

AUSGEOID93

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Background

In 1991 AUSLIG released a grid of geoid ellipsoid separation values (N values) for the Australian region (Kearsley & Govind, 1991). These N values were in terms of WGS84 and were computed on a 10 minute grid (about 20 km) using

- The OSU89A global geopotential coefficients, produced by Professor Richard Rapp, Ohio State University, USA.
- The 1980 Australian Gravity data base from the Australian Geological Survey Organisation.
- Techniques and software developed by Dr A.H.W. Kearsley, Associate Professor University of NSW (Holloway, 1988). This software was converted and enhanced by AUSLIG, to run on a Personal Computer.

This grid of N values was known as AUSGEOID91 and has been used widely by both Government and private sectors.

AUSGEOID93

AUSLIG has now produced an updated Australian geoid known as AUSGEOID93. This new grid is the result of new information and improved software.

- The OSU91 global geopotential model is used.
- The limits of the grid have been extended in some areas.
- The process used to produce the N values has been refined to:
 1. Provide deflections of the vertical as well as N values.
 2. Improve the format of the output files
 3. Eliminate duplication of points
 4. Improve the handling of large data sets
 5. Handle situations where the gravity data is sparse or non-existent

Quality Control

AUSGEOID93 was computed in large blocks which were subsequently merged, prior to separating the data into individual 1:250,000 and 1:1,000,000 topographic map areas.

As each area was computed, a three dimensional surface plot and a contour diagram of the grid was produced to visually check for unexpected jumps or trends. Figures 1 and 2 show these diagrams for the combined data set. In addition a utility program was used to report on the maximum, minimum and average values of:

- The short-wavelength contribution to the N value
- The change in the short-wavelength contribution between each 'ring' computation.

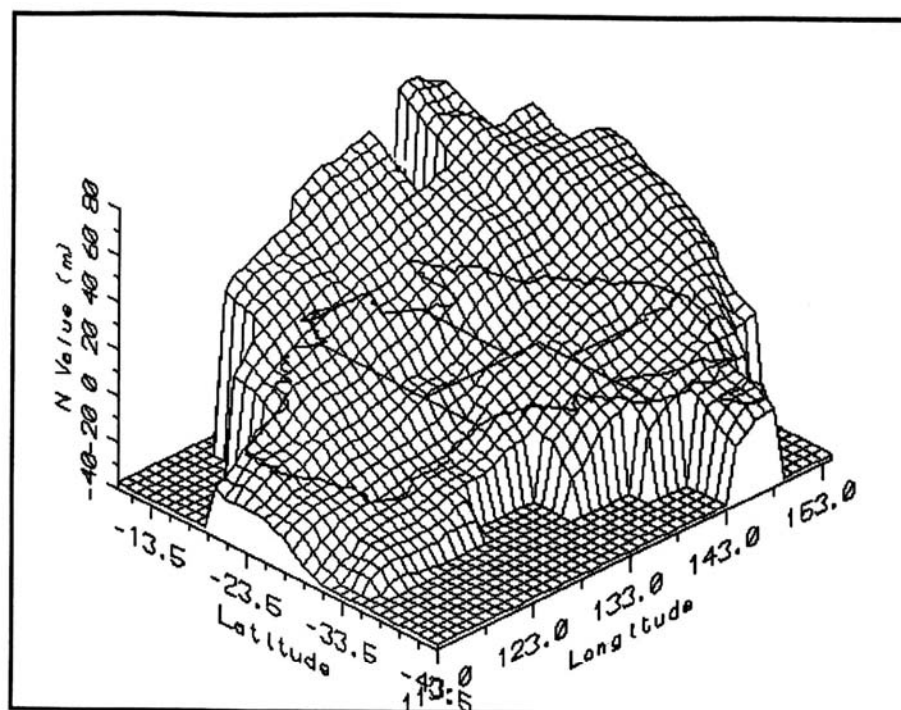


Figure 1 - AUSGEIOD93 surface plot

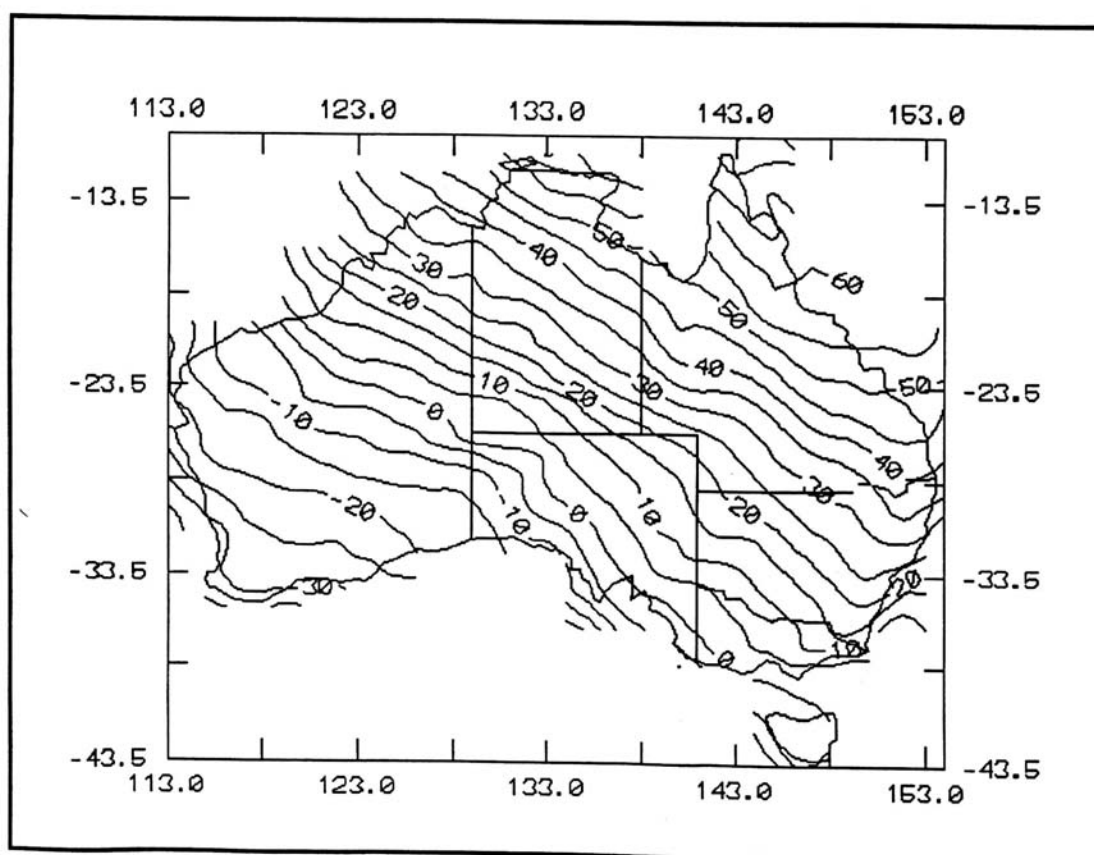


Figure 2 - AUSGEIOD93 contour diagram

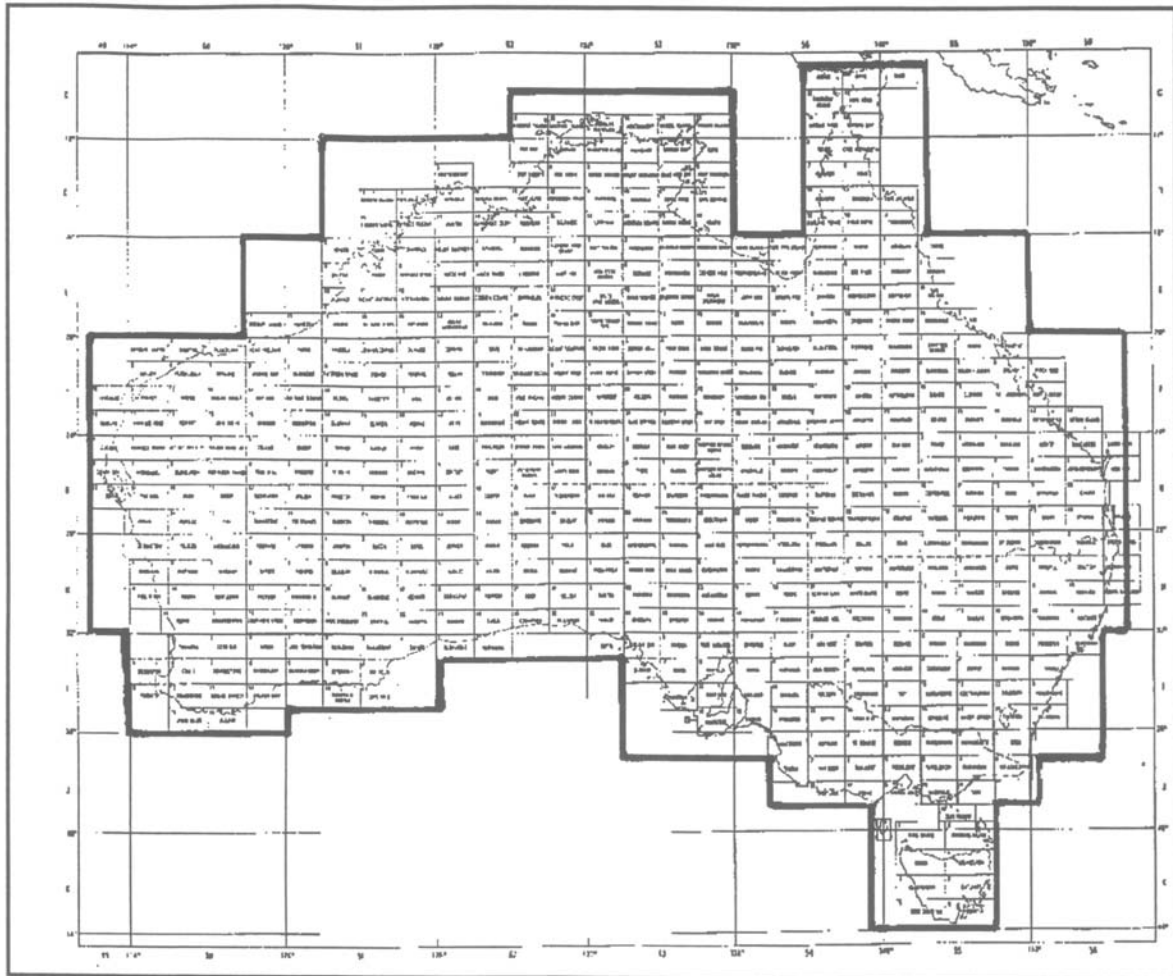


Figure 3 - AUSGEIOD93 coverage

AUSGEIOD93 versus AUSGEIOD91

To determine if AUSGEIOD93 really was an improvement over AUSGEIOD91 comparisons were made between the two data sets in terms of their absolute values and their slope.

Absolute N Value Comparison

Seven of the Australian Fiducial Network sites have absolute point positions in terms of WGS84, produced by DMA's GASP software (Malys and Jensen, 1989). The ellipsoidal heights from these solutions were reduced to geoidal values by subtracting an N value interpolated firstly from the AUSGEIOD93 grid and subsequently from the AUSGEIOD91 grid. The resulting geoidal heights for each site were compared with each other and with the known Australian Height Datum (AHD) value. Figure 4 shows that with one minor exception AUSGEIOD93 gave a consistently better comparison with AHD. It is also interesting that the derived AHD values are within $\frac{1}{4}$ metre of the known AHD. This confirms comparisons obtained by Cudlip et al (1993), who found that the derived AHD values agreed to about 0.3 m of the known AHD values at three bench marks in the Simpson Desert.

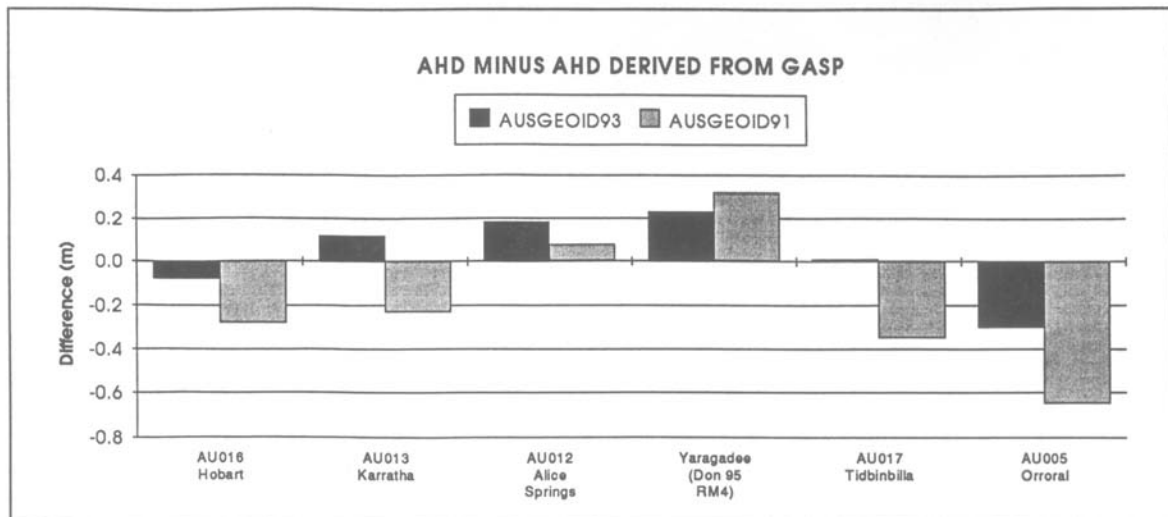


Figure 4 - Comparison of absolute N values

Difference in N Value Comparison

A selection of spirit levelled GPS baselines were used for this comparison. The baselines were located in Western Australia and New South Wales and used a variety of proprietary software packages for the reduction. These baselines were chosen because the data was readily accessible. Additional and possibly more accurate baselines may have given better comparisons with AHD, but the samples shown were sufficient to test the AUSGEIOD data.

For each baseline the difference in WGS84 ellipsoidal height was reduced to a difference in geoidal height by subtracting the difference in N value interpolated firstly from the AUSGEIOD93 grid, and subsequently from the AUSGEIOD91 grid. Figure 5 shows that once again AUSGEIOD93 gave consistently better results.

Although not intended as a test of GPS heighting, these results seem to confirm the claim that GPS can be used to obtain difference in AHD height with an accuracy of 2 to 5 parts per million (Kearsley 1988). It must be remembered that the comparisons in figure 5 include not only the uncertainty in the N value, but also the GPS result and the AHD heights used.

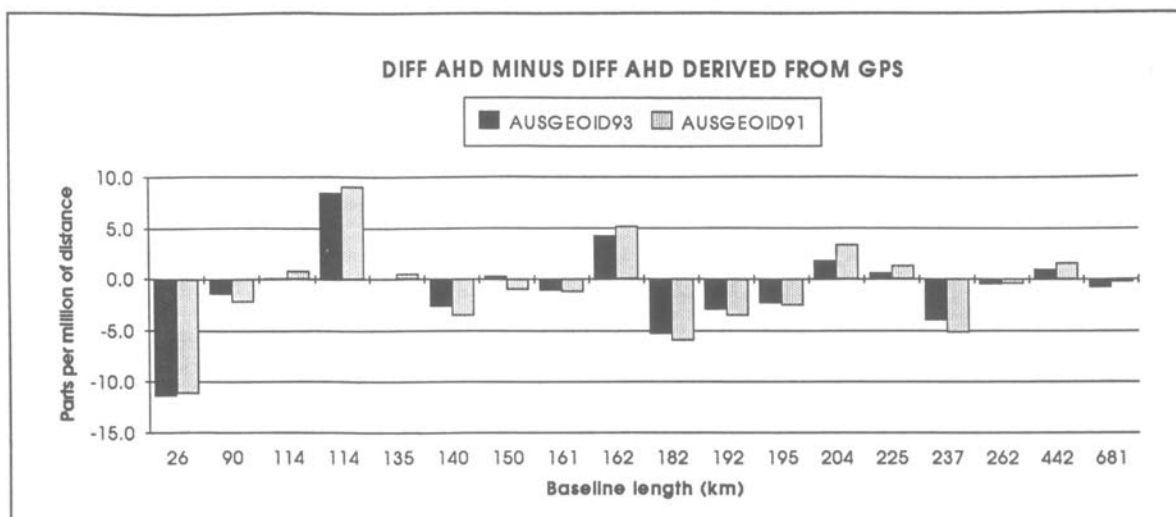


Figure 5 - Comparison of relative N values

Interpolation of N Values

N values may be interpolated for particular locations using AUSLIG's Windows™ interpolation software WINTER. This software will interpolate N values and deflections of the vertical from the AUSGEOD93 grid, in either interactive or batch mode, using either bilinear or bicubic interpolation.

Geoid record indicator									
/		WGS84 N Value (metres)							
		Latitude (deg, min & seconds)							
		Longitude (deg, min & secs)							
GEO	51.29 S10 10	0.000 E129	0	0.000	2.39	-2.60			
GEO	51.00 S10 0	0.000 E129	0	0.000	5.49	-3.06			
Deflection of the vertical in the meridian (secs)									
where the deflection is defined as									
'astro' latitude minus 'geodetic' latitude									
(positive north, negative south)									
Deflection of the vertical in the prime vertical (secs)									
where the deflection is defined as									
('astro' longitude minus 'geodetic' longitude)*Cos(latitude)									
(positive east, negative west)									

Figure 6 - AUSGEOD93 data format

Summary

AUSGEOID93 supersedes the earlier AUSGEOI91 model. The data is available as ASCII files covering standard National Topographic map areas (1:250,000 or 1:1,000,000), with a 10' overlap on each side. These files may be downloaded from AUSLIG's bulletin board or for large areas can be supplied on floppy disk.

AUSLIG will continue to take advantage of current research and improvements in data to produce upgraded geoid models for Australia.

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THE GEOID IN THE CENTRAL MEDITERRANEAN AREA: DETERMINATION AND COMPARISONS

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1 - Introduction

The work presented is the Italian group contribution to the GEOMED Project, which is a project sponsored by the European Community, aiming at the determination of a gravimetric geoid on the Mediterranean area and of the sea surface topography on sea (by using radaraltimetric measurements).

The computation of the geoid in the area has been splitted in three parts roughly corresponding to the Western, Central and Eastern Mediterranean basins.

The Western geoid has been calculated last year and presented in R.Barzaghi et al. (1992); the present paper refers to the description of the computation of the Central geoid ($30^{\circ} \leq \phi \leq 47^{\circ}$; $8^{\circ} \leq \lambda \leq 20^{\circ}$) and to the comparisons between this and other geoids previously computed by different research groups.

2 - Data preparation and computation of the geoid

The basic data base to compute a gravimetric geoid are the Free-Air anomalies in the area and a digital elevation model necessary to smooth the gravity field.

Unfortunately the global digital elevation models we have at the moment present in the area of interest a lot of discrepancies with the higher resolution local models available in the frame of the different national

geodetic data banks (see for instance W.Fürst et al. (1993) and D.Arabelos (1993)).

Thus a new model has been prepared in the Central Mediterranean area starting from the Italian digital elevation model (including some bathymetry along the coasts) with resolution $7.5'' \times 10''$; the bathymetry of the Mediterranean sea digitized by the University of Thessaloniki from Morelli's map and gridded $6' \times 10'$; BGI point values stations in Africa and Corsica; the global model ETOPO5U with resolution $5' \times 5'$. The method used to obtain the new model is described in R.Barzaghi et al. (1993).

Once the more accurate digital elevation model $5' \times 5'$ was available we applied the remove-restore procedure to the Free-Air gravity anomalies in the area.

First of all the gravity data file Δg_{FA} , coming from different sources (an Italian gravity file of 11831 data, almost regularly distributed; 1197 BGI data in Corsica, Algeria, Libya and Tunisia; 25369 gravity data provided by DMA; the Morelli's gravity maps digitized by the University of Thessaloniki and by the Polytechnic of Milan, for a total of 23703 data in the sea) has been homogenized and both the contribution of the global geopotential model OSU91A Δg_{OSU91A} and of the residual terrain correction Δg_{RTM} (computed from the digital elevation model previously mentioned) have been removed.

To obtain the residual geoid we apply to the reduced gravity anomalies the method known as the Fast Collocation, which consists of a computation of the exact collocation formula by means of a FFT algorithm (see G.P.Bottoni and R.Barzaghi, 1993). The method requires that the residual gravity data are gridded on a regular "rectangular" (in geographic coordinates) area.

Because of the lack of data in the whole North-Eastern triangle (Albania, Yugoslavia and part of Hungary) we filled in the area of lacking residual gravity anomalies with zeros. As a consequence of this approximation is important to remark that the geoid computed in the Adriatic sea is particularly weak.

The last step of the computation consists of the restore procedure, i.e. of adding back to the residual geoid N_R the contributions, in terms of geoid undulation, due to the global model N_{OSU91A} and to the residual terrain model N_{RTM} .

Tab. 1 contains the statistics of the various steps of the computation and

the Fig. 1 shows the pattern of the geoid computed on the prediction grid $30^{\circ} \leq \phi \leq 47^{\circ}$; $8^{\circ} \leq \lambda \leq 20^{\circ}$ with resolution $5' \times 5'$.

	n	E	rms	M	m
Δg_{FA} (mgal)	62100	16.08	45.63	270.17	-163.60
$\Delta g_{FA} - \Delta g_{OSU91A}$ (mgal)	62100	0.14	27.68	225.70	-224.35
$\Delta g_{FA} - \Delta g_{OSU91A} - \Delta g_{RTM}$ (mgal)	62100	-1.11	27.02	183.14	-260.66
N_R (m)	29725	-0.13	1.15	2.44	-4.83
$N_R + N_{RTM}$ (m)	29725	-0.11	1.12	3.46	-4.73
$N_R + N_{RTM} + N_{OSU91A}$ (m)	29725	38.82	7.03	50.66	21.58

Table 1 - Main statistics obtained in the computation of geoid (n = number of data, E = average, M = maximum value, m = minimum value)

3 - Comparisons

In order to check our geoid we have performed some comparisons with other geoids computed in the framework of different projects.

First of all because of the overlapping between the geoids computed by the GEOMED project in the Western and in the Central part of the Mediterranean, we have tested their homogeneity.

The overlapping area has bounds $35^{\circ} \leq \phi \leq 43^{\circ}$; $8^{\circ} \leq \lambda \leq 10^{\circ}$ and contains 2425

knots where we have computed the two geoids. In the Tab. 2 the statistics of the two geoids and their differences are shown.

	n	E (m)	rms (m)
WESTERN GEOID	2425	45.280	2.803
CENTRAL GEOID	2425	44.980	2.735
DIFFERENCES	2425	0.300	0.428

Tab.2 - Statistics for the Western and Central geoids in the overlapping area

Although the average and the rms of the differences are not so small, looking at Fig.2, where the pattern of the differences is represented, we may observe that the higher values are concentrated on land.

This consideration comes out also if we restrict the comparison test zone in sub-areas including only sea or different parts of land (Tab. 3).

	BOUNDS	n	E (m)	rms (m)
SEA	$37^{\circ} \leq \phi \leq 39^{\circ}; 8^{\circ} \leq \lambda \leq 10^{\circ}$	576	0.005	0.23
TUNISIA	$35^{\circ} \leq \phi \leq 37^{\circ}; 8^{\circ} \leq \lambda \leq 10^{\circ}$	576	0.500	0.44
SARDINIA	$39^{\circ} \leq \phi \leq 41^{\circ}; 8^{\circ} \leq \lambda \leq 10^{\circ}$	576	0.180	0.36
CORSICA	$41^{\circ} \leq \phi \leq 43^{\circ}; 8^{\circ} \leq \lambda \leq 10^{\circ}$	576	0.490	0.43

Tab. 3 - Statistics of the Western and the Central geoid differences for overlapping subareas.

As we can see the differences indicate that, although the comparison test area is close to the boundaries of the zones in which the geoids have been computed and for this reason the solutions obtained are weaker, anyway they show a good agreement on sea.

The second comparison we have done is between our geoid and the one computed

by J.P.Barriot (1987) in the Mediterranean.

The same comparison has been performed in the Western area and has been presented in M.A.Brovelli (1993).

First of all we recall briefly the main characteristics of the geoid computed by J.P.Barriot; the gravimetric data base is built up starting from data coming from different sources: a data base of mean values $6' \times 10'$ in Europe provided by W. Torge, point values in Africa provided in by BGI, gridded values $6' \times 10'$ obtained by inversion of radaraltimetric data (Seasat) on sea and on the Atlantic ocean. All these values have been interpolated on a regular grid $6' \times 10'$. The global geopotential model adopted is the OSU81 and the method used for the computation is the Stokes-Pizzetti with a cap of 6° , without any topographic correction.

In order to compare this geoid with that of GEOMED project we have gridded it on a regular grid $5' \times 5'$ by bicubic spline functions.

The main statistics for the two geoids and for the differences between them are shown in Tab. 4 and the features of the differences are plotted in Fig. 3.

	n	E (m)	rms (m)	M (m)	m (m)
N_{JPB}	29725	39.10	7.60	52.55	19.66
N_{GEOMED}	29725	38.82	7.03	50.66	21.58
$N_{JPB} - N_{GEOMED}$	29725	-0.28	1.08	4.10	-4.15

Tab. 4 - Statistics in the Central Mediterranean for the geoid provided by J.P.Barriot (N_{JPB}) and the geoid computed by the Geomed group (N_{GEOMED})

As we can see from the figure, the main differences are located on mountainous areas and in Corsica. As for Corsica the high differences are due to the fact that in the computation of the geoid done by J.P.Barriot the gravimetric data of this area have not been included.

AS for the other mountainous regions like the Alpi, the Appenini etc., we think that the large discrepancies are due to topographic corrections.

The last remark is not surprising if we observe, for instance looking at the Geomed geoid, that in the reduced gravity data a big noise is still present (compare the statistics of $\Delta g_{FA} - \Delta g_{OSU91A}$ and of $\Delta g_{FA} - \Delta g_{OSU91A} - \Delta g_{RTM}$ in Tab. 1) and as a consequence the Residual Terrain correction could not represent the real high frequency terrain contribution due to the smoothing implied by the 5' x 5' averaging of topographic heights.

By the way we realize that on sea a reasonably good agreement between the two geoids has been reached and this is a comforting result, as we are mainly interested on marine geoid. Referring to the land geoid the next step will be its recomputation by using block mean values of Δg to reduce the topographic noise, due to the very low resolution of the height information.

In any case, in order to check the computed geoid, we have compared it also with Italgeo 90, the Italian gravimetric geoid (B.Benciolini et al, 1991) shown in Fig. 4.

The database used in the Italgeo90 geoid consists of 23193 gravimetric observations almost gridded 3' x 3' provided by the Institute of Physics of the University of Lecce (M.T.Carrozzo et al., 1981a; E.Cassano, 1983). These data have been previously reduced by a global geopotential model by using the IFE88E2 computed by the University of Hannover. The residual terrain correction applied to the data has been done employing as high resolution terrain the national digital elevation model 7.5" x 10", built up by the Institute of Physics of the University of Lecce (M.T.Carrozzo et al, 1981b; M.T.Carrozzo et al.,1982) and as reference terrain the global digital elevation model computed by the Technical University of Graz TUG87. The residual geoid has been calculated by zone collocation applied on areas of 2° x 3°.

On the points where Italgeo 90 has been computed we have predicted by bicubic splines interpolation the Geomed geoid.

Comparing the two geoids, as it is clear also looking at Fig. 1 and Fig. 4, we observe that the general features seem to be quite similar, although the Geomed one is much smoother. The statistics of the computed geoidal heights and of the differences (Tab. 5) show that not only a bias is present (due probably to the different geopotential model and digital elevation model adopted), but also the rms of the differences is significantly high. As discussed previously we attribute mainly this effect to the poor residual terrain correction applied.

	n	E (m)	rms (m)
$N_{\text{Italgeo90}}$	4170	44.69	4.90
N_{GEOMED}	4170	43.48	4.23
$N_{\text{Italgeo90}} - N_{\text{GEOMED}}$	4170	1.21	1.61

Tab. 5 - Statistics for Italgeo90 and GEOMED geoid in Italy

The last comparison we have done is between our geoid and the geoidal heights observed in the points of the Italian GPS traverse. This one (Fig. 5) is the last part of the European traverse which runs from Trömsö in Norway to Noto in Sicily; in Italy the extension of the traverse is of 1700 km and the points are about 30 km far away one from the other. The GPS measurements have been done in differential mode by using in part single and in part double frequencies receivers (G.Birardi, 1991).

In order to compare the values obtained from the GPS campaign with the GEOMED undulations, we have interpolated by bicubic splines the geoidal heights from the knots of our 5'x5' computation grid to the points belonging to the traverse. The same interpolation has been done also for the Italian geoid Italgeo90.

Fig. 6 shows the pattern of the three geoids (GPS, GEOMED and Italgeo90) along the points of the traverse. The inhomogeneous behaviour of the three curves in different part of the traverse suggests to distinguish different zones.

In the Northern part of the traverse, corresponding to the first points, the high differences between the GPS and GEOMED geoidal heights are due both to the poor topographic correction applied in the computation of the geoid and to the fact that these points are very close to the boundaries of the area where we have data.

In fact, looking at Tab. 6, where the main statistics of the differences between GEOMED and GPS and between Italgeo90 and GPS values are indicated, we may observe that for the first zone, from point 2 to point 8, both a high bias and rms in the differences $N_{\text{GEOMED}} - N_{\text{GPS}}$ are present. For the same points the agreement between the Italgeo90 and the GPS undulations are significantly better.

points of the traverse	$N_{\text{Italgeo90}} - N_{\text{GPS}}$		$N_{\text{GEOMED}} - N_{\text{GPS}}$	
	E (m)	rms (m)	E (m)	rms (m)
2 - 8	-0.814	0.117	-4.427	0.801
9 - 35	-0.340	0.297	-2.096	0.252
36 - 47	-0.297	0.487	-0.822	0.582
49 - 63	0.966	0.540	-0.731	0.569

Tab. 6 - Statistics for the differences between Italgeo90 and GPS geoid and GEOMED and GPS geoid in the points of the Italian branch of the European GPS traverse.

In the second zone, from the point 9 to the point 35, a higher bias is still present in the differences $N_{\text{GEOMED}} - N_{\text{GPS}}$, but the rms is very small, even lower to the one of the differences $N_{\text{Italgeo90}} - N_{\text{GPS}}$. This is due to the fact that the zone considered is flat and in this case the topographic noise does not affect the solution.

The third zone, from point 36 to point 47, shows an rms a little higher for the differences $N_{\text{GEOMED}} - N_{\text{GPS}}$. This can be attributed to the presence of mountains in the area (Appennini).

Finally in the last part of the traverse there are no significant differences

in the rmss.

Referring to the GEOMED undulations the quite high rms found is due, as usual, to the topographic noise; for the Italian geoid the responsible of disagreement from the GPS heights is due in this case to the digital elevation model used in the computation of the geoid (TUG87) which, particularly in the Southern part of Italy, large distortions.

As conclusion we may point out that the main problem both for the Italgeo90 and the GEOMED geoid is related to the topographic correction and to the digital elevation model adopted. Because of the mountainousness of Italy these aspects should be more deepened in the next computations of the geoid.

4 - Conclusions

The comparisons between the computed GEOMED geoid and other solutions obtained by using other methods show that on land both the lack of data in the North-East triangle (ex-Yugoslavia) and the coarseness of data on land affect the solution. In order to improve it the next step will be to recompute the geoid by using block mean values of Δg in order to reduce the topographic noise, due to the very low resolution of the height information. In this way we can make the resolution of on solution poor but noiseless on land so as to avoid propagation of significant errors into the marine area. By the way the main goal of the Geomed project being to compute the geoid on sea this has been reached with a reasonable accuracy which can be roughly estimated as ± 20 cm, as we had been the mentioned comparisons. Another indirect test on the effectiveness of the method on sea is to calculate the correlation coefficient between the radaraltimetric adjusted surface obtained from SEASAT, GEOSAT (ERM 1-22), ERS1 (ERM 1-11) minus the global model OSU91A and the residual geoid $N_R + N_{RTM}$ in the points corresponding to satellite measurements.

The value we got ($\rho = 74$ %) shows that the two surfaces carry a strong common signal and seems to indicate that at least on sea our solution provides some meaningful results. The fact that ρ is not higher is also positive because we must also take into account the effect of SST which provided a signal of 40 cm r.m.s. against the around 72 cm of the signal $N_R + N_{RTM}$.

Figure captions

- Fig. 1 - Contour lines of the total geoid ($e = 1$ m)
- Fig. 2 - Contour lines of the differences between the Western and the Central Geomed geoids in the overlapping area ($e = 0.2$ m)
- Fig. 3 - Contour lines of the differences between the geoid provided by J.P.Barriot and the GEOMED geoid ($e = 0.5$ m)
- Fig. 4 - Contour lines of Italgeo90 geoid ($e = 0.25$ m)
- Fig. 5 - The Italian branch of the European GPS traverse
- Fig. 6 - Comparison between the geoidal heights GPS, GEOMED and Italgeo90 along the Italian branch of the European GPS traverse.

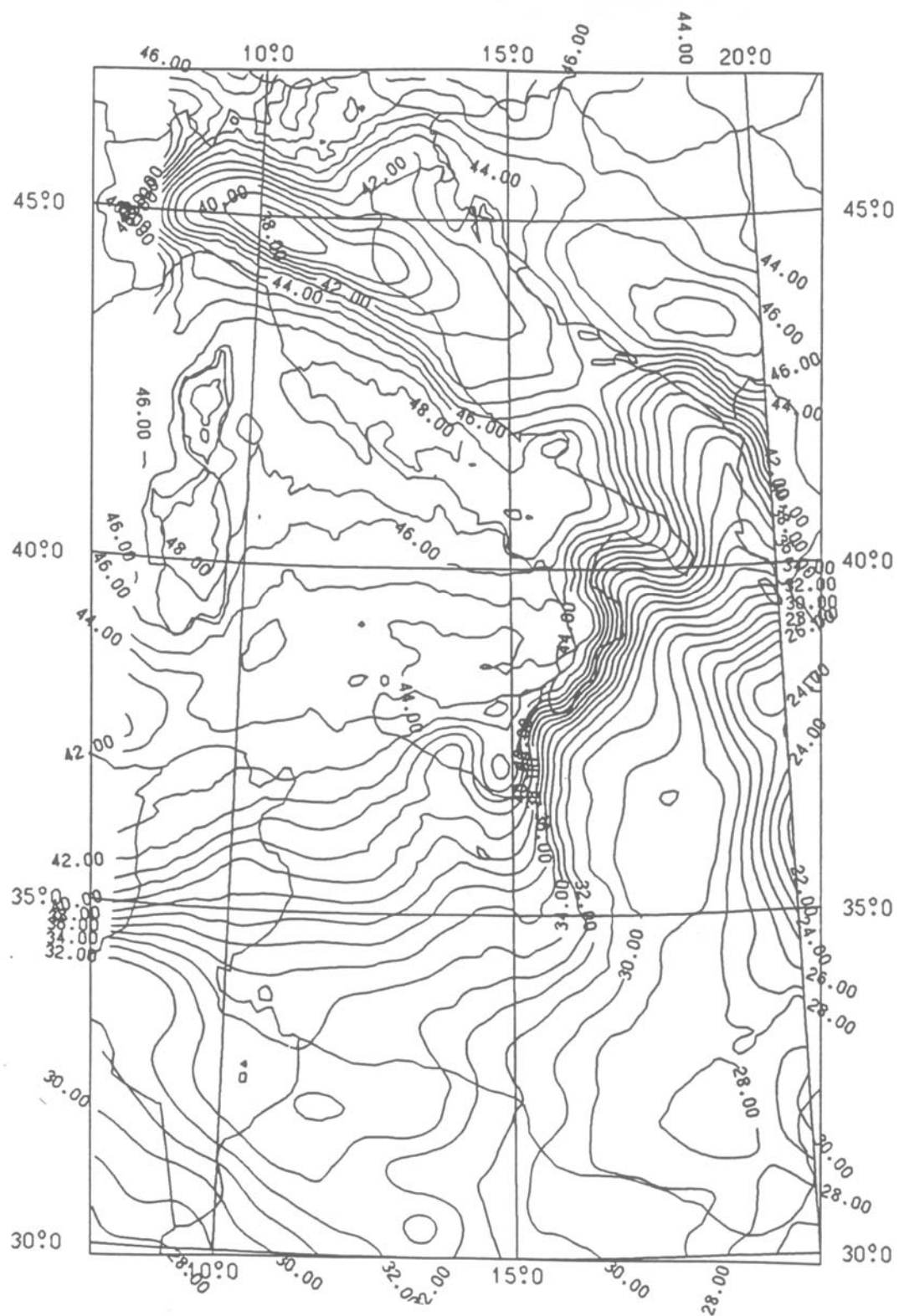


Fig. 1

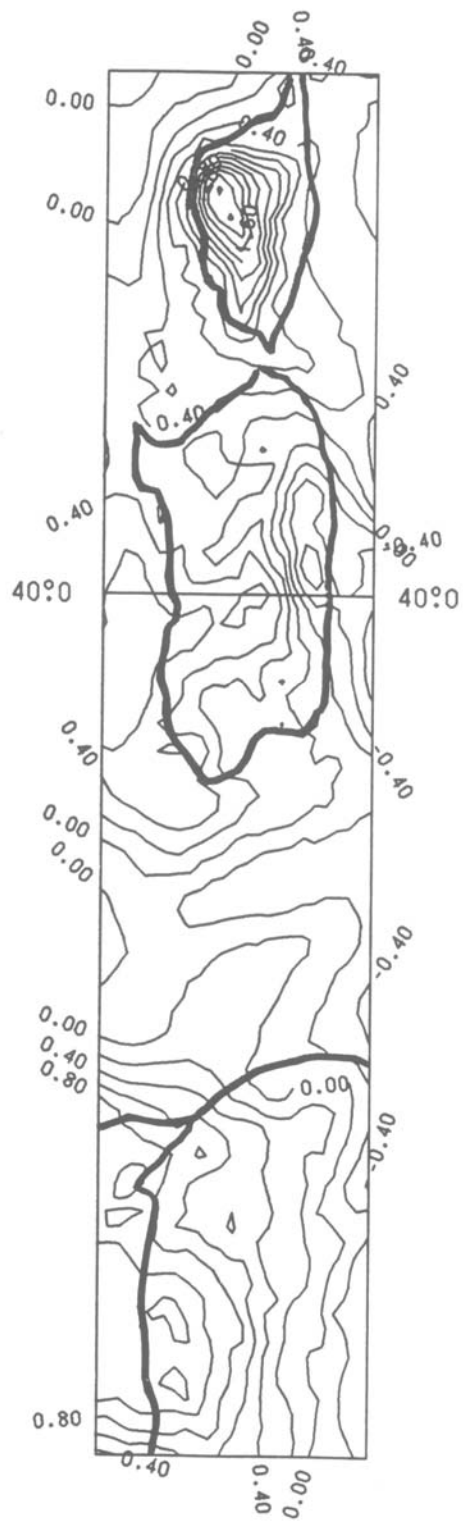


Fig. 2

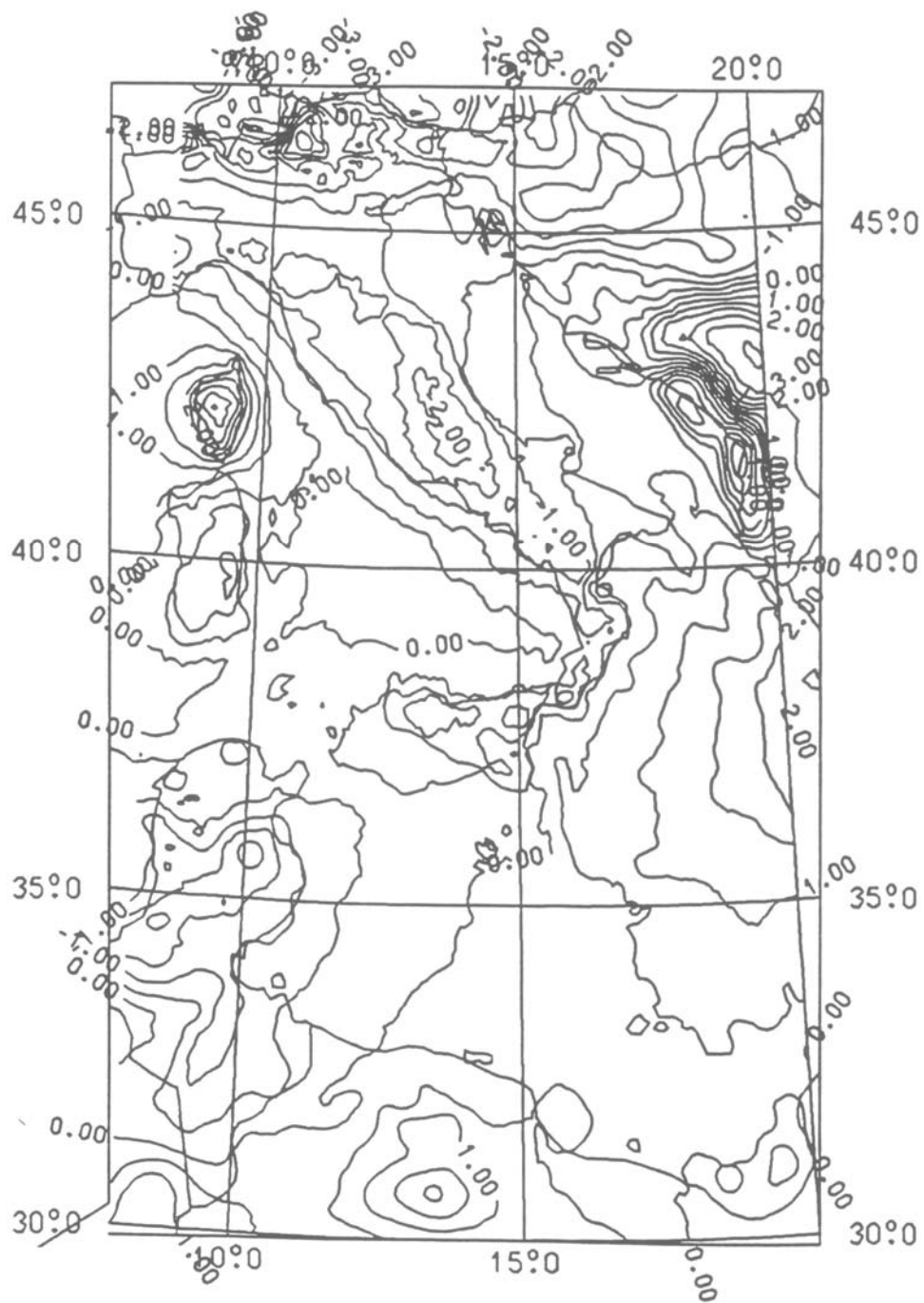


Fig. 3

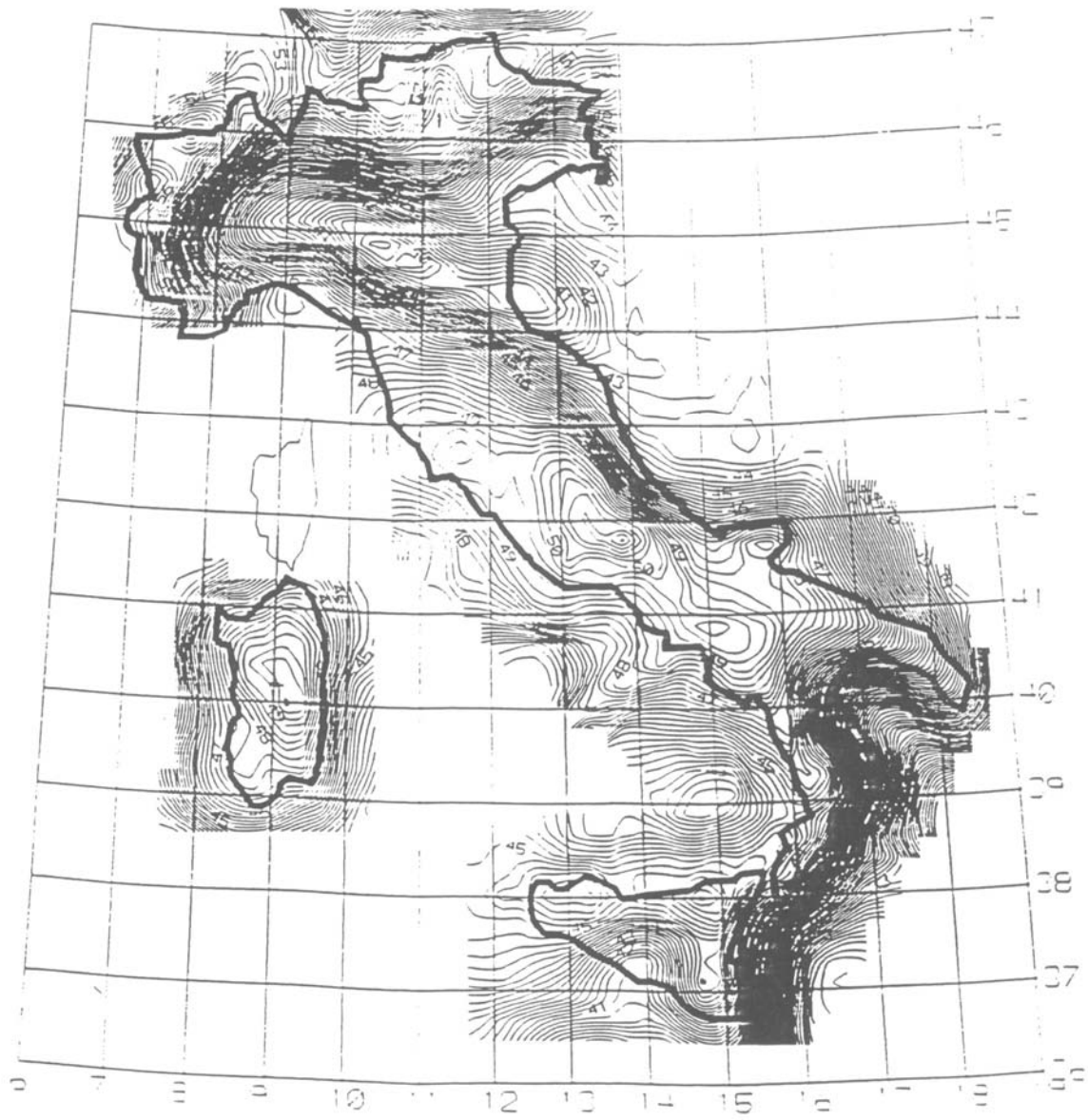


Fig. 4

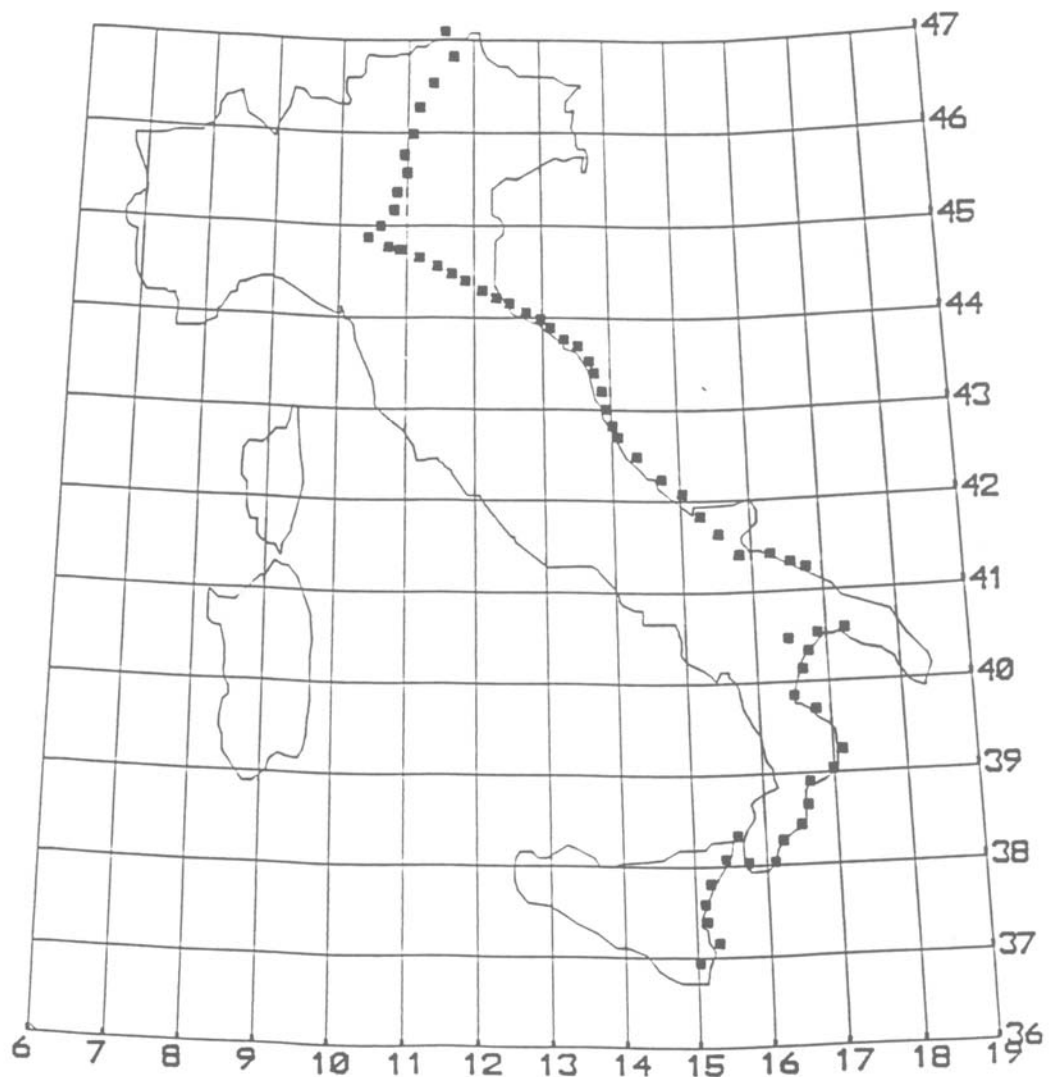


Fig. 5

COMPARISON N(GPS),N(ITALGEO),N(GEOMED)

EUROPEAN TRAVERSE (BRENNERO-NOTO)

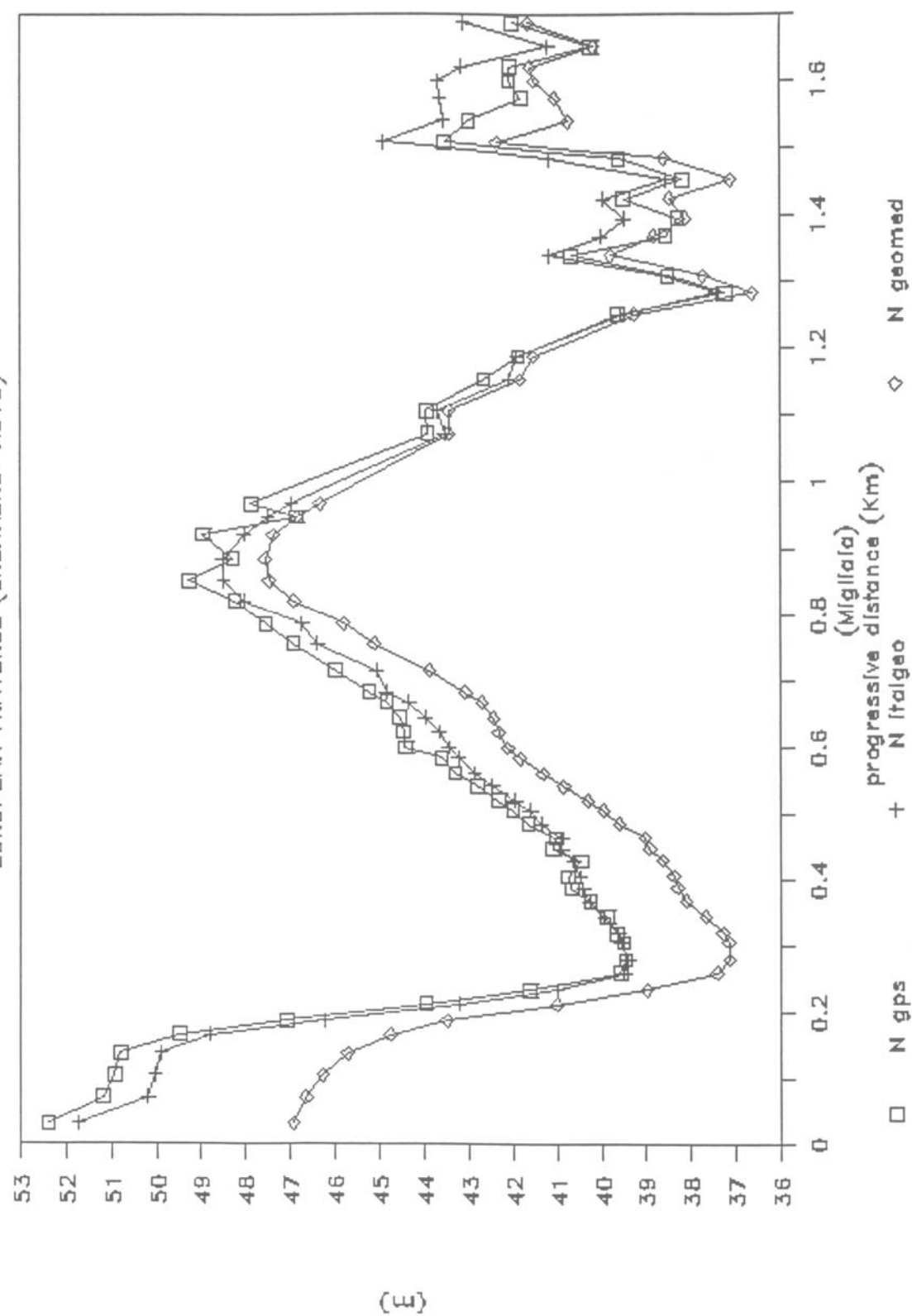


Fig. 6

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A METHOD FOR GROSS-ERROR DETECTION IN GRAVITY DATA BANKS

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Summary. A method for detecting gross-errors in data banks has been developed by using the least-squares collocation. A comparison of the difference between the observed and the predicted value with a critical value may be used to find outliers in an iterative process. The method has been applied to 1651 free-air gravity anomalies in a $2^\circ \times 2^\circ$ test area of central Spain. To compute reduced anomalies a geopotential model complete to degree and order 360 and topographic information have been considered. Results show a good quality of the gravity anomalies checked.

1. Introduction

When one is interested in gravity field it is necessary to be sure that data sets do not contain gross-errors. The search of them is the first task before using data in computations. Several methods have been proposed by other authors (Bureau Gravimetrique International 1989). For large numerical data bases semi-automatic methods which identify gross-errors are used followed by a detailed analysis of each value. Some methods predict a value in each point from the closest points stored in the data file. A comparison of the difference between the observed and the predicted value with the error estimate can then be used to identify a possible gross-error (Tscherning, 1990).

2. Gross-error detection by using the method of collocation

A gravity anomaly in a point P, Δg_p , is predicted from a set of values Δg_i , $i = 1, \dots, n$ in the neighbourhood spaced as regularly as possible in all

direction by,

$$\hat{\Delta}_{g_P} = \sum_{i=1}^n a_i \Delta_{g_i} \quad (1)$$

where,

$$a_i = \left\{ c_{iP} \right\}^T \left\{ c_{ij} + d_{ij} \right\}^{-1} \quad (2)$$

with,

- c_{iP} : The covariance between the observation i and the predicted value at P
- c_{ij} : The covariance matrix of the observations
- d_{ij} : The covariance matrix of the observation errors (associated to the points i and j); in practise it is a diagonal matrix

The error estimate is then computed as,

$$\sigma_P^2(\Delta_{g_P} - \hat{\Delta}_{g_P}) = c_o - \left\{ c_{iP} \right\}^T \left\{ c_{ij} + d_{ij} \right\}^{-1} \left\{ c_{jP} \right\} \quad (3)$$

A measurement is rejected or considered suspected if,

$$\left| \Delta_{g_P} - \hat{\Delta}_{g_P} \right| > k \left\{ \sigma_P^2(\Delta_{g_P} - \hat{\Delta}_{g_P}) + \sigma_P^2(\Delta_{g_P}) \right\}^{1/2} \quad (4)$$

where k is a constant generally having the value 3, and $\sigma_P^2(\Delta_{g_P})$ is the error variance of the observation Δ_{g_P} .

From equation (3) and (4) it is observed that gross-errors are easily found when c_o is as small as possible, and therefore equation (3) should be applied when data have been smoothed by removing the long-wavelengths by taking into account a geopotential model, and the local gravity field irregularities caused by topographic relieve.

We should be aware that a gross-error can contaminate data in the neighbourhood, therefore, the detection and removal of gross-errors should be a process with several iterations.

The method that we have used works in an iterative way with two disjoined sets of observations. This will be described in section 4.

3. Available Data

3.1 Gravity Data

The free-air gravity anomalies used were supplied by Instituto Geográfico

Nacional (IGN 1983). For our computations we have chosen a number of 1561 gravity anomalies in a test area whose limits are: $40^{\circ} \leq \varphi \leq 42^{\circ}$ and $-5.5^{\circ} \leq \lambda \leq -3.5^{\circ}$.

3.2 Spherical Harmonic coefficients

For this work several spherical harmonic coefficient sets were available, OSU89B (Rapp and Pavlis 1990), OSU91A (Rapp et al. 1991) and IFE88E2 (Basic et al. 1990). To choose the most adequate for our area, we compared observed gravity anomalies Δg_i to the gravity anomalies implied by each model $\Delta g_{i,mod}$ for all points in the area. As shown in Table 1 the best model was IFE88E2 because it gives the best mean and the lower standard deviation. It is no surprise that IFE88E2 fits the gravity anomalies over the area in question slightly better because this model was tailored to detailed gravity data over Europe, while other models have only used 30' x 30' mean gravity anomalies over the area.

Table 1. Comparison of gravity anomaly observations to geopotential model implied anomalies (units are mgal)

MODEL max. degree	IFE88E2 (360)	OSU89B (360)	OSU91A (360)
mean $\left(\Delta g_i - \Delta g_{i,mod} \right)$	-1.43	-2.44	-7.13
st. dev $\left(\Delta g_i - \Delta g_{i,mod} \right)$	16.06	17.08	17.73
max $\left(\Delta g_i - \Delta g_{i,mod} \right)$	92.71	94.35	89.15
min $\left(\Delta g_i - \Delta g_{i,mod} \right)$	-50.22	-54.80	-59.48

3.3 Topographic Data

Topographic data were available on a 6"x 9" grid provided by Defense Mapping Agency Aerospace Center (private communication). We used the RTM reduction (Forsberg 1984) with 30' x 30' mean height grid for the reference surface. Residual topography out to a distance of 110 km was taken into account.

4. How the method works

The goal of this method is to detect gross-errors by comparing observed and predicted values as shown in section 2 working in an iterative way in order to avoid that doubtful points could contaminate the predicted values. In each step, the predictions are made without taking into account the doubtful points detected in the previous step.

From the data in the test area, two subsets have been selected so that they contain different points, named A and B, with 825 and 826 observed gravity anomalies respectively. Table 2 shows some statistical results in both sets.

Table 2. Statistical summary of edited gravity anomalies

	Mean (mgal)	Standard Deviation (mgal)	Maximum Value (mgal)	Minimum Value (mgal)
SET A (825 points)				
Δg	8.18	28.60	122.15	-62.44
$\Delta g - \Delta g_{IFE}$	-1.37	15.88	91.89	-50.22
$\Delta g - \Delta g_{IFE} - \Delta g_{RTM}$	1.79	9.04	32.05	-22.43
SET B (826 points)				
Δg	8.11	29.48	135.26	-66.96
$\Delta g - \Delta g_{IFE}$	-1.49	16.24	92.71	-47.14
$\Delta g - \Delta g_{IFE} - \Delta g_{RTM}$	1.61	8.95	32.41	-23.89

To compute predicted values we have used the remove-restore technique considering the spherical harmonic coefficients set, IFE88E2, and the gravity field effect of topography by the Residual Terrain Modelling (RTM).

Firstly a part of the two data sets was selected holding one value in each cell of 3' x 4' covering the area (squares at 41° latitude) namely A* and B*.

Two covariance functions have been computed from the reduced anomalies in each subset by using Tscherning's program EMPCOV which have been fitted to two model ones C_{A^*} and C_{B^*} by using program COVFIT (Knudsen 1987).

For the analytical approximation of the empirical covariance functions we have used the covariance function model of gravity anomalies given by (Tscherning and Rapp 1974),

$$C(P,Q) = \sum_{n=N+1}^{\infty} \frac{A(n-1)}{(n-2)(n+24)} \left(\frac{R_B^2}{r \cdot r'} \right)^{n+2} P_n(\cos\psi) \quad (5)$$

Table 3 shows some characteristics of the empirical and synthetic covariance functions.

Table 3. Characteristics of covariance functions in a first step

SET	Empirical Function		Synthetic Function			
	C_o (mgal ²)	ψ_o (arc min)	C_o (mgal ²)	ψ_o (arc min)	order	$R_B - R_E$ (Km)
A*	87.75	23	76.00	23	221	-3.3
B*	86.75	22	78.00	23	221	-3.3

C_o : Anomaly Variance, ψ_o : First zero length

By using collocation (section 2) predicted anomalies have been obtained in points of set A from data in set B* with the covariance function C_{B*} , and in points of set B from data in set A* and the covariance function C_{A*} . The program GEOCOL (Tscherning 1985) has been used in this part of the work.

By comparing observed and predicted values in each set and considering eq.(4), we can detect in a first step what we have called suspected errors.

By removing the suspected points from the original data sets A and B, and selecting new points in each 3' x 4' cells two new covariance functions have been calculated and new predicted values have been computed in all points of the original sets A and B respectively.

If the observations detected by considering eq. (4) are again the same appeared in the previous step, we can confirm that they are definitively gross-errors because, in this second step, the doubtful points have not been taken into account, and therefore, they could not have contaminated the last

predicted values.

On the other hand a few new doubtful points could have appeared. A reason could be that they had been masked in the first step. In our work they have been considered as gross-errors.

Table 4 shows a statistical summary of the validated data in sets A and B.

Table 4. Statistical summary of validated gravity anomalies

	Mean (mgal)	Standard Deviation (mgal)	Maximum Value (mgal)	Minimum Value (mgal)
SET A (816 points) Δg	8.05	28.52	122.15	-62.44
SET B (808 points) Δg	7.92	29.39	135.26	-66.96

5. Conclusions

The aim of this work was to establish a method to detect gross-errors in data banks by using collocation in an iterative process to avoid that doubtful points could have contaminate the predicted values.

The smoother data are by removing long-wavelengths and local irregularities caused by topography, the easier to find a gross-error is, so we have reduced the data by considering a geopotential model and topographic effects.

This method has been used in a test area in Central Spain resulting nearly 2% of rejected points showing the high quality of the observations.

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Impact of airborne gravimetry on geoid determination - the Greenland example -

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Abstract

This note illustrates the importance which airborne gravimetry can have on geoid determinations in previously unsurveyed areas. Using preliminary gravity data from the Greenland 1991-92 airborne gravity project, a new geoid has been determined by spherical FFT methods. The geoid based on airborne gravity data has changed the previously best available geoid model by up to 8 m.

Introduction

Airborne gravimetry has in the years 1991-92 successfully been used for mapping the gravity field over the entire Greenland subcontinent. The gravity measurements have been carried out by the US Naval Research Laboratory and Naval Oceanographic Office in cooperation with KMS, NOAA and DMA (Brozena, 1991), and have provided a nearly uniform data coverage over the previously unknown interior parts of Greenland (Fig. 1).

The gravity data have been measured using a modified LaCoste and Romberg marine gravimeter, with Eötvös corrections and vertical accelerations determined by long-range interferometric GPS techniques (Brozena et al., 1989). A Bell BGM-3 or BGM-5 gravimeter was additionally used for gravity measurements, but these data have not been used in the present preliminary investigation. A precise pulse-limited radar altimeter was used for independent elevation determinations over the ocean border areas, and have for the first time demonstrated direct airborne measurements of geoid variations (Brozena et al., 1993), equivalent to satellite altimetry, albeit at a lower accuracy.

The survey was flown at a nominal elevation of 4100 m with a typical aircraft speed around 450 km/h. The individual data tracks have been adjusted in a cross-over analysis, minimizing the r.m.s. cross-over errors of both the airplane radar heights and measured gravity.

Based on this cross-over adjustment, the gravity free-air anomaly data aloft are estimated to have an error around 5 mgal, with a spatial resolution of 10-15 km. The internal error estimate is supported by upward continuation of ground truth data (Forsberg and Brozena, 1992; Forsberg et al., 1993). At the center of the Greenland ice cap, around the GRIP ice-drilling site at an elevation of 3200 m, airplane subtracks have been directly surveyed in a special ground gravity survey in 1993. The r.m.s. comparison errors for these tracks are around 6 mgal, whereas the comparison results to upward continued surface data in the coastal regions are more typically around 9-10 mgal, including the errors of the upward continuation. The upward continuation process is quite uncertain, due to lack of sufficient gravity data and accurate digital terrain models.

Accuracies around the 5 mgal-level makes airborne gravimetry a very well-suited data type for geoid determination, as such accuracies corresponds to geoid errors at the few-decimeter level. Large-scale airborne gravimetry may thus be a very efficient tool for mapping the geoid over unsurveyed, continental regions, where geoid errors might otherwise be at the level of several meters.

2. Geoid model of Greenland from surface data

In the Greenland example, a geoid made from the airborne measurements exclusively is compared to the hitherto used gravimetric geoid model GGEIOD92, based on surface gravity data only. Because the surface gravity data coverage of Greenland is very sparse (fig. 2), and mainly limited to the ice-free coastal regions, the geoid in the interior of Greenland is mainly controlled by the used long-wavelength reference field OSU91A, which again is based essentially on satellite tracking information from GEM-T2.

The GGEIOD92 geoid model has been computed by spherical Fourier techniques, using the remove-restore method, i.e. the spherical Fourier method is applied on the residual

$$T^c = T - T_{ref} - T_{RTM}$$

where T_{ref} is the reference potential OSU91A, and T_{RTM} the terrain effect. The terrain effects have been computed using a 5' x 10' digital terrain model (fig. 3), together with a similar grid of ice thicknesses derived from radar soundings. A reference height surface of 100 km resolution has been used in the RTM reduction for definition of the reference topography. Since the thickness of the Greenland ice cap is only known with a limited accuracy, and data lack completely from many areas such as the marginal ice zones, this and the general lack of resolution and accuracy of the used terrain models results in quite large geoid errors. Terrain effects on gravity stations have been computed using the approximation

$$\Delta g^c = \Delta g - 2\pi g \rho (h - h_{ref}) + c$$

where the terrain correction c is only computed in areas of sufficiently dense terrain models, and the density ρ takes into account whether the computation involves ice (0.92) or rock (2.67). Terrain effects on the geoid, used in the "restore" part, have been computed by spherical FFT.

The spherical FFT technique used for the gravity to geoid conversion is based on Stokes' formula

$$N_p = \frac{R}{4\pi\gamma} \iint \Delta g S(\psi) \cos\phi d\phi d\lambda$$

When the spherical argument ψ is expressed through the sin-formula

$$\sin^2 \frac{\Psi}{2} = \sin^2 \frac{\Delta \phi}{2} + \sin^2 \frac{\Delta \lambda}{2} \cos \phi_p \cos \phi$$

where $\phi = \phi - \phi_p$ and $\Delta \lambda = \lambda - \lambda_p$, then by the approximation

$$\cos \phi_p \cos \phi = \cos^2 \frac{\phi_p + \phi}{2} - \sin^2 \frac{\phi_p - \phi}{2} \approx \cos^2 \phi_m - \sin^2 \frac{\Delta \phi}{2}$$

the resulting expression may be approximated by a convolution in latitude and longitude (Strang van Hess, 1990), which is readily evaluated by Fourier methods

$$N = (\Delta g \cos \phi) * S = \mathcal{F}^{-1}(\mathcal{F}(\Delta g \cos \phi) \mathcal{F}(S))$$

where $*$ is the convolution operator, S Stokes' function, and $\mathcal{F}$ the two-dimensional Fourier transform.

The convolution above will be exact along the latitude parallel ϕ_p , and by using a sequence of reference latitude parallels and a linear interpolation scheme, a virtually exact FFT procedure may be carried out in geographical coordinates on the sphere (Forsberg and Sideris, 1993). In the present case a 9-band spherical FFT solution was found to be sufficient.

The GGEIOD92A solution is shown in Fig. 4. The application of the terrain and reference field reductions changed the mean value and standard deviation as follows:

Unit: mgal	Mean	Std.dev.
Original free-air data	15.29	38.43
RTM-reduced data	19.05	32.27
RTM-reduced - OSU91A	0.54	19.12

3. A geoid model from airborne gravity data

An alternative geoid model has been computed from airborne gravity data only. This geoid has been computed without any use of terrain data to avoid the ice-margin aliasing errors. The "airborne geoid" has been computed from downward continued airborne gravity data, again using OSU91A as a reference field and a multi-band spherical FFT geoid prediction. The effect of removing OSU91A is shown below (based on preliminary LCR gravity data, with a correction applied for a major instrumental bias problem in the 1992 survey):

Unit: mgal	Mean	Std.dev.
Airborne free-air data	25.2	35.3
Airborne data - OSU91A	-1.8	27.8

The downward continuation of the airborne gravity measurements has been carried out by prediction of vertical gravity gradients by Wiener filtering

$$T_{zz} = \mathcal{F}^{-1}[k \mathcal{F}[\Delta g] \frac{1}{1+ck^2}]$$

where k is the radial wavenumber. The last term represents the short-wavelength attenuation factor assuming the noise in the airborne data to be white, and the PSD of the gravity field to follow Kaula's rule. The scaling constant c has been matched to the resolution of the airborne gravity data, i.e. 15 km. The predicted gradients are shown in Fig. 5. Using the gradient the airborne data has been downward continued by

$$\Delta g^* = \Delta g_{4100m} - T_{zz}H$$

Due to the filtering used the higher order terms will only have minor influence, and the linear approximation may be considered sufficient. The downward continuation yielded maximal corrections to the gravity data around 30 mgal.

From the downward continued free-air data the new "airborne" geoid has been computed by a 9-band spherical FFT, using a 5' x 10' grid completely analogously to the GGEOID92A computation. The final geoid, after restoral of OSU91A, is shown in Fig. 6. The computed "airborne" geoid is technically a downward continued height anomaly geoid (ζ^*), and not the classical geoid (N). Similarly GGEOID92A may be considered as height anomalies (ζ), but since the differences are small (dm-level), the loose general term of geoid has been adopted here.

Fig. 7 shows differences between the new "airborne" geoid and the GGEOID92A. The differences are seen to be major, especially in north Greenland with more than 8 m difference. The major difference in north Greenland has later been identified as due to an error in the terrestrial data. The W-E ice cap traverse across the northern Greenland ice cap, carried out by the British North Greenland Expedition in 1952, has probably been offset by several tens of mgal. Due to lack of other data this error has gone unnoticed until recently, and illustrates the danger of relying on too sparse and isolated data for continental geoid determinations. Because no other data is in the neighborhood of the faulty data, the error may by gridding affect a large region, and have a large impact even on global mean anomaly data bases.

To perform an overall evaluation of the geoids is difficult. As an example the fits of 12 geoid heights, derived from a GPS survey in 1991 in very rugged topography of central West Greenland, are given below. The fit of the airborne geoid and GGEOID92A is not considered significantly different, as the GPS-derived geoid undulations from local sea-level observations probably have an error of similar magnitude.

R.m.s. fit of geoid to GPS/levelling data in Disko area

OSU91A	1.89 m
GGEOID92 - surface data only	0.71 m
Geoid from airborne gravity only	0.84 m

4. Conclusions

By the example from Greenland it is shown how airborne gravity may completely change the geoid situation in a large, continental area. Geoid models based on sparse terrestrial data may have large errors, in Greenland up to 5 m or more. A single erroneous source in the data base generated geoid errors in previous computations up to 8 m.

The Greenland airborne gravity survey may be considered a model for future coverage of other inaccessible areas, notably the Arctic and the Antarctic, and long-range airborne gravimetry over land areas may together with ocean satellite altimetry provide a future global gravity field model of high resolution. Airborne gravity will complement future satellite geopotential missions through the higher resolution, but on the other hand satellite missions may control the longer wavelengths, and eliminate hard-to-avoid minor biases in the airborne gravity surveys.

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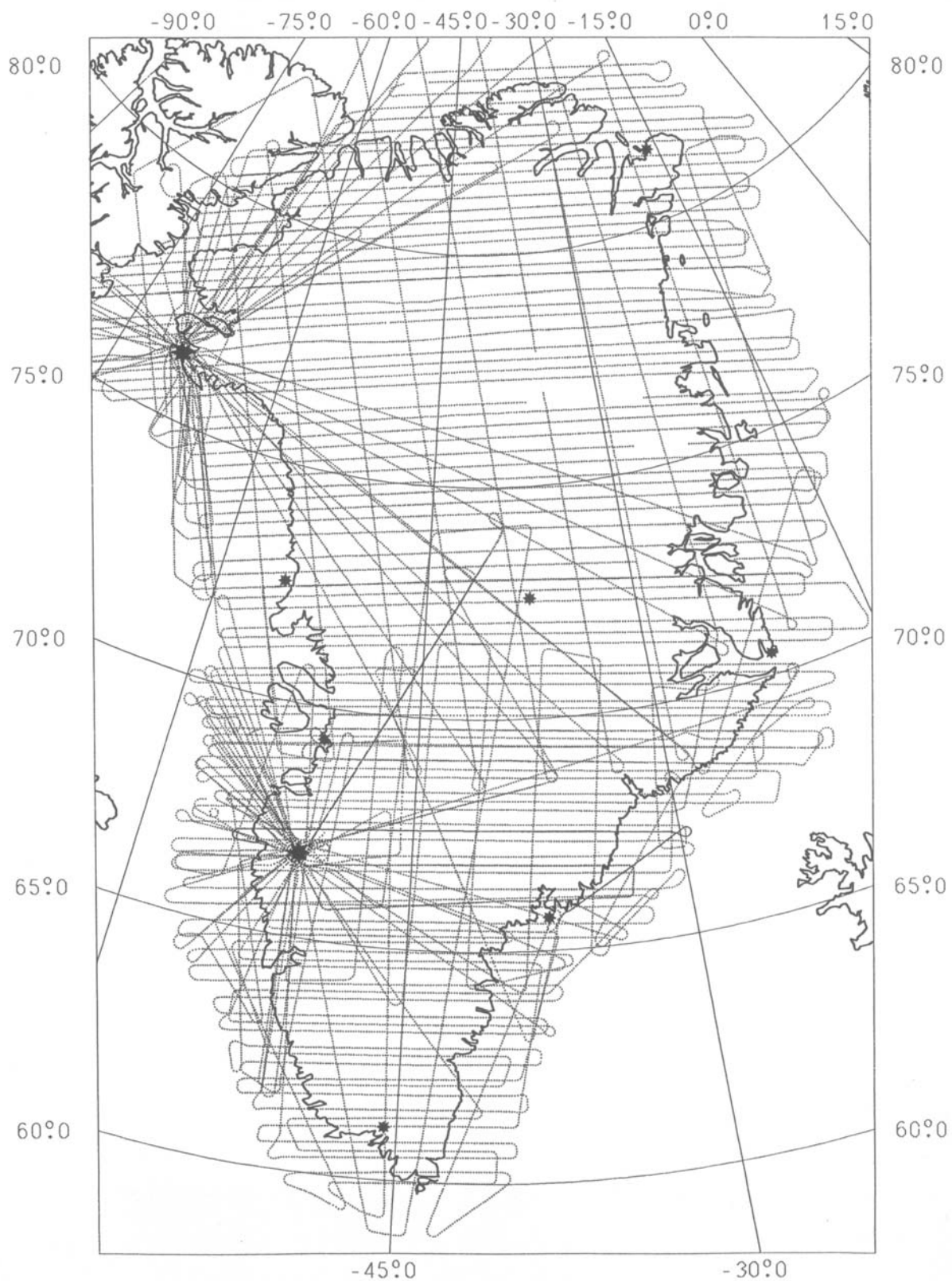


Fig. 1. Flight tracks Greenland Aerogeophysics Project 1991-92. Stars show locations of reference GPS sites.



Fig. 2. Greenland surface gravity points, status July 1993.

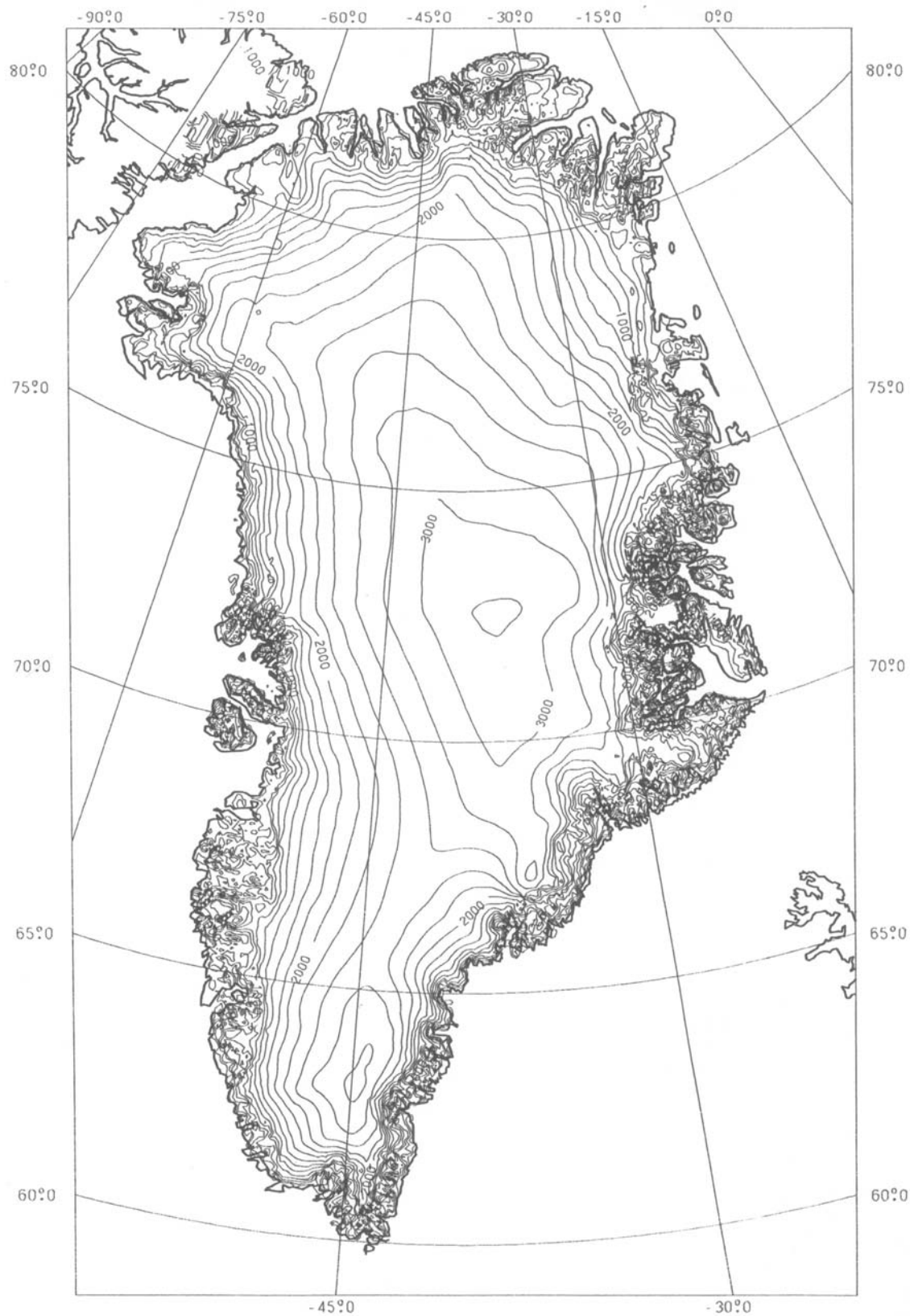


Fig. 3. Surface height contours, 200 m contour interval.

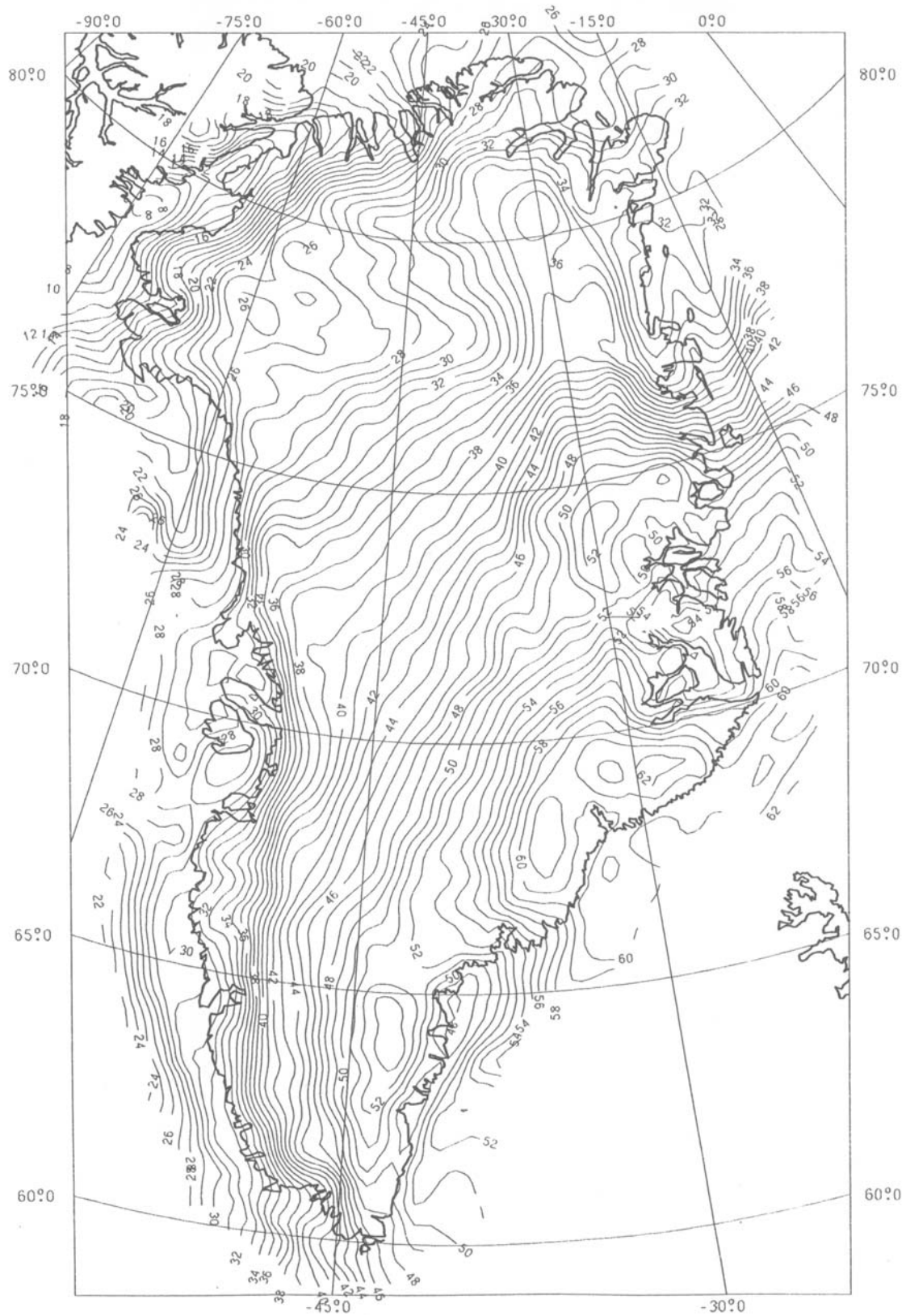


Fig. 4. GGEIOD92, contour interval 1 m.

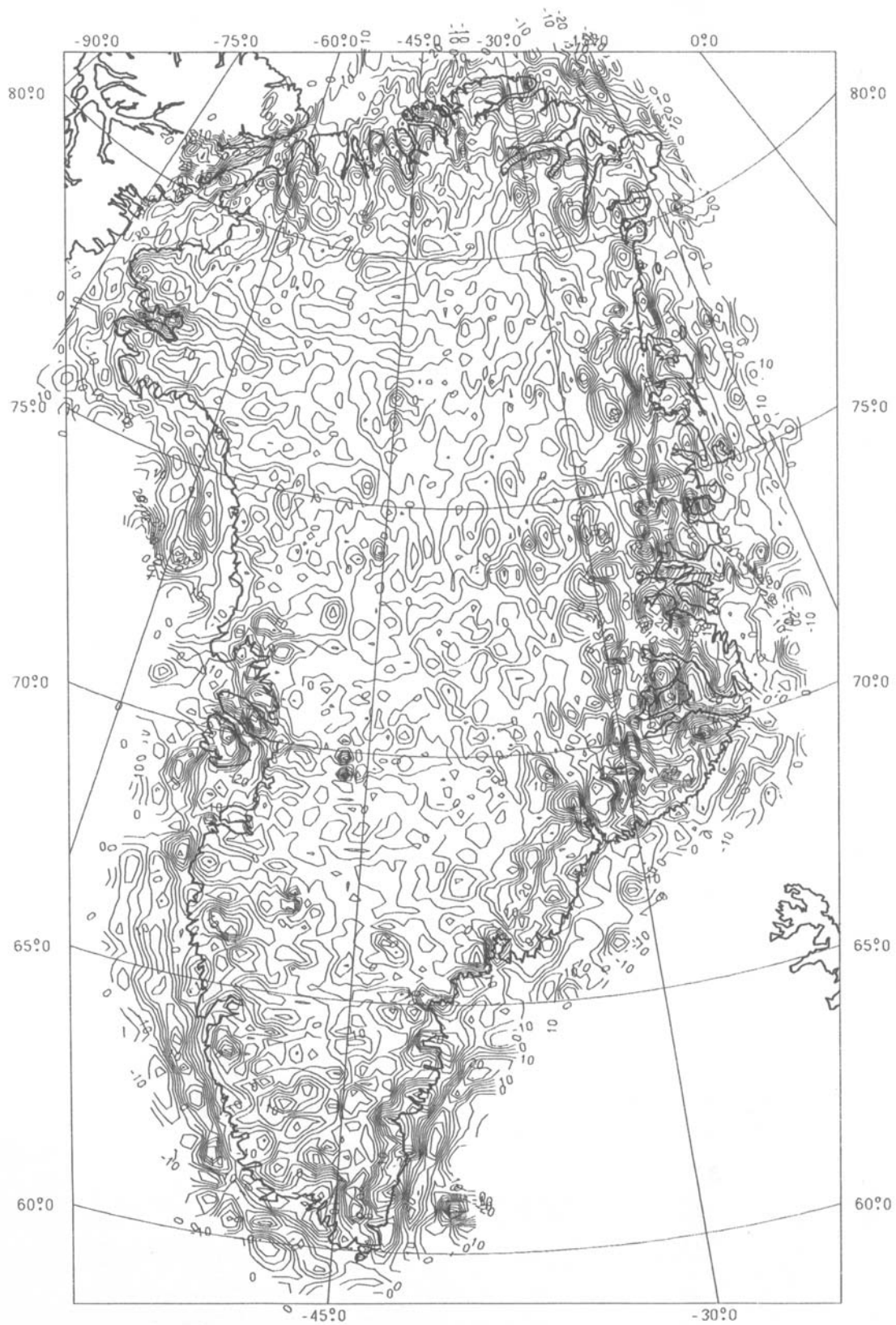


Fig. 5. Vertical gravity gradients from Wiener filtering. Contour interval 5 E (1 E = 0.1 mgal/km)

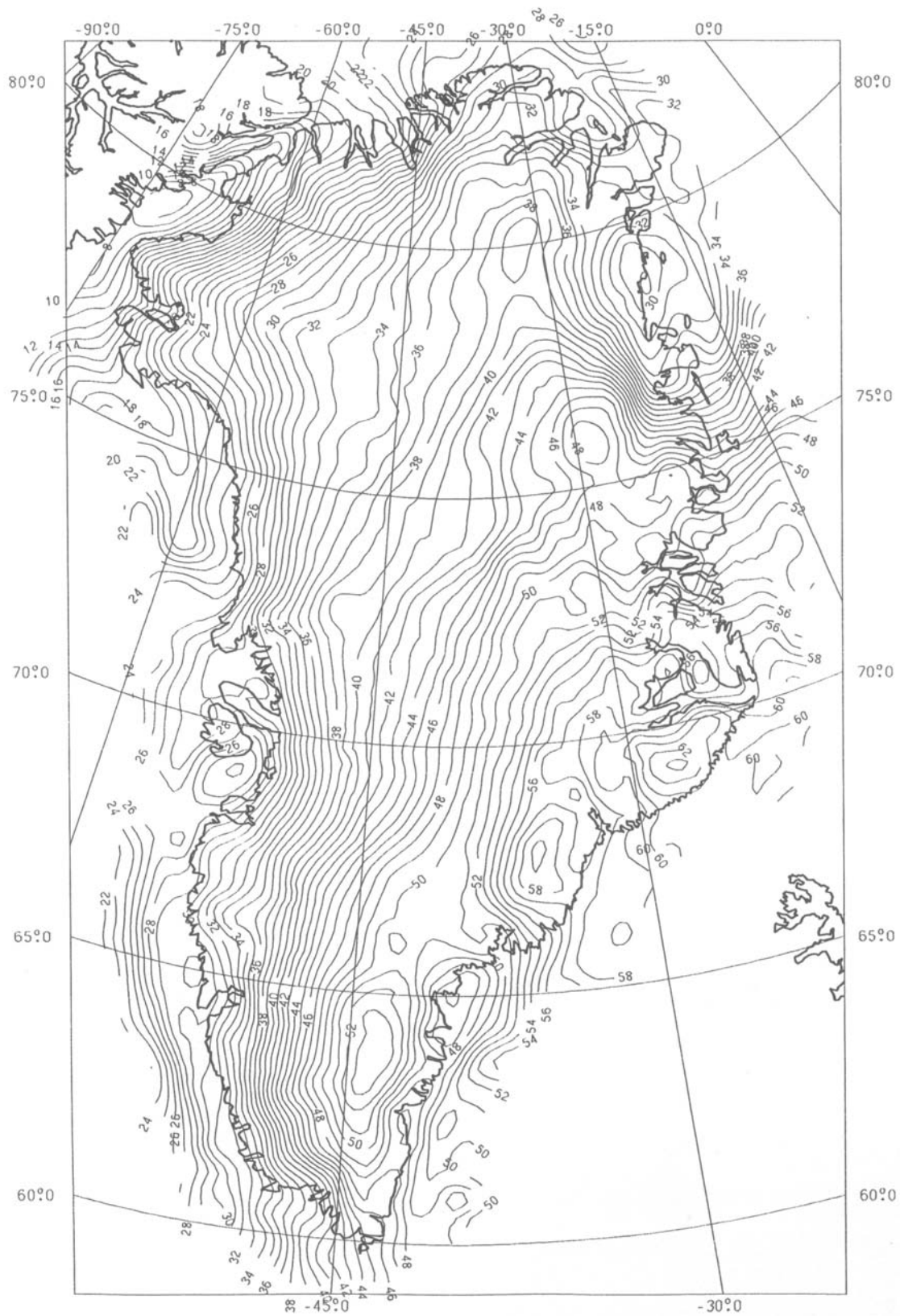


Fig. 6. Geoid computed from airborne gravity data alone. Contour interval 1 m.

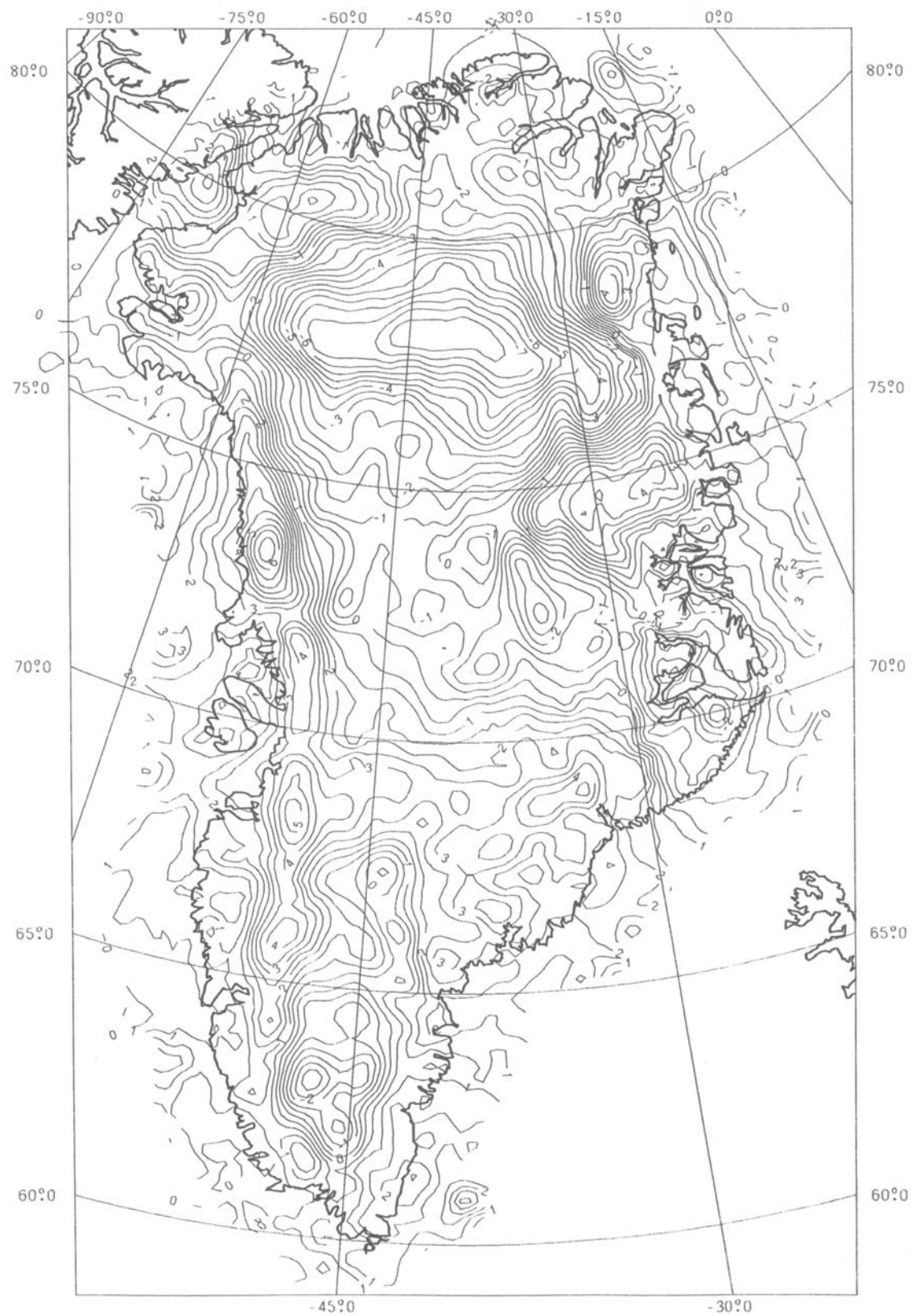


Fig. 7. Differences between GGEIOD92 and airborne geoid. Contour interval 0.5 m.

Progress on Geoid Determination in Europe

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Abstract

We discuss developments in the field of geoid determination for the European area in recent years.

In May 1992, a Workshop was convened in Prague on this subject, the Proceedings of which have recently appeared. There was an unusually broad participation by scientists involved in geoid determination from all parts of Europe. The Workshop, and the contacts it created, has been a great stimulus for geoid determination studies not only at the Subcommittee's Computing Centre, IfE Hannover, where data collection is proceeding very well, but also for national efforts, which are now underway e.g. in Romania and the Baltic states.

Here we shall explain the importance of data availability and completeness of coverage (both gravity anomalies and terrain data) for successful geoid determination, and the uses to which such a geoid may be put.

Introduction

After its formation at the IAG International Geoid Commission Meeting in Milan in 1990, the Subcommittee for the Geoid in Europe has developed a number of activities. In the task description for the Subcommittee, the then President of the International Geoid Commission, Prof. Richard H. RAPP, spoke of "coordination and communication", and referred to the work of the Institut für Erdmessung of the University of Hannover.

In the past three years, much has happened. The Hannover group at the Institut für Erdmessung, University of Hannover, under Prof. W. TORGE serving as the Computing Centre of the Subcommittee, has indeed been very active; compiled data, both gravimetric and terrain related, processed and computed successively improved geoid, and reported on their results on various international meetings. These computations, as well as the present status of data collection, will be reported on elsewhere in this meeting.

The Prague Workshop

The most important event has been the First Continental Workshop on the Geoid in Europe, in May 1992. The Proceedings have recently appeared; a goodlooking, over 500

page thick, volume, professionally prepared Research Institute for Geodesy, Topography and Cartography in Prague. Special mention must be made here of the work done by Dr. Petr HOLOTA in getting this volume out, but even more as head of the Local Organizing Committee in making the Workshop, in the view also of most participants, a great success.

Of course, part of the reason for this success, and for recent progress in European geoid determination in general, is the changed political situation on the continent. Thanks to this, the task of data collection by the Hannover Computing Centre has been easier recently, and they have obtained various materials from countries in Eastern Europe, which would not previously have been possible; and there are prospects of more coming.

There was a very broad participation in the Prague Workshop, also from Eastern European countries, made possible by generous financial support by both IUGG and IAG.

There was an animated panel discussion, which was very useful and went into such matters as the various horizontal and vertical datums used in the different countries, which have to be unified for geoid computation; the role of terrain information (digital terrain models for terrain effect computation), ways of disseminating geoid results, and problems of data classification.

As for the "technical" proposal for geoid determination by the Hannover group, this caused less discussion and was accepted: Resolution 5' by 3', area covered 25°W to 60°E, 35° to 85°N; although Hannover will use Spherical FFT for its computation, the exploration of alternative solution methods was also broadly supported.

A number of *recommendations* were passed by the gathering; of those, I wish to mention that of letting the International Geoid Service (now established in Milan, cf. below) handle the dissemination of the geoid to be produced in Hannover, and that it shall be freely available for scientific purposes; that the problem of collecting, homogenizing, producing and disseminating Digital Terrain Models shall be addressed by IAG Section III and its services (referring to the Bureau Gravimétrique International and International Geoid Service); and that EUREF and UELN shall undertake to unify the horizontal and vertical datums in Europe in preparation of the geoid determination.

WEEGP

A project which is related to the European geoid determination, because it seeks to compile largely similar data, is the *West-East European Gravity Project* (WEEGP), an undertaking under the leadership of Dr. J. Derek FAIRHEAD of the University of Leeds. It seeks to collect and unify all gravity data available on the European continent, database it, and provide a number of products derived from it to its participants/sponsors, which are both oil companies and national gravimetric agencies.

While the primary focus of WEEGP is on geophysical prospecting, much of their data collecting effort would overlap that of the European geoid computation, and fortunately it has been possible to reach an understanding that the Hannover group could have access to suitably gridded gravity data from WEEGP for this purpose.

International Geoid Service

This service, the establishment of which was noted with satisfaction by participants in the Prague Workshop, started operations on September 1, 1992. At the moment the service is quite active, having already published its first Bulletin.

The tasks of the IGeS are many; summarizingly we can mention the collection and dissemination of data and software related to geoid determination; the organization of courses in geoid determination; issuing an information bulletin; and others. For this term, however, it was chosen to concentrate on a smaller number of tasks, of which I would especially mention the *International Technical School for the determination and use of the geoid*, planned in Milan on October 25 to 29, 1993.

There has been expressed interest in this kind of training by at least one geoid researcher from Eastern Europe, so this is worth giving more publicity. I am especially happy to have noticed that in a number of countries in the former East, notably Romania, Latvia and Estonia, interest in precise geoid determination has "woken up", possibly in conjunction with interest in, and the possibility to use, the Global Positioning System for setting up a national geodetic framework much quicker than would be possible by traditional techniques, and probably much quicker also than they could have dreamed of just a few years ago! It will not be necessary to underline the importance of this for the inventarization and exploitation of a country's natural resources as well as for administrating land ownership.

Theoretical considerations

Here we shall shortly elaborate on the need for complete data coverage, both qua gravity and qua terrain data, for the determination of an accurate geoid.

Essentially, the need for global data coverage is the result of the use of the well known STOKES equation for geoid determination, which is a *surface integral* (contrary to e.g. astrogeodetic geoid determination which is based on a line integral):

$$\bullet \quad N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\psi) d\sigma.$$

Any gravimetric technique of geoid determination is - implicitly or explicitly - evaluating this integral, and as the STOKES function is of global support, must have a **global full areal coverage** of gravity anomaly Δg data.

Fortunately, it is possible to get partly around this limitation, if one has a *geopotential model* or *spherical harmonic expansion* representing the long-wavelength (crude) features of the Earth's gravity field, or equivalently, the geoid. It is then possible to modify the above formula for computing the *residual* geoid undulation δN , which means that the kernel function S will be replaced by one of more *local support*. Then one only needs Δg information from a *cap* of limited radius around the point of computation.

A useful global reference model for this purpose can be obtained from satellite tracking

data. Unfortunately the shortest wavelengths "felt" by satellites are some 10° , which corresponds to a spherical harmonic expansion to degree 36. Much more detailed global reference models, given to degree and order 360, do exist, but are to a large extent already based on terrestrial data, both gravity anomalies and (on the oceans only) satellite altimetry. This means that those institutes constructing these models are dependent upon the availability, worldwide, of terrestrial gravity data, and in areas where such data is lacking, the models cannot be relied upon.

This situation will not really change until satellite missions capable of sensing the Earth's gravity field at much higher resolution become a reality. Alternatively, one can of course use locally collected GPS positioning data, on points of known levelled height, to constrain a "local cap" geoid solution, amounting to a differential technique.

With digital terrain data, needed to evaluate the *terrain effect* on gravity anomalies going into the geoid computation, the situation is just as tricky, but in a different way. What we are talking about is the use of an *improper boundary*, one that is "wrinkled" more than the spacing of the gravimetric points *on it* that go into the geoid solution. Under these circumstances the requirements for applying the boundary value problem technique - or any equivalent technique - are simply not fulfilled.

The only solution: one *must*, first of all, *make the terrain as smooth as* the grid of gravity data points used in the geoid computation. This is what is done in "terrain reduction", or more accurately, "residual terrain modelling".

This is not the place to go into more detail, but I want to make the following warnings:

- Terrain effects are *biased*, i.e. always negative. They represent the attraction upward of a neighbouring hill, or the *lacking* attraction downward of a neighbouring valley. Don't believe that averaged gravity anomaly and terrain elevation values are any help!
- The major part of the terrain effect is *very* short wavelength only, and I mean *very*. Think of 50 to 100 m, not kilometres, which is the typical gravity point spacing. Digital Terrain Models with even 500 m spacing are of little use for *this* particular purpose, as they completely *miss* the high frequency *bulk* of the effect, which in mountainous areas can *not* be ignored! (Of course such crude terrain models are useful for reducing point gravity measurements to local mean terrain level.) Cf. also VERMEER and FORSBERG (1992).

Conclusion

In the course of just a few years the geoid has evolved from an esoteric concept occupying a small global brotherhood of theoreticians into a highly practical, and urgently needed, tool for interfacing the newer satellite geodetic techniques with existing, traditionally based practices and frameworks. Europe has been no exception; Eastern Europe has now a possibility to leap over one generation and move into the satellite age directly. But *the geoid is a prerequisite* for this. We have come a long way, but there is also a lot of work ahead of us.

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AVAILABILITY OF ALTIMETER DATA STATUS DECEMBER 1993

Prepared by C.C.Tscherning, Geophysical Department,
Haraldsgade 6, DK-2200 Copenhagen N, Denmark.

Several satellites have carried a Radar Altimeter: GEOS-3, SEASAT, GEOSAT, ERS-1 and recently Topex-Poseidon. The usefulness of theses data for gravity field modelling have been demonstrated in numerous papers. However, we should always remember that the altimeter determined heights are heights of the sea-surface, and not of the geoid. They can only to a certain approximation be used directly as gravity field data.

In this brief note, information about the availability of the data has been collected.

A major part of the geodetically relevant data are available at the JPL Physical Oceanography Distributed Active Archive Center (PO.DAAC), operated by the NASA Center JPL in Pasadena, California. A detailed description is found in JPL Publication 90-49, Rev. 4, June 15, 1993.

In order to obtain data, write to:
User Services Office, JPL Physical Oceanography DAAC, M/S 300-320,
Jet Propulsion Laboratory,
4800 Oak Drive,
Pasadena, CA 91109, USA.

The following datasets are available from DAAC:
GEOS-1 Altimeter Geophysical Data Record (GDR). Contains sea-surface heights, sea state, wind speed, Swiderski ocean-tide height, Cartwright solid-tide height. Data volume: 170 Mbytes.

SEASAT Altimeter GDR. Contains height above reference ellipsoid, significant wave height, automatic gain control, backscatter coefficient, quality flags. Data volume 350 Mbytes.

Topex/Poseidon Altimeter Merged GDR. Contains: Satellite altimeter range and position, significant wave height, automatic gain control or backscatter coefficient, ionospheric, atmospheric and ocean correction, quality flags. Volume: CD-ROM, more than 200 MB per 10 day cycle. Also Topex altimeter sensor data records are available on special request.

JPL also distributes software for data analysis and display.

Data from GEOSAT are distributed by the U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Oceanographic Data Center. However, non-US scientists will have to submit their request for the data through the scientific officer at their country's embassy in the US. More information are available from :

NODC, User Services Branch, NOAA/NESDIS E/OC21, 1825
Connecticut
Avenue, NW, Washington D.C. 20235, USA,
Phone: 202 606-4549, Fax: 202 606-4586,
e-mail: services@nodc2.nodc.noaa.gov

The following data are available:

Geosat T2 GDR from the Exact Repeat Mission (Nov. 86 - Dec. 89).
Orbits are based on GEM T2. Data Volume: 6 CD-ROM Discs.

Geosat GDR's from the Geodetic mission South of 30 degrees southern latitude. Data Volume: 2 CD-ROM Discs.

Geosat Crossover differences from the Geodetic Mission (April 1985-Sept. 1986). (Important for ocean variability studies). Data Volume: 8 CD-ROM Discs.

Remember not to contact NODC directly for Geosat data if you are not residing in the USA !

ERS-1 data are available from ESA/ESRIN.

ERS Help Desk, ESRIN,

Via Galileo Galilei,

I-00044 Frascati, Italia.

Phone: 39 694180600, Fax: 39 694180510,

e-mail: helpdesk@ers.esrin.esa.it

If you are a registered user, you will have the possibility to survey the available data using telnet.

A variety of altimeter products are available:

Altimeter Fast delivery product (near real time 1 Hz data). 220 Mbytes per month. It is also available as an off-line product.

Altimeter Ocean Product, processed data similar to GDR's. Range to surface, satellite height, wind speed at nadir, significant wave height. At present data with orbit information with a height standard deviation of 0.15 m are available.

Quick-look Ocean Product records. This is an upgrade of the fast delivery data.

Detailed information is available in ERS User Handbook, ESA SP-1148, Revision 1, Sept. 1993.

Both at ESA/ESRIN and at the National Geophysical Data Center, 325 Broadway, E/GC4, Dep. 894, Boulder Colorado 80303-3328, USA are a number of further products available, such as gravity data, digital topography and bathymetry, spherical harmonic coefficients and sea surface topography. You should write to these services in order to be on their mailing list for further information. But please remember, that the data are not necessarily distributed free of charge. A CD-ROM with altimeter data typically costs between 10 and 20 US

Dollars.

Also, remember that if you are or become a principal or co-investigator with a satellite mission (such as ERS-2), then you generally will be receiving the data free of charge.

References: Cheny, R.E, B.C. Douglas, R.W. Agreen, L.L. Miller and D.L. Porter: The Complete Geosat Altimeter GDR Handbook, NOAA Manual NOS NGS 7, Rockville, MD. 1991.

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JPL: JPL Seasat project, Altimeter Geophysical Specifications. JPL 622-226, 1980.

Is the determination of the geoid a reasonable problem?

M.A. Brovelli, F. Sansó

1. Introduction: geoid and inverse problems

The determination of the geoid (also called by C.F. Gauss the mathematical figure of the earth (cf. H. Moritz §2.2)) is one of the oldest formulations of the main problem of physical geodesy which is at one and the same time to find the "figure of the earth" and its gravity field in space. The geoid should be that particular equipotential surface of the gravity field, which best interpolates the sea surface.

This definition is indeed idealized and, although nowadays it is possible by means of satellite radar altimetry to connect the geometric measurements referring to different oceans, it seems extremely difficult to identify precisely the geoid through this surface which is known to be systematically separated from an equipotential by a quantity called sea surface topography (SST).

In practice what we usually consider as reference surface for the definition of a height system, is in reality a piece of equipotential surface often physically referred to a gauge point and these pieces do not belong all to the same equipotential, so that a height datum problem can be established (Rummel and Teunissen, 1988; Heck and Rummel, 1990; Pavlis 1991).

If we were able to properly connect all these different origins through space measurements we would have an unified definition of a reference equipotential surface which we could call geoid.

This surface would be very close to the stationary (i.e. suitably time averaged) surface of the oceans, as the SST can amount at most to some meters for physical reasons.

Since in addition the density of water is very close to 1 g cm^{-3} , in correspondence to this part of the earth surface it is possible to formulate the knowledge of the gravity field, slightly above and below the sea surface, by computing at most small and precise corrections.

The problem becomes more critical in continental areas where gravity measurements can supply, e.g. by solving a suitable B.V.P., the separation between actual geopotential surfaces and corresponding surfaces of the normal potential, also called height anomalies, which can be significantly different specially in mountainous areas from geoid undulations, i.e. the same quantities computed at the geoid level (Heiskanen and Moritz, cap. 13; Sjoberg).

By using different observations on the surface one can define several kinds of "geodetic boundary value problems" the solution of which, even if difficult, constitutes a so called direct problem in potential theory: its solution provides the

gravity potential on the surface itself and outside. If one likes to know the gravity potential inside the masses, for instance to reconstruct the shape of the geoid below continents, one is obliged first of all to know the mass distribution from the external surface of the earth down to the geoid itself, and secondly to make a so called backward continuation of the exterior gravity potential which is a so called "inverse problem" of potential theory, particularly difficult because it always displays a character of instability and requires a suitable regularization to provide any sensible solution.

We can distinguish different situations according to the possible hypotheses one can make on the available data. Particularly we can distinguish three problems:

- 1) in the first case we assume that the internal mass density of the earth is fully known. This case, to which we shall refer as the Newton case, is the only one to be not a "true" inverse problem, since in this case its solution is just a matter of calculus; yet this is interesting to establish sufficient physical conditions under which it seems sensible to state the problem of determining the geoid, for instance requiring that this surface be simply connected and smooth;
- 2) in the second case we assume that the mass density ρ is known between the actual surface of the earth S and some internal surface S_0 , e.g. an internal sphere, in such a way that it is a priori known that the geoid S_G has to lay within these two; in this case the problem is reduced to a pure backward continuation problem;
- 3) since in case 2), above, it is clear that we require more information than the one strictly needed, as we provide the knowledge of ρ between S_G and S_0 , it seems conceptually interesting to try to understand whether and how we can say that we know ρ only between S and S_G ; this can in fact be achieved if we assume ρ to be known as a function of the natural coordinates w (the geopotential itself), ϕ and λ .

The determination of the geoid as an inverse problem the discussed for instance (Sansó Madrid; Engel et al., 1993) in this paper we want to restrict our attention only on the question 1) above, which in spite of its formal simplicity requires a closer analysis in our advice.

2. Is the geoid a single regular surface? The Newton case

In this paragraph we shall assume first that $\rho(P)$, the mass density function, is perfectly known everywhere inside the body of the earth in order to define clearly our problem illustrating it also by counter examples.

If ρ is perfectly known, then the gravity potential $W(P)$ is provided by the Newton's integral (B being the generating body on which $\rho > 0$)

$$W(P) = \int_B \frac{\rho(Q)}{r_{PQ}} dB_Q + \frac{1}{2} \omega^2 (x_P^2 + y_P^2) \quad (2.1)$$

which is defined everywhere throughout the space. ⁽¹⁾ Let us remark that (2.1) presupposes that the axis z is taken along the rotation axis. The centrifugal term may have an effect on the shape of the equipotential surface only well far away from the body B , if this is earth-like. In fact assuming a field like the simple central field with the same mass as the earth and the same angular velocity, i.e.

$$\omega = \frac{M}{r} + \frac{1}{2}\omega^2 r^2 \cos^2 \phi \quad (2.2)$$

we find that W attains a minimum at the equator at about 70 times the earth's radius; naturally the bijectivity of potential values and closed simple equipotential surfaces is broken from there on. On the other hand it is clear that close to the sphere with radius R , equal to the mean radius of the earth, we will have only a small perturbation of the purely spherical potential, so that equipotentials will be almost spheres. By this argument we shall go on now mainly paying attention to the gravitational part of W .

Now it is time to look into the values of the potential W inside the masses.

If we define for instance the geoid as the largest equipotential surface no part of which is lying outside the masses, ⁽²⁾ we must first of all be sure that there exists one such surface: in fact take a body of uniform density like that shown in Fig. 2.1, and it will be clear that no equipotential surface can lie completely inside the masses.

For instance, if on the contrary the body B (on which $\rho > 0$ by definition) is convex it is easy to see that since $g_\nu < 0$ at all points external to B and on its surface S , then its potential W is decreasing in moving outward.

Let us examine for one moment W (which is continuous as it will be clarified in a minute) on the surface S and more precisely let M be the point of S where W attains its maximum, W_M ; since the tangential gradient of W at M is zero we have that $\underline{g}_M$ (which cannot be zero on S) is pointing inside B along the normal and W is therefore also increasing in that direction.

-
- (1) Let us remark that for the sake of simplicity of formulas in this paragraph we adopt the convention of multiplying masses, and then also densities, by the gravitational constant G : this will modify the numerical value of masses, so that for the Earth for instance $M \sim 4 \cdot 10^{20} \text{ cm}^3 \text{ s}^{-2}$, but it will not modify their ratios.
- (2) Let us observe that if one part of the surface of the body B is an equipotential and the rest of the surface is higher, then this, at least intuitively, becomes automatically the geoid according to this definition.

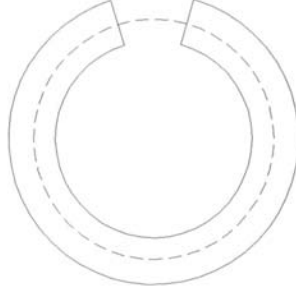


Figura 2.1 - A body with non-internal equipotential surfaces (two concentric spheres with a hole).

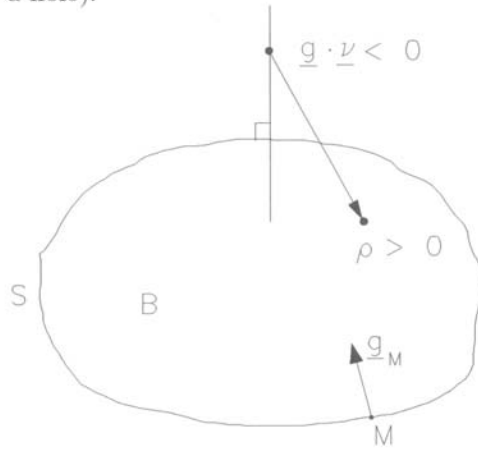


Fig. 2.2 - The potential of a convex body is decreasing at the exterior along normals to S .

Therefore the equation

$$W(Q) = W_M \quad (2.3)$$

can have, and has in fact, solutions only inside the body B , giving rise to a surface which is the geoid according to our definition (cf. Fig. 2.3).

Since $W_0 > W_M$, $W_P < W_M$ along any radius OP we have at least one Q such that $W(Q) = W_M$.

Naturally such a surface does not need to be regular and even not simply connected as it is illustrated by the example of a homogeneous sphere of radius 1, with a small point mass for instance on the x axis at the point $x = 0.9$.

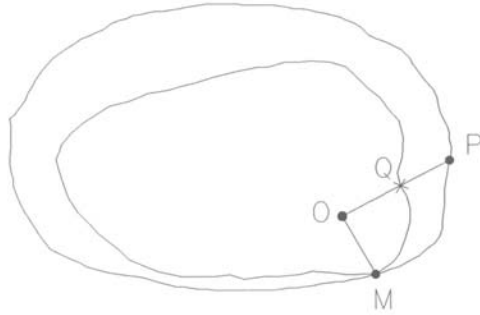


Fig. 2.3 - The "geoid" as defined here has to lay within masses.

In fact the (purely gravitational) potential of the homogeneous sphere is

$$W_0 = \begin{cases} \frac{M}{R} \frac{1}{2} [3 - \frac{r^2}{R^2}] & r \leq R \\ M \cdot \frac{1}{r} & r \geq R \end{cases} \quad (2.4)$$

as shown along the x axis in Fig. 2.4.

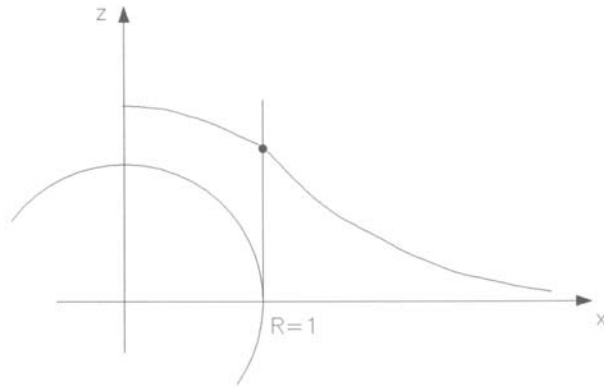


Fig. 2.4 - The potential of a homogeneous sphere.

If we then add to W_0 in (2.4) a point mass ϵ we get, along the same section,

$$W = W_0 + \frac{\epsilon}{|r - 0.9|}$$

which is shown in Fig. 2.5, where one realizes for instance that the equipotential $W = \bar{W}$ has a non-connected structure.

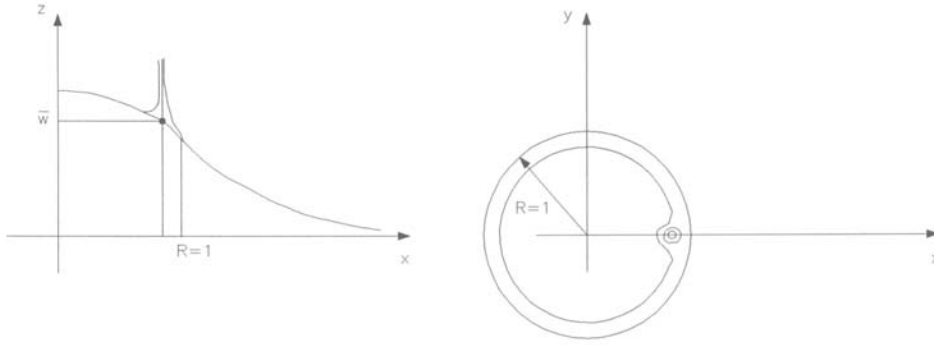


Fig. 2.5 - Example of a multi-sheet equipotential surface.

Since the earth is not strictly convex we need to examine more closely the question in order to be able to give at least sufficient conditions for the geoid to exist and to be a single regular surface; we will see that such conditions can be given in a simple form and they are also likely to be satisfied.

Before continuing it seems appropriate to define precisely the nature of the regularity of the function W inside B . Naturally this depends exclusively on the hypotheses we want to make for $\rho(P)$ and here we shall assume the most natural of such hypotheses, namely that $\rho(P)$ is purely a measurable, essentially bounded function on B ; this is mathematically expressed by the relation $\rho \in L^\infty(B)$ and it is known that this implies also

$$\rho \in L^p(B) \quad \forall p > 1 . \quad (2.5)$$

Then, from well known properties of generalized potentials (cf. [Miranda]) one can state the following Lemma:

Lemma : if ρ is in $L^\infty(B)$ then W is in $H^{2,p}(B)$ ($\forall p > 1$), i.e. it has $L^p(B)$ second derivatives; this implies also that $W \in C^{1,\alpha}(B)$, i.e. that the first derivatives of W are Holder continuous throughout B with any exponent, $\alpha < 1$.

A fortiori this implies that W together with its first derivatives are all continuous functions throughout the earth's body and this is what we need in the next paragraph.

3. Conditions for the geoid to be a single surface

We now try to look at conditions that guarantee the existence of the geoid as a simple surface; for this reason we try to find first of all an internal surface S_0 and suitable conditions implying that the geoid $\bar{S}$ is lying between S and S_0 and on the same time that the potential is in fact decreasing when we move outward from S_0 to S along a suitable family of lines.

For this purpose let us first of all stipulate that instead of knowing the mass density $\rho(P)$, which is indeed a stringent condition, we rather know a lower bound for ρ , i.e.

$$\rho(P) \geq \underline{\rho}(P) \quad ; \quad (3.1)$$

for the sake of simplicity we shall assume that $\underline{\rho}(P)$ is function of r_P only so that we can compute explicitly the corresponding potential

$$\underline{W}(P) = \int_0^R k(r, s) \underline{\rho}(s) \, ds \quad , \quad (3.2)$$

$$k(r, s) = 4\pi \cdot \begin{cases} \frac{s^2}{r} & s \leq r \\ s & s > r \end{cases} \quad , \quad r < R$$

which generalizes (2.4).

Let us observe that by the above hypotheses we are placing ourselves in a so-called "spherical approximation", i.e. we are disregarding the effect of the flattening of the earth's ellipsoid to make the computations elementary: the same computations however, just with a little more trouble, could be repeated for a family of ellipsoids, though it will be clear in the context that this perturbation should not affect significantly our conclusions.

We further remark that by (3.2) we are implicitly assuming $\rho(P) = 0$ within the topographic masses

We can also note that, for $r_P > R$, $\rho(P) > 0$ only within the "topographic masses" which can extend in space by less than

$$\epsilon \cdot R, \quad \text{with } \epsilon \cong 0 \, (10^{-3}) \quad .$$

We shall further assume that an upper bound is known for ρ , when $r_P \leq R$, which can be stated in the form

$$\rho(P) \leq (1 + p) \underline{\rho}(r_P) \quad , \quad (3.3)$$

maintaining, together with (2.7) that $\rho(P)$ has to stay within an interval of amplitude $p\rho(r_P)$ as shown schematically in Fig. 3.1

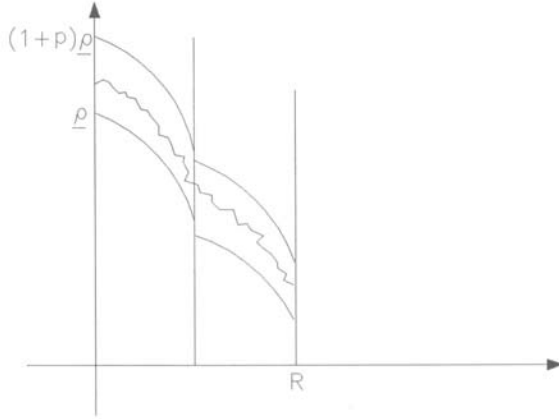


Fig. 3.1a - The mass density depth plot with bounds.

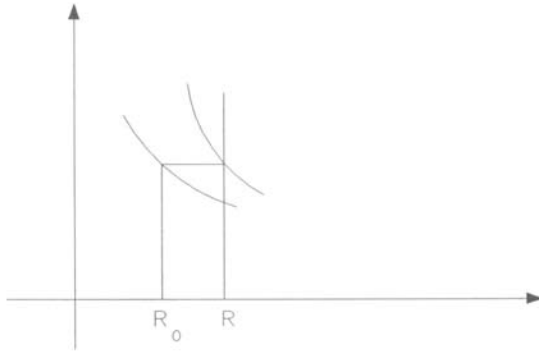


Fig. 3.1b - The geometrical meaning of R_0 .

Due to so-called lateral variations of the mass density we know that in order to be realistic a bound of this kind requires values of p as high as 10 or even 20 percent; as a matter of fact this parameter will dominate our subsequent computations.

Let us now observe that if

$$\underline{M} = 4\pi \int_0^R \underline{\rho}(r) r^2 dr$$

and if M_t denotes an upper bound for topographic masses, derived for instance from an upper bound

$$\rho(P) < \bar{\rho}_t \quad , \quad r > R$$

in the topography, then the potential W on the external surface can be majorized as follows (note that on S , $r \geq R$)

$$W|_S \leq (1+p)\frac{M}{R} + \frac{M_t}{R} \quad ; \quad (3.4)$$

in this expression $M_t \ll pM$ so that the first perturbing term of the main part, $\frac{M}{R}$, is $p\frac{M}{R}$, while the influence of topography is negligible. By using (3.2) we want to find the radius

$$R_0 = R - \Delta R$$

such that (cf. Fig. 3.1)

$$\underline{W}(R_0) = (1+p)\frac{M}{R} \quad . \quad (3.5)$$

In this rough computation it is enough to use a linear approximation, setting

$$\underline{W}(R_0) = \underline{W}(R) + \frac{\partial \underline{W}}{\partial r}(R)(-\Delta R) = \frac{M}{R} + \frac{M}{R^2}\Delta R \quad , \quad (3.6)$$

which inserted into (3.5) gives

$$\frac{\Delta R}{R} \simeq p \quad , \quad R_0 \simeq (1-p)R \quad . \quad (3.7)$$

Now it is obvious that along any radius, as shown in Fig. 3.2, moving from S_0 (the sphere of radius R_0) to S we must somewhere meet the value $W_M = \max_S W$; in fact

$$W(P_1) \geq \min_{S_0} W \geq \underline{W}(R_0) = (1+p)\frac{M}{R} \geq \max_S W = W_M \geq W(P_2)$$

and since W is a continuous function the equation

$$W(Q) = W_M$$

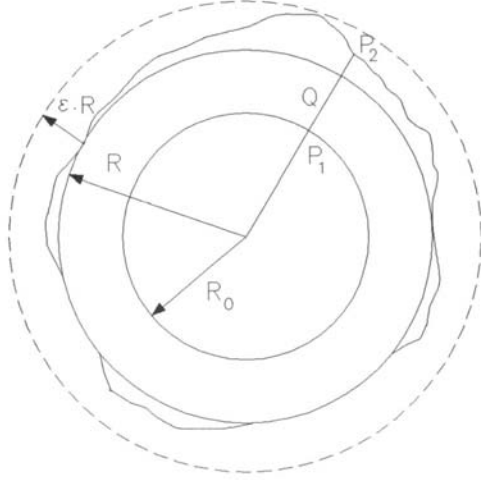


Fig. 3.2 - Uniqueness of the geoid point between the sphere S_0 and the external surface.

must have at least one solution between P_1 and P_2 .

We shall now look at conditions that guarantee that $\frac{\partial W}{\partial r} < 0$ for $r > R_0$, so that the above root is unique and then we will have proved that the geoid $\bar{S}$ is a single surface lying between S_0 and S . Assume for the moment that P is any point between the two spheres of radius R_0 and R , i.e. $R_0 < r_P = r < R$.

As a matter of fact we can write

$$W(P) = \underline{W}(P) + \delta W(P)$$

where the "anomalous" potential δW is generated by the difference between actual density and reference density $\underline{\rho}(P)$, i.e.

$$\delta W(P) = \int_B \frac{\delta \rho(Q)}{|\underline{r}_{PQ}|} dB_Q = \int_{B_R} \frac{\delta \rho(Q)}{|\underline{r}_{PQ}|} dB_Q + \int_{B_t} \frac{\delta \rho(Q)}{|\underline{r}_{PQ}|} dB_Q \quad . \quad (3.8)$$

In the sphere $B_R(r \leq R)$ we have

$$0 \leq \delta \rho(Q) \leq p \underline{\rho}(Q) \quad (3.9)$$

and within the topographic masses, represented by the set B_t contained in the layer $R \leq r \leq (1 + \epsilon)R$,

$$0 \leq \delta \rho \leq \bar{\rho}_t \quad . \quad (3.10)$$

Therefore we can write

$$\frac{\partial W(P)}{\partial r} = \frac{\partial \underline{W}(P)}{\partial r} + \frac{\partial \delta W(P)}{\partial r} \quad , \quad (3.11)$$

where, as one can verify from (2.7)

$$\frac{\partial W(P)}{\partial r} = -\frac{M(r)}{r^2} \quad (r = r_P) \quad (3.12)$$

$$M(r) = 4\pi \int_0^r \underline{\rho}(s) s^2 ds \quad ,$$

and, following (3.8), (cf. Fig. (3.3))

$$\begin{aligned} \frac{\partial \delta W}{\partial r} &= \int_{B_R} \frac{(\underline{r}_Q - \underline{r}_P) \cdot \underline{e}_P}{r_{PQ}^3} \delta \rho(Q) dB + \int_{B_t} \frac{(\underline{r}_Q - \underline{r}_P) \cdot \underline{e}_P}{r_{PQ}^3} \delta \rho(Q) dB = \\ &= I_1 + I_2 \end{aligned} \quad (3.13)$$

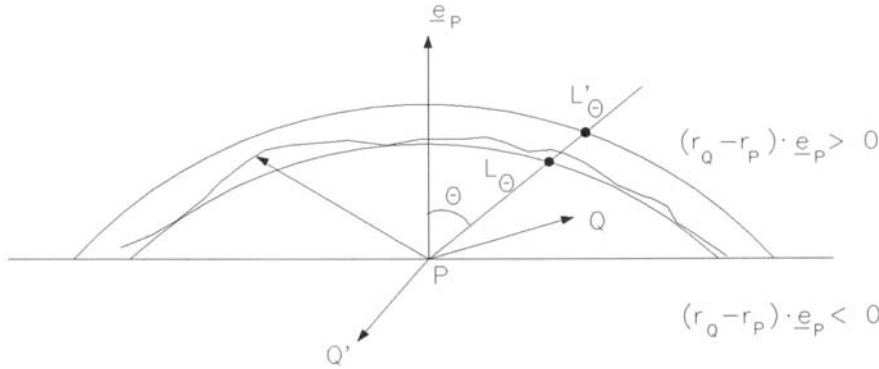


Fig. 3.3 - body that can give a positive contribution to $\frac{\partial \delta W}{\partial r}$; note that $\underline{e}_P = \underline{r}_P/r$ and that $r_{PQ} < L_\theta$ in B_R , $L_\theta < r_{PQ} < L'_\theta$ in B_t and $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \lambda < 2\pi$.

As shown in Fig. 3.3 the only part that can give a positive contribution to I , is that with $0 \leq \theta \leq \frac{\pi}{2}$, so that, also recalling (3.9) and the fact that $\underline{\rho}$ is decreasing,

$$\begin{aligned} I_1 &\leq 2\pi \cdot p \underline{\rho}(r) \int_0^{\frac{\pi}{2}} d\theta \sin \theta \int_0^{L_\theta} \frac{l \cos \theta}{l^3} l^2 dl = \\ &= 2\pi p \underline{\rho}(r) \cdot \frac{1}{3} \left[\frac{R^3}{r^2} - \frac{(R^2 - r^2)^{\frac{3}{2}}}{r^2} - r \right] \end{aligned} \quad (3.14)$$

With a further majorization, equation (3.14) is easily rearranged as

$$I_1 \leq \frac{1}{2} \frac{p\rho(r)}{r^2} \cdot [V_R - V_r] \quad (3.15)$$

where V_R, V_r are the volumes of the spheres of radius R and r respectively.

As for the second integral of (3.13) we can use a similar reasoning, only using the bound (3.10) and the fact that B_t is contained within the two spheres of radius R and $(1 + \epsilon)R$, so that

$$I_2 \leq \frac{\frac{1}{2}\bar{\rho}_t}{r^2} [V_{(1+\epsilon)R} - V_R] \simeq \frac{\frac{1}{2}\bar{\rho}_t V_R \cdot 3\epsilon}{r^2} \quad (3.16)$$

Collecting (3.11), (3.12), (3.15), (3.16) one can see that

$$\frac{\partial W(P)}{\partial r} < -\frac{1}{r^2} [M(r) - \frac{p}{2}\rho(r)(V_R - V_r) - \frac{3}{2}\epsilon\bar{\rho}_t V_R] \quad (3.17)$$

so that the condition

$$\frac{\partial W(P)}{\partial r} \leq 0$$

is satisfied if

$$M(r) \geq \frac{p}{2}\rho(r)(V_R - V_r) + \frac{3}{2}\epsilon\bar{\rho}_t V_R \quad (3.18)$$

Instead of (3.18) we can impose the more stringent condition

$$M(R_0) \geq \frac{p}{2}\rho(R_0)(V_R - V_r) + \frac{3}{2}\epsilon\bar{\rho}_t V_R \quad (3.19)$$

or, writing

$$\bar{\rho}_0 = \frac{M(R_0)}{V_{R_0}}$$

for the mean density within the sphere S_0 ,

$$\bar{\rho}_0 \geq \frac{p^2}{2}(3 + 3p + p^2)\rho(R_0) + \frac{3}{2}\epsilon(1 + p)^3\bar{\rho}_t \quad (3.20)$$

Since

$$\frac{p^2}{2}(3 + 3p + p^2) < 1$$

for $p < 0.618$ and for instance

$$\begin{aligned}\frac{p^2}{2}(3 + 3p + p^2) &= 0.594 \text{ for } p = 0.5 \\ &= 0.580 \text{ for } p = 0.3\end{aligned}$$

since $\epsilon \simeq 10^{-3}$ and also very likely

$$\bar{\rho}_0 \geq \bar{\rho}_t \text{ ,}$$

we may conclude that (2.25) can hardly be disregarded for realistic density values.

We may conclude that $\frac{\partial W}{\partial r} < 0$ between S_0 and the outer sphere or radius R .

It is finally easy to get convinced that a fortiori this inequality holds when P is in the topographic masses, due to the thinness of the volume they occupy.

As a conclusion we can claim that any distribution satisfying bounds like (3.1) and (3.3) with an uncertainty parameter p for instance smaller than 50 % and with as crust density majorized by the average density $\bar{\rho}_0$, will produce a regular geoid in the layer between S_0 and S .

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The European geoid: a comparison between Fast Fourier and Fast Collocation approaches.

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Introduction

The estimation of the geoid over large areas has been performed frequently using the Fast Fourier Transform method. As it is well known, this technique allows the use of large gravity data sets in a fast and efficient way (Denker, 1989; Forsberg, 1990). Furthermore new implemented versions take into account in a proper way spherical corrections, in order to reduce distortions due to the planar approximation used (Forsberg and Vermeer, 1992).

The Fast Collocation approach (Bottoni and Barzaghi, 1991), which has been recently proposed, is an alternative method for geoid computations with large data sets; it is based on collocation applied to gridded homogeneous data , the same used in the FFT procedure. Due to the Toeplitz/Toeplitz structure of the covariance matrix , a fast method for computing the collocation solution can be used, in order to overcome the principal drawback of collocation, i.e. the large amount of storage memory and the huge amount of CPU time required. Recent refinements of the technique (Barzaghi and others, 1992) allow a more rigorous approach, which takes into account that data are distributed on a regular geographical grid.

Hence, the two methods are equivalent in terms of input data base and, although they are based on different theoretical and numerical approaches, FFT can be considered as the frequency domain counterpart of FC.

What we want to test in this paper, is exactly the numerical homogeneity of FFT and FC when applied to a common test field. Furthermore we would like to discuss the different performances, if any, of the two methods in terms of reliability of the obtained solutions; to do so, internal compatibility of the estimated geoids has been checked and an external comparison with a GPS/levelling geoid has been carried out.

The gravity data base

Gravity values have been supplied and processed by the IFE of Hannover; in the following we briefly describe the procedure adopted to get the data that we used in our computations.

The original gravity data base was obtained by merging BGI and IFE gravity files over a large area covering the central part of Europe (Denker and Torge, 1993). The standard remove/restore technique has been used to reduce the data. The geopotential model that was subtracted is the OSU89B complete up to degree and order 360; the RTM reduction was computed using a reference topography which was constructed through a 15'x15' moving average filter. Finally, the reduced data were gridded using least squares interpolation (a modified version of R. Forsberg's GEOGRID program). The outcome of this procedure was a 60"x100" grid of reduced gravity values.

From this file, we extracted, in the area $46 < \varphi < 56$, $0 < \lambda < 22$, two coarser sets of data to be used in the computations:

(A) 3'x5' gridded gravity values averaged (window size = 3'x3') from the 60"x100" grid with the following statistics

$$\begin{aligned} n &= 52800 & E(\Delta g_r) &= 0.11 \text{ mGal} & \sigma(\Delta g_r) &= 11.12 \text{ mGal} \\ \text{Max}(\Delta g_r) &= 124.11 \text{ mGal} & \text{min}(\Delta g_r) &= -78.15 \text{ mGal} \end{aligned}$$

(B) 3'x5' gridded gravity values selected from the 60"x100" grid with the following statistics

$$\begin{aligned} n &= 52800 & E(\Delta g_r) &= 0.11 \text{ mGal} & \sigma(\Delta g_r) &= 11.27 \text{ mGal} \\ \text{Max}(\Delta g_r) &= 127.16 \text{ mGal} & \text{min}(\Delta g_r) &= -79.50 \text{ mGal} \end{aligned}$$

In this way we could verify the response of the two methods to the different frequency content of the input data (the difference between the two files gives an idea of the neglected high frequency part ; $E(\Delta g_r) = 0.00 \text{ mGal}$, $\sigma(\Delta g_r) = 0.47 \text{ mGal}$, $\text{Max}(\Delta g_r) = 8.66 \text{ mGal}$, $\text{min}(\Delta g_r) = -8.87 \text{ mGal}$).

FFT and FC computations

Three different solutions have been derived using the two data sets mentioned above. FFT computations have been carried out on a SNI/Fujitsu S400/40; the CPU time required is about twenty seconds.

The characteristics of these geoid estimates are listed in the following:

(a) FFT-1

- input gravity data : Δg_r from data base (A)
- zero padding technique to reduce edge effects
- analytical transform of the kernel function

(b) FFT-2

- input gravity data : Δg_r from data base (B)
- zero padding technique to reduce edge effects
- analytical transform of the kernel function

(c) FFT-3

- input gravity data : Δg_r from data base (A)
- zero padding technique to reduce edge effects
- numerical transform of the kernel function via FFT

As mentioned before, in this way we tested the internal reliability of the method with respect to the frequency content of the input data. The statistics of the three solutions and of their differences are collected in Tab. 1, while the contour plots of FFT-1, (FFT-1)–(FFT-2) and (FFT-1)–(FFT-3) are in Fig. 1, Fig. 2 and Fig. 3 respectively.

	n	E(m)	σ (m)	Max(m)	min(m)
FFT -1	52800	0.03	0.34	3.51	-1.21
FFT -2	52800	0.03	0.34	3.52	-1.23
FFT -3	52800	0.07	0.31	3.16	-1.04
FFT-1 – FFT-2	52800	~0.00	~0.00	0.06	-0.06
FFT-1 – FFT-3	52800	-0.04	0.04	0.39	-0.33
FFT-2 – FFT-3	52800	-0.04	0.04	0.41	-0.34

Tab. 1 - FFT solution statistics

The statistics prove that FFT-1 is practically equivalent to FFT-2 and that there are higher discrepancies between these solutions and FFT-3; this is obviously related to the different method used in transforming the kernel function (this approach to the solution has been proposed by Tziavos (1993) and may yield better results).

The FC procedure has been similarly applied to data sets (A) and (B). The empirical covariance functions of the two sets are practically equivalent; the one related to data set (A) is plotted in Fig.4. together with the best fit model covariance function, which is of the type

$$C_{\Delta g_{\Delta g}}(P, Q) = \sum_{i=2}^m \alpha \sigma_i^e \left(\frac{R^2}{r_P r_Q} \right)^{i+2} P_i(\cos \psi_{PQ}) + \sum_{i=2}^m \sigma_i \left(\frac{R^2}{r_P r_Q} \right)^{i+2} P_i(\cos \psi_{PQ})$$

$$\sigma_i^e = \text{error degree variances} \quad \sigma_i = \frac{A(i-1)}{(i-2)(i+4)}$$

$$\alpha = 0.296 \quad m = 340 \quad R = R_E - 4613 \text{ m} \quad P_i(\bullet) = \text{Legendre polynomials}$$

$$R = \text{mean Earth radius} \quad \psi = \text{spherical distance}$$

The FC version we applied was the one implemented to take into account that data are distributed on a regular geographical grid (R. Barzaghi and others, 1992). The geoid estimates were computed on the same grid as the input data, and the statistics are presented in Tab. 2 (see also Fig. 5 and Fig. 6). Here FC-1 is the solution we got from data set (A) while FC-2 is based on data set (B). FC computations have been performed on a HP730 and the CPU time for getting the estimate was about 6 hours. Obviously the CPU time required for the FC solutions is not comparable with the one implied by FFT; however, one must note that the performances of the two computers used in the computations are extremely different. Anyway, what is relevant is that with FC we are now able to obtain, in a reasonable time, geoid estimates over large areas, so that we have an external check of the FFT procedure and vice versa.

	n	E(m)	σ (m)	Max(m)	min(m)
FC -1	52800	0.04	0.33	3.24	-1.19
FC -2	52800	0.04	0.33	3.25	-1.19
FC - 1 – FC - 2	52800	~0.00	~0.00	0.04	-0.03

Tab. 2 - FC solution statistics

We still can see that there is a good consistency of the two solutions which are also very close to the FFT derived values (see Tab. 1). Furthermore, the differences between FC-1 and FC-2 (see Fig. 6) display a high frequency pattern which is similar to the one in Fig. 2 (FFT-1 – FFT-2). The only difference is that the pattern of Fig. 6 is enhanced with respect to that of Fig. 2: this seems to prove that FC is more sensible than FFT to the frequency content of the input data.

Comparisons among FFT, FC and GPS geoid undulations

We firstly compared FFT with FC, obtaining the results which are summarized in Tab. 3 and 4.

	n	E(m)	σ (m)	Max(m)	min(m)
FC-1 – FFT -1	52800	0.02	0.06	0.64	-0.31
FC-1 – FFT -2	52800	0.02	0.06	0.67	-0.32
FC-1 – FFT -3	52800	- 0.02	0.05	0.84	-0.50

Tab. 3 - FC-1 versus FFT-i statistics

	n	E(m)	σ (m)	Max(m)	min(m)
FC-2 – FFT -1	52800	0.02	0.06	0.62	-0.29
FC-2 – FFT -2	52800	0.02	0.06	0.64	-0.30
FC-2 – FFT -3	52800	- 0.02	0.05	0.84	-0.50

Tab. 4 - FC-2 versus FFT-i statistics

The agreement of FC and FFT is good, also considering that the main differences are at the boundaries of the area (see Fig. 7 and Fig. 8). In particular, the largest discrepancies are on the southern boundary, that is quite natural since this area includes the Alps.

Furthermore, it seems that FFT-3 gives a slightly better agreement with FC (see Fig. 9) results, confirming that FFT with numerically transformed kernel leads to an improvement in the geoid estimate (at least with respect to FC).

Then, an external check for FC and FFT geoids has been carried out using the GPS/levelling European geotraverse. In the area under investigation, we considered 30 points of this traverse and we predicted the residual FC and FFT geoids on these points (to do that we used solutions FC-2. and FFT-3). The statistics of the GPS/levelling data (from which we removed the model and the RTC contributions in terms of geoid undulation), of the predictions and of the differences are listed in Tab. 5.

	n	E(m)	σ (m)	Max(m)	min(m)
GPS/levelling	30	0.10	0.22	0.89	-0.19
FFT(traverse)	30	0.13	0.14	0.63	-0.10
FC(traverse)	30	0.13	0.14	0.64	-0.12
GPS/lev. – FFT	30	-0.03	0.13	0.27	-0.23
GPS/lev. – FC	30	-0.03	0.13	0.28	-0.22

Tab. 5 - Comparisons between FFT,FC geoids and GPS/levelling

As one can see in Fig. 10 (and in a more evident way in Fig. 11) the most relevant discrepancies between GPS/levelling and FC (or FFT) geoids have a clear periodic pattern, with wavelength of about 1000~1200 km and amplitude of nearly 0.25 m.

This is a well known phenomenon and it is probably related to distortions which are present in the global geopotential model. The local geoid estimation procedure is not able to recover such patterns (Crespi and others, 1993); hence, these components are present in the comparisons with GPS derived undulations. Furthermore, a more detailed analysis of Fig. 11 shows that there is a higher frequency signal in the residuals, with an amplitude of about 5~7 cm, which could be related to unmodelled RTC effects.

Conclusions

The Fast Collocation and Fast Fourier Transform methods gave geoid estimates which proved to be practically equivalent. The two approaches still differ in terms of CPU time needed for the computation (FFT is very much faster than FC) but a rigorous and direct comparison was not possible in this paper, since two different computers were used to perform FC and FFT computations.

As expected, being the two solutions very similar, the external tests between FFT or FC and GPS/levelling undulations led to equivalent results.

These comparisons showed a main periodic pattern with the wavelength which is as wide as the area width. This is likely to be related to geopotential model errors, that FC or FFT cannot correct properly, due to the amplitude of the window containing the data.

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Figure captions

Fig. 1 – FFT-1 estimated geoid (m)

Fig. 2 – FFT-1 – FFT-2 (m)

Fig. 3 – FFT-1 – FFT-3 (m)

Fig. 4 – Empirical covariance function and model covariance function of gravity data set (A)

Fig. 5 – FC-1 estimated geoid

Fig. 6 – FC-1 – FC-2 (m)

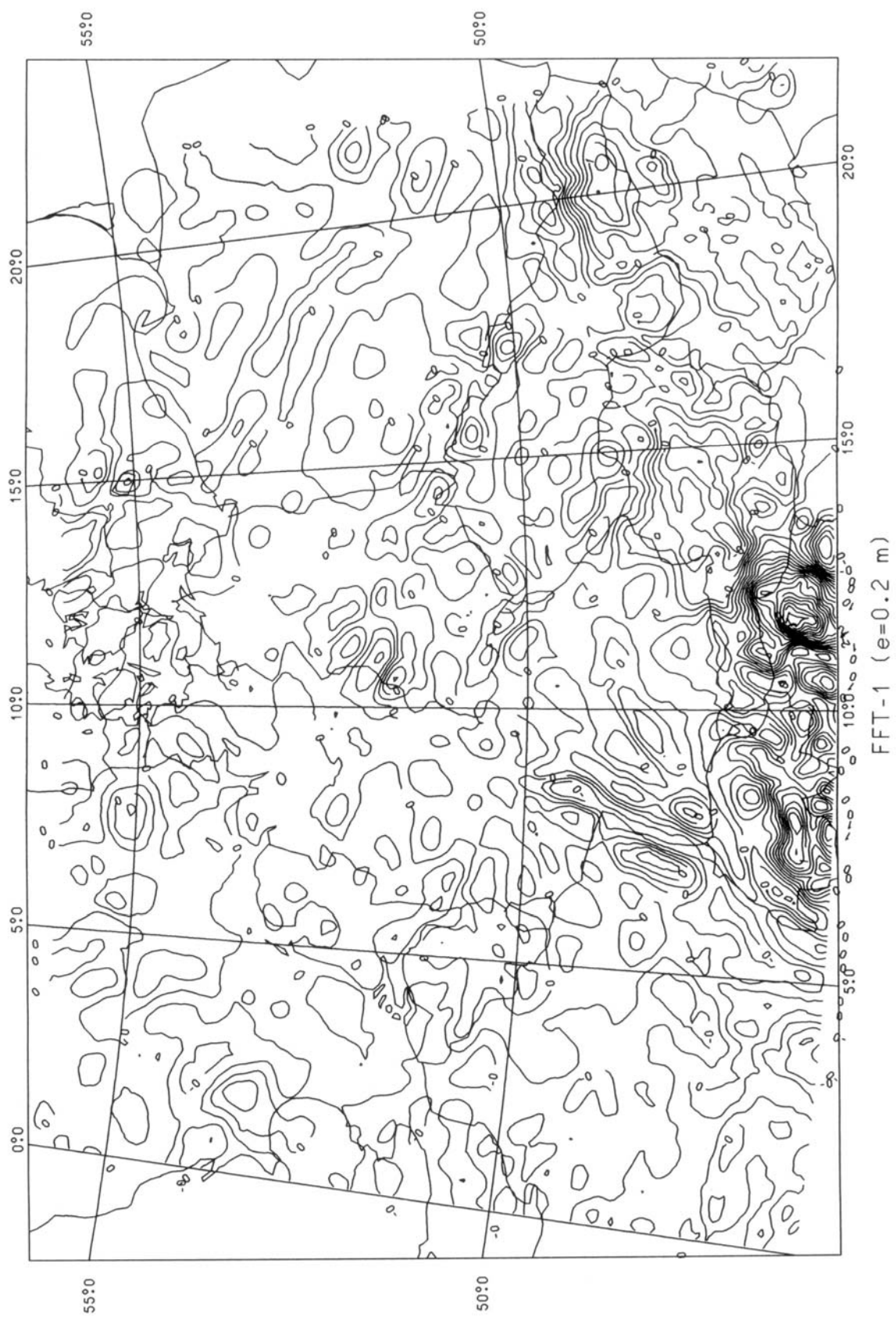
Fig. 7 – FC-1 – FFT-1 (m)

Fig. 8 – FC-1 – FFT-2 (m)

Fig. 9 – FC-1 – FFT-3 (m)

Fig. 10 – Comparisons among geoid undulations (m)

Fig. 11 – Differences between geoid undulations (m)



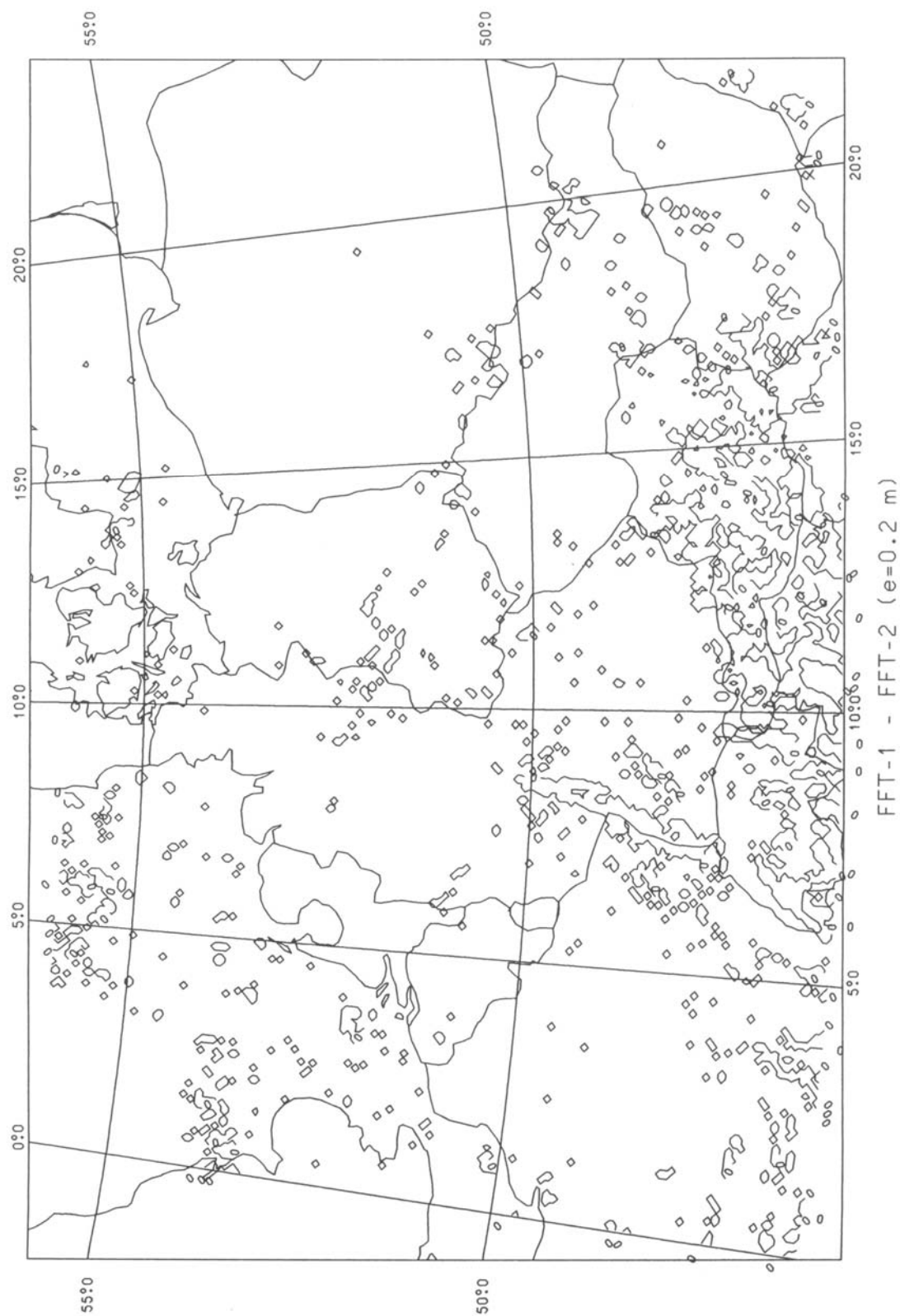


Fig. 2 - FFT-1 - FFT-2 (m)

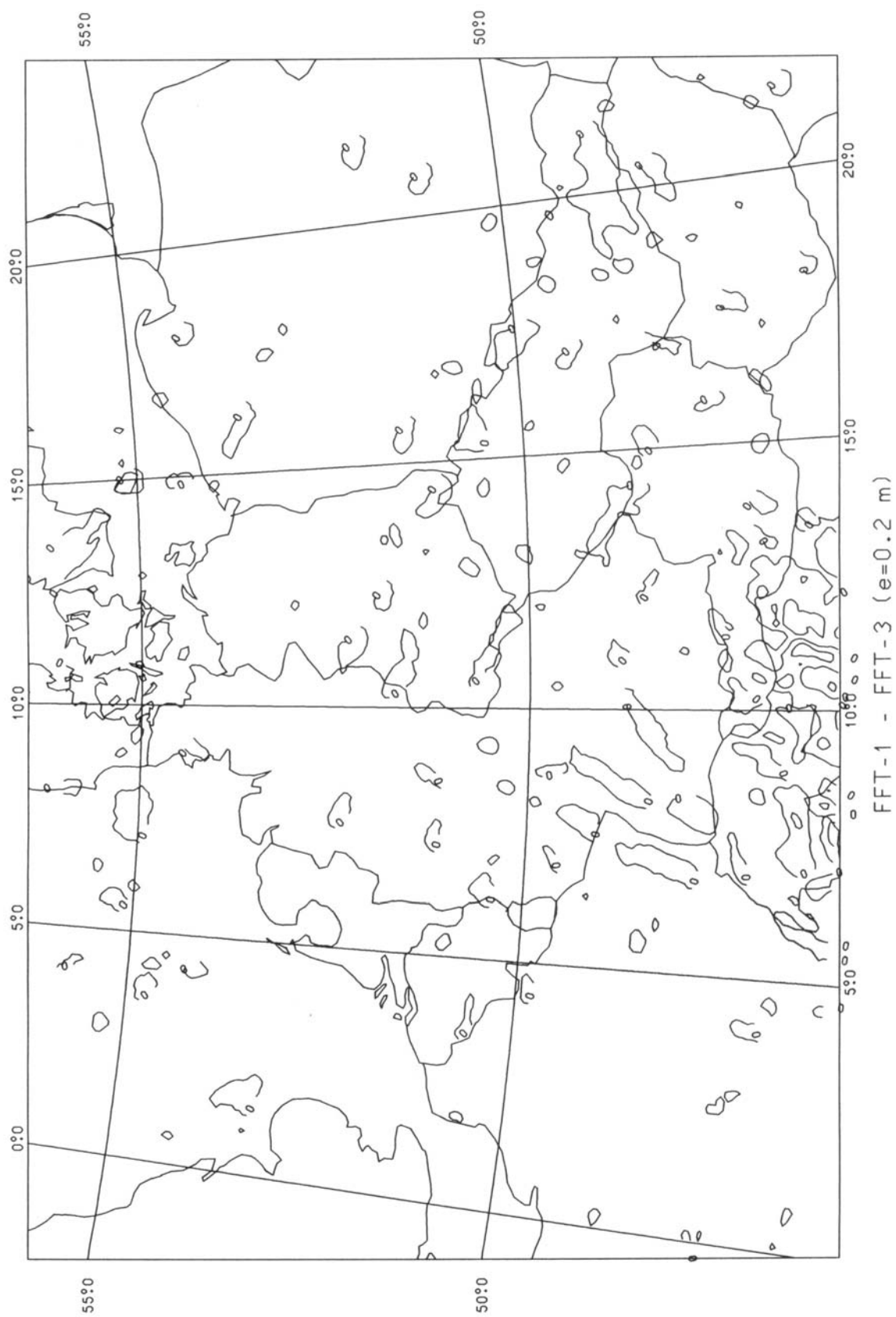


Fig. 3 - FFT-1 - FFT-3 (m)

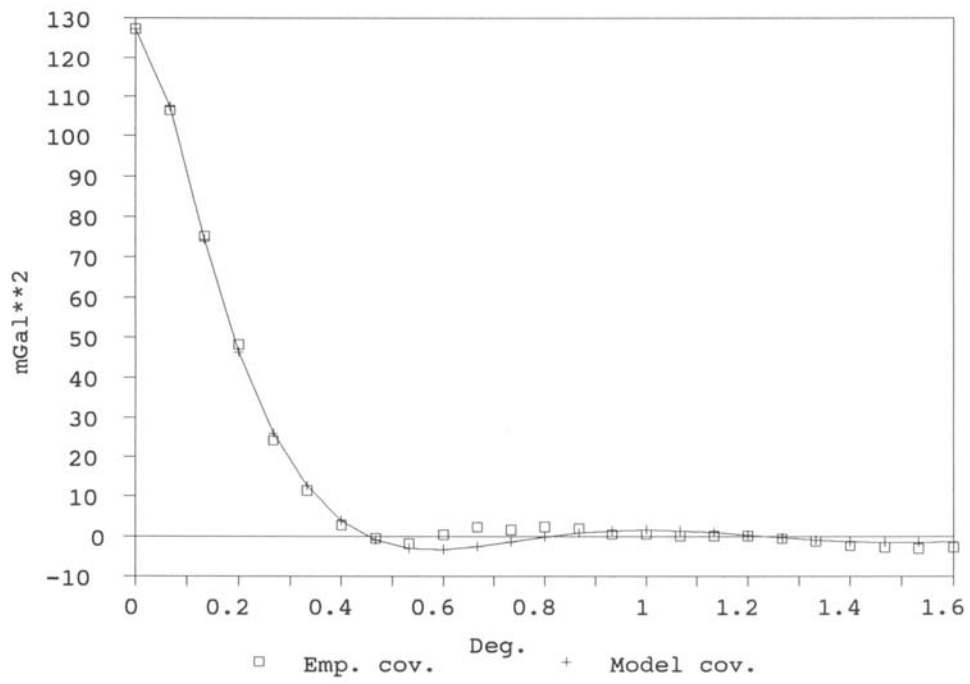
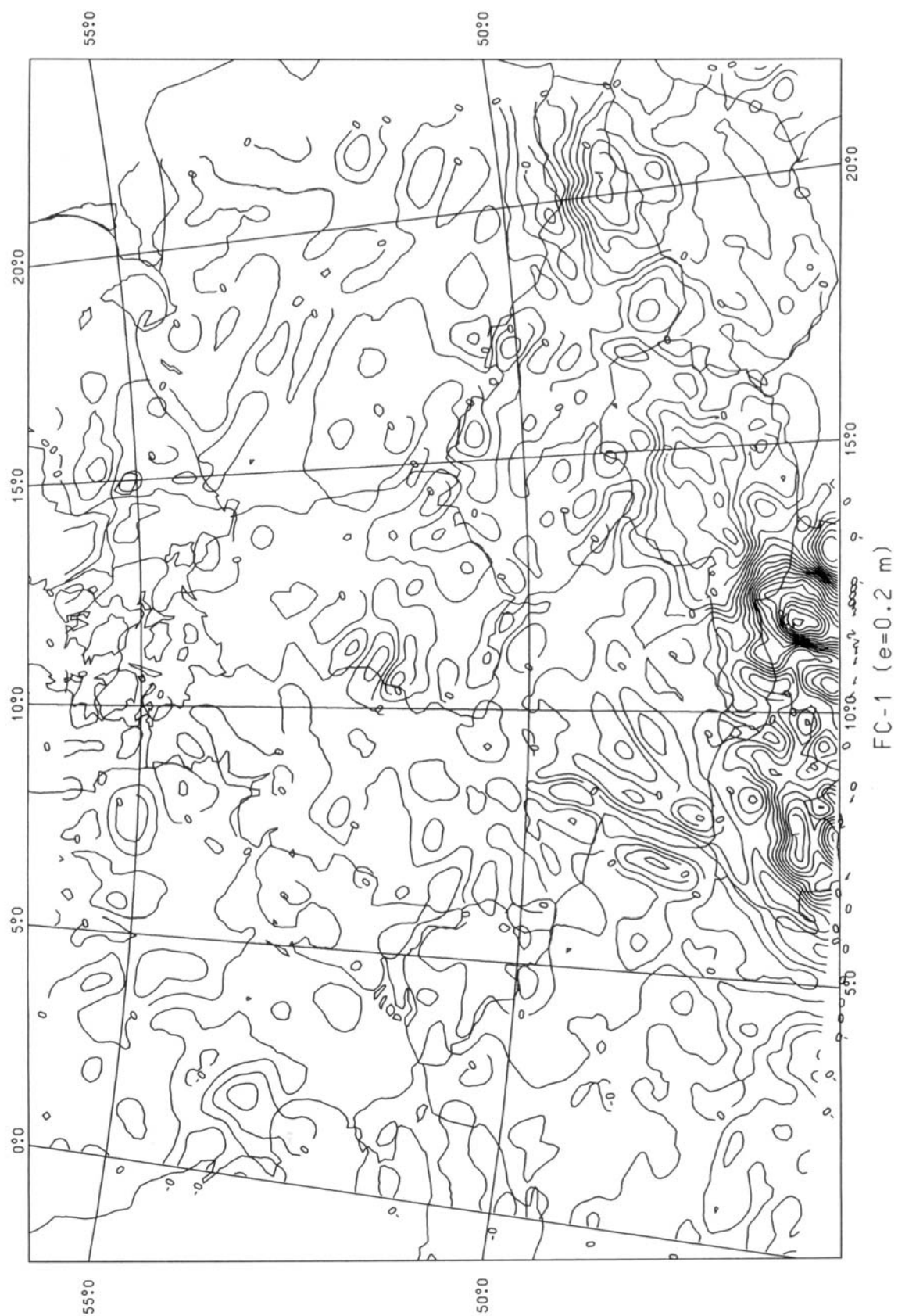


Fig . 4 – Empirical covariance function and model covariance function of gravity data set (A)



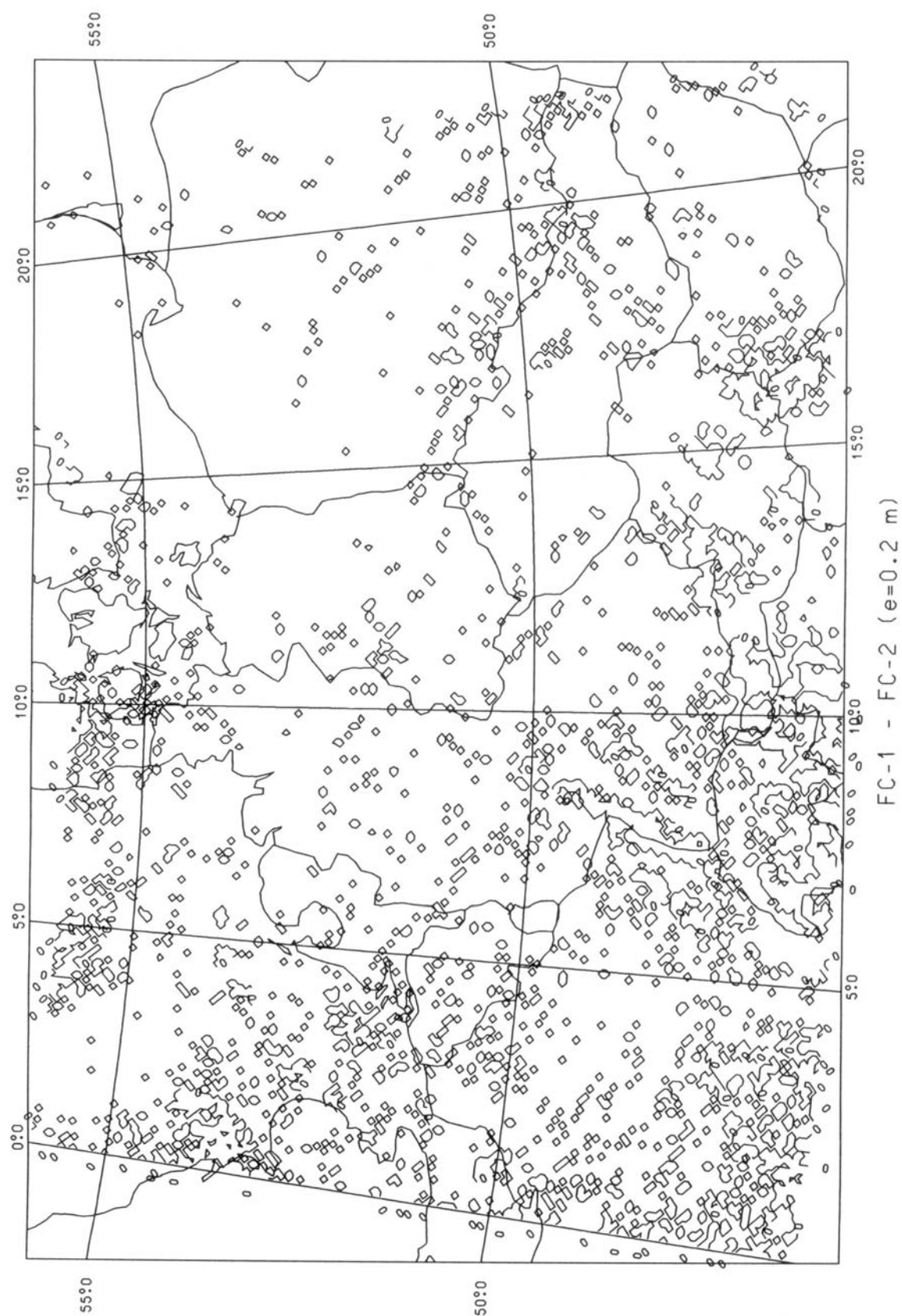


Fig. 6 - FC-1 - FC-2 (m)

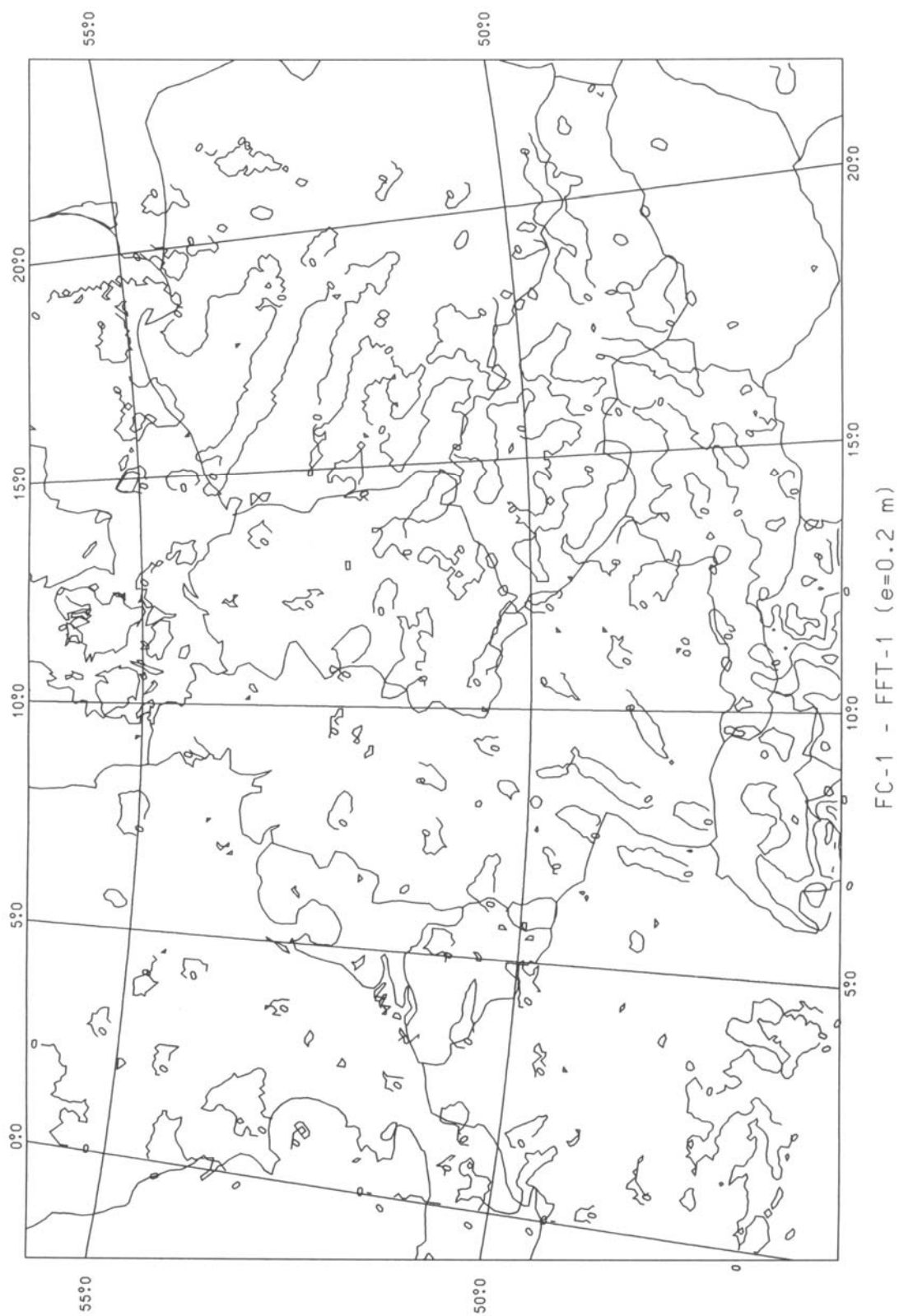


Fig. 7 - FC-1 - FFT-1 (m)

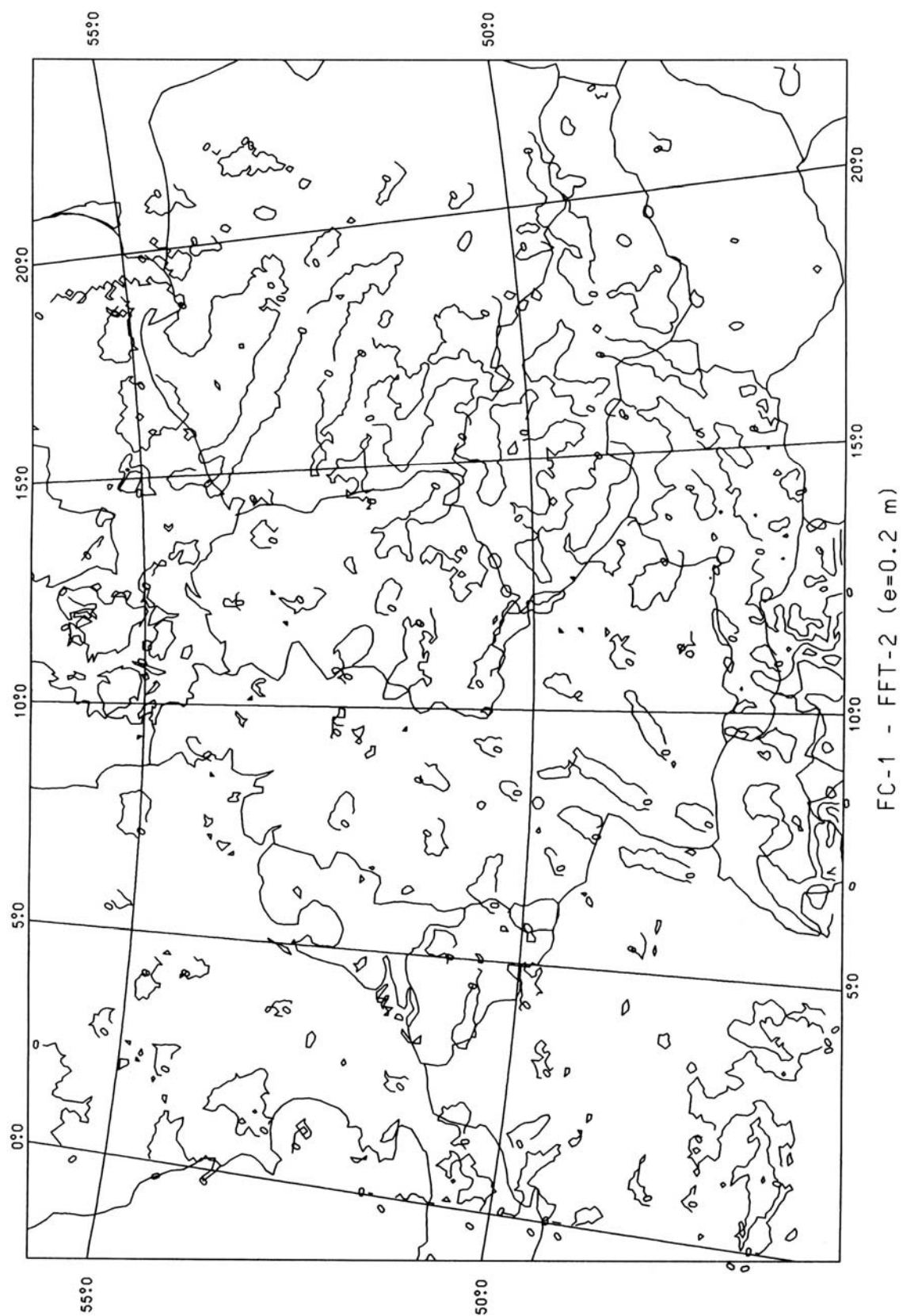


Fig. 8 - FC-1 - FFT-2 (m)

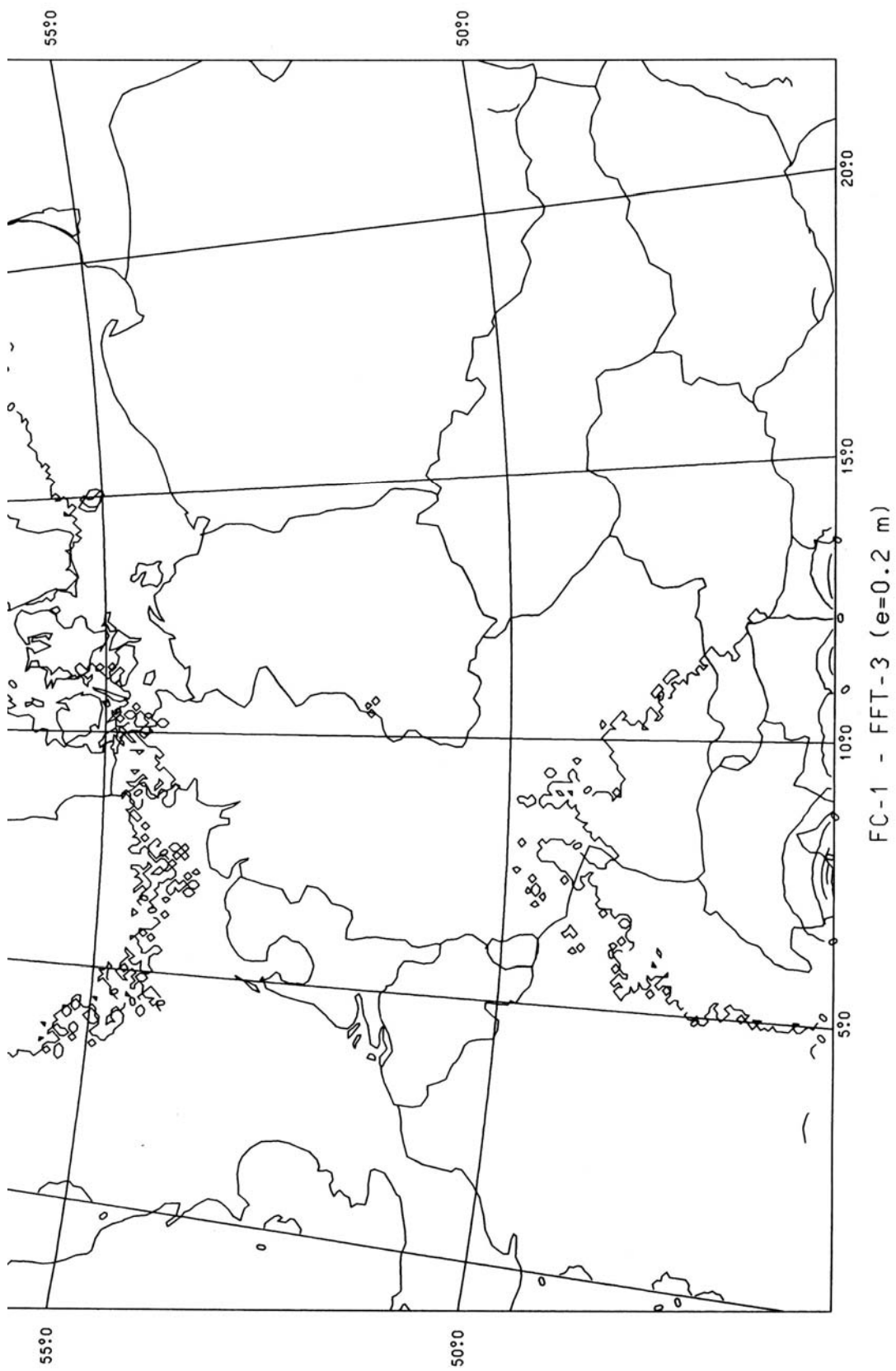


Fig. 9 - FC-1 - FFT-3 (m)

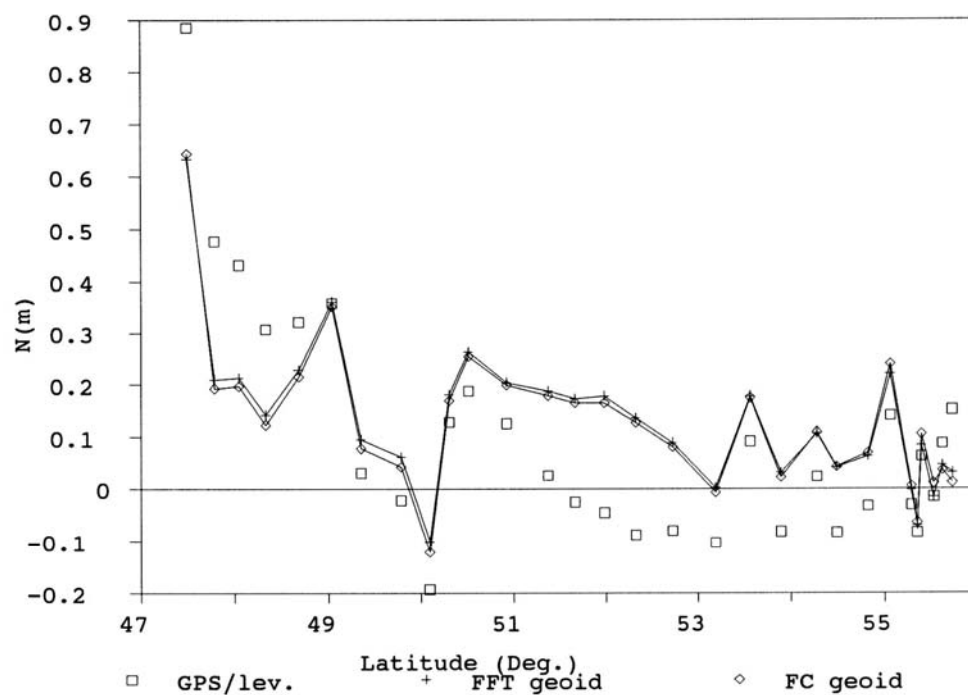


Fig 10 - Comparisons among geoid undulations

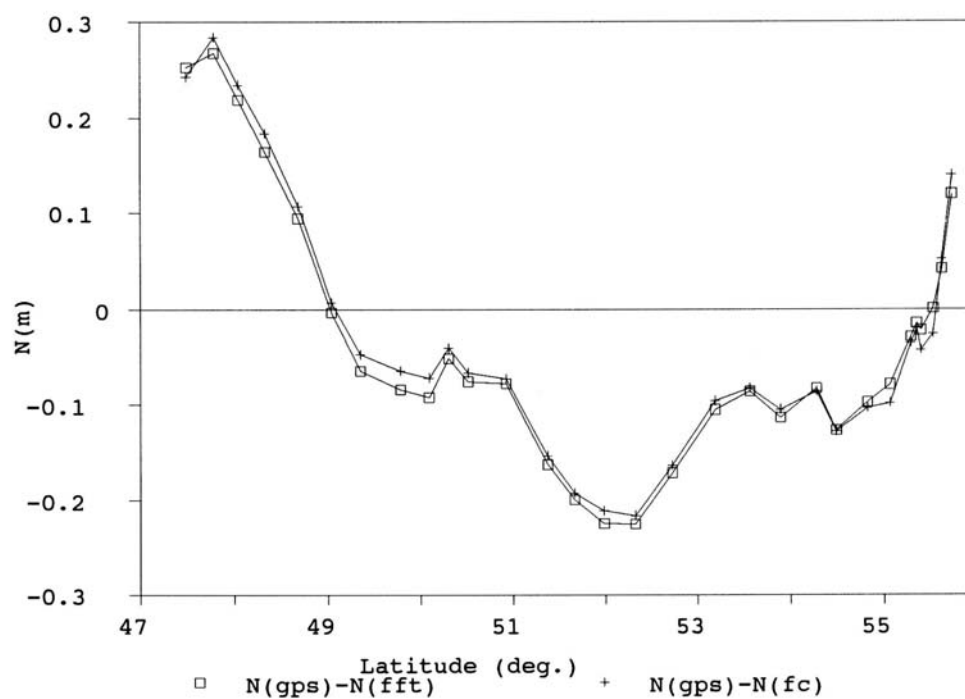


Fig 11 - Differences between geoid undulations

OCEANIC TIDES IN THE MEDITERRANEAN SEA

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1. INTRODUCTION

For many applications, a complete and accurate tidal model of the Mediterranean Sea is needed, no sufficiently accurate model being presently available. In this paper, we show results of a finite element hydrodynamical model to recover the main components of the tidal spectrum in the whole Mediterranean Sea. Notice that a very detailed discussion of the results in terms of accuracy and of tidal phenomenology can be found in Canceill [1993] and Canceill et al. [1994].

2. MODEL SUMMARY AND COMPUTATIONAL CONDITIONS

2.1. Computational procedure

As we wish to only discuss results, we do not recall any detail of the mathematical formulation which was used to turn the classical nonlinear hyperbolic tidal problem into a set of pseudolinear elliptic problems for each component of the tidal spectrum; The reader can refer to Le Provost et al. [1981], and Vincent and Le Provost [1988] for details, especially concerning the way non linear bottom friction is taken into account to dissipate tidal energy. Forcing terms include the tide-generating potential for the astronomical constituents, and second-order advective or frictional terms for the nonlinear harmonics and wave-wave interaction constituents. Tidal loading effects are taken into account in the forcing terms after Francis and Mazzega [1990]. Boundary conditions associated to the problem are a no normal flow condition along land boundaries whereas the sea-surface elevation is specified along marine boundaries. A classical finite element procedure is used to obtain numerical solutions of the problem.

M2 is first computed as a dominant wave through an iterative process [see Le Provost et al. [1981] for details] due to the way nonlinear bottom friction terms are processed. Then, 7 astronomical components (S2, N2, K2, O1, K1, P1, Q1, all significant at the 1 cm level) are computed taking into account fully linearized bottom friction coefficients derived from the M2 velocity field. Last, the knowledge of M2 allowing the computation of nonlinear forcing terms and the determination of the bottom friction coefficients, computation of M4 has then been realized.

Hereafter we restrict our presentation to the M2, S2, K1 and O1 components.

2.2. Model running conditions

Classical use of the model is to run it with the no normal flow condition along the whole land boundary. Hereafter, we cut the land boundary in sections separated by bound points where we specify the sea-surface elevation and release the no normal flow condition. This has been done in order to compensate for drawbacks essentially coming from misknowledge of bathymetry. The selected tidal data used as constraints were chosen among the most trustful within the full data set. For M2, the sea-surface elevation was thus specified at 5 sites, each site having been selected so that each oscillating basin could be influenced by one given datum. Other adequate tidal data were used in the same sense for all the other waves.

For all semi-diurnal computations, the Chezy drag coefficient has been chosen as $2.5 \cdot 10^{-3}$, which corresponds to the mean value generally admitted for the dissipative quadratic friction law. On the other hand, the Chezy coefficient has been set to $5.0 \cdot 10^{-3}$ for all the diurnal components, which was found the appropriate value to best simulate those waves.

Last of all, we have always considered the problem of the tide propagating from the Atlantic ocean to the Mediterranean Sea through the Strait of Gibraltar regarded as open.

2.3. Finite element mesh and bathymetry

The triangular finite element grid that was generated (not shown) has a typical elementary size of 10 to 20 km over the whole basin. This is quite smaller than typical mesh sizes for any of the previous models even when limited to specific areas.

The basic bathymetric chart is derived from the ETOPO5 data base available from the National Oceanic and Atmospheric Administration (NOAA) in Boulder, Colorado. This chart has been jointly used with the International Bathymetric Chart of the Mediterranean provided by the British Oceanographic Data Center to check and correct coastline files that were found inaccurate by 15 km along Africa.

2.4. In situ data for model validation

Tide gauge data from Polli [1959], the International Hydrographic Organization [1979], Purga et al. [1979], Mosetti and Purga [1989], Rickards [1985] or provided by R. Ray (personal communication, 1992) were assembled. The final data set only includes tidal harmonic constituents derived from records longer than fifteen days (one month should have surely been better but would have substantially reduced the data set). In case of several analysis for a given gauge station, the most recent analysis has been retained. Nevertheless, the quality of the data set probably

comparisons have to be taken with caution. Up to 84 tide gauge stations were used in the validation process: The exact number actually varies for each wave and will be reported in the appropriate sections.

To provide a complete accuracy test, we will compare model and in situ data for both individual components and total tide prediction.

3. THE RESULTING M2 COTIDAL MAP

Among the well known features of the mediterranean M2 tide, the produced maps confirm that amplitudes are generally small (less than 10 cm), except near Gibraltar, in the northern Adriatic Sea near Trieste and in the Gulf of Gabes where they can reach 30 to 50 cm.

Comparing our solution with those by Sanchez et al. [1992] and Dressler [1980], the major discrepancy we have identified lies in the presence of a well defined amphidromic pattern in the occidental basin. Neither Dressler [1980] nor Sanchez et al. [1992] display such an amphidrome. This amphidrome is numerically robust as varying boundary conditions by a few centimeters and degrees at the Strait of Gibraltar does not suppress it but only moves it slightly (such an experiment roughly simulates what would occur when considering data errors of about 2 or 3 centimeters and about 5 degrees for the input Atlantic tidal wave). The other main amphidrome in the Strait of Sicily is present in all three global models. On the other hand, south of Crete at the African coast an amphidrome is appearing in our model only. In the Aegean Sea and the high amplitude area in the Gulf of Gabes, performing extended comparisons with both other models is difficult or even not possible, these zones being not modelled or with insufficient resolution by the previous models.

When comparing to local models, our solution is in very good agreement with Polli's Adriatic Sea model [1959] as well as with Mosetti and Purga's [1989] and Molines' [1992] Strait of Sicily and tunisian shelf models. The classical anticlockwise amphidrome in the channel-shaped Adriatic Sea located in the middle of the Ancona - Sestrice axis and the amplitude increase to 27 cm in Trieste are adequately reproduced. For the Aegean sea, comparison with Livieratos and Zadro's model [1977] shows differences in the location of the amphidrome: Whereas it is located at 35.75° north and 26.2° east approximately in their model, ours is displaced northward by about 100 km, which leads to different tidal patterns in the whole basin. However, Livieratos and Zadro show that their solution is sensitive to open boundary conditions at the south of their basin.

Also, the solution has been compared with 84 in situ tide gauge data scattered over the whole domain. Discrepancies between model and data are characterized by a standard deviation of 1.2 cm for the amplitudes and 12 degrees for the phases. 68% of the amplitude differences are within 1 cm whereas 75% of the phase differences are within 15 degrees. The computed

checking of the solution shows that high phase differences are essentially located in zones of low amplitudes so that it has no major impact on in-time predictions. It is especially the case in the Strait of Sicily for which amplitudes are always recovered within 1 cm whereas a somewhat large phase lag of -20° possibly due to the uncertainty on the bathymetric field apparently occurs in the tidal propagation.

4. THE S2 COMPONENT

The typical S2 amplitude is comprised between 2.5 and 7.5 cm except in limited areas north of the Adriatic Sea, north of the Aegean Sea, near Gibraltar and above all on the tunisian shelf where the amplitude reaches 36 cm at Gabes (see Figure). Maps show that the input/output ratio in the area of the Gulf of Gabes is larger for S2 than for M2: The resonance effect for S2 is thus stronger for S2 than for M2. As already mentioned by Molines [1992], the interpretation of such a phenomenon could be that the Gulf has a free oscillation period closer to the S2 period than to the M2 one. The amphidrome located in the western basin is slightly shifted northward toward the spanish coast in comparison with the M2 amphidromic system. Similarly, the Strait of Sicily amphidrome is also shifted northward. An amphidrome clearly appears south of Crete on the african coast, that is again northward compared to M2.

84 tide gauge were examined for comparison. It shows that more than 80% of the amplitude differences between model and data is within 1.0 cm. As for M2, a global negative phase delay between model and data is observed; the phase difference distribution exhibits a standard deviation of 15 degrees with the largest values occuring in areas of weakest amplitudes.

5. THE K1 WAVE

The K1 map (see Figure) displays a unique amphidrome rotating anticlockwise and located near Linosa island in the Strait of Sicily. The K1 amplitude is higher than 2.5 cm in the whole western basin, with a noticeable amplification north of the Adriatic Sea. The model has been compared with 84 tide gauge stations. 55% of the data are located in regions where the K1 amplitude is less than 5 cm. For these data, discrepancy with the model is between -0.5 and 1.5 cm for the amplitudes whereas phase differences are scattered in a ± 20 degrees band with larger values up to 30 degrees near the amphidrome. In the remainder of the domain, the solution is underdamped; model amplitudes and data differ from -0.5 cm to 3 cm with the largest discrepancies in the northern Adriatic Sea where the K1 tide is maximum; phase discrepancies are essentially comprised between -10 and 10 degrees. Underdamping of the K1 tide could not be suppressed by increasing the Chezy coefficient. A possible reason could be the transfer of kinetic energy into potential energy in areas where internal dissipation

6. THE O1 COMPONENT

The O1 tidal pattern (see Figure) is slightly different from the K1 pattern. An amphidromic system has been evidenced east of Kerkenna island: It is thus shifted to the south in comparison with K1 amphidrome. Convergence of cotidal lines near 19° east along the Lybian coast has not been noticed for K1 and indicates that a possible virtual amphidrome exists over Lybia. As for K1, cotidal lines have a specific convergence pattern west of Sicily. This could be the signature of trapped diurnal waves associated with the strong diurnal currents existing in this region.

O1 Amplitudes increase westward from 1.5 to 3.0 cm in the whole western basin including the Tyrrhenian Sea and from zero to 2 cm in the eastern basin. Once again, the Adriatic Sea is the place where O1 is amplifying up to 6.5 cm in Trieste, 1 cm more than the gauge value. Among the 77 gauge stations selected for individual comparisons, 50% show a ± 0.5 cm difference with the model, and 30% with a 0.5 to 1.5 cm difference. As for K1, the solution is thus underdamped. A phase discrepancy between -20 and 20 degrees is obtained for 80% of the data.

7. CONCLUSION

A new set of 9 major oceanic tidal constituents has been determined in the Mediterranean Sea. Comparisons with a large set of tide gauges provide with the following estimated accuracy: Better than ± 1.2 cm for the amplitudes of M2, ± 1 cm for S2 and ± 0.5 cm for the other components excepted K1 which is unexplainably underdamped; a ± 15 degrees uncertainty is reached for the Greenwich phases. For the 49 stations allowing comparisons in terms of total predictions using the 8 major tidal waves (M2, S2, N2, K2, K1, O1, P1, Q1), model and observations fit to approximately 3 cm.

This model has been used to compute a set of corrections for GEOSAT ERM (Exact Repeat Mission) data from November 1986 to October 1989. It is now used by ESA and NASA/CNES respectively for the ERS-1 and TOPEX/POSEIDON data. It is expected that this will improve significantly the potential of such data for the Mediterranean Sea.

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EFFORT TOWARD A GEOID MODEL FOR SOUTH AMERICA

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GETECH (Geophysical Exploration Technology) of the University of Leeds has been involved since 1986 in continental scale gravity and magnetic compilation projects funded by oil and mineral industries. One of those projects, the "South America Gravity Project" (SAGP, 1988-1991) was sponsored by 14 international petroleum companies and supported by many national organizations within South America. SAGP was able to compile over 500 000 gravity stations onshore and 1 800 000 offshore. Since the end of the project in 1991 several major gaps have been infilled, particular in central-north Brazil, through a special project.

A current research study being carried out by EPUSP (Escola Politécnica-Universidade de São Paulo) and GETECH is using the SAGP and the new Brazilian data to improve the geoid model of South America. This study is being done at the University of Leeds with the financial support of CNPq (Brazil)/Royal Society (England). Links with the International Geoid Commission of IAG (International Association of Geodesy) and contacts with many geodesy groups in different institutes with profitable and important exchange of experiences are being made.

To generate a South American Geoid model the complete onshore SAGP gravity file has been organized in a way to facilitate the computations. After that, mean gravity anomalies (Bouguer and Free Air) and mean heights have been computed for blocks of 5'x5' where at least two points were available within a block. Using this file of 5' mean anomalies, new mean values will be derived for block sizes of 10', 20', 30' and 60'. These new mean values for the different block sizes will be the basic data together with digital terrain and geopotential models to compute geoidal heights. The modified Stokes integral will be used to derive the short wavelength component of the geoidal height. The long wavelength component will be computed using a low order geopotential model, e. g., GEMT2.

It is also intended to use the marine gravity data around the South America continent to estimate mean anomalies (Free Air) in block sizes as onshore and in this way improve the data distribution for the numerical integration.

It is expected that the results to be obtained will be an important contribution to the knowledge of geoidal heights on a regional scale. Further studies should be undertaken in each country to derive detailed models with the participation of national organizations.