

FOREWORD

This Special Issue of the IGeS Bulletin collects paper presented at the G7 session of the XXVI EGS Assembly (Nice, 25-30 March 2001).

The G7 session was on “Regional and local gravity field approximation”, which is a central subject in Physical Geodesy. More than twenty papers were presented at that session, both on numerical and theoretical aspects of gravity field estimation.

Since the topics of the G7 session fit the aims of the IGeS Bulletin and a remarkable amount of papers were submitted for publication to the Conveners of the session, a special issue was realized due also to the relevance and quality of the papers.

Ilias N. Tziavos

Riccardo Barzaghi

HIGH ACCURATE GEOID USING ADAPTED REFERENCE FIELD AND SEISMIC MOHO DEPTHS

Hussein A. Abd-Elmotaal

Civil Engineering Department, Faculty of Engineering,
Minia University, Minia 61111, Egypt

Norbert Kühtreiber

Section of Physical Geodesy, Institute of Theoretical Geodesy
Graz University of Technology, Graz, Austria

ABSTRACT

The traditional remove/restore technique for geoid determination has two main disadvantages. The first is the assumption of the behaviour of the compensation of the topographic masses through an isostatic hypothesis (usually, Airy-Heiskanen model). The second is the removal of a part of the effect of the topography and its compensation twice. To overcome the first disadvantage, the seismic Moho depths, representing the real compensating masses, have been used instead of assuming an isostatic hypothesis. In order to avoid the double consideration of the effect of the topographic-isostatic masses, the effect of these masses for the used fixed data window has been subtracted from the reference field yielding an adapted reference field. This adapted reference field has been used for the remove/restore technique. The results show that using the adapted technique with Airy-Heiskanen isostatic model gives the best reduced anomalies. The absolute difference between the geoid computed by the adapted reference field with seismic Moho depths and GPS geoid shows nearly a linear trend. All geoids, however, have practically the same precision.

1. INTRODUCTION

The paper attempts one trial to improve the accuracy of the geoid. In modern geodesy, the geoid computation is done by applying the well-known remove/restore technique. The effect of the topographic-isostatic masses is removed from the source gravitational data and then restored to the resulting geoidal heights. The conventional way of removing this effect faces a theoretical problem. A part of the influence of the topographic-isostatic masses is removed twice as it is already included in the global reference field. This leads to some double consideration of that part of the topographic-isostatic masses. A possible way to overcome this difficulty in computing geoidal heights from topographic-isostatic reduced gravity anomalies is to adapt the used reference field due to the effect of the topographic-isostatic masses for a fixed data window.

Another drawback of the traditional geoid computation technique is the use of an isostatic model to produce the compensating masses. In practical applications, the Airy-Heiskanen isostatic model is usually used. To overcome this disadvantage, the seismic Moho depths, representing the real compensating masses, have been used instead of assuming an isostatic hypothesis.

The necessary harmonic analysis of the topographic-isostatic potential is given. The adapted reference field is used in the remove/restore process. A wide comparison among geoids computed by the adapted reference field with both Airy-Heiskanen isostatic model and seismic Moho depths and a geoid computed by the traditional remove/restore technique is made.

2. THE ADAPTED REMOVE/RESTORE TECHNIQUE

Within the well-known remove/restore technique, the effect of the topographic-isostatic masses is removed from the source gravitational data and then restored to the resulting geoidal heights. Figure 1 shows schematically the conventional gravity reduction for the effect of the topographic-isostatic masses. The short-wavelength part depending on the topographic-isostatic masses is computed for a

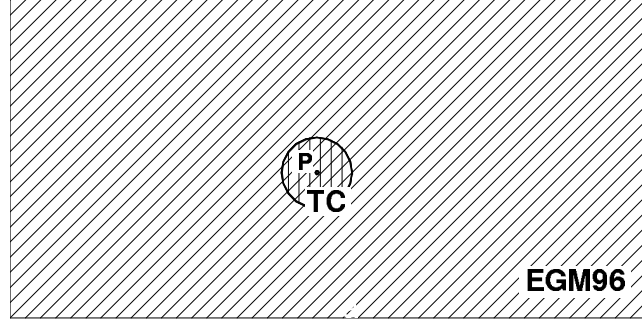


Fig. 1. The traditional remove/restore technique.

point P for the masses inside the circle (say till 167 km around the computational point P). Removing the effect of the long-wavelength part by a global reference field (e.g., EGM96) implies removing the influence of the global topographic-isostatic masses, shown as a rectangle in Fig. 1. The double consideration of the topographic-isostatic masses inside the circle is then seen.

A possible way to overcome this difficulty in computing geoidal heights from topographic-isostatic reduced gravity anomalies is to adapt the used reference field due to the effect of the topographic-isostatic masses for a fixed data window. Figure 2 shows the advantage of the adapted remove/restore technique schematically. We concentrate on the influence of the topographic-isostatic masses in the reduction process. Consider a measurement at point P , the short-wavelength part depending on the topographic-isostatic masses is now computed by using the masses of the whole data area (small rectangle). The adapted reference field is reduced by the influence of the topographic-isostatic masses of the data window (area). Thus, removing the long-wavelength part by using this adapted reference field does not lead to a double consideration of a part of the topographic-isostatic masses.

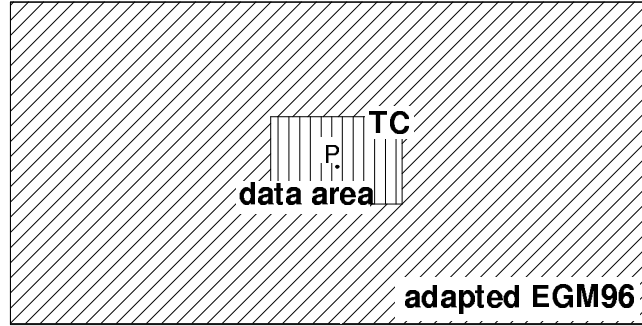


Fig. 2. The adapted remove/restore technique (window technique).

3. HARMONIC ANALYSIS OF THE TOPOGRAPHIC-ISOSTATIC POTENTIAL

The topographic-isostatic potential T_{TI} (compensating masses either produced by an isostatic model or using the seismic Moho depths) can be easily defined as the potential of all disturbing masses out of an ideal crustal layer of uniform density ρ_0 and thickness T_0 , superimposed underlying material of constant density ρ_1 . The potential T_{TI} can be written as:

$$T_{TI}(P) = G \iiint_v \frac{\delta\rho_Q}{\ell_{PQ}} dv_Q, \quad (1)$$

where G is the Newton's gravitational constant, $\delta\rho_Q$ denotes the density contrast, ℓ_{PQ} is the spatial distance between P and Q and dv_Q is the volume element at Q .

It is known that T_{TI} is harmonic outside the earth's surface and its spherical harmonic series is convergent outside a sphere completely enclosing the earth (Heiskanen and Moritz, 1967, p. 60).

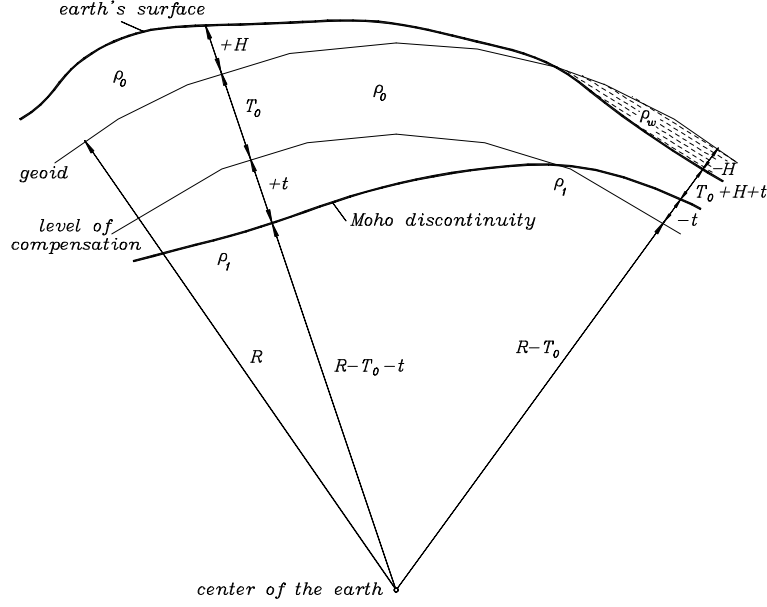


Fig. 3. Topographic masses and their compensation.

Outside that sphere, the convergent series representation of the reciprocal distance can be used (ibid., p. 33)

$$\frac{1}{\ell_{PQ}} = \sum_{n=0}^{\infty} \frac{r_Q^n}{r_P^{n+1}} P_n(\cos \psi_{PQ}), \quad (2)$$

where r is the modulus of the radius vector, $P_n(\cos \psi)$ is the Legendre polynomial of degree n and ψ_{PQ} is the spherical distance between P and Q . The Legendre polynomial may be expressed as

$$P_n(\cos \psi_{PQ}) = \frac{1}{2n+1} \sum_{m=-n}^n \bar{R}_{nm}(P) \bar{R}_{nm}(Q). \quad (3)$$

Thus the topographic-isostatic potential can be represented by

$$T_{TI}(P) = G \sum_{n=0}^{\infty} \frac{1}{(2n+1)r_P^{n+1}} \sum_{m=-n}^n \bar{R}_{nm}(P) \left\{ \iiint_v \delta \rho_Q r_Q^n \bar{R}_{nm}(Q) dv_Q \right\} \quad (4)$$

with

$$dv = r^2 dr d\sigma,$$

where $d\sigma$ is the spherical surface element.

To calculate the volume integral between the parentheses of (4), we will confine ourselves to the spherical approximation. Then its contribution due to the topographic masses outside the geoid and ocean water inside the geoid, see Fig. 3, is

$$\iiint_v = \iint_{\sigma} \int_{r=R}^{R+H} \delta \rho_Q r_Q^{n+2} dr_Q \bar{R}_{nm}(Q) d\sigma_Q, \quad (5)$$

and its contribution due to the compensated masses is

$$\iiint_v = \iint_{\sigma} \int_{r=R-T_0}^{R-T_0-t} \delta \rho_Q r_Q^{n+2} dr_Q \bar{R}_{nm}(Q) d\sigma_Q, \quad (6)$$

where R is the radius of the mean earth's sphere, T_0 is the normal crustal thickness, H is the topographic height (+) or ocean bottom depth (-), and t is the root (+) or antiroot (-).

The integration of (5) and (6) with respect to r is straightforward and is expressed by (Sünel, 1985, p. 5)

$$\iiint_v = \frac{R^{n+3}}{n+3} \iint_{\sigma} \delta\rho_Q \left[\left(1 + \frac{H_Q}{R}\right)^{n+3} - 1 \right] \bar{R}_{nm}(Q) d\sigma_Q \quad (7)$$

$$\iiint_v = \frac{R^{n+3}}{n+3} \left(1 - \frac{T_o}{R}\right)^{n+3} \iint_{\sigma} \delta\rho_Q \left[\left(1 - \frac{t_Q}{R - T_o}\right)^{n+3} - 1 \right] \bar{R}_{nm}(Q) d\sigma_Q. \quad (8)$$

Hence, the harmonic coefficients of the topography and its isostatic compensation as well as the harmonic series expansion of the topographic-isostatic potential can be expressed, by inserting (7) and (8) into (4), as (Hanafy, 1987, p. 80):

$$T_{TI}(P) = \frac{GM}{r_P} \sum_{n=0}^{\infty} \left(\frac{R}{r_P}\right)^n \sum_{m=-n}^n \bar{T}_{nm} \bar{R}_{nm}(P), \quad (9)$$

where

$$\begin{aligned} \bar{T}_{nm} = & \frac{R^3}{M(2n+1)(n+3)} \iint_{\sigma} \left\{ \delta\rho_Q \left[\left(1 + \frac{H_Q}{R}\right)^{n+3} - 1 \right] + \right. \\ & \left. + \delta\rho_Q \left(1 - \frac{T_o}{R}\right)^{n+3} \left[\left(1 - \frac{t_Q}{R - T_o}\right)^{n+3} - 1 \right] \right\} \bar{R}_{nm}(Q) d\sigma_Q, \end{aligned} \quad (10)$$

where M denotes the mass of the earth, given by

$$M \doteq \frac{4\pi R^3}{3} \rho_M, \quad (11)$$

where ρ_M denotes the mean earth's density

$$\rho_M = 5.517 \text{ g/cm}^3.$$

For the practical determination of the harmonic coefficients of the topographic-isostatic potential, (10) may be written as

$$\begin{aligned} \bar{T}_{nm} = & \frac{3 \Delta\phi \Delta\lambda}{4\pi \rho_M (2n+1)(n+3)} \sum_i^{\phi} \sum_j^{\lambda} \left\{ \rho_{ij} \left[\left(1 + \frac{H_{ij}}{R}\right)^{n+3} - 1 \right] + \right. \\ & \left. + \Delta\rho_{ij} \left(1 - \frac{T_o}{R}\right)^{n+3} \left[\left(1 - \frac{t_{ij}}{R - T_o}\right)^{n+3} - 1 \right] \right\} \left\{ \begin{matrix} \cos m\lambda_j \\ \sin m\lambda_j \end{matrix} \right\} \bar{P}_{nm}(\cos \theta_i) \cos \phi_i, \end{aligned} \quad (12)$$

where $\sum$ denotes the summation along ϕ and λ , $\Delta\phi$ and $\Delta\lambda$ are the grid sizes of the used Digital Height and Moho Models in the latitude and the longitude directions, respectively, $\Delta\rho$ is always given by

$$\Delta\rho = \rho_1 - \rho_o, \quad (13)$$

and ρ is given by

$$\begin{aligned} \rho &= \rho_o & \text{for } H \geq 0, \\ \rho &= \rho_o - \rho_w & \text{for } H < 0, \end{aligned} \quad (14)$$

and ρ_w is the density of the ocean's water.

In case of the Airy-Heiskanen isostatic model, the thickness of the root/antiroot t is determined by applying the principle of mass balance, which can be written in the spherical approximation as (Rummel et al., 1988, p. 3)

$$\rho_o R^3 \left[\left(1 + \frac{H}{R} \right)^3 - 1 \right] = (\rho_1 - \rho_o)(R - T_o)^3 \left[1 - \left(1 - \frac{t}{R - T_o} \right)^3 \right]. \quad (15)$$

This condition can be written for the thickness of the root/antiroot t as follows:

$$\frac{t}{R - T_o} = 1 - \left\{ 1 - \frac{\rho}{\rho_1 - \rho_o} \left(1 - \frac{T_o}{R} \right)^{-3} \left[\left(1 + \frac{H}{R} \right)^3 - 1 \right] \right\}^{\frac{1}{3}}, \quad (16)$$

where ρ is given by (14).

4. Source Data

4.1. Gravity Data

The gravitational data set for this investigation is a set of free-air gravity anomalies at 3671 stations in Austria and neighbour countries. The data distribution within Austria is homogeneous (nearly on a grid of 6×6 km). The gravity data outside Austria (almost on a grid of 16×16 km) have been included to correct the edge effect in the computed gravimetric geoid for Austria.

4.2. Digital Height Models

Two different Digital Height Models are available for this investigation. A coarse model of $90'' \times 150''$ resolution in the latitude and the longitude directions, respectively, and a fine model of $11.25'' \times 18.75''$ resolution. The fine DHM covers the window $44.75^\circ \text{ N} \leq \phi \leq 50.25^\circ \text{ N}$; $7.75^\circ \text{ E} \leq \lambda \leq 19.25^\circ \text{ E}$. The coarse model covers the window $40^\circ \text{ N} \leq \phi \leq 52^\circ \text{ N}$; $5^\circ \text{ E} \leq \lambda \leq 22^\circ \text{ E}$.

The coarse digital height model has been created by integrating the Austrian fine DHM with GTOPO30 ($30'' \times 30''$) and global bathymetry model provided by the Naval Oceanographic Office ($1' \times 1'$). Figure 4 shows the coarse digital height model used for this investigation. It shows the high mountainous structure of the Alps.

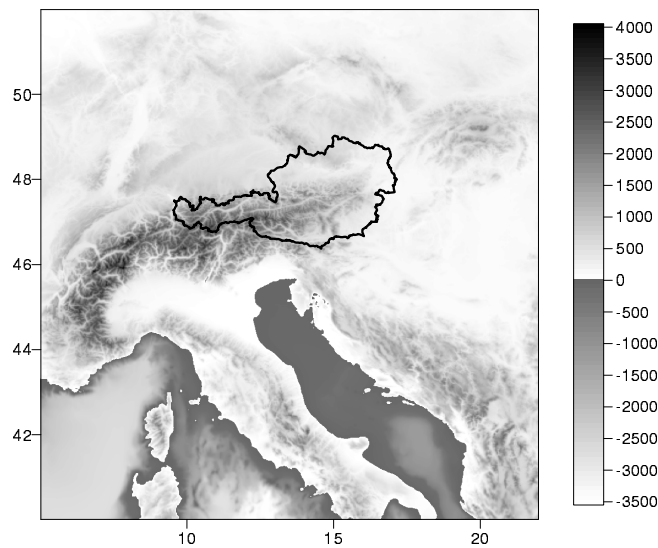


Fig. 4. The coarse ($90'' \times 150''$) digital height model.

43. Seismic Moho Data

A coarse Digital Moho Model ($90'' \times 150''$) has been created by interpolating the Geiss (1987) Moho data ($35^\circ \text{ N} \leq \phi \leq 50^\circ \text{ N}; 5^\circ \text{ W} \leq \lambda \leq 25^\circ \text{ E}$) with the Moho depths computed from the complex harmonic coefficients provided by Čadek and Martinec (1991) ($N_{max} = 70$). The coarse digital Moho model covers the window $40^\circ \text{ N} \leq \phi \leq 52^\circ \text{ N}; 5^\circ \text{ E} \leq \lambda \leq 22^\circ \text{ E}$. A program to compute the seismic Moho depths from the complex harmonic coefficients has been developed. The Geiss (1987) seismic Moho set has a $12' \times 20'$ resolution in the latitude and the longitude directions, respectively. A Kriging interpolation with an error variance equals 2 km^2 was used to create the coarse digital seismic Moho model. Figure 5 shows the coarse digital seismic Moho model for the data window. It shows the deeper Moho depths under the Alps.

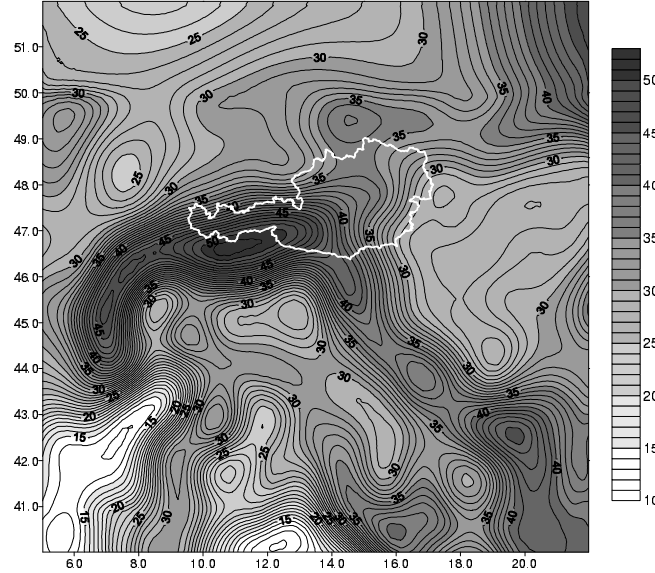


Fig. 5. The coarse $90'' \times 150''$ digital seismic Moho model for the data window.
Contour interval: 1 km

The fine ($11.25'' \times 18.75''$) digital seismic Moho model has been created by interpolating the coarse Moho model using Kriging interpolation with zero error variance. The fine digital seismic Moho model covers the window $44.75^\circ \text{ N} \leq \phi \leq 50.25^\circ \text{ N}; 7.75^\circ \text{ E} \leq \lambda \leq 19.25^\circ \text{ E}$.

44. Reference Field

The EGM96 global geopotential earth model represents the used reference field for the classical long-wavelength gravity reduction.

Two different adapted reference fields are computed within this investigation. The first adapted reference field has been created by subtracting the topographic-isostatic potential coefficients, computed by (12) using the Airy-Heiskanen isostatic model (t is computed by (16) with (14)), from the EGM96 coefficients. This adapted reference field will be called in the sequel *Airy adapted reference field*. Here, the following parameters set has been used,

$$\begin{aligned} T_0 &= 30 \text{ km}, \\ \Delta\rho &= 0.2 \text{ g/cm}^3. \end{aligned} \tag{17}$$

The second adapted reference field has been created by subtracting the topographic-isostatic potential coefficients, computed by (12) using the seismic Moho depths, from the EGM96 coefficients. This adapted reference field will be called in the sequel *Moho adapted reference field*. Here, the following parameters set has been used,

$$\begin{aligned} T_0 &= 32 \text{ km}, \\ \Delta\rho &= 0.2 \text{ g/cm}^3. \end{aligned} \tag{18}$$

The used digital height and Moho models for both adapted reference fields are the coarse ($90'' \times 150''$) digital height and Moho models covering the full window $40^\circ \text{ N} \leq \phi \leq 52^\circ \text{ N}$; $5^\circ \text{ E} \leq \lambda \leq 22^\circ \text{ E}$. The window topographic-isostatic potential coefficients has been computed starting from $n = 15$ to $N_{max} = 360$. The starting value of $n = 15$ is connected to the window-size and also comes in agreement with the known geophysical evidence that the lower potential coefficients have sources deeper than the Moho depths.

4.5. GPS Benchmarks

Figure 6 shows the distribution of the available GPS benchmarks with known orthometric heights in Austria. It shows that most of the stations are located in the eastern part of Austria. Only few stations are located at the mountainous western part of Austria. Figure 6 shows, more or less, a good distribution of the GPS benchmarks in Austria.

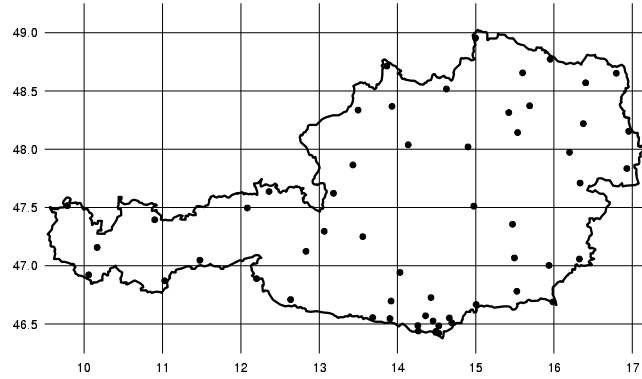


Fig. 6. GPS benchmarks with known orthometric heights.

5. Gravity Reduction

The geoid computation in this investigation is done through the remove/restore technique. This implies that the effect of the reference field and the effect of the topography and its compensation should be removed from the gravity anomalies. This can be expressed as follows:

$$\Delta g = \Delta g_{obs} - \Delta g_h - \Delta g_{Ref}, \tag{19}$$

where Δg_{obs} stands for the observed free-air anomalies, Δg_h is the effect of topography and its compensation on the gravity anomalies, and Δg_{Ref} is the effect of the reference field on the gravity anomalies. Thus the computed geoid N can be expressed by:

$$N = N_{Ref} + N_{\Delta g} + N_h, \tag{20}$$

where N_{Ref} gives the contribution of the reference field, $N_{\Delta g}$ gives the contribution of the reduced gravity anomalies, and N_h gives the contribution of the topography and its compensation (the indirect effect).

The EGM96 global geopotential model, the Airy adapted reference field and the Moho adapted reference field complete to degree and order 360 have been used to compute the effect of the reference field on the gravity anomalies Δg_{Ref} . The reader who may be interested in the software which can

be used for geopotential computations from spherical harmonic expansions is kindly invited to refer to, e.g., (Rapp, 1982; Tscherning et al., 1983, 1992; Abd-Elmotaal, 1998).

The effect of the topographic-isostatic masses on the gravity anomalies has been computed for the geoid computed by the traditional Airy-Heiskanen model (Airy geoid) using the TC-program (Forsberg, 1984) for all masses within a radius of 167 km around the computational point. For the other two geoids, the topography and its compensation for the whole fixed data window ($40^\circ \text{ N} \leq \phi \leq 52^\circ \text{ N}$; $5^\circ \text{ E} \leq \lambda \leq 22^\circ \text{ E}$) have been considered. For the Moho geoid, the seismic Moho depths were used to compute the compensating masses by the formula

$$t = T_{Moho} - T_o, \quad (21)$$

where t represents the depth of the compensating masses, T_{Moho} represents the Moho depth and T_o is the normal crustal thickness. Hence the TC-program has been modified to compute the effect of the seismic Moho depths (or rather, the seismic compensating masses t) on the gravity anomalies.

A constant density of the topography and crust ρ of

$$\rho = 2.67 \text{ g/cm}^3, \quad (22)$$

has been used with the parameter sets (17), (17) and (18) for *Airy*, *Airy window* (adapted Airy geoid with fixed window) and *Moho window* (adapted Moho geoid with fixed window) geoids, respectively.

Table 1 shows the statistics of the reduced gravity anomalies. Here the subscripts *Airy*, *Airy win* and *Moho win* refer to the method used for the reduction, as stated above.

gravity anomalies	statistical parameters			
	min.	max.	mean	st. dev.
	mgal	mgal	mgal	mgal
Δg_{obs}	-144.29	200.65	9.15	43.25
$\Delta g_{obs} - \Delta g_{Ref} - \Delta g_{Airy}$	-122.70	66.48	-19.46	27.09
$\Delta g_{obs} - \Delta g_{Ref} - \Delta g_{Airy win}$	-85.80	89.32	4.90	26.04
$\Delta g_{obs} - \Delta g_{Ref} - \Delta g_{Moho win}$	0.33	238.02	106.15	49.84

Table 1. Statistics of the reduced gravity anomalies (3671 gravity stations)

Finally, the reduced gravity anomalies are gridded on a $3' \times 3'$ geographical grid using Kriging interpolation. This step is needed for the geoid computation through an FFT technique.

6. Geoid Computation and Comparison

The geoid for Austria has been computed using the remove/restore technique, eqs. (19) and (20). The contribution of the gravity anomalies on the geoid undulation $N_{\Delta g}$ has been computed by 1D-FFT technique using the FFTGEOID program by (Sideris and Li, 1993). The contribution of the topography and its compensation N_h has been computed using the modified TC-program. The EGM96 global geopotential earth model, the adapted Airy reference field and the adapted Moho reference field complete to degree and order 360 have been used to compute the contribution of the reference field on the geoid undulations N_{Ref} .

Figure 7 shows the absolute difference between the geoid computed by using the traditional remove/restore technique with Airy-Heiskanen isostatic model N_{Airy} and the geoid derived from the combination of the GPS and levelling N_{GPS} . It shows that the difference ranges from 2.21 m to 3.64 m with an average of 2.99 m and a standard deviation of 0.36 m. The structure of the differences shows a non-linear long-wavelength behaviour.

Figure 8 shows the absolute difference between the geoid computed by using the adapted remove/restore technique with Airy-Heiskanen isostatic model $N_{Airy win}$ and N_{GPS} . It shows that the

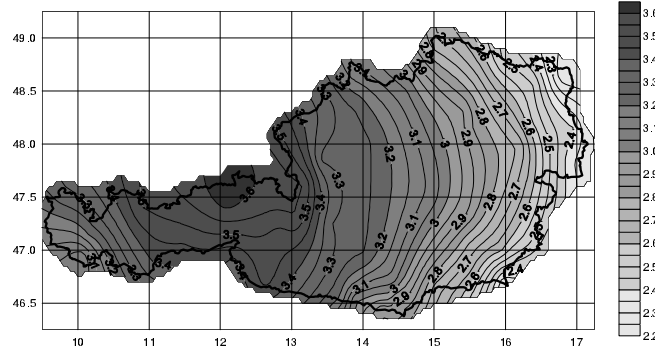


Fig. 7. Absolute geoid difference ($N_{GPS} - N_{Airy}$) for Austria. Contour interval: 5 cm.

difference ranges from -1.24 m to -0.20 m with an average of -0.80 m and a standard deviation of 0.24 m. Figure 8 shows the smallest absolute geoid difference compared to the GPS geoid.

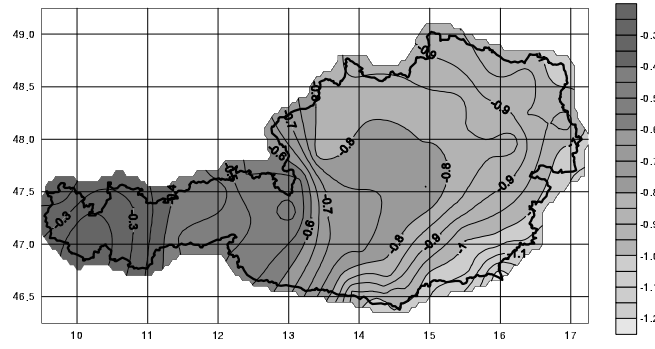


Fig. 8. Absolute geoid difference ($N_{GPS} - N_{Airy\ win}$) for Austria. Contour interval: 5 cm.

Figure 9 shows the absolute difference between the geoid computed by using the adapted remove/restore technique with seismic Moho depths $N_{Moho\ win}$ and N_{GPS} . It shows that the difference ranges from -13.74 m to -10.65 m with an average of -12.16 m and a standard deviation of 0.93 m. Figure 9 shows an approximate linear trend of the absolute geoid difference.

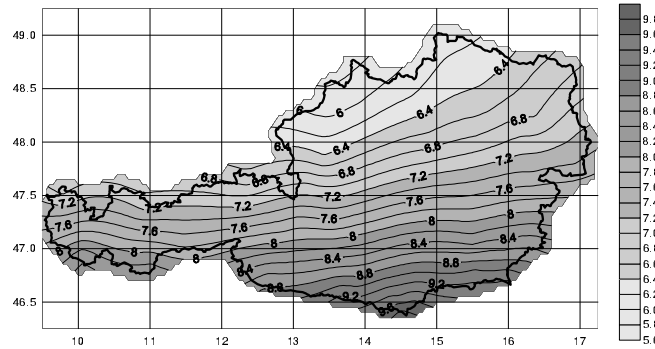


Fig. 9. Absolute geoid difference ($N_{GPS} - N_{Moho\ win}$) for Austria. Contour interval: 20 cm.

The computed geoids N_{Airy} , $N_{Airy\ win}$ and $N_{Moho\ win}$ have been fitted to the GPS geoid N_{GPS} using 56 GPS stations in Austria. A third order polynomial has been removed from the absolute geoid differences. The remaining differences represent the precision of the fitted geoid. Table 2 lists the statistics of the remaining differences at the GPS benchmarks after removing the trend function. Table 2 shows that the standard deviation of the remaining differences for all geoids is practically the same. The Airy window geoid has the smallest standard deviation. The range of the remaining difference for the Moho window geoid is slightly better.

geoidal height	statistical parameters			
	min.	max.	range	st. dev.
$N_{GPS} - N_{Airy}$	-15.4	9.1	24.5	5.0
$N_{GPS} - N_{Airy\ win}$	-15.7	8.9	24.6	4.8
$N_{GPS} - N_{Moho\ win}$	-13.4	9.7	23.1	4.9

Table 2. Statistics of the remaining differences at the GPS benchmarks after removing a third order polynomial trend function. All values are in cm

Table 3 shows the difference between the fitted geoids. It shows that these differences are in the same order of magnitude as the geoid accuracy.

geoidal height	statistical parameters			
	min.	max.	mean	st. dev.
$N_{Airy} - N_{Airy\ win}$	-16.8	12.7	0.0	4.1
$N_{Airy} - N_{Moho\ win}$	-19.0	20.0	0.6	5.7
$N_{Airy\ win} - N_{Moho\ win}$	-14.2	19.7	0.5	3.5

Table 3. Statistics of the differences between the fitted geoids. All values are in cm

7. Conclusion

An adapted method for the geoid computation within the remove/restore technique has been introduced. The problem of the double consideration of a long-wavelength part of the topography and its compensation has been taken into account by adapting the reference field. The seismic Moho depths have also been used representing the compensating masses instead of assuming an isostatic model.

Using the adapted Airy reference field with Airy-Heiskanen isostatic model gives the best reduced anomalies (both centered and minimum variance). The Airy window geoid fits the GPS geoid with a minimum range and standard deviation. The absolute difference between Moho window and GPS geoids shows nearly a linear trend, but with a relatively large range. After removing a trend part from the absolute difference, all geoids, however, have practically the same precision. The fitted Moho window geoid has the smallest range-difference compared to the GPS geoid. The fitted Airy window geoid has the smallest standard deviation.

The difference between the fitted geoids is in the same order of magnitude as the geoid precision. It is believed that using a variable density contrast in case of Moho window geoid will significantly improve its precision.

References

- Abd-Elmotaal, H. (1998) An Alternative Capable Technique for the Evaluation of Geopotential from Spherical Harmonic Expansions. *Bollettino di Geodesia e Scienze Affini*, Vol. 57, pp. 25–38.
- Čadek, O. and Martinec, Z. (1991) Spherical Harmonic Expansion of the Earth's Crustal Thickness up to Degree and Order 30. *Studia geoph. et geod.*, pp. 151–165.
- Forsberg, R. (1984) A Study of Terrain Reductions, Density Anomalies and Geophysical Inversion Methods in Gravity Field Modelling. Report No. 355, Department of Geodetic Science, The Ohio State University, Columbus, Ohio.
- Geiss, E. (1987) A New Compilation of Crustal Thickness Data For The Mediterranean Area. *Annales Geophysicae*, Vol. 5B, No. 6, pp. 623–630.

- Hanafy, M. (1987) Gravity Field Data Reduction Using Height, Density, and Moho Information. Ph.D. Dissertation, Institute of Theoretical Geodesy, Graz University of Technology, Graz.
- Heiskanen, W.A. and Moritz, H. (1967) Physical Geodesy. Freeman, San Francisco.
- Rapp, R.H. (1982) A Fortran Program for the Computation of Gravimetric Quantities From High Degree Spherical Harmonic Expansions. Report No. 334, Department of Geodetic Science, The Ohio State University, Columbus, Ohio.
- Rummel, R., Rapp, R.H., Sünkel, H. and Tscherning, C.C. (1988) Comparison of Global Topographic/Isostatic Models to the Earth's Observed Gravity Field. Report No. 388, Department of Geodetic Science, The Ohio State University, Columbus, Ohio.
- Sideris, M.G. and Li, Y.C. (1993) Gravity Field Convolutions Without Windowing and Edge-Effects. Bulletin Géodésique, Vol. 67, pp. 107–118.
- Sünkel, H. (1985) An Isostatic Earth Model. Report No. 367, Department of Geodetic Science, The Ohio State University, Columbus, Ohio.
- Tscherning, C.C., Forsberg, R. and Knudsen, P. (1992) The GRAVSOFTE Package for Geoid Determination. Presented at the 1st Continental Workshop on the European Geoid, Prague, May 1992.
- Tscherning, C.C., Rapp, R.H. and Goad, C. (1983) A Comparison of Methods for Computing Gravimetric Quantities From High Degree Spherical Harmonic Expansions. Manuscripta geodaetica, Vol. 8, pp. 249–272.

Efforts in predicting the local gravity field

Urs Marti

Federal Office of Topography, Seftigenstrasse 264, CH-3084 Wabern, Switzerland

ABSTRACT

A problem of the adjustment of the new national height system LHN95 [Marti et al. 1998] is the prediction of gravity values on levelling stations without observed gravity. The aim is to predict the surface gravity and the gravity in tunnels and caves with an accuracy of better than 1 mgal ($=10^{-5} \text{ ms}^{-2}$) everywhere in Switzerland, which is about the accuracy we need for levelling purposes.

The principal data set for the prediction and interpolation are the gravity measurements of the Swiss Geophysical Commission (SGPK) which have been gathered and processed during the last few years (fig. 1). The gravity has been measured in that project with a density of about 1 point per km^2 . The second important data set is the airborne gravity set of Switzerland which may be of lower accuracy but helps a lot to determine gravity gradients (Klingel  et al. 1996).

The first results showed that in geologically difficult regions it was not possible to predict the gravity with the requested accuracy out of Bouguer anomalies or isostatic anomalies. Models for the density distribution of the interior of the earth had to be introduced in order to get sufficient results.

The method we use to predict gravity can be summarised as follows: We calculate Bouguer-anomalies for all gravity stations and subtract the influence of local density models. The residuals then usually show a rather smooth behaviour and can be interpolated by means of collocation.

The results showed that with this method it is possible to predict gravity in more than 90% of all cases with an accuracy of better than 1 mgal even in mountainous areas.

1. Terrestrial Gravity Data

Most of the terrestrial gravity data in Switzerland has been measured, collected and re-evaluated in a project of the Swiss Geophysical Commission. These about 30'000 stations (fig. 1) are distributed more densely in flatter areas and in areas of special interest, whereas in mountainous regions, the distribution of the stations is much less dense. The main purpose of this project was the mapping of Bouguer Anomalies at a scale of 1:100'000 [Dumont et al. 1994-2001]. All the stations are referred to the IGSN71 gravity system by connecting them to the new gravity base network SG95 of Switzerland [Arnet et al. 1997]. Some older measurements [Klingel  et al. 1980] have also been transferred into the IGSN71 system.

An additional set of gravity measurements (about 4500 stations) along the first and second order levelling lines is measured since 1953 by the Swiss Geodetic Commission (SGK). Usually the gravity measurements along these lines are performed in the same year as the levelling measurements. On newer lines the density of the gravity stations is much higher than on the older ones. On repeated levelling lines, no significant change of gravity could be observed until now.

Not every levelling benchmark or levelling line has observed gravity values. On such points gravity has to be interpolated by means of the measurements of nearby stations.

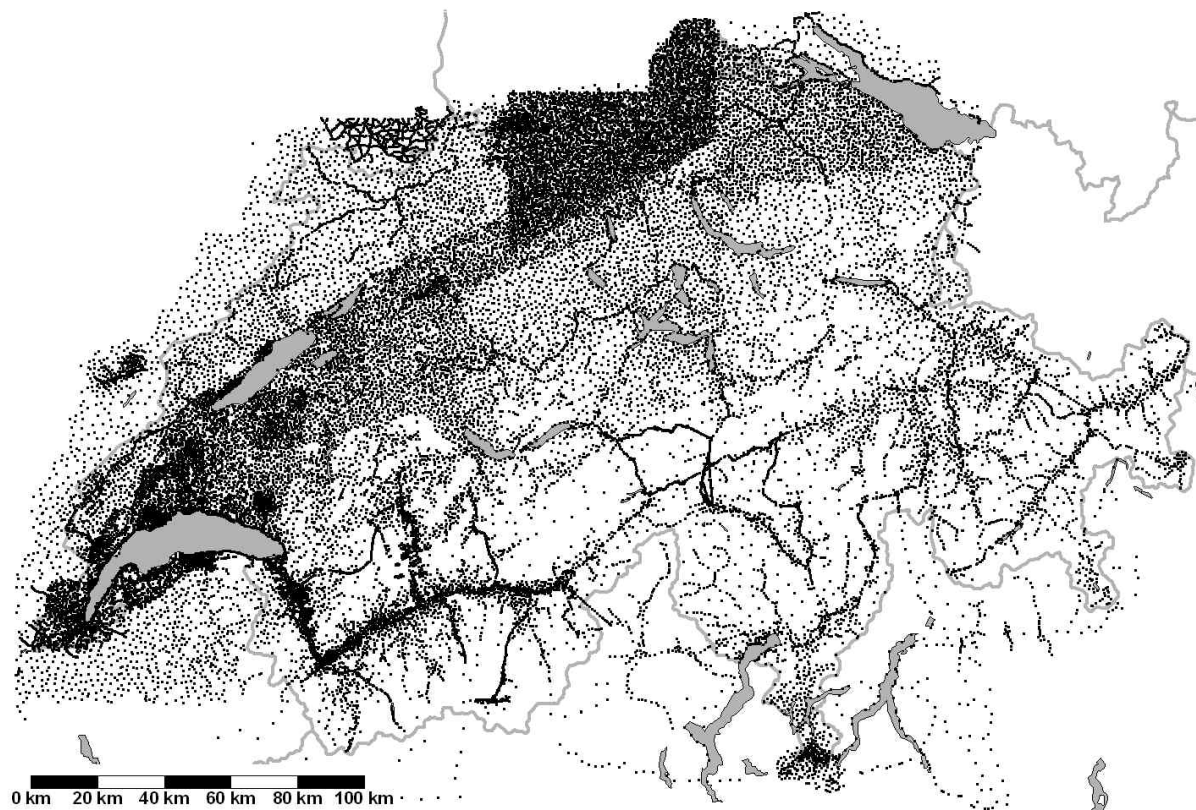


Fig. 1: Terrestrial gravity data of Switzerland at January 2001

2 Airborne Gravimetric Data

In November and December 1992 the whole country has been covered by airborne gravimetric observations [Klingel  et al. 1996] during a project of the Geodesy and Geodynamics Lab (GGL) of the Federal Institute of Technology (ETH) of Zurich. The main characteristics of this survey were the following:

- Flight altitude 5200 m
- Aircraft speed 240 km/h
- Gravimeter sampling rate 1 sec

This resulted in a total of more than 60'000 data points. The differences between upward continued ground data and the airborne Bouguer anomaly give a mean offset of 0.6 mgal and a standard deviation of 11.9 mgal.

This data set may be of lesser quality than the ground data and is not yet included in the prediction of interpolated gravity values but it would be very useful in regions where terrestrial measurements are missing (mountains, glaciers, lakes) and mainly for the purpose of estimating gravity gradients and therefore also for the step of downward continuation of the gravity field (including the determination of the geoid and predicting deflections of the vertical).

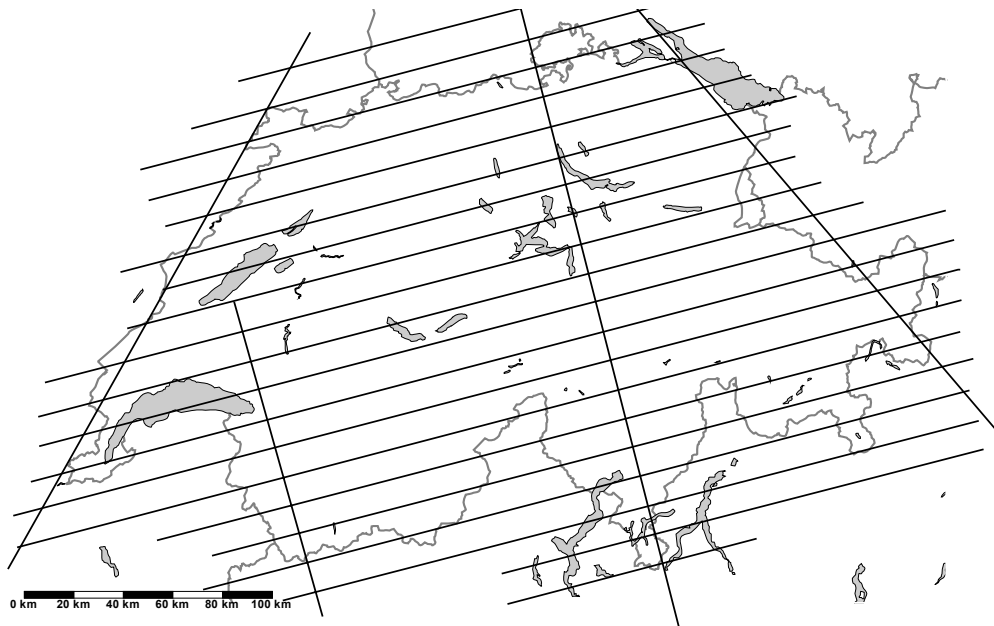


Fig. 2: Aerogravimetric Flight lines 1992

3. The mass models

The mass models used for the gravity prediction are the same as the ones used for the geoid determination in Switzerland [Marti, 1997] and for the calculation of orthometric heights. This means, that a 25 meter DEM and a 5 km model of the Moho depth are the principal models. In addition, models of local density anomalies (such as water masses of lakes, ice masses of glaciers, geological structures and sediments of the largest rivers) are introduced. Most of these models are formed by a grid of vertical rectangular prisms, whereas others consist of irregularly distributed rectangle and triangle prisms. Some other models are constructed by means of irregular polyhedrons (finite elements).

4. The Method of Interpolation

The principle of predicting absolute gravity values is to subtract all known influences of masses from the observed gravity. The remaining residuals should be smooth enough to interpolate. Furthermore we suppose that the remaining residuals do not cause significant vertical gradients (at least at the altitudes we are interested in). Therefore the residuals are interpolated by an empirical 2-dimensional least squares collocation approach based on the Gaussian function of normal distribution.

In a first step, Bouguer anomalies are calculated mainly out of a 25-m Digital Terrain Model with standard density of 2.67 kgm^{-3} . These Bouguer anomalies mainly show the influence of the depth of the crust-mantle boundary (Moho) and of the Ivrea body in the South of the country. Since the Bouguer anomalies show some correlation with the topography, an important vertical gradient is still contained in the data. Bouguer anomalies are therefore not very suitable to be used for gravity interpolation on different altitudes.

In a second step, further effects are eliminated. These are mainly the influences of the Moho depth (instead of an isostatic model) and the Ivrea body. But also smaller effects

such as the water masses of lakes, the ice masses of glaciers and local density anomalies are taken into account. The resulting residuals show a rather smooth behaviour and are not correlated with topography. The effects of local density anomalies which have not been modelled remain clearly visible - mainly the unmodelled sediments of some rivers cause local residual anomalies of a few mgal. The residuals show a rather strong trend from Northwest to Southeast which until now can not be explained by geological or geophysical reasons. Even by varying the density contrast at the Moho this trend can not be eliminated. Since these residuals are not correlated with topography we assume that most of the effects causing vertical gradients are eliminated and we use them for the 2D-interpolation by means of collocation. The absolute gravity value on any station is obtained by adding the effects of all models to the interpolated residual gravity.

5. Testing The Interpolation

A first test was to predict the surface gravity at the gravity stations out of the measurements of their neighbours. This allowed us to eliminate gross errors in the original data set and gave us an idea of the obtainable accuracy. The results are shown in fig. 3 as a histogram, where we see that for about 60% of all points the interpolation gives a result that agrees better than 0.5 mgal with the measured values. For more than 90% of all interpolated values the agreement is better than 1.5 mgal. Except from regions with less dense data coverage, the residuals are smaller than 1-2 mgal in most parts of the country. Near the rivers with unmodelled sediment basins, larger systematic differences in the order of 2-3 mgal may appear.

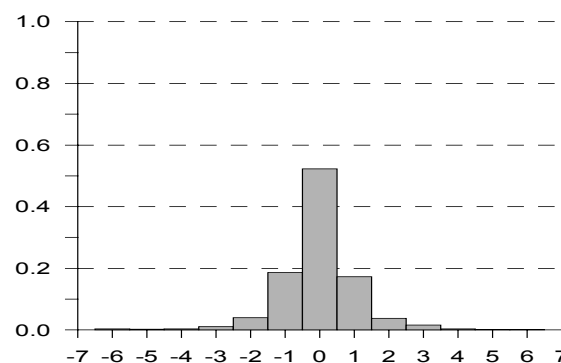


Fig. 3: Histogram of the differences between measured and interpolated surface gravity values in mgal (cross validation of the data set of the SGPK, ~31'000 points).

In a second test the measured gravity values at the levelling benchmarks have been compared with the interpolated values out of the SGPK data set (see fig. 4). The result seems to be slightly less good than in the first test. But again, most of the differences are smaller than 1 mgal. Larger differences appear again near the rivers with sediment basins. The biggest differences of up to 7 mgal are reached on some benchmarks in tunnels. This is not a very bad result but implies all the same, that the residuals used for interpolation still contain components causing vertical gravity gradients. After this test the data set of the levelling benchmarks was combined with the data set of the SGPK. This allows us - also in tunnels - to get results on the 1 mgal level.

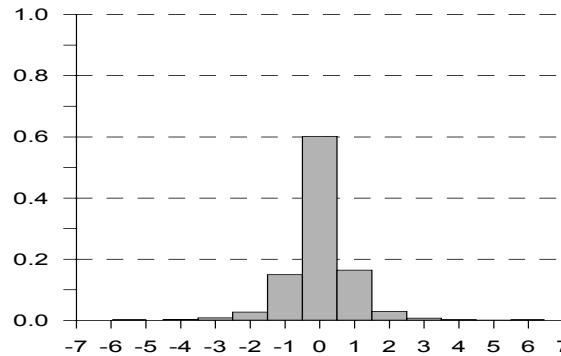


Fig. 4: Histogram of the differences between measured and interpolated surface gravity values at the levelling benchmarks in mgal (interpolation of ~4500 independent stations).

6. Conclusions

The actual amount and distribution of the gravity measurements is now sufficient to interpolate absolute gravity values in most parts of Switzerland with an accuracy of about 1 mgal or better.

Problematic areas can be found in regions with unmodelled local density anomalies (mainly sediment basins of rivers) and at the edges of our models. The reason for this can be found in the fact that density contrasts cause rather large horizontal gravity gradients, which can not be easily represented by measurements in the vicinity. This implies that the density models have to be constructed rather carefully and that it is very important that its geometric position and resolution has to be of high accuracy for the purpose of gravity interpolation. An example for this fact can be seen in fig. 5 where we represent the gravity effect of an artificial body with a typical density contrast of -0.67 kgm^{-3} (standard density to density of quaternary sediments) along a profile. The variation of the horizontal position of the disturbing mass of about 100 meters can change the effect on gravity for several mgal. This results in very difficult interpolation conditions near the boundaries of these masses.

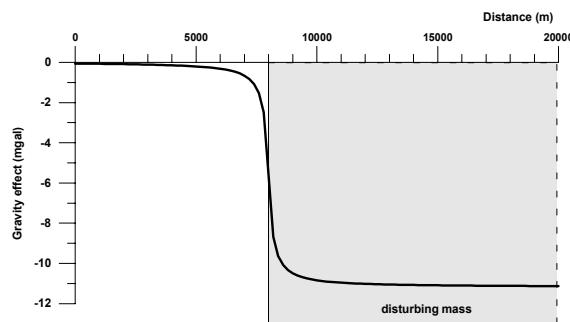


Fig 5: Effect of an artificial disturbing mass on gravity (density contrast $\sigma = -0.67 \text{ kgm}^{-3}$)

Until now our density model was mainly used for the determination of the disturbing potential (geoid) and the prediction of deflections of the vertical. For this purpose this model fulfilled our demands, but it is not sufficient for the task of interpolating surface gravity

values with an accuracy of 1 mgal. The improvement and refinement of the 3-dimensional density model is therefore the point where further efforts and investigation would be very helpful.

In large tunnels gravity can be predicted out of surface observations with an accuracy of only about 3 to 5 mgal. This indicates that the model of the density distribution in the upper crust has to be refined and that our assumptions and calculations of vertical gradients are not sufficient. We hope to increase the accuracy of the estimation of this gradient by including the airborne data in our model and changing from a 2-dimensional to a 3-dimensional interpolation. How much the integration of a global gravity model such as EGM96 can help the estimation of vertical gradients has still to be investigated.

For the moment we solve the problem of interpolating gravity values in tunnels by dense gravity observations in the tunnels themselves and by using only these measurements for the interpolation on the remaining benchmarks. This allows us a reasonable interpolation as well.

As a summary can be said that in 90% of all cases and under normal circumstances, the gravity interpolation on any point in Switzerland is possible with an accuracy in the order of 1 mgal.

On levelling lines in flatter areas with good density models it can even be said that no more gravity measurements are necessary. The interpolated values obtained of the existing data set fulfil all the needs of levelling.

Acknowledgements

We would like to thank the Swiss Geophysical Commission (SGPK), the Swiss Geodetic Commission (SGK) and the Geodesy and Geodynamics Lab (GGL) of the Federal Institute of Technology (ETHZ) for their great efforts of the last years in the field of measuring and evaluating gravity in Switzerland.

References

- Arnet F., E. Klingel  (1997): SG95: Das neue Schweregrundnetz der Schweiz. *Geod tisch-geophysikalische Arbeiten in der Schweiz*, Vol. 54, Z rich 1997.
- Dumont, B., E. Klingel , Ph. Logean, R. Olivier (1994-2001): Gravimetric Atlas of Switzerland 1:100'000 (Bouguer anomaly). Swiss Geophysical Commission.
- Klingel , E., R. Olivier (1980): La nouvelle carte Gravim trique de la Suisse. *G ophysique* Nr. 20. Swiss Geophysical Commission.
- Klingel , E., M. Cocard, M. Halliday and H.-G. Kahle (1996): The Airborne Gravimetric Survey of Switzerland. *G ophysique* Nr. 31. Swiss Geophysical Commission.
- Marti, U. (1997): Geoid der Schweiz 1997. *Geod tisch-geophysikalische Arbeiten in der Schweiz*, Vol. 56, Z rich 1997.
- Marti, U. and D. Schneider (1998): The new Swiss National Height System LHN95. In: *IAG Symposia Volume 118: Advances in Positioning and Reference Frames*. Springer 1998.

Combining Levelling with GPS Measurements and Geoid Information

Urs Marti, Andreas Schlatter and Elmar Brockmann

Federal Office of Topography, Seftigenstrasse 264, CH-3084 Wabern, Switzerland

ABSTRACT

The demand of using GPS for height determination and to replace expensive levelling measurements in regions where not the maximum accuracy is needed, requests a height system in which levelling and GPS in combination with geoid information lead to compatible results. The new kinematic height system LHN95 of Switzerland should fulfil this task.

The geoid of Switzerland CHGEO98 with an accuracy of 3 to 5 cm over the whole country is available since 1998 and all the levelling measurements since 1902 are now available in digital form and have been evaluated in a kinematic adjustment process. More than 120 levelling bench marks are also connected to the national GPS network, which has been re-measured in 1998. This gives us in a first step an excellent data set for comparing and analysing the different results. In a second step we will try to include the orthometric heights obtained out of GPS and the geoid into the adjustment of the levelling network. This should strengthen the solution over long distances.

In addition to these more general investigations two special GPS campaigns have been performed in mountainous regions where we wanted to test the possibilities to replace levelling by GPS measurements. The evaluation revealed promising results but also that much attention has to be paid on the GPS evaluation method. Especially the modelling of the influence of the troposphere plays a major role. Just with standard models it is not possible to get satisfactory results. More sophisticated modelling methods have to be applied.

1. INTRODUCTION

One of the main problems of surveyors today is the accurate height determination. Levelling over longer distances is usually too expensive and too time consuming and therefore the users prefer the much cheaper GPS method. Of course, they have to apply the correction of an accurate geoid or quasigeoid model to get orthometric or normal heights which are usually used as national height systems. But this arises the problem that in general, these heights are not in accordance with the heights obtained out of levelling. The simple reason for this discrepancy is the simple fact that all the implied data sets (GPS, levelling, geoid) show smaller or larger random and systematic errors.

The goal for a national height system therefore has to be a common treatment of the 3 data sets and to reach consistency between them. The simple condition, that the ellipsoidal height has to be equal to the sum of orthometric height and the geoid undulation has to be fulfilled. Only under these circumstances it is possible to replace levelling by cheaper GPS measurements at least in regions where not the highest accuracy is needed. A first step in this direction has to be

- a good connection between the GPS and the levelling networks through many GPS/levelling stations
- an accurate geoid model
- and a detailed analysis of the involved data sets and the differences between them

2. The available data sets

2.1 The national GPS network

The Swiss national GPS network LV95 was originally established between 1989 and 1994. Since then it is systematically densified and consists today of about 240 principal and densification points. 22 of these points form the permanent GPS network AGNES (Automated GPS Network of Switzerland). The whole network was re-measured in 1998 and the results assured us that the accuracy of the network is clearly better than 1 cm in position and about 1 to 3 cm in height [Gubler *et al*, 1999]. The network and its connection to the national levelling network is displayed in fig. 1.

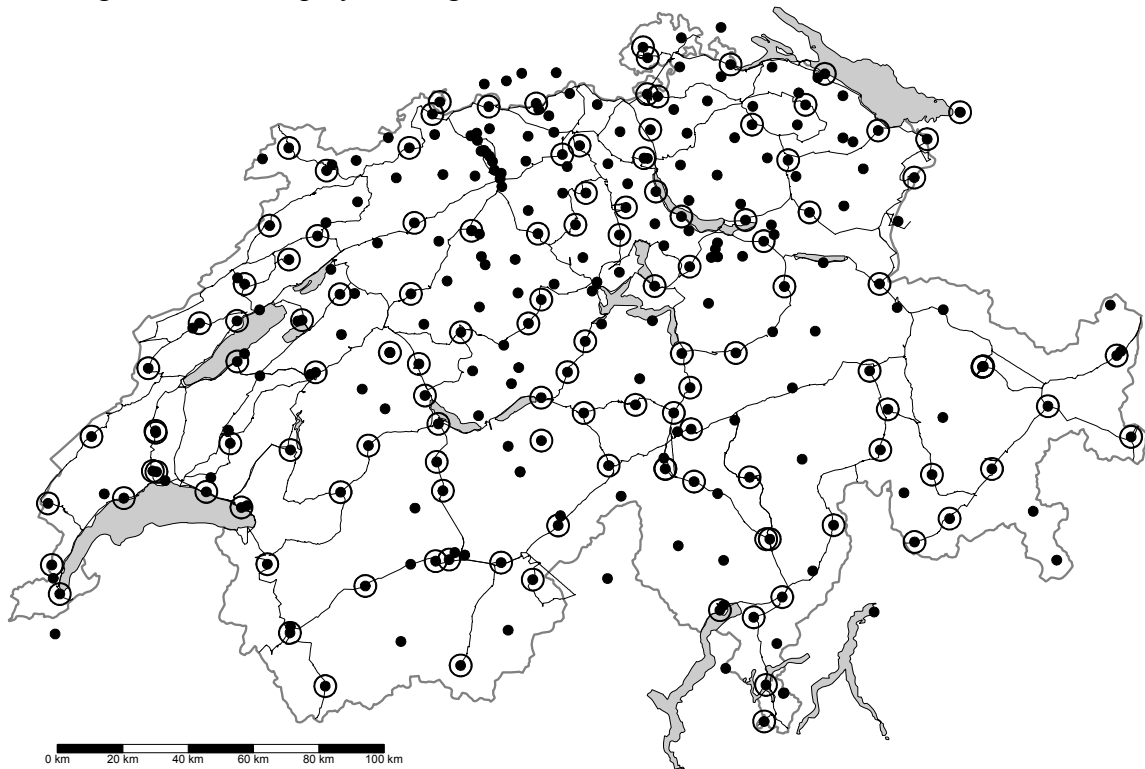


Fig. 1: The national GPS network LV95 (including the permanent network AGNES) and its connection to the levelling network LHN95. GPS/levelling stations are marked with circles.

2.2 The national Levelling network

The national first and second order levelling network (LHN95) has a length of about 4000 km and contains about 11'000 bench marks [Marti *et al*. 2001]. All measurements in this network since 1902 are available in digital form and most of the lines are at least observed in 2 epochs. This allows us to treat LHN95 as a kinematic network and to estimate vertical movements. LHN95 is an orthometric height system but in order to assure the data exchange with neighbouring countries, normal heights are available as well. The orthometric heights are obtained out of the geopotential numbers by division through the mean gravity along the plumbline, which itself is estimated out of surface gravity and a density model of the upper Earth's crust. The accuracy of LHN95 is about 0.7 mm/km or about 1 cm/100 km.

2.3 The geoid of Switzerland

The 3rd data set involved is the geoid model of Switzerland CHGEO98 [Marti, 1997]. It is available also in form of a quasigeoid. For its determination, about 600 astronomically observed deflections of the vertical and 70 connections between GPS stations and levelling bench marks were used. Gravity measurements were only used in the step of downward continuation.

Furthermore the same mass models as in the calculation of orthometric heights were used in the remove/restore step. These are mainly a 25-meter DEM and density models for crustal structures. After reducing the measurements from the effects of topographic masses, the residuals were interpolated by means of least squares collocation. The final geoid was calculated by re-adding the effects of the mass models. The result revealed an accuracy of 2 to 3 cm in the flatter areas and of about 5 cm in the Alps. This could be verified by a comparison with independent GPS/levelling stations and by a comparison with the European gravimetric geoid EGG97 [Denker and Torge, 1997]. These two totally independent solutions showed (at least in flatter areas) an accordance of better than 5 cm.

3. Comparison of the data sets

For about 120 stations all the 3 quantities ellipsoidal height, orthometric height and geoid undulation are available and we can form differences of the orthometric height out of levelling minus orthometric heights out of GPS and the geoid without applying offset or tilt parameters. These differences are displayed in fig. 2. Another comparison on the level of normal heights and height anomalies gave slightly better results since the density distribution in the Earth's crust is of no importance. But, in our case, these differences between the two comparisons are really small (up to 1 cm) and so it is not worth to discuss both of them. In most parts of the country the differences are smaller than 3 cm, which indicates that levelling, the geoid and GPS yield rather consistent results. In flatter areas the agreement is even better than 2 cm. But all the same, there are regions with systematic differences of more than 5 cm. This is mainly the case in the Southeast of Switzerland, but also in the North a systematic trend of the residuals of up to 4 cm exist. The most reasonable explanation up to now is that these differences are caused mainly by local problems of the geoid. It seems less likely that the problem is caused by systematic errors of the levelling because of the small residuals of the adjustment of the levelling network. A systematic trend of the GPS can most probably be excluded as well. The entire GPS network LV95 was re-measured in 1998 [Gubler et al., 1999] and great attention was paid to troposphere modelling and therefore also to height determination. Only small systematic differences of up to 5 cm to the original result of 1995 could be detected which may explain some of the differences in the North but they do not explain the differences in the South and Southeast.

A common adjustment of all the 3 data sets has not been performed yet. The principal problems that have to be resolved first are the detection and elimination of systematic errors, a reasonable weighting between the different types of observations but also to get the full variance-covariance information of the geoid. A first approach for this common adjustment can be found in [Kotsakis et al. 1999].

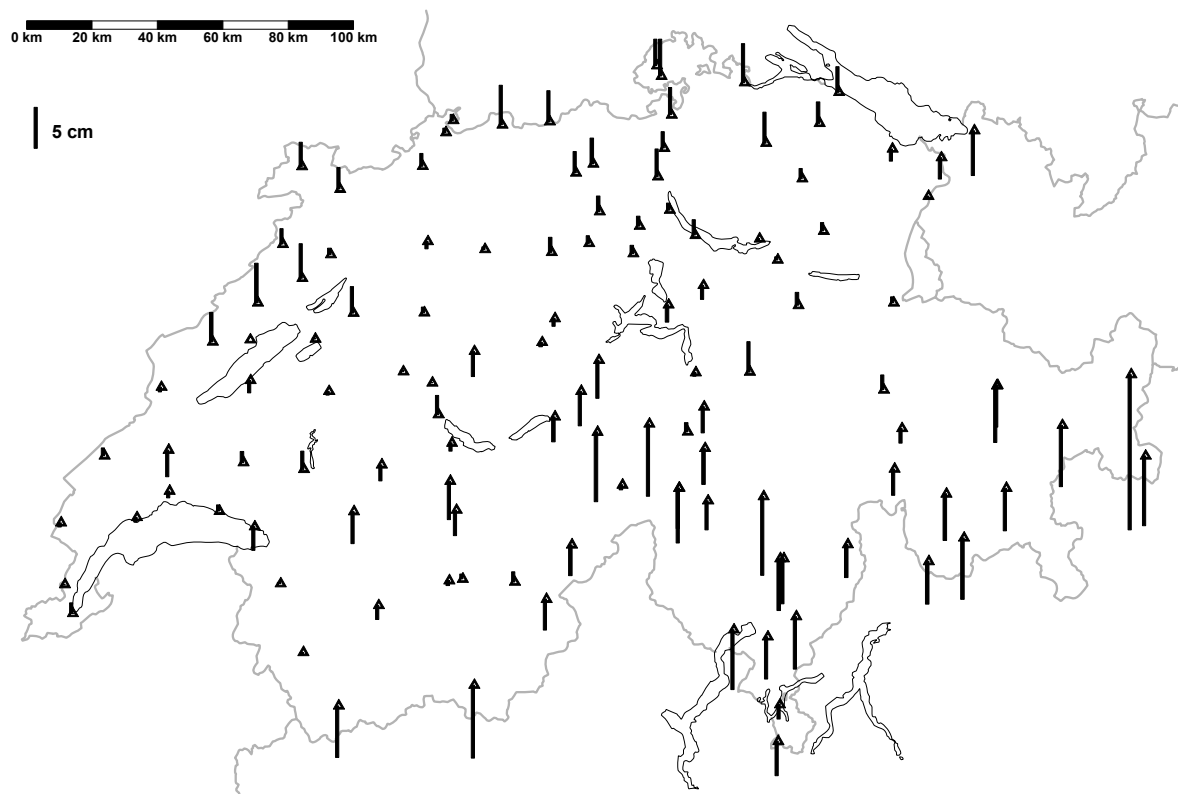


Fig. 2: Differences between orthometric heights out of levelling and orthometric heights out of GPS and geoid information.

4 The Possibilities of Substituting Levelling by GPS

One of the main goals of a common treatment of levelling, GPS and the geoid is to substitute expensive levelling through cheaper GPS measurements at least in regions where not the highest accuracy is necessary. In order to prove the capability of GPS for height determination, two test campaigns were carried out in 1998 on 4 consecutive days: the first one in the Emmental, a pre-Alpine hilly region with height differences of up to 500 metres and the second one at the Sustenpass, in a high Alpine area with height differences of up to 1700 metres. Static GPS measurements were carried out since it was obvious from the beginning that kinematic methods would not give the requested accuracy. But since GPS should remain a cheaper method than levelling, no enormous effort was put into the observations. A total of about 65 levelled stations were observed in day and night sessions of 3 to 12 hours. Mainly the impact of troposphere modelling, cut-off angle, session length, topography and GPS evaluation software was tested.

Alpine-scale height differences observed by GPS are biased mainly by a mismodelling of the troposphere. In collaboration with the Institute of Geodesy and Photogrammetry ETH Zurich (A. Geiger), zenith path delays were computed from meteorological data (ANETZ data) of the Swiss Meteorological Institute (SMI) using the 4-dimensional modelling software called COMEDIE [Hirter, 1998].

Comparisons between GPS and levelling were performed by transforming the GPS ellipsoidal heights into orthometric heights using the geoid model CHGEO98.

The campaigns were designed to answer the following questions:

- The impact of different processing strategies and different troposphere models (different a-priori models, estimation of tropospheric zenith path delays from GPS observations, introduction of zenith path delays from COMEDIE) on the GPS height estimation.
- Impact of data analysis of different GPS processing packages. Commercial packages and the scientific package Bernese 4.1. [Beutler *et al.* 1996] were used.
- Impact of the session length on the height estimates.
- Accuracy of the GPS heights compared with levelling / geoid information.

4.1 The test campaign Emmental

The Emmental measurements were carried out in June 1998 to investigate the possibility of replacing second and third order levelling in hilly regions by GPS and a geoid model. The test network consists of a total profile length of 44 km with a total height difference of about 300 m following a recently re-measured levelling line of the national levelling network LHN95.

The evaluation of this GPS network has been performed with several software packages and by varying as many parameters as possible. Some characteristics of the results can be summarised as follows:

- for baselines longer than 10 km, a L3 solution has to be used; a L1/L2 solution gives no accurate heights.
- to obtain good heights, a weighting of the GPS observations with zenith angle or a higher cut-off angle has to be used
- Troposphere parameter estimation out of GPS is not reliable for sessions shorter than 3 hours
- commercial software packages give comparable results with a standard evaluation with the Bernese software
- The best height repeatability is obtained by using the COMEDIE troposphere model
- using a standard Saastamoinen troposphere model gives a rather good height repeatability but the result shows a height bias compared to the COMEDIE solution

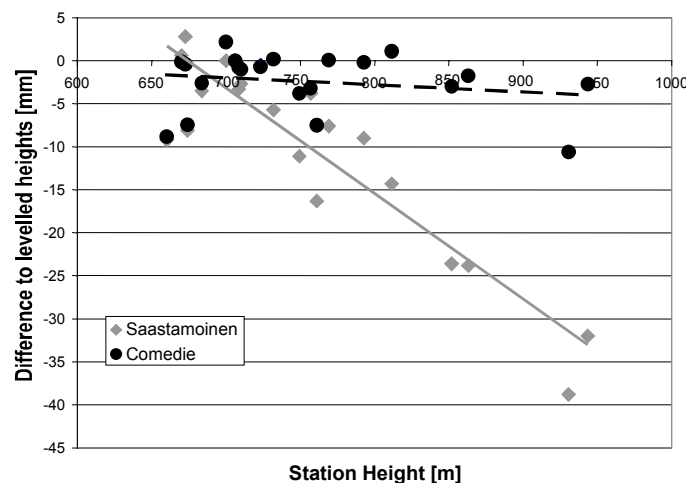


Fig. 3: Results of the Emmental Test Campaign: Comparison of the ellipsoidal heights (GPS) corrected for geoid undulations with orthometric heights from levelling using: a) standard troposphere model (Saastamoinen; solid grey line), b) 4D meteo model (COMEDIE; dashed black line)

After the GPS evaluation the obtained ellipsoidal heights were corrected with the geoid and then compared to the orthometric heights from levelling. The principle results can be seen in fig. 3: If we use a standard meteo model the heights obtained out of GPS and geoid can be biased of up to 10mm/100 m height difference. When we use a more sophisticated meteo model such as COMEDIE, the results are in very good agreement with levelling. The bias disappears completely. This also proves that the GPS repeatability is not a reliable indicator for detecting height biases.

4.2 The test campaign Sustenpass

The test network Sustenpass consists of a profile of 48 km in length with a total height difference of about 1700 m following the pass road. The 30 GPS stations are connected to a recently re-measured national 2nd order levelling line. The observations have been performed on 3 days in July 1998 in 9 sessions with a length between 3 and 12 hours. An important role in the analysis of the result plays the height profile shown in fig. 4 at left, where we have to distinguish between the West (Wyler) and the East (Wassen) side.

The results of the GPS evaluation and the repeatability were not very different from the Emmental campaign and do not need to be discussed here. Because of the big height differences the repeatability was in general worse but it were the same strategies that lead to the best results. The conclusions from the analysis of the GPS heights corrected for the geoid and their comparison with the orthometric heights from levelling are as follows:

- The modelling of the troposphere was much more difficult due to the difficult topography and the distribution of the meteo sites. The results from the two flanks are not homogeneous.
- When standard troposphere models (Saastamoinen) were used, the height estimates are biased by up to 3 mm per 100 m.
- When using the 4D meteo model (COMEDIE), the agreement between GPS heights and levelled heights is better on the west side (diff. < 4 cm) and worse on the east side (diff. < 9 cm) of the pass (see fig. 4). The bias in the height scale can not be eliminated completely even with long observation sessions.
- Of course, the Sustenpass is not only a difficult region for GPS campaigns, but also for levelling and the geoid determination. Therefore it is not sure that the detected biases are a consequence of the GPS results only. The levelling and the geoid as well, are not supposed to be free of systematic errors.

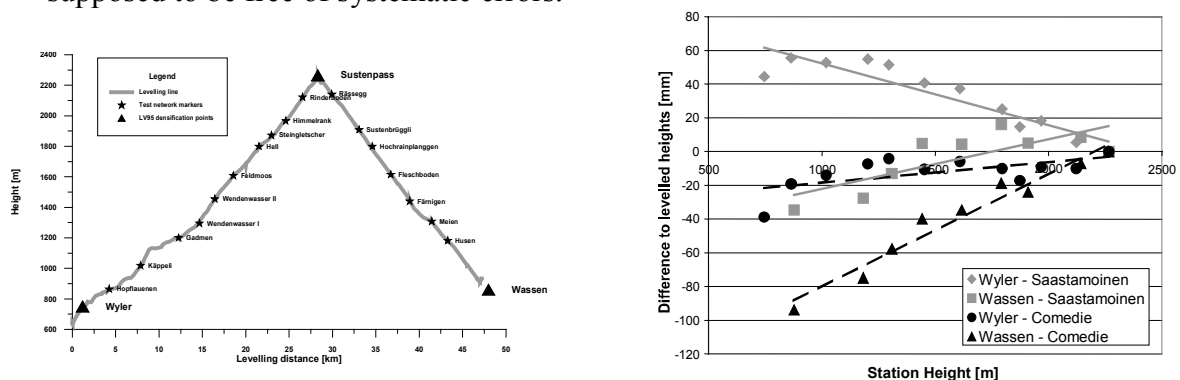


Fig. 4: Test network Sustenpass: Height profile (at left) and comparison of the ellipsoidal heights (GPS) corrected with geoidal undulations with orthometric heights from levelling for the two sides of the Sustenpass (Wyler: western part; Wassen: eastern part) using: a) standard troposphere model (Saastamoinen; solid line), b) 4D meteo model (COMEDIE; dashed line).

4.3 Consequences of the test campaigns

When special attention is paid to the tropospheric corrections, the use of GPS for the densification of height networks in flat and pre-Alpine areas with height differences smaller than 500 m is promising. In order to avoid severe biases in the height scale, standard troposphere models should not be used. Excellent results were obtained when a new 4D meteo model (COMEDIE) using regional meteo data was applied.

In Alpine networks with large height differences, height determination by GPS is still a difficult task. Even the most sophisticated tropospheric models are not yet capable of eliminating systematic influences from tropospheric refraction.

The results of these test campaigns revealed that the substitution of levelling is possible under the following conditions:

- no extreme topography
- relative accuracy demands are not very high (about 1 cm over 30 km).
- special attention is paid to troposphere modelling. A standard model is not suitable.
- even with the most sophisticated troposphere models only suitable for scientific GPS software, it is still a very difficult task to get a good accuracy in high mountainous areas.
- several hours of GPS measurements are necessary to obtain sufficient results.
- appropriate parameter choice in the GPS evaluation.

5 Summary, Conclusions

The national levelling network LHN95 is ready to be used and rigorous orthometric heights are available on all 1st and 2nd order levelling lines. About 120 levelling benchmarks are also connected to the GPS network LV95 and can be used to detect discrepancies between levelling, GPS and the geoid model CHGEO98.

The 3 data sets GPS, levelling and the geoid are in agreement of 2 cm in flatter areas and up to 8 cm in some regions in the Alps. Usually this is sufficient for the daily work of the surveyors. These differences are not randomly distributed over the country but some regional trends seem to exist. They are supposed to be caused mainly by long wavelength errors of the geoid, systematic errors in levelling or GPS are less probable or at least smaller. This implies that it is perhaps worth to consider a new geoid determination including some options that were not included in the geoid model CHGEO98 such as the full integration of gravity measurements, the use of a global model or the integration of the newest GPS/levelling measurements.

Until now, GPS, levelling and the geoid are only compared among each other. A combined adjustment of all 3 data sets has not been performed yet. The main reasons that prevent us from these step are that systematic errors have to be eliminated first, that a reasonable weighting has to be found and that the full variance-covariance information for the geoid is not available.

The two test campaigns in the Emmental at the Sustenpass showed that levelling can be replaced by GPS under some circumstances when accuracy demands are not too high and a accurate geoid or quasigeoid model is available. In regions with extreme height differences it is extremely difficult to bring GPS and levelling into agreement. The main reason for this is the influence of tropospheric refraction on GPS but also on levelling. Therefore much attention has to be paid to troposphere modelling in the GPS evaluation. But even if the GPS results show a good repeatability in height, we can not be sure to get a good agreement with levelling. Depending on the chosen parameters in the GPS evaluation we can get biases in the order of 1 cm per 100 meter height difference. Parameters like the cut-off angle

and the weighting of the observations with their zenith angle are of major importance in the height determination with GPS. Session lengths shorter than 3 hours usually are unsuitable for an accurate height estimation.

References

- Denker, H., and W. Torge (1997): The European gravimetric quasigeoid EGG97. In: *Forsberg, R., M. Feissel, and R. Dietrich (Eds.), Geodesy on the Move, IAG Symposia 119*, pp. 249-254, Springer 1997.
- Gubler, E., D. Schneider, U. Marti, A. Wiget, and U. Wild (1999): "Remeasurement of the Swiss National Geodetic Survey (LV95) and the establishment of an Automated GPS Network in Switzerland (AGNES)". In: *Gubler, E. and H. Hornik (Eds.): Subcommission for the European Reference Frame (EUREF), Publication Nr.7/1, pp.212-215, München 1999*.
- Kotsakis C., M. Sideris (1999): On the adjustment of combined GPS/levelling/geoid networks. *Journal of Geodesy Volume 73/8, pp 412-421*
- Hirter H. 1998: "Mehrdimensionale Interpretation von Meteorologischen Feldern zur Berechnung der Brechungsbedingungen in der Geodäsie". Institut für Geodäsie und Photogrammetrie an der ETH Zürich, Mitteilungen Nr.64. Zürich 1998.
- Marti, U. (1997): Geoid der Schweiz 1997. *Geodätisch-geophysikalische Arbeiten in der Schweiz*, Vol. 56, Zürich 1997.
- Marti, U. and D. Schneider (1998): The new Swiss National Height System LHN95. In: *IAG Symposia Volume 118: Advances in Positioning and Reference Frames*. Springer 1998.
- Marti, U. and A. Schlatter (2001): The new Height System In Switzerland. Proceedings of the IAG Symposium on Vertical Reference Systems, Cartagena, Colombia. Springer, in press.

A COMPARISON BETWEEN COMPUTED AND OBSERVED UNDULATION DEGREE VARIANCES AT THE LOW AND MEDIUM FREQUENCY SPECTRUM.

Doufexopoulou M. G., Pagounis V.N.

National Technical University of Athens
Faculty of Rural and Surveying Engineering

Abstract

In a number of previous investigations in Greece, raw geoid undulations derived from GPS and trigonometric height control were compared to model undulations computed by EGM96 and OSU91A models at various truncations. From a number of 144 control points it was shown that there exist discrepancies between "observed" and computed undulations with coefficients C_{nm} , S_{nm} . These discrepancies were studied in the lateral sense of the geoid variation, where it was found that they associate well with the tectonic background of the region. Several points show discrepancy within low degrees of expansion beyond $n=20$ and the discrepancy for all points is lateral variable. In this work we present an attempt to investigate these discrepancies at six selected control points along the depth by assuming that the depth of a "causal" source upon undulation maybe traced in terms of each spherical harmonic expansion. For this purpose observed undulation degree variance variances C_{nm} , S_{nm} at each point are computed between two frequency bands $n = 20 - 120$ and $n = 120 - 360$ and these are compared with computed undulations variances once from the model coefficients. In addition, power amplitude spectra of observed undulations by Fast Fourier Transform (FFT) and Maximum Entropy Method (MEM) spectral methods are computed by formed numerical series per points (observation signal). Its element of point series is produced by subtracting from the observed N , a computed variable "component" through the models' coefficients at decreasing truncation, with a lag interval $n=2$. On the basis of found differences between observed and model variances and from the performance of spectral methods upon the data we discuss how the content of these physical information about "sources" at depth within observed undulations may become more detectable and useful.

1. Introduction.

Spectral analysis is a group of methods based on statistical moments. These methods are used for various goals related to experimental data [e.g Bath, 1974]. The power spectrum as a useful statistical "tool" was introduced in the late 50 s [Blackmann & Tukey, 1958] in relation to communication signal transfer. Since then it has extensively been used also in Earth disciplines.

There, the spectral methods are used for two different goals: 1) To evaluate statistical measures of data in frequency domain and 2) to profit of the advantages of the FFT algorithm in computing convolution integrals. The second type of use was promoted after the invention of this algorithm [Singleton, R. C., 1967] that provided a fast method to evaluate the convolution integrals met in Physical geodesy [e.g. Stokes and Vening-Meinesz formula terrain correction, etc]. Today the FFT method is widely used in association with the computation of geoid [e.g Arabellos, D., Tziavos, I., 1994]

Hoffman-Wellenhof and Moritz [1986] discussed explicitly that convolution of two functions is equal to their homogeneity. Homogeneity is also assumed for the geoid undulation in this application in the context that the standard modeling of geoid via geopotential models is used as the reference to compare the experimental results. In the application the observed geoid undulations are received as raw differences between geodetic and leveled height: $N=h-H$ at certain test points. The spectral methods are used as a means to evaluate how much of information about the mass-density distribution within the local lithosphere is contained in 6 observed undulations at certain locations. If such information is carried in the observed undulations then it may be depicted in terms of harmonic degrees of spherical expansion. The evaluation is done by using the spectral methods in two ways: a) by comparing experimental spectra and experimental degree variances after the data among the test points [this is a relative comparison that questions the

homogeneity] and b) by comparing the experimental degree variances at each test point with degree variances produced through the coefficients of models OSU91A and EGM96. [This comparison is between the data after each test point in respect to each of the two models. The data by which the experimental spectra and experimental degree variances are computed come from numerical series of which each consecutive term is the observed N from which we eliminate in turn a frequency dependent term which corresponds to a truncated undulations computed by each of the two models coefficients at increasing truncation with lag $n=2$:

$$N_i = N_{\text{obs}} - N_{\text{she}} \quad [1]$$

Where N_i is the term of the data series, $N_{\text{obs}} = h - N$ and n is the truncation degree of the model increasing at lag $n=2$.

It should be noted that the use of spectra and of degree variances is in context to characterize individual point undulations and not in the usual use to evaluate or model the lateral variation of geoid. In this study the experimental degree variances are considered for single points and they represent how the observed undulation varies in spectral sense, when the frequency dependent term by the truncated model expansion is gradually eliminated (see eq. 1).

The power spectrum of a data series is defined equally by any of two methods: as the Fourier transform of the covariance function of the data or as the Fourier transform of the function represented by the data.

In this context the spectral techniques are used here with the data series created after eq. 1.

Spectral analysis related to the global scale representation of quantities like the geoid undulation provides model expressions of their variation through the degree variances. The contributions of Rapp [1971] and Tscherning & Rapp [1974] in this field were followed by the division of the information existing in the geoid according to the wavelength. This has been associated to the terms of degrees of spherical harmonic expansion [Rapp 1977]. This division of the spectrum of the geoid in frequency bands was refined later by Schwarz [1985]. The degree variances of any gravity field quantity are coefficients that express by a model or a “rule” information of how a gravity field quantity is varying along the degrees of spherical harmonic expansion.

Degree variances may be computed through the coefficients C_{nm}, S_{nm} of any spherical expansion model. Alternatively they are modelled as functions varying per degree n of expansion under certain mathematical restrictions [Tscherning-Rapp, Meissl] from data through well known models. Alternatively they are evaluated directly from the coefficients C_{nm}, S_{nm} of any geopotential model. In both methods of evaluation the degree variances per degree n of expansion represent a power amplitude spectrum of the quantity that they represent (e.g. gravity anomaly, geoid). In relation to the modelling of geoid they are related to lateral variations of undulation within a region or in global scale. This relation receives a statistical, analytic or a physical interpretation. Concerning to the physical interpretation the lateral variations of geoid are connected to lateral variations of Newton effects within the lithosphere [e.g. Sandwell et al. 1994].

This study presents the results received from a numerical experiment in Greece that is based on the use of undulation degree variances as an index of the information content carried within observed undulations. We compare the differences from experimental undulation amplitude spectra in the “vertical” sense received by FFT and Maximum Entropy Method [MEM] [e.g. Narayan, R. & Nityananda, R. (1984)] among 6 points as well as experimental undulation degree variances with degree variances computed by the coefficients of spherical harmonic expansion models, OSU91A and EGM96. The experimental spectra by FFT and MEM methods, the 3 dominant eigen-values of the corresponding correlation matrix of data.

The content of information of such undulations on lateral lithosphere variation has been shown recently [Pagounis, V. 2000]

For the needs of these comparisons we created from observed undulation N at single points experimental data series referenced to the two models OSU91A, EGM96 as follows: From the observed undulation N a computed “component” after the coefficients C_{nm}, S_{nm} of a certain model is subtracted. The subtracted part

is variable for each term of the series (variable frequency band) with lag of variation $n=2$. Thus at each tested point undulation a frequency dependent experimental series of geoid signal is produced, composed by 171 terms. For each data series the experimental amplitude spectra and the corresponding experimental undulation variances are computed.

The results of these comparisons carry two types of information: 1) How an unknown depth “source” beneath the test region affects at different locations the variation of the observed undulation in respect to each model [e.g. OSU91A and EGM96] 2) when more causal “sources” at depth exist how these affect each of the observed undulations]. This information is monitored through the experimental spectra and the degree variances among the 6 test points.

The main emphasis is given to compare the degree variances between experimental and computed after the coefficients of the two models. The paper is composed in 3 parts. In the first part we give the background on the formulation of the experimental data series at a single point. The second part deals with the description of the carried numerical experimental and presents the received results while the third part contains comments and conclusions that may be drawn from these comparisons.

2. BACKGROUND OF METHOD AND HOW THE EXPERIMENTAL DATA SERIES ARE PRODUCED.

In this approach observed geoid undulation is:

$$N=h-H \quad (2)$$

Where H is trigonometric or leveled height and h is the height after a satellite positioning system. This raw type of undulation is considered in the study as the output signal of any unknown mass-density that might exist mostly within the local lithosphere beneath the test region.

In a number of previous investigations that used in total 144 such observed undulations [Pagounis, V., 2000] within Greece in various combinations [Doufexopoulou, M. & Pagounis, V., 1997, 1998] it was shown that by a tested methodology the main lateral local lithosphere structure may be depicted [Pagounis, V., 2000]

The present investigation contributes to the mass-density information that may be depicted from such point undulations as an effect in respect to ranges of certain harmonic degrees.

The main idea behind this approach is to separate from each observed undulation signal a model part as expressed by a spherical harmonic expansion at certain degree n or between a range of degrees. This separation has the purpose to enhance the local observed signal by a sort of numerical high pass or band pass filter. The task is achieved by introducing a number of experimental statistical indices [Pagounis, V. 2000] of spectral concept and has been based on lateral variation of observed undulations [Pagounis, V. 2000].

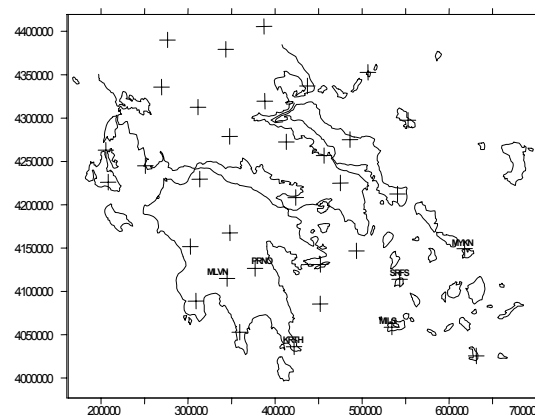


Figure 1 Distribution of control points.

These indices specify which is the rate of expansion degrees of a model that provide enhanced local lithosphere signal within observed undulations without to add “noise” at the signals. Beyond this rate the elimination of high frequencies [in context of structural trend] through the expansion of the model at higher degree than the found one introduces “noise” within the signals, if these are to be used for the initial prospecting of the lithosphere. One of proposed indices the variability index [Doufexopoulou, M. & Pagounis, V., 1998] was used to select the 6 sample points in the present investigation. These 6 trigonometric points belong to two groups, of which 3 are located at the South of Peloponnissos (continental region) and the other 3 are points in islands within the Central part of Aegean [Cyclades].(Fig.1).

For each point undulation two experimental data series are produced, each referenced to one of models OSU91A and EGM96. These models showed in the earlier studies strong similarities in how they adapt to the local lateral variation of observed geoid undulations [Doufexopoulou, M.&Pagounis, V.1998]. Thus a similar performance of these models on the depth variation of undulations is reasonable to be expected. Each experimental series is composed from 171 data, in which each term is:

$$N_n = N_{obs} - (N_n - N_{360}) \quad (3)$$

The term N_n at the left side represents the numbering of the terms and increases in lag of spherical expansion $n=2$ (in total 171 values) because the term N_n within the brackets is variable and increasing at this lag. N_n starts from $n=20$ and increases the $n=360$. N_{obs} are the raw observed undulations after eq. [2]. Note that the index n is not the same between the left and the right side of eq. [3]. The term in brackets ($N_n - N_{360}$) is then the variable contribution from the model component that eliminates from the local undulation signal a sort of frequency contribution coming from the model. This processing operates as frequency dependent numerical filtering applied at the observed undulation data and provides for each test point a data series per model. The starting term of each produced experimental series contains all frequencies from $n= 2-20$ and those beyond 360. By increasing the order of the terms of the series each new term includes slightly higher frequencies within a lag $n=2$ at the time, which exist in observed undulation. By this process 171 terms are prepared from each point signal, which are experimental data. Fig. 2 and Fig.3 show the graphs of data for 2 points (one per each group) for the two used models.

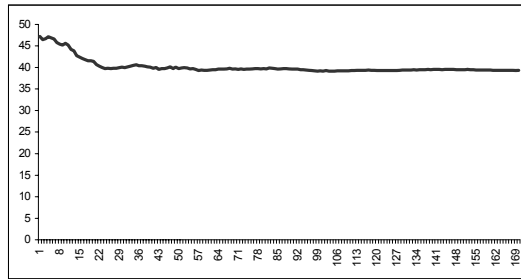


Figure 2 Data series for point KRTN referenced to OSU91A.

3. Description of the experiments and results.

In previous evaluations with the 144 observed undulations as output signals and by applying various tests with undulation computed with coefficients C_{nm} , S_{nm} of models OSU91A, EGM96 it was found that local lithosphere information may be safely modelled through the harmonic expansions OSU91A and EGM96 up to the degree $n=120$. Beyond this degree the addition of more terms in the model undulation N , introduce “noise” to the local “signal” when the differences the type:

$$\delta N_n = N_{obs} - N_n \quad (4)$$

These signal differences are investigated in detail by Pagounis,V. in amplitude “noise” introduced by the models coefficients.

In the present study it was decided to compute for each model two spectral bands per test point A band between degrees n=20-120 where the effect of local lithosphere is expected detectable in the data and a band from n=121 to n =360 within which it is believed that the lithosphere effect is masked by variable

However after the first plots of the prepared series using eq. [2] and from the investigation of the values of the standard deviation of the 2nd and 3rd order eigen-values of the correlation matrices of the experimental series it became apparent that the separation into two bands would be meaningless. It could be seen from the raw data plots that a “flat” part of the signals [related to the start of “noise” of the data series] starts after degree n=100-120.

For each series of test point and each model the FFT and MEM spectra were calculated as well the 3 most significant eigen values of the corresponding correlation data matrix. Finally the standard deviation of the predicted values of the next values in each series was calculated together with the predicted values using academic software, “Chaos data Analyser” which operates in MS-DOS.

The power amplitude spectra by any spectral method show how the power existing in the series of undulation varies per degree of expansion along the low-to higher frequencies (expressed in respect to the degree n). This indication is valid under the supposition that most of the cause of this variation originates from mass-density “sources” beneath the test region. Thus a variation among spectra of points in the distribution of amplitude power of N and at the eigen-values of the correlation matrix express variations due to the effect of the same causal “source” upon the different locations of the points and/or how the different causal “sources” in the region affect each test point. The separation of the two effects is a very difficult task.

The squared values of the terms:

$$\sigma_n(N) = \delta\delta N = (\delta N_{n+2} - \delta N_n)^2 \quad [5]$$

Of each term of experimental series are a type of experimental undulation variance at the point that increases with lag =2 [Doufexopoulou, M. & Pagounis, V., 1998].

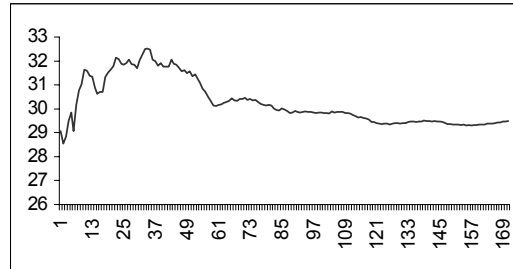


Figure 3 Data series for point MLVN referenced to OSU91A.

These observed variances are comparable to the corresponding degree variances from the coefficients of each of the two models [or any other model] at depth:

$$C_N = \left(\frac{R}{\gamma} \right)^2 \sum_{m=0}^n (C_{nm}^2 + S_{nm}^2) \quad (6)$$

Model degree variances are computed with [Eq.6] with a small program written for the needs of this work.

4. Conclusions and comments.

The results confirm what was expected from the investigation of the lateral variation [Doufexopoulou, M. & Pagounis, V. 1998, Pagounis, V. 2000]. The model of reference is an irrelevant choice for the present experiment. Only for one point (KRTN) a different data series was produced between the two reference

models [EGM96A, OSU91A]. Data series at all points referenced to EGM96 provide higher values from these after OSU91A. This is observed along the initial $\frac{1}{4}$ of the total number of the terms of series at all points. This rate corresponds until about degree $n=70$ and an interpretation is that for the test area the EGM96 coefficients spread the spectral power along the degrees of expansion more “smoothly”. Due to the irrelevance of the reference model only minor differences between FFT spectra of the data referenced to each model for the same point are detected. It seems though that some spectra referenced to OSU91A carry more amplitude power than those referenced to EGM96. The FFT spectra for all points become “flat” beyond the range $n=60-70$. This leads to conclude that the local observed undulations contain local information than cannot be depicted by the model undulations at the range of degrees $n=20-70$. Amplitude spectra produced with the MEM show a very sharp peak at very low degrees and then become quite flat near to zero power for all points. This could be expected due to the poor resolution of the analysed data (raw initial values) and is possibly a reason that this spectral method “failed”.

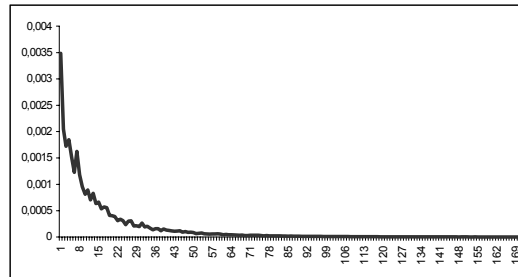


Figure 4 The computed degree variances for point SRFS (EGM96).

The correlation matrices at depth for the data created after the 6 undulations have strong similarities in the main shape for both models, indicating again that the observed undulations carry local mass-density information. The plots of the 3 dominating eigen-values of the correlation matrix have only eigen value of significant size for the first one. The second and third eigen-value drop near to zero. Among the 6 points the first eigen-value is higher for the group of the 3 “continental” points [MILS, KRTH, PRNO] than for the points within Aegean. This indicates that beneath the 3 “continental” points the causal “sources” at depth affect more those undulations. The effect may be due either to a bigger depth of the “source” or due to the location of these points nearby a “source”. This result is valid for both models of reference OSU91A and EGM96.

The comparison between experimental degree variances and computed after the coefficients C_{nm} , S_{nm} of the two models showed that the computed degree variances between the two models differ only in scale of power amplitude but they vary in the same way along the degrees. The comparison shows also that degree variances from the two models depict power amplitude until about degree=110 and beyond this degree they do not provide information. This result is in agreement with previous studies in this region.

In contrary, the experimental degree variances for the 6 tested points, show that observed undulations carry local information on mass-density effects at depth and that this information is depicted in detail along the range of degrees [20-80]. The information is prominent about degrees $n=40-60$ or 70 and varies among the test points. At this range the amplitude level is much higher than the power at the surrounding degrees. For most points a considerable peak around the “noisy” part of spectrum may be detected beyond degree $n=70$ around $n=130$. This information could not be depicted in the model degree variances by any of models.

It may be argued that these results show a possibility to detect the presence of local causal effects from lithosphere within the observed undulations, at the low and medium frequency part of the spectrum. At this stage of investigation there was no attempt to interpret this information.

The computed amplitude experimental spectra by the FFT and the 3 dominating eigen-values of the correlation matrices at depth show again that signals at each point carry different information and that these differences may be due to local lithosphere effects. However a separation of causes effects is quite difficult for two reasons:

- 1) There were no results of solution of the forward problem. A solution of the forward problem under probable scenarios may shed more information on this or into similar experiments, which actually aim to approach numerically an “inversion” of the observed geoid.
- 2) Beneath the test area there are various causal “sources” at various depths (main and secondary faults, a subducting slab effect on geoid and an island arc, strong variations in the sea depth).

The variety of “sources” within a small geographical extend interfere as combined effects upon undulations in very complicated way. It is believed that more powerful and detailed statistical tests upon the same data or use of more dense test points in this area for which experimental series have narrower lag (e.g. at each degree) might be useful to extract the content of geoid information related to geo-features at depth. However the present results support the belief that observed undulations when treated as signals do have information about causal “sources” at depth and that a better separation of these “sources” may contribute in geodynamical modelling.

References

Arabelos, D., Tziavos, I., Geoid Solution in the Mediterranean Sea by different methods, in Mare Nostrum, GeoMed Report - 4, Thessaloniki, 1994.

Bath, M., Spectral Analysis in Geophysics, Elsevier Co., Amsterdam, 1974.

Blakman & Tukey The measurement of power spectra, 1959.

Doufexopoulou, M., Pagounis, V., “First numerical results in detecting heterogeneities in regional gravity field using GPS undulations”, EGS XXII, Vienna, 1997.

Doufexopoulou, M., Pagounis, V., “Investigation of tectonic traces upon the regional geoid without the use of gravity”, Reports of the Finnish Geodetic Institute, 98:4, 1998.

Doufexopoulou, M., Pagounis, V., “Investigation of raw variance and variability spectra from non gravimetric geoidal signals”, Bolletino Di Geofisica Teorica Ed Applicata, bg/133, 2000.

Hofmann-Wellenhof, B., Moritz, H., Introduction to spectral Analysis in Mathematical and Numerical techniques in Physical geodesy [ed. H. Suenkel], Lecture Notes in Earth Sciences, 1986 Springer.

Jenkins, G., Watts, D., Spectral Analysis and its Applications. Holden day Pub., 1968.

Meissl, P., A study of covariance functions related to the Earth's gravitational Potential. Dept of Geodetic Science, Ohio State University, Report No. 151, 1981.

Narayan, R., Nityananda, R., Maximum entropy- flexibility versus fundamentalism, Cambridge University Press, 1984.

Pagounis, V.N., A contribution of geodetic height data processing for local geodynamics. Application in Greek area. Ph. D Thesis National Technical University, Athens. Faculty of Rural & Surveying Engineering, 2000.

Rapp, R., Improved models for potential coefficients and anomaly degree variances, Journal of Geophysical Research, Vol. 78, 1973.

Rapp, R., Geoid Information by wavelength, Bulletin geodesique No. 44, 1977.

Schwarz, K-P., Data Types and their spectral properties, in Proceedings of Beijing Summer School in "Local Gravity Field Approximation" Vol.1, Ed K-P Schwarz, 1985.

Sandwell, D.T., Yale, M., Smith, W.H.F., ERS1 geodetic mission reveals detailed tectonic structures, Eos Trans, AGU, 75 (44), p.155, 1994.

Sideris, M., Computation of Gravimetric Terrain Corrections Using Fast Fourier Transform Techniques, University of Calgary, UCSE Reports No. 20007, 1984.

Singleton, R.C On computing the Fast Fourier Transform. Communications of the ACM, 1967 Vol. 10, No. 10, pp. 617-634.

Tscherning, C., Rapp, R., Closed covariance expressions for gravity anomalies, geoid undulations and deflections of the vertical implied by anomaly degree variance models, Dept of geodetic Science Rep. No. 108, OSU, 1974.

Non-stationary noise filtering of gravity data using fast spectral techniques

C. Kotsakis and M.G. Sideris

Department of Geomatics Engineering, University of Calgary
2500 University Drive N.W.
Calgary, Alberta, Canada, T2N 1N4

Abstract

An important problem in the practical use of optimal spectral methods in gravity field modelling is the stationarity assumption for the data noise and the underlying unknown signals. Such a restriction is required, according to the standard Wiener-Kolmogorov estimation theory, in order to obtain signal approximation algorithms of simple convolution structure that can be evaluated very efficiently through fast Fourier transform (FFT) techniques. Often, the observation errors in the input data have significant spatial variations in their statistical behaviour, thus making the noise stationarity assumption unrealistic for many practical situations. Also, a stochastic interpretation of the true values of the various gravity field signals as random variables with similar statistical parameters is rather questionable, since they describe physical phenomena that are not random (probabilistic) and certainly do not have a uniform (stationary) behaviour over their domain. The aim of this paper is to present a spectral Wiener-type optimal filter which can be used in geodetic estimation problems with arbitrary deterministic signals that are masked by non-stationary observation errors.

1. Introduction

The method of Wiener filtering is a well known and efficient spectral tool that can be used for geodetic data 'de-noising' in the frequency domain. Its framework is based on the Wiener-Kolmogorov linear prediction theory for stationary random fields in the presence of stationary additive noise (KAILATH, 1974), and thus it is closely related to the method of least-squares collocation with random observation errors (MORITZ, 1980). The application of the Wiener filter in geodesy, either as an independent practical tool for data pre-processing or as an integral component of a more general linear estimation methodology (i.e. input-output systems theory), has primarily focused on problems related to optimal spectral gravity field modelling. For some practical applications, see SIDERIS (1996), PAWLOWSKI AND HANSEN (1990), TZIAVOS ET AL. (1996). A detailed discussion on the use of the Wiener-Kolmogorov filtering theory in gravity field estimation, and its relationship with other linear approximation techniques traditionally used in geodesy, can be found in SANSONO AND SIDERIS (1997).

In order to employ the classic Wiener filtering algorithm with noisy geodetic data, a stationarity assumption has to be made for both the true/unknown signal and the external random errors. Such restrictions become quite problematic for many gravity field applications, since (i) the underlying true signals cannot easily admit a stochastic interpretation (thus making the stationarity assumption meaningless) and, most importantly, (ii) the additive data noise does not usually follow a spatially uniform statistical behaviour; see KOTSAKIS AND SIDERIS (2001) for more discussion. In this paper we will outline a convolution-based algorithmic procedure that can be used in geodetic estimation problems regardless of the spatio-statistical properties of the underlying signals and the data noise. Our analysis will cover the simple case where an unknown deterministic field is observed under the masking of non-stationary random errors, and the desired output corresponds to an improved ('de-noised') linear interpolating model of the noisy input data. An important point in our approach is that the sampling resolution of the data will be taken into account within the optimization procedure, resulting in a resolution-dependent noise filter. A numerical example, using a synthetic two-dimensional gravity anomaly grid, has also been included to demonstrate the performance of our optimal noise filter under non-stationary additive noise, at different sampling resolution levels.

2. Methodology – Problem formulation

The main problem that is studied in this paper is the frequency-domain estimation of an unknown deterministic field $g(x, y)$ using its noisy gridded samples $d(nh_x, mh_y)_{n,m \in \mathbb{Z}}$ according to the linear observation equation

$$d(nh_x, mh_y) = g(nh_x, mh_y) + v(nh_x, mh_y) \quad (1)$$

where $g(nh_x, mh_y)$ are the true signal values, $v(nh_x, mh_y)$ is a non-stationary noise sequence, and $h_x \times h_y$ corresponds to the sampling resolution level. The unknown signal will be assumed to have compact support over the real plane, and thus the integer sampling indices n and m in Eq. (1) can be practically restricted within a finite range $0 \leq n \leq (N-1)$ and $0 \leq m \leq (M-1)$. The Gauss-Markov stochastic model for the observation errors is

$$E\{v(nh_x, mh_y)\} = 0 \quad (2a)$$

$$E\{v^2(nh_x, mh_y)\} = \sigma_v^2(nh_x, mh_y) = \sigma_v[(nh_x, mh_y)(nh_x, mh_y)] \quad (2b)$$

$$E\{v(nh_x, mh_y)v(kh_x, lh_y)\} = \sigma_v[(nh_x, mh_y)(kh_x, lh_y)] \quad (2c)$$

The symbol $\sigma_v^2(\cdot)$ denotes the noise variance at a single data point, whereas $\sigma_v[(\cdot)(\cdot)]$ corresponds to the noise covariance (CV) between two data points. The Fourier transform of the noiseless signal grid will be denoted by $\bar{G}(\omega_x, \omega_y)$ and it is given by the summation formula (DUDGEON AND MERSEREAU, 1984)

$$\bar{G}(\omega_x, \omega_y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} g(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \quad (3)$$

where the overbar symbol will be used to indicate a periodic function. We will also use the notation $\bar{V}(\omega_x, \omega_y)$ for the Fourier transform of the input data noise, which is defined as follows:

$$\bar{V}(\omega_x, \omega_y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} v(nh_x, mh_y) e^{-i(nh_x\omega_x + mh_y\omega_y)} \quad (4)$$

Two special properties will be imposed a-priori in the estimation procedure, namely linearity and translation-invariance. The reason for introducing the second property is to obtain a signal estimate that is independent of the reference system used to describe the position of the data points. Note that the shift-invariance condition has often been applied in the theoretical formulation of optimal estimation methods using errorless data (SANSO, 1980; KOTSAKIS, 2000a), although its justification is not altered by the noise presence in the observations. Taking into account these two assumptions, the signal estimation formula will have the general convolution-type expression

$$\hat{g}(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d(nh_x, mh_y) \xi_h(x - nh_x, y - mh_y) \quad (5)$$

where $\xi_h(x, y)$ is a filtering kernel that needs to be determined in some optimal sense. The subscript h is used to indicate that the estimation kernel will generally depend on the data resolution level $h_x \times h_y$.

The signal error produced by the filtering formula in Eq. (5) can be decomposed into two components, i.e.

$$e(x, y) = g(x, y) - \hat{g}(x, y) = e_h(x, y) + e_v(x, y) \quad (6)$$

where $e_h(x, y)$ is the part of the total estimation error caused from the use of discrete data with finite resolution (aliasing error), and $e_v(x, y)$ is the additional part due to the noise presence in the signal samples. In the absence of noise from the input data, the best we can obtain is just an *interpolated model* $\tilde{g}(x, y)$ for the unknown field that will depend on the true signal values at the given spatial resolution. We will assume that such a noiseless signal model is given in terms of a linear and translation-invariant formula, as follows:

$$\tilde{g}(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} g(nh_x, mh_y) \phi_h(x - nh_x, y - mh_y) \quad (7)$$

where $\varphi_h(x, y)$ is some interpolating kernel that generally depends on the data resolution. The noise-dependent estimation error will be measured with respect to such a linear interpolating model for the unknown field, i.e.

$$e_v(x, y) = \tilde{g}(x, y) - \hat{g}(x, y) \quad (8a)$$

whereas the (pure) aliasing error is

$$e_h(x, y) = g(x, y) - \tilde{g}(x, y) \quad (8b)$$

The actual choice of the interpolating kernel $\varphi_h(x, y)$ is irrelevant for the purpose of this paper, and it can be optimized by minimizing a suitable functional of the aliasing error component $e_h(x, y)$; see KOTSAKIS (2000a, b). The unknown filtering kernel in Eq. (5) will be determined through the familiar Wiener-Kolmogorov *mean-square-error* (MSE) criterion

$$P_{e_v}(\omega_x, \omega_y) = E \left\{ \left| E_v(\omega_x, \omega_y) \right|^2 \right\} = \text{minimum} \quad (9)$$

where $E_v(\omega_x, \omega_y)$ is the Fourier transform of the noise-dependent error term $e_v(x, y)$. It is easy to show that the corresponding optimal estimation filter will finally be given by the formula

$$\Xi_h(\omega_x, \omega_y) = \frac{|\overline{G}(\omega_x, \omega_y)|^2}{|\overline{G}(\omega_x, \omega_y)|^2 + \overline{P}_v(\omega_x, \omega_y)} \quad \Phi_h(\omega_x, \omega_y) = \overline{W}(\omega_x, \omega_y) \quad \Phi_h(\omega_x, \omega_y) \quad (10)$$

where $\Xi_h(\omega_x, \omega_y)$ and $\Phi_h(\omega_x, \omega_y)$ are the Fourier transforms of the filtering kernel $\xi_h(x, y)$ and the ‘reference’ interpolating kernel $\varphi_h(x, y)$, respectively. The auxiliary term $\overline{P}_v(\omega_x, \omega_y)$ shown in the last equation corresponds to the ‘power spectral density (PSD)’ noise quantity

$$\overline{P}_v(\omega_x, \omega_y) = E \left\{ \overline{V}(\omega_x, \omega_y) \overline{V}^*(\omega_x, \omega_y) \right\} = E \left\{ \left| \overline{V}(\omega_x, \omega_y) \right|^2 \right\} \quad (11)$$

where E denotes the usual expectation operator. The result in Eq. (10) indicates that the estimation procedure can be decomposed into two individual steps which are connected in a linear cascading manner (see Figure 1). The first step, expressed by the filter component $\overline{W}(\omega_x, \omega_y)$, has the role of ‘de-noising’ the discrete data using information on the average behaviour of the input noise and the unknown field at the given resolution level. The second filter component $\Phi_h(\omega_x, \omega_y)$ is solely used to obtain a continuous representation for the output signal $\hat{g}(x, y)$ based on an a-priori selected interpolating/modelling kernel $\varphi_h(x, y)$. For more mathematical details and derivations, see KOTSAKIS AND SIDERIS (2001).

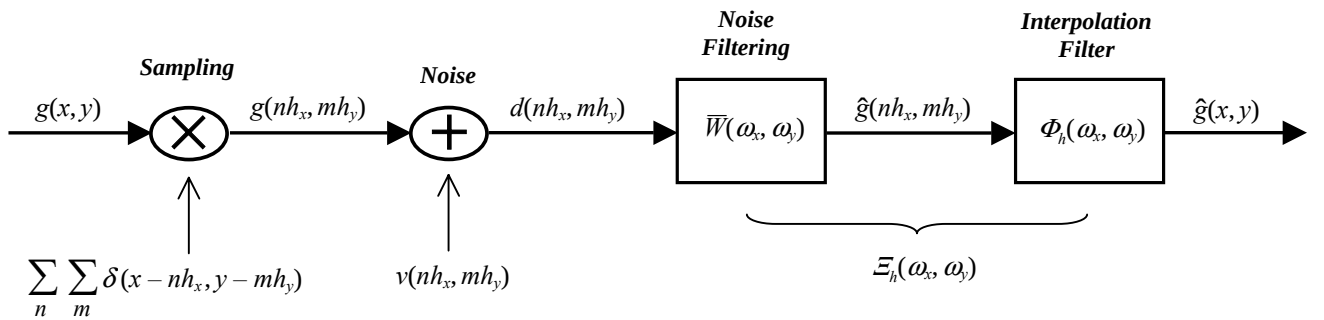


Figure 1. The cascading structure of the optimal linear estimation filter.

3. The Wiener-like structure of the optimal estimation filter

The structure of the optimal estimation filter in Eq. (10) is very similar to the classic Wiener filter, since they are both defined in terms of a certain signal-to-noise ratio (SNR) expression. However, there do exist conceptual differences between the two filtering schemes because in our formulation: (i) the unknown field has been modelled as a deterministic (instead of stochastic) signal, and (ii) the additive data noise has not been restricted to being stationary. Thus, it is important to clarify what is the exact meaning of the two frequency-domain terms that appear in the expression of our SNR-type optimal noise filter $\bar{W}(\omega_x, \omega_y)$. From Eq. (10), we have that

$$\bar{W}(\omega_x, \omega_y) = \frac{\frac{1}{NM} |\bar{G}(\omega_x, \omega_y)|^2}{\frac{1}{NM} |\bar{G}(\omega_x, \omega_y)|^2 + \frac{1}{NM} \bar{P}_v(\omega_x, \omega_y)} = \frac{\bar{A}(\omega_x, \omega_y)}{\bar{A}(\omega_x, \omega_y) + \bar{B}(\omega_x, \omega_y)} \quad (12)$$

where NM is the total number of points in the input data grid. The two auxiliary functions $\bar{A}(\omega_x, \omega_y)$ and $\bar{B}(\omega_x, \omega_y)$, in the last equation, correspond to the Fourier transforms of two associated sequences which have the CV-like expressions (for a proof see KOTSAKIS AND SIDERIS, 2001)

$$a(nh_x, mh_y) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} g(kh_x, lh_y) g((k+n)h_x, (l+m)h_y) \quad (13a)$$

and

$$b(nh_x, mh_y) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v[(kh_x, lh_y) ((k+n)h_x, (l+m)h_y)] \quad (13b)$$

The sequence in Eq. (13a) is the discrete (spatio-statistical) CV function of the true deterministic signal at the given data resolution level, and thus the term $\bar{A}(\omega_x, \omega_y)$ in Eq. (12) is just the power spectrum of the true signal values $g(nh_x, mh_y)$. The sequence in Eq. (13b), on the other hand, does not exactly correspond to the noise CV function and, as a result, the quantity $\bar{B}(\omega_x, \omega_y)$ in Eq. (12) should not in general be viewed as the data noise PSD (such an interpretation is possible only when the data noise is stationary). The sequence $b(nh_x, mh_y)$ can be perceived as a ‘mean’ CV function of the random observation errors. Its value at the origin gives an average indication of the noise level at every data point of the input grid, i.e.

$$b(0, 0) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v[(kh_x, lh_y) (kh_x, lh_y)] = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \sigma_v^2(kh_x, lh_y) \quad (14)$$

whereas its values at the other points correspond to ‘averages’ of the noise covariance over pairs of data points with coordinate difference equal to (nh_x, mh_y) . The numerical evaluation of the optimal noise filter $\bar{W}(\omega_x, \omega_y)$ in practice can take place only at discrete frequency values using the discrete Fourier transforms (DFTs) of the two CV-type sequences $a(nh_x, mh_y)$ and $b(nh_x, mh_y)$; more details on the properties and practical implementation of the optimal noise filter can be found in KOTSAKIS AND SIDERIS (2001).

4. Numerical experiment

A numerical experiment was performed to test the noise filtering component of the optimal estimation kernel that was previously derived. A deterministic signal $g(x, y)$, assumed to represent some local gravity anomaly field, was initially synthesized using a truncated Fourier series expansion with a record length of 200×200 km. The continuous signal was sampled at various resolution levels to obtain noiseless gridded values

$g(nh_x, mh_y)$. Four different sampling resolutions were selected, namely 0.5×0.5 , 1×1 , 2.5×2.5 and 5×5 km. All signal grids at each resolution level were partitioned into four equal blocks/quadrants, labeled as northwest (NW), northeast (NE), southwest (SW) and southeast (SE). The simulated data noise, which is going to be added to the true signal values, will have different statistical behaviour in each of the four grid quadrants.

A zero-mean noise sequence was added to the samples of the true signal in order to create the input data $d(nh_x, mh_y)$ at every resolution level. The noise values originated from a non-stationary and uncorrelated Gaussian stochastic process, using the routines for random number generation of the MATLAB™ software package. The noise variance was constant within each quadrant (NW, NE, SW and SE) of the data grids, with its values set to 144 mGals^2 , 9 mGals^2 , 144 mGals^2 and 49 mGals^2 , respectively. The sample statistics of the simulated noise values in the four different quadrants of the data grids are shown in Table 1, for some selective resolution levels.

Table 1. Statistics of the simulated noise values in the four quadrants of the data grids (in mGals).

Data resolution (in Km)	0.5×0.5				1.0×1.0			
Grid quadrants	NW	SW	NE	SE	NW	SW	NE	SE
Max	49.61	55.64	12.47	27.37	53.15	43.04	11.45	25.88
Mean	-0.04	0.02	0.02	-0.01	0.03	0.01	0.02	-0.08
Min	-52.61	-46.34	-12.14	-27.14	-52.71	-47.87	-10.24	-28.58
Std	11.99	12.05	3.00	6.98	12.07	12.08	2.95	7.01

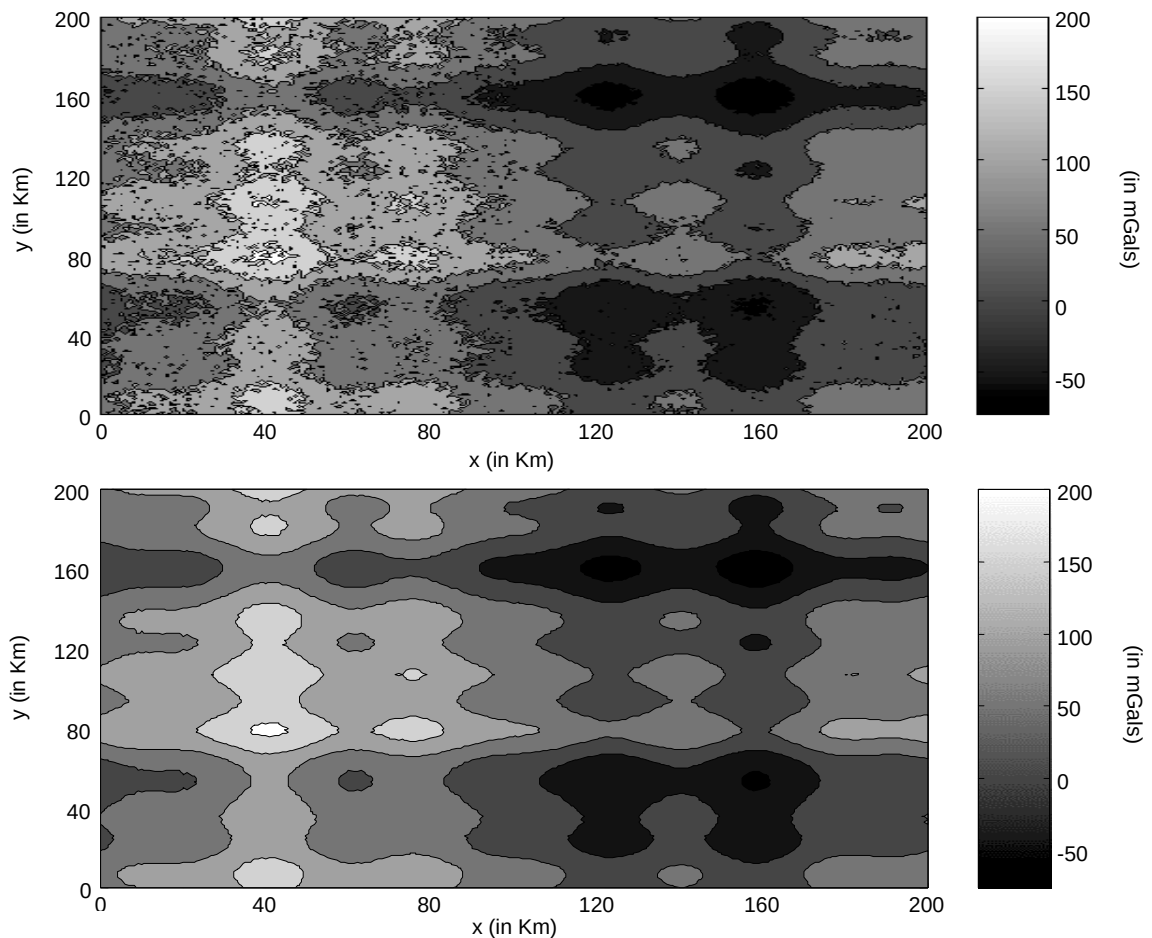


Figure 2. Noisy gravity anomaly values (top plot) and estimated/filtered gravity anomaly values (bottom plot), for the data resolution level (1×1) km. Observe that the input noise is much stronger in the western parts of the data grids.

The optimal noise filter $\overline{W}(\omega_x, \omega_y)$ was computed through a fast Fourier transform (FFT) algorithm at each resolution level $h_x \times h_y$, according to the SNR expression given in Eq. (12). It was then multiplied by the FFT of the noisy gridded data $d(nh_x, mh_y)$ and the result was finally transformed back to the space domain as an estimated ('de-noised') signal sequence $\hat{g}(nh_x, mh_y)$. An example of the original noisy data grid and the filtered signal values is given in Figure 2. The statistics of the differences between the true signal samples and the estimated signal values, for every resolution level used, are also given in Table 2 below.

Table 2. Statistics of the differences between the true and the filtered signal values (in mGals).

Data resolution (in Km)	0.5 × 0.5	1.0 × 1.0	2.5 × 2.5	5.0 × 5.0
Max	3.09	3.51	5.53	6.26
Mean	0.00	0.01	-0.05	0.10
Min	-3.25	-4.00	-4.86	-8.01
Std	0.66	0.91	1.43	2.26

It is interesting to observe that the output estimation error of the filtered signal values is decreasing, as the data resolution increases. Such a result is not surprising and it just confirms the (already well-known from signal analysis theory) fact that oversampling leads to noise reduction in the final signal estimate.

5. Conclusions

We have presented a modification of the classic Wiener filtering algorithm which allows us to work with deterministic fields that are masked by additive non-stationary noise. The informal similarities of our estimation framework with the Wiener filtering formalism stem from the initial assumption in Eq. (5) that the optimal signal estimate should be linear and translation-invariant. This led to a convolution SNR-type computational scheme that can always be implemented very efficiently using FFT techniques. Of special importance in our derivations was the decomposition of the total estimation error into an aliasing component and a noise-dependent component. A detailed discussion on this subject, along with some comments on the problems encountered when we attempt to apply a 'one-step' optimization of the total signal estimation error, can be found in KOTSAKIS AND SIDERIS (2001). The presented methodology can be proven a useful tool in various geodetic estimation problems of local and/or regional scale, such as the optimal spectral geoid determination from noisy gridded gravity data.

References

- DUDGEON D.E., MERSEREAU R.M. (1984) Multidimensional digital signal processing. Prentice-Hall, Englewood Cliffs.
- KAILATH T. (1974) A view of three decades of linear filtering theory. IEEE Trans Info Theory, **IT-20**(2): 146-181.
- KOTSAKIS C. (2000a) The multiresolution character of collocation. J Geod, **74**(3-4): 275-290.
- KOTSAKIS C. (2000b) Multiresolution aspects of linear approximation methods in Hilbert spaces using gridded data. PhD thesis, Dept. of Geomatics Engineering, University of Calgary, UCGE Report No. 20138, Calgary, Alberta.
- KOTSAKIS C., SIDERIS M.G. (2001) A modified Wiener-type filter for geodetic estimation problems with non-stationary noise. J Geod, **75**(11-12) (to appear).
- MORITZ H. (1980) Advanced Physical Geodesy. Herbert Wichmann Verlag, Karlsruhe.
- PAWLOWSKI R.S., HANSEN R.O. (1990) Gravity anomaly separation by Wiener filtering. Geoph, **55**(5): 539-548.
- SANSO F. (1980) The minimum mean square estimation error principle in physical geodesy (stochastic and non-stochastic interpretation). Boll Geod Sci Affi, **39**(2): 112-129.
- SANSO F., SIDERIS M.G. (1997) On the similarities and differences between systems theory and least-squares collocation in physical geodesy. Boll Geod Sci Affi, **54**(2): 173-206.
- SIDERIS M.G. (1996) On the use of heterogeneous noisy data in spectral gravity field modelling methods. J Geod, **70**(8): 470-479.
- TZIAVOS I.N., SIDERIS M.G., LI J. (1996) Optimal spectral combination of satellite altimetry and marine gravity data. Report of the Finnish Geodetic Institute, **96**(2): 41-56, Presented at the XXI EGS General Assembly, Hague, May 6-10.

Gravity Field Recovery from Torsion Balance Data Using Collocation and Spectral Methods

Gy. Tóth, Sz. Rózsa, J. Ádám, and I. N. Tziavos

Department of Geodesy and Surveying, Budapest University of Technology and Economics,
Budapest, Hungary, and
Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki,
Greece

Abstract

14 235 torsion balance data for Hungary are presently available at the Eötvös Loránd Geophysical Institute (ELGI) in digital form. These data are of good quality and we have used these data to recover gravity and geoid for two test areas in Hungary. Least squares collocation and numerical integration by spectral methods were used for the computations. We have compared our predictions to gravity anomalies and geoid undulations based on the HGTUB2000 gravimetric geoid solution as well as to GPS/leveling data. The comparison shows that using only gravity gradient data sufficient accuracy can be achieved in gravity field recovery. It reaches the level of 1-2 mGal in terms of standard deviation between the observed and the predicted quantities.

1 Introduction

Nowadays there is an increasing interest in measuring gravity gradients of the Earth's gravity field. Such data are being measured from various mobile (air, marine and satellite) platforms (see Bell et al. (1997) and Pawlowski (1998)). One major advantage of gradient measurements is their relative insensitivity to platform accelerations which constitutes a major problem for airborne gravimetry. Moreover, for space gradiometric devices (e.g. the GOCE gradiometer, Visser (1999)) the exponential gravity signal decrease for increasing altitude is counteracted by the signal amplification at shorter wavelengths, which is the result of differentiating twice the gravity potential (Rummel et al., 1993).

Historically, the first gradiometric measurements were collected by the terrestrial gradiometric device invented by R. Eötvös at the end of the 19th century (Selényi, 1953). In this respect Hungary has an advantageous position, since more than 60 000 torsion balance measurements have been collected between 1901 and 1967 by different surveys (Völgyesi, 2001). The main part of the original recording of these surveys is stored at Loránd Eötvös Geophysical Institute (ELGI). Even in time of Eötvös these data have been used for various geodetic tasks besides the main purpose of collecting the torsion balance measurements, exploration for natural resources.

In this paper we use two different techniques for gravity field recovery from terrestrial gravity gradiometry (Eötvös torsion balance data); least squares collocation and numerical integration by spectral methods. Least squares collocation is particularly well suited for gradient data since it

Area A	W_{xz}	W_{yz}	$2W_{xy}$	W_{Δ}	t_{xz}	t_{yz}	$2t_{xy}$	t_{Δ}
min	-16.3	-22.9	-29.1	-43.2	-10.3	-11.4	-36.7	-28.2
max	42.5	27.6	32.3	33.7	8.5	13.3	22.4	16.2
mean	8.5	0.6	4.3	0.5	0.0	-0.3	0.2	-0.1
std	± 6.9	± 6.4	± 8.2	± 10.0	± 2.3	± 2.6	± 5.0	± 4.8
corr	0.300	0.419	0.488	0.455				
Area B	W_{xz}	W_{yz}	$2W_{xy}$	W_{Δ}	t_{xz}	t_{yz}	$2t_{xy}$	t_{Δ}
min	-38.4	-65.8	-175.3	-324.3	-42.3	-38.4	-167.9	-292.6
max	75.9	67.4	299.7	216.0	35.1	38.9	257.7	163.1
mean	13.1	-8.6	2.1	-7.8	1.5	-2.4	-1.4	-5.5
std	± 15.1	± 17.3	± 51.6	± 65.9	± 7.9	± 8.4	± 44.6	± 56.4
corr	0.649	0.708	0.927	0.918				

Table 1. Statistics of gravity gradients and curvatures for test areas A and B (number of data: 752 and 691 resp.; all figures are in Eötvös Units, $1 \text{ E.U.} = 10^{-9} \text{ s}^{-2}$). t denotes the topographic effect and the linear correlation coefficients between gravity and topographic gradients are also shown.

can handle them alone and in various combinations. For example: all of the measured gradients, mixed vertical-horizontal type $\{W_{xz}, W_{yz}\}$ and pure horizontal type combination $\{W_{\Delta} = W_{yy} - W_{xx}, 2W_{xy}\}$ can be included in gravity field recovery by least squares collocation. Examples for its application in the literature are GGSS gravity gradient data (Jekeli, 1993) processing in Arabelos and Tziavos (1992) and Arabelos and Tscherning (1999). The spectral methods can handle large amount of gradient data, but analytical solution for the integration kernel is required for pure horizontal type of torsion balance observables $\{W_{yy} - W_{xx}, 2W_{xy}\}$. Gravity anomalies and geoid heights were recovered for two areas of different topography in Hungary by these two methods.

The outline of the paper is as follows: Firstly we describe our data and data preprocessing. Secondly an outline of the different methods is presented with special emphasis on the covariance function determination. Thirdly, numerical results will be presented and discussed, with some conclusions drawn and recommendations.

2 Data preparation

2.1 Torsion balance data

The source of the terrestrial gravity gradient data for the computations was the database of ELGI, now containing 14 235 point values (average station distance is 1-3 km). This data set includes all the gradients $\{W_{xz}, W_{yz}\}$ and the so-called curvatures $\{W_{yy} - W_{xx}, 2W_{xy}\}$ and the horizontal geodetic coordinates of the stations. Also, terrain effects on all the gradients are available for each measurement station. Since no height information is associated with these data, we have interpolated station heights from a $500 \text{ m} \times 500 \text{ m}$ DTM to be used in the least squares collocation.

Two areas with different characteristics of the terrain has been selected for the gravity field recovery. Area A is of very smooth topography (minimum height: 79 m, maximum: 120 m). Its size is approximately $60 \times 40 \text{ km}$. Moderate hills can be found in the test area B of size $25 \times 22 \text{ km}$ (minimum and maximum heights of the terrain are 113 m and 320 m, respectively). The location of the above two test areas are shown in Fig. 1. The basic statistics of the measured gradients and their topographic effects for the two test areas are shown in Table 1. By looking at the numbers there seems to be a difference between the two areas since the correlation coefficients are very high in area B, especially for the pure horizontal gradient combinations (curvatures). This confirms the well-known fact that torsion balance measurements are very sensitive to the near horizontal topographic masses.

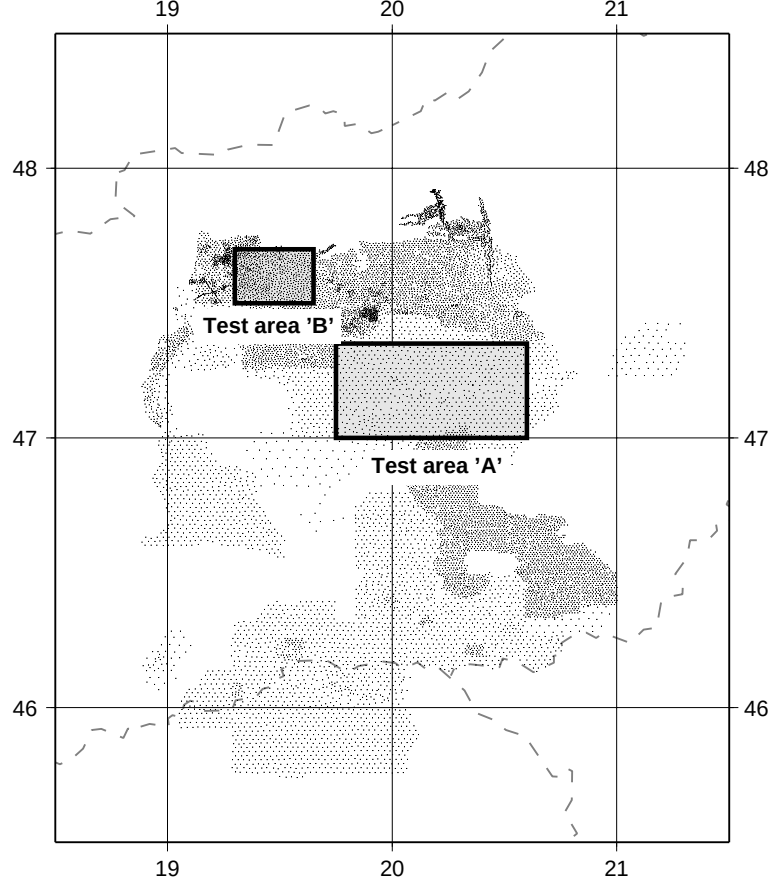


Fig. 1. Distribution of torsion balance measurements in Hungary. The locations of two test areas A and B are shown. Geodetic coordinates are in the GRS80 system.

The accuracy of gradients are estimated to be about $\pm 1 - 2$ E.U. , while for the curvature data these numbers are generally higher, $\pm 2 - 4$ E.U. according to Völgyesi (2001).

2.2 Data preprocessing

As a general rule, all known gravity signals should be removed from the data to obtain a field which is statistically more homogeneous before collocation prediction is attempted. This will make the residuals more smooth, easily interpolable, and with zero mean. The first known gravity gradient signals that have been removed were the normal value of the gradients U_{xz} and U_{Δ} , expressed by the formulas (Hein, 1981)

$$U_{zx} = \frac{1}{M} \gamma_e \beta \sin 2\varphi \approx 8.12 \sin 2\varphi \quad [\text{E.U.}] \quad (1)$$

and

$$U_{\Delta} = \gamma \left(\frac{1}{M} - \frac{1}{N} \right) \approx 10.26 \cos^2 \varphi \quad [\text{E.U.}] \quad (2)$$

where γ_e is the normal gravity at the equator, β denotes gravity flattening and M and N are curvatures in the meridian and prime vertical directions, respectively at the point with latitude φ and normal gravity γ .

The second known signal that has been removed from the gradients was a reference field represented by a geopotential model. In the present study the effect of the EGM96 field has been removed. Thirdly, the topographic effect on the torsion balance measurements has been removed. This produced about 60% signal decrease e.g. for W_{Δ} for test area B.

Finally, since it is not possible to remove a localized trend by applying these corrections only, we decided to take into account a local trend as well. This procedure was similar to the one used by Hein and Jochemczyk (1979) for the torsion balance data in Germany when modelling local covariance functions from torsion balance gradients. Three such local trend models - as functions of the position - were applied:

- Simple bias
- Linear trend function

$$f_1(\phi, \lambda) = c_1 + c_2(\phi - \phi_0) + c_3(\lambda - \lambda_0) \quad (3)$$

- Quadratic trend function

$$f_2(\phi, \lambda) = c_1 + c_2(\phi - \phi_0) + c_3(\lambda - \lambda_0) + c_4(\phi - \phi_0)^2 + c_5(\phi - \phi_0)(\lambda - \lambda_0) + c_6(\lambda - \lambda_0)^2 \quad (4)$$

In these equations $c_1, \dots, c_6$ and ϕ_0, λ_0 are constants whereas ϕ and λ are geodetic latitude and longitude, respectively.

3 Gravity field recovery

3.1 Least squares collocation

Covariance function determination

Empirical covariance functions were determined for the two test areas from differently reduced gravity gradients. Only residual gravity gradients were used for the covariance function estimation, i.e. no other gravity information has been included. The reason for this was that although $1' \times 1.5'$ mean gravity anomalies were available to us over both areas we wanted to assess the quality of the recovery of the gravity anomalies from torsion balance measurements only.

Next step was to approximate the empirical values by a model. The covariance model used was the Tscherning-Rapp Model 2 with $B = 4$ in the denominator as described in (Tscherning, 1994), Eq.(19). Thus the degree variances of the model were

$$\sigma_\ell^2(T) = \frac{A}{(\ell - 1)(\ell - 2)(\ell + 4)} \quad \text{for } \ell \geq 3. \quad (5)$$

Three combinations of reduced gradients were used for covariance fitting:

- Mixed horizontal-vertical combination $\{W_{xz}, W_{yz}\}$
- Pure horizontal combination $\{2W_{xy}, W_\Delta\}$
- All the measured gradients $\{W_{xz}, W_{yz}, 2W_{xy}, W_\Delta\}$

Empirical and theoretical autocovariance functions for the test area A are shown in Fig. 2.

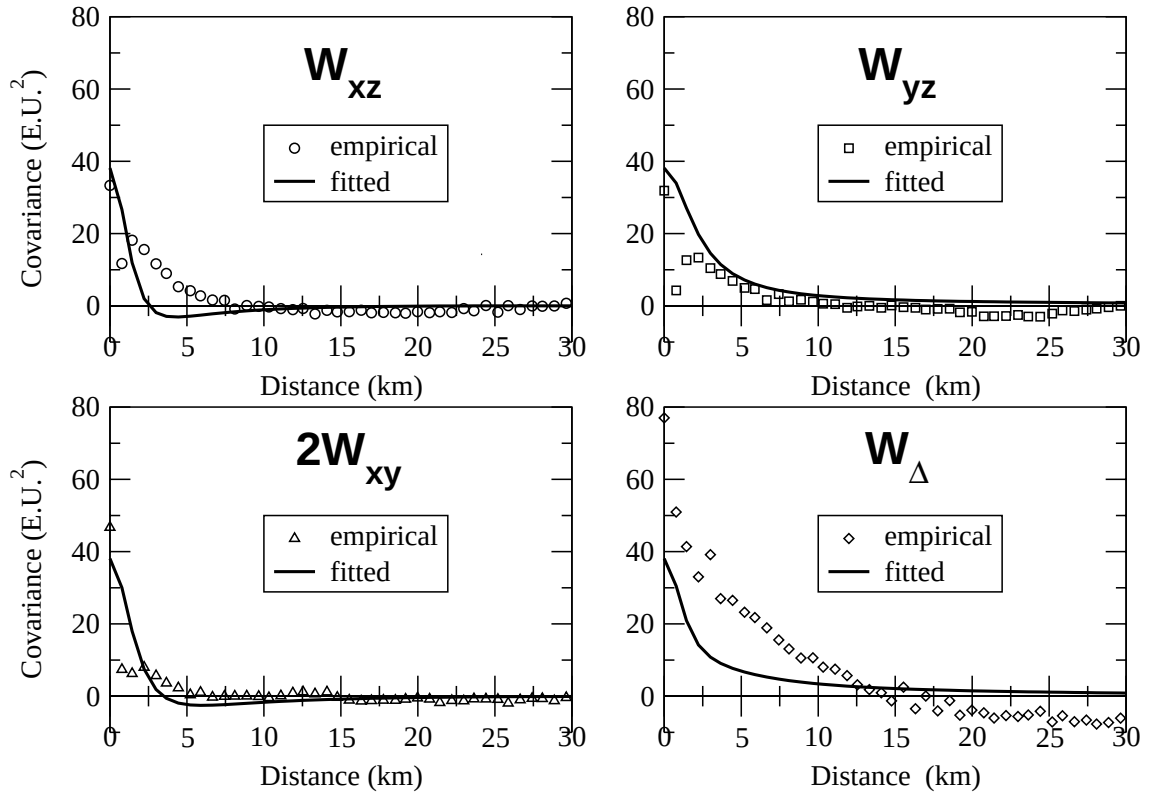


Fig. 2. Autocovariance functions for the test area A when all gradients are considered. A linear trend was removed from the residual gradients.

Gravity and geoid prediction

Since our aim was to compare the predicted gravity field gradients with the actual data, two such quantities - gravity anomalies and geoid heights - were computed by least squares collocation for the three different combinations of the torsion balance measurements (see above). Our previous test have shown (Tóth, 2001) that the linear trend model is the best, all the following computations were restricted to this choice of the trend removal. This means altogether $2 \times 3 \times 2 = 12$ combinations (the two test areas, the three combinations of the reduction, and for gravity and geoid).

The prediction points were the $1' \times 1.5'$ grid points of the HGTUB2000 gravimetric geoid solution (Tóth and Rózsa, 2000). The prediction area was slightly smaller than data area to avoid the possible edge effects. Geoid height predictions has also been performed on the GPS/Levelling points of the OGPSH Hungarian GPS network. There were 6 such points for area A and only one point for area B.

3.2 Numerical integration by spectral methods

It is possible to recover various gravity field quantities by numerical integration of the gravity gradients if the closed analytical integral formulas are available. The closed integral formulas (for both horizontal and mixed horizontal/vertical derivatives) has been developed by van Gelderen and Rummel (2000) for the theoretical solution of the geodetic boundary value problem. Working with these new formulas will be elaborated in a forthcoming paper. In the present paper a more simple procedure was followed.

Area A	min	max	mean	sd
$\{W_{xz}, W_{yz}\}$	-2.14	4.77	1.38	± 1.37
linear trend removed	-4.54	2.09	0.00	± 1.05
$\{2W_{xy}, W_{\Delta}\}$	-12.64	-6.55	-10.20	± 1.08
linear trend removed	-2.01	2.64	0.00	± 0.85
$\{W_{xz}, W_{yz}, 2W_{xy}, W_{\Delta}\}$	0.87	6.92	4.07	± 0.89
linear trend removed	-3.09	1.62	0.00	± 0.68
Area B	min	max	mean	sd
$\{W_{xz}, W_{yz}\}$	-11.98	3.31	-5.31	± 3.96
linear trend removed	-3.37	4.08	0.00	± 1.67
$\{2W_{xy}, W_{\Delta}\}$	-14.51	2.65	-6.50	± 4.48
linear trend removed	-5.75	7.85	0.00	± 2.73
$\{W_{xz}, W_{yz}, 2W_{xy}, W_{\Delta}\}$	-22.68	-3.67	-15.36	± 4.55
linear trend removed	-5.46	6.71	0.00	± 2.15

Table 2. Statistics of gravity anomaly differences in mGal for test areas A and B.

Vassiliou (1986) derived plane integration formulas and spectral relations between mixed-type gravity gradients T_{xz} , T_{yz} and gravity disturbances δg . The integral formula is the following:

$$\delta g(x, y) = -\frac{1}{2\pi} \iint \left[\frac{T_{xz}(x_1, y_1)(x - x_1) + T_{yz}(x_1, y_1)(y - y_1)}{(x - x_1)^2 + (y - y_1)^2} \right] dx_1 dy_1 \quad (6)$$

The connection between the spectra of gradients and the gravity disturbance is determined by the following equation

$$\mathcal{F}\{\delta g(x, y)\} = -\frac{j u}{2\pi q^2} \mathcal{F}\{T_{xz}(x, y)\} - \frac{j v}{2\pi q^2} \mathcal{F}\{T_{yz}(x, y)\}, \quad (7)$$

where $\mathcal{F}\{\cdot\}$ denotes Fourier transform, u and v are the spatial frequencies (in cycles per distance unit), j is the imaginary unit and $q^2 = u^2 + v^2$. If the $(2/R)T$ term, which describes the difference between the gravity anomaly and the disturbance is neglected on the residuals, Eq. (7) is the approximate spectral connection between the gravity gradients and the gravity anomalies.

This approximate formula was used in the spectral domain to perform numerical integration in planar approximation. To achieve this, residual gravity gradients were interpolated onto a 1.5 km $\times$ 1.5 km planar grid for area A, thus two 26 $\times$ 42 grids with 1092 points each were used for the spectral computations. In the FFT procedure 100% zero padding has been applied on the input grids to avoid circular convolution, see Sideris and Li (1993).

4 Results and conclusions

As it was explained in the previous section, residual gravity anomalies and geoid heights were recovered for two selected test areas with different characteristics of the terrain in Hungary. These residuals were compared to the recent HGTUB2000 gravimetric geoid solution, i.e. they were compared with residual Faye anomalies of this gravimetric solution, reduced to the EGM96 gravity and topography. This comparison is valid, since both values are topographically reduced ones. We have considered also to remove a linear trend from these differences by least squares fitting. In Figs. 3 and 4 the gravity anomaly differences are shown for the two test areas when all the gradients were included in the collocation solution. The statistics of gravity prediction differences for the

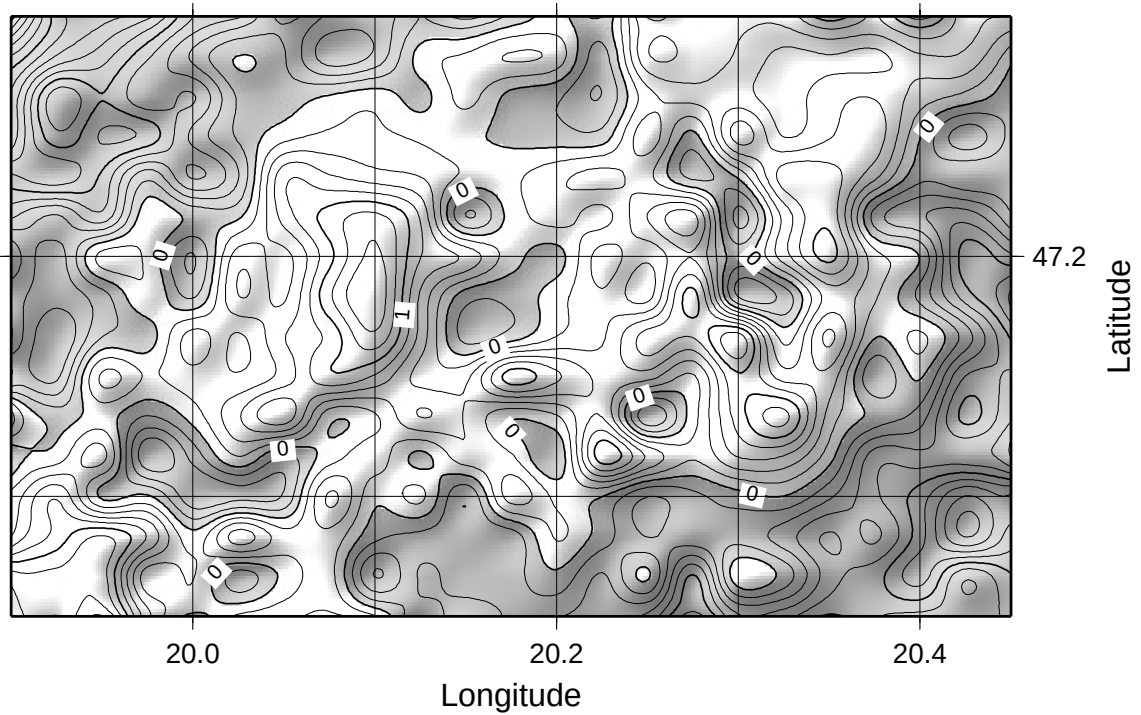


Fig. 3. Gravity anomaly differences from the collocation solution. Test area A, contour interval is 0.2 mGal, all gradients are used for gravity recovery. Linear trend was eliminated from the differences.

collocation solution are shown in Table 2 for different combination of gradients.

Corresponding numbers for the geoid height predictions are presented in Table 3.

The comparison with GPS/levelling derived geoid heights at 6 points showed after bias removal the standard deviation of differences ± 0.110 m, with maximum 0.154 m, and minimum -0.138 m.

Looking at the above numbers it can be concluded that for the area of very smooth topography the $\{2W_{xy}, W_{\Delta}\}$ combination proved to be better than the $\{W_{xz}, W_{yz}\}$ combination in terms of the standard deviation of differences. For this area it can be seen that the best results are obtained when all of the gradients are used in a combination solution, producing as low standard deviation of gravity and geoid height differences as ± 0.7 – 0.9 mGal and ± 7 – 11 mm, respectively.

For the area with moderate hills the topographic effect seems to be dominant and maybe this is the reason why the results are better with the $\{W_{xz}, W_{yz}\}$ combination. The prediction of gravity anomalies produced ± 1.7 – 3 mGal residuals in terms of standard deviation of differences and ± 8 – 15 mm in geoid heights with respect to our previous gravimetric geoid solution.

Numerical integration by planar FFT was done over area A and Table 4 shows the statistics of differences between various solutions. The area of comparison was restricted to that part of the grid where the collocation result were also available. These final comparisons have been done on a 150 m grid. It can be concluded from these results that collocation and spectral methods agree at the 0.4 mGal level when predicting gravity anomalies from mixed horizontal/vertical gravity gradients.

5 Recommendations and outlook

Our numerical tests for two selected areas in Hungary demonstrated the fact that torsion balance measurements can advantageously be applied for gravity field determination. The preferred method of using these data is in combination with other gravity field quantities such as gravity anomalies, deflections of the vertical, geoid heights derived from GPS/levelling data. Least squares collocation,

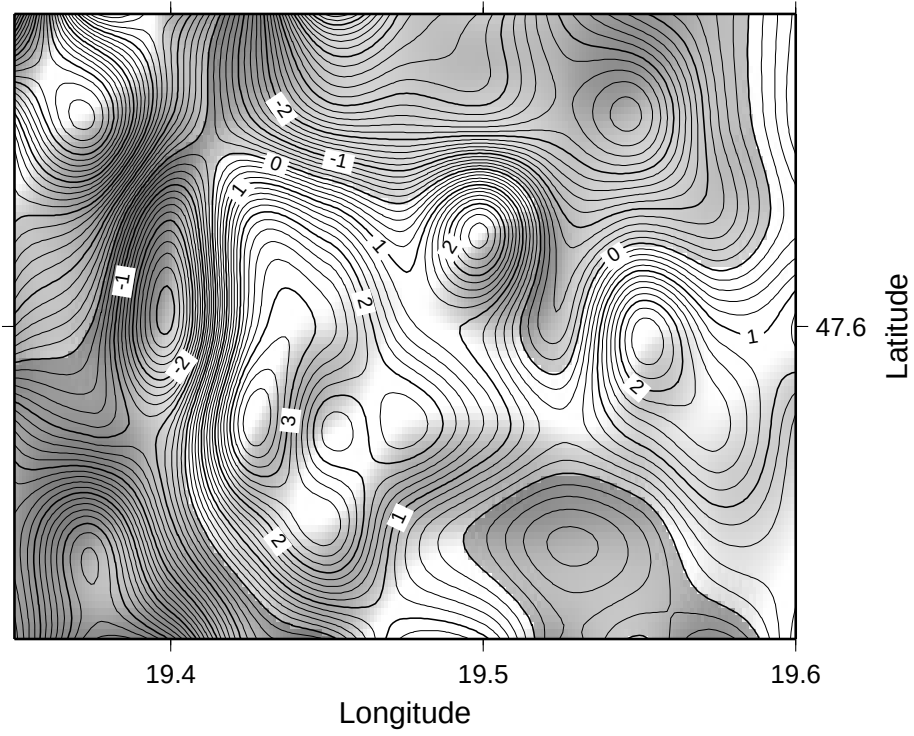


Fig. 4. Gravity anomaly differences from the collocation solution. Test area B, contour interval is 0.2 mGal, all gradients are used for gravity recovery. Linear trend was eliminated from the differences.

Area A	min	max	mean	sd
$\{W_{xz}, W_{yz}\}$	-0.791	-0.656	-0.736	± 0.036
linear trend removed	-0.014	0.018	0.000	± 0.007
$\{2W_{xy}, W_{\Delta}\}$	-0.818	-0.741	-0.778	± 0.024
linear trend removed	-0.014	0.015	0.000	± 0.006
$\{W_{xz}, W_{yz}, 2W_{xy}, W_{\Delta}\}$	-0.665	-0.573	-0.602	± 0.023
linear trend removed	-0.043	0.016	0.000	± 0.011
Area B	min	max	mean	sd
$\{W_{xz}, W_{yz}\}$	-0.764	-0.687	-0.707	± 0.015
linear trend removed	-0.059	0.018	0.000	± 0.015
$\{2W_{xy}, W_{\Delta}\}$	-1.054	-0.876	-0.989	± 0.039
linear trend removed	-0.019	0.040	0.000	± 0.014
$\{W_{xz}, W_{yz}, 2W_{xy}, W_{\Delta}\}$	-1.127	-0.795	-0.980	± 0.075
linear trend removed	-0.012	0.024	0.000	± 0.008

Table 3. Statistics of geoid height differences in meters for test areas A and B.

Area A	min	max	mean	sd
Collocation - HGTUB2000	-5.16	2.27	-1.37	± 1.04
FFT - HGTUB2000	-2.33	6.38	1.51	± 1.14
Collocation - FFT	-4.11	-1.65	-2.88	± 0.35

Table 4. Gravity differences in mGal for test area A between different solutions

though numerically tedious, can effectively be used for such combination solutions, and we plan to integrate our available torsion balance data with these other gravity field quantities to recover geoid heights accurately for Hungary.

Our tests have showed that it is essential to carefully deal with terrain effects on gradients, especially when using pure horizontal second order derivatives of the potential, before using such data for gravity field determination. Covariance function fitting can be problematic in areas with pronounced topography, so using different simple trend models to remove systematic effect might be important.

Using numerical integration for recovering gravity field quantities from the terrestrial gradient measurements is still in its infancy. To our knowledge only line integration has been used to recover deflections of the vertical, gravity anomalies or geoid heights (Völgyesi, 2001). van Gelderen and Rummel (2000) has recently shown that the solution of the geodetic boundary value problem (GBVP) is possible with different gravity field quantities including torsion balance observations. After looking at their solution, however, it seems that it will be difficult, if at all possible, to find closed expressions to some of the kernel functions involved. Anyhow, this method is also interesting since it also allows combining different parameters of the gravity field in a least squares sense. The practical application of these integral formulas is still an open direction of research for the future.

Acknowledgements. The authors gratefully acknowledge the financial support of the Hungarian Scientific Research Fund (Project No. T-020177), the support of the Geodesy and Geodynamics Research Group of the Hungarian Academy of Sciences and the support of the Greek-Hungarian Intergovernmental Scientific and Technology Cooperation Fund. The comments by G. Strykowski on the manuscript are also greatly appreciated.

References

- Arabelos, D. and Tscherning, C., Gravity Field Recovery from Airborne Gradiometer Data Using Collocation and Taking into Account Correlated Errors, *Phys. Chem. Earth (A)*, 24, 19–25, 1999.
- Arabelos, D. and Tziavos, I., Gravity Field Approximation Using Airborne Gravity Gradiometer Data, *Journal of Geophys. Res.*, 97, 7097–7108, 1992.
- Bell, R., Anderson, R., and Pratson, L., Gradiometer spinning onto Gulf trends, *The American Oil & Gas Reporter*, 1997.
- Hein, G., Untersuchungen zur terrestrischen Schweregradiometrie, in *Veröffentlichungen der Deutschen Geodätischen Kommission*, vol. C 264, DGK, München, 1981.
- Hein, G. and Jochemczyk, H., Reflections on Covariances and Collocation with Special Emphasis on Gravity Gradient Covariances, *Manuscripta Geodaetica*, 4, 45–86, 1979.
- Jekeli, C., A review of gravity gradiometer survey system data analyses, *Geophysics*, 58, 508–514, 1993.
- Pawlowski, B., Gravity gradiometry in resource exploration, *The Leading Edge*, 1998.
- Rummel, R., Sansó, F., van Gelderen, M., Brovelli, M., Koop, R., Migliaccio, F., and Schrama, E., Spherical Harmonic Analysis of Satellite Gradiometry, in *Netherlands Geodetic Commission New Series*, vol. 39, NGC, 1993.
- Selényi, P., Roland Eötvös Gesammelte Arbeiten, 1953.
- Sideris, M. G. and Li, Y. C., Gravity field convolutions without windowing and edge effects, *Bull. G éod.*, 67, 107–118, 1993.
- Tóth, G., Gravity prediction by Eötvös torsion balance data in Hungary, *to be published in Periodica Polytechnica*, 2001.
- Tóth, G. and Rózsa, S., New Datasets and Techniques - an Improvement in the Hungarian Geoid Solution, *Paper presented at GGG2000 Conference, Banff, Alberta, Canada*, 2000.
- Tscherning, C., Geoid Determination by Least-squares Collocation using GRAVSOF, in *International School for the Determination and Use of the Geoid*, pp. 135–164, Milan, october 10-15, 1994.
- van Gelderen, M. and Rummel, R., A General Least Squares Solution of the Geodetic Boundary Value Problem, *Submitted to publication in Journal of Geodesy*, 2000.
- Vassiliou, A., Numerical Techniques for Processing Airborne Gradiometer Data, in *UCSE Report*, vol. 20017, Dept. of Surveying Engineering University of Calgary, Canada, 1986.
- Visser, P., Gravity field determination with GOCE and GRACE, *Advances in Space Research*, 23, 771–776, 1999.
- Völgyesi, L., Geodetic applications of torsion balance measurements in Hungary, paper presented at the EGS XXVIth General Assembly, Nice, France, 25-30 April, 2001, 2001.

ON THE INDIRECT EFFECT OF HELMERT'S CONDENSATION METHODS

M. Kuhn^{1,2}

¹Currently at: Department of Spatial Sciences, Curtin University of Technology,
GPO Box U1987, Perth WA 6845, Australia
e-mail: kuhn@vesta.curtin.edu.au

²Deutsches Geodätisches Forschungsinstitut, Marstallplatz 8, 80539 München, Germany

ABSTRACT

Geoid determination by Stokes' formula requires that there are no masses outside the geoid and that gravity observations refer to the geoid. Both Helmert's 1st and 2nd methods of condensation satisfy these requirements by condensating the topographical masses on a layer below the geoid or directly on the geoid. For computing the direct as well as the indirect effect also a precise density model for the topographical masses is necessary. In this contribution the indirect effect for Helmert's condensation methods is studied for different density models and numerical investigations are done for two different test areas.

1. INTRODUCTION

It is common in geodesy to use mathematical reductions for the effect of topographical masses in order to satisfy the assumption of Stokes's boundary value problem that there are no masses outside the geoid and the gravity observations refer to the geoid (e.g. Heiskanen and Moritz 1967). This was the original intention of Helmert's method of condensation (Helmert 1884). Applying Helmert's model, implies the computation of three different correction terms (e.g. Wichiencharoen 1982, Martinec 1998). These are the so-called direct topographical effect on gravity, the primary indirect topographical effect on potential, which is called here indirect effect on potential, and the secondary indirect topographical effect on gravity. All these effects are computed using Newton's volume integral which means they are directly influenced by the density distribution of the topographical masses.

The indirect effect on potential of Helmert's 2nd method of condensation has been studied by several authors. Heiskanen and Moritz (1967), Wichiencharoen (1982), Vaniček and Kleusberg (1987), Wang and Rapp (1990), Heck (1993) and others have used the planar approximation of the geoid. Spherical approximation of the geoid has been employed e.g. by Martinec (1998), Sjöberg and Nahavandchi (1999), Vaniček, et al. (1999), Kuhn (2000a) among others. Due to the small magnitude of the indirect effect, Helmert's 2nd method of condensation has also been intensively studied within a geoid determination, in the so-called Stokes-Helmert technique, by many authors (e.g. Wichiencharoen 1982, Martinec and Vaniček, 1994, Vaniček and Martinec 1994, Martinec et al. 1993, Sjöberg and Nahavandchi 1999, Vaniček et al. 1999).

Usually the topographical effects are computed using a mean crustal density of $\rho_0 = 2670 \text{ kg/m}^3$. The real density distribution of the topographical masses however can differ from ρ_0 due to geological formations by more than 10%. Especially in mountainous areas these differences can result in significant influences on the geoidal height. Therefore, in the last few years the use of more sophisticated density models within a precise geoid determination can be observed (e.g. Martinec 1993, Marti 1997, Martinec 1998, Kuhn 2000a 2000b, Huang et al. 2001).

This paper studies both the indirect effect on potential of Helmert's 1st and 2nd method of condensation and the influence of different density information. Numerical investigations are made for two different test areas.

2. THE INDIRECT EFFECT ON GEOID DETERMINATION

The indirect effect on potential within the Stokes boundary value problem is the difference between the gravitational potential of the topographical masses and the compensated masses of the actual used compensation model (Heiskanen and Moritz 1967). Using Helmert's model in general the topographical masses, generating the gravitational potential V^t are condensed on a surface layer generating the gravitational potential V^c . Following this idea Helmert (1884) suggested the decomposition of V^t by

$$V^t = V^c + \delta V^h, \quad (1)$$

where δV^h is called the residual topographical potential (e.g. Martinec 1998). Taking V^t and the potential V^g generated by the masses below the geoid, the gravitational potential of the Earth can be represented by

$$V = V^g + V^t. \quad (2)$$

Following these assumptions the gravity potential of the Earth W may be expressed either by

$$W = V^g + V^t + \Phi \quad \text{or by} \quad W = V^g + V^c + \delta V^h + \Phi \quad (3)$$

where Φ is the centrifugal potential of the Earth. For Helmert's model the gravity potential of the Earth is given by

$$W^h = V^g + V^c + \Phi \quad (4)$$

where V^t is approximated by V^c . Introducing the normal gravity potential U , generated by a geocentric biaxial ellipsoid spinning with the same angular velocity as the Earth, the disturbing potential of Helmert's model becomes

$$T^h = W^h - U \quad (5)$$

T^h is harmonic outside the geoid (e.g. Martinec 1998) and therewith for a spherical approximation of the geoid, Stokes' formula (e.g. Heiskanen and Moritz 1967) can be applied to determine the disturbing potential T_0^h given on the geoid at point P_0 . Using Brun's formula (e.g. Heiskanen and Moritz 1967) and neglecting terms of the order $O(N^2)$ the separation between the geoid given in Helmert's model (co-geoid) and the reference ellipsoid is given by

$$N^h = \frac{T_0^h}{\gamma_Q} \quad (6)$$

where γ_Q is the normal gravity at the corresponding point P_Q on the reference ellipsoid. Finally applying the indirect effect on geoid here simply called the indirect effect

$$\delta N^h = \frac{\delta V_0^h}{\gamma_Q} \quad (7)$$

which is the indirect effect on potential δV_0^h , given on the geoid, divided by the normal gravity γ_Q , the geoid height is derived by

$$N = N^h + \delta N^h = \frac{T_0^h + \delta V_0^h}{\gamma_Q} \quad (8)$$

Note N^h describes the equipotential surface with the gauge value W_0 within Helmert's model of the Earth, called in general co-geoid (e.g. Wichiencharoen 1982). For this model the topographical masses are approximated by the condensed masses following the idea given in Eqn. (4). The separation between the co-geoid and the geoid is given by the indirect effect (Eqn. 7) which represents the difference by the potential effect of the residual topographical masses δV_0^h .

In order to compute the indirect effect the gravitational potentials V^i and V^c have to be computed. The first term is given by Newton's volume integral

$$V^i(\Omega, r) = G \cdot \int_{\Omega'} \bar{\rho}(\Omega') \cdot \int_{r'=r_g(\Omega')}^{r_g(\Omega')+H(\Omega')} \frac{r'^2}{L(r, \psi, r')} \cdot dr' d\Omega', \quad (9)$$

where G is Newton's gravitational constant. Here (Ω, r) denotes the geocentric spherical coordinates and the integration is performed over the topographic masses bounded by the radius of the geoid $r_g(\Omega)$ and the Earth's surface given by the radius function $r_g(\Omega)+H(\Omega)$, where H stands for the topographical height. $L(r, \psi, r')$ is the distance between the computation point (Ω, r) and an integration point (r', Ω') given by

$$L(r, \psi, r') = \sqrt{r^2 + r'^2 - 2 \cdot r r' \cos \psi} \quad (10)$$

and ψ is the angular distance between both points. The 3-D density function $\rho(\Omega, r)$ is simplified by the mean value, averaged along the topographical column of height $H(\Omega)$ as given e.g. in Martinec (1998) by

$$\bar{\rho}(\Omega) = \frac{1}{H(\Omega)} \cdot \int_{r=r_g(\Omega)}^{r_g(\Omega)+H(\Omega)} \rho(\Omega, r) \cdot r^2 \cdot dr. \quad (11)$$

The gravitational potential V^c is given by Newton's surface integral

$$V^c(\Omega, r) = GR_0^2 \cdot \int_{\Omega_0} \frac{\kappa(\Omega')}{L(r, \psi, r')} \cdot d\Omega', \quad (12)$$

where $\kappa(\Omega)$ is the corresponding surface density value derived from the 3D-density function $\rho(\Omega, r)$ and R_0 is the mean radius of the geoid in spherical approximation. The Newton kernel $1/L(r, \psi, r')$ is only weakly singular and the singularity can be removed mathematically (e.g. Martinec 1998).

In accordance with Martinec and Vaniček (1994) the superscript “ h ” denotes values occurring for Helmert's model. In principle the above described formulae can be used for any other compensation model which satisfies the assumption for Stokes' formula. In this case the values with the superscript “ h ” have to be replaced by the corresponding value for the actual used compensation model (e.g. Kuhn 2000a).

3. HELMERT'S 1st AND 2nd CONDENSATION METHOD

Helmert (1884) suggested two different methods to compensate the topographical masses by a surface layer. His aim was to create a harmonic continuation of the gravity potential of the Earth, from the Earth's surface down to the geoid. In the 1st method of condensation the topographical masses are condensed on a surface layer “parallel” to the geoid, located in a depth of $D = 21 \text{ km}$ below the geoid, whereas in the second method of condensation the condensation layer is located directly on the geoid (Figure 1). These are two simple models for the regularisation of the topographic masses which don't have a physical interpretation such as the well known isostatic compensation models, but they will fulfil the assumption for Stokes' formula.

The surface density $\kappa(\Omega)$ of the condensed masses, derived in general from the 3-D density function $\rho(\Omega, r)$, can be chosen in many different ways. In this paper the actual density distribution of the topographical masses $\rho(\Omega, r)$ is modelled by a laterally variable density function $\bar{\rho}(\Omega)$, according to Eqn. (11). In order to leave the mass of the Earth unchanged the surface density $\kappa(\Omega)$ is chosen by the principle of conservation of topographical masses. In spherical approximation of the geoid we have to deal with the masses of tesseroids

$$M_{Tess}(\Omega) = \frac{1}{3} \cdot \left[(R_0 + H(\Omega))^3 - R_0^3 \right] \cdot (\sin \varphi_2 - \sin \varphi_1) \cdot \Delta \lambda \bar{\rho}(\Omega) \quad (13)$$

and the corresponding mass of the surface element of a tesseroid

$$M_{F_{Tess}}(\Omega) = (R_0 - D)^2 \cdot (\sin \varphi_2 - \sin \varphi_1) \cdot \Delta \lambda \kappa(\Omega). \quad (14)$$

The tesseroid is the spherical body illustrated in Figure 1 bounded by the geographical coordinate lines $\lambda_1, \lambda_2, \varphi_1, \varphi_2$, and the radii R_0 and $R_0 + H$.

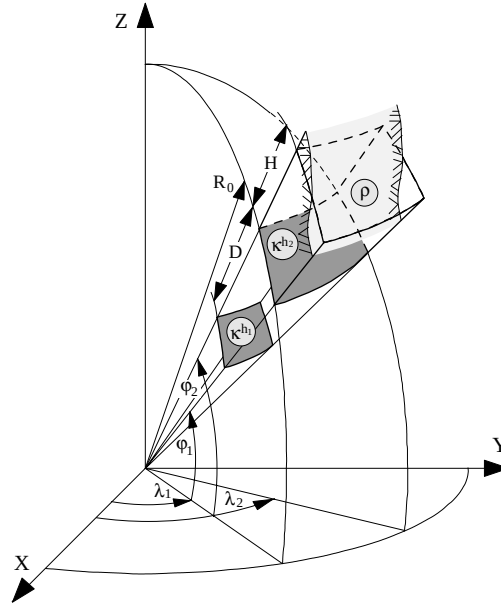


Fig. 1: Helmert's model of condensation in spherical approximation

The equality of the masses in Eqn. (13) and (14) yields the condition for the surface density of Helmert's 1st model of condensation by (e.g. Kuhn 2000a),

$$\kappa^{h_1}(\Omega) = \left(\frac{R_0}{R_0 - D} \right)^2 \cdot \left[1 + \frac{H(\Omega)}{R_0 - D} + \frac{H^2(\Omega)}{3 \cdot (R_0 - D)^2} \right] \cdot \bar{\rho}(\Omega) H(\Omega) \quad (15a)$$

with $D = 21 \text{ km}$. In the second method of condensation the surface layer is located directly on the geoid ($D = 0$) and yields the surface density

$$\kappa^{h_2}(\Omega) = \left[1 + \frac{H(\Omega)}{R_0} + \frac{H^2(\Omega)}{3 \cdot R_0^2} \right] \cdot \bar{\rho}(\Omega) H(\Omega) \quad (15b)$$

which is also given by e.g. Martinec Vaniček (1994). Further simplifications of the Eqn. (15a) and (15b) can be done by approximating the tesseroid by a prism of the same mass as it is done e.g. by Anderson (1976) or Kuhn (2000a), which results in the simple expression for the surface densities of the 1st and 2nd method of condensation

$$\kappa^{h_1}(\Omega) = \left(\frac{2 \cdot R_0 + H(\Omega)}{2 \cdot (R_0 - D)} \right)^2 \cdot \bar{\rho}(\Omega) H(\Omega), \quad (16a)$$

$$\kappa^{h_2}(\Omega) = \left(\frac{2 \cdot R_0 + H(\Omega)}{2 \cdot R_0} \right)^2 \cdot \bar{\rho}(\Omega) H(\Omega), \quad (16b)$$

The relative error using Eqn. (16a) and (16b) instead of Eqn. (15a) and (15b) reaches $1.7 \cdot 10^{-7}$ for a maximal height of 10 km which can be neglected for all practical purposes.

Finally in a planar approximation of the geoid, terms of the order H/R are neglected, and the surface density is given by the well known formula $\kappa^{h_1}(\Omega) = \kappa^{h_2}(\Omega) = \bar{\rho}(\Omega) H(\Omega)$.

4. NUMERICAL INVESTIGATIONS

To investigate the behaviour of the indirect effect of Helmert's methods of condensation two different test areas are chosen. Test area I is the region of Baden-Württemberg (south-west part of Germany). This region has mainly moderate mountains with the highest elevation of about 1500 m located in the south-west. In the western part the big valley of the Rhine can be seen (Figure 2). Test area II is the region of central Europe (Figure 3), which contains many different topographical formations, from great flat areas over different mountainous regions up to very high mountains in the European Alps with elevations of nearly 5 km.

In order to determine the indirect effect for test area I a 30'' x 50'' Digital Terrain Model (DTM) developed by the German Federal Agency for Cartography and Geodesy (Bundesamt für Kartographie und Geodäsie, BKG), has been used. For Europe and the influence of the global topography the global 5' x 5' DTM (JPG95E) developed by the NASA/Goddard Space Flight Centre (GSFC) in different resolutions is employed.

All potential effects represented by Eqn. (9) and (12) are determined by the sum of the effects from different geometrical bodies such as tesseroids and prisms for the topographical masses as well as spherical surface elements of the condensed masses (e.g. Kuhn 2000a).

In Figures 4 and 5 the indirect effect is illustrated for Helmert's 1st and 2nd method of condensation computed with the constant density $\rho_0 = 2670 \text{ kg/m}^3$ for the topographical masses and the surface densities according to Eqn. (15a) and (15b).

Both results are showing a good correlation with topography with a smoother structure in Figure 4 as in Figure 5. This is due to the farer location of the condensation layer with respect to the computation point. Another effect coming from the location of the surface layer are the higher magnitudes for the 1st method of condensation. They are going up to more than one metre in the region of the highest mountains whereas they are reaching only one decimetre in the case of Helmert's 2nd method of condensation (Table 1). Even for the highest mountains in the European Alps the indirect effect of Helmert's 2nd method of condensation (not illustrated here) reaches only some decimetres whereas they are reaching several metres for the 1st method of condensation which can also be seen in Table 1.

	Test Area I		Test Area II	
[m]	$\delta N_{\rho_0}^{h_1}$	$\delta N_{\rho_0}^{h_2}$	$\delta N_{\rho_0}^{h_1}$	$\delta N_{\rho_0}^{h_2}$
Mean	0.390	0.034	0.024	0.038
Min	-0.244	0.024	-0.673	0.023
Max	2.123	0.130	4.315	0.533

Table 1: Statistic of the indirect effect derived with $\rho_0 = 2670 \text{ kg/m}^3$.

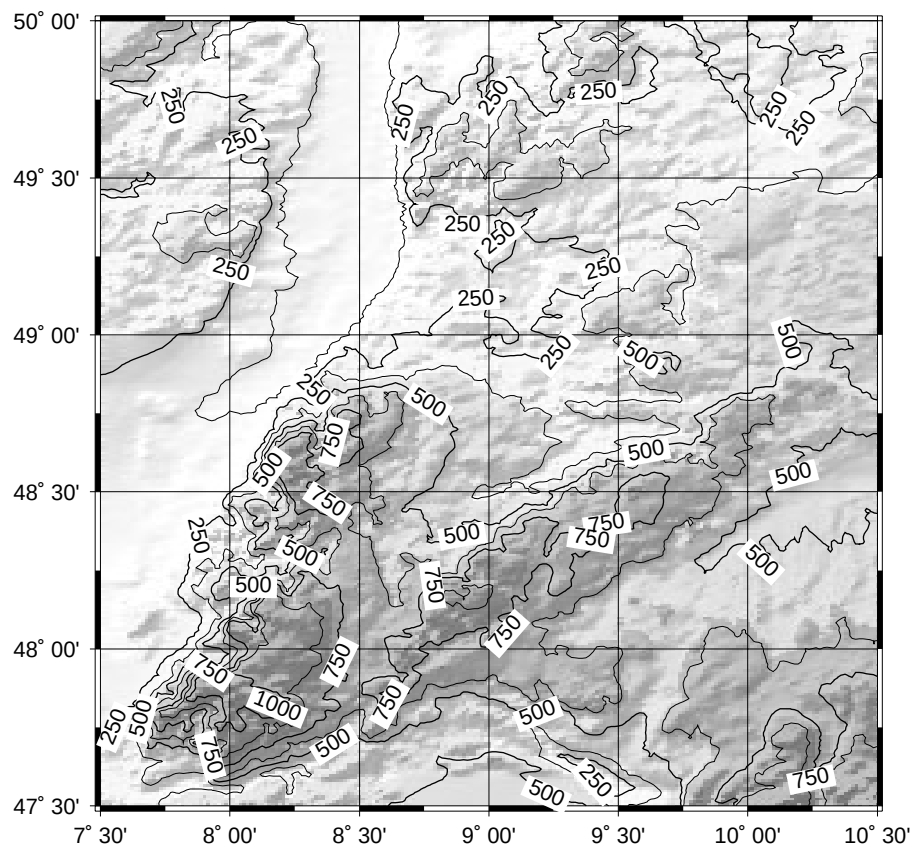


Fig. 2: Heights in test area I. Contour interval: 125 m.

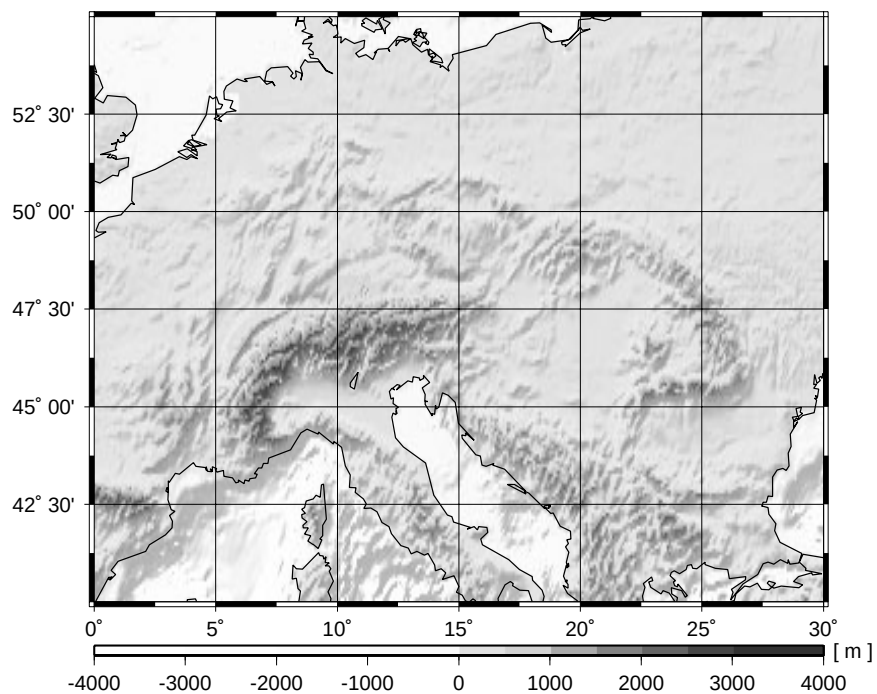


Fig. 3: Heights in test area II [m]

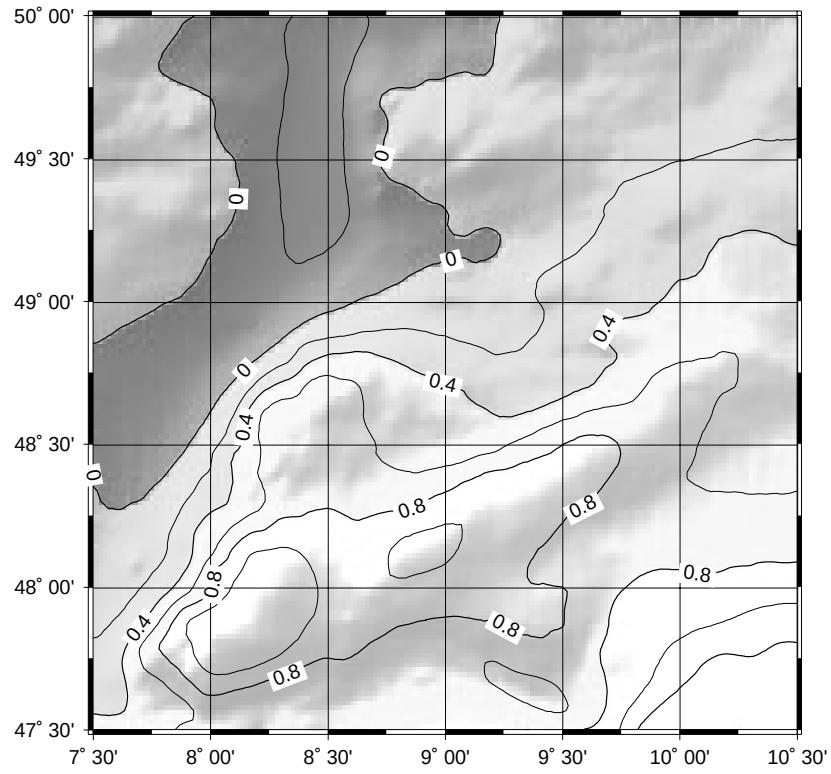


Fig. 4: Indirect effect $\delta N_{\rho_0}^{h_1}$ in test area I for Helmert's 1st method of condensation with $\rho_0 = 2670 \text{ kg/m}^3$. Contour interval: 0.2 m.

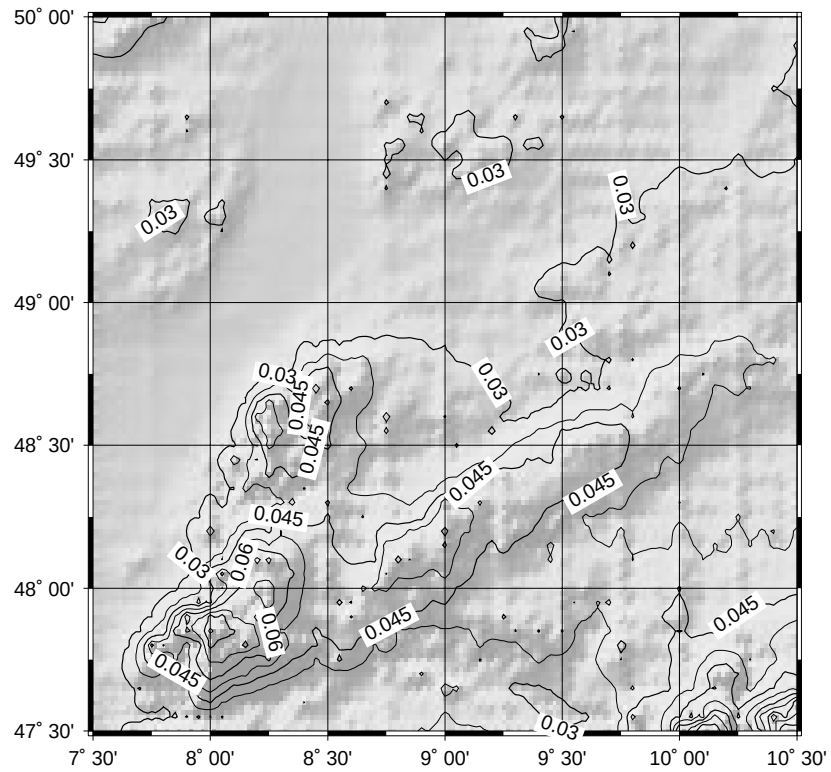


Fig. 5: Indirect effect $\delta N_{\rho_0}^{h_2}$ in test area I for Helmert's 2nd method of condensation with $\rho_0 = 2670 \text{ kg/m}^3$. Contour interval: 0.0075 m.

As mentioned at the beginning, taking only a constant density for the topographical masses is only a rough approximation of the real density distribution. Therefore in test area I a laterally variable density model is derived from a geological map and additional geophysical information. The main density anomalies are big sedimental layers in the valley of the Rhine and the so-called Molasse basin (south-east) with a mean density of 2420 kg/m^3 which differ from ρ_0 of about 10%.

Deriving density models from the information of geological maps, however, has to be acted with a little caution. They normally represent only the surface rock type and have no or only few information about the continuation with depth. Also the density of the same rock type can differ by a great amount. However in this paper the main intention is not to model the density distribution in an absolute correct way but to have a look on the influence on the indirect effect coming from different density models. For the same reason in test area II an analytical density model based on the isostatic model of Pratt-Hayford in spherical approximation is used. Here the density of the topographic masses as well as the compensation masses, which are not considered here, are laterally variable with respect to the topographical heights $H(\Omega)$. In spherical approximation of the geoid for the principle of mass conservation, the density is given by (e.g. Kuhn 2000a)

$$\bar{\rho}(\Omega) = \frac{D}{D + H(\Omega)} \cdot \left[1 - \frac{H(\Omega)}{R_0} - \frac{2}{3} \cdot \frac{(D - H(\Omega)) \cdot H(\Omega)}{R_0^2} \right] \cdot \rho_0,$$

where $\rho_0 = 2670 \text{ kg/m}^3$ is the mean crustal density for $H = 0$, and $D = 100 \text{ km}$ is the depth of compensation. Using this density model, the differences with respect to ρ_0 can reach nearly 5% for the highest elevation in the Alps. Using these two density models the results for the indirect effects are similar to those illustrated in Figures 4 and 5, which is also shown by statistical values in Table 2.

	Test Area I		Test Area II	
[m]	$\delta N_{\rho}^{h_1}$	$\delta N_{\rho}^{h_2}$	$\delta N_{\rho}^{h_1}$	$\delta N_{\rho}^{h_2}$
Mean	0.381	0.034	0.030	0.034
Min	-0.255	0.024	-0.662	0.024
Max	2.163	0.134	4.200	0.128

Table 2: Statistic of the indirect effect derived with the described density model.

More interesting is to have a look on the differences occurring with respect to the constant density model. In Figures 6 and 7 the differences are illustrated for Helmert's 1st and 2nd method of condensation within test area I. Once again the results for Helmert's 1st method of condensation are much more smoother than the results for the second method of condensation. The correlation with topography already seen in Figures 4 and 5 is still remaining. The statistical values of these results are given in Table 3.

	Test Area I		Test Area II	
[m]	$\delta N_{\rho}^{h_1} - \delta N_{\rho_0}^{h_1}$	$\delta N_{\rho}^{h_2} - \delta N_{\rho_0}^{h_2}$	$\delta N_{\rho}^{h_1} - \delta N_{\rho_0}^{h_1}$	$\delta N_{\rho}^{h_2} - \delta N_{\rho_0}^{h_2}$
Mean	-0.009	0.000	0.005	0.001
Min	-0.116	-0.005	-0.114	0.000
Max	0.072	0.008	0.012	0.016

Table 3: Statistic of the differences in the indirect effect using different density models.

The maximal differences of about ± 1 decimetre in Figure 6 occurring in the region of the highest mountains whereas they are always less than 1 centimetre in Figure 7. In Figure 6 it can be seen that

the differences for Helmert's 1st method of condensation are occurring nearly everywhere in the test area whereas they are nearly zero for great parts in the case of the 2nd method of condensation. Only in the region of the highest mountains they reach several millimetres.

The differences for test area II are illustrated in Figures 8 and 9. Here, mainly the structure of the central Alps can be seen. The differences are reaching once again more than 1 decimetre for Helmert's 1st method of condensation. In the case of Helmert's 2nd method of condensation the maximal differences are only 16 mm (Table 3).

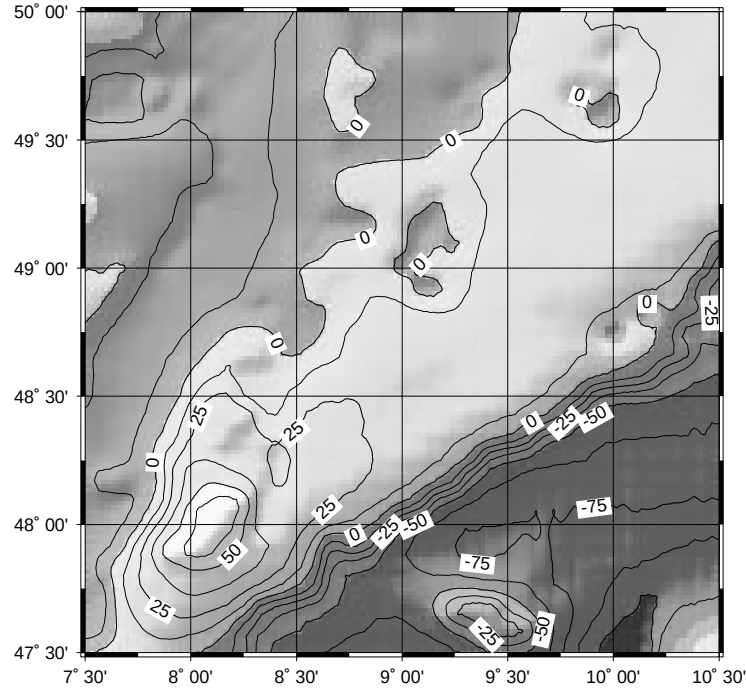


Fig. 6: Difference $\delta N_{\rho}^{h_1} - \delta N_{\rho_0}^{h_1}$ in test area I. Contour interval: 12.5 mm.

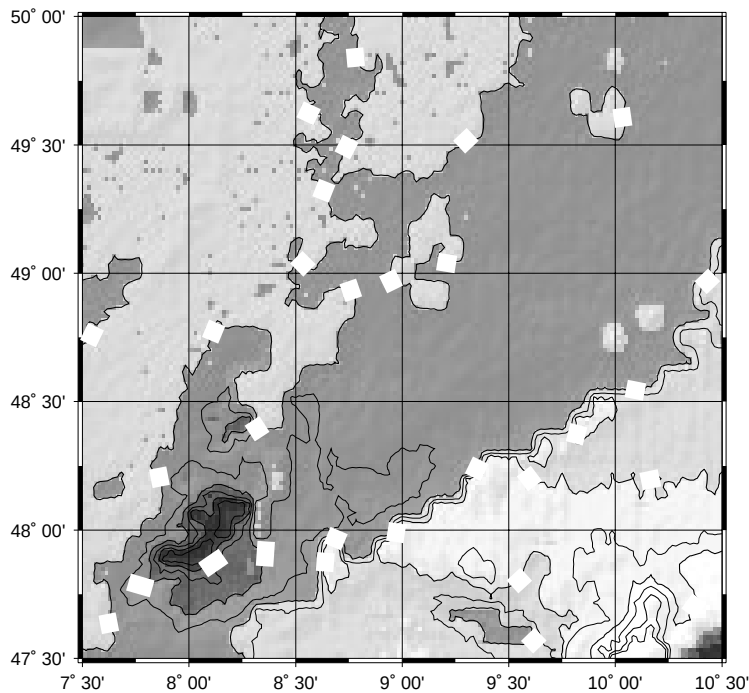


Fig. 7: Difference $\delta N_{\rho}^{h_2} - \delta N_{\rho_0}^{h_2}$ of in test area I. Contour interval: 0.5 mm.

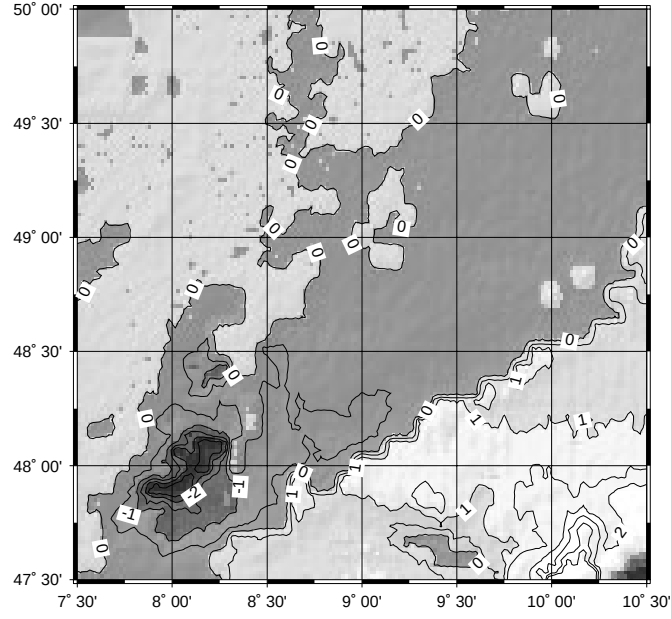


Fig. 8: Difference $\delta N_{\rho}^{h_1} - \delta N_{\rho_0}^{h_1}$ in test area II. Contour interval: 12.5 mm.

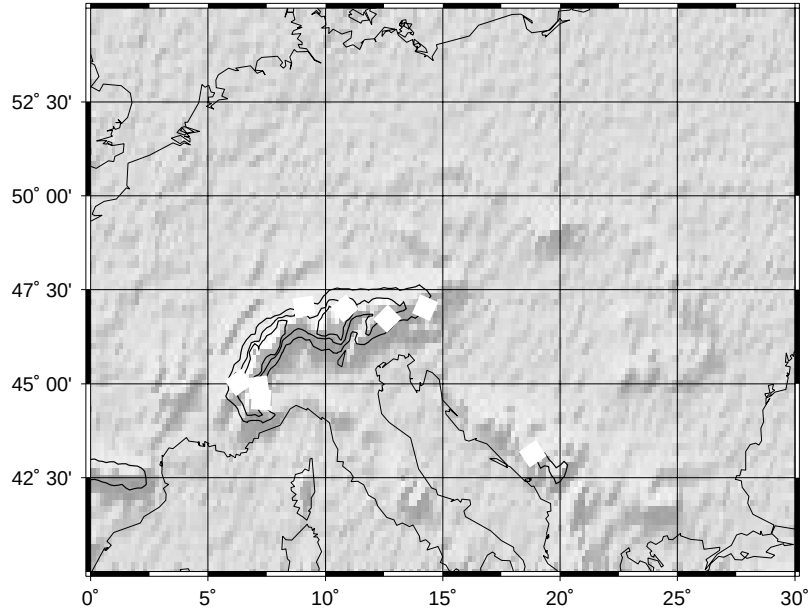


Fig. 9: Difference $\delta N_{\rho}^{h_2} - \delta N_{\rho_0}^{h_2}$ in test area II. Contour interval: 2 mm.

5. DISCUSSION AND CONCLUSIONS

It has been shown that a more sophisticated density model for the topographical masses instead of a constant density model is necessary for computing the indirect effect within Helmert's models of condensation if an accuracy of one centimetre is to be obtained. The differences in the indirect effect occurring in high-mountain areas by using different density models can reach more than ± 1 decimetre in the case of Helmert's 1st method of condensation and more than ± 1 centimetre for Helmert's 2nd method of condensation. Regarding only Helmert's 2nd method of condensation, which is used in the Stokes-Helmert technique, for flat or moderate mountainous areas the choice of

the density distribution is of minor importance in the view of a one centimetre accuracy. As it can be seen in Figures 7 and 9, for the major part of the two test areas a specific choice of the density model may be neglected and a constant density model is applicable. Only in regions of the highest mountains a more sophisticated density model has to be considered where the effects can reach more than one centimetre.

The indirect effect itself, however, has to be taken into account for both methods. Regarding an accuracy of one decimetre or less the indirect effect for Helmert's 2nd method of condensation has to be considered only in high-mountain regions.

Making a comparison with other compensation models such as the isostatic models, the 1st method of condensation yields similar results as the isostatic model of Pratt-Hayford (Kuhn 2000a, 2000b). The 2nd method of condensation however yields results that are not comparable with an other compensation model.

As mentioned before, the development of a 3-D density model by using the information of geological maps is questionable, because they normally do not give all the necessary information with depth. For determining reliable lateral variable density values, it would be more appreciable to use the information of geology together with a gravimetric method such as the method of Nettleton (1939) or the method described in Parasnis (1962).

ACKNOWLEDGEMENT

This contribution has been developed during a two year stay of the author at the Deutsches Geodätisches Forschungsinstitut (DGFI), Munich, Germany in the BMBF funded project EVAMARIA No. 03F0193A/0.

REFERENCES

- Anderson, E.G. (1976): The Effect of Topography on Solutions of Stokes' Problem. Unisurv S14, University of New South Wales, Kensington, Australia.
- Heck, B. (1993): A revision of Helmert's second method of condensation in geoid and quasigeoid determination. Paper presented at the 7th Int. Symposium of Geodesy and Physics of the Earth, IAG Symposia Proceedings, No. 112, Springer 1993, pp. 246-251.
- Heiskanen, W., Moritz, H. (1967): Physical Geodesy. Freeman and Company, San Francisco, p.364. 1967.
- Helmert, F.R. (1884): Die mathematischen und physikalischen Theorien der höheren Geodäsie. Vol. 2, Leipzig, B.G. Teubner (reprinted in 1962 by Minerva GMBH, Frankfurt/Main).
- Huang, J.; Vaniček, P.; Pagiatakis, S.D.; Brink, W. (2001): Effect of topographical density on geoid in the Canadian Rocky mountains. Journal of Geodesy, Vol. 74, pp. 805-815.
- Kuhn, M. (2000a): Geoidbestimmung unter der Verwendung verschiedener Dichtehypothesen. DGK Reihe C Heft 520.
- Kuhn, M. (2000b): Density Modelling for Geoid Determination. In M.G. Sideris (ed.): Gravity, Geoid and Geodynamics 2000, International Association of Geodesy Symposia, Vol. 123, pp 271-276.
- Marti, U. (1997): Geoid der Schweiz 1997. Geodätisch-Geophys. Arbeiten der Schweiz SGK, Band 65.
- Martinec, Z. (1993): Effect of lateral density variations of topographical masses in view of improving geoid model accuracy over Canada. Final report under DSS contract No. 23244-2-4356/01-SS, Geodetic Survey of Canada, Ottawa, June, 1993.
- Martinec, Z. (1998): Boundary-value Problems for Gravimetric Determination of a Precise Geoid. Lecture Notes in Earth Sciences, No. 73, Springer 1998.
- Martinec, Z.; Matyska, C.; Grafarend, E.W.; Vaniček, P.(1993): On Helmert's 2nd condensation method. Manuscripta geodaetica, Vol. 18, pp. 417-421.
- Martinec, Z.; Vaniček, P. (1994): The indirect effect of Stokes-Helmert's technique for a spherical approximation of the geoid. Manuscripta geodaetica, Vol. 18, pp. 417-421.
- Nettleton, L.L. (1939): Determination of density for reduction of gravimetric observations. Geophysics, Vol. 4, no. 3, pp.176-183.
- Parasnis (1962): Principles of Applied Geophysics. 4th edn., Chapman & Hall, London, 1986.

- Sjöberg, L.E.; Nahavandchi, H. (1999): On the indirect effect in the Stokes-Helmert method of geoid determination. *Journal of Geodesy*, Vol. 73, pp. 87-93.
- Vaniček, P.; Kleusberg, A. (1987): The Canadian geoid – Stokasian approach. *Manuscripta geodaetica*, Vol. 12, pp. 86-98.
- Vaniček, P.; Kleusberg, A.; Chang, H.; Fashir, H.; Christou, N.; Hofman, M.; Kling, T.; Arsenault, T. (1987): The Canadian Geoid. *Dep. of Surv. Eng. Rep.*, Vol. 129, University of New Brunswick, Fredricton, N.B.
- Vaniček, P.; Martinec, Z. (1994): The Stokes-Helmert scheme for the evaluation of precise geoid. *Manuscripta geodaetica*, Vol. 19, pp. 119- 128.
- Vaniček, P.; Huang, J., Novák, P.; Pagiatakis, S.D.; Véronneau, M.; Martinec, Z.; Featherstone, W.E. (1999): Determination of boundary values for the Stokes-Helmert problem. *Journal of Geodesy*, Vol. 73, pp. 180-192.
- Wang, Y.M.; Rapp, R.H. (1990): Terrain effects on the geoid undulation computations. *Manuscripta geodaetica*, Vol. 15, pp. 23-29.
- Wichiencharoen, C. (1982): The indirect effects on the computation of geoid undulation. *OSU Rep. No. 336*.

Terrain correction by means of simple spatial density model

Mario Brkić

Ministry of Defense, Institute for Defense Studies, Research and Development,
P.P. 252, Bijenička 46, HR-10000 Zagreb, Croatia
Email: mario.brkic@morh.hr

Tomislav Bašić

Faculty of Geodesy, University of Zagreb,
Kačićeva 26, HR-10000 Zagreb, Croatia

ABSTRACT

Simple spatial density model was implemented in the planar terrain correction (tc) approximation. The methods of exact numerical integration and Fast Fourier Transform for the tc quadratic approximation were redefined to include, besides height and surface density digital models, the subsurface horizon's depth and density digital models as well. In accordance with this, the terrain correction was computed for various resolutions of the digital models in the chosen area of the Republic of Croatia. The redefined methods were compared with numerical integration and FFT methods in which only surface variable density model was utilized. The relationship between the redefined spatial and spectral domain methods was investigated, as well as their sensitivity to the chosen density model. The superior time efficiency of the FFT was confirmed. It was shown that the terrain correction with proposed spatial density model could be used in solving physical geodesy problems.

1. INTRODUCTION

Terrain correction (tc), the effect of topographic irregularities in respect to the Bouguer's plate, is usually used in solving geodetic BVP, and in topographic reductions. In doing so, one can use planar approximation of terrain correction, where density is usually assumed as constant and representative for the test area, or given by the surface density digital model. How does the tc planar approximation look like if the density is defined as a spatial function, $\rho = \rho(x,y,z)$? In which way should the available geological information be utilized in that case? Some already suggested answers (Brkić and Bašić, 1997), given in space domain, under spherical approximation and discrete density change at subsurface discontinuities, have found practical application (Čolić et al., 1996).

The goal of this work is to propose a tc formulation in which density is defined as a simple spatial function.

2. TC PLANAR APPROXIMATION

The basic integral formula for the terrain correction in planar approximation is given by (see e.g. Tziavos, 1993):

$$tc(x_P, y_P, z_P) = G \iiint_E \int_{h_P}^h \rho \frac{(z - h_P)}{[(x - x_P)^2 + (y - y_P)^2 + (z - z_P)^2]^{3/2}} dx dy dz \quad (1)$$

where h_P is the height of the station P , and h the height of topography (Fig 1).

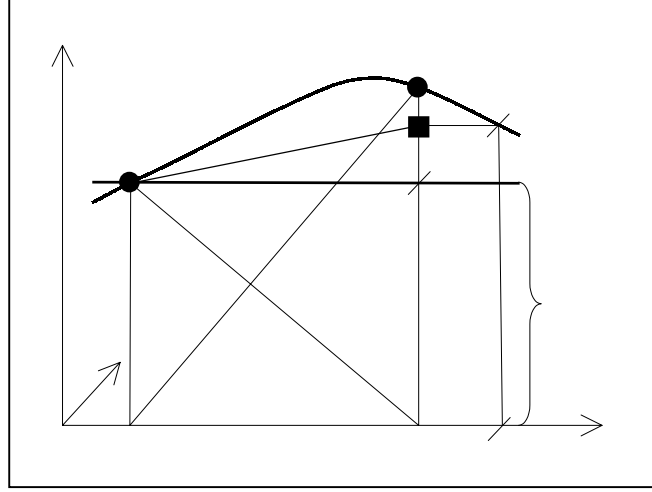


Fig. 1. Tc planar approximation.

Terrain correction may be calculated by space or spectral domain methods, such as exact numerical integration or Fast Fourier transform (see e.g. Schwarz et al., 1990), or their combination (see e.g. Tziavos and Andritsanos, 1998). In this work the integral (2) is utilized for the *exact numerical integration NII* method. Prism representation of irregular topographic masses is assumed in the space domain.

$$\begin{aligned} \iiint \frac{z}{(x^2 + y^2 + z^2)^{3/2}} dx dy dz = & x - x \ln(y + (x^2 + y^2 + z^2)^{1/2}) - \\ & y \ln(x + (x^2 + y^2 + z^2)^{1/2}) - \\ & z \operatorname{arctg}\left(\frac{x}{z}\right) + \\ & z \operatorname{arctg}\left(\frac{xy}{z(x^2 + y^2 + z^2)^{1/2}}\right) \end{aligned} \quad (2)$$

The planar approximation (1) can be integrated with respect to z , and expanded into series (*ibid.*), which may result in the so-called *2D quadratic approximation formula*:

$$tc(x_p, y_p) = G \iint_E \rho \left\{ \frac{1}{s_0} \left[\frac{1}{2} \left(\frac{\Delta h}{s_0} \right)^2 - \frac{3}{8} \left(\frac{\Delta h}{s_0} \right)^4 \right] \right\} dx dy, \quad (3)$$

where $s_0^2 = (x-x_p)^2 + (y-y_p)^2$, $\Delta h = h - h_p$, $z_p = z_p$ (see Fig. 1). In practice, the series expansion convergence criterion must be met ($|\Delta h/s_0| < 1$). This quadratic approximation formula is the starting point for the spectral domain formulation. With the following notation:

$$\begin{aligned} r_3(x, y) &= s_0^{-3}(x, y) \\ r_5(x, y) &= s_0^{-5}(x, y) \end{aligned} \quad (4)$$

eq. 3 double integrals may be rewritten in convolution form:

$$tc(x_p, y_p) = \frac{1}{2} G \left[(\rho h^2) * r_3 - 2h_p(\rho h) * r_3 + h_p^2 \rho * r_3 \right] - \frac{3}{8} G \left[(\rho h^4) * r_5 - 4h_p(\rho h^3) * r_5 + 6h_p^2(\rho h^2) * r_5 - 4h_p^3(\rho h) * r_5 + h_p^4 \rho * r_5 \right] \quad (5)$$

Using following notation of Fourier transformations:

$$\begin{aligned} R_3(u, v) &= \mathbf{F}\{r_3(x, y)\} \\ R_5(u, v) &= \mathbf{F}\{r_5(x, y)\} \\ D(u, v) &= \mathbf{F}\{\rho(x, y)\} \\ DH_i(u, v) &= \mathbf{F}\{\rho(x, y)h^i(x, y)\} \quad i = 1, 2, 3, 4 \end{aligned} \quad (6)$$

and since space domain convolutions are spectral domain multiplications, one can obtain 2D quadratic approximation spectral domain formula (FFT2):

$$tc(x_p, y_p) = \frac{1}{2} G \left[\mathbf{F}^{-1}\{DH_2 \cdot R_3\} - 2h_p \mathbf{F}^{-1}\{DH \cdot R_3\} + h_p^2 \mathbf{F}^{-1}\{D \cdot R_3\} \right] - \frac{3}{8} G \left[\mathbf{F}^{-1}\{DH_4 \cdot R_5\} - 4h_p \mathbf{F}^{-1}\{DH_3 \cdot R_5\} + 6h_p^2 \mathbf{F}^{-1}\{DH_2 \cdot R_5\} - 4h_p^3 \mathbf{F}^{-1}\{DH \cdot R_5\} + h_p^4 \mathbf{F}^{-1}\{D \cdot R_5\} \right] \quad (7)$$

Similarly to (Tziavos, 1993), in this work the FFT2 formula was numerically evaluated by application of the 100% zero-padding technique, on both data and kernel functions, simultaneously with the direct DFT computation of the kernel functions spectra. In this way the singularity problems of kernel functions were overcome. Zero-padding is also known as superior to window tapering in elimination of circular convolution errors. The above NI1 and FFT2 formulas are needed as a point of reference in numerical experiments.

3. SPATIAL DENSITY MODEL

Imagine the simplest case in which density changes in a linear manner between the surface and some subsurface horizon:

$$\rho(x, y, z) = \rho_h(x, y) + b(x, y)(h - z), \quad (8)$$

where $\rho_h(x, y)$ denotes surface densities, $b(x, y)$ vertical density gradient, and h topography height, see Fig. 2.

Both density $\rho_h(x, y)$ and its vertical gradient are generally different for each prism. Surface densities $\rho_h(x, y)$ are given in a form of a digital model. Vertical density gradient $b(x, y)$ should in practice be calculated with the help of digital models of horizon's depths and densities:

$$b(x, y) = \frac{\rho_{horizon}(x, y) - \rho_h(x, y)}{h - z_{horizon}}. \quad (9)$$

In other words, vertical density gradient $b(x, y)$ is a critical parameter in the spatial density model, as well as in tc definition. Note that in the case when the horizon is very close to the earth's surface, the gradient may destroy tc solution. This may be considered as a major limitation of the proposed density model.

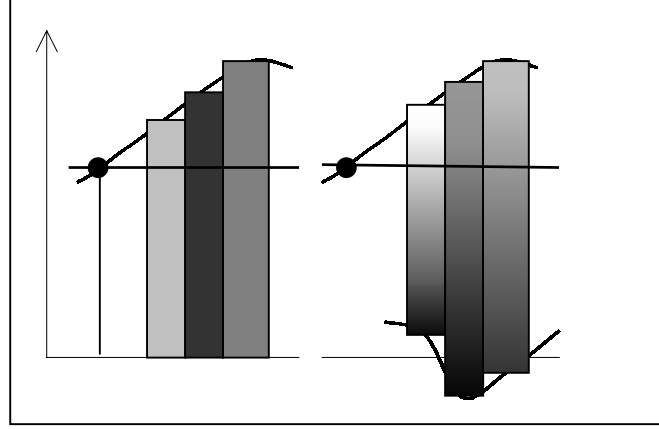


Fig. 2. Prism representation: surface variable densities (left) and simple spatial density model (right).

On the other hand, the proposed density model makes sense where linear variation of density in respect to depth is evident. The horizon does not have to be a discontinuity in the Earth's interior, but can simply be a set of measurements of depths and densities. Also, this formulation can be used for aggregation of a few close and discrete layers into one with continuous density change between the top of topography and the lowest horizon.

4. TC BY MEANS OF SPATIAL DENSITY MODEL

The planar approximation of terrain correction with presumed spatial density model (8) may be written as

$$tc(x_p, y_p, z_p) = G \iiint_E \int_{h_p}^h \frac{(z - h_p) [\rho_h(x, y) + b(x, y)(h - z)]}{[(x - x_p)^2 + (y - y_p)^2 + (z - z_p)^2]^{3/2}} dx dy dz \quad (10)$$

Beside the integral (2), the space domain *exact numerical integration* (NI4) also requires the following integral:

$$\begin{aligned} \iiint \frac{z^2}{(x^2 + y^2 + z^2)^{3/2}} dx dy dz = & -\frac{3xy}{2} + \frac{x^2}{2} \arctg\left(\frac{y}{x}\right) - \\ & \frac{y^2}{2} \arctg\left(\frac{y}{x}\right) + \\ & \frac{z^2}{2} \arctg\left(\frac{xy}{z(x^2 + y^2 + z^2)^{1/2}}\right) - \\ & \frac{y^2}{2} \arctg\left(\frac{xz}{y(x^2 + y^2 + z^2)^{1/2}}\right) - \\ & \frac{x^2}{2} \arctg\left(\frac{yz}{x(x^2 + y^2 + z^2)^{1/2}}\right) + \\ & xy \ln(z + (x^2 + y^2 + z^2)^{1/2}). \end{aligned} \quad (11)$$

NI4 numerical integration demands the usage of integral (2) two times per prism, and once of integral (11). This is the reason for significant decrease in time efficiency of space domain terrain correction calculation.

In the same way as previously, the new tc formula (10) can be integrated, expanded into series, and rewritten as *2D quadratic approximation formula*:

$$tc(x_p, y_p) = \underbrace{G \iint_E \rho_h \frac{1}{s_0} \left[\frac{1}{2} \left(\frac{\Delta h}{s_0} \right)^2 - \frac{3}{8} \left(\frac{\Delta h}{s_0} \right)^4 \right] dx dy}_{(3)} + G \iint_E b \left[\frac{1}{6} \left(\frac{\Delta h}{s_0} \right)^3 \right] dx dy \quad (12)$$

where $\Delta h = h - h_p$. In expansion of $[1 + (\Delta h/s_0)^2]^{1/2}$ the convergence condition is assumed to be fulfilled. Note that the first term of tc expression is the previous eq. 3; the second term denotes the contribution of the assumed simple spatial density model.

The only difference from the previous convolution formula is the last line:

$$\begin{aligned} tc(x_p, y_p) = & \frac{1}{2} G [(\rho_h h^2) * r_3 - 2h_p(\rho_h h) * r_3 + h_p^2 \rho_h * r_3] - \\ & \frac{3}{8} G \left[(\rho_h h^4) * r_5 - 4h_p(\rho_h h^3) * r_5 + \right. \\ & \left. 6h_p^2(\rho_h h^2) * r_5 - 4h_p^3(\rho_h h) * r_5 + h_p^4 \rho_h * r_5 \right] + \\ & \frac{1}{6} G [(bh^3) * r_3 - 3h_p(bh^2) * r_3 + 3h_p^2(bh) * r_3 - h_p^3 b * r_3]. \end{aligned} \quad (13)$$

With the following Fourier transforms:

$$\begin{aligned} R_3(u, v) &= \mathbf{F}\{r_3(x, y)\} \\ R_5(u, v) &= \mathbf{F}\{r_5(x, y)\} \\ D(u, v) &= \mathbf{F}\{\rho_h(x, y)\} \\ DH_i(u, v) &= \mathbf{F}\{\rho_h(x, y)h^i(x, y)\} \quad i = 1, 2, 3, 4 \\ BH_i(u, v) &= \mathbf{F}\{b(x, y)h^i(x, y)\} \quad i = 1, 2, 3 \\ B(u, v) &= \mathbf{F}\{b(x, y)\} \end{aligned} \quad (14)$$

and by application of the convolution theorem, the *2D quadratic approximation spectral domain tc formula (FFT4)* is as follows:

$$\begin{aligned} tc(x_p, y_p) = & \frac{1}{2} G [\mathbf{F}^{-1}\{DH_2 \cdot R_3\} - 2h_p \mathbf{F}^{-1}\{DH \cdot R_3\} + h_p^2 \mathbf{F}^{-1}\{D \cdot R_3\}] - \\ & \frac{3}{8} G \left[\mathbf{F}^{-1}\{DH_4 \cdot R_5\} - 4h_p \mathbf{F}^{-1}\{DH_3 \cdot R_5\} + \right. \\ & \left. 6h_p^2 \mathbf{F}^{-1}\{DH_2 \cdot R_5\} - 4h_p^3 \mathbf{F}^{-1}\{DH \cdot R_5\} + h_p^4 \mathbf{F}^{-1}\{D \cdot R_5\} \right] + \\ & \frac{1}{6} G [\mathbf{F}^{-1}\{BH_3 \cdot R_3\} - 3h_p \mathbf{F}^{-1}\{BH_2 \cdot R_3\} + 3h_p^2 \mathbf{F}^{-1}\{BH \cdot R_3\} - h_p^3 \mathbf{F}^{-1}\{B \cdot R_3\}]. \end{aligned} \quad (15)$$

This formula differs from the FFT2 in four direct and four inverse 2D Fourier transforms; a considerable increase of computing effort is thus not expected.

In practical calculation of the FFT4 terrain correction, 100% zero-padding of heights, densities, density gradients, their multiplications, and kernel functions is utilized, simultaneously with direct DFT computation of the kernel functions spectra.

5. DATA

Test area of the Republic of Croatia, defined by y: 5601000-5690000 m and x: 5045000-5120000 m, characterized with good reliability of input digital models, was chosen for examination of tc formulations. NI1 and FFT2 methods for tc computation utilize digital terrain model and surface density digital model. In addition, NI4 and FFT4 make use of digital depth model of PT base of sediments and PT density digital model (Fig. 3 and Fig. 4). The PT refers to Mesozoic's sediments that contain limestone, dolomite, sandstone and basite. Original digital models were described in (Čolić et al., 1996) from which 0.5 and 1 km grid spacing digital models were interpolated for the needs of the present work (Tab. 1 and Tab. 2). Since the absolute accuracy of data was unknown, all the data was assumed as errorless, so all the tc solutions are thus relative in nature.

DM	$\Delta = 0.5 \text{ km}$			
	h [m]	ρ_h [gcm ⁻³]	d_{PT} [m]	ρ_{PT} [gcm ⁻³]
mean	157.224	2.115	1948.047	2.415
rms	± 170.994	± 2.121	± 2487.269	± 2.425
min.	93.000	1.980	-550.811	1.913
max.	831.000	2.800	6834.144	2.800

Table 1. Statistics of 0.5 km input digital models.

DM	$\Delta = 1 \text{ km}$			
	h [m]	ρ_h [gcm ⁻³]	d_{PT} [m]	ρ_{PT} [gcm ⁻³]
mean	157.547	2.116	1950.426	2.416
rms	± 171.709	± 2.121	± 2490.112	± 2.426
min.	94.000	1.980	-550.811	1.924
max.	816.000	2.800	6796.903	2.801

Table 2. Statistics of 1 km input digital models.

Apparently we are dealing with geologically exciting and peculiar structures (see Fig. 5): extreme negative values of vertical density gradient b originate in density drop on small depth scales (which are about 50 m).

Previously shown tc formulations are implemented in TC_NI and TC_FFT Fortran programs, based on TCSUMD and TCFFTDTP programs, which were provided by kindness of Prof. Tziavos.

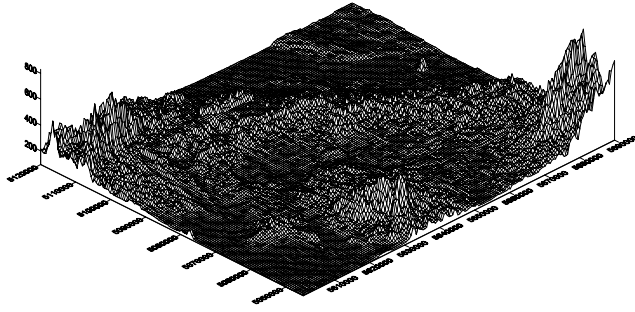


Fig. 3. Digital terrain model.

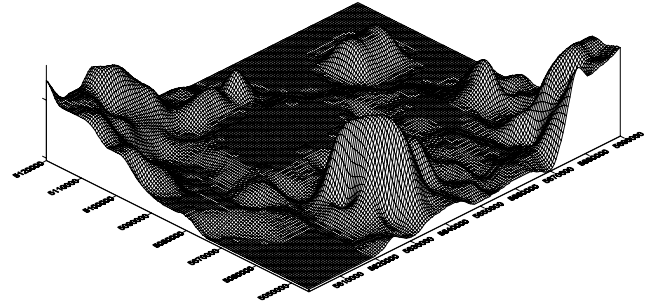


Fig. 4. Surface density digital model.

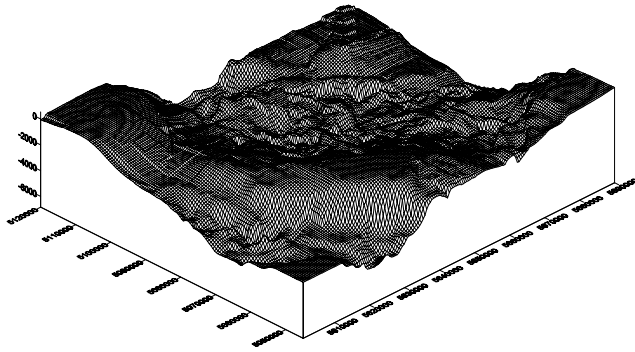


Fig. 5. PT depths digital model.

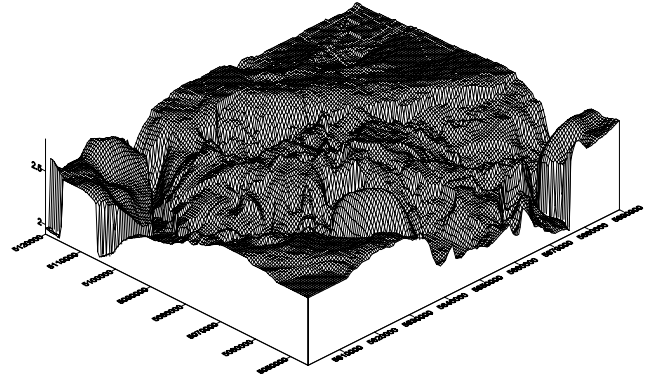


Fig. 6. PT density digital model.

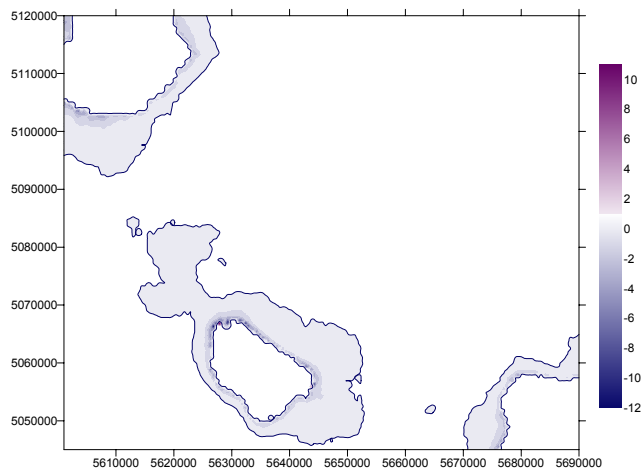


Fig. 7. Vertical density gradient b [$\text{gcm}^{-3}\text{km}^{-1}$].

6. NUMERICAL EXPERIMENTS

The first cycle of numerical experiments involved calculation of NI1 and FFT2 terrain corrections, in subsequent comparisons used as a reference. Because of long duration of NI1 tc calculation (approx. 4 months of dedicated CPU time on HP Vectra VL computer), the 0.1 km grid spacing was not investigated.

Since the tc accuracy calculated with 0.5 km grid spacing has decreased by about 0.5 mgals in respect to 0.1 km tc solution (Tziavos et al., 1988), the following comparisons should be considered as relative, but indicative and, therefore sufficient for this purpose. While Tab. 3 shows basic NI1 and FFT2 tc statistics, Fig. 8 and Fig. 9 indicate strong correlation of tc solution with the actual digital terrain model.

tc [mgal]	NI1		FFT2	
	Δ [km]		Δ [km]	
	0.5	1.0	0.5	1.0
mean	0.111	0.094	0.100	0.083
rms	± 0.324	± 0.276	± 0.282	± 0.231
st.dev.	± 0.304	± 0.260	± 0.264	± 0.216
min.	0.001	0.001	0.001	0.001
max.	7.163	4.111	6.176	3.458

Table 3. Terrain correction by means of NI1 and FFT2 methods.

The second cycle of tc computations involved PT digital models. Since the 0.1 km tc computation lasts for about 8 months, it was not feasible to exactly determine dependence of accuracy to grid spacing. Both terrain correction statistics (Tab. 4) and Fig. 10 and Fig. 11 demonstrate results very close to the first cycle.

tc [mgal]	NI4		FFT4	
	Δ [km]		Δ [km]	
	0.5	1.0	0.5	1.0
mean	0.111	0.093	0.100	0.082
rms	± 0.323	± 0.275	± 0.282	± 0.231
st.dev.	± 0.304	± 0.259	± 0.263	± 0.215
min.	0.001	0.001	0.001	0.001
max.	7.163	4.111	6.157	3.435

Table 4. Terrain correction by means of NI4 and FFT4 methods.

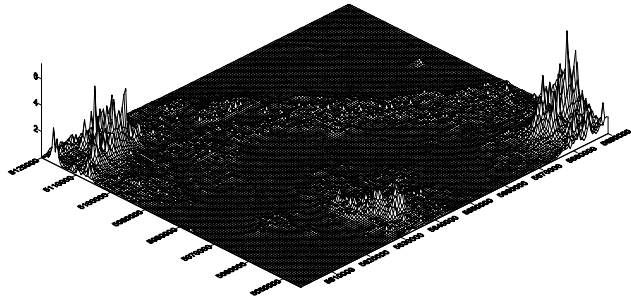


Fig. 8. NI1 terrain correction [mgal].

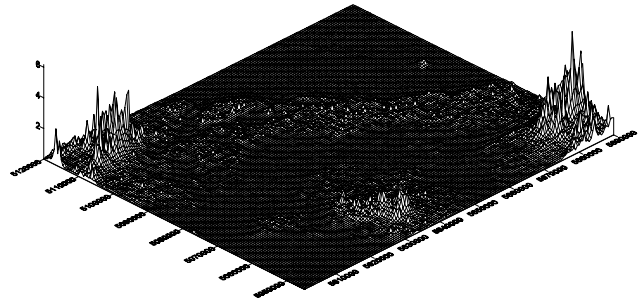


Fig. 9. FFT2 terrain correction [mgal].

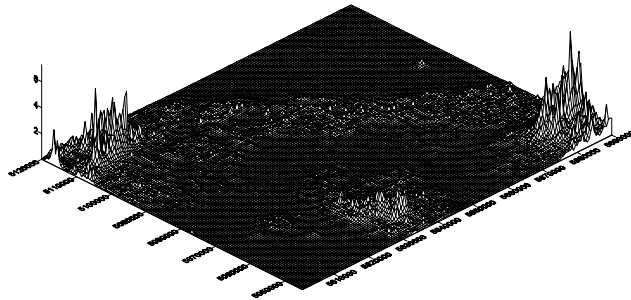


Fig. 10. NI4 terrain correction [mgal].

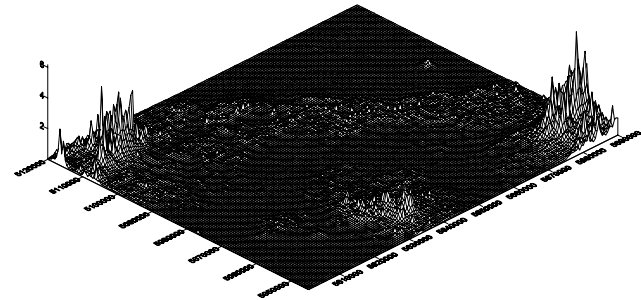


Fig. 11. FFT4 terrain correction [mgal].

Differences between surface and spatial density formulations are given in Tab. 5 and Fig. 12 and 13. These differences are correlated with actual digital heights and density models. It is evident from statistics and figures, that the differences show larger sensitivity of numerical integration method to inclusion of horizon's data.

The shape of the vertical density gradient b is clearly recognized on Fig. 12. Despite very small rms and mean differences, the effect of spatial density model may reach 0.5 mgals, so it is worthwhile to emphasize the importance of reliability of input digital models.

tc [mgal]	NI4 - NI1		FFT4 - FFT2	
	Δ [km]		Δ [km]	
	0.5	1.0	0.5	1.0
mean	-0.001	-0.001	0.000	0.000
rms	± 0.009	± 0.009	± 0.001	± 0.001
st.dev.	± 0.009	± 0.009	± 0.001	± 0.001
min.	-0.384	-0.362	-0.078	-0.034
max.	0.476	0.128	0.021	0.005

Table 5. Terrain corrections difference.

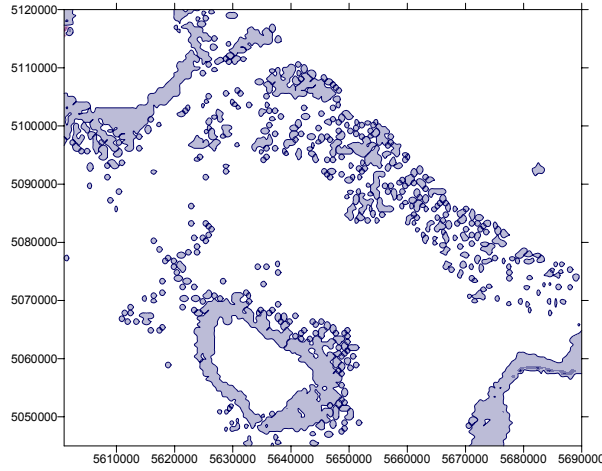


Fig. 12. NI4-NI1 tc difference [mgal].

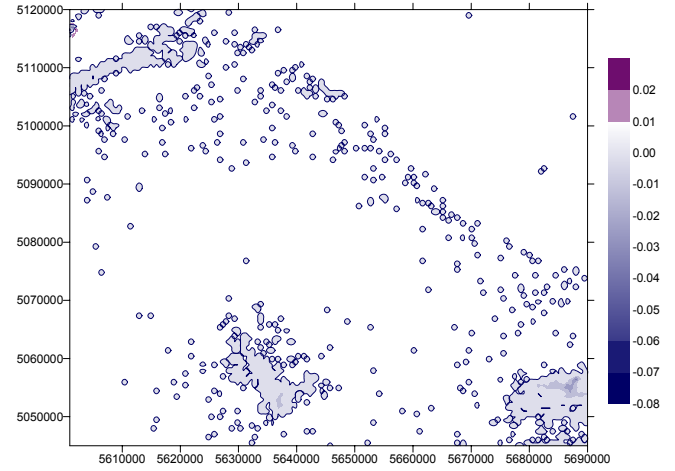


Fig. 13. FFT4-FFT2 tc difference [mgal].

Differences between two domains are given in Tab. 6. Fig. 14 shows a different sensitivity of NI1 and FFT2 methods to inclusion of surface density model. This difference is still strongly correlated to digital terrain model. The effect of the addition of PT digital models is pointed out in Fig. 15.

tc [mgal]	FFT2 - NI1		FFT4 - NI4	
	Δ [km]		Δ [km]	
	0.5	1.0	0.5	1.0
mean	-0.011	-0.011	-0.011	-0.011
rms	± 0.044	± 0.047	± 0.047	± 0.049
st.dev.	± 0.043	± 0.046	± 0.045	± 0.048
min.	-0.987	-0.713	-1.006	-0.730
max.	0.093	0.069	0.432	0.410

Table 6. Terrain corrections difference.

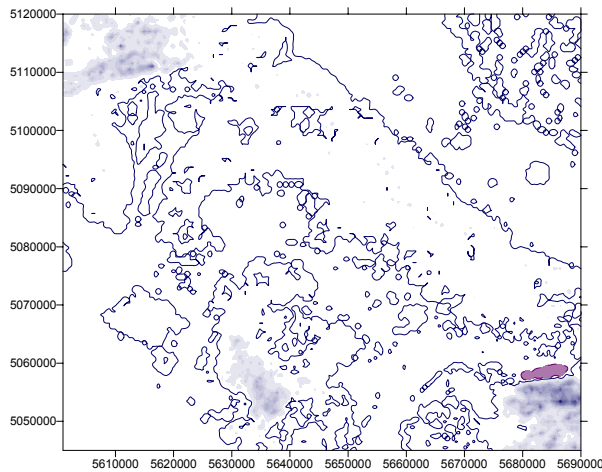


Fig. 14. FFT2-NI1 tc difference [mgal].

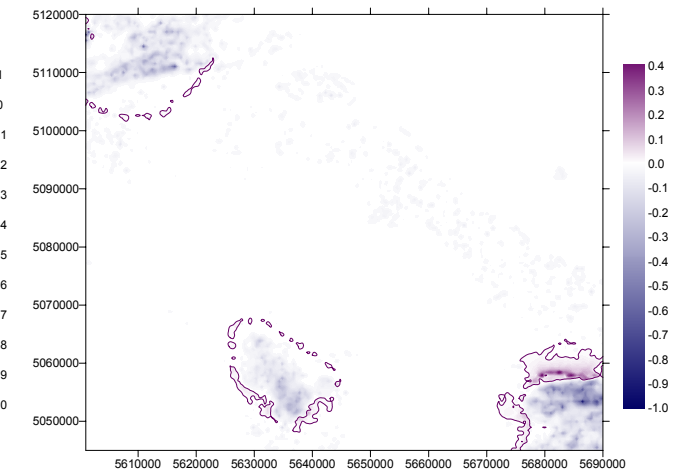


Fig. 15. FFT4-NI4 tc difference [mgal].

7. CONCLUSION

The main objective of this work was to take account of spatial density model in terrain correction formulation. The proposed simple linear density function was implemented into terrain correction space and frequency domain definition. The contribution of the proposed spatial density model in NI4 and FFT4 terrain correction was estimated by comparison to NI1 and FFT2 tc with surface non-constant densities. The NI4-NI1 differences can be considerable, and are much stronger correlated to vertical density gradient b than FFT4-FFT2 tc differences. Since both rms discrepancies are insignificant, all methods can be equally applied in solving of geodetic BVP problems.

The next objective was to examine the relationship between space and frequency domain methods. Naturally, the discrepancy between FFT and NI methods is smaller if the smallest grid spacing is used. Although FFT4-NI4 extreme differences can be significant, the use of FFT4 as an alternative to NI4 is acceptable, since the rms accuracy is under 0.5 mgal. FFT4 better time efficiency contributes to this point.

The use of the proposed NI4 and FFT4 methods seems proper if the presumed density model is geologically correct and digital models are reliable. Unrealistic data, numerical instabilities of the proposed spatial density model (in case of surface and horizon closeness), and inconsistency of digital models, can result in falsely high values and changes of vertical density gradient, whose structure can be followed in tc solutions and comparisons. This is especially obvious in the space domain method: NI4 is more sensitive to actual density model formulation than FFT4. Moreover, it can be shown that in respect to the DM corruption with random errors, NI4 method may give unrealistically high or even completely destroyed tc solutions.

Taking into account the computing time efficiency, let us point out the substantial effort in application of NI4 method. The exact numerical integration NI4 lasted 0.68 h and 9.67 h, for 1 and 0.5 km grid spacing and our test area respectively, on HP Vectra VL PC. This was roughly twice as long as in the case of the NI1 method. Exactly because of NI4 method inefficiency it was impossible to estimate tc for those grid spacing from which one can expect more accurate solutions. On the other hand, considerable timesavings arise from the use of FFT4 method: computations last only 4.56 and 23.99 seconds, which confirms the superior time efficiency of spectral methods.

Acknowledgements. The authors wish to express their sincere gratitude to I. N. Tziavos of University of Thessaloniki for providing us essential software as well as useful references.

References

- Brkić M., Bašić, T. (1997): Three Layer Earth's Crust Approximation in Topographic Reductions: a Theory and Practice, *Annales Geophysicae, XXII European Geophysical Society General Assembly*, Vienna
- Čolić, K., Bašić, T., Pribičević and B., Medak, D. (1996): A geodetic gravimetric method for better modelling of geological structures in the test area of the Croatia, *Geologica Croatica*, 49/2, 211-215

- Tziavos, I. N. (1993): Numerical Considerations of FFT Methods in Gravity Field Modelling, *Report Nr.188, Wissenschaftliche arbeiten der fachrichtung vermessungswesen der Universität Hannover*.
- Tziavos, I. N. and Andritsanos, V. D. (1998): Recent advances in terrain correction computations, 2nd Continental Workshop on the Geoid in Europe, Budapest, March 10-14, 1998.
- Tziavos, I. N., Sideris, M.G., Forsberg, R. and Schwartz, K.P. (1988): The effect of the terrain on airborne gravity and gradiometry, *Journal Of Geophysical Research*, 93(B8), 9173-9186.
- Schwarz, K. P., Sideris, M. G. and Forsberg, R. (1990): "The use of FFT techniques in physical geodesy", *Gephys. J. Int* 100, 485-514

AIRBORNE GRAVITY SURVEY OF GREENLAND'S CONTINENTAL SHELF

A.V. Olesen, R. Forsberg and K. Keller

KMS, National Survey and Cadastre, Rentemestervej 8, DK-2400 Copenhagen, Denmark

ABSTRACT.

During four field campaigns in 1998, 1999, 2000 and 2001 National Survey and Cadastre - Denmark (KMS) has done airborne gravity measurements along the coast of Greenland, from 59 to 85 degrees north and 75 degrees west to 0 degree, covering the continental shelf. The aim has been to bridge the gap between the existing terrestrial data and gravity derived from satellite altimetry in the open ocean. In the northern part with permanent ice cover almost no gravity information existed before these measurements.

The gravity system consisted of a LaCoste & Romberg S gravimeter updated to airborne use, multiple GPS receivers, a laser altimeter and a low cost IMU unit. The system was installed in a Twin-Otter aircraft equipped with autopilot and additional fuel tank.

Flight conditions were generally good and a high accuracy has been obtained in the processing of the data. Crossover analysis and independent comparisons to shipborne and ice surface gravity measurements indicate a noise level around 2 mGal at 6 km resolution. Our results show that with careful processing the need for crossover adjustment of airborne gravity measurements can be eliminated.

The paper outlines the survey, processing methods, evaluation of results and the impact of the new data on gravimetric geoid models in the coastal region. Our measurements represent a contribution to the "Arctic Gravity Project", an international effort to compile a grid of all available and releasable gravity data for the Arctic region, in support a.o. of the upcoming satellite gravity field missions.

1. INTRODUCTION

The US Naval Research Laboratory pioneered large-scale airborne gravity surveys a decade ago, see Brozena (1991). The development in the quality of airborne gravimetry in recent years is closely linked to the better performance of the Global Positioning System. Today airborne gravimetry is a truly operational tool for gravity mapping and geoid determination, see Forsberg et al. (1996).

Airborne gravimetry is a fast, viable and economic method for gravity mapping. Some of the biggest advantages are the uniform and seamless coverage of land and sea, and the ability to cover remote and otherwise inaccessible areas. Also the bias free behaviour of

airborne gravity data obtained by spring type gravimeters is an important point for geodetic applications, see Olesen et al (2000).

2. BACKGROUND OF THE AIRBORNE GRAVITY SURVEYS

A number of airborne gravimetry campaigns have been done on a regular basis since 1998, covering the coastal region of Greenland, Svalbard and vicinity, and the Baltic Sea. All operations have been done with the same airplane, a DHC-6 Twin-Otter, a general-purpose charter airplane of Greenlandair, Nuuk. The measurements have all been low and slow, mostly flown at 500 ft above sea level at 130 knots airspeed, giving high-resolution

measurements with essentially no need for analytical downward continuation.

The measurements around Greenland were performed during four field campaigns in 1998, 1999, 2000 and 2001. The purpose of the surveys has been to complement the existing on-shore gravity coverage, mainly established by KMS by helicopter-based conventional gravimetry in the years 1991-97. The airborne surveys cover a relatively narrow band along the coast of Greenland, see Figure 1. The surveys were dedicated to provide gravity data bridging across the inner continental shelf areas, where data from satellite altimetry are less reliable than in the open sea. In North and North-East Greenland wider shelf areas have been surveyed (Forsberg et al., 1999); these are regions north of the coverage of the ERS radar altimetry, or areas covered by near-permanent sea-ice. The

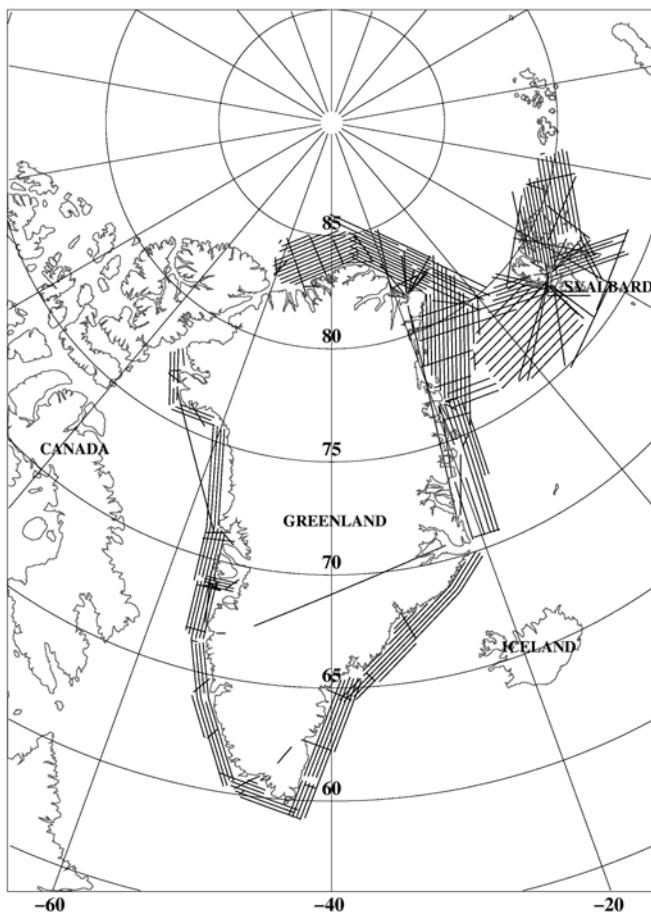


Fig. 1. Ground tracks for the Greenland surveys. Additional tracks around Svalbard flown in cooperation with Statens Kartverk, Univ. of Bergen and a Norwegian oil company are also shown.

airborne gravity surveys contribute to an ongoing effort to map the gravity and geoid of the entire Arctic region ("Arctic Gravity Project", cf. www.nima.mil/gandg/agp). The US National Imagery and Mapping Agency, NIMA, provided economic support for the Greenland surveys.

Use of the free space in the aircraft, allowed us to make joint side-by-side flight test in 1998 with a strapdown inertial gravity system from University of Calgary/Intermap (Glennie et al., 2000).

The overall dimensions of the survey are 3000 km from south to north and a total of 50,000 line kilometres of filtered data. The total flown distance is somewhat larger, approximately 65,000 kilometres. In most areas the line spacing is 10 nautical miles.

3. THE KMS AEROGRAVIMETRY SETUP

The KMS airborne gravimetry system is based on a ZLS-modified LaCoste and Romberg gravimeter (S-99, owned by University of Bergen), where the basic corrections for non-

gravitational and Eötvös effects is provided by kinematic long-range carrier-phase GPS positioning. The S-99 gravimeter has proven as an extremely stable and reliable field instrument, with typical bias drifts less than 1 mGal per week, thus making this type of system especially useful for geoid determination in previously unsurveyed areas.

The KMS gravimeter assembly is heritage from the AGMASCO project (Forsberg et al, 1996). The aircraft power converter and system control are in the KMS system mounted in a single rack, together with GPS units and a data logger. Numerous GPS receivers are usually flown (Ashtech, Trimble and Javad makes), fed by beam-splitters on two antennas (forward and aft). A custom-made inertial measurement unit, consisting of 3 Litef fibre-optics gyros and 3 Schaewitz accelerometers,



Fig. 2. Internal view of aircraft installation looking forward in cabin. Ferry tank to the left, LCR gravimeter to the right, and electronics rack in background.

provide aircraft attitude. The rack system and IMU have been custom-built by a small Danish engineering company (Greenwood Engineering). Since 2000 a medium grade integrated INS/GPS system (Honeywell H764G) has been flown as well, with the aim to provide improved attitude as well as gravity recovery redundancy. A single person operates the system in-flight, and a field crew of 2 to 3 geodesists makes for small and efficient survey operations.

For ocean ice sheet applications an Optech laser altimeter is used to map surface heights, whenever conditions permit (low fog is frequent in the arctic). Over the ocean the laser may be used as an independent source of vertical acceleration estimates, with a fit to GPS accelerations after filtering typically better than 1 mGal r.m.s. (Olesen et al., 1997).

4. GPS PROCESSING

To obtain acceleration estimates suitable for airborne gravimetry the relative position accuracy must be at the centimeter level, whereas the requirements for absolute positioning are more relaxed, and is determined by the free air gradient (0.3086 mGal/m). 0.5 m will be satisfactory for most applications. Normally this is obtained routinely with careful use of commercial GPS processing software for baselines up to several hundred kilometers.

Occasionally the GPS receiver may lose lock on the GPS carrier wave, e.g. due to ionospheric scintillation, multipath or satellite constellation changes and artificial jumps can be introduced in the position estimates. It is of crucial importance to identify such jumps and remove the effects on the resulting gravity. Due to the dynamics of the flight it is not possible to identify the jumps directly in the raw position estimates, but fortunately analysis of the high frequency part of the unfiltered gravity signal has proven to be a valuable help in the search for cycle-slip induced position jumps. The INS unit provides a similar possibility for finding GPS errors.

During 1999 and especially 2000 and 2001 the sunspot activity was quite high causing periods with unsettled ionosphere. Now and then we have had problems tracking the GPS signal, especially the L2 frequency, but it has been a minor problem causing only a few percent data loss.

5. NON-LINEAR GRAVITY SENSOR RESPONSE

Optimal modeling of the sensor response is of crucial importance in airborne gravimetry, due to the extremely high noise level. The gravimeter response to vertical accelerations was analyzed, with special attention on possible non-linearity. Figure 3 below shows the observed calibration factor for the gravimeter as a function of beam velocity and position.

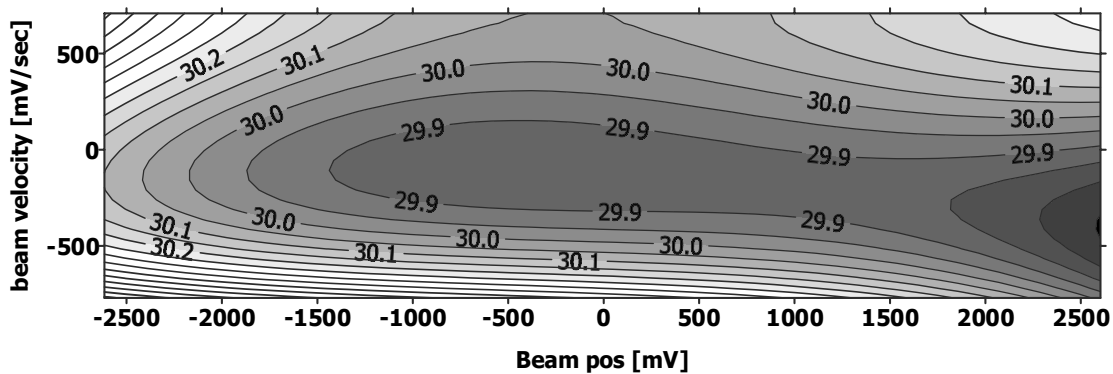


Fig. 3. Observed non-linearity of beam response

This so-called k-factor relates beam velocity with unbalanced forces acting on the beam and is normally assumed to be a constant, see Valiant (1991).

The introduction of a non-constant k-factor improves the modeling of the sensor response compared to a constant k-factor, and thereby reduces the noise signal left in the unfiltered gravimeter output. It is especially important to reduce the noise induced by the aircraft phugoid motion, since the spectral content of this noise overlaps with the gravity signal. An improved sensor modeling allows a shorter final low-pass filter to be applied and thereby enhances the resolution of the system.

Calibration factors for the platform horizontal accelerometers have been determined by a FFT technique, based on the frequency dependent behaviour of the platform, see LaCoste (1967) and Olesen et al (1997).

6. GRAVITY DATA REDUCTION

A proper synchronization of the involved data streams is obviously important. Both the gravimeter and GPS have a high frequency signal in common, the vertical acceleration of the aircraft. This signal is used to synchronize the data by cross correlation, see Olesen et al (1997).

The gravity sensor is mounted on a 2-axis gimbaled platform. The platform is kept horizontal by torque motors. A feedback loop with two horizontal accelerometers and two gyros gives the input signal to the torque motors. The gyros control the short-term behaviour of the platform, while the horizontal accelerometers control the long-term level. In the absence of horizontal accelerations the platform is driven so that the accelerometer outputs are zero. That is the condition for the platform to be orthogonal to the gravity vector and gyro drift is automatically compensated. Details of the operation principle of the LCR gravimeter can be found in Valiant (1991).

The LaCoste & Romberg meter uses a combination of two internal measurements - spring tension and beam velocity - to obtain the relative gravity variations. The basic gravimeter observation equation for relative gravity y is of the form

$$y = sT + kB' + C \quad (1)$$

where T is spring tension, s the scale factor, B' the velocity of the heavily damped beam and the factor k the beam velocity/acceleration scale. A beam-type gravimeter like the S-meter is sensitive to horizontal accelerations even when the platform is leveled, and a cross-coupling correction C is computed in real time by the gravimeter control computer.

Free-air gravity anomalies at aircraft level are (omitting second order terms) obtained by

$$\Delta g = y - y_0 - h'' + \delta g_{eotvos} + \delta g_{tilt} + g_0 - \gamma_0 - (h - N) \frac{\partial \gamma}{\partial h} \quad (2)$$

where h'' is the GPS vertical acceleration, δg_{eotvos} the Eötvös correction (computed by the formulas of Harlan, 1968), y_0 the base reading, g_0 the apron gravity value, γ_0 normal gravity, h the GPS ellipsoidal height and N the geoid undulation (EGM96 used throughout). The platform off-level correction δg_{tilt} is expressed as

$$\delta g_{tilt} = \sqrt{y^2 + A_x^2 + A_y^2 - a_x^2 - a_y^2} - y \quad (3)$$

where a and A denotes horizontal kinematic aircraft accelerations and horizontal specific forces measured by the platform accelerometers, respectively.

7. FILTERING

Low pass filtering plays a fundamental role in airborne gravity processing. The objective of the filtering is both to account for the difference in inherent filtering from the data acquisition, and to remove the high frequency part of the gravity signal. The gravimeter data acquisition system uses a 1 sec. boxcar filter on 200 Hz data, whereas the inherent filtering of the accelerations derived from GPS positions depends on the GPS processing software, and the algorithm applied for differentiation. This difference in filtering has no impact on the linear terms in the processing (the vertical GPS accelerations and the output from the vertical gravimeter sensor) due to the heavy final filtering, but may have a big influence on the nonlinear terms (mainly squared horizontal accelerations derived from GPS and the platform horizontal accelerometers). A short initial filtering, approximately 3 seconds, seems to mask this difference in inherent filtering. An improper initial filtering

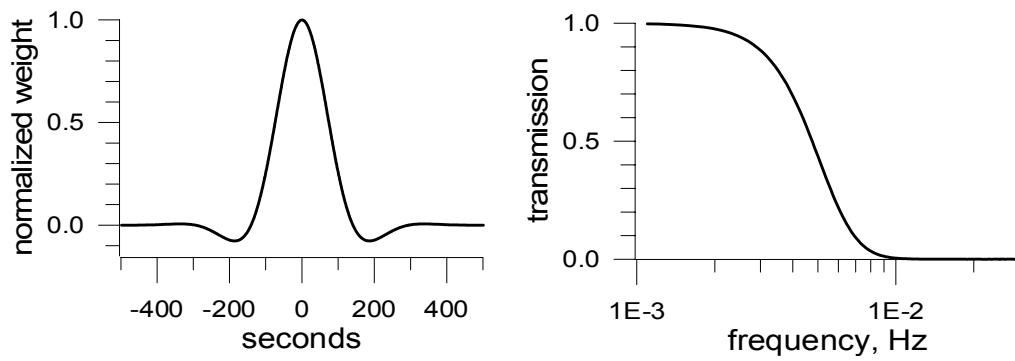


Fig. 4. Normalized convolution filter (left) and transfer function (right)

may introduce biases into the resulting gravity anomalies and thereby degrade the usefulness of the data set for geodetic purposes.

Air turbulence causes accelerations that often reach hundred thousand mGal, while we are looking for gravity anomalies of magnitude a few mGal. The low signal to noise ratio mainly affects the short wavelengths, therefore low pass filtering allows recovery of a useful signal. The filter design ultimately boils down to a trade-off between resolution and measurement accuracy.

The final filter applied to the actual gravity data is a multistage Butterworth filter with a half power point at 200 seconds, see figure 4. With an aircraft speed of 60 m/sec this filter gives a 12 km full Fourier wavelength resolution. It is common practice to use the half wavelength as a definition of resolution; in this case the resolution is 6 km.

8. DATA VALIDATION

The resulting free air anomalies for North-East Greenland shelf area are shown in Figure 5 as a contour plot. It illustrates the roughness of the gravity field around Greenland. The final processed data set comprised 479 crossovers and an analysis of the differences in the crossing points is shown in Table 1. The RMS difference of 2.8 mGal indicates a noise

Unit: mGal	no of crossings	RMS difference	max. difference
1998, 1999, 2000 and 2001	479	2.8	9.4
1998 subset	84	2.6	6.0
1999 subset	77	2.8	8.4
2000 subset	97	2.7	8.9
2001 subset	64	2.4	6.8

Table 1. Crossover statistics

level around 2 mGal (2.8 divided by square root 2), when the noise is assumed uncorrelated from track to track. It should be emphasized that no bias or tilt adjustment has been applied to the track data to obtain these numbers, as experience has shown that such an adjustment may actually degrade the data (in spite of improvements in RMS crossover statistics, cf. Forsberg et al., 1999). Any crossover adjustment will by nature distribute point errors at crossing points into along-track corrections, and thus provide a way for short-period errors to leak into longer wavelengths. For proper processed aerogravity data obtained with a nearly drift-free gravimeter, there is no physical justification to apply such a crossover adjustment. The crossover results for the 1998, 1999, 2000 and 2001 subsets of the data

show the same constant noise level for the four years, which indicates that 2 mGal noise level can be anticipated also for future work under similar not very favorable conditions, that is: long baselines for DGPS, unsettled ionosphere and a pretty rough gravity field.

For independent quality control the gravity values at existing surface data points were predicted from the airborne data, using a weighted-mean algorithm. Comparisons were only made for data points within 2 km from the airborne tracks. No downward continuation algorithms were applied to the airborne data due to the low flight altitude. Table 2 shows comparisons to Canadian gravity data collected on the sea ice in Lincoln Sea and to recent high quality marine data distributed along most of the east and west coast. The standard deviation for both comparisons is in a nice agreement with the error estimate from the cross over analysis, taking

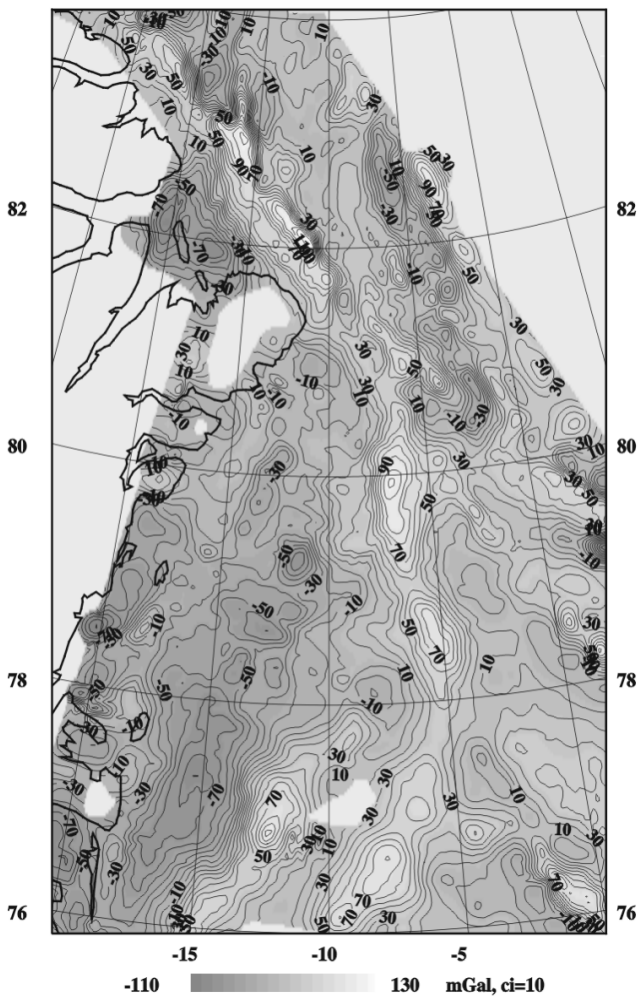


Figure 5. Free air anomalies of NE-Greenland shelf area, illustrating the roughness of the field. 10 mGal contour interval.

into account that the surface data as well as the prediction algorithm contribute to the noise. Also the table shows that the airborne data has little or no bias error. The data are therefore very suitable for geoid determination, where biases have an especially large impact, due to the inherent low-pass filtering nature of geoid determination.

Unit: mGal	max. dist.	no of points	max. diff.	mean diff.	stdv
KANUMAS marine data	2 km	2417	15.1	0.1	2.9
	1 km	1178	13.7	0.1	2.7
Canadian ice data	2 km	27	2.6	0.6	1.4
	1 km	12	2.2	0.4	1.3

Table 2. Comparison to surface data within 2 km from airborne tracks

9. IMPACT ON GEOID MODELLING

The geoid is determined from the airborne data by conventional methods of physical geodesy. With gravity errors at 2 mGal, typical relative geoid errors are at the 20-30 cm level for 18 kilometers (10 nautical miles) track spacing, as based on collocation error studies (Forsberg et al., 2000). To illustrate the impact of the new data on geoid models, a new geoid incorporating the airborne data was compared with the previous best geoid, computed by exact the same methods.

A traditional remove-restore technique was used for the computations, with the geoid signal constructed by subdividing it into three parts

$$N = N_1 + N_2 + N_3 \quad (4)$$

where the first part comes from a spherical harmonic expansion complete to degree and order 360 (EGM96), the second part from the topography, and the third part from residual gravity (i.e., gravity anomalies minus the global field model and gravimetric terrain effects). Terrain effects have been removed in a consistent "RTM" terrain data reduction scheme using rectangular prisms as basic integration element. Residual gravity anomalies are converted into residual height anomalies by spherical FFT (Strang van Hees, 1990). The method in principle evaluates Stokes' integral

$$N_3 = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_3 S(\psi) d\psi \quad (5)$$

through a series of band wise two-dimensional FFT operations of form

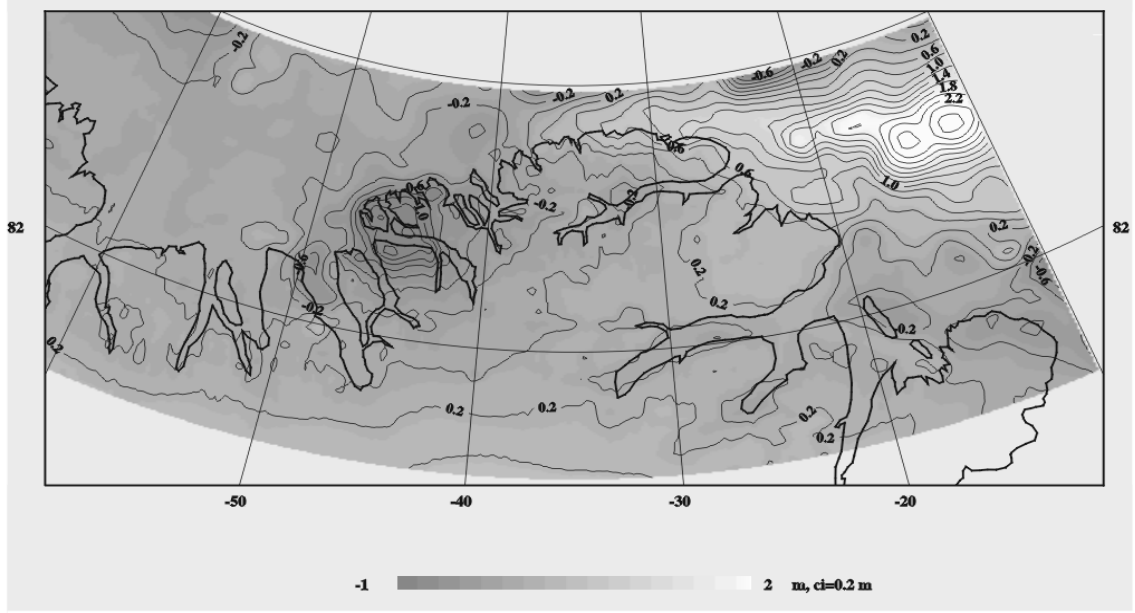


Fig. 6. Geoid differences in North Greenland

$$N_3 = S_{ref}(\Delta\varphi, \Delta\lambda) * [\Delta g_3(\varphi, \lambda) \sin\varphi] = F^{-1}[F(S_{ref})F(\Delta g \sin\varphi)] \quad (6)$$

where S_{ref} is a modified Stokes' kernel function, and $*$ and F the two-dimensional convolution and Fourier transform, respectively, for details see Forsberg and Siderius (1993). In the actual implementation of the method, the data are gridded by least-square collocation, and a zero-padding is used to limit the periodicity errors of the FFT.

Figure 6 shows the difference between the geoid models in the northern part of Greenland, where little data were available before the airborne measurements, causing the old geoid to be essentially identical to EGM96. Some major differences up to 2 meters are seen, illustrating the need for new data in this area to improve global models (e.g. EGM96).

10. CONCLUSION

The KMS airborne gravity system based on a LaCoste and Romberg S gravimeter and a Twin-Otter has proven to be a reliable tool for regional gravity mapping even under difficult logistic and operational conditions. It showed 2 mGal accuracy at 6 km resolution, and it should be noticed that this performance is obtained in an area with quite steep gradients in the gravity field. This performance was obtained repeatedly during four consecutive years with periods of unsettled ionosphere.

A careful calibration of the gravimeter on in-flight data, together with modeling of non-linearity in the sensor response resulted in a data quality in terms of accuracy and resolution

that is suitable for geodetic use. Especially the bias free properties of the resulting gravity data is an important point for geodetic applications, e.g. for geoid determination.

ACKNOWLEDGEMENT. US National Imagery and Mapping Agency, NIMA provided financial support for the airborne gravity measurements. Also thanks to Arne Gidskehaug, University of Bergen, Norway, for cooperation with the field measurements in Greenland.

REFERENCES

- Brozena, J. M.: The Greenland Aerogeophysics Project: Airborne gravity, topographic and magnetic mapping of an entire continent. In: *From Mars to Greenland: Charting Gravity With Space and Airborne Instruments*. IAG symposium series 110, pp. 203-214, Springer Verlag, 1991.
- Forsberg, R. and M. G. Sideris: Geoid computations by the multi-band spherical FFT approach. *Manuscripta geodaetica*, 18, 82-90, 1993.
- Forsberg, R., K. Hehl, U. Meyer, A. Gidskehaug, L. Bastos: Development of an airborne geoid mapping system for coastal oceanography (AGMASCO). *Proc. Int. Symp. of Gravity, Geoid and Marine Geodesy (GraGeoMar96)*, Tokyo, 1996.
- Forsberg, R., A. V. Olesen and K. Keller: Airborne Gravity Survey of the North Greenland Shelf 1998. Kort - og Matrikelstyrelsen Technical Report no. 10, 1999.
- Forsberg, R., A. V. Olesen, K. Keller: Airborne gravity survey of the North Greenland continental shelf. To appear in IAG proceedings volume of Gravity, Geoid and Geodynamics conference, Banff, 2000.
- Glennie, C L, K. P. Schwarz, A. Bruton, R. Forsberg, A. Olesen, K. Keller: A Comparison of Stable Platform and Strapdown Airborne Gravity. *Journal of Geodesy*, 74, 2000.
- Harlan, R. B.: Eotvos corrections for airborne gravimetry, *J. Geophys. Res.*, 3, 4675-4679, 1968.
- LaCoste, L. B. J.: Measurement of Gravity at Sea and in the Air. *Reviews of Geophysics*, 5, 477-526, 1967.
- Olesen A. V., R Forsberg, A. Gidskehaug: Airborne gravimetry using the LaCoste & Romberg gravimeter – an error analysis. In: M.E. Cannon and G. Lachapelle (eds.). *Proc. Int. Symp. on Kinematic Systems in Geodesy, Geomatics and Navigation*, Banff, Canada, June 3-6, 1997, Publ.Univ. of Calgary, pp. 613-618, 1997.
- Olesen, A. V., R. Forsberg, K. Keller, A. Gidskehaug: Airborne Gravity Survey of Lincoln Sea and Wandel Sea, North Greenland. *Chemistry and Physics of the Earth*, Vol 25 A, pp 25-29, 2000.
- Strang van Hees, G.: Stokes formula using fast Fourier techniques. *Manuscripta geodaetica*, vol. 15, pp. 235-239, 1990.
- Valliant, H.: The LaCoste & Romberg air/sea gravimeter: an overview. In: *CRC Handbook of Geophysical Exploration at Sea*, Boca Raton Press, 1991.

High-Resolution Geoid Computation by Combining Shipborne and Multi-Satellite Altimetry Data in the Eastern Mediterranean Sea

G.S. Vergos¹, F.A. Bayoud¹ and M.G. Sideris¹ and I.N. Tziavos²

¹Department of Geomatics engineering, University of Calgary, 2500 University Drive N.W., Calgary, Alberta, Canada, T2N 1N4

²Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki, Greece

ABSTRACT

The eastern part of the Mediterranean sea, where many geoid solutions exist, is very interesting from the geophysical and geodynamic points of view since it is the place where the Eurasian plate overthrusts the subducting African one. In this study, a new high-resolution multi-satellite (GEOSAT-GM, ERS1-GM, T/P) altimetric geoid is computed and validated, and then combined with shipborne gravity data to obtain the final model. For the low frequency part of the gravity spectrum, we used and compared the results of two geopotential models, EGM96 and GPM98b, complete to degree and order 360 and 1800, respectively. The solutions are compared with T/P data as control points, and with the existing GEOMED solution. We focus on the use of the Multiple Input Multiple Output System Theory (MIMOST) method for the efficient and optimal combination of the heterogeneous data, in which special emphasis is paid to the prediction error estimates. This study is part of a general one aiming at the computation of a high-resolution, high-accuracy geoid for the entire region using altimetry, marine, and land gravity data.

KEYWORDS

Geoid determination, marine geoid, satellite altimetry, sea surface heights, shipborne gravimetry, combination, MIMOST.

1. INTRODUCTION

The Mediterranean Sea has been the spotlight of many geoid computations in the recent years due to its geophysical and geodynamical characteristics. With the advent of altimetry, it became a natural laboratory for geoid and Sea Surface Topography (SST) determinations. In the past ten years, many studies have been conducted, aiming to determine a high-accuracy and high-resolution geoid. Among the many studies, we mention the GEOMED Project, Arabelos and Tziavos (1996), Tziavos et al. (1998), Andritsanos et al. (2000b), Vergos (2000), among others. Our study is focused on the Eastern part of the Mediterranean Sea ($31.5^{\circ} \leq \varphi \leq 36.5^{\circ}$ and $27.5^{\circ} \leq \lambda \leq 35.5^{\circ}$) close to the island of Cyprus and the Middle East coastline. Recent analysis of geodetic data, collected in the past two decades, indicate that the eastern part of the Mediterranean Sea has one of the region's largest horizontal motions with respect to a stable Europe (Smith et al., 1994; Le Pichon et al., 1995).

The marine gravity data for the area under study are characterised by their insufficient accuracy. This study aims, using a remove-restore method, at the determination of a high-accuracy and high-resolution geoid by combining all available data sources. Furthermore, we want to investigate whether the combined use of satellite altimetry and shipborne gravity data will improve the geoid modelling compared to the gravimetric geoid only.

Altimetric Sea Surface Heights (SSHs) from the GEOSAT and ERS1 Geodetic Missions (GMs) are used for the computation of the altimetric geoid solution. The land and marine free air gravity anomalies used in this study come from the Bureau Gravimétrique International (BGI, 2000) database.

The gravimetric geoid was computed by the 1D-FFT Stokes convolution (Haagmans et al., 1993; Sideris and She, 1995) using the available free-air gravity anomalies. Shipborne gravity and altimetry data are used for determining a high-resolution ($5' \times 5'$) and high accuracy geoid by optimally combining them with the Multiple Input Multiple Output System Theory (Bendat and Piersol, 1986; Sideris, 1996; Andritsanos et al., 2000a). The above three solutions were referenced to two geopotential models, namely EGM96 (Lemoine et al., 1996), complete to degree and order 360, and GPM98 (Wenzel, 1998), complete to degree and order 1800. As control points and for the validation of our solutions TOPEX/POSEIDON (T/P) SSHs from the 3rd year of the satellite's mission were used. The 3rd year of the T/P mission was selected so that a common observation period between satellites (T/P and ERS1) would be available.

2. ALTIMETRIC GEOID

With the advent of space geodesy and especially during the last fifteen years, satellite altimetry provided the first extensive, long-term and accurate measurement of the Earth's oceans. Satellite altimetry proved to be an accurate and economical, compared to shipborne gravimetry, method to model the oceans and determine the gravity field over large water areas. The geodetic missions of GEOSAT and ERS1 altimetric satellites provide a vast amount of data with resolutions of 2-4km at the equator (Smith et al., 1994). These data cover the Earth's oceans densely and offer precise information due to the accurate models used for the geophysical and instrumental corrections and the modeling of the satellite orbit. Thus, it is possible to compute a marine geoid with an accuracy close to a few centimeters. The altimetric data sets used in this study were GEOSAT-GM newly realized SSHs referred to JGM-3 (Joint Gravity Model 3) orbits provided by NOAA (1997). The observation period of this data set was from March 30th, 1985 to September 30th, 1986. Additional altimetric data were those of the mapping phase of ERS1-GM SSHs from AVISO (1998). The observation period of these altimetric data extends from April 10th, 1994 to March 21st, 1995. Finally, the 3rd year of T/P SSHs distributed by AVISO (1998) were used. The period of observation of this last data set was from October 16th, 1994 to October 8th, 1995. Due to the limited test area the validation tests were first carried out in a larger area ($30^{\circ} \leq \varphi \leq 48^{\circ}$ and $0^{\circ} \leq \lambda \leq 40^{\circ}$) in order to compute representative outcomes.

2.1 VALIDATION OF GEOSAT-GM DATA AND GEOID SOLUTION

The GEOSAT-GM SSHs were provided to us in the usual Geophysical Data Record (GDR) format, so a first preprocessing step to correct the SSHs due to the geophysical and instrumental errors, was needed. The models and methods used for the aforementioned corrections are the same as those described in the GEOSAT-GM handbook (1997). In the present study, the signal of the quasi-stationary part of the Dynamic Ocean Topography (DOT) was not taken into account due to the lack of external information about it. Thus, we assumed that the corrected SSHs (174546 point values) can be considered as geoid heights (N). GEOSAT-GM data cover not only sea but some continental areas as well, hence the first test had to deal with the removal of these data. It is well-known that satellite altimetry SSHs give some erroneous values close to the coastline due to the scattering of the radar altimeter pulse from the shallow sea bottom; for that reason, data close to the coastline were removed, as well. A $1' \times 1'$ bathymetric model for the Mediterranean Sea was used (Sandwell, 1996) and an interpolation of depth values was carried out at the points where SSHs from GEOSAT-GM were available. Then, data points with depths

greater than -200m were neglected (see Table 1). According to this test, 25855 point values (14.82%) were finally removed (see Table 1).

max	min	mean	std
240.151	-286.336	29.553	± 13.323
240.151	-286.336	28.858	± 13.428

Table 1. GEOSAT-GM data before and after the bathymetry test. Unit: [m].

From Table 1, even after the bathymetry test, some extreme minimum and maximum values are still evident, thus indicating the presence of outliers. After the bathymetry test the long wavelength part of the geoid heights was removed by subtracting the contribution of EGM96 (Lemoine et al., 1996) and GPM98b (Wenzel, 1998) geopotential models from the SSHs. The statistics of the so derived residual geoid heights (N_{res}) are summarized in Table 2.

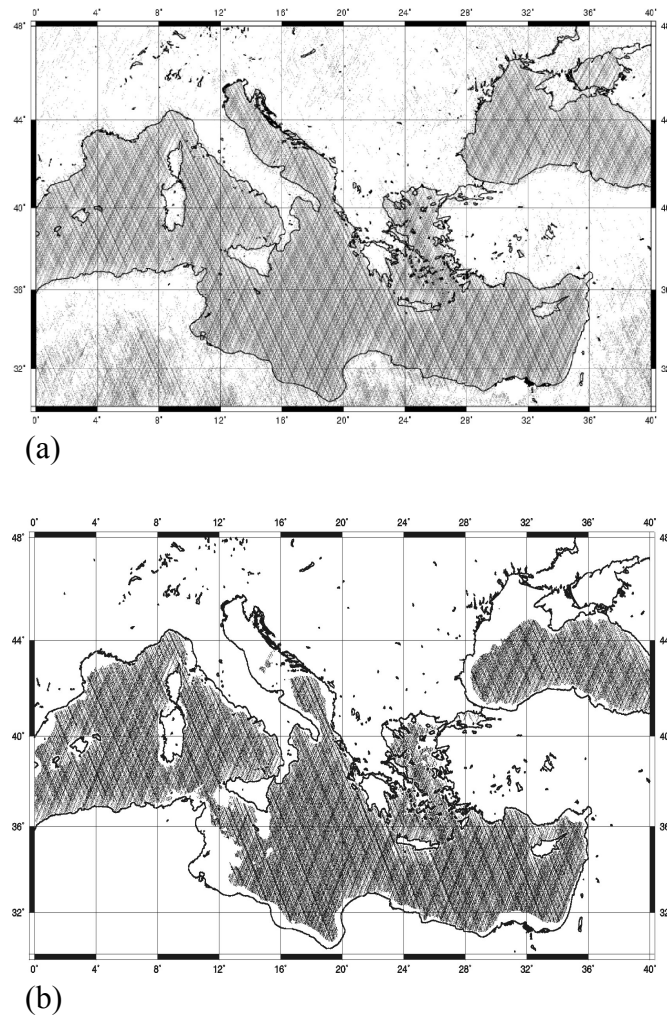


Figure 1. GEOSAT-GM data distribution before (a) and after (b) the bathymetry test.

	max	min	mean	std
N_{res} (EGM96)	212.341	-324.884	-0.473	±2.737
N_{res} (GPM98b)	212.823	-324.557	-0.502	±2.648

Table 2. GEOSAT-GM residual undulations. Unit: [m]

The residual geoid heights in Table 2 indicate a number of unexpected values. This is due to the presence of blunders and/or systematic errors. In order to remove these large values, an additional 3rms test for blunder detection was performed for the entire Mediterranean Sea. This resulted in the removal of 1926 residual geoid heights (1.3%) in the case of EGM96 and 1379 residual geoid heights (0.93%) in the case of GPM98b. To summarize, during the two validation tests (bathymetry and 3rms) 27781 GEOSAT-GM geoid heights (15.9%) reduced to EGM96 geopotential model and 27324 geoid heights (15.6%) reduced to GPM98b were eliminated. The remaining residual geoid heights in the area under study ($30^{\circ} \leq \varphi \leq 38^{\circ}$ and $27^{\circ} \leq \lambda \leq 36^{\circ}$) are 24563 and 24621 referenced to EGM96 and GPM98b fields, respectively. The statistics of these residual geoid heights in the test area are given in Table 3.

	max	min	mean	std
N_{res} (EGM96)	1.482	-1.490	-0.337	±0.321
N_{res} (GPM98b)	1.527	-1.816	-0.764	±0.265

Table 3. GEOSAT-GM residual undulations in the area under study, after the 3rms test. Unit: [m]

Comparing Tables 2 and 3, a refinement of the GEOSAT residual geoid heights in terms of minimum, maximum and standard deviation values is obvious. To these residual geoid heights, a bias and tilt crossover adjustment was applied for the minimization of the orbital errors. Comparing the residual geoid heights before and after the adjustment, insignificant improvement, for the case of EGM96, and slight deterioration, for the case of GPM98, in terms of the standard deviation and the mean value of the differences were detected. This can be mainly attributed to the improved and high accuracy orbits used for the present release of the GEOSAT-GM GDRs (JGM3). We believe that this is a good indication that the remaining radial orbit errors in the GEOSAT-GM data are very small and a crossover adjustment is not needed to reduce it. On the other hand, crossover adjustment can be very useful in reducing the influence of oceanic phenomena contaminating geodetic mission data, especially in regions with high sea surface variability (SSV) located in purely oceanic regions and not in a closed sea area like the Mediterranean Sea. For these reasons, the residual geoid heights after the 3rms test and before adjustment were used for the subsequent geoid determination. In the next step the point residual geoid heights were interpolated onto a $5' \times 5'$ grid. Then, as a final step of the GEOSAT-GM data processing, the contribution of EGM96 and GPM98b geopotential models to the residual geoid heights was restored and the final geoid height solutions of GEOSAT-GM altimetry data were calculated (see Table 4).

	max	min	mean	std
N (EGM96)	37.977	0.669	18.263	±8.443
N (GPM98b)	37.873	0.671	18.477	±8.383

Table 4. The final geoid solution from GEOSAT-GM altimetry data. Unit: [m]

2.2 VALIDATION OF ERS1-GM DATA AND GEOID SOLUTION

The preprocessing step for the ERS1-GM SSHs included, as with the case of GEOSAT-GM, the correction of the data due to the geophysical and instrumental errors according to the models and methods described in AVISO handbook (AVISO, 1998). The total number of corrected ERS1-GM SSHs available for the Mediterranean Sea was 105105 point values. Then the contribution of the EGM96 and GPM98b geopotential models was removed from the corrected SSHs. The statistics of the derived residual geoid heights are shown in Table 5.

	max	min	mean	std
N_{res} (EGM96)	1.572	-2.062	-0.223	± 0.347
N_{res} (GPM98b)	2.832	-1.816	-0.216	± 0.467

Table 5. ERS1-GM residual undulations. Unit: [m]

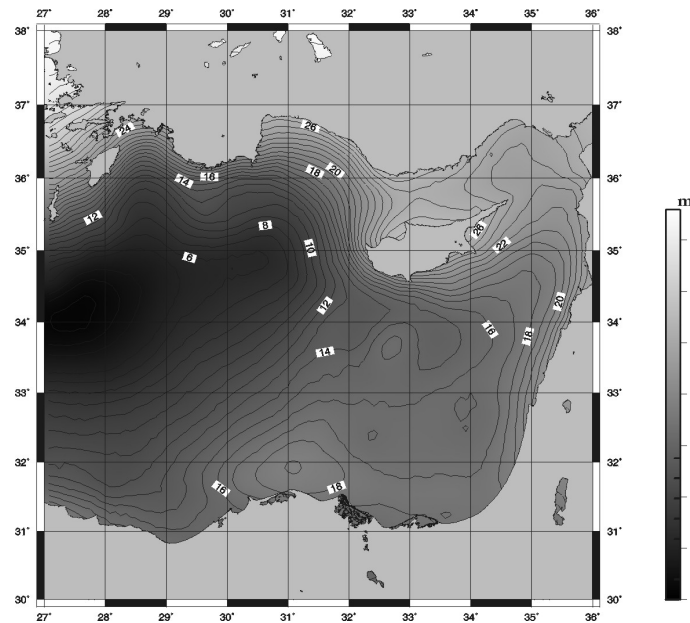


Figure 2. The final geoid solution from GEOSAT-GM to GPM98 reference filed.

In a further step, and in order to detect the presence of blunders, a 3rms test was applied to the residual geoid heights during which 866 (0.0087%) for EGM96 and 35 (0.00033%) point residual geoid heights were removed. The statistical results of this numerical test for the area under study are summarized in Table 6. As in the case of GEOSAT-GM N_{res} , a crossover adjustment was applied with the same results as the ones discussed before.

	max	min	mean	std
N_{res} (EGM96)	1.098	-1.181	-0.264	± 0.315
N_{res} (GPM98b)	1.014	-1.436	-0.653	± 0.253

Table 6. ERS1-GM residual undulations in the area under study, after the 3rms test. Unit: [m]

The data after the 3rms test were then used for the prediction on a $5' \times 5'$ grid. The contribution of the geopotential models was restored in these gridded residual geoid heights. The statistics of the final ERS1-GM geoid height solutions are given in Table 7. The final altimetric geoid solutions are depicted in Figure 3.

	max	min	mean	std
N (EGM96)	37.717	0.669	18.263	± 8.443
N (GPM98b)	37.958	0.712	18.594	± 8.417

Table 7. The final geoid solution from ERS1-GM altimetry data. Unit: [m]

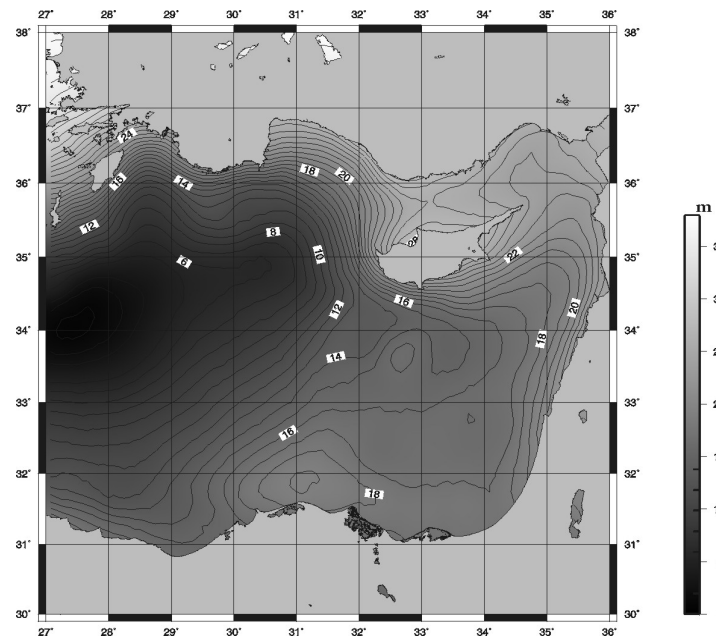


Figure 3. The final geoid solution from ERS1-GM to GPM98 reference field.

3. GRAVIMETRIC GEOID

The gravity data used were sea and land free-air gravity anomalies in the area under study. 18618 marine free-air gravity anomalies and 724 land free-air gravity anomalies, for the island of Cyprus, were used (BGI, 2000). The statistics of this dataset are reported in Table 8 and the data distribution is depicted in Fig. 4.

	max	min	mean	std
Δg	140.000	-239.000	-63.595	± 58.105

Table 8. Statistics of gravity data. Unit: [mGal]

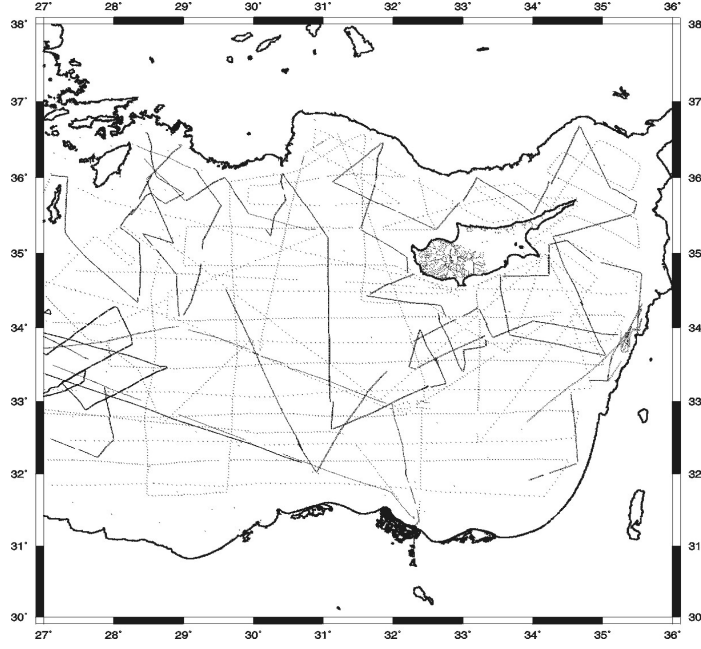


Figure 4. Distribution of gravity data in the area under study.

All these gravity data values were first reduced to EGM96 and GPM98b reference surfaces (Table 9). Then these residual gravity anomalies were gridded on a $5' \times 5'$ grid and the computation of the residual geoid heights was carried out by applying the 1-D FFT spherical Stokes convolution (Haagmans et al., 1993; Sideris and She, 1995):

$$N^{gr} = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} \mathcal{F}_1^{-1} \left\{ \sum_{\phi=\phi_1}^{\phi_a} \mathcal{F}_1\{S\} \mathcal{F}_1\{\Delta g \cos \phi\} \right\} \quad (1)$$

Where N^{gr} is the gravimetric geoid height, R is the mean earth radius, γ is the normal gravity, Δg denotes gravity anomaly, and $S(\psi)$ is Stokes' function. The statistics of these residual geoid heights are reported in Table 10.

	max	min	mean	std
N_{res} (EGM96)	160.552	-113.760	-8.887	± 18.430
N_{res} (GPM98b)	158.032	-93.616	-8.240	± 11.033

Table 9. Statistics of reduced gravity anomalies to EGM96 and GPM98b reference fields. Unit: [mGal]

	max	min	mean	std
N_{res} (EGM96)	-0.865	-4.001	-2.469	± 0.521
N_{res} (GPM98b)	-0.848	-3.127	-2.210	± 0.445

Table 10. Statistics of residual gravimetric geoid undulations. Unit: [m]

The final gravimetric geoid solutions were then calculated by restoring the contribution of the reference models EGM96 and GPM98b to geoid height residuals. The statistics of these solutions are given in Table 11. Finally, we depict the gravimetric geoid height solutions for both reference fields in Figures 5 and 6.

	max	min	mean	std
N (EGM96)	30.196	-0.037	13.484	± 6.256
N (GPM98b)	29.654	-0.139	13.220	± 6.183

Table 11. The final gravimetric geoid solution to EGM96 and GPM98b reference fields. Unit: [m]

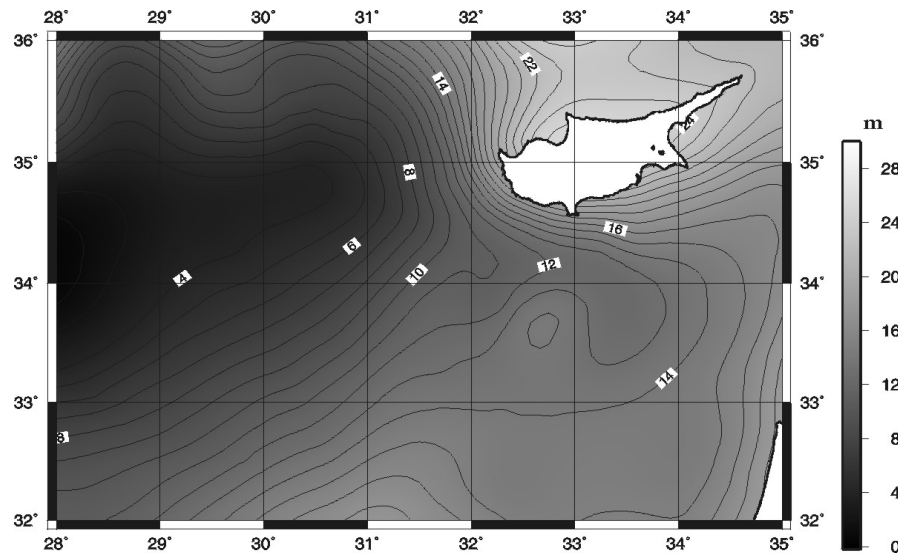


Figure 5. The final gravimetric geoid solution to EGM96 reference field.

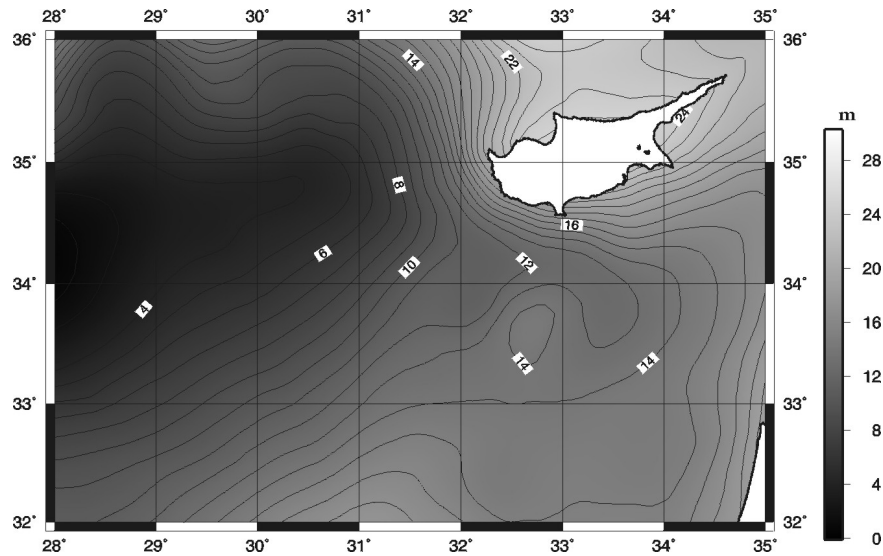


Figure 6. The final gravimetric geoid solution to GPM98 reference field.

4. Combined method and geoid solution

Having determined the altimetric and gravimetric geoid height solutions, combined ones were computed using the Multiple Input Multiple Output System Theory (MIMOST) as presented in Andritsanos et al. (2000a), Andritsanos et al. (2000a). The data used for the combination method are the residual altimetric geoid heights from ERS1-GM and GEOSAT-GM reference to EGM96 and GPM98b fields and the corresponding data from the gravimetric geoid solutions.

Due to the lack of specific information about the errors in both the altimetric and the gravimetric solution, simulated noises were used as input error. Randomly distributed fields (white noise) were generated in MATLAB[®] using 5cm standard deviation for the altimetry derived geoid heights and 10cm standard deviation for the gravimetric one. The final solution from the combination method, as well as the output error PSD function, were calculated according to the following equations:

$$\hat{N}_o = \begin{bmatrix} H_{\hat{N}_g} & H_{\hat{N}_a} \end{bmatrix} \left(\begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} - \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \right)^{-1} \begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix}^{-1} \begin{bmatrix} N_{og} \\ N_{oa} \end{bmatrix} \quad (2)$$

$$P_{\hat{e}\hat{e}} = \left\{ \begin{bmatrix} H_{\hat{N}_g} & H_{\hat{N}_a} \end{bmatrix} \left(\begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} - \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \right) - \begin{bmatrix} \hat{H}_{\hat{N}_o N_{og}} & \hat{H}_{\hat{N}_o N_{oa}} \end{bmatrix} \right. \\ \left. \begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} \right\} \left(\begin{bmatrix} H^*_{\hat{N}_g} \\ H^*_{\hat{N}_a} \end{bmatrix} - \begin{bmatrix} \hat{H}^*_{\hat{N}_o N_{og}} \\ \hat{H}^*_{\hat{N}_o N_{oa}} \end{bmatrix} \right) + \begin{bmatrix} \hat{H}_{\hat{N}_o N_{og}} & \hat{H}_{\hat{N}_o N_{oa}} \end{bmatrix} \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \begin{bmatrix} H^*_{\hat{N}_g} \\ H^*_{\hat{N}_a} \end{bmatrix} \quad (3)$$

where $\hat{N}_o$ is the spectrum combined geoid solution, N_g and N_a are the spectra of the pure gravimetric and altimetric signals respectively, N_{og} and N_{oa} are the spectra of gravimetric and altimetric observations, m_g and m_a are the input noises, H_{xy} is the theoretical operator that connects the pure input y and output signals x , $\hat{H}_{x_o y_o}$ is the optimum frequency impulse response function, $\hat{e}$ is the output noise and P_{xy} are PSD functions (Andritsanos et al, 2000a). The statistics of the final geoid height solution from the optimal combination of the heterogeneous heights referenced to the EGM96 and GPM98b geopotential models can be seen in Table 12 and are depicted in Figure 7.

	max	min	mean	std
N (EGM96)	32.210	0.613	15.036	±6.461
N (GPM98b)	32.042	1.103	15.301	±6.210

Table 12. The combined geoid solution to EGM96 and GPM98b reference fields. Unit: [m]

5. COMPARISON OF THE COMPUTED GEOID HEIGHT SOLUTIONS WITH T/P SSHS AND GEOMED

The previously described geoid height solutions were compared with stacked T/P SSHs from the 3rd year of the satellite mission, which are considered as geoid heights when neglecting the SST signal. T/P

SSHs are known for their high accuracy and they can be safely used as control points for our solutions. Additionally, we present the comparison between the geoid solution from the combination method and the GEOMED geoid for the Mediterranean Sea. From Table 13 and Figure 8, we can see that the differences with GEOMED are at the $\pm 56\text{cm}$ and $\pm 63\text{cm}$ level when using EGM96 and GPM98b as reference fields for our combined solution. This can be mainly attributed to the different reference fields used (GEOMED uses OSU91A as a reference field) and the more accurate altimetry data used in the present study.

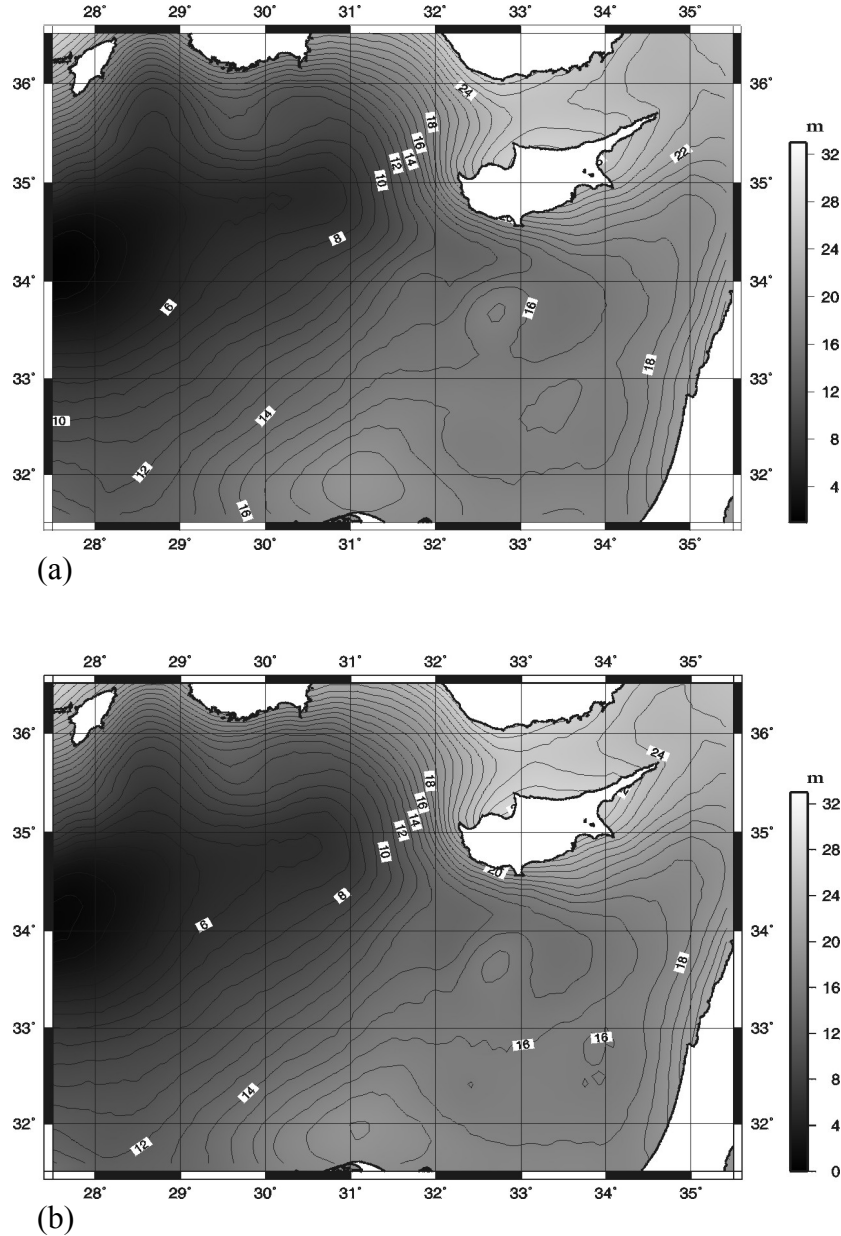


Figure 7: The final geoid height solutions from the MIMOST method to EGM96 (a) and GPM98b (b) reference surfaces.

N_{dif}	max	min	mean	std
(EGM96)	2.621	-0.778	0.984	± 0.559
(GPM98b)	3.370	-0.694	0.625	± 0.635

Table 13. Statistics of the differences between GEOMED and the combined solution. Unit: [m]

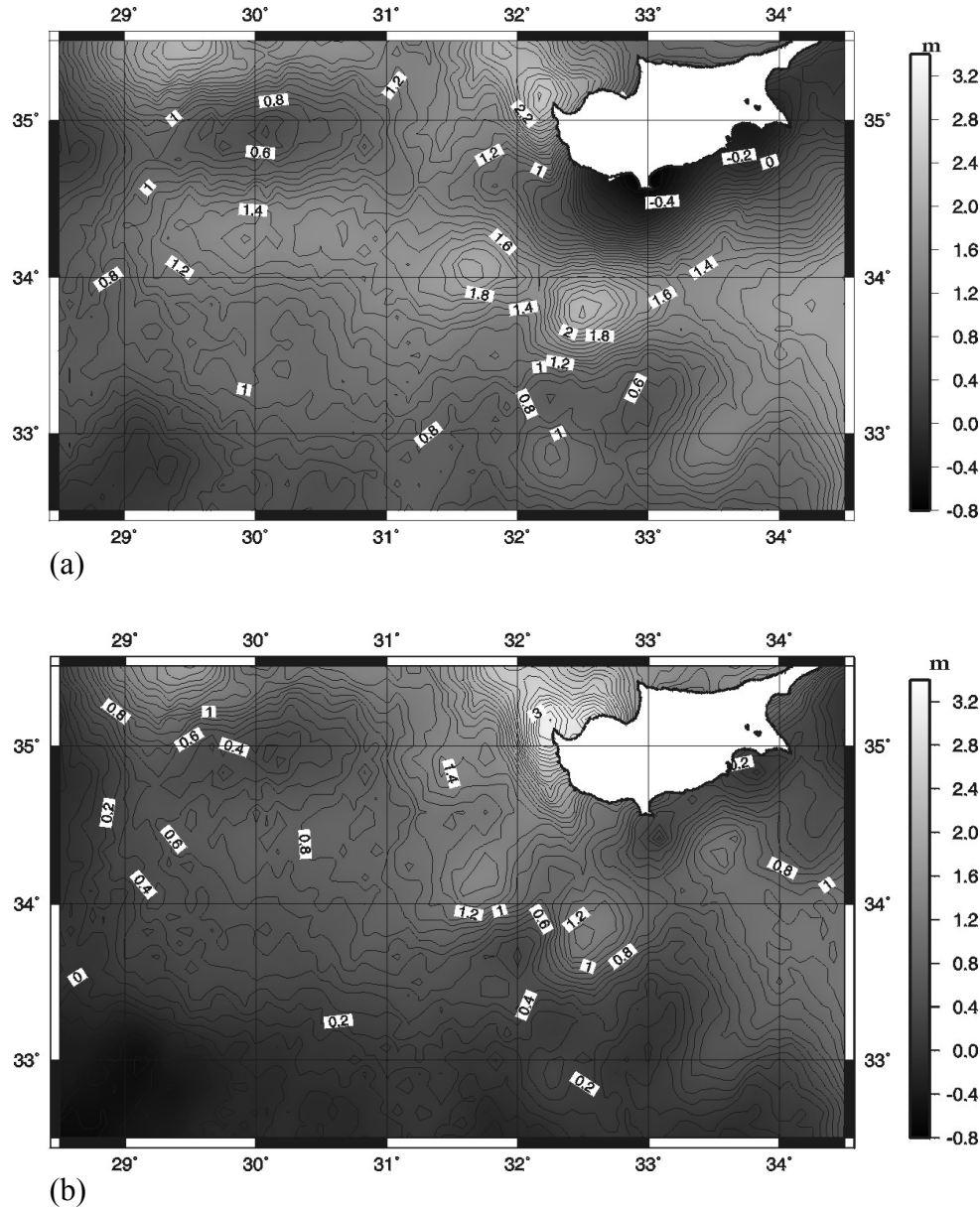


Figure 8. Geoid height differences between GEOMED and the final combined solutions from the MIMOST method for EGM96 (a) and GPM98b (b) reference surfaces.

For the comparisons between the aforementioned solutions and the stacked SSHs from the 3rd year of the T/P mission the computed differences were minimized by using a four-parameter transformation model:

$$SSH_{T/P} = N - b_0 \cos\phi \cos\lambda - b_1 \cos\phi \sin\lambda - b_2 \sin\phi - b_3 \quad (4)$$

where the parameters b_0 , b_1 , b_2 and b_3 were calculated by the least squares technique and N is the altimetric, gravimetric or combined geoid height depending on the solution under consideration. Comparing the standard deviation of the differences between the different geoid height solutions and the stacked T/P heights, it is easily concluded that the altimetric solutions are superior to the gravimetric ones. When comparing the pure altimetric geoid heights with T/P SSHs, the accuracy is at the level of ± 8 for both satellites (see figures 9). The differences for the gravimetric solutions on the other hand reach the level of ± 20 cm in terms of standard deviation (see figure 10). The comparison between the optimally combined geoid heights and the stacked T/P SSHs reach an accuracy close to ± 7 cm in terms of standard deviation of the differences for GPM98b but ± 29 cm for EGM96 (see figure 11).

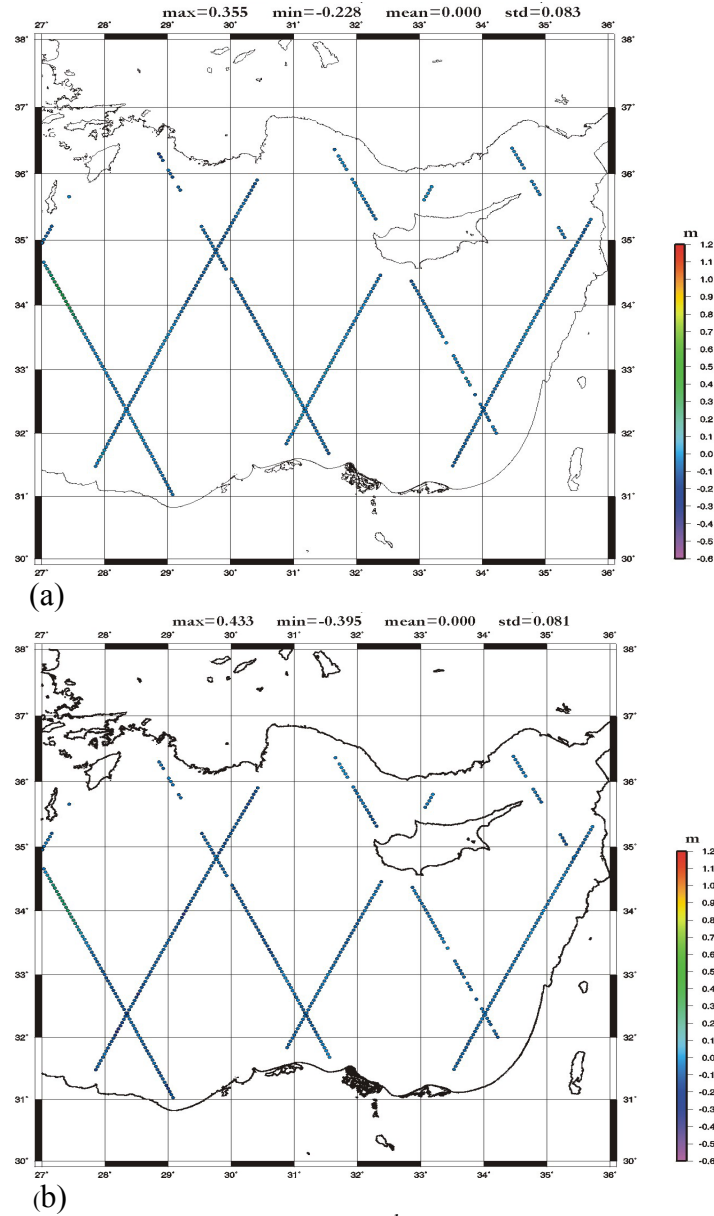


Figure 9. Differences between stacked T/P SSHs (3rd year) and ERS1-GM (a) and GEOSAT-GM (b) geoid height solutions (GPM98b reference field). Unit: [m]

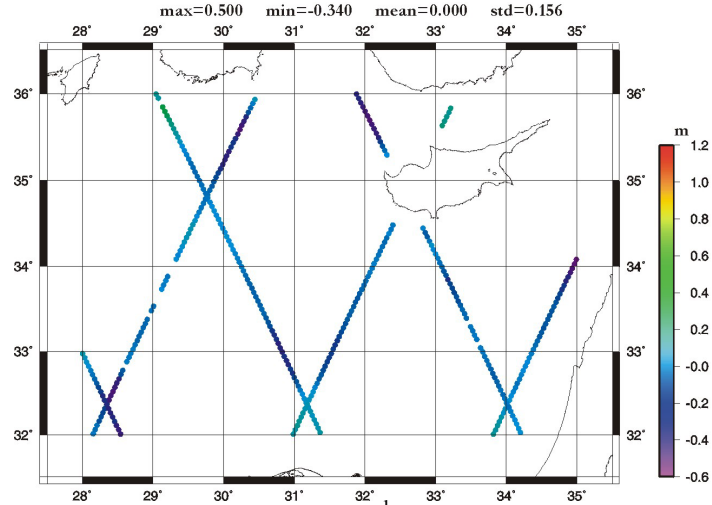


Figure 10. Differences between stacked T/P SSHs (3rd year) and the gravimetric geoid height solution to GPM98b reference field. Unit: [m]

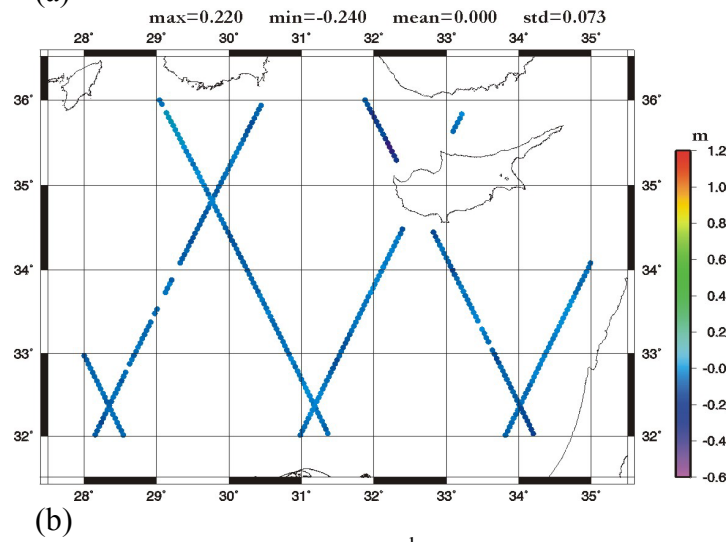
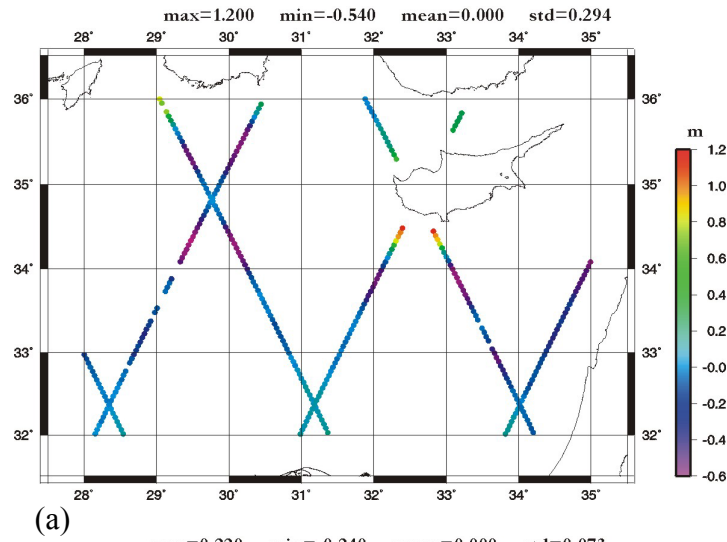


Figure 11: Differences between stacked T/P SSHs (3rd year) and the final geoid height solutions from the MIMOST method to EGM96 (a) and GPM98b (b) reference models. Unit: [m]

6. CONCLUSIONS – FUTURE PLANS

Eight local geoid solutions were computed for the Eastern part of the Mediterranean Sea close to the island of Cyprus. Satellite altimetry, marine and land gravity data were used in a combined solution for the optimal determination of the geoid in the area under study.

The well-known remove-restore method was used for the altimetric and gravimetric geoid determination when the gravimetric residual geoid heights are determined using the 1-D FFT method. Finally the Multiple Input Multiple Output System Theory was used to optimally combine the available data sets.

Based on these results the determination of a purely altimetric geoid is possible with an accuracy close to $\pm 5\text{cm}$ while the determination of a purely gravimetric one has a lower accuracy close to $\pm 18\text{cm}$. The differences between T/P SSHs and the gravimetric geoid heights can be mainly attributed to the unavailable SST signal and the poor accuracy of the available gravity data. Nevertheless the achieved accuracy of the gravimetric geoid is better compared to a study by Vergos (2000) for an area northwest of the present one, where the available gravity data came from the digitization of the Morelli's maps (Behrend et al. 1996) and had even lower accuracy.

As far as the combined solution is concerned, it is evident that the use of altimetry data, at least when using GPM98 as a reference field, improve by $\pm 9\text{cm}$ the accuracy of geoid determination as compared to the gravimetric only solution. This can be attributed not only to the use of altimetry data but to the noise level used for the different data sets (5cm for the altimetric and 10cm for the gravimetric ones) which give more weight to the altimetric geoid heights in the combination algorithm. Throughout this research, GPM98b proved to give better agreement, compared to EGM96, with our control datasets. This is an indication that GPM98b is more appropriate, at least for the Mediterranean Sea, to be used as the reference field when geoid modeling is concerned.

A detailed study of the different solutions, with more emphasis on the error spectrum, and an investigation of the frequency contents of the two geopotential models is needed and will be carried out. In order to obtain a higher resolution and higher accuracy geoid solution in the area under study, a significant improvement of both land and sea gravity databases is necessary. It is expected that the forthcoming gravity field dedicated satellite missions will contribute to the improvement of such databases.

Acknowledgements

The financial assistance provided to the first two of the authors by the GEOIDE Network of Centers of Excellence is gratefully acknowledged. We extensively used the Generic Mapping Tools Version 3.1 (Wessel and Smith, 1998) in displaying our results.

References

- Andritsanos, V.D., M.G. Sideris and I.N. Tziavos. *A survey of gravity field modelling applications of the Input-Output System Theory (IOST)*. To appear in IGeS Bulletin, 2000a.
- Andritsanos, V.D., M.G. Sideris and I.N. Tziavos. *Quasi-stationery Sea Surface topography estimation by the multiple Input-Output method*. Accepted for publication in Journal of Geodesy, 2000b.
- AVISO User Handbook – Corrected Sea Surface Heights (CORSSHs). AVI-NT-011-311-CN, Edition 3.1, 1998.
- Arabelos, D. and I.N. Tziavos. *Combination of ERS1 and TOPEX altimetry for precise geoid and gravity recovery*. Geophys. J. Int., 125, pp. 285-312, 1996.

- Behrend, D., H. Denker, and K. Schmidt. *Digital gravity data sets for the mediterranean sea derived from available maps*. Bulletin d' information, No 78, pp. 31-39, BGI, 1996.
- Bendat, J.S. and Piersol A.G. *Random data: Analysis and Measurement Procedures*. Second edition. John Wiley and Sons, New York, 1986.
- Bureau Gravimétrique International. *Shipborne gravity data for the Eastern Mediterranean Sea*. Personal communication, 2000.
- The Geosat Altimeter JGM-3 GDRs Handbook. Internet Resources: <http://www.ibis.gdrl.noss.gov/SAT/gdrs/geosat-handbook>, 1997.
- GEOMED group. *Mare Nostrum – GEOMED*. Report No. 4, Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki, Greece, 1993. Eds.: D. Arabelos and I.N. Tziavos
- Haagmans, R., E. de Min and M. van Gelderen. *Fast evaluation of convolution integrals on the sphere using 1D FFT, and a comparison with existing methods for Stokes' integral*. Manuscr. Geod. 18, pp. 227-241, 1993.
- Lemoine, F.G., et al. *The development of the joint NASA GSFC and NIMA geopotential model EGM96*, NASA Technical Paper, 1998 – 206861, 1998.
- National Oceanographic and Atmospheric Administration - NOAA. *The GEOSAT-GM Altimeter JGM-3 GDRs*. 1997.
- Pichon Le, X.J. Angelier, N. Chamot-Rooke, S. Lallemand, R. Noomen, G. Veis. *Geodetic determination of the kinematics of Central Greece with respect to Europe: Implications for Eastern Mediterranean Tectonics*. Journal of Geophysical Research, Vol. 100, 12265-12690, 1995.
- Sandwell D. *Data Base Description for Digital Bathymetric Data Base-Variable Resolution (DBDB-V)*. Version 1.0, Naval Oceanographic Office, Stennis Space Center, March, 1996
- Sideris, M.G. *On the use of heterogeneous noisy data in spectral gravity field modeling methods*. Journal of Geodesy, 70, pp. 470-479, 1996.
- Sideris, M.G. and B.B. She. *A new high-resolution geoid for Canada and part of the U.S. by 1D-FFT method*. Bull. Geod. 69:92-108, 1995.
- Smith D.E., R. Kolenkiewicz, J.W. Robbins, P.J. Dunn, M.H. Torrence. Horizontal crustal motion in the Central and Eastern Mediterranean inferred from Satellite Laser Ranging measurements. *Geophysical Research Letter*, 21, 1979 – 1982, 1994b.
- Smith, W.H.F. and D.T. Sandwell. *Bathymetric prediction from dense satellite altimetry and sparse shipboard bathymetry*. Journal of Geophysical Research, 99, No. B11, 21803-21824, 1994.
- Tziavos, I.N., M.G. Sideris and R. Forsberg. *Combined satellite altimetry and shipborne gravimetry data processing*. Marine Geodesy, 21, pp.299-317, 1998.
- Vergos, G.S., *The contribution of satellite altimetry to gravity field modeling – A case of study in the Eastern Mediterranean*. Diploma thesis, School of Rural and Surveying Engineering, Faculty of Engineering, Aristotle University of Thessaloniki, Department of Geodesy and Surveying, 2000.
- Wenzel, H.G. *Global models of the gravity field of high and ultra-high resolution*. In Lecture Notes of IAG's Geoid School, Milano, Italy, 1999.
- Wessel, P., and W.H.F. Smith. *New improved version of Generic Mapping Tools released*. EOS Trans. Amer. Geophys. U., vol. 79 (47), pp. 579, 1998.

Combination of Multi-Satellite Altimetry and Shipborne Gravity Data for Geoid Determination in a Coastal Region of Eastern Canada

G.S. Vergos, R.S. Grebenitcharsky and M.G. Sideris

Department of Geomatics engineering, University of Calgary, 2500 University Drive N.W., Calgary, Alberta, Canada, T2N 1N4

ABSTRACT

The combination of satellite altimetry and shipborne gravity data for optimal geoid determination in the region of Newfoundland, Eastern Canada, is investigated and compared with existing geoid solutions in the area under study. In this study, a new high-resolution multi-satellite (GEOSAT-GM, ERS1-GM, T/P) altimetric geoid is computed and validated, and then combined with shipborne gravity data to obtain the final model. The newly released 1998 GEOSAT-GM data referring to JGM-3 will be used, thus higher accuracy is expected. T/P data are used as control points for the resulting geoid, which is then compared to existing geoid solutions (GSD solutions) in the test area. We will also investigate the use of a global Sea Surface Topography (SST) model to test how well it models the stationery part of the SST. We will focus on the use of the Multiple Input Multiple Output System Theory (MIMOST) method for the efficient and optimal combination of heterogeneous data. Finally two geopotential models, namely EGM96 and GPM98b, are implemented to test how well they model the long wavelength part of the geoid.

KEYWORDS

Geoid determination, marine geoid, satellite altimetry, sea surface heights, shipborne gravimetry, sea surface topography, combination, MIMOST.

1. INTRODUCTION

The eastern coast of Canada is an area of extensive research activities relevant to geoid determination and gravity field modelling. The Geodetic Survey Division (GSD) of Natural Resources Canada (NRCan) in collaboration with universities like the ones of Calgary and New Brunswick generate new geoid solutions almost every five years, taking advantage of the new available and more accurate data sets. The specific area under study is very advantageous since it offers a great amount of shipborne gravity data, with very good resolution and coverage close to the coastline, collected during the past decades. This enables us to overcome difficulties and uncertainties of geoid determination close to the coastline. Additionally, since it is a part of the Atlantic Ocean, global models for various gravity field and oceanography related quantities, such as ocean circulation and SST models, can be safely used. As it is well known such models are not generated for closed sea areas, just like the Mediterranean Sea, hence they are not usually used in these cases. With the advent of satellite altimetry a vast amount of high-accuracy and high-resolution data were available for use in gravity field determination. Our study is focused in the Eastern coastal region of Canada ($43^{\circ} \leq \phi \leq 57^{\circ}$ and $298^{\circ} \leq \lambda \leq 312^{\circ}$) including the island of Newfoundland in the Labrador Sea and the St. Lawrence gulf.

In this study we focus, using the well-know remove-restore method, on the combined use of multi-satellite altimetry and shipborne gravity data for a high-accuracy and high-resolution ($3' \times 3'$) marine geoid determination in the area under study. We determine, using the available datasets, purely altimetric and gravimetric geoid solutions and then using the Multiple Input Multiple Output System Theory (MIMOST) (Bendat and Piersol, 1986; Sideris, 1996; Andritsanos et al., 2000a) for the efficient combination of heterogeneous data, we compute the final geoid solutions. We also want to investigate

whether the combined use of satellite altimetry and shipborne gravity data will improve geoid modelling compared to the gravimetric geoid only.

Altimetric Sea Surface Heights (SSHs) from the ERS1 and the newly released (NOAA 1997) GEOSAT Geodetic Missions (GMs) are used for the computation of the altimetric geoid solution. Stacked 3rd year TOPEX/POSEIDON (T/P) SSHs are used as control points for the validation of our solutions. The 3rd year of the T/P mission was selected so that a common observation period between two satellites (T/P and ERS1-GM) would be available. The marine free air gravity anomalies used in this study were kindly provided to us by Mr. M. Véronneau and were generated by the Geological Survey of Canada (Véronneau 2001). The gravimetric geoid was computed by the 1D-FFT Stokes convolution (Haagmans et al., 1993; Sideris and She, 1995) using the available marine free-air gravity anomalies. The above three solutions were referenced to two geopotential models, namely EGM96 (Lemoine et al., 1996), complete to degree and order 360, and GPM98 (Wenzel, 1999), complete to degree and order 1800. For the determination of the Sea Surface Topography (SST) signal we used the model from the simultaneous EGM96 adjustment (Lemoine et al. 1998). As control for our gravimetric geoid solutions the KMS99 global marine gravity field (Andersen and Knudsen 1998) was used. Comparisons with the official Canadian geoid GSD95 (Véronneau 1997) were performed in order to assess the differences with our solutions due to the different reference fields (in the computation of GSD95, OSU91A geopotential model was used as a reference field) and the more accurate data used.

2. ALTIMETRIC GEOID

Almost 73% of our planet's surface is covered by water and contrary to the immense size of water areas our knowledge of the processes taking place in the earth's oceans was little until the late 1970's. With the advent of the space era and the launch of the Seasat altimetric satellite in late 1970's the potentials of satellite altimetry to monitor the oceans became evident. It was the first time that a satellite was put in orbit with its primary goal being the monitoring of the oceans. The satellite managed to collect only 42 hours of data which were though more than that collected during the last 100 years of earth-based observations. Satellite altimetry is an increasing in accuracy method and provides a dense, long-term, repeated in period and accurate monitoring of the earth's oceans. It provides an economical tool, compared to shipborne gravimetry, to model the oceans and determine the gravity field over great water areas. The geodetic missions of GEOSAT and ERS1 altimetric satellites provide a vast amount of data with resolutions of 2-4km in the equator (Smith et al., 1994), which cover the Earth densely and offer precise data sets due to the accurate models used for the geophysical and instrumental corrections and the approximation of the satellite orbit. Because of the improved models used in the processing of altimetry data a marine geoid determination with an accuracy close to a few centimeters is feasible. The altimetric data sets used in this study were GEOSAT-GM newly released SSHs referred to JGM-3 (Joint Gravity Model 3) orbits and were provided by NOAA (1997). The observation period of this data set was from March 30th, 1985 to September 30th, 1986. Additional altimetric data were those of the mapping phase of ERS1-GM SSHs from AVISO (1998). The observation period of these altimetric data extends from April 10th, 1994 to March 21st, 1995. Finally, the 3rd year of T/P SSHs distributed by AVISO (1998) were used. The period of observation of this data set was from October 16th, 1994 to October 8th, 1995.

2.1 VALIDATION OF GEOSAT-GM DATA AND GEOID SOLUTION

An initial preprocessing step in the validation of the GEOSAT-GM SSHs was their correction due to the instrumental and geophysical errors, included in the SSH observations, in order to get corrected SSHs.

The GEOSAT-GM data were provided to us in the usual Geophysical Data Record (GDR) format and the models and methods used for the aforementioned corrections are the same as those described in the GEOSAT-GM handbook (1997). The total number of corrected SSHs available in the area under study were 95348 point values. As a next step we corrected the GEOSAT-GM SSHs for the SST signal. The SST model used in this study comes from the simultaneous EGM96 geopotential model adjustment (Lemoine et al. 1998) and was kindly provided to us by Dr. N. Pavlis. The model comes as a set of spherical harmonic coefficients of the SST complete to degree and order 20. Using the model we computed SST values on the points where GEOSAT-GM SSHs were available and we removed them from the SSHs signal. These SSHs are considered as geoid heights (N). The statistics of the original and corrected for the SST signal GEOSAT-GM SSHs are presented in Table 1.

max	min	mean	std
330.383	-311.601	6.174	± 19.206
331.039	-310.989	6.816	± 19.310

Table 1. Original and corrected for the SST signal GEOSAT-GM data. Unit: [m].

The available GEOSAT-GM data records cover not only sea but some continental areas as well, thus the first test had to deal with the removal of the continental data. It is well-known that satellite altimetry SSHs give some erroneous values close to the coastline due to the scattering of the radar altimeter pulse from the shallow sea bottom; for that reason, data close to the coastline were removed as well. A 1'x1' bathymetric model for the area under study was used (Sandwell, 1996) and an interpolation of depth values was carried out at the points where SSHs from GEOSAT-GM were available. Following this interpolation process data points with depths greater than -50m were neglected as suspected to contain blunders. According to this test, 6619 point values (6.94%) were finally removed (see Table 2).

max	min	mean	std
331.039	-310.989	8.849	± 17.959

Table 2. GEOSAT-GM data after the bathymetry test. Unit: [m].

From Tables 1 and 2 for the SSHs before and after the bathymetry test some very large minimum and maximum values are still present, thus indicating the presence of outliers. On the other hand we can see an improvement of the standard deviation value at the level of 1.5m. After the bathymetry test the long wavelength part of the geoid heights was removed by subtracting the contribution of EGM96 (Lemoine et al., 1998) and GPM98b (Wenzel, 1998) geopotential models from the SSHs. The statistics of the so derived residual geoid heights (N_{res}) are summarized in Table 3.

	max	min	mean	std
N_{res} (EGM96)	338.818	-316.654	-1.892	± 10.252
N_{res} (GPM98b)	338.892	-316.768	-2.050	± 10.249

Table 3. GEOSAT-GM residual undulations. Unit: [m]

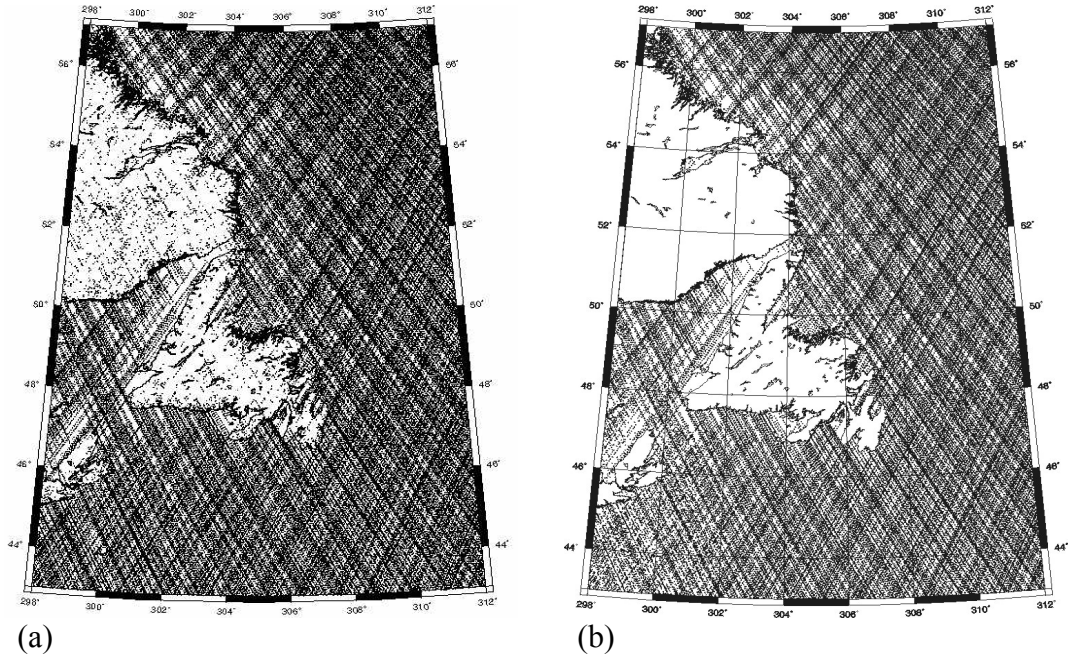


Figure 1. GEOSAT-GM data distribution before (a) and after (b) the bathymetry test.

The residual geoid heights in Table 3 indicate a number of unexpected values. This is due to the presence of blunders and/or systematic errors. To remove these large values, an additional 3rms test for blunder detection was performed for the GEOSAT-GM residual geoid heights. After the 3rms test 7225 (8.14%) in the case of EGM96 and 10293 (11.6%) in the case of GPM98b residual geoid heights were removed. To summarize, during the two validation tests (bathymetry and 3rms), 13844 GEOSAT-GM data points (14.52%) reduced to EGM96 geopotential model were removed and 16921 data points (17.74%) reduced to GPM98b were eliminated.

The remaining data in the area under study ($43^{\circ} \leq \varphi \leq 57^{\circ}$ and $298^{\circ} \leq \lambda \leq 312^{\circ}$) are 81504 and 78427 residual geoid heights referenced to EGM96 and GPM98b respectively. The statistics of these residual geoid heights in the test area are given in Table 4.

	max	min	mean	std
N_{res} (EGM96)	0.720	-0.721	0.141	± 0.195
N_{res} (GPM98b)	0.581	-0.581	0.010	± 0.194

Table 4. GEOSAT-GM residual undulations after the 3rms test. Unit: [m]

To verify the effectiveness of the bathymetry mask and that of the 3rms validation we compare the statistics of the GEOSAT-GM data before and after the tests (Tables 1 and 4 respectively). From these two tables it is evident that the extreme maximum and minimum values are removed and the residual geoid heights, in terms of the standard deviation values, are at the $\pm 19.5\text{cm}$ level. To these residual geoid heights a crossover adjustment was applied for the minimization of the orbital errors. Comparing the residual geoid heights before and after the adjustment insignificant improvement, for the case of EGM96, and deterioration, for the case of GPM98, in terms of the standard deviation and the mean value of the differences was detected. This fact can be mainly attributed to the improved and high accuracy orbits used for the present release of GEOSAT-GM GDRs (JGM3). For this reason, the

residual geoid heights after the 3rms test and before adjustment were used for the subsequent geoid determination.

The point residual geoid heights were interpolated onto a 3'×3' grid. Then, during the last step of the remove-restore method, the contribution of EGM96 and GPM98b geopotential models to the residual geoid heights was restored and the final geoid height solutions of GEOSAT-GM altimetry data were calculated (see Table 5).

	max	min	mean	std
N (EGM96)	32.480	-20.539	5.918	±13.482
N (GPM98b)	32.481	-20.549	5.925	±13.496

Table 5. The final geoid solution from GEOSAT-GM altimetry data. Unit: [m]

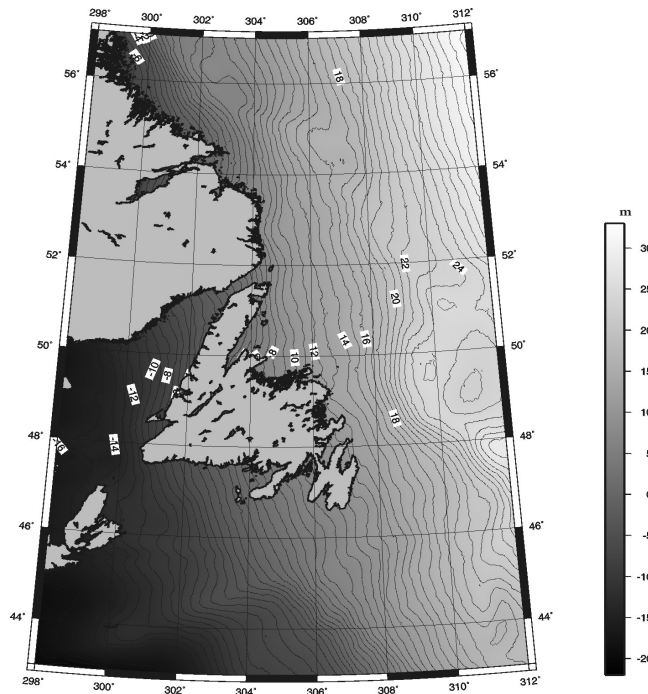


Figure 2. The final geoid solution from GEOSAT-GM to GPM98 reference filed.

2.2 VALIDATION OF ERS1-GM DATA AND GEOID SOLUTION

The ERS1 satellite altimetry SSHs were provided to us in the usual GDR format, thus a first preprocessing step, as in the case of GEOSAT data, for the correction due to the geophysical and instrumental errors was needed. This was done according to the models and methods described in the AVISO handbook (AVISO, 1998) for the ERS1-GM data. A total number of 56053 ERS1-GM corrected SSHs for the area under study were used. ERS1 data do not refer to land or close to the coastline regions hence the bathymetry mask validation test was not needed in this case. Following again the remove-restore method the contribution of the EGM96 and GPM98b geopotential models was removed from the corrected SSHs. The statistics of ERS1-GM residual SSHs are shown in Table 6.

	max	min	mean	std
SSH_{res} (EGM96)	0.852	-1.459	-0.421	±0.343
SSH_{res} (GPM98b)	1.203	-2.269	-0.555	±0.347

Table 6. ERS1-GM residual undulations. Unit: [m]

As with the case of GEOSAT-GM data we then removed the signal of the SST. To do so we interpolated, from the spherical harmonics SST model, SST values on the points where ERS1-GM SSHs were available. The statistics of these residual geoid heights are presented in Table 7. From the such derived residual geoid heights and in order to detect the presence of blunders a 3rms test was applied. During this test 500 (0.009%) for EGM96 and 1984 (0.035%) for GPM98b data points were removed. The statistics of the ERS1-GM data after the 3rms test are summarized in Table 8. A crossover adjustment, with the same results as the ones discussed for the case of GEOSAT-GM data, was applied for the ERS1-GM residual geoid heights.

	max	min	mean	std
N_{res} (EGM96)	1.569	-0.594	0.213	±0.200
N_{res} (GPM98b)	1.942	-1.527	0.079	±0.257

Table 7. ERS1-GM residual undulations after the correction for the SST signal. Unit: [m]

	max	min	mean	std
N_{res} (EGM96)	0.831	-0.594	0.205	±0.186
N_{res} (GPM98b)	0.633	-0.635	0.072	±0.198

Table 8. ERS1-GM residual undulations after the 3rms test. Unit: [m]

Thus, the data after the 3rms test and before the adjustment, were used for the interpolation on a 3'×3' grid. Finally the contribution of EGM96 and GPM98b geopotential models was restored in these gridded residual geoid heights. The statistics of the final geoid height solutions from ERS1-GM altimetric mission are given in Table 9 and the solution referenced to GPM98b is depicted in Figure 3.

	max	min	mean	std
N (EGM96)	32.440	-20.484	6.019	±13.446
N (GPM98b)	32.457	-20.483	5.992	±13.472

Table 9. The final geoid solution from ERS1-GM altimetry data. Unit: [m]

3. GRAVIMETRIC GEOID

The gravity data used were shipborne free-air gravity anomalies covering the area under study. 160481 marine free-air gravity anomalies for the Labrador Sea and the St. Lawrence gulf were used (Véronneau

2001). The statistics of the gravity data are presented in Table 10 and the data distribution is depicted in Fig. 4.

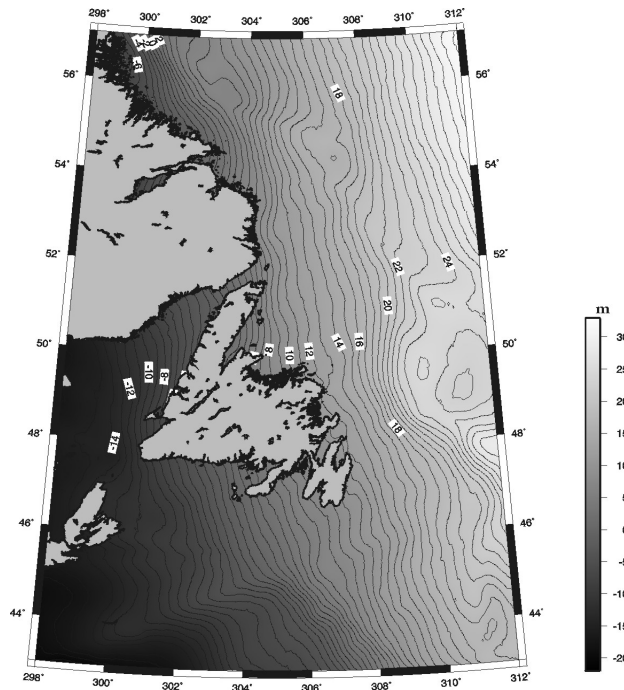


Figure 3. The final geoid solution from ERS1-GM to GPM98reference field.

	max	min	mean	std
Δg	133.900	-103.500	0.249	± 31.054

Table 10. Statistics of gravity data. Unit: [mGal]

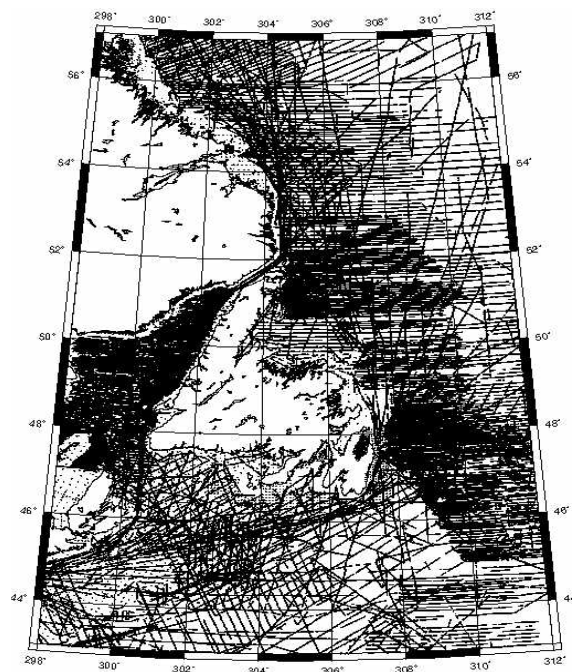


Figure 4. Distribution of gravimetric data.

From Fig. 4 it can be seen that the data distribution is irregular, showing very dense coverage of the St. Lawrence gulf and the eastern part of Newfoundland reaching a resolution of 15". On the other hand the data distribution in the southern and north-eastern part of the area under study is sparser reaching a resolution of 6'-7'. For that reason we decided to set the resolution of our final gravimetric geoid solution to 3'×3' so that it will compensate the sparser distributions and be in agreement with the altimetric solutions previously determined. The data come from shipborne campaigns held during the last 30 years, thus varying in accuracy. They are referenced to GRS67 and since the altimetry data refer to GRS80 a transformation of the available gravity data to GRS80 was needed. Following Li (1996) and Wenzel (1999) the formulas used for the transformation are:

$$\Delta g_{\text{GRS80}} = \Delta g_{\text{GRS67}} + \gamma_{\text{GRS67}} - \gamma_{\text{GRS80}} \quad (1)$$

$$\gamma_{\text{GRS67}} = 978031.8(1+0.0053024\sin^2\varphi-0.0000059\sin^2(2\varphi)) \quad (2)$$

$$\gamma_{\text{GRS80}} = 978032.7(1+0.0053024\sin^2\varphi-0.0000058\sin^2(2\varphi)) \quad (3)$$

The so referenced to GRS80 free-air gravity anomalies were first reduced to EGM96 and GPM98b reference surfaces (Table 11). Then a 3rms test for blunder detection was performed during which 1929 (0.012%) referenced to EGM96 and 2856 (0.019%) referenced to GPM98 residual gravity anomalies were removed (see Table 12). These residual gravity anomalies were gridded on a 3'×3' grid and the computation of the residual geoid heights was carried out by applying the 1-D FFT spherical Stokes convolution (Haagmans et al, 1993; Sideris and She, 1995).

$$N^{\text{gr}} = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma} \mathcal{F}_1^{-1} \left\{ \sum_{\varphi=\varphi_1}^{\varphi_a} \mathcal{F}_1\{S\} \mathcal{F}_1\{\Delta g \cos \varphi\} \right\} \quad (4)$$

Where N^{gr} is the gravimetric geoid height, R is the mean earth radius, γ is the normal gravity, Δg denotes gravity anomaly, and $S(\psi)$ is Stokes' function. The statistics of these residual geoid heights are reported in Table 13.

	max	min	mean	std
N_{res} (EGM96)	52.365	-74.653	-3.784	±10.564
N_{res} (GPM98b)	48.033	-124.605	-3.760	±6.183

Table 11. Statistics of reduced gravity anomalies to EGM96 and GPM98b reference fields. Unit: [mGal]

	max	min	mean	std
N_{res} (EGM96)	31.287	-31.301	-3.550	±9.812
N_{res} (GPM98b)	16.675	-17.089	-3.322	±4.628

Table 12. Statistics of reduced gravity anomalies to EGM96 and GPM98b reference fields after the 3rms test. Unit: [mGal]

	max	min	mean	std
N_{res} (EGM96)	1.315	-1.858	-0.473	± 0.293
N_{res} (GPM98b)	0.344	-0.560	-0.177	± 0.116

Table 13. Statistics of residual gravimetric geoid undulations. Unit: [m]

The final gravimetric geoid solutions were then determined following the last step of the remove-restore method by restoring the contribution of the reference models EGM96 and GPM98b to geoid height residuals. The statistics of these solutions are given in Table 14. Finally, we depict the gravimetric geoid height solutions for both reference fields in Figures 5 and 6.

	max	min	mean	std
N (EGM96)	28.730	-22.476	0.299	± 13.105
N (GPM98b)	31.893	-20.862	5.709	± 13.759

Table 14. The final gravimetric geoid solution to EGM96 and GPM98b reference fields. Unit: [m]

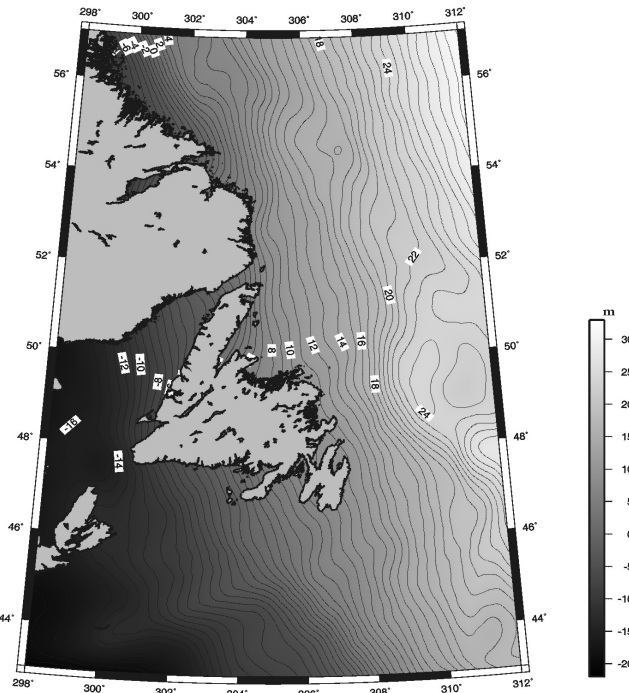


Figure 5. The final gravimetric geoid solution to EGM96 reference field.

4. COMBINED METHOD AND GEOID SOLUTION

Having determined the altimetric and gravimetric geoid height solutions, combined ones were computed using the Multiple Input Multiple Output System Theory (MIMOST) as presented in Andritsanos et al. (2000a), Andritsanos et al. (2000b). The data used for the combination method are the altimetric residual geoid heights from ERS1-GM and GEOSAT-GM reference to EGM96 and GPM98b fields and the corresponding gravimetric residual geoid heights.

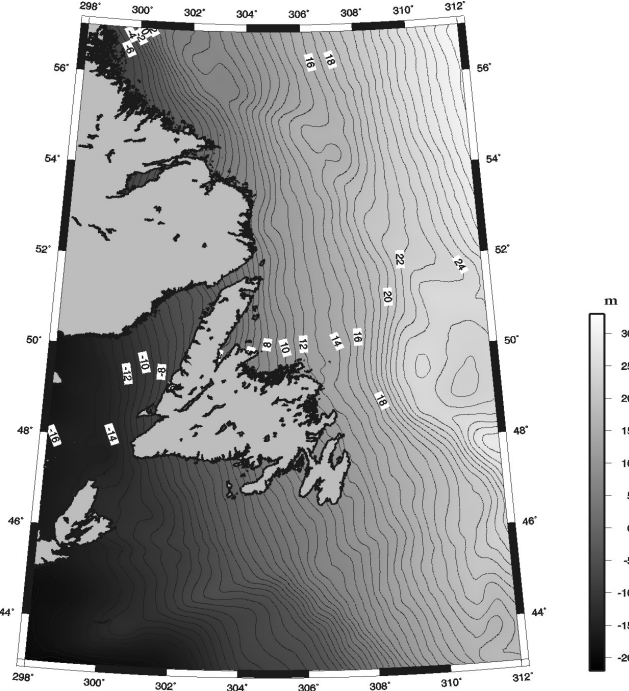


Figure 6. The final gravimetric geoid solution to GPM98 reference field.

Due to the lack of specific information about the errors in both the altimetric and the gravimetric solution, simulated noises were used as input error. Randomly distributed fields (white noise) were generated in MATLAB[®] using 5cm standard deviation for the altimetry derived geoid heights and 10cm standard deviation for the gravimetric one. The final solution from the combination method, as well as the output error PSD function, were calculated according to the following equations:

$$\hat{N}_o = \begin{bmatrix} H_{\hat{N}_{N_g}} & H_{\hat{N}_{N_a}} \end{bmatrix} \left(\begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} - \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \right)^{-1} \begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} \begin{bmatrix} N_{og} \\ N_{oa} \end{bmatrix} \quad (5)$$

$$P_{\hat{e}\hat{e}} = \left\{ \begin{bmatrix} H_{\hat{N}_{N_g}} & H_{\hat{N}_{N_a}} \end{bmatrix} \left(\begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} - \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \right) - \begin{bmatrix} \hat{H}_{\hat{N}_o N_{og}} & \hat{H}_{\hat{N}_o N_{oa}} \end{bmatrix} \right. \\ \left. \begin{bmatrix} P_{N_{og}N_{og}} & P_{N_{og}N_{oa}} \\ P_{N_{oa}N_{og}} & P_{N_{oa}N_{oa}} \end{bmatrix} \right\} \left(\begin{bmatrix} H^*_{\hat{N}_{N_g}} \\ H^*_{\hat{N}_{N_a}} \end{bmatrix} - \begin{bmatrix} \hat{H}^*_{\hat{N}_o N_{og}} \\ \hat{H}^*_{\hat{N}_o N_{oa}} \end{bmatrix} \right) + \begin{bmatrix} \hat{H}_{\hat{N}_o N_{og}} & \hat{H}_{\hat{N}_o N_{oa}} \end{bmatrix} \begin{bmatrix} P_{m_g m_g} & 0 \\ 0 & P_{m_a m_a} \end{bmatrix} \begin{bmatrix} H^*_{\hat{N}_{N_g}} \\ H^*_{\hat{N}_{N_a}} \end{bmatrix} \quad (6)$$

where $\hat{N}_o$ is the spectrum combined geoid solution, N_g and N_a are the spectra of the pure gravimetric and altimetric signals respectively, N_{og} and N_{oa} are the spectra of gravimetric and altimetric observations, m_g and m_a are the input noises, H_{xy} is the theoretical operator that connects the pure input y and output signals x , $\hat{H}_{x_o y_o}$ is the optimum frequency impulse response function, $\hat{e}$ is the output noise and P_{xy} are PSD functions (Andritsanos et al, 2000a). The statistics of the final geoid height solution from the optimal combination of the heterogeneous residual geoid heights referenced to the EGM96 and GPM98b geopotential models can be seen in Table 15 and are depicted in Figure 7.

	max	min	mean	std
N (EGM96)	24.612	-15.809	5.028	± 10.175
N (GPM98b)	25.302	-15.625	5.294	± 10.368

Table 15. The combined geoid solution to EGM96 and GPM98b reference fields. Unit: [m]

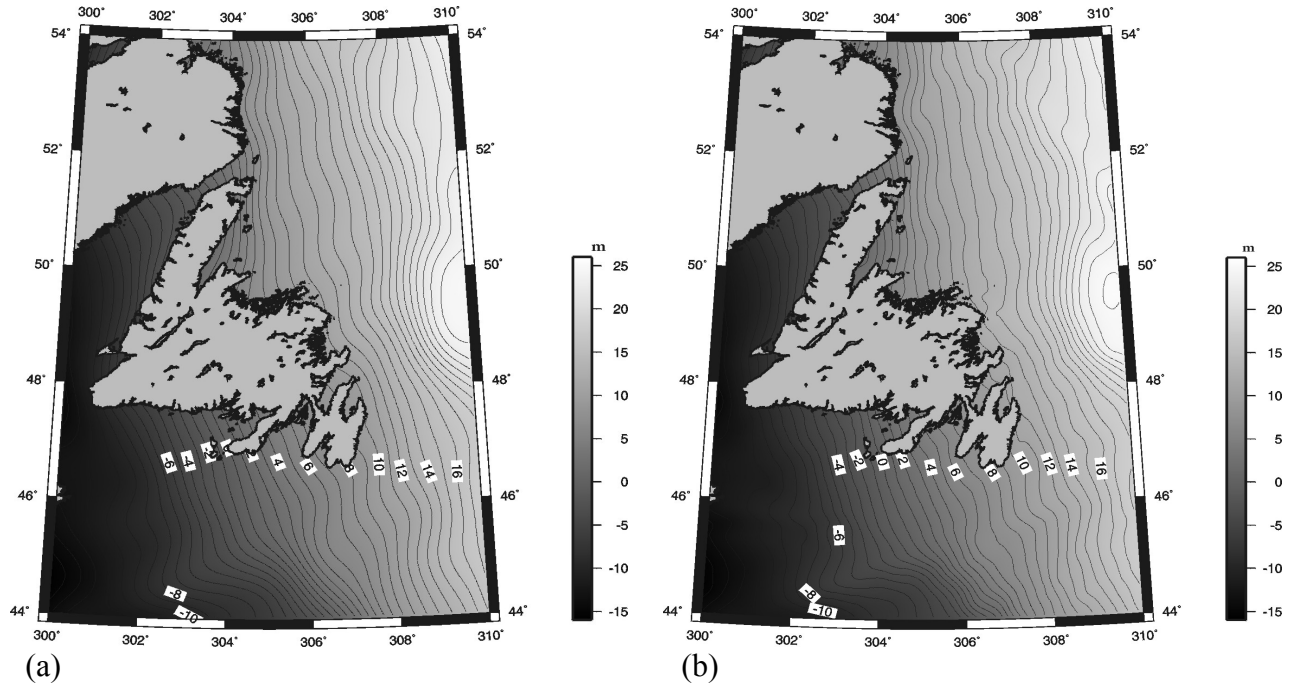


Figure 7. The final geoid height solutions from the MIMOST method for EGM96 (a) and GPM98b (b) reference surfaces.

5. COMPARISON OF THE COMPUTED GEOID HEIGHT SOLUTIONS WITH T/P SSHS, GSD95 AND KMS99

The previously determined geoid height solutions were compared with GSD95, the official geoid of Canada. Especially for the gravimetric solutions we use KMS99 altimetry derived global gravity field for the accuracy assessment. Then we compare our solutions with stacked T/P SSHs from the 3rd year of the satellite mission, which are considered as geoid heights when neglecting the SST signal. T/P SSHs are known for their high accuracy and they can be safely used as control points for our solutions. From Figure 8 we can see that the differences between GSD95 and ERS1-GM, referenced to EGM96, altimetry solution are at the $\pm 26\text{cm}$ level. These differences ($N_{\text{GSD95}} - N_{\text{ERS1-GM}}$) present a large minimum value (-2.187cm), which is mainly attributed to the enhanced data used in the ERS1-GM solution and to the different reference fields used, since GSD95 refers to OSU91A. From Figure 9 the differences between the gravimetric geoid solution referenced to EGM96 and KMS99 geoid are at the $\pm 29\text{cm}$ level indicating that our gravimetric geoid is close to the altimetry derived KMS99 one. Still the superiority of altimetry data is evident though.

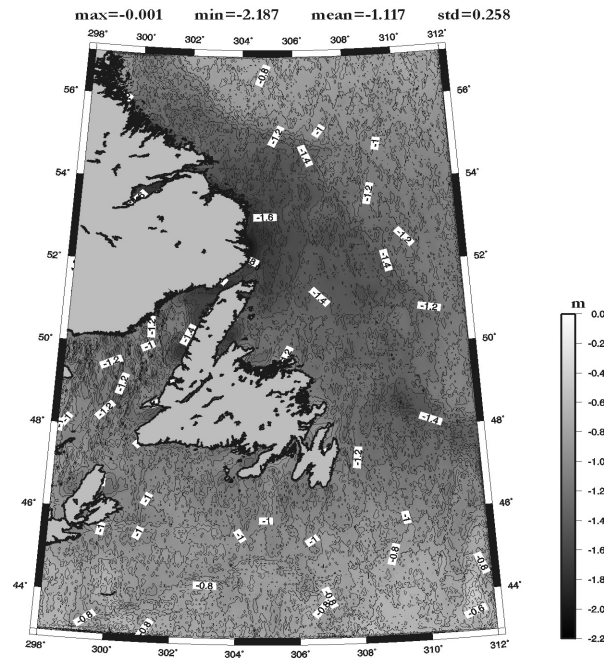


Figure 8. Geoid height differences between GSD95 and ERS1-GM geoid solution to EGM96 reference field.

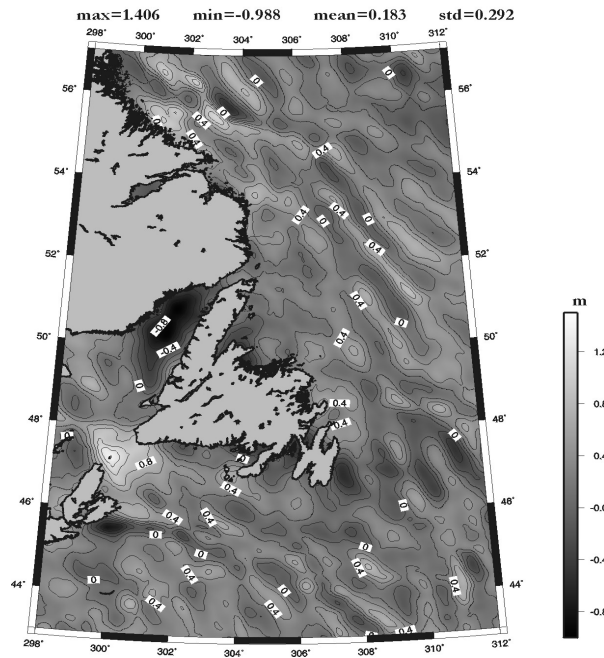


Figure 9. Geoid height differences between KMS99 and gravimetric solution referenced to EGM96 model.

The differences between GSD95 and the geoid solution from the combination method are presented, with their statistics, in Figures 10a and 10b. We can see that the combined use of altimetry and shipborne gravity data improves the agreement with GSD95, especially close to the coastline, and this can be attributed to the fact that the shipborne data were used in the GSD95 geoid modeling as well. Nevertheless the differences due to the different fields used are evident especially in the case of GPM98b where the standard deviation is at the $\pm 50\text{cm}$ level.

For the comparisons between the aforementioned solutions and the stacked SSHs from the 3rd year of the T/P mission the computed differences were minimized by using a four-parameter transformation model:

$$SSH_{T/P} = N - b_0 \cos\phi \cos\lambda - b_1 \cos\phi \sin\lambda - b_2 \sin\phi - b_3 \quad (7)$$

where the parameters b_0 , b_1 , b_2 and b_3 were calculated by the least squares technique and N is the altimetric, gravimetric or combined geoid height depending on the solution under consideration. The statistics of the differences, after bias and tilt fit, between T/P SSHs and the ERS1-GM, GEOSAT-GM and gravimetric solutions, are presented in Tables 16, 17 and 18 respectively.

	max	min	mean	std
T/P_{SSHs} - N_{EGM96}	0.562	-0.574	0.000	±0.095
T/P_{SSHs} - N_{GPM98b}	0.557	-0.481	0.000	±0.094

Table 16. Geoid height differences between T/P SSHs and ERS1-GM altimetric solutions after bias and tilt fit. Unit: [m]

	max	min	mean	std
T/P_{SSHs} - N_{EGM96}	0.858	-0.450	0.000	±0.109
T/P_{SSHs} - N_{GPM98b}	0.840	-0.420	0.000	±0.105

Table 17. Geoid height differences between T/P SSHs and GEOSAT-GM altimetric solutions after bias and tilt fit. Unit: [m]

	max	min	mean	std
T/P_{SSHs} - N_{EGM96}	0.962	-0.895	0.000	±0.253
T/P_{SSHs} - N_{GPM98b}	0.822	-0.797	0.000	±0.222

Table 18. Geoid height differences between T/P SSHs and the gravimetric solutions after bias and tilt fit. Unit: [m]

Comparing the standard deviation of the differences between the different geoid height solutions and the stacked T/P heights, it is easily concluded that the altimetric solutions show better agreement with T/P SSHs compared to the gravimetric ones. The agreement of the pure altimetric geoid heights with T/P SSHs is at the level of ±9 for ERS1-GM and ±10 for GEOSAT-GM. The differences for the gravimetric solutions on the other hand reach the level of ±25cm in terms of the standard deviation value. Comparing the use of the two different reference fields we can conclude that they give almost the same results for the satellite altimetry geoid solutions when for the gravimetric ones the solution to GPM98b gives 3cm better agreement with T/P SSHs.

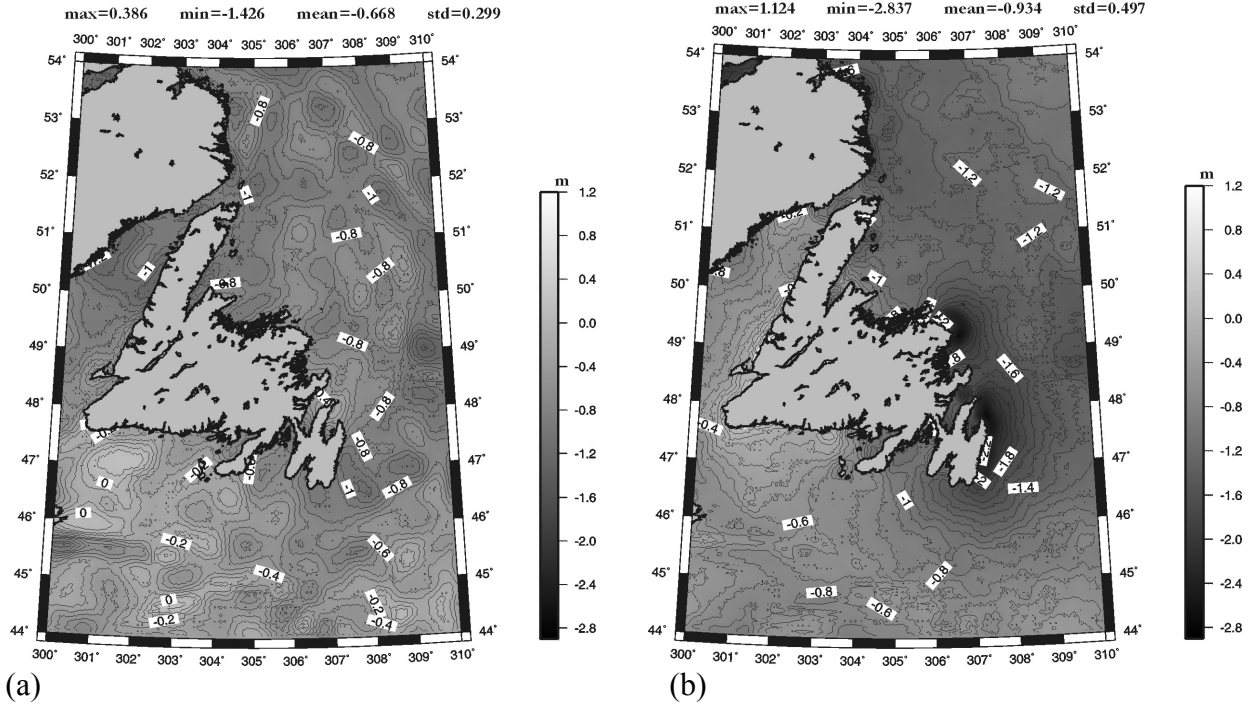


Figure 10. Geoid height differences between GSD95 and the combined solutions referenced to EGM96 (a) and GPM98b (b) fields.

For the differences between the stacked T/P SSHs with the geoid heights from the optimal combination method an accuracy close to $\pm 22\text{cm}$ and $\pm 20\text{cm}$ in terms of standard deviation of the differences, for EGM96 and GPM98b respectively, is achievable (see Figure 11). Comparing to the same statistics for the gravimetric solutions, we can see that the combined use of satellite altimetry and shipborne gravity data improves the agreement with T/P SSHs by 3cm for both EGM96 and GPM98b.

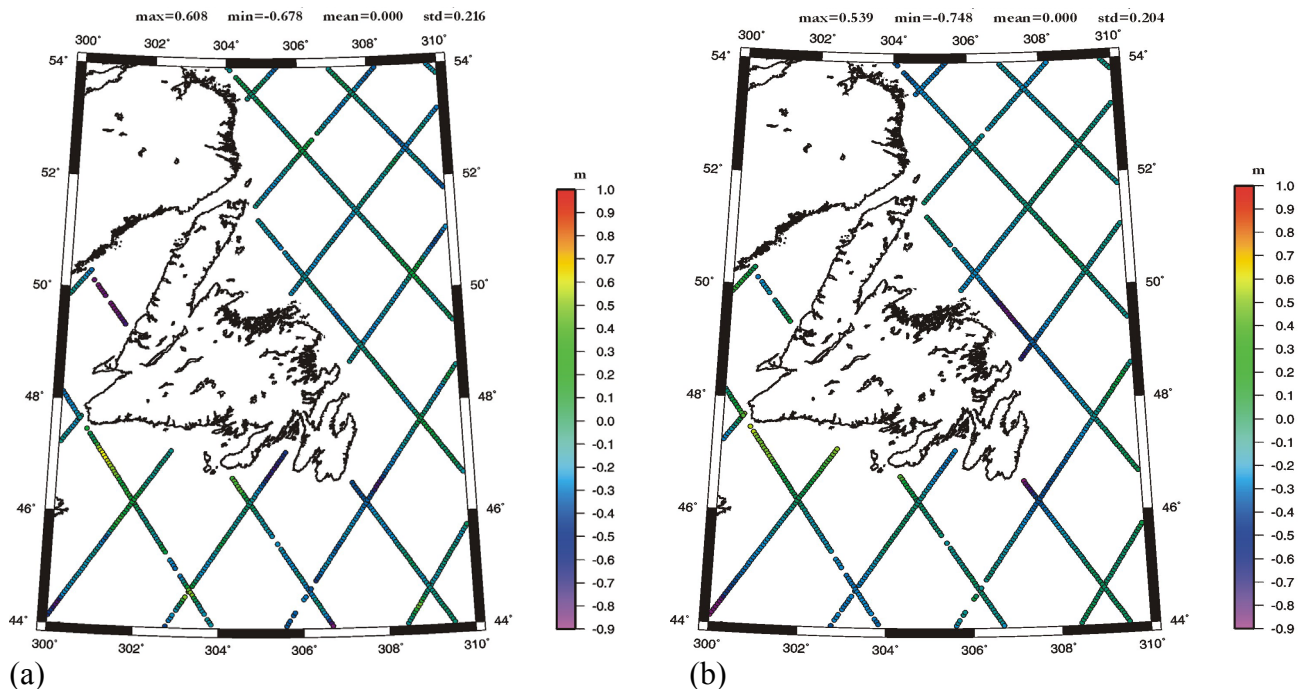


Figure 11. Differences between the final geoid height solutions from the MIMOST method for EGM96 (a) and GPM98b (b) reference models and the stacked T/P SSHs (3rd year).

6. CONCLUSIONS – FUTURE PLANS

Eight local geoid solutions were computed for the Eastern part of Canada close to the island of Newfoundland in the Labrador Sea. We determined pure altimetric and gravimetric geoid height solutions for the area under study using two geopotential models, namely EGM96 and GPM98b, as reference fields. The MIMOST method for the optimal combination of heterogeneous data was then used to determine the final geoid solutions. Throughout this study the well-known remove-restore method is used for the geoid determination when the gravimetric residual geoid heights are computed using the 1-D spherical FFT method.

Based on these results the determination of a purely altimetric geoid is possible with an accuracy close to $\pm 7\text{cm}$ while the determination of a purely gravimetric one has a lower accuracy close to $\pm 20\text{cm}$. The differences between T/P SSHs and the gravimetric geoid heights can be mainly attributed to the poorer accuracy of the available gravity data, at least compared to that of T/P SSHs. The differences between GSD95 and the ERS1-GM altimetric solution to EGM96 model are mainly due to the different reference fields used and to the more accurate data used in the present study.

As far as the combined solution is concerned, it is evident that the use of altimetry data, both for EGM96 and GPM98 reference fields, improves by $\pm 3\text{cm}$ the accuracy of geoid determination as compared to the gravimetric only solution. This can be attributed not only to the more accurate altimetry data but to the noise level used for the different data sets (5cm for the altimetric and 10cm for the gravimetric ones) which give more weight to the altimetric geoid heights in the combination algorithm. The comparison of our combined solution with GSD95 gives better agreement close to the coastline when compared to the differences of the satellite only geoid models. This is an indication that the use of shipborne gravity data enhances the altimetric only solutions, since the latter lack data close to the coastline and in shallow ocean areas. Throughout this research, GPM98b proved to give better or equal agreement, compared to EGM96, with our control datasets. This is an indication that it is more appropriate to use GPM98b as the reference field, at least for the test area, when geoid modeling is concerned.

A detailed study of the different solutions, with more emphasis on the error spectrum, and an investigation of the frequency contents of the two geopotential models is needed and will be carried out. Additionally special emphasis will be paid on the coastline when combining land and marine gravity data for geoid determination and towards the investigation for a numerical solution of the fixed Altimetry Gravimetry Boundary Value Problem (AGBVP). In order to obtain a higher resolution and higher accuracy geoid solution in the area under study, an improvement of the marine gravity databases is necessary.

Acknowledgements

The financial assistance provided to the first two of the authors by the GEOIDE Network of Centers of Excellence is gratefully acknowledged. We extensively used the Generic Mapping Tools Version 3.1 (Wessel and Smith, 1998) in displaying our results.

References

Andersen, O.B. and P.Knudsen. *Global marine gravity field from the ERS1 and GEOSAT geodetic mission altimetry*. J. Geophys. Res., Vol. 103, No. C4, pp. 8129 (97JC02189), 1998.

- Andritsanos, V.D., M.G. Sideris and I.N. Tziavos. *A survey of gravity field modelling applications of the Input-Output System Theory (IOST)*. To appear in IGeS Bulletin, 2000a.
- Andritsanos, V.D., M.G. Sideris and I.N. Tziavos. *Quasi-stationary Sea Surface topography estimation by the multiple Input-Output method*. Accepted for publication in Journal of Geodesy, 2000b.
- AVISO User Handbook – Corrected Sea Surface Heights (CORSSHs). AVI-NT-011-311-CN, Edition 3.1, 1998.
- Bendat, J.S. and Piersol A.G. *Random data: Analysis and Measurement Procedures*. Second edition. John Wiley and Sons, New York, 1986.
- The Geosat Altimeter JGM-3 GDRs Handbook. Internet Resources: <http://www.ibis.gdrl.noss.gov/SAT/gdrs/geosat-handbook>, 1997.
- Haagmans, R., E. de Min and M. van Gelderen. *Fast evaluation of convolution integrals on the sphere using 1D FFT, and a comparison with existing methods for Stokes' integral*. Manuscr. Geod. 18, pp. 227-241, 1993.
- Li, J. *Detailed marine gravity field determination by combination of heterogeneous data*. UCGE Reports No. 20102, Department of Geomatics Engineering, The University of Calgary, Calgary, Alberta, Canada, 1996.
- Lemoine, F.G., et al. *The development of the joint NASA GSFC and NIMA geopotential model EGM96*, NASA Technical Paper, 1998 – 206861, 1998.
- National Oceanographic and Atmospheric Administration - NOAA. *The GEOSAT-GM Altimeter JGM-3 GDRs*. 1997.
- Sandwell D. *Data Base Description for Digital Bathymetric Data Base-Variable Resolution (DBDB-V)*. Version 1.0, Naval Oceanographic Office, Stennis Space Center, March, 1996.
- Sideris, M.G. *On the use of heterogeneous noisy data in spectral gravity field modeling methods*. Journal of Geodesy, 70, pp. 470-479, 1996.
- Sideris, M.G. and B.B. She. *A new high-resolution geoid for Canada and part of the U.S. by 1D-FFT method*. Bull. Geod. 69:92-108, 1995.
- Smith, W.H.F. and D.T. Sandwell. *Bathymetric prediction from dense satellite altimetry and sparse shipboard bathymetry*. Journal of Geophysical Research, 99, No. B11, 21803-21824, 1994.
- Vergos, G.S., *The contribution of satellite altimetry to gravity field modeling – A case of study in the Eastern Mediterranean*. Diploma thesis, School of Rural and Surveying Engineering, Faculty of Engineering, Aristotle University of Thessaloniki, Department of Geodesy and Surveying, 2000.
- Véronneau, M. *Shipborne gravity data for the area of Newfoundland, Canada*. Gravity data from the Geological Survey of Canada, personal communication, 2001.
- Véronneau, M. *The GSD95 Geoid Model for Canada*. Gravity Geoid and Marine Geodesy, Proceedings of international symposium, Tokyo, Japan, 1996. Springer Vol. 1997, pp. 573-580, 1997.
- Wenzel, H.G. *Global models of the gravity field of high and ultra-high resolution*. In Lecture Notes of IAG's Geoid School, Milano, Italy, 1999.
- Wessel, P., and W.H.F. Smith. *New improved version of Generic Mapping Tools released*. EOS Trans. Amer. Geophys. U., vol. 79 (47), pp. 579, 1998.

ALTIMETRIC DATA FROM GEOSAT, TOPEX/POSEIDON, ERS-1 AND ERS-2 SATELLITES IN THE GENERATION OF A HIGH RESOLUTION GRID OF GRAVITY ANOMALIES BY INVERSION OF AZORES REGION

J. Calvao

Faculdade de Ciências da Universidade de Lisboa, E-mail : jcalvao@fc.ul.pt

M. J. Sevilla

Facultad de Matematicas, Universidad Complutense de Madrid, E-mail: maast01@sis.ucm.es

Abstract

Satellite altimetry from Geosat (18 months) and ERS-1 (10 months) geodetic missions provide data with a very dense coverage, allowing a very detailed recovery of the gravity field. Stacked data from TOPEX/Poseidon (6 years in a 10 day repeat cycle), ERS-2 (4 years in a 35 day repeat cycle) and Geosat ERM (2 years in a 17 day repeat cycle) are used to define a precise reference frame where the previously mentioned data are adjusted.

A remove-restore procedure is used to obtain residual sea surface heights eliminating the contributions of the low (with the global geopotential model EGM96 as reference field) and high (the effect of the topography/bathymetry is computed using the RTM correction with the local accurate bathymetric model AZDTM98 and the global model JGM95E) frequencies to the sea surface heights.

A validation procedure is then applied, comparing each value with an estimation obtained using least squares collocation and a grid is produced. Finally an efficient method based on Fast Fourier Transform is used to obtain the residual gravity anomalies by inversion of the mentioned grid. After adding the contributions to the gravity anomalies from the global geopotential model and from the topography/bathymetry, the result is compared with adjusted gravity data obtained by gravimetric surveys.

1. Introduction

The 3° by 3° area under study, located on the Azores region (limited by $38^{\circ} < \phi < 40^{\circ}$ and $331^{\circ} < \lambda < 333^{\circ}$) have been subjected to systematic bathymetric surveys, in the aim of FARA and RIDGE projects. This data was complemented with topographic data on the islands, which resulted on the high precision 1'x1' digital terrain model AZDTM98 (Catalão and Sevilla, 2000). The test area was chosen due to its high mountains and deep sea, with pronounced height differences and their influence on the computed gravity anomalies has been analysed. With some improvements, the processing used follows the one described by Andersen and Knudsen in the derivation of the KMS99 grid of gravity anomalies.

2. Data Selection and Editing

The altimetric observations, with an along-track spacing of about 7 km, were extracted in a wider 5° by 5° area. The primary focus was on the recovery of the short wavelength gravity field information, so not all corrections are relevant. Correction for ionospheric and dry tropospheric path delay provided by NOAA and ESA were applied. Near land, where wet tropospheric correction is normally non-existent, this correction was computed using meteorological information by spline interpolation. Although this computed correction is less accurate, it allows to retain data points near the coast. Solid Earth tide correction was applied and the NAO tide model (Matsumoto et al., 2000) was used to compute the ocean and load tide contribution. Inverse barometer correction was also applied. All data sources applied the orbits based on JGM-3 gravity model. If the standard deviation of the height observation exceeded 0.3 m or any of the applied corrections were absent, the data point was removed. The repeating ground tracks of the Geosat, Topex/Poseidon and ERS-1/2 altimeters offer the possibility of improving the accuracy and resolution of gravity anomalies along widely spaced tracks, allowing the recovery of short wavelength gravity anomalies.

3. Data Processing

Data from both geodetic phases were interpolated every 0.5 seconds to obtain a better along track resolution and all data were merged. To reduce the effects of sea surface variability and orbit errors, each track was fitted to the corresponding EGM96 geoid profile using tilt and bias parameters (figure 1).

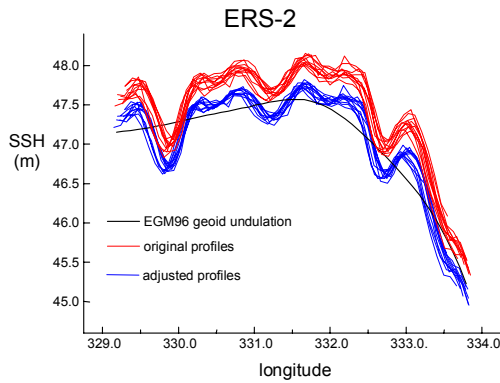


figure 1: fitting ERS-2 tracks to EGM96 geoid

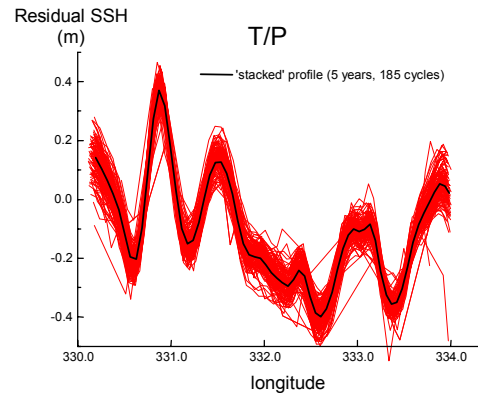


figure 2: stacking TOPEX/Poseidon data

EGM96 model was used as reference field; gravitational effects of the topography and bathymetry are the dominant sources of local gravity field variations, so numerically removing this effects in the remove-restore technique leads to a smoother residual field. There are a number of different methods for computing topographic corrections using a high resolution digital terrain model. In the RTM method, only local high frequency topographic irregularities are taken into account by referring all elevations to a smooth mean elevation surface. Two digital terrain models were used, the above mentioned AZDTM98 and the 5'x5' JGM95E (figure 3).

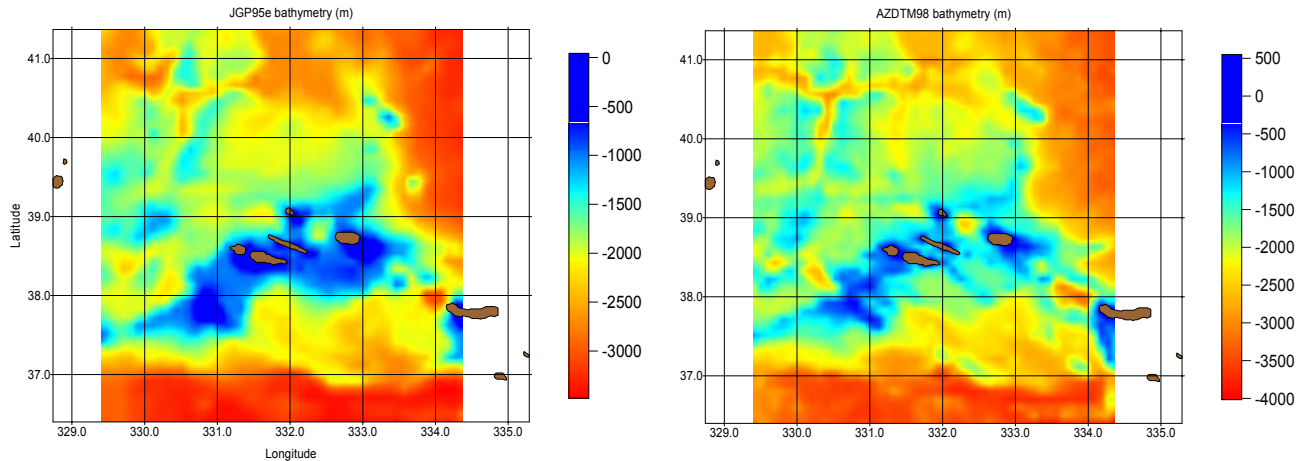


figure 3: bathymetric models used to compute terrain effects

To reduce the number of points, data from the repeat missions were stacked (figure 2) and data from all sources were merged. A cross-over adjustment was carried out on the residual data using bias and tilt parameters. Each observation was then compared with an estimated height obtained using least squares collocation to detect gross errors and the fitting to the geoid and the cross-over adjustment were repeated.

At the cross-overs, sea surface height differences reflect model errors and ocean variability; to avoid artefacts in the geoid grid due to bad cross-overs, this differences between ascending and descending tracks were eliminated, distributing it by both arcs. An Akima spline (Akima, 1970), which was designed to combat large oscillations, was used to interpolate the new tracks, so that at each cross-over, both arcs cross exactly at the same height (figure 4)

Due to the large amount of altimetric data, it has become increasingly important to have efficient methods of deriving gravity anomalies. The fastest methods have been based on spectral techniques. If an

FFT based method is used, one problem is that of gridding the data. The edited and adjusted altimeter data were interpolated on a regular 2' by 2' grid using collocation and introducing a covariance function that filters sea surface variability. To avoid spectral leakage, a cosine taper was applied in the outer 0.5° of the grid. The gravity anomalies were derived from geoid undulation using FFT techniques, where a Wiener filtering function was used to reduce noise influence. Finally, the contributions of the low and high frequencies were added again.

4. Results and conclusions

Three data sets of gravity anomalies were produced: without RTM correction, with RTM correction with JGM95E model and with RTM correction with AZDTM98 model:

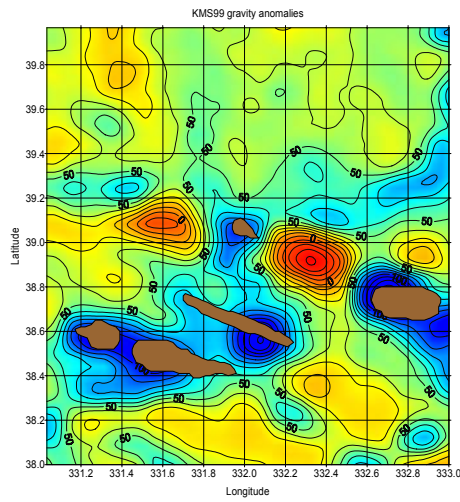


figure 4: KMS99 gravity anomalies

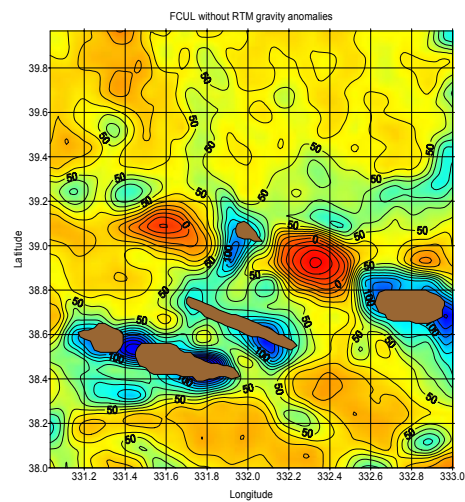


figure 5: gravity anomalies without terrain correction

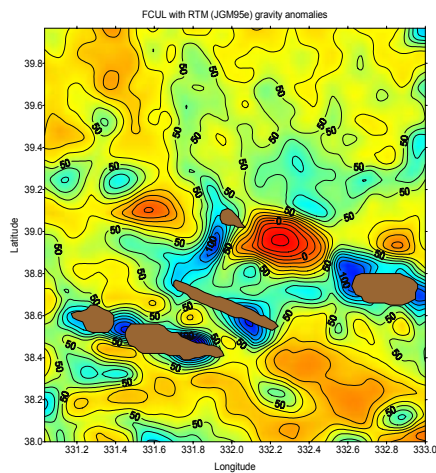


figure 6: gravity anomalies with terrain correction (JGP95E)

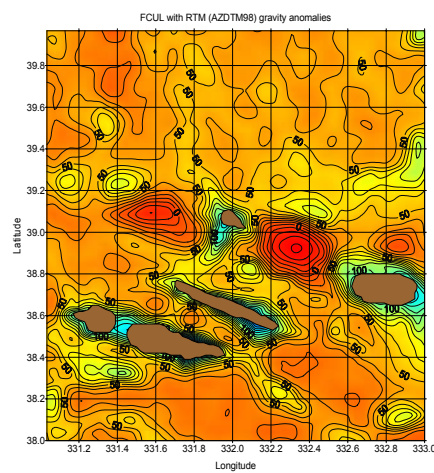


figure 7: gravity anomalies with terrain correction (AZDTM98)

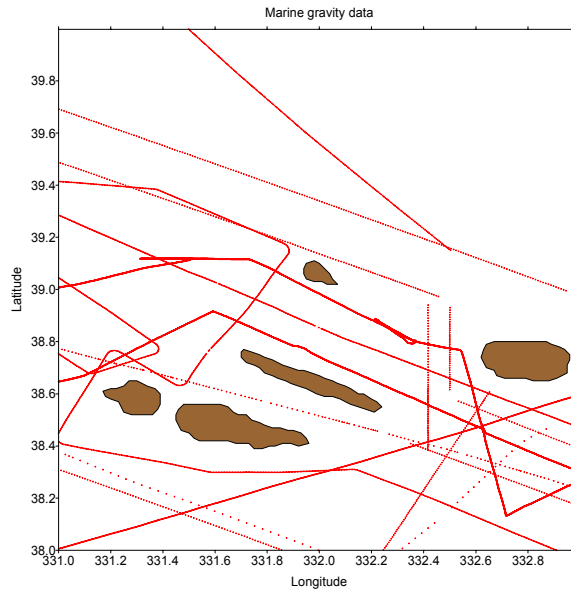


figure 8: distribution of marine gravity data

These grids were compared with adjusted marine gravity data and the results are shown in the following table:

	KMS99	without RTM	with RTM (AZDTM98)	with RTM (JGP95E)
#	7296	7296	7296	7296
mean	0.22	1.18	3.51	2.58
std	10.01	9.59	7.61	10.50
min	-51.07	-36.83	-31.17	-32.85
max	39.03	31.53	27.45	32.30

table 1: comparison of the different data sets

The best results were obtained with the inclusion of RTM correction in the procedure of computing the gravity anomalies using an accurate model of the bathymetry/topography. In general, the accuracy decreases with the proximity to coast so the procedures that were used seem to result in a better agreement with marine observations.

References

- Andersen, O.B. and P. Knudsen (1998), "Global marine gravity field from ERS-1 and Geosat geodetic mission altimetry." *Journal of Geophysical Research*, Vol. 103, No. C4, pp. 8129-8137.
- Arabelos, D. and I.N. Tziavos (1998), "Gravity-field improvement in the Mediterranean Sea by estimating the bottom topography using collocation." *Journal of Geodesy*, Vol. 72, pp. 136-143.
- Dahl, O.C. and R. Forsberg (1998), "Geoid models around Sognefjord using depth data." *Journal of Geodesy*, Vol. 72, pp. 547-556.
- Catalão, J. and M.J. Sevilla (1998), "Geoid studies in the north-east Atlantic (Azores-Portugal)." In *Geodesy on the Move*, Proceedings of the IAG Scientific Assembly, Eds. R. Forsberg, M. Feissel, R. Dietrich, Rio de Janeiro, Sept. 3-9, 1997, Springer-Verlag, pp. 269-274.
- Catalão, J. and M.J. Sevilla (2000), "The effect of high precision bathymetric model on geoid computation." *International Geoid Service*, Bulletin N. 10, pp. 91-99. Milan (2000).

- Fernandes, M.J., L. Bastos and J. Catalão (1998), "The role of dense ERS altimetry in the determination of the marine geoid in Azores." Presented at the *Fourth International Symposium on Marine Positioning, INSMAP 98*, 30 November - 4 December 1998, Melbourne, Florida.
- Forsberg, R. (1984), "A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling." *Dept. of Geodetic Science and Surveying*, Rep. No. 355, The Ohio State University, Columbus, Ohio.
- Heiskanen, W.A. and H. Moritz (1967), *Physical Geodesy*. W.H. Freeman and Company, San Francisco
- Lemoine, F.G. et al.(1997), "The development of the NASA GSFC and NIMA Joint Geopotential Model" In *Proceedings of the International Symposium on Gravity, Geoid and Marine Geodesy, GRAGEOMAR*, Eds. J. Segawa, H. Fujimoto and S. Okubo, The University of Tokyo, Tokyo, Sept. 30 - Oct. 5, Springer-Verlag, pp. 461-469.
- Lourenço, N. et al. (1999), "Morpho-tectonic analysis of the Azores Volcanic Plateau from a new bathymetric compilation of the area." *Marine Geophysical Researches* (in press).
- Sandwell, D. and W.H.F. Smith (1997), "Marine gravity anomaly from Geosat and ERS1 satellite altimetry." *Journal of Geophysical Research*, Vol. 102, No. B5, pp. 10039-10054.
- Sevilla, M.J. (1995), "A new gravimetric geoid in the Iberian peninsula" In "New Geoids in the World". BGI Bull. 77, IgeS Bulletin N. 4, pp. 163-180.
- Tziavos, I., M.G. Sideris and H. Sünkel (1996), "The effect of surface density variation on terrain modelling - A case study in Austria." In *Proceedings of the EGS XXI General Assembly, Session G7, Techniques for Local Geoid Determination*, Eds. I. Tziavos and M. Vermeer, The Hague, The Netherlands, May 6-10, pp. 99-110.

Altimetry-gravimetry boundary value problems with smoothness conditions in coastal regions

R.S. Grebenitcharsky and M.G. Sideris

Department of Geomatics engineering, University of Calgary, 2500 University Drive N.W., Calgary, Alberta, Canada, T2N 1N4

ABSTRACT

Altimetry-gravimetry boundary value problems (AGBVPs) are investigated in coastal regions from a theoretical and an application point of view. A numerical solution of AGBVPs for local geoid approximation using compatibility conditions along the coastline is discussed. The existence and uniqueness of the solution of different types of AGBVPs including compatibility conditions along the coastline are discussed. Variational methods for solving differential equations are applied. The effects of smoothness assumptions on the data are analysed in the case of combination of different types of data. Because of using different types of data, irregularities along the coastline could be expected. These irregularities are investigated using the property of wavelets to give information about the distribution of different frequencies in space. The effect of such irregularities on the solution with compatibility conditions is investigated and conclusions are drawn.

KEYWORDS

Geoid, altimetry-gravimetry boundary value problems, satellite altimetry, gravity

1. INTRODUCTION

Compatibility conditions in altimetry-gravimetry boundary value problems are usually used to ensure the well-posedness of a boundary value problem (Sacerdote and Sanso, 1987; Svensson, 1988). The uniqueness and existence of the solution of AGBVPs depend upon the smoothness assumptions along the coastline. The addition of compatibility conditions along the coastline may cause the problem to become well-posed but it is questionable if such conditions upon the data are realistic for practical use (Svensson, 1988).

Another application of compatibility conditions along the coastline is related to the regularization of the solution in the neighbourhood of the coastline. According to Svensson (1988), unless the compatibility conditions are recognized, the mixed problems cannot be solved with a higher degree of smoothness. Additional constraints on the data for the regularization of the solution across the coastline were derived in Sacerdote and Sanso (1987). Getting a higher degree of smoothness along the coastline is necessary for one of the most often used methods for the solution of AGBVPs – the variational method for solving differential equations. It requires both the boundary surface and the boundary conditions (observations) to be smooth enough, i.e., to have continuous derivatives up to a certain order. At the same time, irregularities at the coastline exist because different types of data with different observation procedures, accuracy and resolution are used. To overcome these problems, compatibility conditions along the coastline can be used.

The regularization of data along the coastline will smooth the solution but what is the magnitude of this effect from an application point of view? The purpose of this paper is *to investigate the influence of smoothness compatibility conditions along the coastline on the solution of AGBVPs*. A numerical experiment in the region of Newfoundland (Eastern Canada) is conducted to estimate the influence

compatibility conditions have on the smoothing of the solution of AGBVPs. Gravity anomalies on land and at sea are used, and the effect on final geoid solution of smoothing the data (boundary conditions) is estimated.

2. AGBVPs – GENERAL FORMULATION AND WEAK SOLUTION

The general formulation for an AGBVP according to Lehman (1999) is:

$$(1) \quad \begin{cases} \nabla^2 T = 0 \\ L_o(T) = b_o \quad \text{on} \quad \Gamma_o \quad (\text{ocean} / \text{sea}) \\ L_L(T) = b_L \quad \text{on} \quad \Gamma_L \quad (\text{land}) \\ T = O(|r|^{-2-n}), \quad |r| \rightarrow \infty \end{cases}$$

where:

T is the disturbing potential;

∇^2 is the Laplacean differential operator;

L_o is a linear functional of T at sea (ocean);

L_L is a linear functional of T on land;

Γ_o is the boundary surface at sea;

Γ_L is the boundary surface on land;

r is the geocentric radius vector of a point outside the boundary surface; and

$n+2$ is the power of decreasing the inverse geocentric radius vector length .

According to Rektorys (1977), a weak (generalized) solution of the AGBVP exists and it consists of a functional u satisfying the stable boundary conditions (related to geoid heights N_o or disturbance potential T_o as measurements at sea) plus a “refinement” of u based on the unstable conditions (gravity disturbances δg_o at sea or gravity anomalies Δg_L on land as measurements). The term “weak” (generalized) shows the fact that the solution satisfies Laplace’s differential operator and boundary condition not in the sense of classical pointwise functions but in the sense of generalized functions.

According to Rektorys (1977), we can expect that the weak solution for Laplace’s operator has generalized derivatives of all orders. The higher the degree of derivatives included in the compatibility conditions, the closer the weak solution is to the classical solution of the problem. According to (Sacerdote and Sanso, 1987), infinitely differentiable solutions can be expected only for points, that are far from the coastline (inner regularity). Additional constraints must be imposed on the data along the coastline. In this way, the irregularities between the data along the coastline need to be regularized introducing compatibility conditions. Actually, we can assume smoothness conditions for derivatives up to infinite degree along the coastline.

Finally, the conclusion given in Rektorys (1977) that “if all data of the considered problem (including the boundary surface) are sufficiently smooth, then the weak solution is the classical solution of the considered problem” is the underlying reason for investigating the smoothing procedure along the coast line for AGBVPs.

3. COMPATIBILITY CONDITIONS AND REGULARITY OF THE WEAK SOLUTION FOR DIFFERENT TYPES OF AGBVPs

In the previous two paragraphs it has been explained that compatibility conditions along the coastline are necessary to increase the regularity of the data and to have a weak solution which is closer to the classical solution of AGBVPs. The form of the compatibility conditions can be found in Svensson (1988) and (Sacerdote and Sanso, 1987). In both cases, they are given in an implicit form. Before getting them in an explicit form, a preliminary investigation of the effect of smoothing along the coastline is necessary. It can be shown that the compatibility conditions in Svensson (1988) correspond to the requirement that the data and their derivatives be consistent along the coastline. Compatibility conditions could be simply substituted by conditions on data and their derivatives along the coastline only.

For both types of AGBVPs, we have the following data on the boundary: geoid heights (disturbing potential values) at sea and gravity anomalies on land for AGBVP I, and gravity disturbances at sea together with gravity anomalies on land for AGBVP II. In Svensson (1988), it has been shown for AGBVP I and AGBVP II that, with the introduction of additional compatibility conditions on the coastline, both problems become normal solvable (satisfying the Fredholm alternative). To achieve this, he introduced new linear pseudodifferential differential operators A and C such that:

$$A = (-\nabla^2 + I/4)^{1/2} - 3/2 \quad (2)$$

$$C = (-\nabla^2 + I/4)^{1/2} + 3/2 \quad (3)$$

$$AC = CA = -\nabla^2 - 2, \quad A + 3 = C \quad (4)$$

This allows not only the problem to become normal solvable, but also increases the regularity along the coastline. It is evident from the interpretation of the compatibility conditions (Svensson, 1988) that they are closely related to the agreement of functionals and their derivatives along the coastline. This fact will allow us to use wavelets as multiscale differential operators later on to smooth data along the coastline. The physical meaning of A and C is given by the table for important pseudodifferential operators on the sphere in Svensson (1983, p.18). In fact, A and C can be interpreted as gravity anomaly and gravity gradient operators. Gravity disturbances could be represented as $C(I-C)$, where I is identity operator.

Using in addition new pseudodifferential operators E , F , L and M , which define the following mappings: $\bar{H}_{1/2}(\Gamma_o) \xrightarrow{E} H_{1/2}(\Gamma)$, $\bar{H}_{-1/2}(\Gamma_l) \xrightarrow{F} H_{1/2}(\Gamma)$, $\bar{H}_s(\Gamma_l) \xrightarrow{L} H_s(\Gamma)$, $\bar{H}_{s-2}(\Gamma_o) \xrightarrow{M} H_s(\Gamma)$, a new form of AGBVP I and AGBVP II is possible. Here the notations H_i and $\bar{H}_i$ denote Sobolev spaces H of i^{th} order and their closers, respectively. It is assumed that $T \in H_s$ and $\Delta g \in H_{s-1}$, where s is the order of generalized derivatives used in the definition of Sobolev spaces. Pseudodifferential operators E , F , L and M are basically extensions from land and sea to the entire sphere Γ . The simplest extension could be to, e.g., fill the missing data with zeros (Eskin, 1981).

The boundary conditions (observations) for AGBVP II in spherical approximation are

$$\left| \begin{array}{ll} -\frac{\partial T}{\partial r} - \frac{2}{R}T = \Delta g_l & \text{on } \Gamma_l \\ -\frac{\partial T}{\partial r} = \delta g_o & \text{on } \Gamma_o \end{array} \right. \quad (5)$$

The new (transformed) boundary conditions for AGBVP II then are

$$\left| \begin{array}{ll} AT = \Delta g_l & \text{on } \Gamma_l \\ (A+2)(A+3)AT = (-\nabla^2 - 2)\delta g_o & \text{on } \Gamma_o \\ (A+2)SL\Delta g_l + (A+2)SM(-\nabla^2 - 2)\delta g_o + (A+2)T_l = 0 & \text{on the coastline} \end{array} \right. \quad (6)$$

S is the Stokes operator;
 T_l is the first degree term of T ;
 Δg_l is the gravity anomaly on land; and
 δg_o is the gravity disturbance at sea.

We assume the following conditions related to data and their gradients along the coastline:

$$\left| \begin{array}{ll} \Delta g_l = \delta g_o & \text{on the coastline} \\ \nabla^2(\Delta g_l) = \nabla^2(\delta g_o) & \text{on the coastline} \end{array} \right. \quad (7)$$

where ∇ is the gradient operator. Gravity disturbances on the coastline are considered equal to the gravity anomalies, which is possible of course if the sea surface topography is known with enough accuracy.

Now we will show that conditions (7) are closely related to the condition on the coastline in (6) and that, infact, the coastline condition (6) is satisfied under the assumption of eq. (7) and $T_l=0$. As usual, we assume that the first degree term T_l is zero, i.e., we place the coordinate system in the centre of the Earth. Let $P=L+M$. The operator P could be simply interpreted as merging data on land and at sea. From eqs. (7) and (4) we can immediately obtain

$$AC\Delta g_l = AC\delta g_o \quad \text{on the coastline.} \quad (8)$$

Applying the L (or $P-M$) and then the $(A+2)S$ operators to both sides of (8), we obtain

$$LAC\Delta g_l = (P - M)AC\delta g_o \quad \text{on the coastline,} \quad (9a)$$

$$(A + 2)SL(AC - I + I)\Delta g_l = (A + 2)S(P - M)AC\delta g_o \quad \text{on the coastline} \quad (9b)$$

or, equivalently

$$(A + 2)S\{L(AC - I)\Delta g_l - PAC\delta g_s\} + \{(A + 2)SL\Delta g_l + (A + 2)SMAC\delta g_s\} = 0. \quad (9c)$$

Taking into account that on the coastline we have $LAC\Delta g_l = LAC\delta g_o$ we have:

$$(I - I)\{(A + 2)SL\Delta g_l + (A + 2)SMAC\delta g_o\} = 0. \quad (10)$$

It is clear that the equation in brackets could have any value. Now we can assume that

$$(A + 2)SL\Delta g_l + (A + 2)SMAC\delta g_o = -(A + 2)T_l. \quad (11)$$

Using (4) we see that the condition (11) is equal to the compatibility condition (eq. 6) along the coastline in Svensson's representation of the AGBVP II. Thus it makes sense to use conditions (7) for smoothing the data along the coastline. Wavelets can then be used as multiscale differential operators to investigate the influence of smoothness compatibility conditions on data.

4. APPLICATION OF WAVELETS FOR DETECTING IRREGULARITIES ALONG THE COASTLINE AND AS MULTISCALE DIFFERENTIAL OPERATOR

The properties of wavelets to give not only the frequencies of a signal but also their spatial distribution in different scales can be used to detect discrepancies between different data along the coastline. We can use a wavelet decomposition and reconstruction to place smoothness compatibility conditions on the data and the boundary along the coastline. After the decomposition up to certain level, we could eliminate irregularities along the coastline in the high frequency part of the decomposition. This is equivalent to putting constraints on the n^{th} derivatives (i.e., we use smoothness conditions). The necessary and sufficient condition for a wavelet transform to be an n -order multiscale differential operator is the corresponding wavelet to have n vanishing moments (Mallat, 1998). It is known that Daubechies wavelets of order N have $N-1$ vanishing moments. Wavelet decomposition and reconstruction let us restrict the vertical, horizontal and diagonal coefficients up to certain level for every level of decomposition. The threshold value is determined as an average value of all detail coefficients, excluding the values along the coastline. These constraints mean that we have implicitly used additional smoothness conditions on the derivatives of the solution. Restricting the detail

coefficients on the coastline means that we force the horizontal gradients of the signal both on land and at sea to be equal and to have as values, the average values for every level of decomposition. These restrictions are only for points which are close to the coastline. The signal far from the coastal region remains unchanged. Using this procedure we can expect that the irregularities along the coastline will be smoothed and this smoothing will affect the coastal region. A numerical experiment has been conducted to investigate the effect of smoothness compatibility conditions on the data and on the final geoid solution.

5. NUMERICAL EXPERIMENT INVESTIGATING THE INFLUENCE OF COMPATIBILITY CONDITIONS ON THE SOLUTION OF AGBVP II

Gravity data on land and gravity data derived from altimetry (provided by KMS – solution KMS99) at sea have been used in the region of Newfoundland, Eastern coast of Canada, for the numerical solution of AGBVP II, applying smoothness conditions along the coastline using wavelets. To show more details along the coastline, some results are presented in a smaller window (see Figure 1).

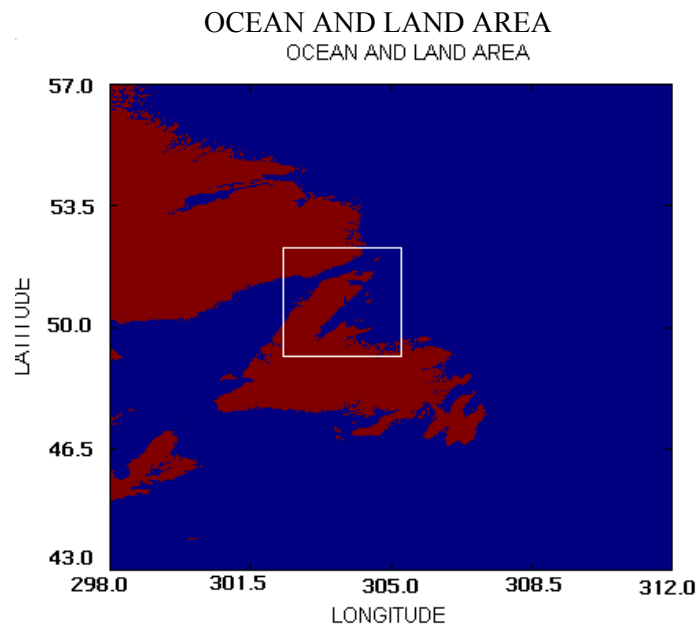


Figure 1. Area under investigation.

In the numerical experiment, “Daubechies 4” wavelet decomposition up to third level has been used to detect existing irregularities between both data types and to place additional compatibility conditions on the data along the coastline.

Figure 2 shows the ability of “Daubechies 4” wavelets to detect irregularities between different types of data along the coastline. It is obvious that the greatest values of wavelet coefficients are along the coastline. After this, an investigation of the influence of these conditions on the final geoid solution in the coastal region is conducted. The statistics for the original and smoothed data and for their differences are presented in Table 1, Table 2 and Table 3.

max	min	mean	rms	std
97.255	-67.812	-0.927	9.649	9.649

Table 1. Original gravity anomaly residuals referenced to EGM96. Unit: [mGal].

max	min	mean	rms	std
83.894	-65.542	-0.924	9.645	9.601

Table 2. Smoothed gravity anomaly residuals referenced to EGM96 Unit: [mGal].

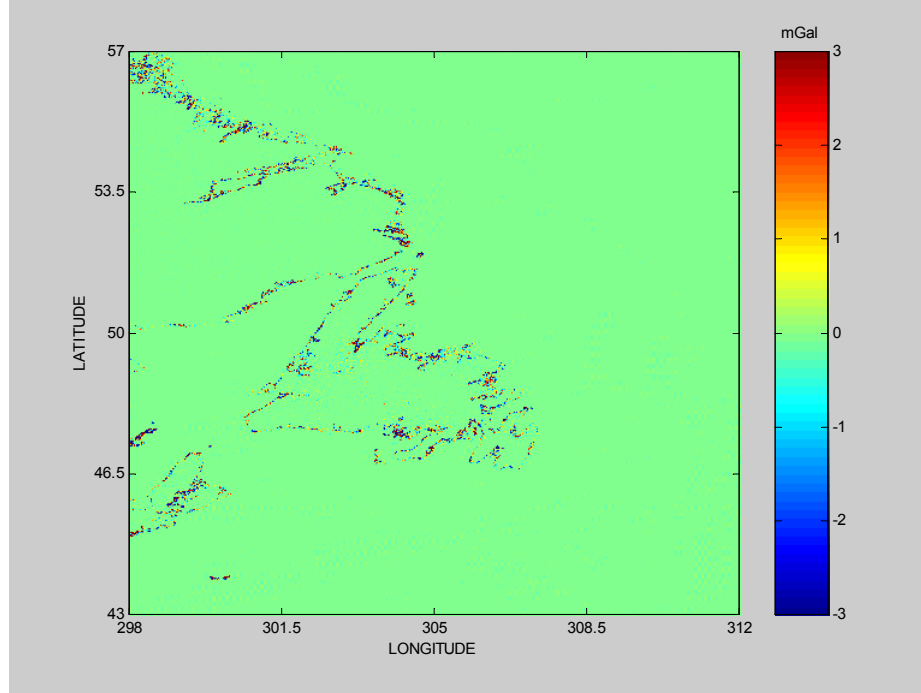


Figure 2. Magnitude of diagonal detail coefficients after the first level of “Daubechies 4” wavelet decomposition.

max	min	mean	rms	std
42.489	-38.537	0.001	1.761	1.761

Table 3. Residual gravity anomaly differences (smoothed-original). Unit: [mGal].

Smoothed gravity anomaly residuals are a result of placing compatibility conditions along the coastline using “Deubeshie 4” wavelet decomposition and reconstruction. It is evident that the mean, rms and standard deviation are not changed significantly. In general, the signal is not changed except along the coastline. The discrepancies between the land and KMS gravity data are clearly visible along the coastline (see Figure 3). Wavelets have been applied to smooth these discrepancies. It is evident that the irregularities have been smoothed along the coastline, but for the areas away from the coastline there are no changes. Applied such a smoothing procedure influences only the coastal area. The differences between smoothed and original data are given in Figure 5.

The differences in the entire area range between -39 mGal and 43 mGal. The colour graphs are only for the zoomed area and have been chosen to show more details along the coastline. It is clear that the corrections to the original data to get smoother data along the coastline are concentrated along the coastline. Both original and smoothed data were used to determine the geoid heights in the area under study. The differences between these two solutions give us information about the influence of the compatibility conditions on the final geoid solution for the region of investigation. The statistics for the final geoid solutions using original and smoothed gravity anomalies are given in Table 4, Table 5 and Table 6.

max	min	mean	rms	std
0.711	-0.845	-0.045	0.189	0.183

Table 4. Original residual geoid heights. Unit: [m].

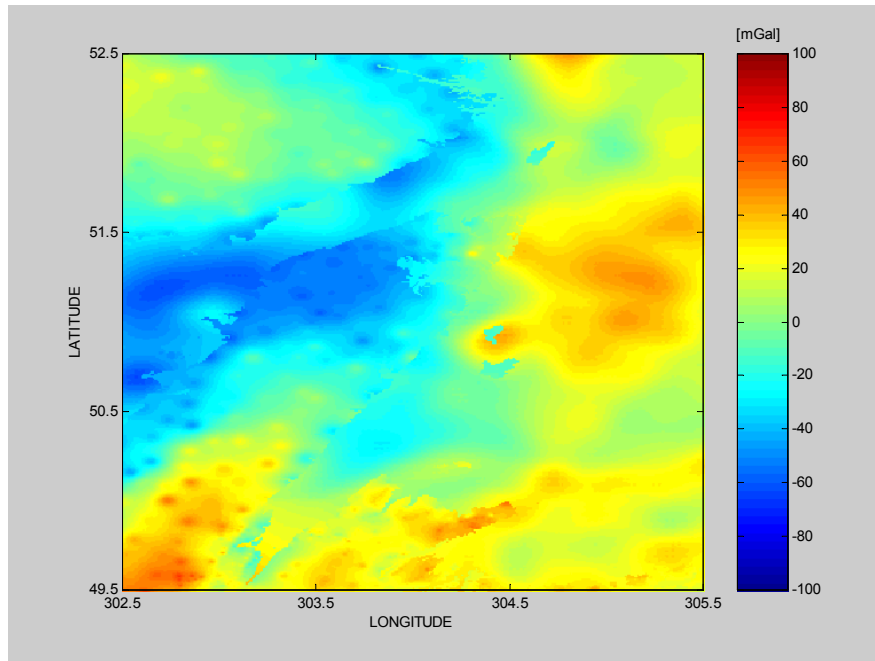


Figure 3. Original land gravity and gravity data from altimetry in the area under study.

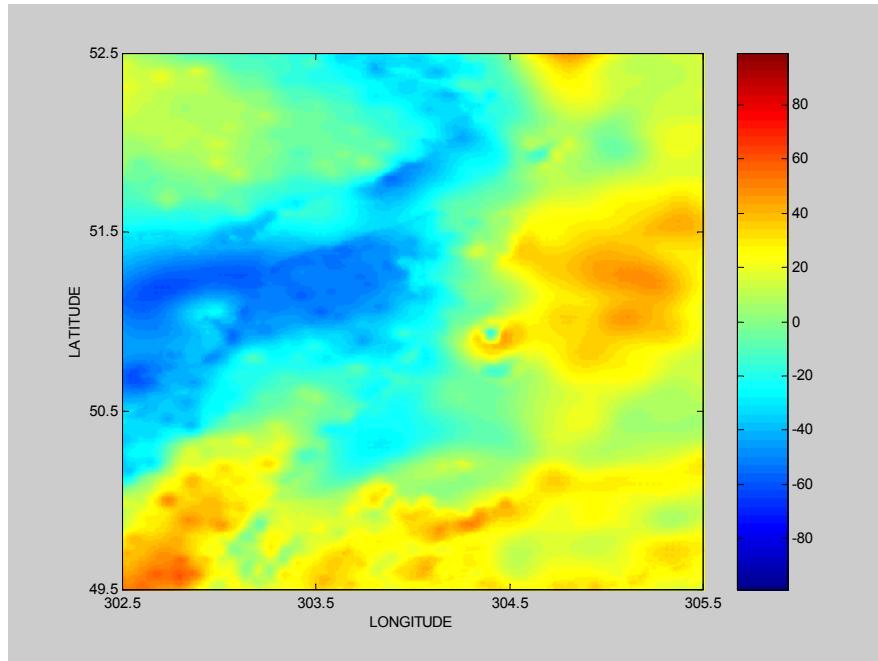


Figure 4. Smoothed along the coastline land gravity and gravity data from altimetry in the area under study.

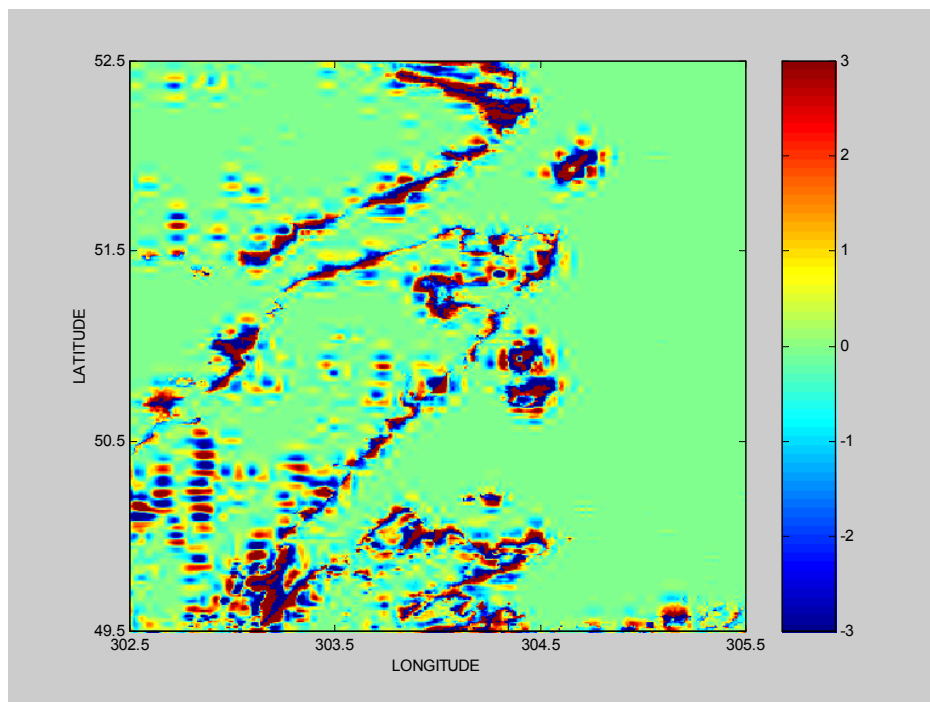


Figure 5. Differences between original and smoothed land gravity and gravity data from altimetry in the area under study.

max	min	mean	rms	std
-----	-----	------	-----	-----

0.719	-0.844	-0.045	0.188	0.183
-------	--------	--------	-------	-------

Table 5. Smoothed residual geoid heights. Unit: [m].

max	min	mean	rms	std
0.045	-0.050	0.000	0.001	0.001

Table 6. Residual geoid heights differences (smoothed-original). Unit: [m].

The differences between the residual geoid heights derived from original residual gravity anomalies and residual geoid heights derived from smoothed residual gravity anomalies show that the smoothing along the coastline does not change the statistics of the entire area. The effect of smoothness is only along the coastline. The magnitude of the effect of the smoothness conditions is between -5 and $+5$ cm. There are greater values than these but they are due to edge effects. Smoothing again does not change the mean and rms of the geoid results in the area under study but only the values along the coastline (see Figures 6 and 7).

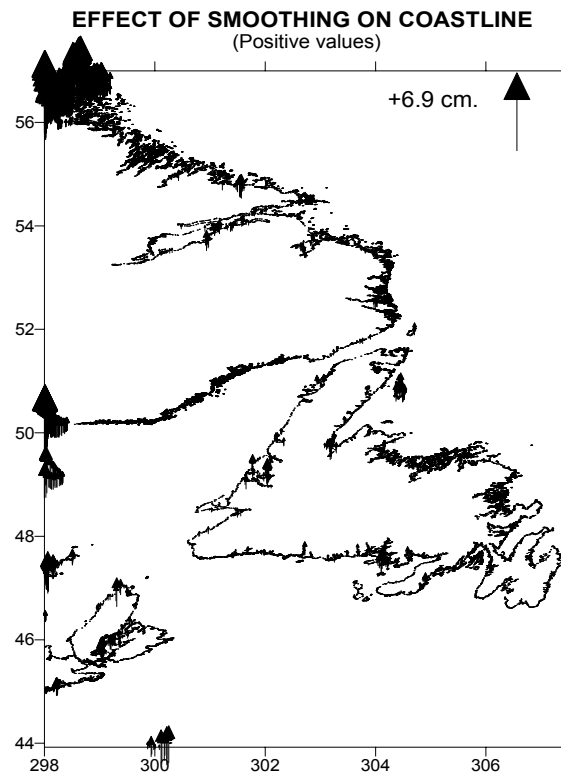


Figure 6. Positive effect of smoothness conditions on the points along the coastline.

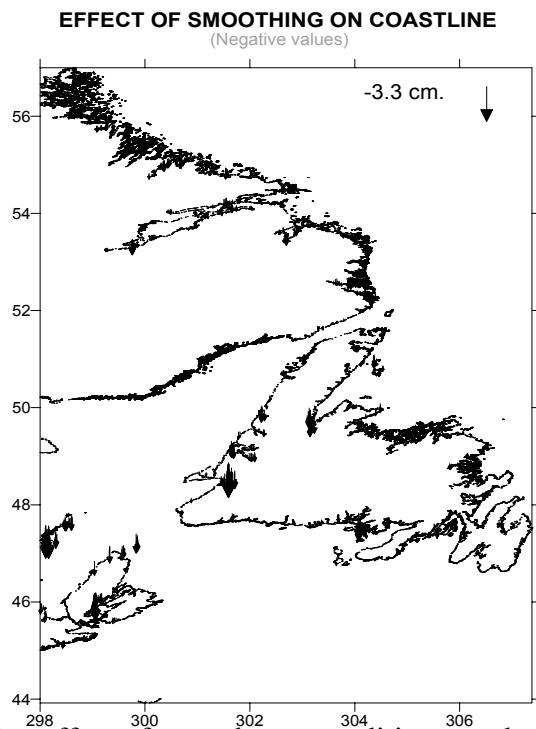


Figure 7. Negative effect of smoothness conditions on the points along the coastline.

6. CONCLUSIONS

Wavelet transforms using wavelets with n -vanishing moments can be used successfully for detecting irregularities along the coastline and for smoothing data (compatibility conditions along the coastline).

To smooth the data along the coastline, we should restrict the detail coefficients in every level of decomposition to a certain value. The threshold value could be the average of all detail coefficients, excluding the values along the coastline.

The differences between smoothed and original gravity anomalies were -39 mGal and 43 mGal for entire area. After the smoothing procedure, the greatest differences were located mostly along the coastline. For areas away from the coastline the differences are negligible, i.e., the used smoothing procedure has influence only along the coastline.

The effect of smoothness on the final geoid solution along the coastline is between -5 cm and +5 cm in our test area. There is no effect on the regions away from the coastline, except for edge effects.

Although the smoothness effect is not large in this area, the final geoid solution using smoothness conditions along the coastline is theoretically closer to the classical solution.

A numerical solution of an AGBVP should take into account the compatibility conditions along the coastline, especially for cm-level geoid determination.

A possible solution of the AGBVP can be given with application of wavelet base functions (Goswami and Chan, 1999) and cosine base functions (Mallat, 1998). The main advantage of this type of solution is that the compatibility conditions could be established in the space domain using the spatial distribution of the frequency content of the data. Using wavelets, the solution could be found using multiresolution analysis (MRA) and for cosine base functions the advantage of local scaled cosine base functions could be used.

The area under study is characterized by a flat topography. It is expected that in regions with mountains along the coastline or in areas of rougher gravity field in general the smoothness effect will be larger and should be investigated.

REFERENCES

- Eskin G (1981) Boundary value problems for elliptic pseudodifferential equations, Tran. of Mathematical Monographs, 52, American Mathematical Society.
- Goswami, J and Chan A (1999) Fundamentals of Wavelets, - Theory, Algorithms, and Applications, John Wiley&Sons, USA.
- Lehmann R (1999) Boundary-value problems in the complex world of geodetic measurements. J Geod 73: 491-500.
- Mallat S (1998) A wavelet tour of signal processing. Academic press, USA.
- Rektorys K (1977) Variational methods in mathematics, science and engineering, D. Reidel Publishing Company, Dordrecht/ Boston.
- Sacerdote F and Sanso F (1987) Further remarks on the altimetry-gravimetry problems., Bull Geod: 61, 65-82.
- Svensson L (1983) Pseudodifferential operators – New Approach to the boundary problems of physical geodesy, Manuscr Geod: 8, 1-40.
- Svensson S L (1988) Some remarks on the altimetry-gravimetry problem. Manuscr Geod: 13, 63-74.

A NEW TIDAL MODEL FOR THE AEGEAN SEA

S. Spatalas

Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Univ. Box 440, 540 06 Thessaloniki,
Greece
spatalas@eng.auth.gr

ABSTRACT

A new tidal model has been determined for the Aegean Sea based on the analysis of sea level data derived from tide gauge stations located at the surrounding Greek territory and the complex of the Aegean islands. The method used was a linear interpolation technique based on the inverse of the square of the distance of each evaluation point from the tidal station as an appropriate weight. The computation of the amplitude and phase lag of the main tidal constituents M₂, S₂, O₁, and K₁ has been carried out and the relevant maps have been derived. The validity of the model has been tested by correcting data of the GEOSAT and ERS-1 altimetric missions.

1. INTRODUCTION

Ocean tides have a significant effect on the altimeter data, especially in open sea, and tidal models are of main importance in marine geoid modeling. In closed sea areas such as the Mediterranean, global tidal models are not valid, since they fail to represent the reality.

The Aegean sea is a very interesting basin from the tidal point of view, due to its shape, its anomalous bathymetry, and the large number of the islands within it. In this paper, a new tidal model for the Aegean Sea is proposed, concerning the main tidal constituents M₂, S₂, O₁, K₁. Furthermore the corresponding tidal maps are presented. The resulting parameters for diurnal and semidiurnal waves were obtained from the analysis of original tide gauge data. The tidal maps were created by interpolation of the tidal parameters (amplitude and phase lag) and the validation of the model is carried out by correcting altimeter data of the GEOSAT and ERS-1 altimetric missions.

The following sections describe: (a) the data used, (b) the analysis of the tidal data, (c) the tidal model developed, (d) the procedure of the tidal correction of the altimeter data, and (e) the evaluation of the correction through the crossover and collinear analysis of the altimeter data.

2. DATA USED

The data available in the Aegean Sea consist of tide gauge and satellite altimetry ones.

Tide gauge data

Tidal records from 7 floating type tide gauge stations (hourly values) from 1986 until 1987, and 2 digital-type tide gauge stations were analyzed. These floating type tide gauge stations are evenly distributed in the Aegean Sea, belonging to and controlled by the Hellenic Navy Hydrographic Service (H.N.H.S.) (Arabelos and Spatalas 1992, Spatalas 1995). Data have been digitized from analog recordings by H.N.H.S. There are many gaps in these data files and consequently the recording period for some stations is shorter than two years.

Since 1989, two other digital-type tide gauge stations have been installed and maintained by the Department of Geodesy and Surveying, University of Thessaloniki (Spatalas, 1995), also the location of one of them coincides with that of one of the floating-type stations (harbor of Thessaloniki) for comparison purposes, while the second one is installed in Platamon, southern of the city of Katerini. Table 1 summarizes the tidal data used.

Table 1. The actual time interval covered by the available tide gauge data

Station	Latitude (φ)	Longitude(λ)	Years	Days	Readings
Thessaloniki	40.636500	22.917500	2.0	728	17472
Alexandroupolis	40.833333	25.883333	2.0	728	15168
Platamon	39.946000	22.718000	1.1	391	5376
Hios	38.366666	26.150000	2.0	724	14736
Piraeus	37.933333	23.616667	2.0	720	15888
Syros	37.433333	24.950000	2.0	728	14838
Leros	37.170300	26.885100	2.0	728	16752
Souda	35.483330	24.083333	1.1	391	7728

The rms error of these data, according to the data accuracy estimation method by Wenzel (1976), varies from ± 1.8 to ± 3.9 cm.

Satellite altimeter data

GEOSAT DATA

A number of 3843 sub-satellite points from 142 tracks is available for the test area covering the time interval from November 9, 1986 to December 16, 1987. The GEOSAT altimeter data belong to the ERM unclassified operational period of GEOSAT and cover the area $36^{\circ} \leq \varphi \leq 41^{\circ}$, $23^{\circ} \leq \lambda \leq 28^{\circ}$. The contribution of the geopotential model IfE88 was subtracted (Wenzel 1998).

ERS-1 (ERM) DATA

After a validation procedure applied to the ERS-1 (ERM) data in order to detect and remove outliers from the altimeter data (mainly points on land or close to the coasts) a number of 208 sub-satellite points from 4 tracks is available for the test area covering

the time interval from November 26, 1991 to November 29, 1991. The contribution of the OSU 91 geopotential model was subtracted. The mean value and the s.d. of the reduced altimeter heights are 1.24 and 0.75 m, respectively.

3. TIDAL DATA ANALYSIS

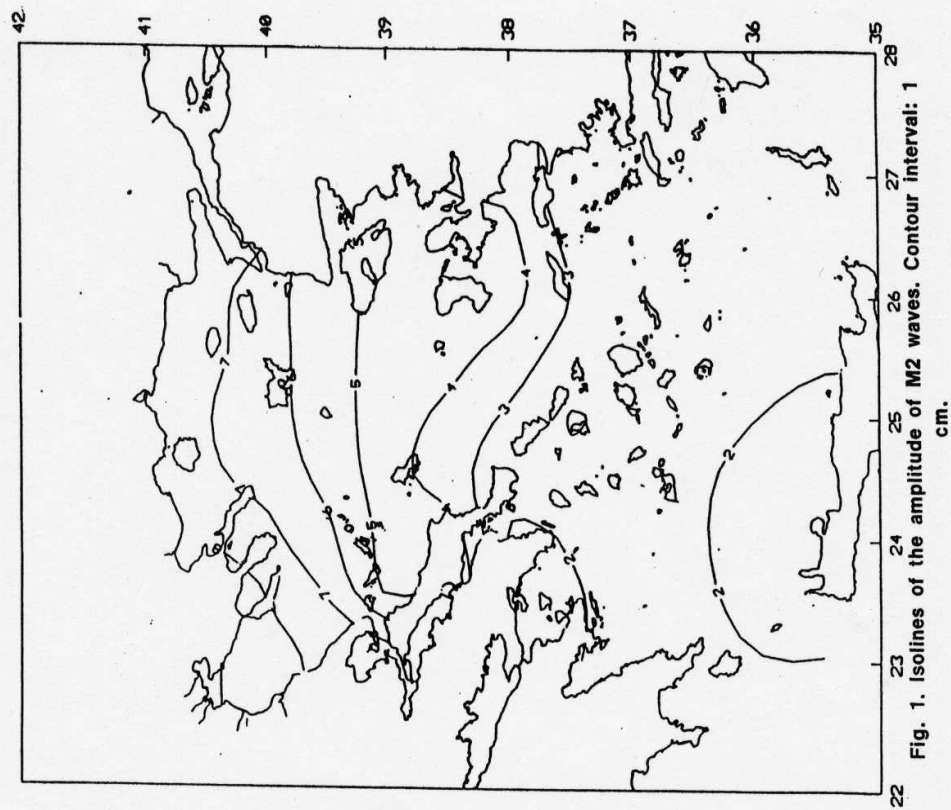
For the analysis of the tidal data, two FORTRAN programs were used: The ETERNA software (H. G. WENZEL, pers. comm. 1990) and the SV software (A. P. Venedikov, pers. comm. 1991). To filter out non-tidal long-period frequencies, two low-pass filters were used: Pertzev's low-pass filter (59 coefficients) in the case of ETERNA and a 48 hour low-pass filter Venedikov (1983), in the case of SV. In both cases, the Cartwright-Tayler-Edden tidal potential development was used (Cartwright and Edden 1973). The poor quality and the limited time interval of the available tide gauge data do not permit the estimation of parameters of the long period waves. For this reason, parameters were computed only for the diurnal and semidiurnal waves. According to the experiments conducted, the results of the data analysis, using the ETERNA and SV software programs, are in a good agreement. The analysis of the data show that the maximum amplitude for the M2 dominant constituent was computed in Thessaloniki (North Aegean) and it was equal to 9 cm, while the minimum value of the amplitude for the M2 constituent was computed in Souda (South Aegean) and it was equal to 1 cm. The s.d. of the estimation of the diurnal and semidiurnal waves was ± 1.38 cm and ± 2.99 cm respectively. The estimation of the accuracy was carried out by a least squares method.

4. THE TIDAL MODEL

Based on the tidal parameters of these 8 tide gauge stations, obtained from the analysis of the tidal data, the tidal model was created by interpolating the parameters at every point of the Aegean Sea and taking into account the narrow tidal effect in this sea area. The method used was a linear interpolation technique based on the inverse of the square of the distance of each evaluation point from the tidal station as an appropriate weight (Fig. 1-8). The maximum value of the tidal correction applied to altimetric data was of the order of 10 cm for the northern region of the Aegean sea, especially for the Thermaikos gulf. The minimum value, at the 2 cm level, is located in the sea area of northern Crete.

Figures 1-8 present contour maps of the amplitude and the corresponding phase difference of M2, S2, O1 and K1 constituents.

Tidal model for the Aegean sea.
M2 (amplitude)



Tidal model for the Aegean sea.
M2 (phase)

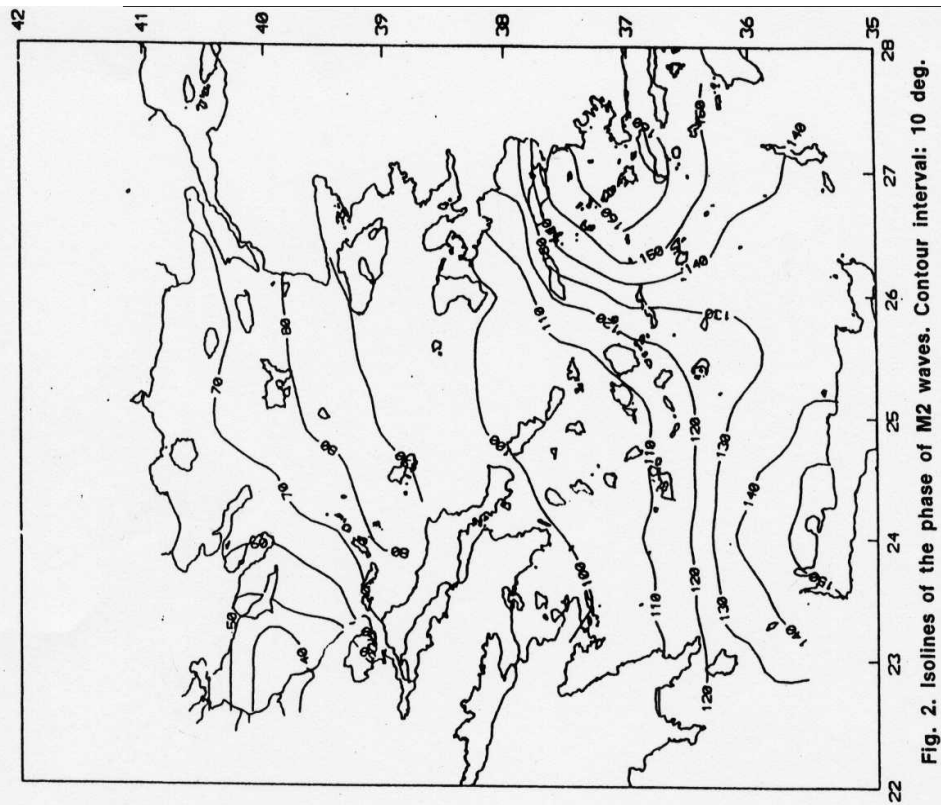


Fig. 2. Isolines of the phase of M2 waves. Contour interval: 10 deg.

Fig. 1. Isolines of the amplitude of M2 waves. Contour interval: 1 cm.

Tidal model for the Aegean sea.
S2 (amplitude)

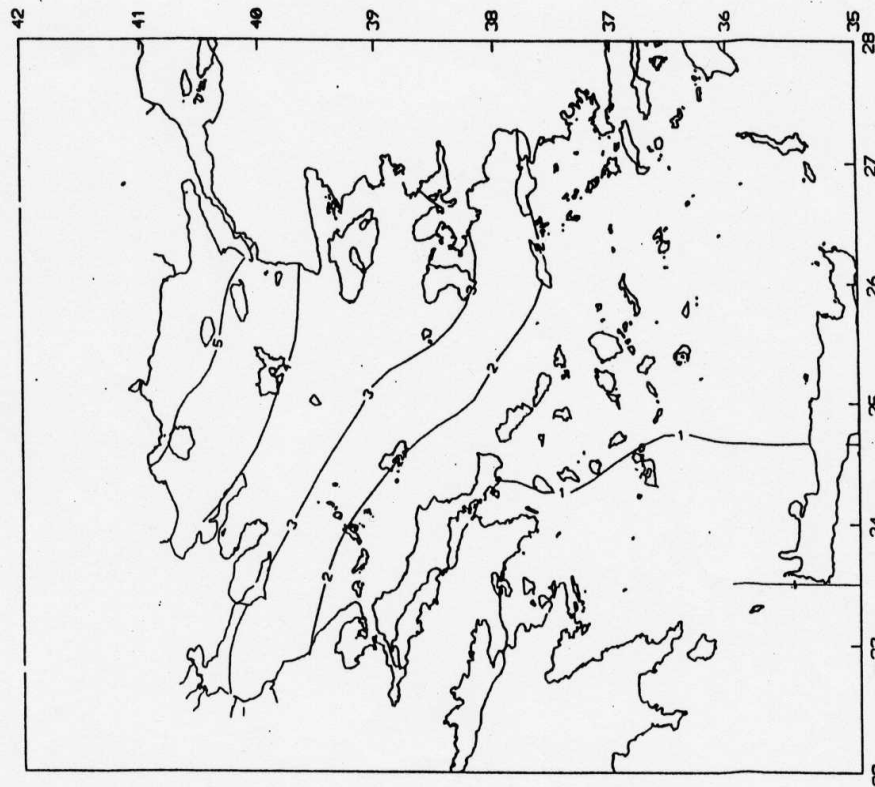


Fig. 3. Isolines of the amplitude of S2 waves. Contour interval: 1 cm.

Tidal model for the Aegean sea.
S2 (phase)

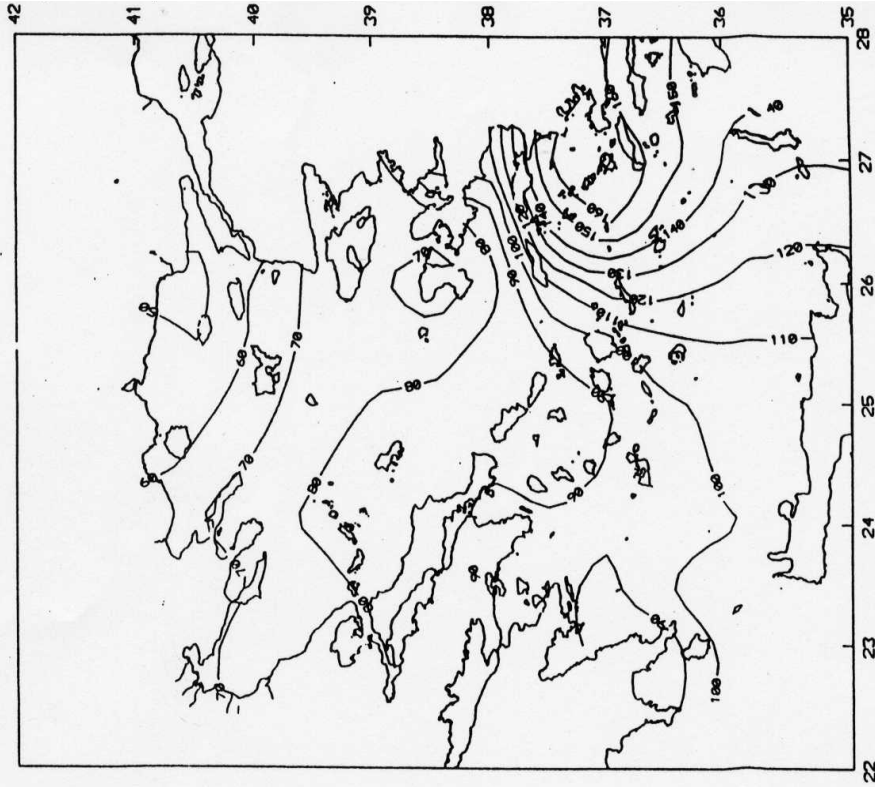


Fig. 4. Isolines of the phase of S2 waves. Contour interval: 10 deg.

Tidal model for the Aegean sea.
O1 (amplitude)

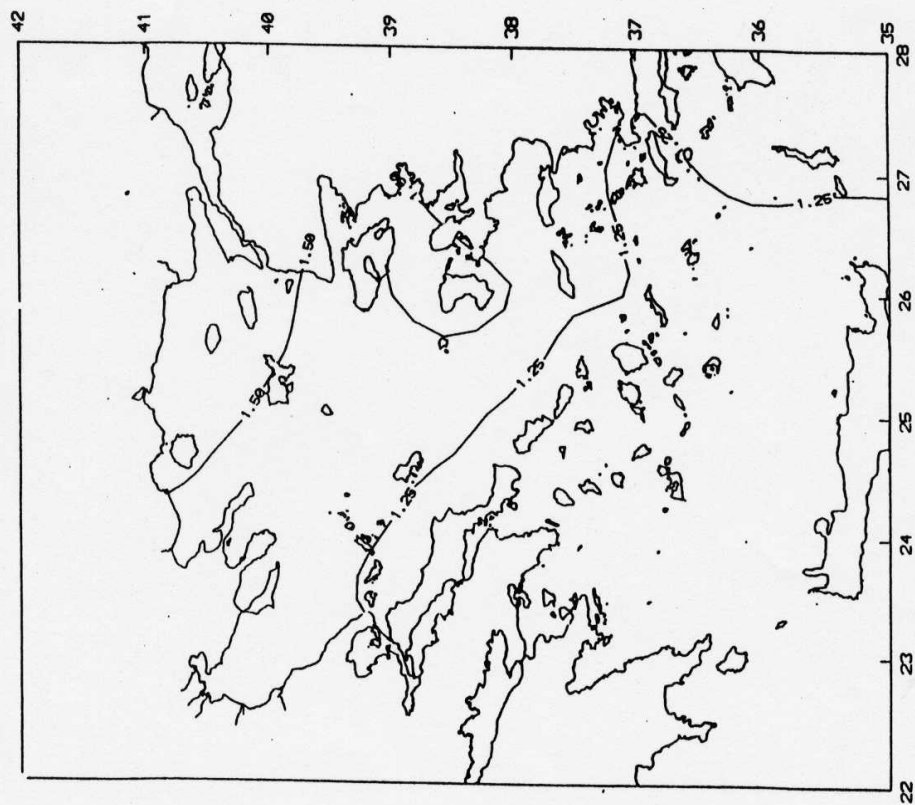


Fig. 5. Isolines of the amplitude of O1 waves. Contour interval: 1 cm.

Tidal model for the Aegean sea.
O1 (phase)

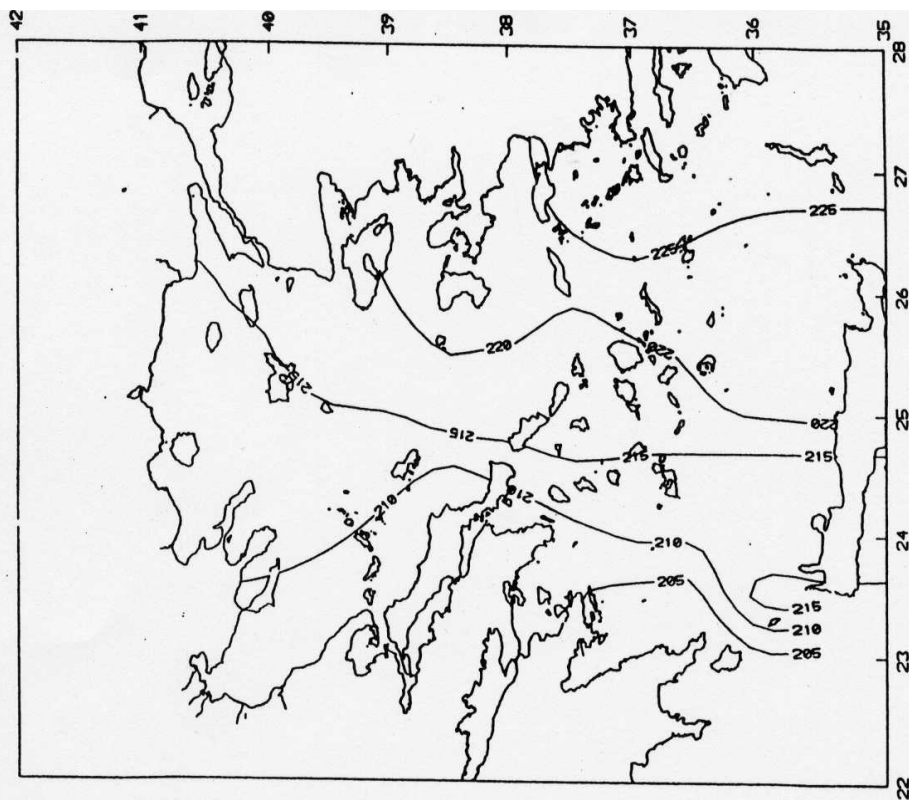


Fig. 6. Isolines of the phase of O1 waves. Contour interval: 10 deg.

Tidal model for the Aegean sea.
K1 (amplitude)

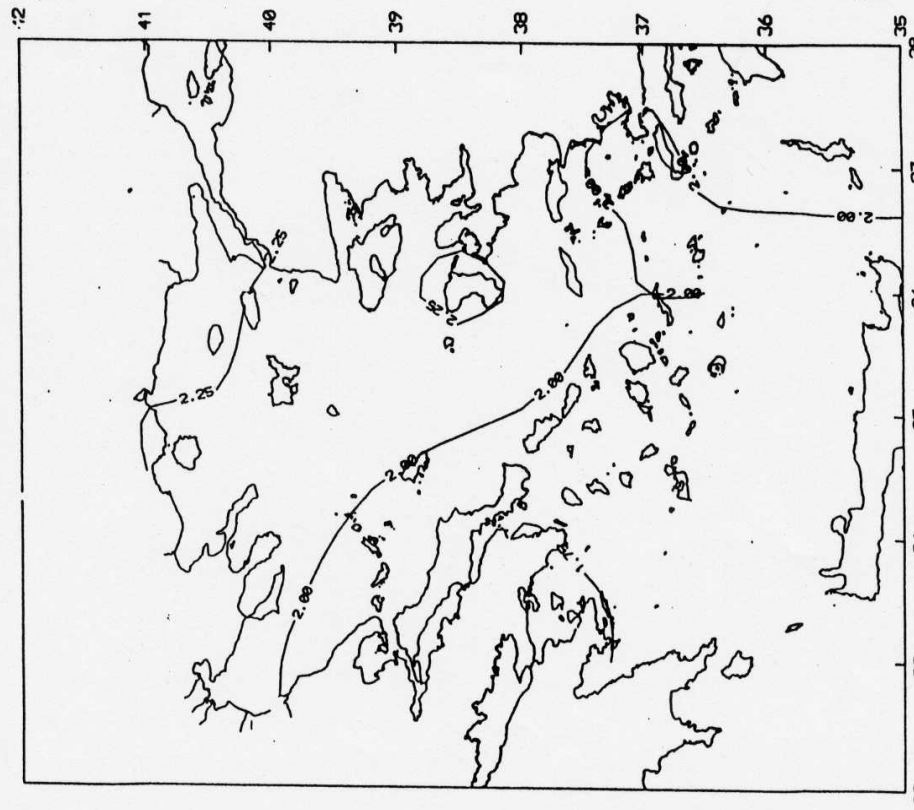


Fig. 7. Isolines of the amplitude of K1 waves. Contour interval: 1 cm.

Tidal model for the Aegean sea.
K1 (phase)

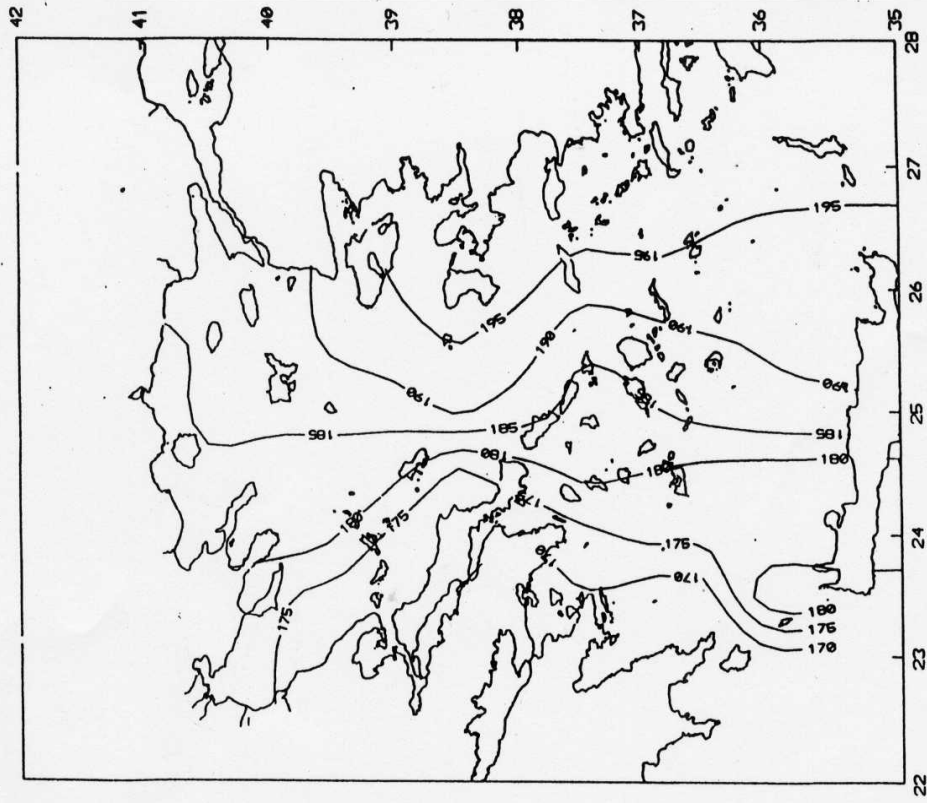


Fig. 8. Isolines of the phase of K1 waves. Contour interval: 10 deg.

5. VALIDATION OF THE TIDAL MODEL

The validity of the model has been tested by correcting altimetric data of the GEOSAT and ERS-1 missions. The altimetric data were studied before and after the tidal correction.

The altimetry data tidal correction, in the case of GEOSAT, was carried out in two ways: (1) By directly using tidal corrections interpolated from the original data and (2) By computing the corresponding corrections from tidal parameters interpolated at the time of each sub-satellite point from the parameters of the 8 tide gauge stations.

Concerning the first case only the data of the floating type stations were used, since the data of the digital type station (Platamon) cover a different time period. The interpolation was performed in two steps. During the first step, tidal corrections were interpolated at the 7 tide gauge stations at the time of each sub-satellite point. Then, these tidal corrections values were computed in the position of each sub-satellite point. The inverse distance weighted mean was used as interpolation method. The mean sea level value computed from the analysis of the data at each station was subtracted from the original tidal recordings.

Following the second way, amplitude and phase values were interpolated in each sub-satellite point using the inverse distance weighted mean algorithm. In each sub-satellite point, simultaneously with the interpolated parameters, tidal corrections were computed employing a FORTRAN program based on the formulas given by Wenzel (1976).

In the case of ERS-1 altimeter data the first method could not be applied due to the different time period covered by the original tide gauge and altimeter data.

The effectiveness of the tidal correction was evaluated by comparing the statistics of the crossover and collinear analysis of the altimeter data before and after the tidal correction.

Crossover analysis of GEOSAT altimeter data (with and without the tidal correction)

A bias crossover adjustment was performed to the altimeter data before and after the application of the tidal correction. The GEOSAT altimeter data cover the area $36^{\circ} \leq \varphi \leq 41^{\circ}$, $23^{\circ} \leq \lambda \leq 28^{\circ}$. The contribution of the geopotential model IfE88 was subtracted. The results of the corresponding crossover analysis are summarized in Table 2.

Table 2. Results of the crossover analysis of the GEOSAT altimeter data in the Aegean Sea ($36^{\circ} \leq \varphi \leq 41^{\circ}$, $23^{\circ} \leq \lambda \leq 28^{\circ}$) before and after the tidal corrections

	Without Tidal correction		With tidal correction (directly)		With tidal correction M2 only		With tidal correction Semi diurnal	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
A	5.510	±2.381	5.509	±2.375	5.517	±2.377	5.513	±2.376
B	-3.760	±4.033	-3.783	±4.026	-3.748	±4.021	-3.748	±4.019
C	-0.009	±0.072	0.000	±0.047	-0.007	±0.063	-0.006	±0.061

A= original observations (GEOSAT-IfE88)

B= crossover differences before the adjustment

C= crossover differences after bias adjustment

These results show that the main improvement in terms of the standard deviation of the bias-adjusted crossover differences, appear in the case of the direct tidal correction (it decreases from 7.2 cm to 4.7 cm). It should be noted, that in the case of the direct correction, other ocean effects (currents, atmospheric pressure etc) beside tides are been taken into account as well (see Arabelos and Spatalas 1992). On the other hand, the main improvement in terms of the standard deviation of the non-adjusted crossover differences appears when the tidal correction is made by taking into account the four semidiurnal constituents (it decreases from 4.033 to 4.019 m). It is also remarkable that the main effect is the contribution of M2 waves since the improvement due to the addition of the three other semidiurnal constituents is of the order of a few mm.

Collinear analysis of GEOSAT altimeter data (with and without tidal correction)

The results of the tidal correction of altimeter data could be assessed also through their collinear analysis. More detailed about the collinear procedure are given by Arabelos and Spatalas (1992) and Arabelos et al. (1994). For the altimeter data described before, the statistics of the collinear analysis are summarized in Table 3. The statistics concern the mean value and the standard deviation of the height differences between collinear tracks before and after tidal correction. There are 71 ascending and 71 descending tracks resulting in four main arcs further on called as A, B, C, and D.

Table 3. Results of the collinear analysis of the GEOSAT (ERM) altimeter data in the Aegean Sea ($36^{\circ} \leq \varphi \leq 41^{\circ}$, $23^{\circ} \leq \lambda \leq 28^{\circ}$) before and after tidal corrections

Arc	Without		With tidal		With tidal		With tidal	
	Tidal correction		correction (directly)		correction M2 only		correction Semi diurnal	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
A	2.53	±0.96	2.52	±0.91	2.53	±0.96	2.53	±0.96
B	7.87	±1.47	7.87	±1.44	7.87	±1.47	7.87	±1.46
C	4.18	±1.13	4.17	±1.13	4.19	±1.13	4.19	±1.13
D	7.77	±1.33	7.78	±1.33	7.78	±1.33	7.78	±1.33

The main improvement in terms of the standard deviation of the height differences between collinear tracks appears in arc A (a decrease equal to 5 cm) in the case of the direct tidal correction. Minor improvements appear in the other cases.

Collinear analysis of ERS-1 altimeter data (with and without tidal correction)

The crossover analysis of the available ERS-1 (ERM) altimeter data was not possible to be carried out due to the lack of intersection points between ascending and descending arcs in the area under consideration. For this reason only a collinear analysis was performed and the results are given in Table 4.

Table 4. Results of the collinear analysis of the ERS-1 (ERM) altimeter data in the Aegean Sea before and after tidal correction

Arc	Without Tidal correction		With tidal correction Semi diurnal	
	Mean (m)	Std. Dev. (m)	Mean (m)	Std. Dev. (m)
A	1.32	± 0.75	1.30	± 0.75
B	1.15	± 0.51	1.13	± 0.49

From the results of Table 4 an improvement in the standard deviation of the discrepancies in arc B is apparent. There is also a 2 cm improvement in terms of the “mean” value for both arcs.

6. CONCLUSIONS

A new tidal model has been determined for the Aegean Sea based on the analysis of sea level data derived from tide gauge stations. The computation of the amplitude and phase lag of the main tidal constituents M2, S2, O1, K1 has been carried out and the relevant maps have been derived. The validity of the model was performed by using GEOSAT (ERM) and ERS-1 (ERM) altimeter data. Altimeter data were analyzed before and after applying tidal corrections in the Aegean Sea. The tidal corrections were computed either by interpolating original tide gauge data reduced to mean sea level or by interpolating tidal parameters derived from the analysis of the original data. Future improvement of the model is expected by using additional tide gauge data.

REFERENCES

- Arabelos, D. and S. D. Spatalas, Tidal effects on satellite altimetry data in a closed sea area, Proceedings of the First Continental Workshop for the Geoid in Europe “Towards a precise Pan-European geoid for the nineties”, Prague, Czechoslovakia, May 11-14, pp. 214-223, 1992.
- Arabelos, D., S. D. Spatalas, I. N. Tziavos, M. J. Sevilla, C. de Toro and R. Vieira, Tidal corrections of the ERS-1 satellite altimeter data in the Mediterranean Sea. MARE NOSTRUM, GEOMED Rep. 4, (eds. Arabelos and Tziavos), pp. 149-170, Thessaloniki, March 21-22, 1994.
- Cartwright, D.E. and A.C. Edden, Corrected tables of tidal harmonics. Geoph. J. R. Astr. Soc. 33, 253-264, 1973.
- Spatalas, S.D., The effect of different instrumental errors and the sea-tides on astrogeodetic observations. PhD Thesis, Thessaloniki 1995 (in Greek).
- Venedikov, A.P.: Information on a program for Earth tide data processing. Bull. d’ Inform. Marees Terrestres. Bruxelles, No 90, 1983, 5903-5920.

Wenzel, H.-G.: Zur Genauigkeit von gravimetrischen Erdgezeitenbeobachtungen. Wiss. Arb. der Lehrstühle für Geodäsie, Photogrammetrie und Kartographie an der technischen Universität Hannover, Nr. 67, Hannover 1976.

Wenzel, H.-G., Ultra hochauflösende Kugelfunktionsmodelle GPM98A und GPM98B des Erdschwerefeldes. In: W. Freden (ed); Progress in Geodetic Science. Proceedings Geodätische Woche 1998, October 12-17, 1998, Kaiserslautern. Shaker Verlag.