda_i, u_i)$ solving the relations (15), for u and $\cos \beta$.

Note that, as both u and β depend on both h and ϕ , the surfaces $h=\text{constant}$ and $\phi=\text{constant}$ are not transformed into surfaces $u=\text{const}$ or $\beta=\text{const}$, that means: a geodetic grid with $\Delta \lambda=\text{const}$, $\Delta \phi=\text{const}$ (typically used in global computations) is not transformed into a regular ellipsoidal grid with constant $\Delta \beta=$ and $\Delta \lambda$ (only the spacing $\Delta \lambda=\text{const}$ is preserved, as λ is the same in the two coordinate systems)

2) the $\bar{Q}_{nm}$ functions are computed using (3). It has been verified that a quadratic interpolation between three fixed heights gives $\bar{Q}_{nm}$ with sufficient approximation. Thus the computation of them for each value of u can be avoided: a subroutine computes the $\bar{Q}_{nm}$ functions at two constant values $u_1=1000\text{m}$, $u_2=4000\text{m}$ (the third value is fixed: for $u=b$ $\bar{Q}_{nm}=1 \forall n,m$) and the coefficients to be used in the interpolation. Another subroutine then interpolates at the point height u_i the proper values of $\bar{Q}_{nm}$ for each n,m .

3) as in spherical harmonic manipulators, the normalized Legendre functions $P_{nm}(\theta_e)$ and their derivatives $\frac{\partial P_{nm}(\theta_e)}{\partial \theta_e}$ are computed for the ellipsoidal value of $\theta_e=\pi/2-\beta$ using recursive relations.

4) the normal gravity $\gamma=\gamma(\phi,h)$ and its derivative are approximated by the relations (Heiskanen Moritz (1995)):

$$\gamma = 9.78032677 \frac{(1 + 0.00193185135 \sin^2 \phi)}{\sqrt{1 - e^2 \sin^2 \phi}} - h \cdot 0.3086 \cdot 10^{-5}$$

$$\frac{\partial \gamma}{\partial h} = -\frac{2\gamma_a}{a} (1 + f_2 \sin^2 \phi + f_4 \sin^4 \phi) \cdot (1 + f + m - 2f \sin^2 \phi)$$

5) finally, the values of ζ , Δg and ξ , η are computed through (16) and (17)

5 The source code

ELGRAM source code has been developed in FORTRAN77 language, compiled and tested on a UNIX HP Workstation. It is organized in a main program (ELGRAM) and 8 subroutines; the main program defines (as usual) parameters values, file names, variables, vectors dimensions, reads input data, calls subroutines for the different computation steps and writes on files the output required.

The parameter $n_{\max}$ represents maximum order and degree achievable in the expansions, and it is used to define 9 matrix $(n_{\max}+1) \times (n_{\max}+1)$ and 5 vectors, with dimension that can be seen in Table 1, and it is now set at 400 correspondingly to the coefficients file EGM96ee (derived as said in par.1 from global model EGM96); if desired, it can be easily raised, when new higher degree ellipsoidal model will be available.

ce(n,m), se(n,m)	ellipsoidal coefficients	(0:nmax,0:nmax)
qnm1(n,m), qnm2(n,m), qnmu(n,m) anm(n,m), bnm(n,m)	values of Q_{nm} at three fixed heights and interpolation coefficients	
pol(n,m), der(n,m) pnn(0:nmax), dnn(0:nmax), radq(n)	values of Legendre polynomials and derivatives	
sinml(n), cosml(n)	recursive values of $\sin(m\lambda)$ $\cos(m\lambda)$	(2*nmax+1) (nmax)

Table 1

(We remember that a max ellipsoidal degree $N_{\max}^e=400$ corresponds to a spherical global model of max degree $N_{\max}=360$, as explained in par.1)

The same values of the fundamental constants a , e^2 , ω^2 , kM (Tab.2) as adopted by f477 have been used in ELGRAM to compare values of potential functionals computed at the same test points :

$1/f = 298.257222101$
$KM = 3.986005 \cdot 10^{14}$
$\omega^2 = 7.292115 \cdot 10^{-5}$
$a = 6378137.0$

Table 2

As seen in fig.1 the program starts asking interactively for:

- 1) Name of global model coefficients file (the only one available at the moment is EGM96ee)
- 2) Choice between computation on sparse (0) or gridded (1) points
- 3) a) name of file containing n , ϕ , λ , h , of sparse points
b) number of points
or
a) constant value of h
b) limits and steps of the grid
c) type of output
- 4) maximum degree of the computation (always lower than $n_{\max}$)
- 5) name of output file.

```

global model (coefficients file) ?
sparse point (0) or grid (1) ?

sparse points ==> Points file (n,phi,lambda,h)?
                  number of points ?

      grid      ==> value of h (constant) ?
                  limits and step of the grid ?
                  (phi_min phi_max lambda_min lambda_max Delta phi Delta lambda)
                  output: n,phi,lambda,values (0) or grid (1) ?
                  if (1) ==> choose functional: 1) geoid undulation
                                                  2) gravity anomalies

max degree ?
output file ?

```

Fig.1

The program then reads the model coefficients from coefficients file and, after that, for every point (ϕ, λ) read from the point file (or computed on knots of a regular grid), it performs the computational steps described in par.3 going through the following subroutines:

qnm012	computes Q_{nm} for each n,m up to maximum degree and order ngra
intqnm	computes the coefficients a_{nm} b_{nm} c_{nm} for the successive quadratical interpolation at given height u
geotoel	performs the coordinate transformation $(\phi,h)=>(\beta,u)$
intqnm2	evaluates $Q_{nm}(u)$ at ellipsoidal height u
polleg dscml	compute surface harmonics $Y_{nm}(\beta,\lambda)$
Presint sintell	produce the final values of the functionals via (16) (17)

On the output file a header contains

- 1) Name of the model file
- 2) Max degree used
- 3) Name of the input point file and number of computed points,
or $\phi_{\min}$ $\phi_{\max}$ $\lambda_{\min}$ $\lambda_{\max}$ $\Delta\phi$ $\Delta\lambda$ in4 the gridded case

then follow the output values:

- a) n ϕ λ h β du ζ Δg ξ η in one line for each point,
or
- b) gridded values of selected functional from north to south, from west to east; (organized in blocks at constant ϕ , variable λ)

6 Comparing ellipsoidal and spherical synthesis

As mentioned ELGRAM has been tested by comparing its results with 'spherical' results yielded by f477 (Rapp(1982)): height anomalies ζ , gravity anomaly Δg , anomalous potential gradient $|\nabla T|$, and vertical deflection components ξ, η , have been compared at several points.

In the following are reported some results of comparisons performed on regularly spaced points ($\Delta\lambda=\Delta\phi=0.5^\circ$) along a quarter of the reference meridian $\lambda=0^\circ$ and along a quarter of the parallel $\phi=45^\circ$ (at variable heights).

The following plots display the satisfactory values obtained for both the absolute and relative differences of height anomalies, $|\zeta_s - \zeta_e|$ and $|\zeta_s - \zeta_e|/\zeta_s$

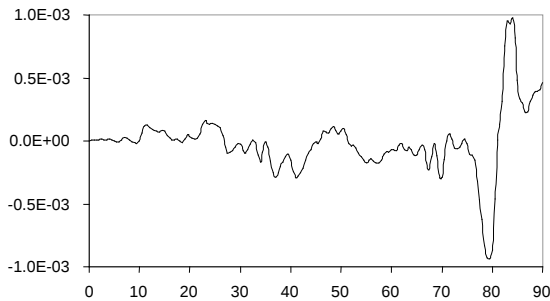


Fig.1 Height anomalies absolute differences (m) along a quarter of parallel $\phi=45^\circ$

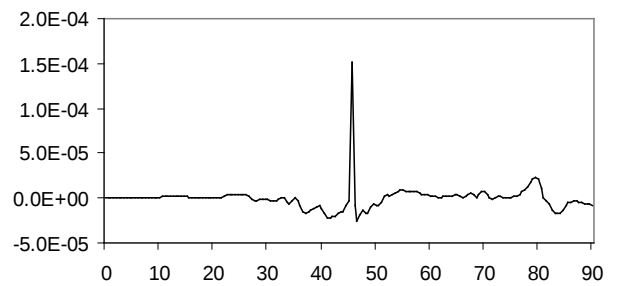


Fig.2 Height anomalies relative differences along a quarter of parallel $\phi=45^\circ$

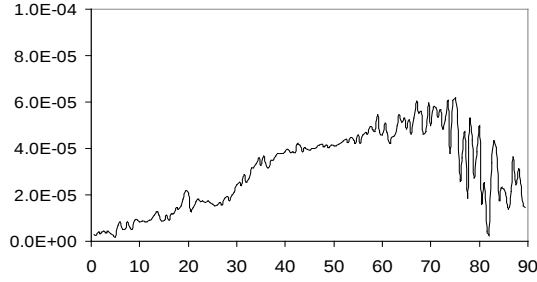


Fig.3 Height anomalies absolute differences (m) along a quarter of reference meridian $\lambda=0^\circ$

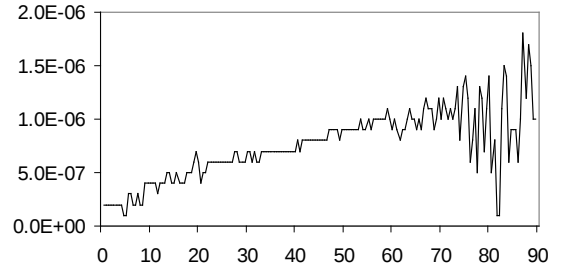


Fig.4 Height anomalies relative differences along a quarter of reference meridian $\lambda=0^\circ$

The spike of relative errors at $\phi=45^\circ$, $\lambda=0^\circ$ in Fig.2 is explained by the very small value of ζ at that point.

On the contrary, when we came to Δg we didn't get small errors, and this is easily understood remembering the different approximations used in the two manipulators, that lead to Δg values which are not comparable. However, as said in par.1, we compared also $|\nabla T|$ values, and we obtained very satisfactory results, as can be deduced from the following plots:

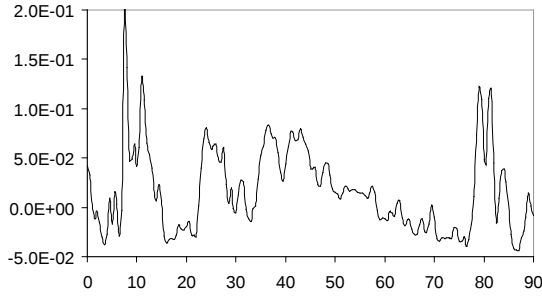


Fig.5 Absolute differences of $|\nabla T|$ (mGal) along a quarter of parallel $\phi=45^\circ$

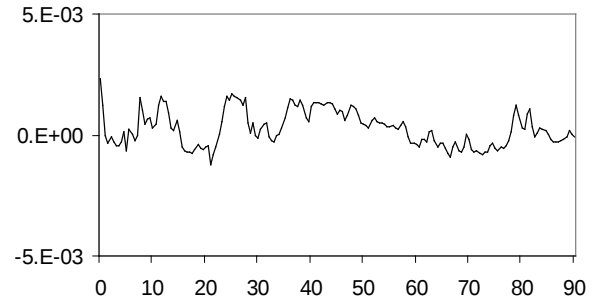


Fig.6 Relative differences of $|\nabla T|$ along a quarter of parallel $\phi=45^\circ$

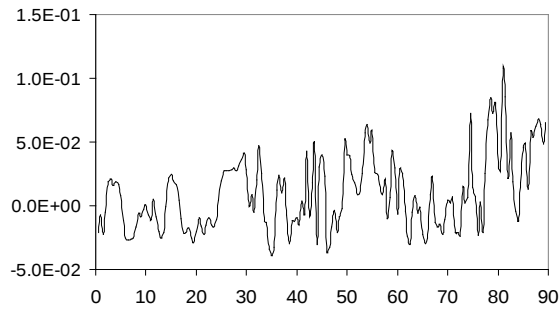


Fig.7 Absolute differences of $|\nabla T|$ (mGal) along a quarter of reference meridian $\lambda=0^\circ$

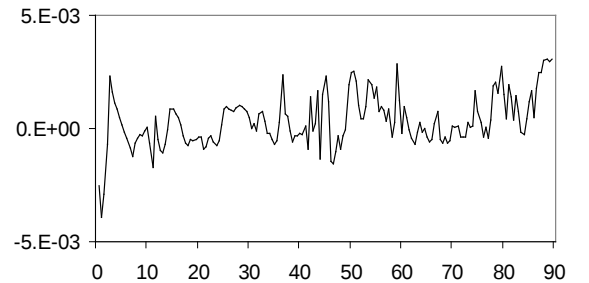


Fig.8 Relative differences of $|\nabla T|$ along a quarter of reference meridian $\lambda=0^\circ$

In this way we have the proof that horizontal derivatives of T in eqs (13) and (14) are properly computed.

The comparisons of ξ, η on the same test points are represented in Fig. 9 to 16 confirming the good agreement between ellipsoidal and spherical model manipulator, in the sense that the maximum error is of the order of the smallest noise in the measurements.

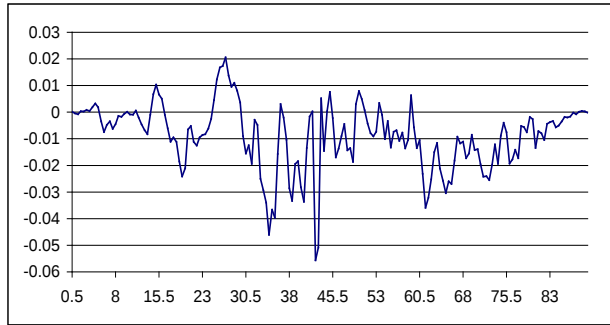


Fig.9 Absolute differences of ξ (sec) along a quarter of reference meridian $\lambda=0^\circ$

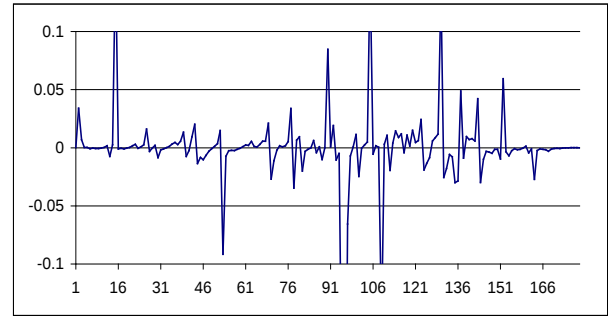


Fig.10 relative differences of ξ along a quarter of reference meridian $\lambda=0^\circ$

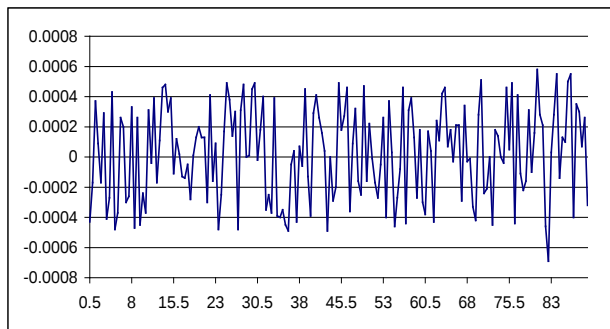


Fig.11 absolute differences of η (sec) along a quarter of reference meridian $\lambda=0^\circ$

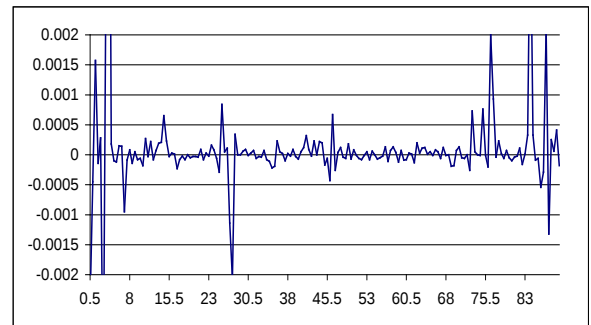


Fig.12 relative differences of η along a quarter of reference meridian $\lambda=0^\circ$

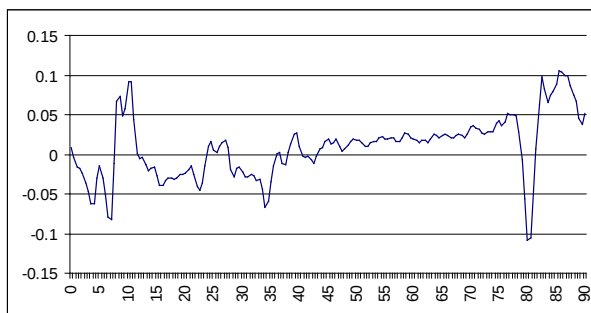


Fig.13 Absolute differences of ξ (sec) along a quarter of reference parallel $\phi=45^\circ$

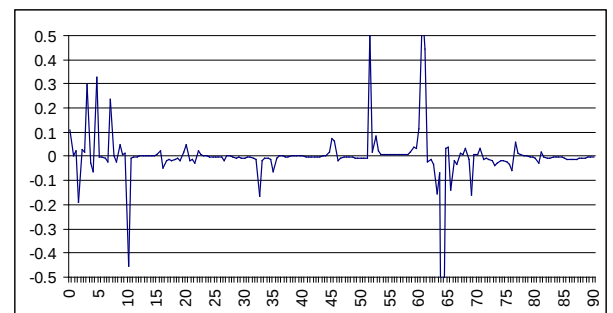


Fig.14 Relative differences of ξ along a quarter of reference parallel $\phi=45^\circ$

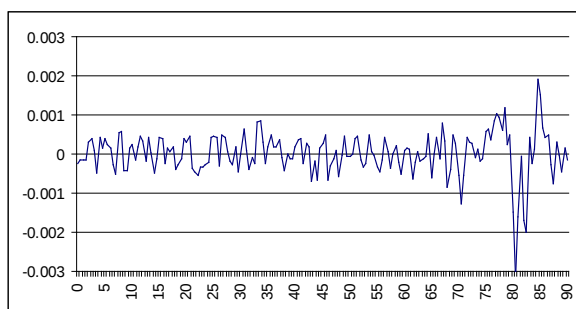


Fig.15 Absolute differences of η (sec) along a quarter of reference parallel $\phi=45^\circ$

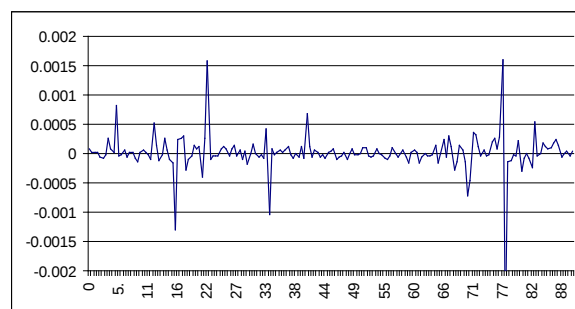


Fig.16 Relative differences of η along a quarter of reference parallel $\phi=45^\circ$

7 Conclusions and further developments

Tests performed on the new ellipsoidal gravity manipulator ELGRAM have got a very positive results; at present the software is able to compute at gridded or sparse points the basic functionals of the anomalous potential ζ , Δg , ξ , η .

We remember that this means that ELGRAM is able to perform the synthesis of ζ , Δg and ξ , η in the topographic layer ($h < 6700\text{m}$), given an ellipsoidal geopotential model.

The software is now freely available through the I.Ge.S. web, where one can find the source code ready to be compiled (main program and all subroutines), the input files for execution of some examples (including those reported in this paper), the point files used for testing the code along a quarter of parallel and of meridian and the corresponding output files for checking results and for training. The ellipsoidal coefficients file computed from EGM96 is also given to allow the use of ELGRAM software.

It is still necessary to test it up to order and degree 1440; this will require a careful optimization of routines and the introduction of FFT along parallels to save computing time.

References

- Cruz, J.Y. (1986) Ellipsoidal Corrections to Potential Coefficients obtained from Gravity Anomaly Data on the Ellipsoid, Report N.371, Dept.of Geodetic Science and Surveying, Ohio State University.
- Gleason, D.M. (1988) Comparing Ellipsoidal Corrections to the Transformation between the Geopotential Spherical and Ellipsoidal Spectrums, Man. Geod. N.13-2, pp114-129.
- Gleason, D.M. (1989) Some Notes on the Evaluation of Ellipsoidal and Spherical Harmonic Coefficients from Surface Data, Man.Geod.N.14, pp110-116.

- Heck, B. (1965) On the linearized Boundary Value Problems of Physical Geodesy, Report N.407, Dept. of Geodetic Science and Surveying, Ohio State University.
- Heiskanen, W.A. and Moritz, H. (1990) Physical Geodesy, Reprint, Institute of Physical Geodesy, Technical University, Graz, Austria.
- Jekeli, C. (1988) The Exact Transformation between Ellipsoidal and Spherical Harmonic Expansions, *Man.Geod.N.13*, pp106-113.
- Lemoine, F.G., Kenyon, S.C., Factor, J.K., Trimmer, R.G., Pavlis, N.K., Chinn, D.S., Cox, C.M., Klosko, S.M., Luthcke, S.B., Torrence, M.H., Wang, Y.M., Williamson, R.G. Pavlis, E.C., Rapp, R.H., and Olson, T.R., (1998) The Development of the Joint NASA GSFC and the NIMA Geopotential Model EGM96, NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt.
- Petrovskaya, M. Vershkov Andrei N. (2000) Simplified relations between the ellipsoidal and spherical harmonic coefficients of the external earth's potential *Bollettino di Geodesia e Scienze Affini N.1, 2000 anno LIX*, pp57-72.
- Rapp 1982 A Fortran Program for the computation of gravimetric quantities from high degree spherical harmonic expansions, Report N.334, Dept.of Geodetic Science and Surveying, The Ohio State University, Columbus, 1982.
- Rapp, H.R. and Pavlis N.K. (1990) The Development and Analysis of Geopotential Coefficient Models to Spherical Harmonic Degree 360, *Journal of Geophysical Research*, Vol.95, N.B13, pp21,885-21,911.
- Sona, G. (1995) Numerical Problems in the Computation of Ellipsoidal Harmonics, *Journal of Geodesy Vol.70* pp117-126.
- Thong, N.C. and Grafarend, E.W. (1989) A Spheroidal Harmonic Model of the Terrestrial Gravitational Field, *Man.Geod.N.14.*, pp285-304.
- Tscherning C.C., Knudsen, P., Forsberg, R. (1991) Description of the GRAVSOFTE package, Geophysical Institute, University of Copenhagen, Technical Report.
- Wenzel, G. (1999) Global Models of the Gravity Field of High and Ultra-High Resolution, Lecture Notes for the International Geoid School, I.Ge.S.

A NOTE ON THE MOLODENSKY G_1 TERM

Ameer Amod

Chief Directorate: Surveys and Mapping
Mowbray, Cape Town, South Africa

Charles L. Merry

Department of Geomatics, University of Cape Town
Rondebosch, Cape Town, South Africa

Abstract

This paper investigates the evaluation of the Molodensky G_1 term using the methods of quadrature, two dimensional (2D) convolution and the 2D fast Fourier transform (FFT). The G_1 term has been evaluated from grid resolutions of 5', 3', 2' and 1' using these techniques for the south western Cape, a coastal province of South Africa. The results of the three approaches do not differ significantly, however the quadrature approach uses an excessive amount of computational time. A comparison between the method of quadrature with respect to 2D convolution and 2D FFT do not show statistics that vary significantly. Grid differencing between the coarser resolution grids and the finer resolution grid of one minute shows that the root mean square (RMS) difference has a tendency to converge as the grid sizes being compared become smaller. Finally, we examine the G_1 contribution to the quasi-geoidal heights and note that the contribution increases as the grid size becomes smaller.

1 INTRODUCTION

Molodensky's first order solution for the height anomaly includes a correction term referred to as the G_1 term. The evaluation of the G_1 term has proved to be demanding on computational time and memory, requiring as it does an extensive numerical integration involving height differences and gravity anomalies. With the implementation of the fast Fourier transform (FFT) in gravity field modelling, the G_1 term can be efficiently evaluated by a two-dimensional (2D) convolution in the spatial domain and by a 2D FFT in the spectral domain [10]. In this paper we evaluate the three approaches (quadrature, 2D convolution and 2D FFT) using a test area in the southwestern Cape province of South Africa - an area of mountainous and rugged topography. The results of the three methods differ significantly in respect of computational time – for a grid resolution of one minute consisting of approximately 44 000 grid points the method of quadrature utilised about 54 hours of computational time as opposed to the convolution and FFT approaches which take less than one minute each [1]. The density of the grid is an important contributor to the computation time; however, too coarse a grid will lead to degraded results. We carry out a further evaluation to determine an optimal grid size.

2 MATHEMATICAL MODELS

Molodensky's first order solution for the height anomaly can be considered to be the integral sum of two parts: Stokes' formula and a correction term:

$$\zeta = \frac{R}{4\pi\gamma} \iint_E \Delta g \cdot S(\psi) d\sigma + \frac{R}{4\pi\gamma} \iint_E G_1 \cdot S(\psi) d\sigma \quad (1)$$

where Δg is the free air gravity anomaly and $S(\psi)$ is Stokes' function.

The G_1 correction term can be expressed on the plane by [10]:

$$G_1 = \frac{R^2}{2\pi} \iint_E \frac{h(x, y) - h(x_p, y_p)}{l^3(x, y)} \Delta g(x, y) dx dy \quad (2)$$

where the horizontal distance l , between the two points is given by:

$$l(x, y) = \begin{cases} [(x_p - x)^2 + (y_p - y)^2]^{1/2} & , \text{ for } x_p \neq x \text{ or } y_p \neq y, \\ 0 & , \text{ for } x_p = x \text{ and } y_p = y \end{cases}$$

$h(x_p, y_p)$ is the normal height of the point of interest, $h(x, y)$ and $\Delta g(x, y)$ represent the normal height and the free air gravity anomaly at the dummy point in the integration.

The practical evaluation of the G_1 term is an exercise in numerical integration (quadrature). The G_1 term can be determined using the expression:

$$G_1 = \frac{1}{2\pi} \sum \frac{h - h_p}{l^3} \Delta g dx dy \quad (3)$$

where $dx = R d\lambda \cos \varphi_p$ and $dy = R d\varphi$, $d\varphi, d\lambda$ correspond to the spatial resolution of the data, which are generally available in gridded form. Numerical integration of (3) can be quite time consuming for areas which are large in extent or where there is a high spatial resolution of the data. The principal limitation is that the summation has to be repeated for each and every point at which the G_1 contribution is required. From a computational point of view this can be numerically intensive, requiring large amounts of memory and computational time.

The availability of gridded height and gravity anomaly data, the development of the FFT algorithm and the convolution form of many geodetic integrals, have led to

attractive alternatives in the computational methods. The integral (2) is a convolution integral and as such can be efficiently evaluated via the FFT. Equation 2 can be expressed in convolution form as follows [10]:

$$G_1 = \frac{1}{2\pi} \left\{ \left[h(x, y) \Delta g(x, y) \right]_p * r(x_p, y_p) - h(x_p, y_p) \left[\Delta g(x_p, y_p) * r(x_p, y_p) \right] \right\} \quad (4)$$

where $*$ denotes the two dimensional convolution operator, and $r(x_p, y_p)$ is given by:

$$r(x_p, y_p) = \begin{cases} \left[(x_p - x)^2 + (y_p - y)^2 \right]^{-3/2} & , \text{ for } x_p \neq x \text{ or } y_p \neq y, \\ 0 & , \text{ for } x_p = x \text{ and } y_p = y \end{cases}$$

Convolution in the space domain can be replaced by multiplication in the frequency domain by simply multiplying the spectra. The G_1 correction term can be evaluated in the frequency domain by:

$$G_1(x, y) = \frac{1}{2\pi} \left[F^{-1} \left\{ F \{ h(x, y) \Delta g(x, y) \} F \{ r(x, y) \} \right\} - h(x, y) F^{-1} \left\{ F \{ \Delta g(x, y) \} F \{ r(x, y) \} \right\} \right] \quad (5)$$

where F and F^{-1} denote the two-dimensional direct and inverse Fourier transforms respectively [10].

When dealing with real data there is usually some type of limited spatial extent. Implementation of the FFT assumes periodicity of the data. If the data are just ‘fed’ into the FFT algorithm, a cyclic convolution will result since the gridded data set is treated as if it has a toroidal geometry. To avoid cyclic convolution a fifty percent pad is generally applied to the data sets, that is, double data length in each dimension. Usually a zero pad is applied. A discontinuity exists where the data transitions into the padded region. This generates a Gibbs phenomenon and a ‘ringing effect’ can be seen around the circumference if a synthesis is done after the FFT analysis [4], [8]. To remove this effect windows are applied to the data which smooth out the existing discontinuities at the boundaries of the data sets, thus eliminating from the spectrum any frequencies which are not really in the data. Windowed data sets are smoothly truncated to zero at the data edges so that the periodic extension of the data becomes continuous. A popular choice for such a window and employed in our analysis is the split cosine filter given by [5]:

$$w(n) = \begin{cases} 1.0 & 0 \leq |n| \leq \alpha \frac{N}{2} \\ 0.5 \left[1.0 + \cos \left(\pi \frac{n - \alpha \frac{N}{2}}{(1 - \alpha) \frac{N}{2}} \right) \right] & \alpha \frac{N}{2} \leq |n| \leq \frac{N}{2} \end{cases} \quad (6)$$

The split cosine filter suppresses the signal content at the edges of the data sets by applying a cosine window at the edges. Data occurring in the middle are multiplied by a value of '1' hence there is no attenuation of the data at the middle of the data sets.

Calculation of the G_1 term requires a dense digital elevation model (DEM), together with free air gravity anomalies at every point of the DEM. This is not always feasible and several authors have suggested that the linear dependence of free air anomalies on elevation can be used to replace the G_1 term with a correction that relies solely on the DEM - the so-called terrain correction [9], [10]:

$$TC = \frac{G\rho}{2} \iint_E \frac{\{h(x,y) - h(x_p, y_p)\}^2}{l^3(x,y)} dx dy \quad (7)$$

As with the G_1 term, the terrain correction can be evaluated using a 2D convolution:

$$TC = \frac{G\rho}{2} \{ h^2(x_p, y_p) * r(x_p, y_p) - 2h(x_p, y_p) [h(x_p, y_p) * r(x_p, y_p)] + h^2(x_p, y_p) [o(x_p, y_p) * r(x_p, y_p)] \} \quad (8)$$

where $o(x_p, y_p) = 1$ for all grid points.

The terrain correction is always positive (G_1 can be positive or negative) and is only an approximation to the G_1 term, as it relies upon a linear relationship between free air anomalies and elevation. We prefer to use the G_1 term, but have included a calculation of the terrain correction in this paper, for the sake of completeness.

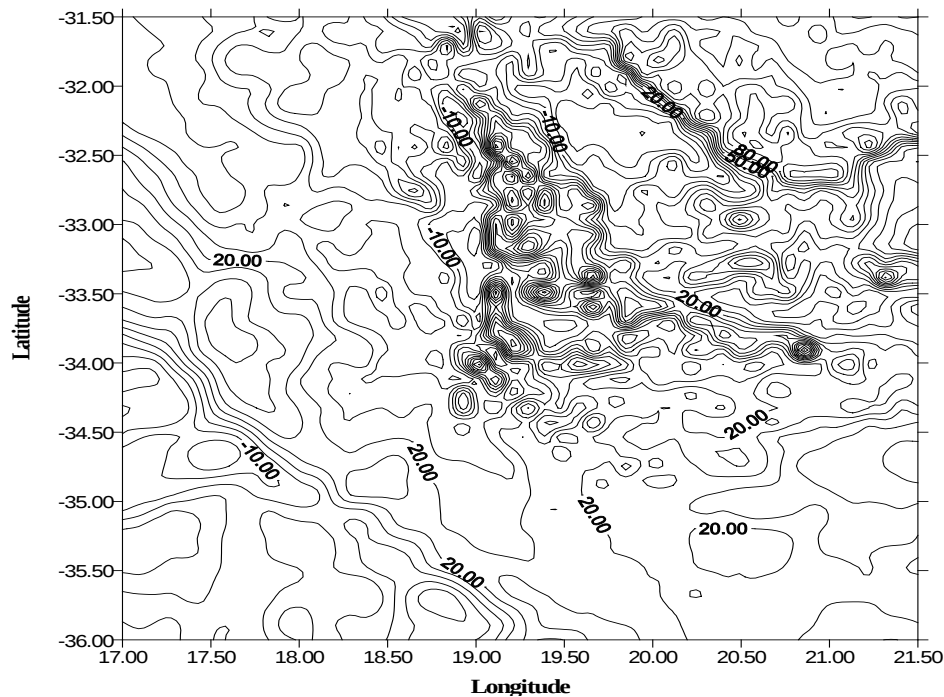
3 DESCRIPTION OF DATA

The computation of the G_1 term requires a consistent data set of free air gravity anomalies and a concomitant DEM. Free-air gravity and elevation data are required on both land and sea as the test area is a coastal province. The land point gravity data are taken from the University of Cape Town's gravity database [12] which has been compiled from a variety of sources over many years, and has been substantially improved by data provided by the South African Council for Geoscience in 1997.

The data are irregularly spaced and some interpolation is required to obtain a consistently gridded data set. The interpolation process simply extracts the Bouguer anomalies of a particular test area and this scattered data is interpolated onto a one minute grid using the Kriging method contained in the software package *Surfer* [6]. Bouguer anomalies are used for the interpolation as they vary smoothly and are thus easier to interpolate. The Bouguer anomalies are converted to free air anomalies using the Bouguer gradient of 0.1119 milligal per metre (mgal/m) and a one-minute DEM. The DEM was compiled from a series of 1:50 000 topographic maps via manual interpolation [7].

Sea gravity data are not always available and are often unreliable and free air gravity anomalies predicted from satellite altimetry were used instead. A data set created using the ERS-1 and Geosat satellite altimeters was obtained from the Danish National Survey and Cadastre [2]. These data, which are on a 3.75' grid, were re-sampled onto a 1' grid using Kriging.

In order to evaluate the effect of using grids of varying density, grids of size five, three and two minutes were obtained by decimating the original one minute grid. Figures 1 and 2 below are plots of the free air anomalies and the DEM of the test area, while Table 1 gives an indication of the statistics for the one-minute gridded data.



**Figure 1: Free Air Gravity anomalies – south western Cape
(Contour Interval: 10mgal)**

	FREE AIR ANOMALIES (mgal)	HEIGHTS (m)
Maximum	161.54	2180.00
Minimum	-47.82	0.00
Mean	18.05	409.51
Standard Deviation	25.67	533.54

**Table 1: Statistics of the Free air anomaly and Height
data for the south western Cape**

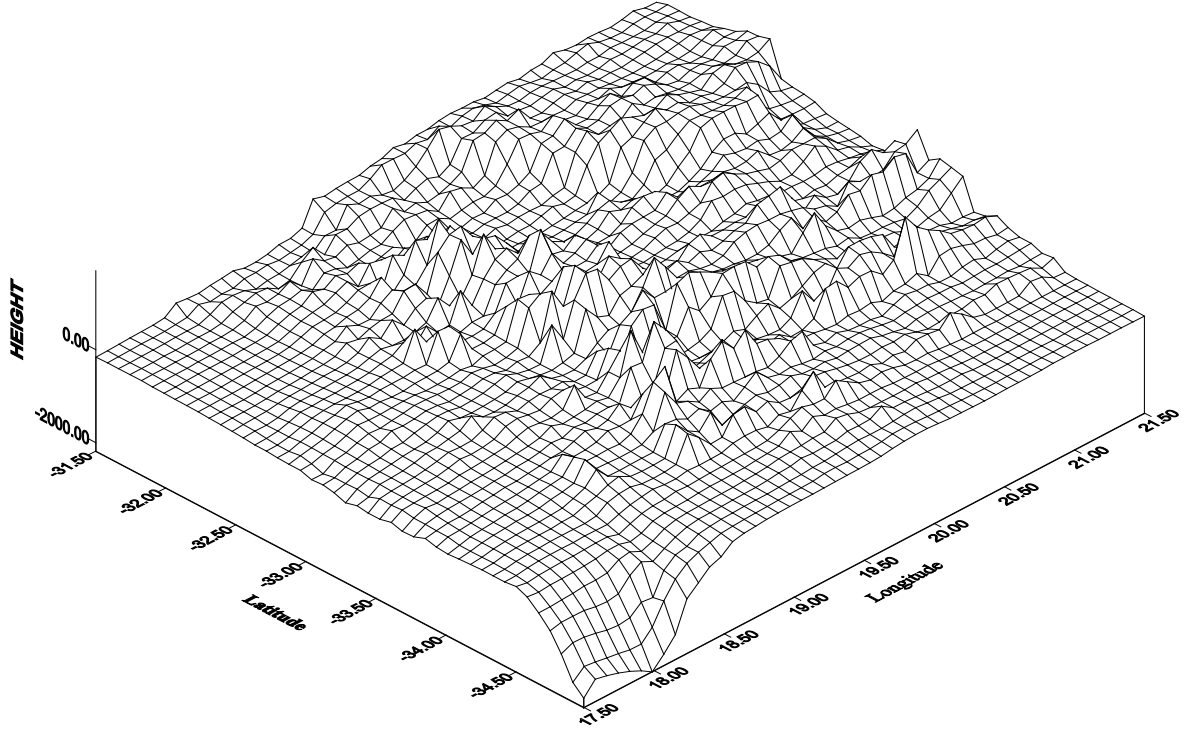


Figure 2: Digital Elevation Model – South Western Cape

4 NUMERICAL INVESTIGATIONS

4.1 Summary of Results

The G_1 term was computed using the techniques of quadrature, 2D convolution and 2D FFT for all four grid sizes (1', 2', 3', 5').

For the quadrature approach (equation 3), the numerical integration was implemented using the *MATLAB* software package [3]. The integration was extended to a distance of 10km only, as the contribution of the distant zones falls away rapidly.

Equation 4 was used to determine the G_1 term for the 2D convolution approach. The two-dimensional convolution operator, `conv2`, available in the *MATLAB* software package was used to evaluate this expression. The advantage of the `conv2` function is that it automatically applies zero padding to the periphery of the data sets, effectively increasing the grid sizes from (p, q) to (2p-1, 2q-1) [3]. As with the quadrature method, the convolution kernel was set up in such a way as to only use data points within 10km of the computation point.

The implementation of equation 5 uses the *MATLAB* two-dimensional FFT operator `fft2`. To reduce discontinuities at the edge of the data sets, a split cosine taper was

applied to the gridded data sets of heights and free air anomalies. As in the convolution approach, zero padding was applied to the periphery of the data sets. To maintain consistent dimensions for the purpose of the matrix operations involved, the entire kernel function was used in the evaluation of the G_1 contribution for each grid point and not just the points within 10km of the computational point.

We now examine the G_1 results for the varying grid sizes for which the G_1 term was computed. These results are summarised in Table 2. These statistics are computed after 10% of the data along the circumference of the border were removed, to reduce aliasing effects. Computations were executed on a WINDOWS NT platform with 64 Megabytes of random access memory (RAM) on a Pentium II with a 466 Megahertz processor.

GRID DIMENSION 45 x 31 (1 395 data points)					
AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 5'$					
	MAX. (mgals)	MIN. (mgals)	MEAN (mgals)	STD DEV. (mgals)	COMP. TIME (in seconds)
Quadrature	6.235	-4.284	0.516	1.008	166.31
Convolution	6.270	-4.335	0.530	1.020	0.27
FFT	6.553	-4.388	0.693	1.085	1.16
GRID DIMENSION 75 x 51 (3 825 data points)					
AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 3'$					
Quadrature	15.870	-8.067	0.568	1.611	1 192
Convolution	16.746	-8.301	0.630	1.701	0.73
FFT	17.601	-9.245	1.021	1.961	2.29
GRID DIMENSION 111 x 75 (8 325 data points)					
AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 2'$					
Quadrature	21.545	-13.843	0.836	2.355	6 169
Convolution	21.545	-13.942	0.863	2.385	1.82
FFT	22.184	-15.131	1.281	2.653	4.53
GRID DIMENSION 181 x 241 (43 621 data points)					
AREA $-34.5^0 < \varphi < -31.5^0$, $17.5^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 1'$					
Quadrature	40.720	-36.883	0.985	3.671	187 200
Convolution	40.363	-36.750	1.012	3.711	13.89
FFT	40.838	-38.874	1.453	3.988	45.91

Table 2: Summary of the G_1 term in respect of Evaluation Methods

Examining Table 2, we see that each computational method reveals similar values for the maximum, minimum, mean and standard deviation for the different grid sizes. The maximum value for the G_1 contribution increases (Figure 3) while the minimum

value decreases (Figure 4) as the data sets become more dense. This is indicative of the nature of the G_1 term which in principle mirrors topography. For grids with a low spatial resolution, where rugged and mountainous details of the topography are not well defined we would expect the magnitudes of the maximum and minimum values of the G_1 contribution to be smaller than for grid resolutions where the features of topography are well depicted.

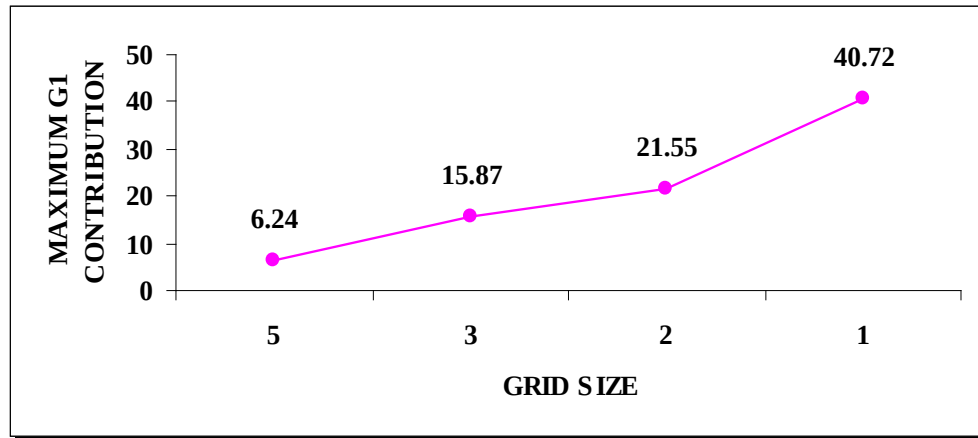


Figure 3: Maximum G_1 Contribution (mgals)

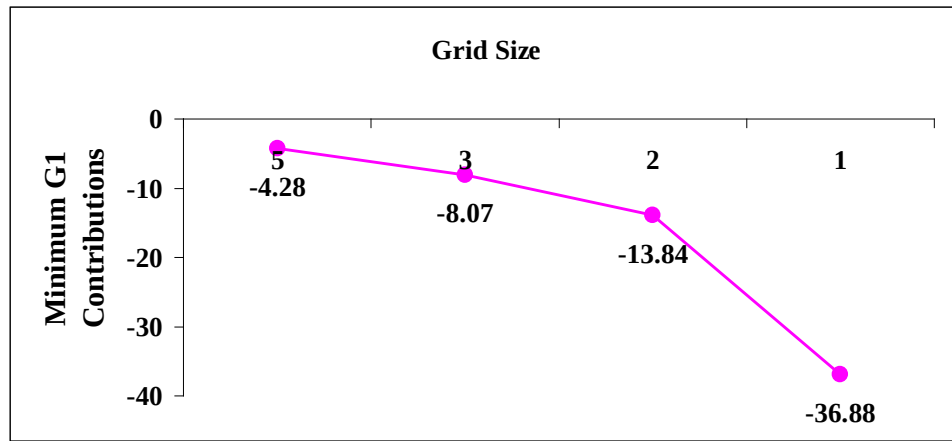


Figure 4: Minimum G_1 Contribution (mgals)

Contour plots of the G_1 contribution for a grid size of one minute evaluated using the methods of quadrature, convolution and FFT are shown in Figures 5, 6 and 7. The plots have been generated using the *Surfer* software [6]. We see that the each method of computation reveals similar plots for the G_1 contribution. The contour plot of the FFT method shows some of the edge effects along the north-eastern border. The plots

further show the correlation of the G_1 term with topography. In areas of rugged and mountainous topography (see Figure 2) we see that the G_1 contribution is quite significant.

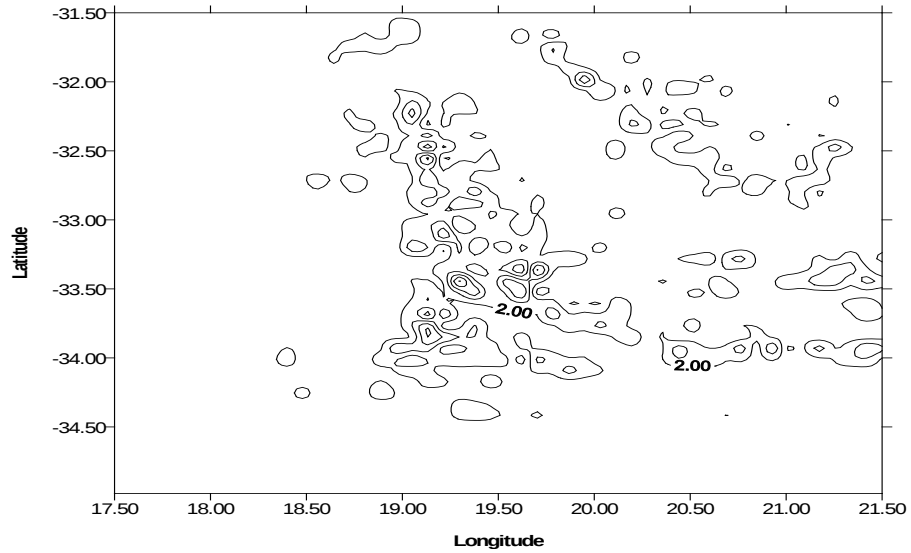


Figure 5: G_1 term via Quadrature (contour interval: 5mgal)

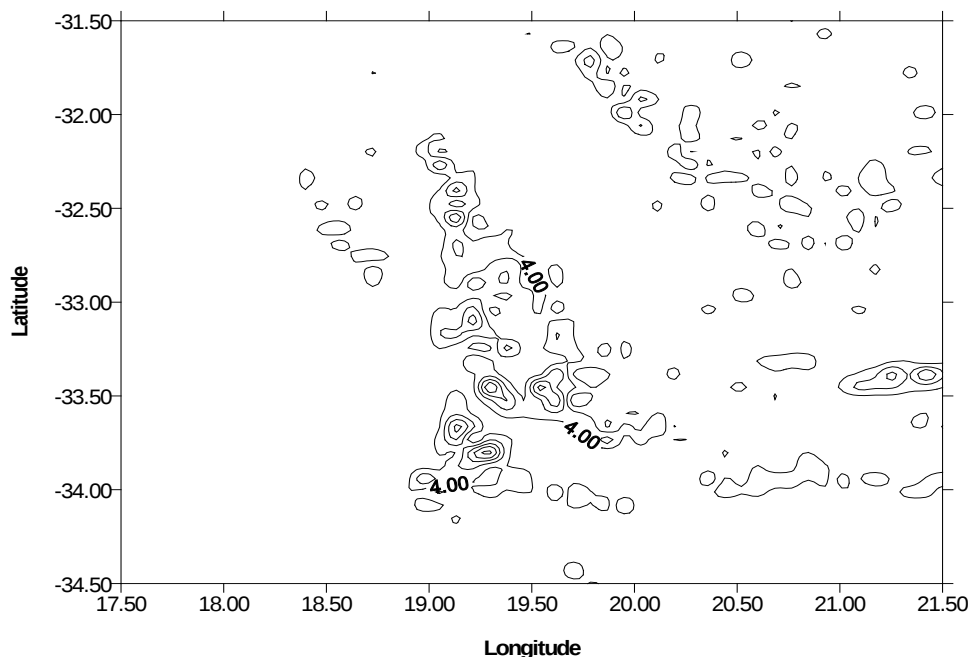


Figure 6: G_1 term via Convolution (contour interval: 5mgal)

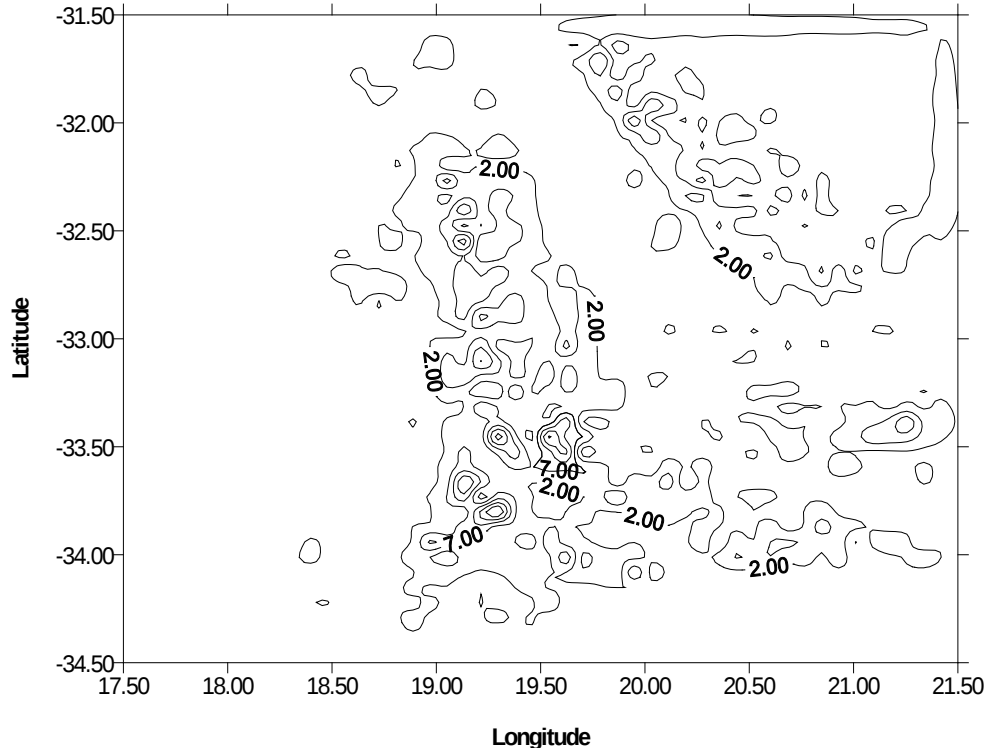


Figure 7: G_1 term via FFT (contour interval: 5mgal)

Numerical integration is demanding in terms of the computational time required for the evaluation of the G_1 term. For a small grid consisting of 1 395 data points (five minute grid), the evaluation of the G_1 contribution by quadrature takes approximately three minutes while the methods of convolution and FFT require about 30 and 60 seconds respectively.

For high resolution grids the time requirement is even more demanding. On a one minute grid consisting of approximately 44 000 data points the quadrature method utilised about 52 hours of computational time for the evaluation of the G_1 term while the convolution and FFT methods took under a minute for the same computation. A further simulation (not shown in Table 2) was carried out on a $\frac{1}{4}$ minute grid consisting of approximately 62 000 points. The convolution and FFT methods utilised 43 and 90 seconds of computational time respectively, while the quadrature approach took a startling 96 hours! The relative times (Pentium II, 466MHz processor) for the one minute grid are shown in Figure 8.

It would appear from the figure that the 2D convolution is more efficient than the FFT. However, it must be remembered that the convolution uses a kernel that extends over a radius of 10km, while the FFT solution uses all data points. In order to assess the comparative efficiency of the two methods, the computation using convolution on a one-minute grid was repeated using a fully-sized kernel. The computation time increases from 14 seconds to 300 seconds, significantly greater than the 46 seconds taken by the FFT method. This result highlights the computational efficiency of using FFT methods in gravity field modeling.

The 10km radius was implemented in order to improve computation times for the quadrature and convolution methods. It is as well to check that this truncation does not introduce serious errors in the final result. On the same one-minute grid, comprising some 44000 points, the RMS difference in the G_1 term between the result using a full kernel and a 10km radius kernel is 0.6mgal, with a maximum discrepancy of 2.9mgal. These discrepancies are not significant in the context of height anomaly computation, and it makes computational sense to restrict the convolution kernel to a limited radius.

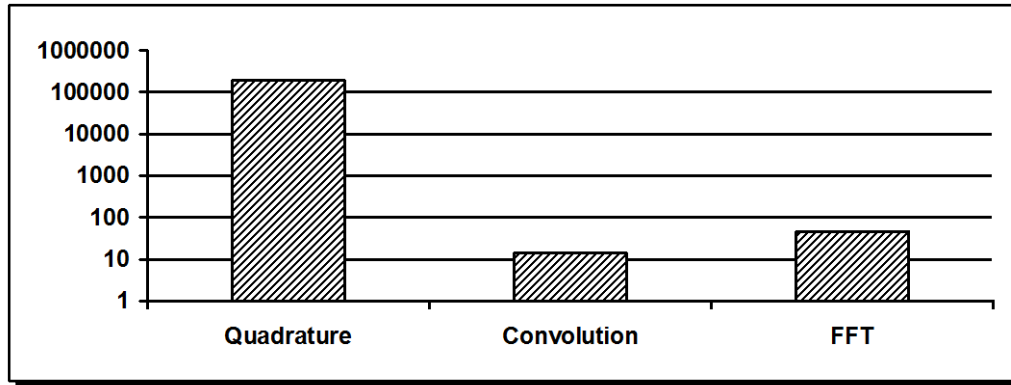


Figure 8: Computational Time (seconds)

4.2 Comparison of Results

In the previous section we summarised the results obtained using the different methods and compared their computational efficiency. In this section we compare the results, using the quadrature method as a reference. The differences are summarised in Tables 3 and 4 and Figures 9 and 10. It is apparent that there is a general tendency for the magnitudes of the maximum, minimum, mean and root mean square values to increase from the low resolution grid of five minutes to the higher resolution grid of one minute. This is primarily due to the higher resolution grids reflecting more details in the topography.

The convolution approach produces results that are much more consistent with the results of the quadrature method than are the results obtained using FFT. In particular, note the plot of the RMS differences in Figures 9 and 10. The RMS discrepancy of the FFT method is approximately ten times larger than that of the convolution approach. This is possibly due to the effects of enforced periodicity, spectral leakage and aliasing errors associated with the Fourier transform. These sources of error cannot be completely eliminated but are significantly reduced by applying edge tapering windows as well as zero padding along the periphery of the data sets. However, it should also be remembered that the quadrature and convolution approaches use a 10km distance kernel, and that part of the discrepancy could be due to the neglect of the distant zones in these two methods.

GRID DIMENSION 45 x 31 (1 395 data points) AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 5'$				
	MAX. (mgals)	MIN. (mgals)	MEAN (mgals)	RMS (mgals)
Quadrature – Convolution	0.061	-0.185	-0.014	0.028
GRID DIMENSION 75 x 51 (3 825 data points) AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 3'$				
Quadrature – Convolution	0.523	-0.904	-0.062	0.1542
GRID DIMENSION 111 x 75 (8 325 data points) AREA $-34.5^0 < \varphi < -32.0^0$, $17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 2'$				
Quadrature – Convolution	0.278	-0.462	-0.027	0.066
GRID DIMENSION 181 x 241 (43 621 data points) AREA $-34.5^0 < \varphi < -31.5^0$, $17.5^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 1'$				
Quadrature – Convolution	0.814	-0.625	-0.035	0.092

Table 3: Summary of the Differences between the Quadrature and Convolution Evaluations of the G_1 term

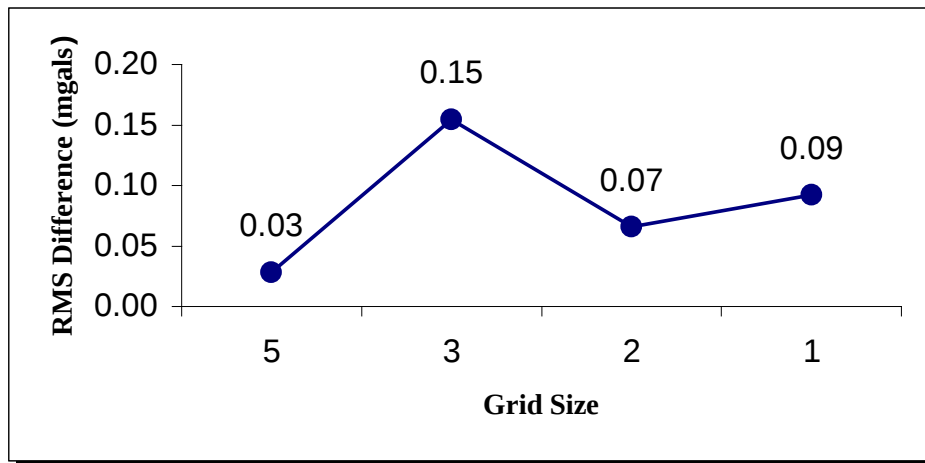


Figure 9: RMS Discrepancy between the Quadrature and Convolution Evaluations of the G_1 term

GRID DIMENSION 45 x 31 (1 395 data points) AREA $-34.5^0 < \varphi < -32.0^0, 17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 5'$				
	MAX. (mgals)	MIN. (mgals)	MEAN (mgals)	RMS (mgals)
Quadrature – FFT	0.280	-0.715	-0.177	0.239
GRID DIMENSION 75 x 51 (3 825 data points) AREA $-34.5^0 < \varphi < -32.0^0, 17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 3'$				
Quadrature – FFT	1.685	-2.587	-0.453	0.699
GRID DIMENSION 111 x 75 (8 325 data points) AREA $-34.5^0 < \varphi < -32.0^0, 17.8^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 2'$				
Quadrature – FFT	2.423	-2.530	-0.446	0.673
GRID DIMENSION 181 x 241 (43 621 data points) AREA $-34.5^0 < \varphi < -31.5^0, 17.5^0 < \lambda < 21.5^0$ $\Delta\varphi, \Delta\lambda = 1'$				
Quadrature – FFT	3.210	-3.236	-0.605	0.765

Table 4: Summary of the Differences between the Quadrature and FFT Evaluations of the G_1 term

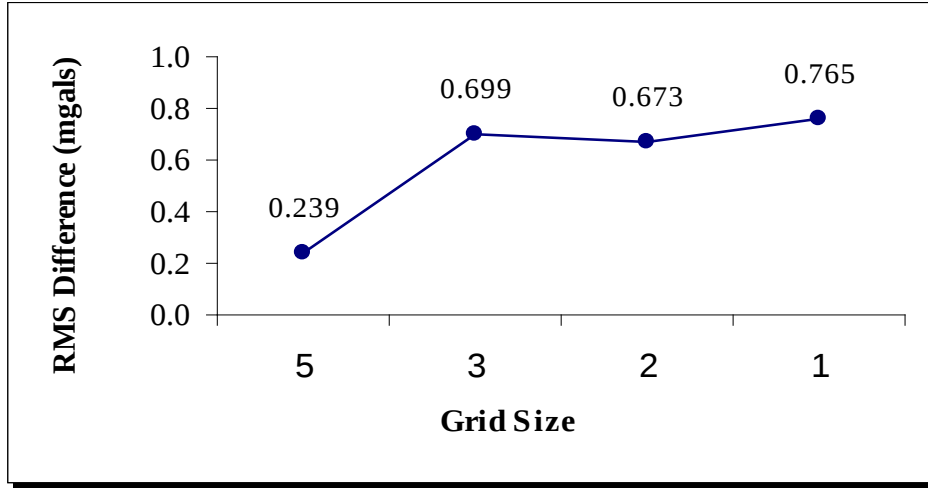


Figure 10: RMS Discrepancy between the Quadrature and FFT Evaluations of the G_1 term

4.3 Influence of Grid Size

The evaluation of the integration inherent in equation 2 requires some form of approximation, whichever method is used. This approximation takes the form of substituting gridded gravity anomalies and height data for their continuous equivalents. Obviously the finer the mesh the closer the approximation. We have used a range of grid sizes in this analysis, from five minutes down to one minute. Although we did carry out an evaluation using a $\frac{1}{4}$ minute grid, the area covered by this grid is too small to provide us with reliable statistics. Nevertheless, we would expect an improvement in accuracy of the computation of the G_1 term as we increase the resolution of the grid.

We have compared the results obtained using the 5', 3' and 2' grids with those obtained using the one-minute grid. The results of this comparison are summarised in Table 6 and Figure 11.

Grid Size Comparison	1' $\leftrightarrow$ 5'	1' $\leftrightarrow$ 3'	1' $\leftrightarrow$ 2'
No. of Common Points	1395	3825	8325
MAX.	30.060	30.880	29.180
MIN.	-28.820	-22.100	-17.520
MEAN	0.840	0.567	0.408
RMS	3.444	3.041	2.471

Table 6: Differences in the G_1 term between 1' and coarser grids (milligals)

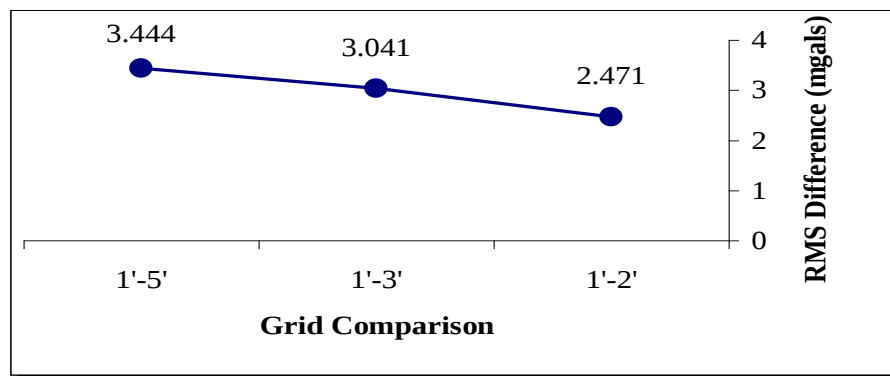


Figure 11: RMS Discrepancies of G_1 term between a 1' grid and 2', 3' and 5' grids

As expected, there is a significant reduction in the mean and RMS discrepancies as the grid size is decreased. However, large individual discrepancies still exist, and it would appear that a grid size even smaller than 1' would be desirable. We have chosen

a particularly rugged area in which to assess the G_1 term, and it is likely that in areas of smoother topography a one-minute grid size would be adequate. This is of practical significance to the determination of the G_1 term in less developed parts of the world, such as Africa. In many African countries a detailed DEM with a sub-minute resolution is just not available; furthermore the computer facilities required to generate a model of the G_1 term using a high resolution grid are also not available.

4.4 Terrain Correction

As mentioned earlier, it has been suggested that the terrain correction, TC (equation 8), could be used as a substitute for the G_1 term, under the assumption that free air gravity anomalies are perfectly correlated with elevation. We have used equation 8 with the same test data on a one minute grid, and the comparison of these results with the equivalent G_1 values from the convolution approach is summarised in Table 7.

	DIFFERENCE (G_1 minus TC)
Maximum	28.4
Minimum	-71.7
Mean	0.4
RMS Difference	4.3

**Table 7: Difference between G_1 term and Terrain Correction
(one minute grid - mgals)**

Although there are some large discrepancies between the two sets of results, the RMS difference is reasonably small. Considering that the terrain correction is always positive and that the G_1 term can take on positive and negative values, the bias (0.4mgals) between them is, at first glance, surprisingly small. It should be borne in mind, however, that the free air anomalies used to compute the G_1 term are themselves computed from interpolated Bouguer anomalies, using a linear gradient with elevation. Hence they bear, *ipso facto*, a strong correlation with elevation, which is the underlying assumption in the derivation of the terrain correction. We should not be surprised that the G_1 and TC values computed by us are similar. Nevertheless these figures do confirm that the terrain correction is generally smaller than the G_1 term, as was also noticed in [10]. We will discuss the impact of using the terrain correction instead of the G_1 term for computing height anomalies in the following section.

4.5 G_1 Contribution to the Quasi-Geoid

The purpose of computing a G_1 term is to produce a refined model of the quasi-geoid. Consequently it is appropriate to investigate the significance of this term in the determination of the quasi-geoid. We have evaluated the second term of equation 1 using a modification of the planar 2D convolution representation from [11]:

$$\zeta_{G_1} = \frac{1}{2\pi\gamma} \left\{ \frac{1}{l} * G_1 \right\} \quad (9)$$

A summary of the contribution to the quasi-geoid is presented in Table 8, for varying grid sizes used in computing G_1 . For a grid with a spatial resolution of 5' the contribution is as much as 15.5cm while the densest grid resolution of 1' yields a maximum contribution of 42.5cm, with an RMS value of 7cm. A graphical illustration (Figure 12) of these results clearly indicate that the statistics of the G_1 contribution show a tendency to increase as the grid resolution becomes higher. This highlights the importance of the G_1 term, especially in modelling high frequency components of the gravity field.

GRID RESOLUTION (minutes)	MAX. (m)	MIN. (m)	MEAN (m)	RMS. (m)
5	0.155	0.029	0.095	0.029
3	0.250	0.045	0.134	0.040
2	0.317	0.057	0.164	0.048
1	0.425	0.055	0.212	0.072

Table 8: Statistical Analysis of the G_1 Contribution to the Quasi-Geoid

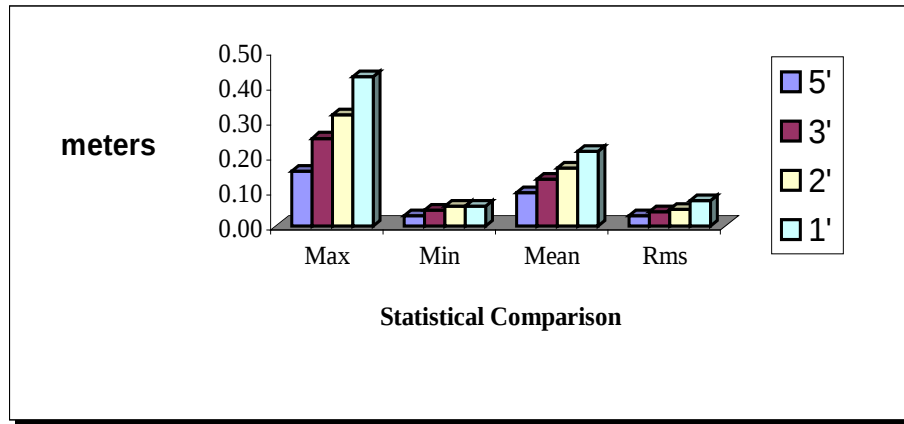


Figure 12: Graphical Illustration of G_1 Contribution to the Quasi-Geoid

We have also determined the contribution of the terrain correction towards the quasi-geoid, replacing G_1 in equation 9 with the terrain correction, TC, described in section 4.4. Very little difference was obtained. The maximum difference is 9cm, with an RMS difference of 2cm. Generally, the TC contribution is smaller than the corresponding G_1 contribution, by an average of 5cm. A comparison with GPS/levelling data for 42 points in the southwest part of the test area shows no significant difference between a quasi-geoid modelled using the G_1 term and one using the TC - both fit the GPS/levelling data with an RMS of 4cm. This is a substantial improvement on a quasi-geoid model using only the first term of

equation 1, where the RMS fit to the GPS/levelling data is only 10cm [1]. This is in sharp contrast to the results of other investigators who have found that the use of the terrain correction in South Africa degrades the results [13].

5 CONCLUSIONS

The G_1 contributions computed using the methods of quadrature, convolution and FFT do not differ significantly. There is close agreement between the quadrature and convolution results, while the FFT results do not agree quite as well. However, this discrepancy (maximum of 3mgals, RMS of 1mgal) is not significant in the context of its contribution to the quasi-geoid.

The differences in computation speed are significant. The quadrature approach is much slower, taking more than 3000 times longer than the convolution and FFT approaches. In the FFT approach the entire distance kernel was used for the computation of the G_1 contribution of a grid point while for the quadrature and convolution methods only points within a radius of 10km of the grid point were considered. Under these conditions the convolution approach is more than three times faster than the FFT method. Where the full grid is used, the FFT approach is faster, by a factor of six. However, the convolution approach, with a limited kernel, provides results that are consistent with the others (RMS discrepancy less than 1mgal). Hence convolution, with a limited kernel, provides the most efficient method of evaluating the G_1 term.

The G_1 term is by nature very sensitive to topography and to grid size. For a coarse resolution grid of five minutes the maximum G_1 contribution is approximately 6mgals while a grid resolution of one minute yields a maximum contribution of 41mgals. As the grid size is decreased the RMS difference in the results (with respect to that for the smallest grid size) reduces in magnitude. Optimally, this RMS difference would reduce to near zero for very small grid sizes. The test area we chose contains rugged and mountainous topography, and it appears that a resolution of better than one minute is required to model the G_1 term in this area. For regions of smoother terrain a one-minute grid size would be adequate.

The terrain correction can be used as an approximation to the G_1 term. Our evaluation showed that although there are some significant individual differences between the two terms, the RMS difference is small (4mgals), with the terrain correction generally being smaller in magnitude than the G_1 term. There is no significant difference in their contribution to the quasi-geoid.

The contribution of the G_1 term to the quasi-geoid is not insignificant and in the test region this contribution reached 42cm, with an RMS contribution of 7cm. Furthermore the use of the G_1 term substantially improved the fit of the quasi-geoid to GPS/levelling data, reducing the RMS discrepancy from 10cm to 4cm.

6. ACKNOWLEDGEMENTS

The gravity anomaly data used for this work are a subset of a compilation of data from various sources. The major source is the South African Council for Geoscience, and additional land data were contributed by the University of Cape Town (UCT). Marine gravity anomaly data were provided by the Danish National Survey and Cadastre. The one-minute digital elevation model (DEM) data were compiled at UCT from maps of the Chief Directorate of Surveys and Mapping (CDSM). Additional $\frac{1}{4}$ minute data were interpolated from the 400m DEM of the CDSM.

This research was carried out while the first author was in the employ of the CDSM and their support is gratefully acknowledged. Financial support for the second author was provided by UCT and the South African National Science Foundation.

7. REFERENCES

- [1] Amod, A (2001), "The Use of Spectral Techniques for Quasi-Geoid Modelling", MSc. (Eng) thesis, University of Cape Town.
- [2] Andersen, O B & Knudsen, P (1998), "Global Marine Gravity Field Data from the ERS-1 and Geosat Geodetic Mission Altimetry", *Journal of Geophysical Research*, 103(C4):8129.
- [3] Anonymous (1997), *MATLAB – the language of technical computing, version 5*, The Mathworks, Inc., Natick, Mass., USA.
- [4] Bracewell, R (1978), *The Fourier Transform and Its Applications*, 2nd. Edn, McGraw-Hill, New York.
- [5] Harris, F J (1978): "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", *Proceedings of the IEEE*, **66**(1).
- [6] Keckler, D (1995), *Surfer for Windows Users Guide*, Golden Software, Golden, Colorado, USA.
- [7] Lamb, A D, Malan, O G & Merry C L (1987), "Application of Image Processing Techniques to Southern African Digital Elevation Models", *South African Journal of Science*, **83**:43-47.
- [8] Milbert, D G (1999), National Geodetic Survey, *Personal Communication*.
- [9] Moritz, H (1980), *Advanced Physical Geodesy*, Herbert Wichmann Verlag, Karlsruhe.
- [10] Sideris, M G and Schwarz K P (1986), "Solving Molodensky's Series by Fast Fourier Transform Techniques", *Bulletin Geodesique*, **60**: 51-63.

- [11] Strang van Hees, G (1990), "Stokes formula using Fast Fourier Techniques", *Manuscripta Geodaetica*, **15**: 235-239.
- [12] van Gysen, H G and Merry C L (1987), "The Gravity Field in Southern Africa", University of Cape Town Department of Surveying Technical Report TR-3.
- [13] van Gysen, H G (1991): "Thin-Plate Spline Quadrature of the inner zone contribution to Stokes' formula", Presented, IAG Symposium on Geoid determination Techniques, XX IUGG General Assembly, Vienna.

A Simple Method to Improve the Geoid from a Global Geopotential Model (or Coarse Geoid Estimation Using Only the Innermost Zone Contribution of Stokes's Formula)

W E Featherstone

*Dept of Spatial Sciences, Curtin University of Technology, GPO Box U1987, Perth WA 6845, Australia;
Tel: +61 8 9266 2734, Fax: +61 8 9266 2703, E-mail: W.Featherstone@curtin.edu.au*

Abstract

This paper presents the results of curiosity-driven experiments to determine how well, if at all, only the innermost zone contribution of Stokes's formula can model the geoid for GPS height transformation. Terrestrial gravity and terrain data are used over Australia, and the results compared with GPS-derived ellipsoidal and Australian Height Datum heights. The results are largely as expected in that the innermost zone contribution to the total geoid height is small. While this approach does not improve upon a regional gravimetric geoid model, it does make a small improvement upon a global geopotential model. Therefore, in areas where regional gravity and terrain data are limited, some small improvements may be made to existing global geopotential models by calculating the innermost zone contribution from localised gravity and terrain data. This is an extraordinarily simple calculation. It may prove to be of value to those who do not have access to regional gravity data (i.e., due to restricted field access or confidentiality) or resources (i.e., personnel, software and high-powered computers) to compile a locally improved geoid model. Accordingly, the method can provide an interim geoid model until the issues of data availability and resources have been resolved to allow the compilation of a regional gravimetric geoid model.

1. Introduction

As is well known, a gravimetric geoid model should be computed using a global integration of terrestrial gravity anomalies via Stokes's formula. This requirement has been alleviated to a large extent by using a high-degree global geopotential model, which provides much of the contribution of the remote zones to the geoid. The contribution of the inner zones to the geoid is provided by a regional integration of terrestrial gravity data in an adapted form of Stokes's integral. Over the last forty years, considerable effort has been directed towards optimally combining a global geopotential model with terrestrial gravity and terrain data over a region.

This paper presents the results of curiosity-driven experiments where the geoid is very coarsely approximated by only the innermost zone contribution of Stokes's formula and the quadratic term of the primary indirect effect. Three specific approaches are considered as follows:

- (1) the innermost zone contribution from terrestrial gravity anomalies only;
- (2) the innermost zone contribution from residual gravity anomalies based on a satellite-only global geopotential model; and
- (3) the innermost zone contribution from residual gravity anomalies based on a combined global geopotential model.

These three coarse approximations will be compared with an Australia-wide set of 1013 co-located GPS-derived ellipsoidal heights and Australian Height Datum (AHD) heights

(Featherstone and Guo 2001), the EGM96S satellite-only and EGM96 combined global geopotential models (Lemoine *et al.* 1998), and the AUSGeoid98 regional gravimetric geoid model of Australia (Featherstone *et al.* 2001). This will give an empirical indication of the relative performance of these approximations of the geoid for the transformation of GPS-derived heights to orthometric heights.

Another motivation for these experiments comes from the many examples where GPS users do not have access to a regional gravimetric geoid model, but do have access to terrestrial gravity data over very limited and/or localised areas (due to restricted field access or confidentiality). In addition, these data may not have been used to construct combined global geopotential models. Moreover, the GPS users may not have the resources (i.e., personnel, software and computers) or access to homogeneous regional gravity data coverage with which to compute a regional gravimetric geoid model. An example of the latter occurs in South East Asia where bordering countries rarely exchange gravity data to allow a regional geoid model to be computed properly. This raises the question as to how well a geoid model can be constructed from a very restricted amount of gravity and terrain data for the transformation of GPS-derived ellipsoidal heights to orthometric heights. This very approximate geoid model from only the innermost zone can be adopted as an interim geoid model until the data and resources become available to properly compile a regional gravimetric geoid model.

2. The Techniques and Their Limitations

It is essential to point out that a geoid model coarsely approximated from only terrestrial gravity data in the innermost zone is theoretically incorrect since it violates the primary requirement for a proper solution of the geodetic boundary-value problem. Essentially, the innermost zone contribution can be conceptualised as a highly truncated form of Stokes's formula, with a correspondingly large truncation error. However, a proportion of this truncation bias can be provided by a global geopotential model, as will be discussed below.

Due to the singularity in Stokes's kernel (when expressed in terms of spherical polar coordinates centred at the computation point), alternative methods are required to practically evaluate Stokes's formula in the innermost zone (eg. Heiskanen and Moritz, 1967, p.120-123). Importantly, however, if the mean gravity anomaly in the innermost zone is small, its contribution to the geoid will also be small. Conversely, in areas outside the innermost zone where the mean gravity anomalies may be large, these will make no contribution to the geoid approximated by only the innermost zone. Accordingly, the geoid approximation from the innermost zone is generally expected to be small.

2.1 The innermost zone and terrestrial gravity anomalies

The first mathematical model experimented with uses an innermost zone contribution evaluated using only the mean terrestrial gravity anomaly (i.e., one referred to the GRS80 ellipsoid) in a square element. Here, the very coarsely approximated geoid height is

$$N_1 = \frac{2a \ln(\sqrt{2} + 1)}{\pi \gamma} \Delta g - \frac{\pi G \rho H^2}{\gamma} \quad (1)$$

where the first term on the right-hand-side is the contribution of the innermost zone to the geoid as given by Haagmans *et al.* (1993, appendix), which is more rigorous for a square

element than the spherical-cap approximation given by Heiskanen and Moritz (1967, p.120-123). In Eq. (1), a is the side-length (in metres) of the square element comprising the innermost zone, Δg is the mean Faye (i.e., terrain-corrected, free-air as an approximation of the Helmert) terrestrial gravity anomaly, and γ is normal gravity on the surface of the normal ellipsoid. The second term on the right-hand-side of Eq. (1) is the quadratic approximation of the primary indirect effect on the geoid (Wichiencharoen, 1982), in which G is the Universal gravitational constant, ρ is the topographic mass-density (assumed constant), and H is the orthometric height of the computation point. This term will also be used for the subsequent geoid approximations.

2.2 A satellite-only global geopotential model and residual gravity anomalies

In this mathematical model, a satellite-only global geopotential model is used to contribute the very long-wavelength components of the geoid. It is also used to compute the gravity anomaly in the innermost zone, which is subtracted from the terrestrial gravity anomaly to yield the residual gravity anomaly. This is necessary to avoid the innermost zone contribution from adding [some of] the long-wavelength components to the geoid solution twice. The residual gravity anomaly is used to compute the innermost zone contribution to the geoid, which is then added to the geoid height implied by the global geopotential model. This is analogous with the remove-compute-restore approach to regional geoid computation. Accordingly, the very coarsely approximated geoid height is given by

$$N_2 = N_{SGGM} + \frac{2a \ln(\sqrt{2} + 1)}{\pi \gamma} (\Delta g - \Delta g_{SGGM}) - \frac{\pi G \rho H^2}{\gamma} \quad (2)$$

where N_{SGGM} is the low-frequency contribution to the geoid from the satellite-only global geopotential model, and Δg_{SGGM} are the gravity anomalies implied by the same degree of expansion of the same global geopotential model. The benefit of using this remove-compute-restore-type approach is that the global geopotential model provides a large proportion of the large truncation bias that results when using only Eq. (1).

2.3 A combined global geopotential model and residual gravity anomalies

In the third mathematical model experimented with, the very coarsely approximated geoid height is given by

$$N_3 = N_{CGGM} + \frac{2a \ln(\sqrt{2} + 1)}{\pi \gamma} (\Delta g - \Delta g_{CGGM}) - \frac{\pi G \rho H^2}{\gamma} \quad (3)$$

where N_{CGGM} is the geoid contribution of a combined global geopotential model and Δg_{CGGM} is the gravity anomaly implied by the same degree of expansion of the same model. The argument presented above regarding the truncation bias is now more valid because the increased degree of expansion of the combined global geopotential model (compared to the satellite-only model) provides a larger proportion of the truncation bias. Accordingly, this mathematical model is expected to deliver the best results. However, this is contingent on terrestrial gravity data in and around the area of interest having been used to construct the combined global geopotential model.

3. Data and Results

The experiments were conducted over Australia, where a reasonably dense coverage of regional terrestrial gravity, terrain and GPS-levelling data are available.

- The Australian Surveying and Land Information Group (now the National Mapping Division of Geoscience Australia) supplied the GPS and spirit levelling data at 1013 points covering most of Australia (Figure 1; Featherstone and Guo, 2001). The spirit levelling data refer to the Australian Height Datum (AHD).

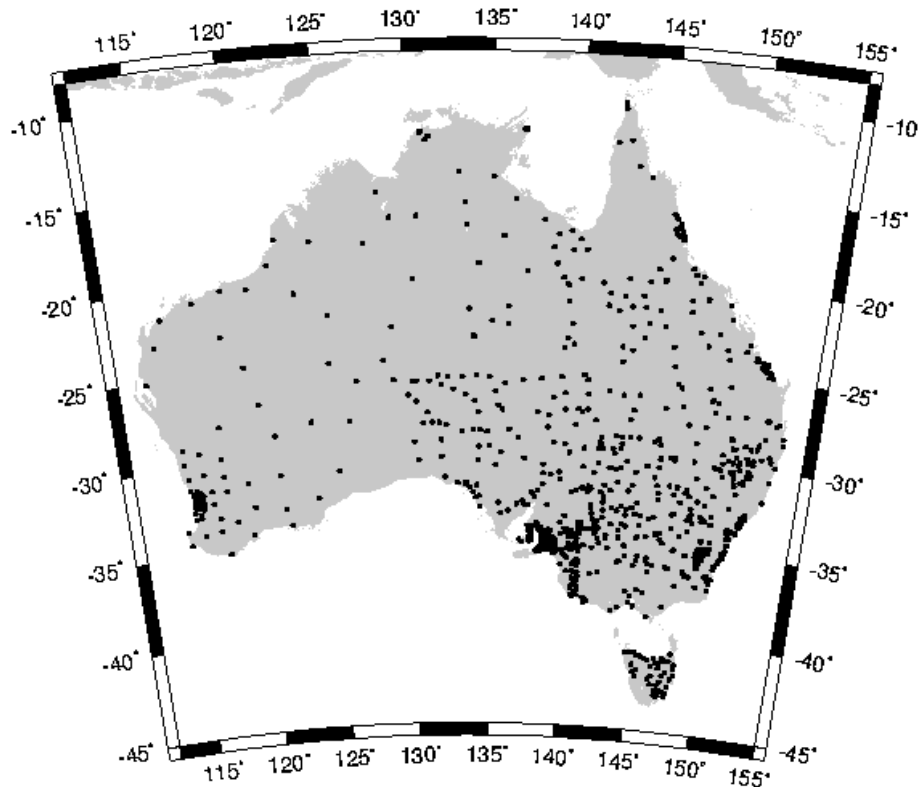


Figure 1. Spatial coverage of the 1013 GPS-AHD stations [Lambert conical projection]

- The terrestrial gravity data used for these experiments are exactly the same as those used to compute AUSGeoid98 (Featherstone *et al.* 2001). These were provided by the Australian Geological Survey Organisation (now Geoscience Australia) and have been supplemented with Sandwell and Smith's (1997) satellite altimeter-derived gravity anomalies in marine areas using least squares collocation (Kirby and Forsberg 1998). Gravimetric terrain corrections have been applied to these data using version 1 of the Australian digital elevation model (Kirby and Featherstone 1999). The resulting set of mean Faye gravity anomalies is given on a 2' by 2' grid over the area bound by $46^{\circ}\text{S} \leq \phi \leq 8^{\circ}\text{S}$ and $108^{\circ}\text{E} \leq \lambda \leq 160^{\circ}\text{E}$ (Figure 2).

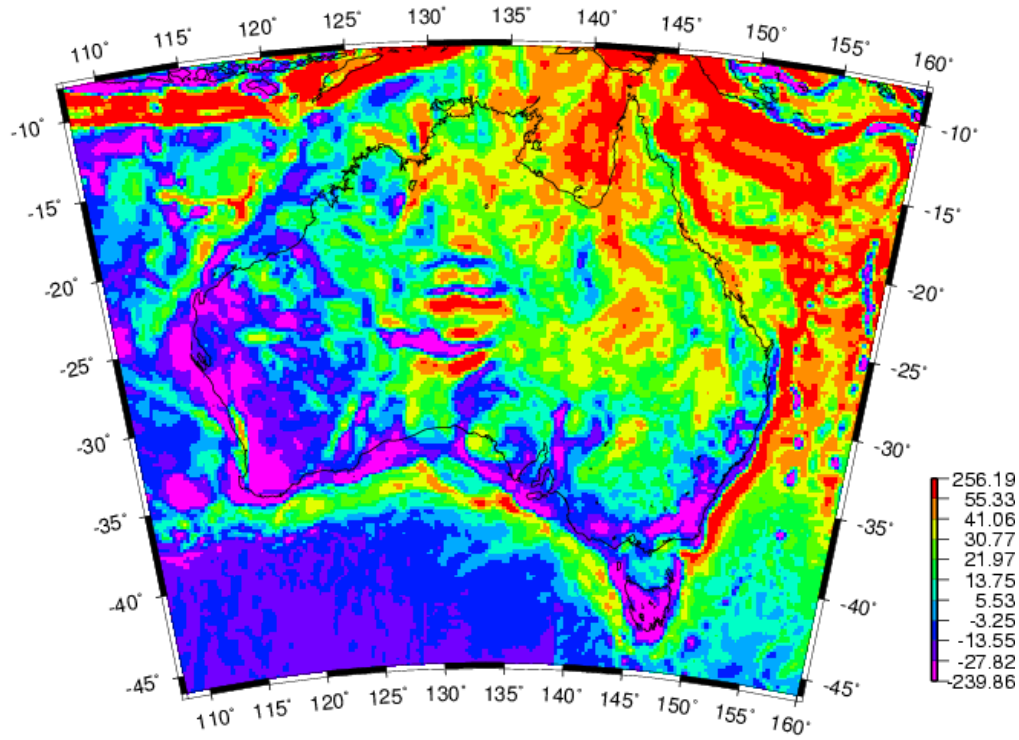


Figure 2. Terrestrial Faye gravity anomalies over Australia [units in mGal]

- Version 1 of the Australian digital elevation model (Carroll and Morse, 1996) was used to compute the approximate primary indirect effect (second term on the right-hand-side of Eqs. (1) to (3)), which was generalised onto the same 2' by 2' grid as the gravity data over land areas. Due to the lack of a digital density model over Australia, a constant topographic mass-density of 2670 kgm^{-3} had to be used in this computation.
- The EGM96S (satellite-only) and EGM96 (combined) global geopotential models (Lemoine *et al.* 1998) were used in Eqs. (2) and (3), respectively. The expansion of EGM96S was truncated at degree 20 in Eq. (2), which is the point beyond which satellite-only global geopotential models become heavily contaminated by noise. This can be demonstrated simply by plotting the error degree variances of the EGM96S coefficients as a proportion of their degree variances. The complete degree-360 expansion of EGM96 was used in Eq. (3).

Table 1 shows the descriptive statistics of only the innermost zone contributions (first term on the right-hand-side of Eqs. (1) to (3)), as well as the approximate primary indirect effect, which is common to all equations. From Table 1, it can be seen that the innermost zone contributions to the geoid from each of the three approximate mathematical models are small and reasonably similar. The approximate primary indirect effect is small, which is to be expected because the tallest mountain in Australia is only 2,228 m high. Given these small values, it is to be expected that Eq. (1), which does not include a global geopotential model and thus has the largest truncation error, will deliver poor results.

<i>Geoid model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
1st term in Eq. 1 with terrestrial gravity anomalies	0.624	-0.488	-0.006	±0.062
2nd term in Eq. 2 with residual gravity anomalies	0.624	-0.505	-0.002	±0.059
2nd term in Eq. 3 with residual gravity anomalies	0.416	-0.597	-0.002	±0.032
Approximate primary indirect effect	0.000	-0.221	-0.002	±0.006

Table 1. Descriptive statistics of the innermost zone contributions from Eqs. (1) to (3) and the approximate primary indirect effect [units in metres]

Table 2 shows the descriptive statistics of the three very approximate geoid models (all terms in Eqs. (1) to (3)). By way of comparison, the AUSGeoid98 regional geoid model [based on the degree-360 expansion of EGM96] is also shown in Table 2. From Table 2, the approximate geoid heights computed from Eqs. (2) and (3) appear reasonably similar to AUSGeoid98, though the maximum and minimum differences are greater than the contribution of the innermost zone (cf. Table 1). This demonstrates the relatively large contribution that the regional integration makes to AUSGeoid98. Also from Table 2, the approximate geoid height computed from Eq. (1) is very different. As stated, this is due to the very large truncation error (up to ~80 m) that results from using only an innermost zone computation of the original Stokes formula (i.e., no inclusion of a global geopotential model to provide an proportion of the truncation bias).

<i>Geoid model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
N_1 (Eq. 1 with terrestrial gravity anomalies)	0.624	-0.488	-0.008	±0.066
N_2 (Eq. 2 with residual gravity anomalies)	75.806	-39.244	10.902	±33.263
N_3 (Eq. 3 with residual gravity anomalies)	85.906	-40.476	10.852	±33.406
AUSGeoid98 (Featherstone <i>et al.</i> , 2001)	80.644	-40.583	10.802	±33.334

Table 2. Descriptive statistics of the geoid models experimented with [units in metres]

Figures 4 and 5 show images of the differences between the AUSGeoid98 regional geoid model (Figure 3), and the very approximate geoid solutions based on EGM96S and EGM96 (Eqs. 2 and 3, respectively). The difference between the very approximate geoid solution based on Eq. (1) and AUSGeoid98 is so small (cf. Table 1) as to be imperceptible in a figure, and would simply resemble Figure 3. As such, it is not shown here.

Figure 4 shows that Eq. (2), based on the satellite-only EGM96S global geopotential model, omits most of the medium wavelength geoid undulations that are provided by regional integration [a one-degree integration radius was used for AUSGeoid98]. Figure 5 shows that Eq. (3), based on the combined EGM96 global geopotential model, omits only some of the medium wavelength geoid undulations that are provided by AUSGeoid98. This is as expected because the higher degree expansion of EGM96 contributes more to the evaluation of the truncation bias at medium wavelengths (cf. Figures 4 and 5).

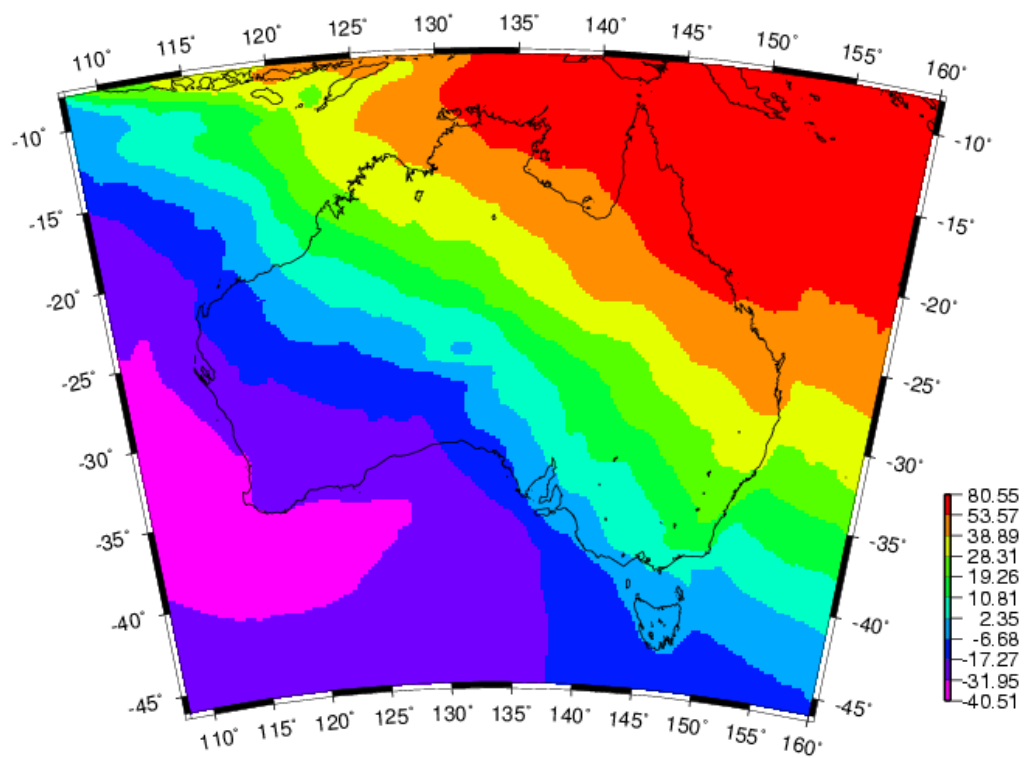


Figure 3. AUSGeoid98 [units in metres]

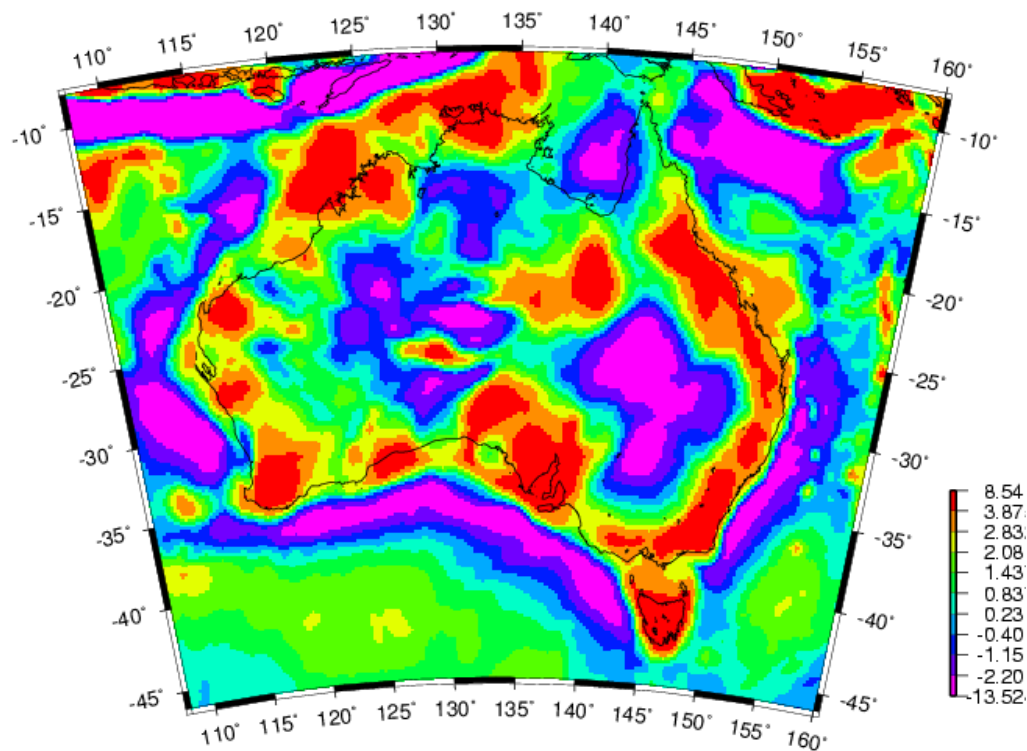


Figure 4. Differences between AUSGeoid98 and the geoid from Eq. 2 [units in metres]

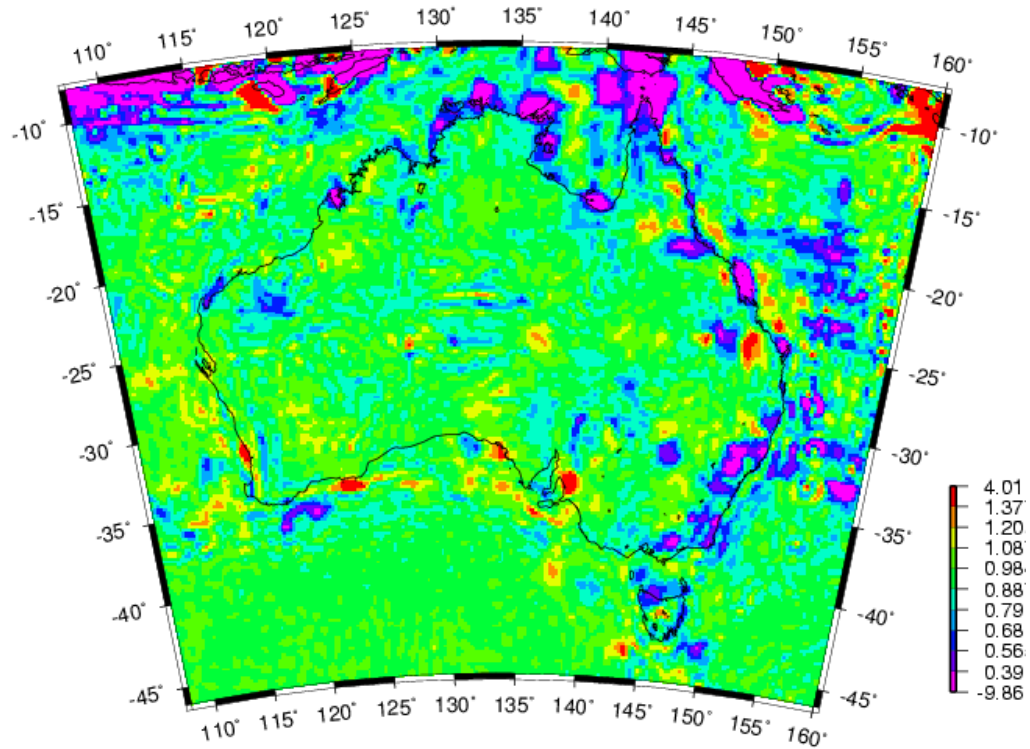


Figure 5. Differences between AUSGeoid98 and the geoid from Eq. 3 [units in metres]

As is customary with most validations of gravimetric geoid models on land, the three very approximate geoid models (Eqs. 1 to 3) were differenced with the 1013 Australian GPS-levelling data (cf. Featherstone and Guo, 2001). By way of comparison, the EGM96S, EGM96 and AUSGeoid98 geoid models were also differenced with the same GPS-levelling data. The results of the fit of each geoid model to the GPS-levelling data are summarised by the descriptive statistics in Table 3. Of course, it is acknowledged that such comparisons are equivocal because of the error budget of the GPS-levelling data.

<i>Geoid model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
N_1 (Eq. 1 with terrestrial gravity anomalies)	71.268	-32.747	11.284	± 23.074
EGM96S truncated to degree 20	6.967	-6.515	1.528	± 2.505
N_2 (Eq. 2 with residual gravity anomalies)	6.767	-6.449	1.521	± 2.470
EGM96 complete to degree 360	3.537	-2.441	-0.015	± 0.441
N_3 (Eq. 3 with residual gravity anomalies)	3.510	-2.464	-0.008	± 0.422
AUSGeoid98 regional geoid model	3.558	-2.572	-0.002	± 0.314

Table 3. Descriptive statistics of the difference between 1013 GPS-AHD data and the various geoid models experimented with [units in metres]

From Table 3, the approximate geoid solution based only on terrestrial gravity anomalies (Eq. 1) is in extremely poor agreement with the GPS-levelling data, which is as

expected. As such, it should never be used. Moreover, given the ready and free availability of global geopotential models that give much better fits (Table 3), it would be pointless to trial this approach any further. Next, it is interesting to observe from Table 3 that the approximate geoid solutions based on the global geopotential models (Eqs. 2 and 3) give marginally better fits to the Australian GPS-levelling data than the global geopotential models alone. Entirely as expected, they do not make as much of an improvement as the AUSGeoid98 regional gravimetric geoid model (Table 3).

4. Concluding Remarks

The results of these curiosity-driven experiments are largely as expected. That is, the contribution of mean terrestrial gravity anomalies from the innermost zone to the geoid is small. Nevertheless, the very coarse approximations of the geoid height given by Eqs. (2) and (3) can add some localised geoid information to a global geopotential model, but as expected, this is not as good as the improvement made by a regional gravimetric geoid model. This very approximate approach may prove to be of some value in areas or countries where access to regional gravity and terrain data is restricted, or where the resources of personnel, software and high-powered computers are not available. This simple method can be adopted to yield an interim geoid solution for use until the issues of data and resources have been resolved.

From the results presented for Australia, Eqs (2) and (3) can be used to make some small improvements to the global geopotential model that, in turn, improve upon the transformation of GPS-derived ellipsoidal heights to orthometric heights. Of these two approaches, Eq. (3) delivers the best results because the high-degree combined global geopotential model provides a larger proportion of the truncation bias. However, this may be contingent on terrestrial gravity data being used in EGM96 (as is the case in Australia). Finally, the calculation of the very approximate geoid height is extraordinarily simple and can be achieved using spreadsheets instead of the need to use high-powered computers, as is the case when computing regional gravimetric geoid models. This simplicity may be of value to GPS users in developing countries.

Acknowledgments

I would like to thank the numerous organisations (Geoscience Australia, NASA, NIMA, Scripps) for providing data and the Australian Research Council for financial support through large grant A39938040 and discovery project grant DP0211827. I would also like to thank Simon Holmes (Curtin University, Australia) and Georgia Fotopoulos (Calgary University, Canada) for useful discussions. Finally, thanks are also extended to the reviewer(s).

References

- Carroll D, Morse MP (1996) A national digital elevation model for resource and environmental management, *Cartography*, 25(2): 395-405.
- Heiskanen WH, Moritz H (1967) *Physical Geodesy*, Freeman, San Francisco, USA, 364pp.
- Featherstone WE, Guo W (2001) A spatial evaluation of the precision of AUSGeoid98 versus AUSGeoid93 using GPS and levelling data, *Geomatics Research Australasia*, 74: 75-105.

- Featherstone WE, Kirby JF, Kearsley AHW, Gilliland JR, Johnston GM, Steed J, Forsberg R, Sideris MG (2001) The AUSGeoid98 geoid model of Australia: data treatment, computations and comparisons with GPS-levelling data, *Journal of Geodesy*, 75(5/6): 313-330.
- Haagmans RRN, de Min E, van Gelderen M (1993) Fast evaluation of convolution integrals on the sphere using 1D-FFT, and a comparison with existing methods for Stokes's integral, *manuscripta geodeatica* 18: 227-241
- Kirby JF, Forsberg R (1998) A comparison of techniques for the integration of satellite altimeter and surface gravity data for geoid determination, in Forsberg R, Feissel M, Deitrich R (eds), *Geodesy on the Move: Gravity, Geoids, Geodynamics, and Antarctica*, Springer, Berlin, Germany, 207-212.
- Kirby JF, Featherstone WE (1999) Terrain correcting the Australian gravity database using the national digital elevation model and the fast Fourier transform, *Australian Journal of Earth Sciences*, 46(4): 555-562.
- Lemoine FG, Kenyon SC, Factor JK, Trimmer RG, Pavlis NK, Chinn DS, Cox CM, Klosko SM, Luthcke SB, Torrence MH, Wang YM, Williamson RG, Pavlis EC, Rapp RH, Olson TR (1998) The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96, *NASA/TP-1998-206861*, National Aeronautics and Space Administration, Washington, USA.
- Sandwell DT, Smith WHF (1997) Marine gravity anomaly from Geosat and ERS 1 satellite altimetry, *J Geophys Res* 102(B5): 10039-10054.
- Wichiencharoen C (1982) The indirect effects on the computation of geoidal undulations, Report 336, Department of Geodetic Science and Surveying, Ohio State University, Columbus, USA.

A geomatic approach to local geoid computation

L. BIAGI, M.A. BROVELLI AND M. NEGRETTI
Politecnico di Milano - Engineering Faculty of Como
Via Valleggio 11, 22100 Como, Italy

Abstract. During the last years Geographic Information Systems have become very powerful tools to study georeferenced data in several different fields. The capabilities of these software packages to store, analyze, display and map data within only one development environment make them interesting also for geodetic applications. Since this approach is novel, at least for our research group, first of all we decided first to apply it instead of procedures which are quite consolidated at the I.Ge.S.

For the sake of clarity, we underline that we use the term “collocation solution” for the geoid obtained via collocation in a national or continental area (e.g.: Italy) and “local geoid computation” for the prediction of the geoid in a point not belonging to the collocation solution grid.

After focusing the target, we must deal with different questions which can be summarized as follows: which GIS package is more suitable for our needs? How to customize it in order to include the geodetic performances we need? Is it better or not to use an interfaced Data Base Management System (DBMS)? In the paper we answer the previous question and we introduce the prototype GEOGRASS.

Keywords. Geomatics, geoid computation, GIS

1 Introduction

Some years ago a team usually co-operating with the I.Ge.S. and which is composed of people originally working on geodesy, gathered to study and then to carry out researches in the field of Geographic Information Systems. As these systems easily allow to archive, manage, process and display geographic data within the same software environment, by using always the same rules and the same command syntax, they seem to be fruitful also for geodetic purposes. Or better, at least, in our opinion the possibility to exploit them for geodetic computation has to be investigated (L. Biagi et al., 1999). This approach was novel for our research group. Therefore, we decided to adopt it in order to implement procedures that are consolidated at the I.Ge.S. Between the latter, the procedure leading to computing the geoidal heights in points not belonging to the collocation grid solution, is very often used. The growing use of GPS positioning method in fact makes necessary the geoid determination in irregularly located points, 'spread points' for short, in order to homogenize the ellipsoidal height coordinate with the more common orthometric one.

Our first aim was to provide, within a GIS environment, the tools suitable for the automatic improvement therefore of this procedure. We are interested in creating a system in which the user is only asked for the point in which he wants the geoid, the window size for the topographic geoidal component computation, the global geopotential model to be used and the system returns the punctual geoidal height.

The basics for this procedure are found in the remove-restore technique (Tscherning in Sansò eds., 1994). The algorithm implemented in effect gets:

$$\hat{N}(P) = \hat{N}^M(P) + \hat{N}^{RTC}(P) + \hat{N}^r(P)$$

where $\hat{N}^M$ is the contribution of the anomalous potential model, estimated in the point P:

$$\hat{N}^M = R \sum_{n=2}^{N_{max}} \sum_{m=-n}^n \left(\frac{R}{r} \right)^{n+1} [C_{nm} Y_{nm}(\lambda_P, \varphi_P)]$$

C_{nm} are the geopotential normalized coefficients of the anomalous potential;

Y_{nm} are the associated Legendre functions;

N_{max} is the maximum degree of the geopotential global model;

$\hat{N}^{RTC}$ is the effect of the residual terrain correction;

$\hat{N}^r$ is the local residual geoid computation:

$$\hat{N}^r = C_{N\Delta g} [C_{\Delta g \Delta g} + \sigma^2 I]^{-1} [\Delta g_r + n_r] = C_{N\Delta g} \hat{S}$$

$\hat{S}$ is the solution vector obtained in the calculation of the collocation grid;

$C_{N\Delta g}$ is the covariance to be estimated between the point P and the points of the collocation grid.

As it is well known the remove-restore method models and computes the geoid by splitting it into the three previously recalled components, corresponding to different wavelength effects.

Therefore the scheme followed for the geoid computation is the same as the “classical” restore procedure, but in this case the computation goes faster, as we do not need to perform the computation entirely. In fact, the solution of the Stokes integral on a grid and the covariance function are already available as previously computed; they play the role of known input variables.

The local geoid computation is obviously dependent on the previously determined collocation solution. Similarly, in the RTC and in the global model computation we must use the same digital terrain model and the geopotential coefficients used in the collocation solution we refer to.

2 The GIS package choice and the DBMS option

As a matter of fact in the geodetic world a GIS geodetic frame (GRAVSOFT) has been built in the last twenty years by the research group led by C.C. Tscherning (Tscherning et al., 1992). Their philosophy is to have different and separate modules allowing the various geodetic computation to easily pass the output of one program as input to another one. We are looking for a solution partly similar but more complete from the point of view of the traditional basic tools available in a GIS.

Therefore, instead of designing and building up a system all over again, we decided to pay our attention to the both commercial and research extant solutions, focusing on the possibility of easy improve the system with tools useful to geodetic computations.

The main characteristics we are interested in are summarized as follows (more details are available in L. Biagi et al., 1999):

- an open GIS with code available, written and writable in a standard language;
- a GIS oriented to raster and site points data model processing (instead of a vector data model);
- a GIS not expensive in order to provide to I.Ge.S. a system which can be freely distributed.

Finally we pitched on GRASS.

This system was originally designed and developed by the Environmental Division of the U. S. Army Construction Engineering Research Laboratory.

Since the first release in 1985, it was distributed with source code, in such a way that users from different sites (education and research institutions, government organizations and private firms) and with various interests (land planners, landscape architects, geographers, geologists,) may customize this software to settle their own questions. Furthermore, as it was (and it is) freely available (General Public License GNU[®] 1989, 1991 Free Software Foundation, Inc., 59 Temple Place - Suite 330, Boston, MA 02111-1307, USA, gnu@gnu.org), the number of users and the types of applications in the research environment have rapidly increased.

Actually the GIS development is supervised by the Development Team, which is based on three main sites: the official home pages are

- <http://grass.itc.it>;
- <http://www.baylor.edu/grass/>

From the point of view of design the software portability is one of the main requirement in GIS development: GRASS runs within a UNIX environment and is written in standard C language. Referring to the functionality, it has more than 200 modules to manage raster, sites points and vector data (lines and surfaces). GRASS, like the other GIS, is not only a useful viewer or analyzer of spatial data but, in some ways, also a DBMS. The GRASS database is based on the UNIX hierarchical directory structure and its access is controlled by few simple rules; it is therefore quite efficient for spatial data handling.

Anyway, this system does not include the possibility of optimizing the management of attribute information, system files and objects interrelationships. These performances can be obtained only by

interfacing a real DBMS to GRASS. The use of a DBMS also provides some auxiliary functionality, which are fundamental when more people work on different projects by using part of the same dataset.

The variety of the information needed for the computation and management of the geoid suggests the use of a DBMS-oriented storing method instead of a file-based one.

This is a simpler and more structured way to implement the most frequently used actions like updating and querying. The choice of a spatial database (instead of other traditional DBMS like, for example, a business administration one) implies the peculiarity of the spatial aspect that must be considered while planning the archive itself, in order to obtain a more efficient management system.

In particular the relational database management systems, that support the most used GIS-engine interfaces, must have extra features such as spatial extensions and an indexing method in order to optimize spatial data management. The same considerations focused on the case of the GIS choice suggest the use of PostgreSQL.

PostgreSQL, which was developed originally within the University of California in the Berkeley Computer Science Department, was sponsored by the Defense Advanced Research Projects Agency (DARPA), The Army Research Office (ARO), the National Science Foundation (NSF) and ELS, Inc. The main characteristics of PostgreSQL are:

- it is an open-source Object-Relational Data Base System (ORDBMS), written in C language;
- it is freely available;
- its transactions have the standard ACID (Atomicity Consistency Isolation Durability) properties (see PostgreSQL documentations).
- it allows to the definition of spatial data (points, lines, polygons);
- it has a b-tree indexing method in the case of monodimensional data;
- it has a r-tree indexing methods in the case of multidimensional data;
- it allows an Open Database Connectivity (ODBC) interface.

Tests have been performed on raster and sites points data by using the GRASS archive, PostgreSQL and other Relational Data Base Management Systems (RDBMS). They testify that the best solution in term of data retrieval time (Biagi et al., 1999) is for the first case simply the use of the GRASS archive, while in case of spread data the performances reached by using a relational database management system (RDBMS) are better. Moreover, in the latter case the comparison of results obtained by using different ORDBMS shows that PostgreSQL is the optimal solution. Moreover, as previously outlined, PostgreSQL follows the main data base integrity standards: these characteristics were not the first analyzed, but obviously have reinforced the choice.

So, we have reached two important items: the geodetic GIS is based on GRASS (GRASS 5.0); the raster data are stored into the GRASS archive; the site points data are managed via PostgreSQL (PostgreSQL 6.5.2).

3 GEOGRASS

The geodetic GIS was created by adding to GRASS a new group of commands characterized by the *gd* (geoid) prefix. The computing kernels of these commands are the original ones, written in FORTRAN; the input-output and the libraries links are written in C by following the standard rules in GRASS.

The original programs used as a basis of the new commands are:

- *tcitge95.f*: program written by R. Forsberg, Danish Geodetic Institute, release April 1991;
- *fastcolc.f*: program written C.C. Tscherning, Danish Geodetic Institute, and modified by R. Barzaghi, DIIAR - Politecnico di Milano, release May 1991;
- *modosup.f*: program written by Oscar L. Colombo, Ohio State University, and modified by R. Barzaghi, DIIAR - Politecnico di Milano, release 1995.

The new commands are *gd.model*, *gd.rtc*, *gd.fastcolc* and *gd.sum*. Next we present a brief description of the implemented commands by following the standard GRASS rules. As an example we compute the geoid in a region around Como (where the I.Ge.S. has its offices); the region boundaries and the resolutions are respectively:

$45^{\circ} 43'50'' \text{ N} \leq \varphi \leq 45^{\circ} 55' 9'' \text{ N} \quad \Delta\varphi = 20''$

$8^{\circ} 59'50'' \text{ E} \leq \lambda \leq 9^{\circ} 11' 10'' \text{ E} \quad \Delta\lambda = 20''$

We computed the geoid on a grid, but the grid points have been handled as spread points. The local computed geoid components and the total interest region geoid are shown in the figures in the next subparagraphs.

3.1 **gd.model**

Provides the computation of the geoid model contribution. Usage:

gd.model input=name output=name coeff=name [ifv=value] [maxdeg=value]

Files:

- input name of existing sites file;
- output name of output sites file;

Parameters:

- coeff name of model's coefficients file;
- ifv inverse flattening value;
- maxdeg maximum degree value.

The command structure is shown in Fig. 1; moreover in Fig. 2 we report the map of the model geoid in the interest region.

3.2 **gd.rtc**

Provides the computation of the geoid residual terrain correction contribution. Usage:

gd.rtc input=name output=name dtmf=name dtmr=name [rc=value]

There are no flags.

Files:

- input name of the existing sites (points) file;
- output name of the output sites file;
- dtmf name of a raster file corresponding to the detailed digital terrain model;
- dtmr name of a raster file corresponding to "reference" digital terrain model (can be obtained by detailed DTM applying moving average).

Parameter:

- rc computation radius (Km): defines the region of interest surrounding the prediction points; the default value is 100km.

The command structure is shown in Fig. 3; the computation results are presented in Fig. 4.

3.3 **gd.fastcolc**

Provides the computation of the local geoid residual contribution: the computation can be done either by applying the fast collocation procedure or by linear interpolating the values belonging to the collocation solution which are nearest to the interest point. The first option is more accurate but takes more computation time. In case of the Italian geoid we proved (Brovelli et al., 2000) that the difference between the two solutions is, for the 99.5 % of the values, less than 4 cm; in the Como area the two solutions differences are always less than 3.9 cm and are larger than 3 cm only for the 3.5% of the value.

Usage:

gd.fastcolc [-i] input=name output=name gravtab=name

Options:

- i choose simple interpolation instead of collocation.

Files:

- input name of existing sites file
- output name of the output sites file
- gravtab name of a PostgreSQL table containing gravity data

The structure of the PostgreSQL table is shown in the Table 1. The command structure is shown in Fig. 5; the collocation geoid component is presented in Fig. 6.

Table 1. The structure of the gravimetric PostgreSQL table

Field	Type	Description
Coord	box	point coordinates
Dg	float8	observed gravity anomaly
N	float8	residual collocation geoid computed via fast collocation
Sol	float8	collocation solution s

3.4 gd.sum

Provides the computation of the geoid by adding the three previously computed contributes. Usage:

`gd.sum input1=name input2=name input3=name output =name`

There are no flags.

Files:

- input1 name of existing sites file (e.g.: output of gd.model)
- input2 name of existing sites file (e.g.: output of gd.rtc)
- input3 name of existing sites file (e.g.: output of gd.fastcolc)
- output name of the output sites file

The command structure and the global local geoid are shown respectively in the Fig. 7 and 8.

4 Conclusions

The basics to build up a geoid-GIS have been set. The GIS we singled out as a frame is not far off from the software environments used by geodesists. In the next the commands for the local geoid computation will be available at the I.Ge.S. web site (<http://www.iges.polimi.it/>). Another possibility will be to provide the computation of the geoid via web.

The future developments consist on introducing more refined interpolation procedures for the local geoid computation which take into account additional local information as new gravity anomaly observations, more detailed digital terrain models and geoid undulations obtained by GPS observations and leveling.

References

- R. Barzaghi, G.P. Bottoni (1993). Fast Collocation *Bulletin Géodésique* Vol. 67, pp. 119-126
- L. Biagi, M.A. Brovelli, G. Salemi (1998). Geodetic Data Management with GIS and DBMS Techniques, *IgeS Bulletin* Vol.8, pp. 113-126.
- L. Biagi, M. A. Brovelli, M. Negretti And C. Saldarini (1999). Indexing Trees Methods And Spatial Ordering For Maps And Geographic Data: an Overview and the Application To The Geodetic GIS Project, In *International Archives of Photogrammetry and Remote Sensing*, Vol. XXXII, Parte 6W7, pp.89-96.
- M. A. Brovelli, M. Negretti, C. Saldarini, (2000)., Il Nuovo Sistema Informativo Geografico Geograss Per La Stima Locale Del Geoide, in print on *Bollettino di Gedesia e Scienze Affini*.
- GNU General Public License (GPL) <http://www.gnu.org/copyleft/gpl.html>
- Official Home of the GRASS GIS, <http://grass.itc.it/>; <http://www.baylor.edu/grass/>.
- PostgreSQL, <http://www.postgresql.org/>
- F. Sansò (edited by) (1994). *International School for the Determination and Use of the Geoid, Lecture Notes*, International Geoid Service, DIIAR Politecnico di Milano.
- C. C. Tscherning, R. Forsberg, P. Knudsen (1992). The GRAVSOFT Package for Geoid Determination, presented at the *Continental Workshop on the European Geoid, Prague, May 1992*

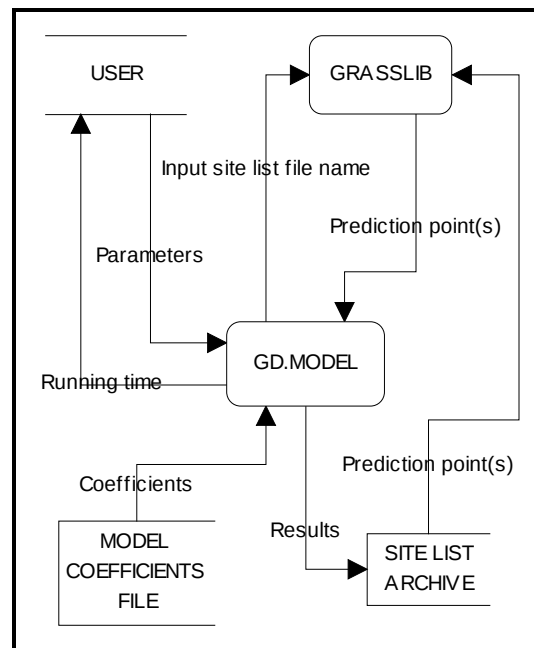
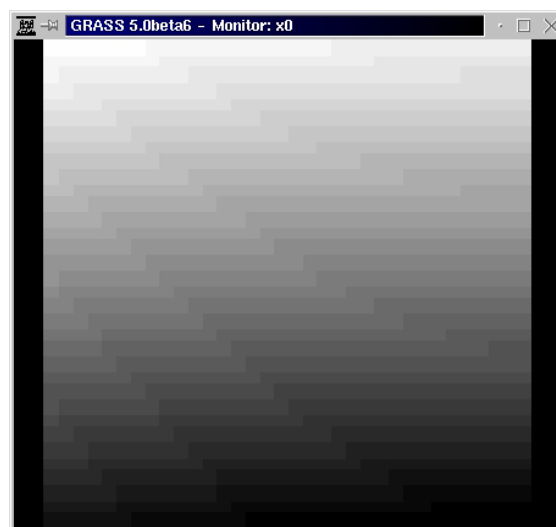


Fig. 1 *gd.model* command Data Flow Diagram



	> 48 m
	> 47 m and < 48 m
	> 46 m and < 47 m
	> 45.7 m and < 46 m

Fig. 2 Global geoid model within the area around Como

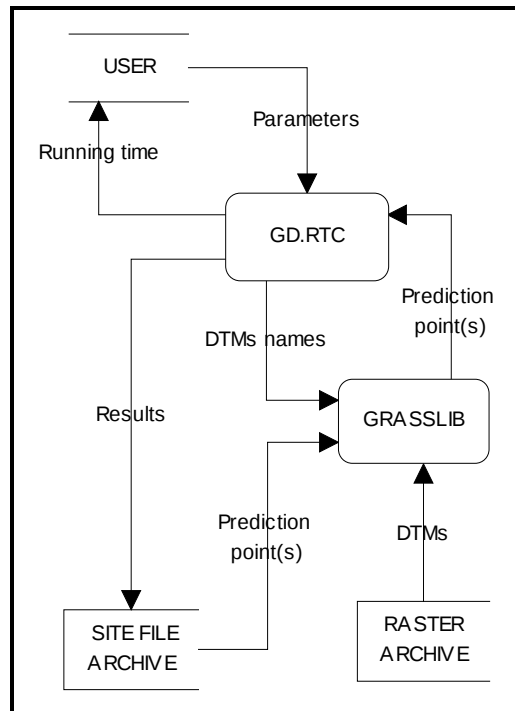
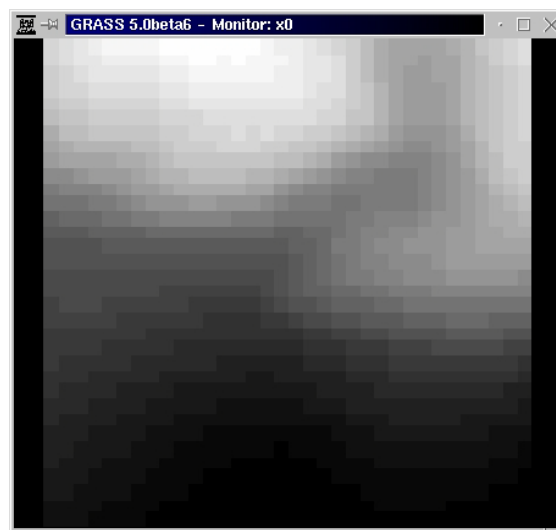


Fig. 3 *gd.rtc* command Data Flow Diagram



	< 0.4 m and > 0.3 m
	< 0.3 m and > 0.2 m
	< 0.2 m and > 0.1 m
	< 0.1 m and > 0 m

Fig. 4 The residual terrain correction within the area around Como

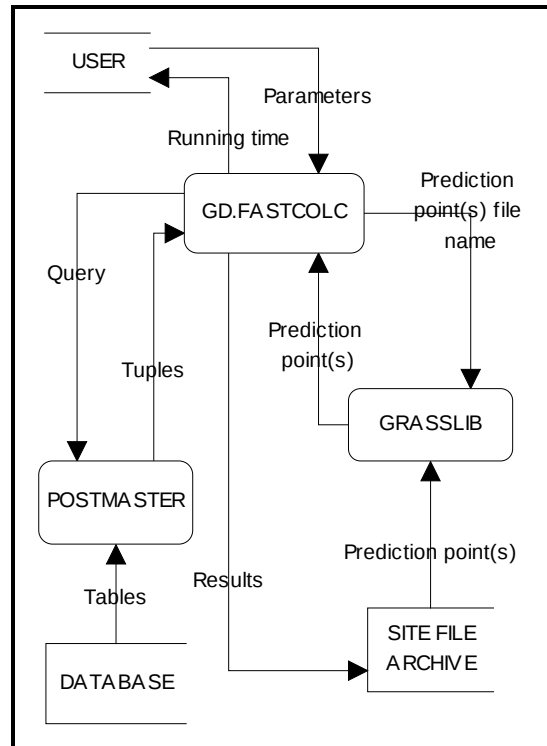
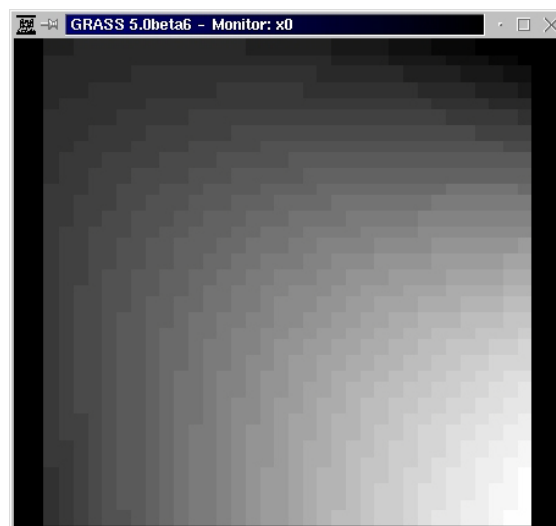


Fig. 5 *gd.fastcolc* command Data Flow Diagram



	< -0.2 m and > -0.22 m
	< -0.1 m and > -0.2 m
	< 0 m and > -0.1 m
	> 0 m and < 0.1 m

Fig. 6 The collocation geoid within the area around Como

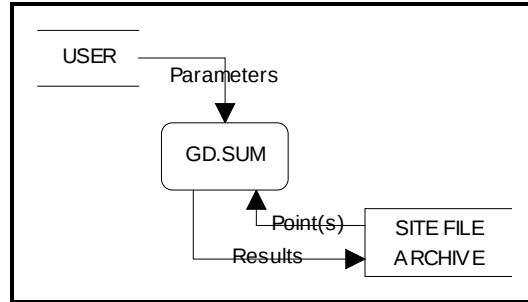
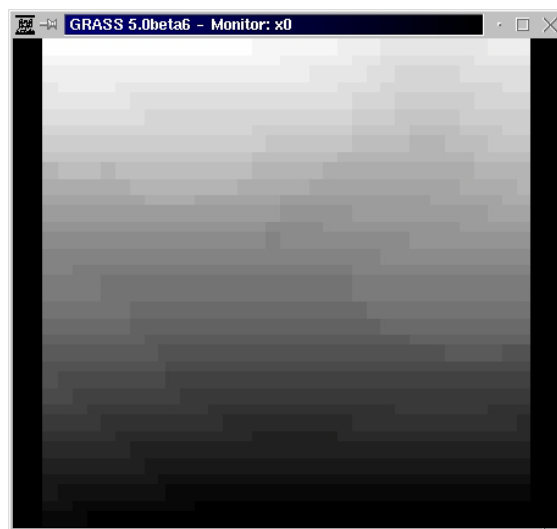


Fig. 7 *gd.sum* command Data Flow Diagram



	> 48 m
	> 47 m and < 48 m
	> 46 m and < 47 m
	> 45.7 m and < 46 m

Fig. 8 The geoid within the area around Como

SCHOOLS
for "The Determination and Use of the Geoid"

A new Geoid School is being now organized in Thessaloniki 30 August - 5 September according to the enclosed program.
In addition another school is being in project for November 2003 in Dubai.

Fernando Sansò

IGeS Geoid School
30 August-5 September, 2002
Thessaloniki, Greece

Organized by The University of Thessaloniki
Dept. Of Geodesy and Surveying

Program:

1st day (30 August 2002):

1400 – 1900 Participants check-in into respective Accommodation and Course Registration

2nd day (31 August 2002):

0830 Opening Ceremony

0900 Break

0930 Lecture 1 – General Theory for geoid computation (prof. F. Sansò)

Lecture 2 – The remove restores concept (prof. F. Sansò)

1300 Lunch

1400 Lecture 3 – Collocation theory (Prof. C.C. Tscherning)

1530 Break

1545 Lecture 4 – Local datum estimation (prof. C.C. Tscherning)

3rd day (2 September 2002):

0830 Lecture 5 – Theory and application of global gravity models (prof. R. Barzaghi)

1000 Break

1015 Lecture 5 (Continues)

1130 Exercises 1 Global Models (prof. R. Barzaghi)

1300 Lunch

1430 Exercises 1 (Continues)

1530 Break

1545 Exercises 2 - Collocation (prof. C.C. Tscherning)

1830 Day Ends

1930 Social Dinner

4th day (3 September 2002):

0830 Exercises 1 - The Practical application of Optimised Ring Integration (prof. C. Tscherning)

1000 Break

1015 Lecture 6 – Practical geoid evaluation by Ring Integration (prof. W. Kearsley)

1230 Lunch

1400 Exercises 2 - Ring integration (prof. W. Kearsley)

1530 Break

1545 Exercises 3 - Ring integration and height datum (prof. W. Kearsley)

1700 Day Ends

5th day (4 September 2002):

0830 Lecture 7 – The theory of terrain correction for gravity anomalies and potential (prof. R. Forsberg)

1000 Break

1015 Lecture 7 (Continues)

1230 Lunch

1430 Exercises 1 (prof. R. Forsberg)

1530 Break

1545 Presentation of the Validation Theory and Software of BGI (prof. R. Forsberg)
1715 Day Ends

6th day (5 September 2002):

0830 Lecture 8 – The FFT Approach to the Geoid Determination (prof. M. Sideris - prof. C. Kotsakis)
1000 Break
1015 Lecture 8 (Continues)
1230 Lunch
1400 Exercises (prof. M. Sideris - prof. C. Kotsakis)
1600 Closing Ceremony
1745 Course Ends

Note: All the software used during the school will be made available on CD for the participants

Activities related to height system in South America

DENIZAR BLITZKOW

Sub-Commission for the Geoid in South America, Escola Politécnica da Universidade de São Paulo, Caixa postal 61548, - 05424-970 São Paulo - SP – Brazil, E-mail: dblitzko@usp.br

ROBERTO TEIXEIRA LUZ

IBGE-DEGED, Av. Brasil, 15671, - 21241-051 Rio de Janeiro – RJ – Brazil

SÍLVIO ROGÉRIO

ABSTRACT

The Working Group III of SIRGAS (Geocentric Reference System for the Americas) and Sub-Commission for Gravity and Geoid in South America SCGGSA are involved in several activities related the problem of Vertical Reference System. The SIRGAS 2000 campaign addressed a special attention to observations on tide gauges in the continent. The results will have an important impact in the connexion of the stations via space techniques. On the ther hand, the SCGGSA carried out many efforts in the last ten years on the improvement of gravity and topographic informations in the continent. The details of the activities carried out and in progress will be presented and some results analysed.