

DESIGN OF A RELATIONAL DATABASE FOR GEODETIC APPLICATIONS

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ABSTRACT.

In this paper the design process of a relational database to store geodetic data is presented. The database structure and the relations among files have been established. Moreover, a communication module to link the database and scientific languages was created.

1. INTRODUCTION

Nowadays, there is an increasing number of data to be used in geodetic computation, therefore the design of a system to store such amounts of data it is a necessary task. In the past the use of sequential files was associated with the fact that scientific routines ran with this kind of files. The use of a database language has improved that because it is much easier to hand files in a database structure than in a sequential form. The database language makes the programming easier than with traditional languages. All normal programming techniques can be used with database language: decision making, looping, sorting, searching, data manipulation.

There is another important advantage. It is possible to make a connection between the database language and a high-level programming language (i.e., FORTRAN). This is very useful because database structures can be done with the database language and a scientific treatment of the data can be used with the programming language.

2. GENERAL DESCRIPTION

The support for the Database System and Users' Application was a Personal Computer with **MS/DOS** operative system because it is one of the most extended in the world.

The next step was to design the database structure. This selection allowed to control all data in the Data Bank, check common points and errors of integrity in the database files and get all the information in a fast and controlled way.

Once the database structure has been designed, our problem was to hand thousands of data. The manager of this database should be fast and give a guarantee on the maintenance of the integrity of the database. The manager selected was dBASE IV because:

- It is a very powerful Database language. It features the choice of a completely menu-driven interface to create database structures and to add, change and delete data items or a set of easy to learn commands.

- It has powerful tools that can make complete programs or to be used as a stepping stone to create your own programs.

- It has a compiler to link programs to form a working integrated system.

The basic structure of the database is shown in Figure 1. There are four well-difference parts:

a) Data collection and storage

The structure of the database allows to add other data coming from different Organizations.

b) Maintenance and validation of data

The internal tools of the database are used to guarantee the integrity of Data Bank.

c) Getting data from the database

The user can get data from the Data Bank in an easy way using the data retrieval methods of the dBASE IV language.

d) User's application

By using scientific software included in this package the user can obtain others types of magnitudes related with those that are in the database.

The three first points are related to the administration of the database while the fourth refers to the scientific software developed.

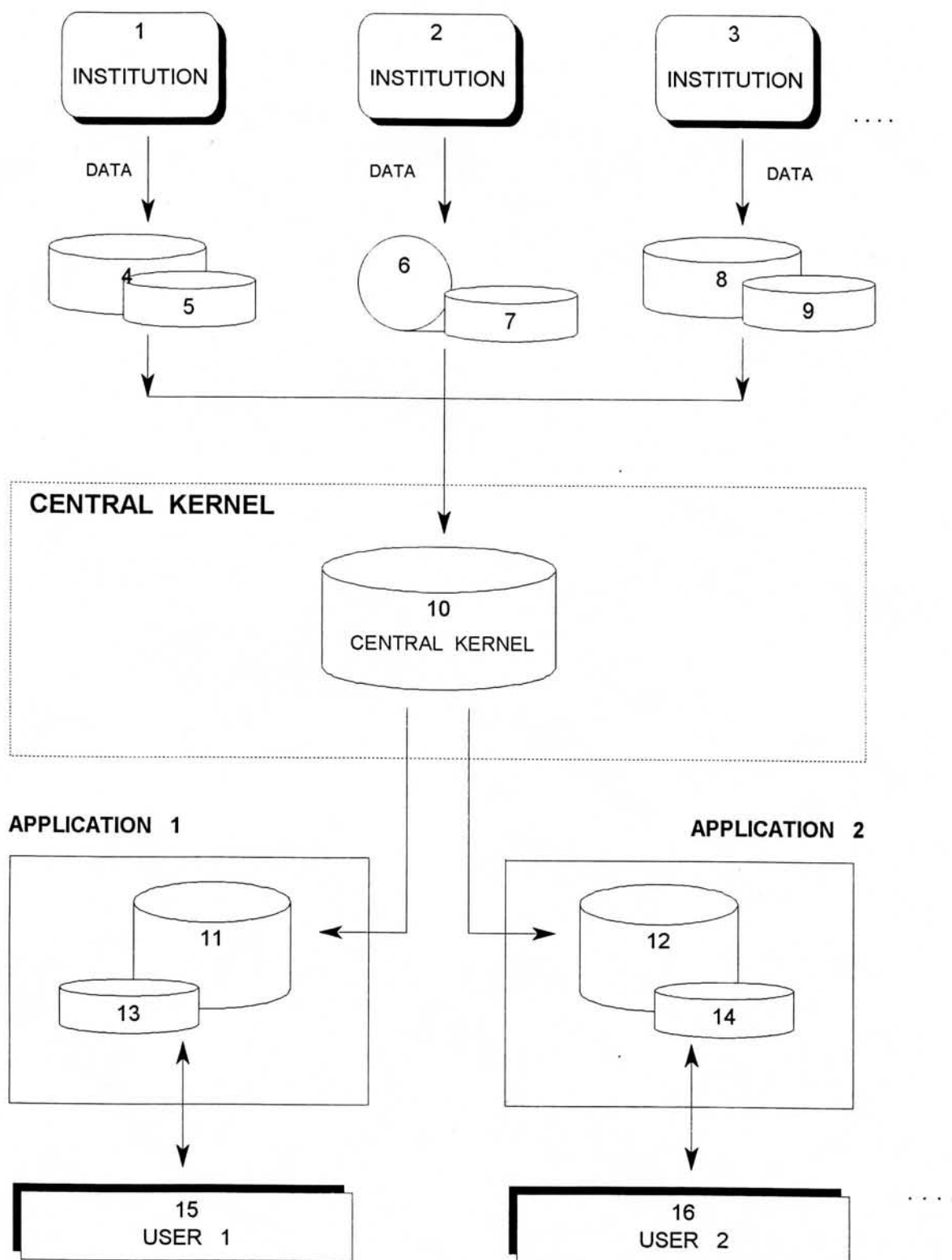


Figure 1

3. MODULES

3.1 Data Model

The different files of the database and the normalization relations can be seen in Figure 2. A brief explanation of the content of the files is given in the sequel. This is the model that has been designed.

[METODOS] Techniques used to get the data. Each observation that appears in the **[OBSERVAC]** and **[OBSERBUS]** files will have associated a code suggesting the technique used to get it.

[CORTERRE] Methods used in the calculation of the topographic correction.

[DETELEVA] Methods used in the determination of the elevation of the point.

[ENTITY] A code is assigned to each organization to identify the origin of data. This code will also keep the specific information of an organism that owing to its structure could not be introduced directly in the **[OBSERVAC]** and **[OBSERVUS]** files.

[COORDENA] Coordinates of the points. The information contained in this file could be used by the user but it cannot be modified.

[OBSERVAC] Observations carried out in the points that appear in the **[COORDENA]** file. In this file each point will have a variable and limitless number of observations. In this way, it is possible to get all the observations related with a given point by saying only the coordinates of the point.

[COORDEUS] Coordinates of the points where the user has done their own observations.

[OBSERVUS] Observations done by the user that correspond to the points of the **[COORDEUS]** file.

[FUNCION] Type of observation which value appears in the **[COORDENA]** and **[OBSERVAC]** files

[SISTPOSI] System of positioning.

[TIPOBSER] Type of observation in the sense of individual or mean value.

[TIPOELEV] Elevation type: land, lake surface, lake bottom, etc.

[PAISES] A code is assigned to each country to identify the origin of data.

[USERS] Users that are authorized to use the application.

[BASICOS] File used to maintain the integrity of the **[COORDENA]** and

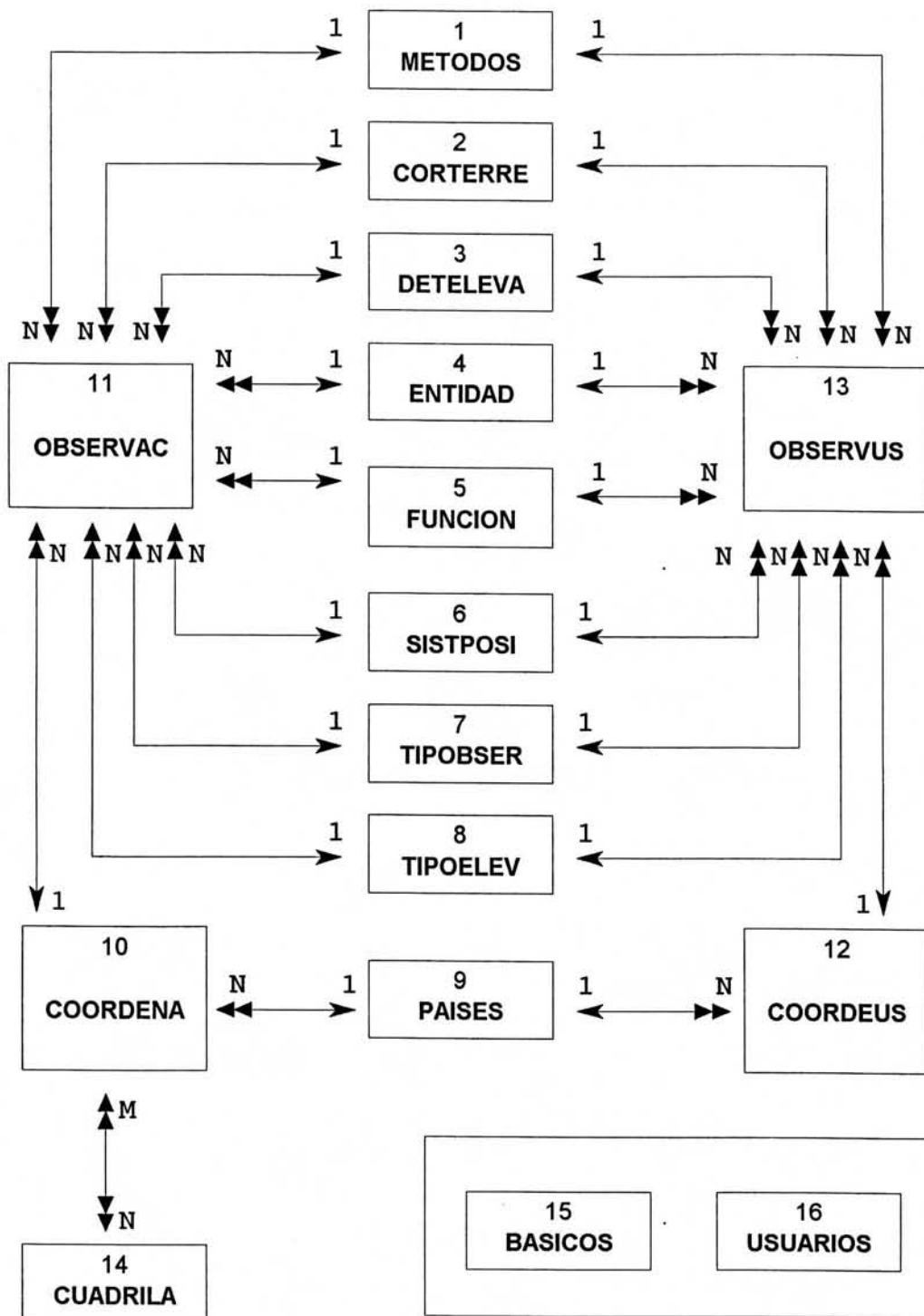


Figure 2

[OBSERVAC] files.

[CUADRILA] The data of this file are used to accelerate the computations in the users' applications.

An example of the structure of [OBSERVAC] file is given in Figure 3.

The data model here detailed has some advantages:

- A database management system like dBASE IV gives us the means to process the data into more useful information. We can store data on files, retrieve information from various related files, change the definition of the files if the necessity arises, and print the information on reports, forms and labels.

- This databank is a relational database with several files related by common fields. These key fields are the link between two related files.

- This structure controls the redundant data, i.e., data that are stored in more than one file in the database.

- One could add many methods, functions, points and observations.

- It allows to study the behaviour of the magnitudes observed in particular areas.

- Moreover the application could support any number of users, permitting a total control on the data introduced by each of them.

3.2 Communication Module

The data model can solve the problem of the data storage and management. Generally, a good database manager does not have routines for scientific computations. Besides, we had many powerful FORTRAN routines that could do all the calculations that we wanted to include in our application. Therefore, a communication module was developed between dBASE IV and FORTRAN.

With the dBASE-FORTRAN communication module is possible to use routines written in FORTRAN language with data stored in dBASE files. In this way, one could separate the functions of both languages, so in each moment the appropriate program is used to obtain the best results: the input/output operations, management, maintenance and administration of data are controlled by dBASE; the scientific computation is carried out with FORTRAN. All the processes are very fast. In each moment the necessary value is sent to the module of communications. This achieves the solicited option and returns the corresponding values to continue the process.

FIELD	FORMAT	LENGTH	DESCRIPTION
CSECUENCIA	Character	15	Record Number
C_OBSERVAC	Character	3	Observation number of the point
CSISTPOSIC	Character	2	System of positioning
CTIPOOBSER	Character	2	Type of observation
PREC_POSIC	Character	2	Accuracy of position
CELEVACION	Character	2	Elevation Code
PREC_ELEVA	Character	2	Accuracy of Elevation
CDETERELEV	Character	2	Determination of the elevation
ELEVACION	Numeric	8.3	Elevation
C_FUNCION	Character	2	Function code
VL_FUNCION	Numeric	8.2	Value of function
COR_TERRE	Numeric	8.2	Terrain Correction
CORTERRE	Character	2	Terrain Correction Code
DENSIDAD	Numeric	4.2	Density
PREC_FUNCIO	Character	2	Accuracy of the function
COR_FUNCIO	Character	2	Correction of the function
C_METODO	Character	2	Function Determination method
CONFIDENC	Character	2	Confidentiality
VALIDACION	Character	2	Qualification
C_ENTIDAD	Character	2	Institution Code
VELOC_SHIFT	Numeric	3	Velocity of the ship (Marine observation)

Figure 3

Once all the problems presented have been overcome, the link between dBASE and FORTRAN allows to give an important improvement in the applications, mainly in two points:

- a) Security and integrity of the data in multiuser environments.

The change of sequential files for relational databases has important advantages like the application can be used in a multiuser environment. The manager database will control the maintenance of files and index files automatically. Then, the users can focus all efforts in the analysis of the data, remaining to the margin of the routine task.

- b) Input/Output process

With dBASE IV the phase of designing the input necessary to produce the desired output is a task that is carried out more easily than with others languages. dBASE IV generates input screens that allow easy entry of the item to be stored. Also, it uses function keys to do the main task: to enter new records onto the file, to change records already on the file, and to delete records from the file.

Moreover, it is possible to use of menus that can be quick and easily used making very fast the operations of data handling.

Everything without losing the capacity of calculation that we had up to now due to all the mathematical routines carried out in the communication module.

4. FUTURE PERSPECTIVES

The conclusion of this paper is a view to the future. Keeping in mind the idea thought at first and taking into account the development of our application the future ways could be:

- a) An emigration toward the graphic environments. Future applications will be developed in graphic environments like OS/2 or WINDOWS. This will allow not only "read" the data but also "look" in graphics, diagrams, etc. which will help us to understand better the quantities under study.

- b) Powerful manager of data. The users' applications should accede easily to more complex databases in remote servers.

- c) Connection between programs.

One could also include in our application software as a word processor, more sophisticated scientific routines, etc., allowing the link between different programs.

Acknowledgements

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Yourdon, E., *The Practical Guide to Structure Systems Design*. Prentice Hall. 1980.

THE IGeS GLOBAL MODELS ARCHIVE

M.A. Brovelli, F. Migliaccio

D.I.I.A.R. - Politecnico di Milano - Milano - Italy

It is well known that the anomalous potential of the gravity field:

$$T = W - U , \quad (1)$$

obtained subtracting the normal potential U from the potential of the earth's gravity field, can be represented as a spherical harmonic expansion:

$$T(r, \vartheta, \lambda) = \frac{GM}{r} \sum_{n=2}^{\infty} \left(\frac{R}{r} \right)^n \sum_{m=-n}^n T_{nm} Y_{nm}(\vartheta, \lambda) \quad (2)$$

where: (r, ϑ, λ) = spherical coordinates;

R = earth radius;

n, m = degree and order of the expansion.

The series (2) is convergent outside a sphere containing the masses that generate T .

A "global model" is by definition a finite parametric model of the anomalous potential T , that is a function of the point P which is harmonic at least in the space external to the masses and which depends on a certain finite number of given parameters.

The global models presently used are in most cases represented by a function of the type (2) truncated at a finite degree N which is defined as the maximum degree of the model. For that function the "parameters" T_{nm} are given.

The determination of the T_{nm} coefficients up to a fixed maximum degree N is, from a theoretical point of view, quite simple if the potential T is supposed to be known on a sphere. Obviously, the real situation is much more complicated, as $T(r, \vartheta, \lambda)$ is not known as a function of (ϑ, λ) at varying altitude.

In fact the data available are of various types and are obtained from different measurements representing different functionals of T . So the total number of data can be very large but their characteristics are quite unhomogeneous. Besides, it is not possible to obtain data from

measurements uniformly distributed over the earth's surface (which is especially true for land measurements) both for political and for geographical reasons.

Presently, the determination of global models of geopotential is based on four different types of data:

- 1) data coming from satellite tracking;
- 2) gravimetric data on land and sea, supplied with point positions;
- 3) radar-altimetric data on sea and ocean, supplied with information on oceanic currents;
- 4) geophysical data, in areas where gravimetric data are not collected.

A particular case of pseudo-global models is represented by the so-called "tailored" models, which are obtained adjusting global models to fit regional data. Obviously, such models are particularly suited to represent the gravity field only in the region of interest, so that they should not be considered as real global models.

Some of the most recent global models have been collected by the International Geoid Service and stored in a directory (public domain) in the I.Ge.S. HP730: in fact it seems to be useful in these days to review the state of the art on this particular item, also considering that a new important effort is being made by DMA-NASA to produce a significantly enhanced new global model, incorporating a much larger and widespread number of data, including data formerly classified. The feeling is that having at hand the best products of today, will facilitate comparisons and assessment of the new foreseen results.

The available models are accessible via "anonymous ftp" from `ipmtf4.topo.polimi.it` in the directory MODEL.

In the following, a list of the available models can be found, with the appropriate references and the names and addresses of the persons to be contacted for information.

OSU91A

The OSU91A potential coefficients model was obtained as a merger of two potential coefficients data sets:

- a potential coefficients model to degree 50 selected on the basis of a combination of the GEM-T2 potential coefficients, surface gravity normal equations and one year of Geosat altimeter data;
- a potential coefficients model from degree 51 to degree 360 calculated using a global set of adjusted anomalies (combination solution with the GEM-T2 potential coefficients and a recent 30' x 30' mean gravity anomaly data set).

Reference:

R.H. Rapp, Y.M. Wang, N.K. Pavlis "The Ohio State 1991 geopotential and sea surface topography harmonic coefficient models", Report n. 410, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio, 1991.

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JGM-1 and JGM-2

These models have been developed by NASA/GSFC Space Geodesy Branch, the Center for Space Research of the University of Texas at Austin and CNES. JGM-1 describes the gravity field up to degree and order 60, JGM-2 up to degree and order 70.

The files available in the IGeS archive are:

JGM-1S (satellite data only)
JGM-2
JGM-2G (gravity data only)
JGM-2S (satellite data only)
JGM.tide (background tide model for JGM-1 and JGM-2)

Reference: R.S. Nerem, F.J. Lerch, J.A. Marshall, E.C. Pavlis, B.H. Putney, B.D. Tapley, R.J. Eanes, J.C. Ries, B.E. Schutz, C.K. Shum,

M.M. Watkins, J.C. Chan, S.M. Klosko, S.B. Luthcke, G.B. Patel, N.K. Pavlis, R.G. Williamson, R.H. Rapp, R. Biancale, F. Nouel "Gravity model development for TOPEX/POSEIDON: joint gravity models 1 and 2", Journal of Geophysical Research, vol. 99, n. C12, pp. 24421-24447, Dec. 15, 1994.

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JGM-3

The model has been developed as a joint solution between the University of Texas at Austin and NASA/GSFC. It describes the gravity field up to degree and order 70.

A presentation of the model was made at the XIX General Assembly of EGS (Grenoble, France, 1994). The paper is still being working.

Reference: B.D. Tapley, M.M. Watkins, J.C. Ries, G.W. Davis, R.J. Eanes, S.R. Poole, H.J. Rim, B.E. Schutz, C.K. Shum, R.S. Nerem, F.J. Lerch, J.A. Marshall, S.M. Klosko, N.K. Pavlis, R.G. Williamson "The JGM-3 gravity model" (manuscript in preparation).

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GEM-T3

The model has been developed by NASA/GSFC from tracking data acquired on 31 satellite orbits in combination with surface gravimetry and satellite altimetry (GEOS-3, SEASAT, GEOSAT). It describes the gravity

field up to degree and order 50.

- References: F.J. Lerch, R.S. Nerem, B.H. Putney, T.L. Felsentreger, B.V. Sanchez, S.M. Klosko, G.B. Patel, R.G. Williamson, D.S. Chinn, J.C. Chan, K.E. Rachlin, N.L. Chandler, J.J. McCarthy, J.A. Marshall, S.B. Luthcke, D.E. Pavlis, J.W. Robbins, S. Kapoor, E.C. Pavlis "Geopotential models from satellite tracking, altimeter and surface gravity data: GEM-T3 and GEM-T3S", Journal of Geophysical Research, vol. 99, n. 82, pp. 2815-2839, Feb. 10, 1994.
- R.S. Nerem, J.F. Lerch, R.G. Williamson, S.M. Klosko, J.W. Robbins, G.B. Patel "Gravity model improvement using the DORIS tracking system on the SPOT-2 satellite", Journal of Geophysical Research, vol. 99, n. B2, pp. 2791-2813, Feb. 10, 1994.
- S.R. Nerem, F.J. Lerch, S.M. Klosko, G.B. Patel, R.G. Williamson, C.J. Koblinsky "Ocean dynamic topography from satellite altimetry based on the GEM-T3 gravity model", Manuscripta Geodaetica, vol. 19, pp. 346-366, Sept. 1994.

Reference person:

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Fax: (301) 286-1760
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GRIM4

It was produced as a cooperative work between GRGS and GFZ/Potsdam and describes the long wavelength gravity field (up to degree and order 72) thanks to the use of space geodetic, altimetric and gravimetric measurements. A paper describing the GRIM4 model will be published in the "Bulletin Geodesique" by the authors:

R. Biancale, G. Balmino, J.M. Lemoine, J.C. Marty, B. Moinot (CNES-GRGS, Toulouse, France);
F. Barlier, Y. Boudon (OCA/CERGA-GRGS, Grasse, France);
P. Schwintzer, C. Reigber, A. Bode, H. Massmann, J.C. Raimondo (GFZ, Potsdam, Germany).

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GFZ93a and GFZ93b

They were produced by the German Processing and Archiving Facility (D-PAF) which operationally processes the ERS-1 data to obtain high precision geophysical products. Among these products are the oceanic geoid heights, based on a high resolution gravity field model complete to degree and order 360. Two solutions for the 360 gravity field were computed, combining long wavelength structures of the GRIM4-C3 gravity field with high resolution features detected from altimeter satellites and terrestrial gravity data. For the GFZ93a model a 9 months mean sea surface from ERS-1 fast delivery data (April to December 1992) is used; for the GFZ93b model a mean sea surface from Ohio State University containing altimeter data from Geosat, Geos3 and Seasat is used. The combination of all data sets is performed by adding all normal equation systems, separately determined for each data set by least squares harmonic analysis.

Reference: Th. Gruber, M. Anzenhofer "The GFZ 360 gravity field model",
in: The European Geoid Determination, Proceedings of Session G3 -
European Geophysical Society XVIII General Assembly, Wiesbaden,
Germany, May 3-7, 1993, R. Forsberg and H. Denker Eds., 1993.

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GPM2

It was produced by the University of Hannover and describes the gravity field up to degree and order 200.

The computational aspects and a description of the used data have been published in German.

Reference:

H.-G. Wenzel "Hochauflösende Kugelfunktionsmodelle für das Gravitationspotential der Erde", Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover, n.137, Hannover, 1985.

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IFE88E2

It was produced by a cooperation of the Institut für Erdmessung, Universität Hannover (Germany), the Kort-og Matrikelstirelsen (denmark) and the Statens Kartverk (Norway). IFE88E2 is a tailored model, complete to degree and order 360. The sources of gravity data used for the model are:

0.5° x 0.5° mean gravity anomalies for Scandinavia;

0.5° x 0.5° mean gravity anomalies for Europe.

IFE88E2 is based on the OSU86F coefficients set, which were used as a start model. The low degree coefficients of OSU86F were not modified due to the limited data collection area and due to the fact that these coefficients are well determined from the analysis of satellite orbit perturbations.

Reference:

T. Basic, H. Denker, P. Knudsen, D. Solheim, W. Torge "A new geopotential model tailored to gravity data in Europe", in "Gravity, Gradiometry and Gravimetry", International Association of Geodesy Symposia, Symposium n. 103 (R. Rummel and R.G. Hipkin eds.), Springer-Verlag, pp. 109-118, 1990.

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R.H. Rapp "Use of altimeter data in estimating global gravity models", Lecture notes in earth sciences, F. Sanso' and R. Rummel eds., vol.50, pp.261-283, Springer-Verlag, 1992.

ACCURATE ORTHOMETRIC HEIGHTS FROM GPS: COMBINED NETWORK ADJUSTMENTS USING GPS, DIFFERENTIAL LEVELING, AND CORRELATED GEOID MODELS

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ABSTRACT

Assigning two adjustable heights (orthometric and ellipsoid) to each control point in a survey allows the network adjustment both to uniquely determine geoid separations, when a geoid model is not present, and to include as observables geoid heights, when one is. By modeling correlated errors for geoid separations, the uniquely-determined separations can be used to construct a geoid model, while overdetermined separations can be used to construct a model of residuals for the primary geoid model. When the residual model predicts residuals for geoid heights at points which have been observed by GPS only, these control points receive adjusted orthometric heights produced by the combination of adjusted GPS (ellipsoid) heights and the improved (adjusted) geoid model. A geoid model can be sequentially improved using this procedure until its variance is less than observational noise, at which point GPS observations will produce orthometric heights which have the same order of accuracy as the GPS observations themselves. An adjustment system designed to achieve this goal, consisting of a combined network adjustment, an undulation model-builder, and a residual model-builder, is described.

1. INTRODUCTION

The primary difficulty involved in integrating GPS measurements with conventional surveying measurements lies in reconciling two horizontal surfaces: the reference ellipsoid, which provides the vertical reference for GPS observations, and the geoid, which provides the vertical reference for terrestrial observations such as differential leveling. Unless the two surfaces are modeled independently in network adjustments, the datum defects between them can force into observational residuals the systematic errors caused by requiring the two surfaces to coincide in the deterministic

model, and can result in height coordinates of reduced accuracy. Introducing a geoid model permits separate modeling of the ellipsoid and geoid surfaces, but unless the geoid heights from the model are included as observables in the network adjustment, any error in the model will be permanently lodged in the adjustment results.

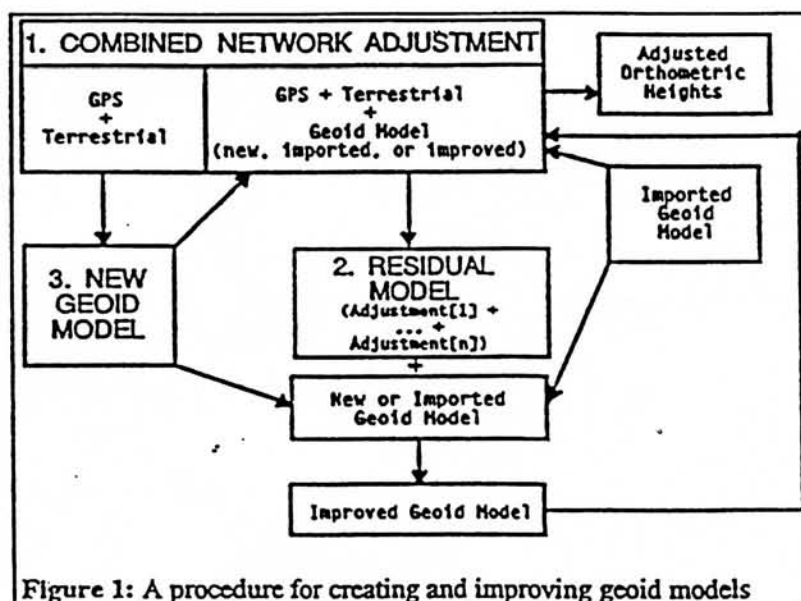


Figure 1: A procedure for creating and improving geoid models

The diagram in Figure 1 illustrates a computation system based upon an adjustment model in which each control point is assigned two adjustable heights: an ellipsoid and an orthometric height. Consequently the ellipsoid surface and the geoid surface are modeled separately. When a geoid model does not yet exist for a network, one may be created for it ("New Geoid Model") using the results of a combined adjustment of GPS and terrestrial observations. When a geoid model does exist, having been either created or imported, geoid heights derived from it may be included in a combined network adjustment which computes residuals for the overdetermined geoid separations. If correlated errors are modeled for the observed geoid heights, the results of this adjustment can be used to generate a residual model which will predict corrections to the geoid model at any point in the region of the network. Each sequential adjustment can therefore serve to improve the original geoid model until the improved model's variance is less than the observational noise in the network. At this point the errors in the improved geoid model will be literally too small to measure, and the adjusted orthometric heights at points observed by GPS only will have the same order of accuracy as the GPS observations themselves.

2. THE ADJUSTMENT MODEL

The adjustment model uses the familiar variation of coordinates technique, with observation equations as expressed below [Uotila (1986)]:

$$Ax + l = v \quad (1)$$

The horizontal coordinate system for the adjustment is either ellipsoidal (ϕ, λ) or rectangular (N,E). The vector of unknowns (x) consists of four possible parameters for each point — the two horizontal coordinates plus ellipsoid height (H) and orthometric height (h) — plus a variable number of nuisance parameters for datum transformations and the like. Using this notation, the geoid separation N at any point is represented by [Bomford (1975), p. 592]:

$$N = H - h \quad (2)$$

The observations are grouped into three main classes: GPS, terrestrial, and geoid. Geoid observations are expressed as geoid heights. Full observational covariance matrices Σ are modeled for each class of observation and are inverted to form the weight matrix P . Forming the covariance matrix for geoid heights is a primary subject of this discussion (see Sections 3 and 5 below), but it can be noted now that it is not always necessary or desirable to form off-diagonal covariant terms for geoid models.

The geometric relationships among the various geodetic surfaces and vertical observations are illustrated in Figure 2. Of the five numbered control points shown on this drawing, all except for the middle point (3) were observed by both GPS and terrestrial measurements. Point 3 was observed by GPS only. The square symbols, which represent observed orthometric height (e.g., by differential leveling) and thus the "observed" geoid, also represent, for purposes of simplification, the "true" location of the geoid, even though there will obviously be some error in the terrestrial observations. (It should be noted that this observed geoid is not an equipotential surface, but rather is the locus of $h = 0$ as defined by reference monumentation and differential leveling; this surface and the equipotential surface are closely related, however, and datum defects between them are easily resolved in the network adjustment by transformations to GPS and/or geoid observations.) Figure 2 shows, in exaggerated form, errors in the geoid model which more or less appear to reflect an undetected local undulation.

Estimating the orthometric height or, what is the same thing, the geoid separation at Point 3 requires applying some model of the geoid surface at that point. The geoid separations at the other four points are uniquely determined by differencing their adjusted ellipsoid and orthometric heights. If an existing geoid model which contains useful information can be imported, it can be combined into the network adjustment from the outset, but if one is not yet available it can be created using the Undulation Model-Builder

described below in Section 5. The following discussion will proceed on the presumption that a geoid model has been introduced into the network

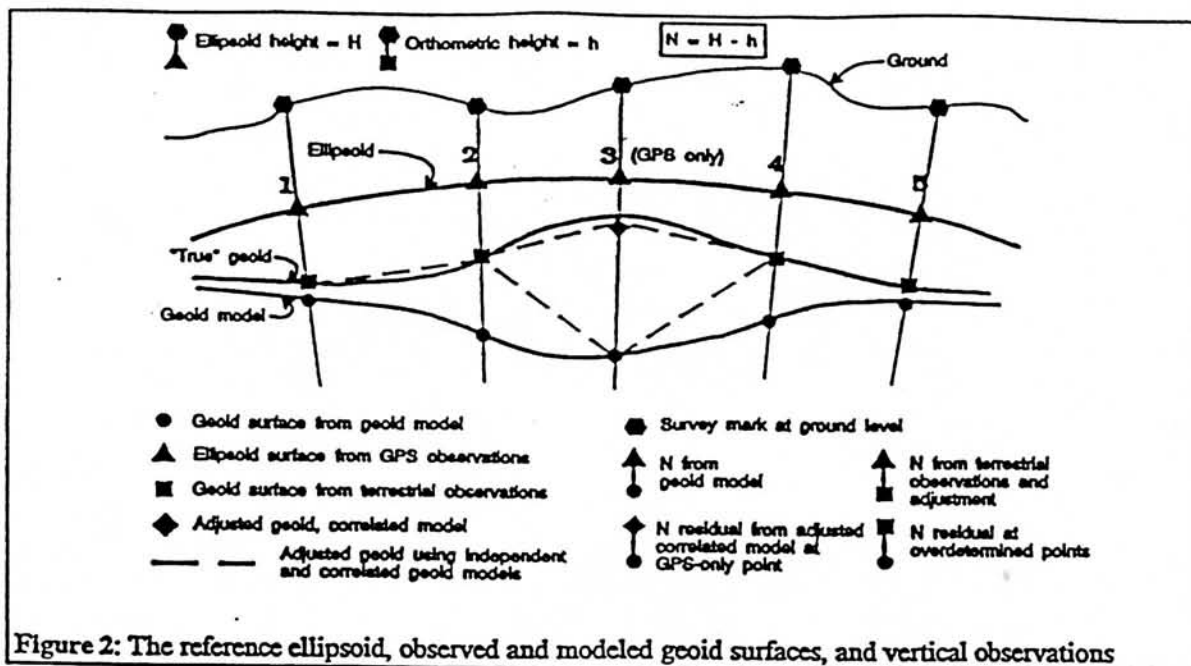


Figure 2: The reference ellipsoid, observed and modeled geoid surfaces, and vertical observations

The usual sequence of adjusting a combined network of this sort will be first to perform minimally constrained adjustments on the GPS and terrestrial portions of the network, for the purposes of establishing accurately their respective observational errors and of assigning appropriate weights. When the geoid model is introduced into the adjustment, it will typically be necessary to apply variance component estimation in a sequential reweighting scheme until the reference variance for the geoid heights approaches unity [Lucas (1985)]. This process requires the presence of redundant overdetermined geoid heights – points observed by both GPS and leveling observations in addition to geoid heights derived from the geoid model – and once it has been carried out the three classes of observation will be normalized.

Since the adjustment must produce unique adjusted geoid separations at the overdetermined points, one or more of the observation classes at those points will have to receive residuals. The class of observation receiving the largest residual will be the class with the largest variance, and hence lowest weight. Typically, for the purpose of this discussion, the geoid model will receive the largest adjustment; if it does not, that fact will imply that the geoid model's variance is less than the observational noise, and therefore that any error in it is too small to be measured by the existing observations. If error in the geoid model is, in fact, too small to be measured, then the network is already in a state to provide accurate orthometric heights from GPS observations, but usually it will be necessary to improve the geoid model (by adjusting it) to bring it to this desired state.

It is notable that introducing as observables geoid heights from a geoid model not only makes it possible to estimate the variance of the geoid model, and therefore to propagate accurate error into the orthometric heights produced at GPS-only points, but also makes it possible to compute a datum transformation for the geoid model as a whole, the transformation parameters being the two deflections in the vertical plus a height constant. By taking advantage of this feature, the geoid model can be transformed to a reference ellipsoid of choice.

If the errors in the observed geoid heights are treated as being independent (uncorrelated), then the adjustment will not assign residuals to observed geoid heights at GPS-only points. This is illustrated in Figure 2 by the dashed ("adjusted geoid") lines at Point 3 which are connected to the surface of the geoid model (i.e., to the circle) at that point. As suggested there, this may produce sharp discontinuities and breaks in the adjusted geoid surface.

However, if the errors in the geoid model are correlated, the observed geoid heights at GPS-only points will in fact be assigned residuals, the magnitude of which will depend jointly upon the degree of correlation between them and the overdetermined points and upon the residuals distributed to the overdetermined points. This is indicated in Figure 2 by the dashed lines at Point 3 connected to the diamond symbol near the "true" geoid surface. As suggested there, this may produce a smoother adjusted geoid surface more closely resembling the "true" geoid.

3. THE TEST NETWORK

A simplified view of the test network is shown in Figure 3. Located in the mountains southeast of Los Angeles, California, the network has 43 control points, all of which were occupied by terrestrial observations, including leveling, and thirteen of which were occupied by GPS. The GPS points are shown as numbered dots. In all of the test adjustments illustrated here, Points 1, 2, and 3 were held fixed in orthometric and ellipsoid height (set to equal values) for the vertical constraints, resulting in a defined geoid separation of zero at these points. Both GPS observations (i.e., the reference ellipsoid) and the geoid model were transformed to this defined system. Inner constraints were used for horizontal constraints.

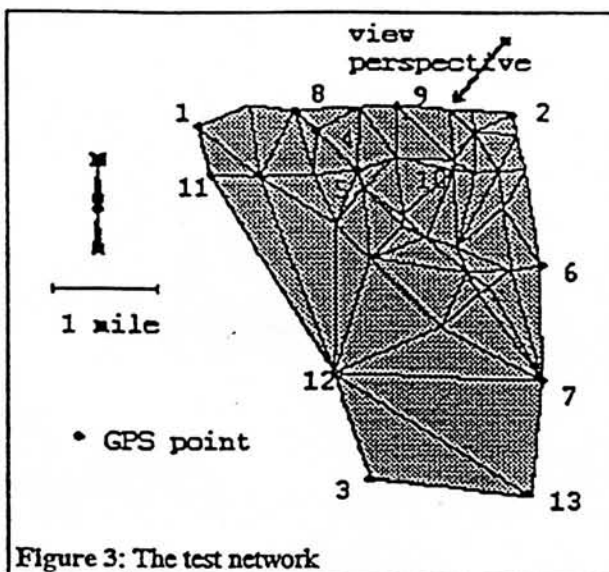


Figure 3: The test network

The network contains 162 GPS observations (54 vectors), 370 terrestrial observations, and (when included in the adjustment) 43 geoid heights. Horizontal precision varies between $1:10^5$ for shorter lines and $1:10^6$ for longer lines. The standard error for differences in orthometric height between control points is less than 0.005 foot (about 1 mm). The standard error for differences in ellipsoid height (GPS vertical observations) ranges between 0.010 and 0.019 foot.

The black lines in Figure 3 do not necessarily represent observations, but rather define the wire-frame used for the 3-D perspectives included in this discussion. All line intersections represent locations of control points. Although most of the control points in this network are in fact terrestrial-only, they will be referred to synonymously as GPS-only points, since the two types are, for the purposes of this discussion, equivalent: at both types it is necessary to use a geoid model to define the local geoid separation. As indicated in Figure 3, the perspective view will be from the northeast, towards the southwest, from elevation angles 15 to 40 degrees above the horizon.

3.1 Adjustment of GPS and Terrestrial Observations Only

The network adjustment of the GPS and terrestrial observations only, carried out for the purpose of estimating the apparent undulations in the observed geoid at the thirteen overdetermined points, is illustrated in Figure 4. The south end of the network is hidden behind the ridge of the undulations. The flat surface shown beneath the geoid surface, at the base of the wire frame, represents a surface parallel to the ellipsoid. The actual location of the reference ellipsoid is in the middle of the wire frame (running through Points 1, 2, and 3) but for purposes of simplification is not drawn there.

The slope in the apparent geoid is steepest in the foreground of Figure 4, between Points 10 and 2. There is about 0.08 foot (2.5 cm) difference in geoid height between these two points, which are about 3900 feet (1.2 km) distant horizontally, producing a local deflection of the vertical on this slope of less than 5 arc seconds. The observational noise in the apparent undulation between these two points is about 0.02 foot (0.5 cm), indicating that the undulation is a real phenomenon. This figure will be referred to again in Section 5, in the discussion of the Undulation Model-Builder.

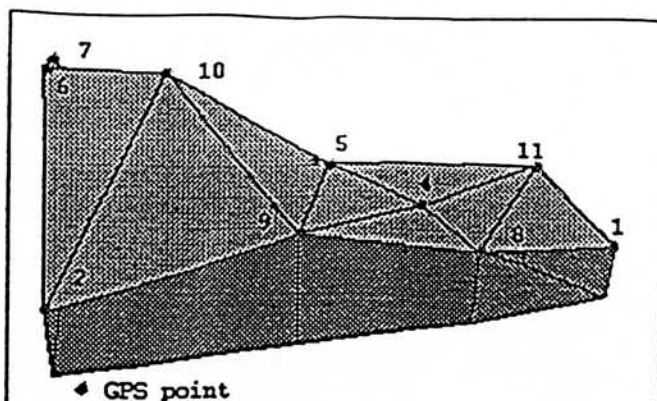


Figure 4: Apparent undulations from GPS and terrestrial observations

3.2 Adding the Geoid Model to the Network

For the combined network adjustment including a geoid model, GEOID90 [Milbert (1991)] was used to provide the geoid heights. GEOID90 is a computer-based geoid model for the continental US distributed by the National Geodetic Survey (NGS). At every control point an estimate of the geoid separation was obtained from GEOID90, then these geoid heights were transformed to the vertical constraints to produce the modeled geoid surface illustrated in Figure 5. The vertical scale is the same as in Figure 4, showing that the geoid surface as modeled by GEOID90 is almost flat (maximum relief is about 0.02 foot or 5 mm). Even the apparent undulation at Point 8 is not quite what it appears to be, as will be seen later.

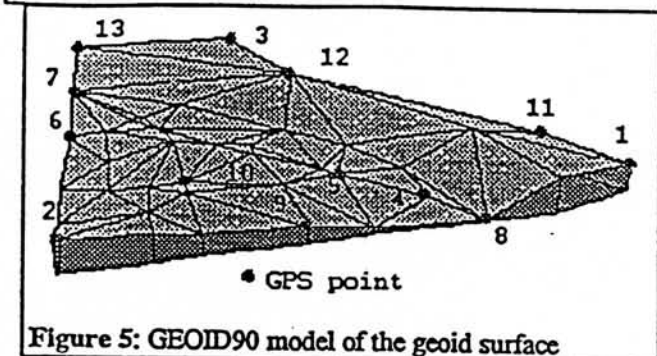


Figure 5: GEOID90 model of the geoid surface

3.2.1 Combined Adjustment Using an Independent Geoid Model. The first combined adjustment described here which includes all three classes of observation models the errors in the geoid heights as independent. As suggested above for Figure 2, sharp discontinuities and breaks in the adjusted geoid surface between overdetermined points and GPS-only points might be expected, and, as illustrated in Figure 6, this is precisely what is seen in the test network.

Figure 6, like all of the wire frame perspectives, shows the overdetermined points by numbered dots, while the GPS-only points (and in this case the transformed geoid model) are represented by line intersections. The steep geoid slope between Points 10 and 2 remains, but GEOID90's surface has been transformed (tilted) to minimize the relief between the geoid model and the overdetermined geoid throughout the network. At those GPS-only points situated between Points 10 and 2 the geoid model has been positioned about 0.04 foot (1.2 cm) below Point 10 and above Point 2. The sharpest discontinuities in the geoid surface can be seen at those GPS-only points immediately adjacent to Points 2 and 10, where the adjusted geoid surface breaks off sharply. In the western portion of the network (to the right of Figure 6), where the overdetermined points do not show as much relief in the geoid surface, the discontinuities are not so sharp.

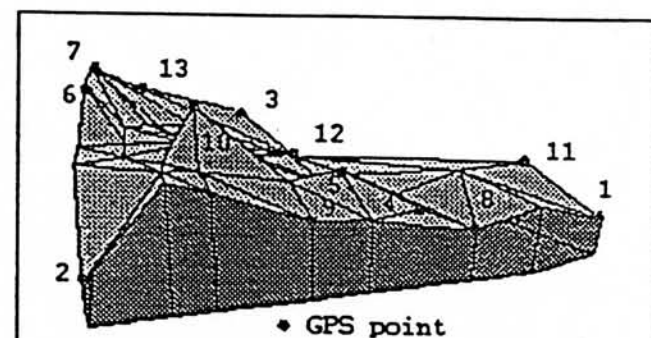


Figure 6: Combined adjustment using an independent geoid model

3.2.2 The Correlated Geoid Model. The next combined adjustment to be reviewed uses a correlated geoid model, but first the computation of its correlated covariance matrix will be described.

The purpose of the correlated covariance matrix is to model the behavior of the short wavelength errors in the geoid model. (Long wavelength errors are neutralized by the datum transformations.) GEOID90, like many other models, uses a combination of a gridded data base and an interpolation algorithm to produce estimates of geoid separations [Milbert (1991)]. The model's errors may be in the data base, in the interpolation algorithm, or in missing data, as when a local undulation within a grid cell may not be reflected in the model. Obviously, the nearer two points are to each other, the more likely it is that the errors in their geoid heights are correlated. Two points which are immediately adjacent to each other (say, 1 millimeter apart) will have a correlation coefficient of 1.0, meaning that their errors in geoid height are perfectly correlated. It is also obvious that, as the distance between points increases, at some distance their errors in geoid height will cease to be correlated (i.e., the correlation coefficient will be zero). As used here, this maximum distance is called the *correlation distance* (d_c) [Jordan (1972)].

When modeling short wavelength errors in the geoid model, the length of the correlation distance will in theory depend upon the spacing of the grid cells in the database. Because GEOID90 uses a spacing of 3 minutes by 3 minutes of geodetic latitude and longitude (approximately 4 miles north and south by 3 miles east and west) its correlation distance should be about 4 miles. Certainly two points farther apart than 4 miles should not be expected to have a correlation between the errors in their geoid heights.

In the computation system being described, the correlation distance may be user-specified, but the default is for the correlation distance to be set at the distance of the diagonal between the map corners of the network. Using this default, all geoid heights in a network will have correlations between their errors, although at points on opposite sides of a network the correlation will approach zero. So long as the points in a network are spaced more closely than the grid cells in the geoid model, the final adjustment results are fairly insensitive to the choice of a correlation distance; a correlation distance which is too short will decorrelate the model, while one which is too long may produce a singular covariance matrix. But when control points are spaced farther apart than the theoretical correlation distance, it is not appropriate to model correlations in the geoid height errors.

For pairs of geoid heights with correlated errors, it remains to define a mathematical function to compute the correlation coefficient between them using as arguments the correlation distance and the point-to-point distance. This function is called the *autocorrelation function* (acf) [Mikhail (1976)].

In the system being described, the acf is defined as follows:

$$C_{N_i N_j} = \frac{\sigma_{N_i N_j}}{\sigma_{N_i} \sigma_{N_j}} = 1.0 - \frac{D_{ij}}{d_c} \quad (3)$$

In this expression, $C_{N_i N_j}$ represents the correlation coefficient between the errors in geoid height at points i and j , while D_{ij} represents the distance between points i and j .

Other acf's could certainly be adopted for modeling residuals in a geoid model, and in fact a different acf, although one used for an undulation rather than a residual model, is described in Section 5. The acf chosen here produces a simple linear interpolation, and was chosen as a conservative strategy which does not impose assumptions about the dominant source of error in the geoid model. The correlation coefficient, which will always be a value between 0 and 1, is here expressed by the ratio of the distance between the two points and the correlation distance.

Before scaling by the variance of the geoid model σ_N^2 , the correlation coefficients represent off-diagonal elements in the cofactor matrix Q_N for the geoid heights, the diagonal terms of which are set at unity. After scaling, the result is the covariance matrix for the observed geoid heights Σ_N .

3.2.3 Combined Adjustment Using a Correlated Geoid Model. The results of the combined adjustment using the correlated geoid model are illustrated in Figure 7.

A comparison between Figures 6 and 7 shows graphically the differences between the adjustment behaviors of the independent and correlated geoid models that were forecast in Section 2 in the discussion of Figure 2. In effect, the correlated covariance matrix has improved the geoid model at those points observed by GPS only. At a purely intuitive level Figure 7 looks more like a geoid surface than Figure 6, just because Figure 7 shows a smooth surface and Figure 6 does not. Anything that the geoid surface can do, the surface of a lake on a very calm day can also do. This subjective impression alone, however, is not enough to demonstrate that the correlated geoid model has in fact produced more accurate results. To demonstrate that this has happened, it is necessary to run some tests for a comparison of the two adjustment strategies.

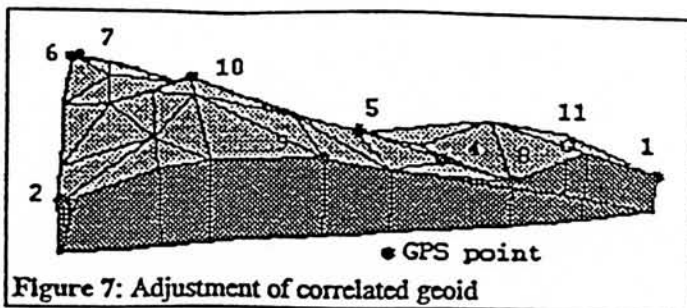


Figure 7: Adjustment of correlated geoid

3.2.4 Test Results. In the tests tabulated here, one or more of the overdetermined points were aliased; i.e., they were treated in test adjustments as though they had been observed by GPS only, and then, following the adjustment, their adjusted orthometric heights were compared against their known orthometric heights as produced by terrestrial observations. This "delta orthometric height" reflects the accuracy of the adjusted geoid model. The next, and quite critical step, compares the propagated error in the orthometric height against the delta orthometric height to see if the propagated error is accurate.

Three test configurations, plus two control configurations, are tabulated here. For each test configuration four combined adjustments were computed using:

- ▷ the uncorrelated & transformed geoid model (U: see Tables 1-3 and 4)
- ▷ the correlated & transformed geoid model (C: see Tables 1-3 and 4)
- ▷ the uncorrelated & untransformed geoid model (UA: see Table 4)
- ▷ the correlated & untransformed geoid model (CA: see Table 4)

The test configurations labeled C and U involve the same ellipsoid and orthometric height constraints at the same three points described earlier in this discussion. Note that because of space limitations the UA and CA tests are not discussed herein and are not tabulated in Tables 1-3, although they are summarized in Table 4. (For these adjustments, fixed orthometric height constraints only — no fixed ellipsoid heights — were applied at Points 1-3, the geoid model was not transformed, and the ellipsoid — i.e., the set of GPS observations — was transformed onto the geoid model.)

Test Point	Apparent Undulation	σ	Geoid Model	Uncorrelated Adjusted Model					Correlated Adjusted Model						
				Geoid Ht N	v_N	σ_N	GPS Ortho	σ_h	Δ_h	Geoid Ht N	v_N	σ_N	GPS Ortho	σ_h	Δ_h
10	0.0774	.022	-0.0063	0.028	.000	.043	0.120	.046	-.048	0.022	.019	.021	0.117	.026	-.043

Table 1: Tests 1U and 1C

Test Point	Apparent Undulation	σ	Geoid Model	Uncorrelated Adjusted Model						Correlated Adjusted Model					
				Geoid Ht N	vN	σ_N	GPS Ortho	σ_h	Δ_h	Geoid Ht N	vN	σ_N	GPS Ortho	σ_h	Δ_h
8	0.0081	.018	-0.0131	0.000	.000	.043	4.038	.045	-.007	0.001	.026	.020	4.035	.024	-.005
9	0.0212	.015	-0.0061	0.021	.000	.042	8.629	.043	.002	0.012	.018	.023	8.636	.025	-.005
10	0.0774	.022	-0.0063	0.028	.000	.041	0.120	.044	-.047	0.022	.021	.022	0.122	.028	-.048
11	0.0281	.014	0.0007	0.005	.000	.045	1.378	.047	-.017	0.007	.025	.022	1.375	.025	-.014
12	-0.0249	.016	-0.0059	0.016	.000	.044	6.298	.046	.040	0.010	.014	.027	6.304	.031	.034
13	0.0397	.019	0.0006	0.048	.000	.048	0.032	.050	.009	0.044	.018	.032	0.034	.034	.008

Table 2: Tests 2U and 2C

Test Point	Apparent Undulation	σ	Geoid Model	Uncorrelated Adjusted Model						Correlated Adjusted Model					
				Geoid Ht N	vN	σ_N	GPS Ortho	σ_h	Δ_h	Geoid Ht N	vN	σ_N	GPS Ortho	σ_h	Δ_h
5	0.0341	.018	-0.0095	0.011	.000	.046	6.580	.047	-.016	0.018	.028	.023	6.569	.024	.000
6	0.0712	.017	-0.0081	0.033	.000	.049	0.993	.051	-.033	0.041	.029	.026	0.978	.028	-.017
8	0.0081	.018	-0.0131	0.000	.000	.049	4.039	.051	-.008	0.001	.025	.020	4.036	.024	-.004
9	0.0212	.015	-0.0061	0.023	.000	.048	8.628	.049	.004	0.017	.021	.024	8.629	.025	.003
11	0.0281	.014	0.0007	-0.001	.000	.052	1.387	.053	-.026	0.002	.020	.022	1.383	.024	-.020
13	0.0397	.019	0.0006	0.034	.000	.055	0.042	.056	-.002	0.044	.018	.033	0.028	.034	.014

Table 3: Tests 3U and 3C

Each test configuration involved a different selection of test points, as follows:

- ▷ Test 1: Point 10 (see Tables 1 and 4)
- ▷ Test 2: Points 8, 9, 10, 11, 12, 13 (see Tables 2 and 4)
- ▷ Test 3: Points 5, 6, 8, 9, 11, 13 (see Tables 3 and 4)

A total of 8 points was tested. Control tests X and Y (see Table 4) were applied to all 8 points: test X used an adjustment of GPS and the geoid model only (no terrestrial observations, and therefore no geoid height residuals), with both the ellipsoid and geoid model transformed; test Y used an adjustment of transformed GPS observations only (adjusted *ellipsoid* heights being compared to known orthometric heights). All tabulated values are expressed in feet.

In Tables 1-3 the column labeled Apparent Undulation shows values derived from the adjustment of GPS and terrestrial data only (no geoid model), as illustrated in Figure 4; these are the differences between adjusted orthometric and ellipsoid heights used to construct the Undulation Model discussed in Section 5. The next column, labeled σ , shows the error propagated into the apparent undulation. The column labeled Geoid Model shows the geoid separation predicted by GEOID90, after transformation into the vertical constraints. The next columns are divided into two areas, one for the uncorrelated model and one for the correlated model, and within each area the following values are shown: Geoid Ht N is the adjusted geoid height; v_N shows the adjustment to the transformed geoid height from GEOID90 (the residuals for the uncorrelated model are, as explained previously, always zero); σ_N is the standard error for the adjusted geoid height; GPS Ortho is the adjusted orthometric height truncated to the four least significant digits; σ_h is the standard error for the adjusted orthometric height; and Δ_h is the difference between the GPS-produced orthometric height and the known, or control, orthometric height produced by differential leveling.

Table 4 summarizes for all tests the mean absolute Δ_h , the mean algebraic (signed) Δ_h , and the mean σ_h for the GPS-produced orthometric height. CORREL MODEL signifies correlated geoid model, TRNSFRM MODEL signifies transformed geoid model, TERREST OBSERV signifies the presence in the test adjustment of terrestrial observations, and NUMBER TEST PTS indicates the number of test points in the adjustment.

TEST LD.	CORREL MODEL	TRNSFRM MODEL	TERREST OBSERV	NUMBER TEST PTS	MEAN		
					abs Δ_h	Δ_h	σ_h
1-C	Y	Y	Y	1	.043	-.043	.026
1-U	N	Y	Y	1	.048	-.048	.046
1-CA	Y	N	Y	1	.029	-.029	.022
1-UA	N	N	Y	1	.059	-.059	.044
2-C	Y	Y	Y	6	.018	-.005	.028
2-U	N	Y	Y	6	.020	-.003	.046
2-CA	Y	N	Y	6	.015	-.006	.024
2-UA	N	N	Y	6	.044	-.003	.055
3-C	Y	Y	Y	6	.010	-.004	.027
3-U	N	Y	Y	6	.015	-.014	.051
3-CA	Y	N	Y	6	.012	-.006	.023
3-UA	N	N	Y	6	.032	-.020	.039
X	Y	Y	N	8	.055	-.051	—
Y	—	—	N	8	.038	-.032	—

Table 4: Summary of test results

In general, the various tests show that for this test network the correlated geoid model produces more accurate GPS-derived orthometric heights than does the uncorrelated geoid model, but that in both cases the propagated errors for the GPS-derived orthometric heights are realistic (the propagated errors are

smaller for the correlated model). This is a critically important result, as it indicates that either technique can produce adjusted heights with accurate error estimation.

The tests also indicate that the adjusted uncorrelated geoid model is superior to the combination of GPS observations and unadjusted geoid model. Interestingly, at least in this test network, when both the ellipsoid and the *unadjusted* geoid model are transformed to the same vertical constraints (test X), there is an apparent degradation in accuracy of orthometric heights compared against adjusted *ellipsoid heights* obtained without using a geoid model at all (test Y). This result might possibly be reversed by choosing a different set of 3 control points for vertical constraints.

Other tests undertaken by the author on other networks generally support the same conclusions: namely, given a sufficient number of precisely overdetermined control points distributed closely enough together to detect local undulations, the correlated geoid model produces more accurate results than the uncorrelated model, and the uncorrelated model produces more accurate results than either the application of an unadjusted geoid model or the use of ellipsoid heights only.

4. THE RESIDUAL MODEL-BUILDER

From the discussion above, it can be seen that when a correlated geoid model is included in a combined network adjustment, residuals to the geoid model are predicted for those points observed by GPS only, and thus an improved geoid model is generated. It should be possible – and is desirable – to use the results of

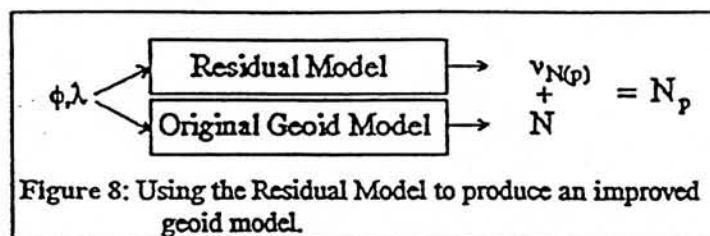


Figure 8: Using the Residual Model to produce an improved geoid model.

such an adjustment to predict residuals anywhere in the region of the network, at any time following the adjustment, for any point desired, such as for a kinematic or fast static GPS point, or for new points added with new observations to the network. By sequentially accumulating residuals at new adjustments (i.e., adjustments with additional observations), the original geoid model in this way would be incrementally improved until its variance was less than the observational noise. The Residual Model-BUILDER discussed here, together with the residual models it constructs, qualifies as such an application and is illustrated schematically in Figure 8. As shown, a horizontal position is submitted to the Residual Model, which automatically extracts an estimate of geoid height N from the original geoid model and, from accumulated adjustments, predicts a residual $v_{N(p)}$, then adds the two values together to produce an improved geoid height N_p , which in turn can be introduced as an observable into a new adjustment.

4.1 The Expanded Adjustment Model

Variation of coordinates (Eq. 1) is not well suited to this kind of post-adjustment prediction, but when the adjustment model is expanded to include both observation equations and condition equations it becomes quite well suited. The expression for this expanded model is [Uotila (1986)]:

$$Ax + Bv + w = 0 \quad (4)$$

The advantage of condition equations (B) is that coordinate parameters are not required in order to express observations. For all cases discussed here, the condition equations (which will be used to predict residuals) are fictitious observations which do not contribute any geometric information or redundancy to the network, and so it is possible immediately to substitute the identities $B = -I$ and $w = I$. Using the residuals v_N computed, at each adjustment, for the observed geoid heights at the overdetermined points, it is possible to produce for each adjustment a vector of Lagrange correlates K for the overdetermined points:

$$K_N = \Sigma_N^{-1} v_N \quad (5)$$

where Σ_N^{-1} is the weight matrix for the correlated geoid heights. The vector of predicted residuals v_p is produced by bringing the weights to the opposite side of the equation:

$$v_p = \Sigma_N K_N \quad (6)$$

For every point at which a predicted residual is desired, the covariant terms between it and the overdetermined points (i.e., the cross-covariance matrix) are computed, and are multiplied by the vector of Lagrange correlates to produce the predicted residual. Because a vector of predicted residuals can be computed for each adjustment in the sequence, the vectors can be summed over the sequence and, as shown in Figure 8, added to the geoid height derived from the original geoid model.

4.2 Test Network Example

To illustrate this procedure, a 20 x 20 rectangular grid (with grid intersections about 200 meters apart) was superimposed over the test network. For each grid intersection, a geoid height was produced from GEOID90, and then rotated to the vertical constraints, as shown in Figure 9.

The view in Figure 9 is to the southwest, as it is for the other perspectives. The information displayed here is exactly the same as in Figure 5, except that the geoid model was here sampled more frequently. The apparent ridge in the modeled geoid surface (varying in height between 3 and 5 millimeters) running north-south through the grid is in fact an artifact of GEOID90's interpolation algorithm [Milbert (1992)]; it is quite small and well within any asserted accuracy for GEOID90. But it is useful for graphical purposes, as it displays clearly the information content from the original geoid model.

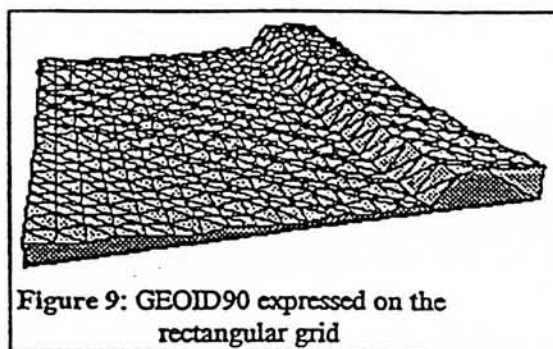


Figure 9: GEOID90 expressed on the rectangular grid

Using the Residual Model-BUILDER with the results from one combined adjustment of the network with the correlated geoid model, a residual model was constructed and residuals were predicted and added to the geoid model, producing the improved geoid model as shown in Figure 10. Here the information content is exactly the same as shown in Figure 7, except that the data points are more densely and regularly distributed. It can be seen from Figure 10 that both the adjustment results and the information content of GEOID90 are expressed in the improved geoid model.

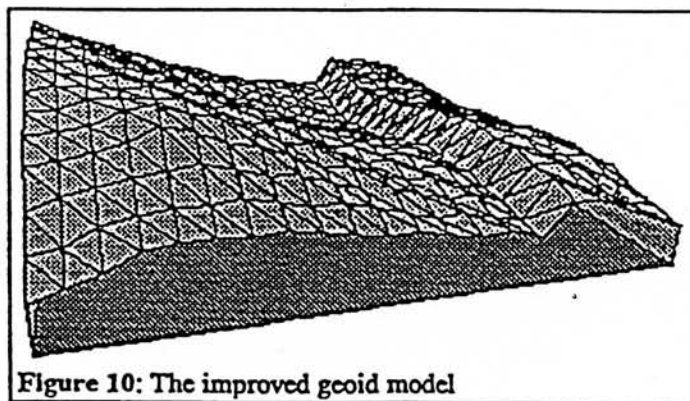


Figure 10: The improved geoid model

5. THE UNDULATION MODEL-BUILDER

Finally, this discussion moves to the construction of a geoid (undulation) model using the Undulation Model-BUILDER with the results of the combined adjustment of GPS and terrestrial observations only (no geoid model), as illustrated in Figure 4. As mentioned in Section 2, the geoid separation is uniquely determined at the thirteen overdetermined points, and this information can be used in conjunction with

least squares collocation (LSC) to create a geoid model. The new geoid model will be employed in the same way as other geoid models: horizontal positions will be submitted to it, and it will return geoid heights.

In this application of LSC the undulations are not directly observed, but they can be computed simply by differencing the adjusted orthometric and ellipsoid heights. A block of (thirteen) derived observation equations for the undulations is formed, with the observations being assigned observed values of zero while the undulations themselves are represented as residuals [Krakiwsky (1990)]. A characteristic feature of LSC is that this vector of residuals is decomposed into the two components of noise and signal. The noise will represent observational error, while the signal will represent modeled undulations:

$$V_{\text{total}} = V_{\text{noise}} + V_{\text{signal}} \quad (7)$$

Corresponding to this decomposition is the decomposition of the covariance matrix for the derived observations into signal and noise components.

$$\Sigma_{\text{total}} = \Sigma_{\text{noise}} + \Sigma_{\text{signal}} \quad (8)$$

Some parts of this puzzle, such as the total residuals (apparent undulations), are already known, and the covariance matrix for the noise can easily be derived from the adjustment as:

$$\Sigma_{\text{noise}} = A_N \Sigma_{\text{FH}} A_N^T \quad (9)$$

where Σ_{FH} represents that portion (hypermatrix) of the covariance matrix for the adjusted unknowns containing information about orthometric and ellipsoid heights, and A_N contains the design coefficients for the derived observations with respect to the adjusted orthometric and ellipsoid heights:

$$A_N = \begin{array}{c|ccccccc} & H_1 & h_1 & H_2 & h_2 & \dots & H_n & h_n \\ \hline N_1 & +1 & -1 & 0 & 0 & \dots & 0 & 0 \\ N_2 & 0 & 0 & +1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ N_n & 0 & 0 & 0 & 0 & \dots & +1 & -1 \end{array} \quad (10)$$

The covariance matrix for the noise is rank deficient, therefore singular, and cannot itself be inverted. The sum of it and the covariance matrix for the signal will not be singular, however, unless apparent undulations are too small to observe (i.e., unless they are smaller than the observational errors).

It remains to determine the total covariance matrix and the covariance matrix of the signal. Using a new autocorrelation function, the cofactor matrix Q for the signal can be generated by setting the variances at unity and the covariance terms equal to the correlation coefficients. The correlation distance d_c is the same one used for the Residual Model-Builder (divided by 5.81 to normalize the two functions; see Figure 11), and the new autocorrelation function is a third-order Markov undulation model [Jordan (1972)]:

$$Q_{ij} = \left[1 + \frac{D_{ij}}{d_c} + \frac{D_{ij}^2}{3d_c^2} \right] e^{-D_{ij}/d_c} \quad (11)$$

The relationship between Eq. 11 and Eq. 3 is illustrated in Figure 11. In this figure, the straight line was generated by Eq. 3 and the curved line by Eq. 11. The correlation coefficient, ranging between zero and 1.0, is shown on the vertical axis, while the correlation distance is represented on the horizontal axis in multiples of one and two.

Because the covariance matrix of the signal is equal to the variance of the signal times the cofactor matrix of the signal:

$$\Sigma_{\text{signal}} = \sigma_{\text{signal}}^2 Q \quad (12)$$

an approximate value for the variance of the signal is estimated and then iterated on until the reference variance for the vector of total residuals approximates unity:

$$\frac{v_{\text{total}}^T [\Sigma_{\text{noise}} + \sigma_{\text{signal}}^2 Q]^{-1} v_{\text{total}}}{r} \approx 1.0 \quad (13)$$

where r is the redundancy number (degrees of freedom) for the derived undulation observations.

Once the covariance matrix for the signal is known, the vector of Lagrange correlates K_{total} for the vector of total residuals may be computed:

$$K_{\text{total}} = \Sigma_{\text{total}}^{-1} v_{\text{total}} \quad (14)$$

At this point the undulation model has been constructed, and predicted undulations N_p can be computed, in the same manner as the predicted residuals, with the equation:

$$N_p = \Sigma_{\text{signal}} K_{\text{total}} \quad (15)$$

Using the same rectangular grid that was used as an example for the Residual Model (Section 4), the Undulation Model is illustrated by Figure 12.

The resemblance between the new geoid model illustrated in Figure 12 and the improved geoid model (produced using the Residual Model-BUILDER) in Figure 10 is clear; in the case of the new geoid model, however, the information from GEOID90 is now absent. This new geoid model can now be used in the same manner described for GEOID90 in the test network, as a starting model which can be improved until the

improved model's variance is less than the observational noise. It will not be possible to verify this until new observations are added to the network and a new adjustment is run, producing new residuals and a new estimate of the variance of the geoid model.

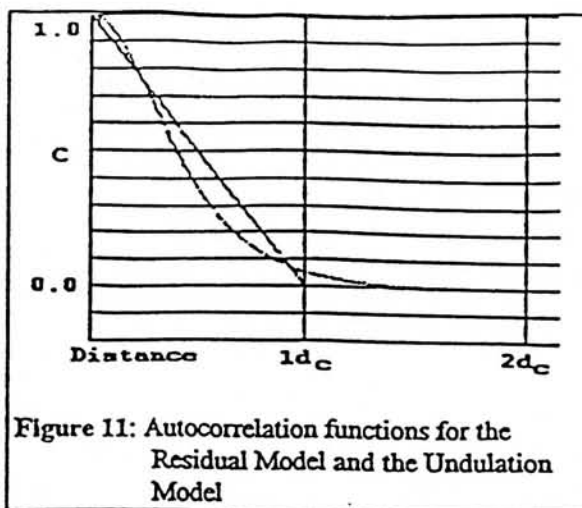


Figure 11: Autocorrelation functions for the Residual Model and the Undulation Model

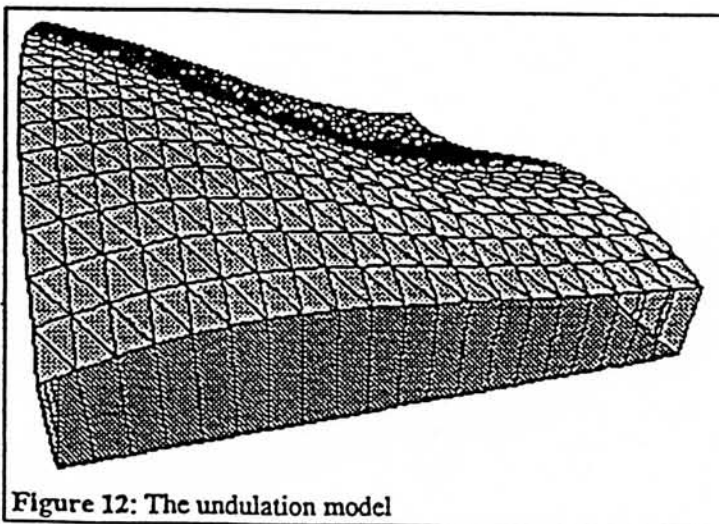


Figure 12: The undulation model

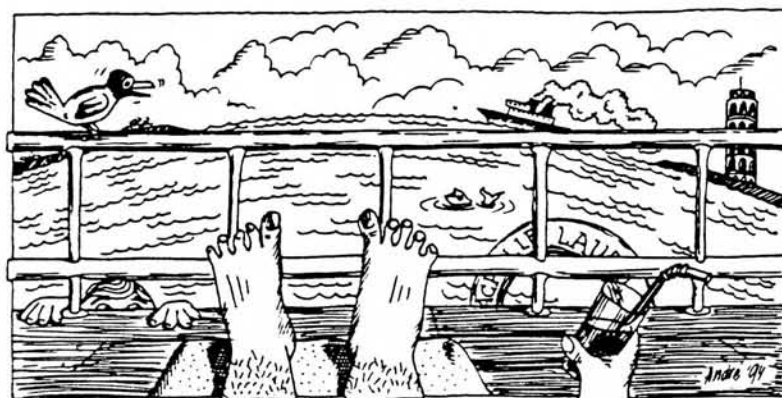
Determination of the radius of the earth.

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Introduction.

Last summer I made a cruise of one week on the Mediterranean with the "Achille Lauro", a big cruiseship with 950 passengers and a crew of 400 people. One day I was sitting on the "Lido-deck", next to the swimming pool and looked to the horizon, or better the skyline, which is the line between sky and sea. The horizon is the horizontal plane, which does not coincide exactly with the skyline if one looks down to the earth from a point above the earth. Occasionally the rail of the ship was on the same height as my eye, so the rail coincided with the skyline. It surprised me to see that the skyline was clearly visible curved against the straight rail. I realized that this must be the curvature of the earth! I was astonished because I had never thought that the curvature of the earth could be seen, without instruments, with the naked eye. I tried to estimate the curvature of the skyline. If I kept my eye such that the skyline coincided with the rail at two points, about 60° left and right from me, then the skyline before me was about 6 mm above the rail. The height of my eye above the sea was about 15 m and the distance to the rail was 3 m. I tried to compute the radius of the earth from these observations and after some puzzling it was possible indeed. The method of computation will be described further on in this paper.

The results of my observations gave estimates of the earth's radius between 4000 and 8000 km. This is not bad if one realizes that the earth's radius is 6400 km, and that these estimates are obtained without any instruments. If one would use this method not from a rolling ship, but from a high mountaintop with view at sea, and one would use a measuring tape and some surveying rods, it must be possible to determine the earth's radius with a precision of 1%. Use of a theodolite would even give one order better. If one repeats this simple method many times on different places, it must be possible to reach a precision of 1 to 1000 or better. If one determines the earth's radius in north-south direction and also in east-west direction one can even determine the flattening of the earth from the difference in curvature. This is an interesting mathematical problem to solve.



This method, described above, to determine the earth's radius has never been used or published before as far as I know. Nowadays better methods are available to determine the earth's radius, using satellites, but it is very interesting to discover a very simple method that gives good results without complex instruments. Even the old Egyptians, Greek or Romans could have used it, but it is remarkable that nobody discovered this method, although they lived around the Mediterranean.

Eratosthenes.

The oldest measurement to determine the radius of the earth dates from about 230 before Christ, by Eratosthenes in Egypt. He measured the north-south distance between Alexandria and Syene (nowadays Aswan) by counting how many days it is travelling by a caravan and how many miles or km one can cover each day. The result was 575 mile or 925 km.

Further he measured the height of the sun at both ends of the line at noon. In Syene was a water-well where the sun was perpendicular at noon in the summer. At the same time he measured the length of the shadow of a tower in Alexandria (figure 1). From this the angle α can be determined, which is the same as the difference in latitude, 7.2° . The earth's radius can be computed as 925 km divided by α in radians, equals 7360 km. It is good luck that this value is not more than 16% off from the modern value 6372 km.

This measurement probably used many years of preparation, but it is not better than the simple method using the curvature of the skyline.

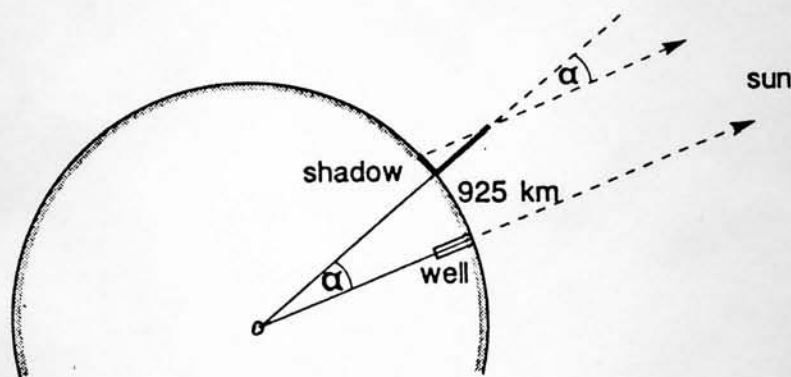


Figure 1: Measurement of the earth's radius by Eratosthenes.

Determination of the radius of the earth from curvature of the skyline.

If we are at a height h above sealevel than the horizontal plane is at eye level. The skyline is the line between sea and sky. This is a little lower than the horizon. The dip of the skyline is the small angle between horizon and skyline, called α , and is dependent on h (figure 2).

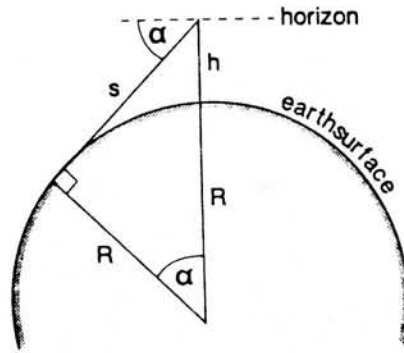


Figure 2: Measurement of the earth's radius by curvature of the skyline.

From figure 2 follows:

$$\cos \alpha = \frac{R}{R + h}$$

or

$$R = h \frac{\cos \alpha}{1 - \cos \alpha}$$

As α is very small we may approximate in the numerator: $\cos \alpha = 1$ and in the denominator: $1 - \cos \alpha = \frac{1}{2} \alpha^2$ (α in radians). So:

$$R = \frac{2h}{\alpha^2} \quad (1)$$

This approximation is possible because h is much smaller than R . If we measure h and α we can compute R . The angle α should be corrected for refraction, that is the curvature of the ray of light by the atmosphere. The measured angle is about 7% smaller than the angle α .

The angle α cannot be measured directly, because the horizon is not visible. Now the horizontal rail of the ship gives us help. If the rail would be exactly at the same height as our eye, then the rail would coincide with the horizon. Now the rail coincides with the skyline, thus something below the horizon. If the distance between our eye and the rail is ℓ , and the distance between the horizon and skyline straight in front of us is a , then (figure 3):

$$a = \ell \alpha \quad (2)$$

If we look left and right of us, by an angle γ , the distance between our eye and the rail becomes bigger proportional to $1/\cos \gamma$. At the cross-point of rail and skyline the distance equals b , so:

$$b = \frac{a}{\cos \gamma} \quad (3)$$

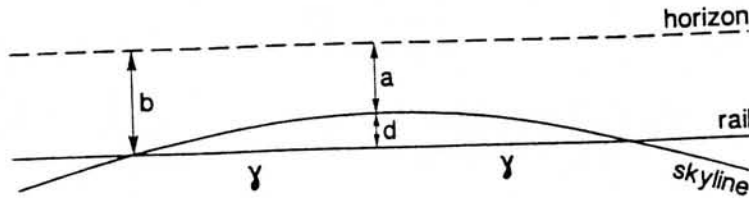


Figure 3: Distances between horizon, skyline and rail of the ship.

The difference $(b-a)$ is the distance between rail and skyline in front of us:

$$d = b - a = \ell \alpha \left(\frac{1}{\cos \gamma} - 1 \right) \quad (4)$$

If we keep our eye at such a height that we see the cross-point at an angle of 60° , so $\cos \gamma = 0.5$ and substitute this in formula (4), we get:

$$d = \ell \alpha \quad \text{or} \quad \alpha = \frac{d}{\ell} \quad (5)$$

This means that we see α directly and we can determine it if we measure d and ℓ . If α and h are known we can determine R with formula (1).

As already written in the introduction it is best to carry out this measurement from a mountain top where we can overview the sea. If the height h above sealevel becomes bigger, than α increases and can be measured more accurately.

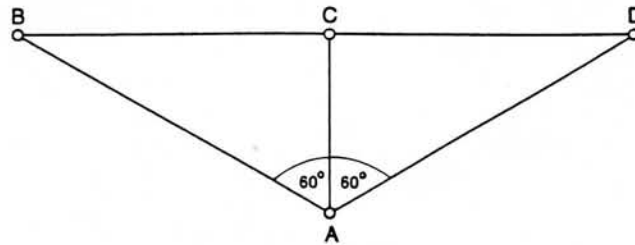


Figure 4: Surveying rods in a triangle to measure the dip of the skyline.

On a mountain we don't need the horizontal rail either. We can use 4 surveying rods which we put in a triangle as indicated in figure 4. On rod A we put a mark at the height of our eye. On B, C, and D a mark at the height of the line of sight between our eye and the skyline. At C we put an other mark at the height of the line of sight between the marks on B and D. This line of sight replaces the horizontal rail. The distance between the two marks on C, divided by the distance AC, is the angle α . This must be increased by 7% for the correction of refraction.

These measurements can be improved by using a theodolite, but this instrument was not available to the old Egyptians. Note that this method is independent of the direction of gravity. It is

is even not necessary to level the theodolite. It is an interesting problem to find out how the dip angle of the skyline can be solved if one measures the vertical angle to the skyline in at least three directions with a non-leveled theodolite (this is left to the reader).

Most of the work for this method is the determination of the height of the mountain. This can be done by levelling or by trigonometric height measurement. Formula (1) gives the radius of the earth. On the other hand one can assume that the radius of the earth is known, 6372 km, and determine the height of the mountain by formula (1). Or one can solve for the refraction if R and h are known both.

Example.

Height mountain	500 m
Distance AC	6 m
Distance marks on C	70 mm
Angle α	$\alpha = \frac{70}{6000} + 7\% = 0.01248 \text{ rad}$
Radius of the earth	$R = \frac{2h}{\alpha^2} = 6417 \text{ km}$

Determination of the radius of the earth by satellites.

The modern method to determine the earth's radius is from laser-distance measurements to satellites. From many stations on earth the distances to satellites are measured with laser-ranging. From these measurements the positions of the stations and the orbit of the satellites can be determined with a precision of a few cm. The mean radius of the earth follows with a precision of one meter. The principle can be explained as follows.

Suppose we have a simplified model with 2 satellites on different heights h_1 and h_2 in a circle orbit (figure 5). From the laser stations the heights h_1 and h_2 are measured. The radii of the orbits are a_1 and a_2 , and the radius of the earth is R , so we have:

$$h_1 = a_1 - R \quad \text{and} \quad h_2 = a_2 - R \quad (6)$$

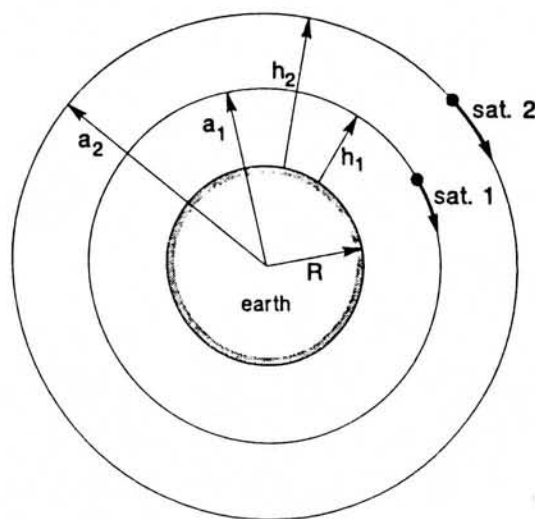


Figure 5: Earth with satellite orbits.

From the third Kepler law follows:

$$\frac{a_1^3}{t_1^2} = \frac{a_2^3}{t_2^2} \quad (7)$$

t_1 and t_2 are the times of revolution of the satellites which can be measured very accurately. From the above three equations a_1 , a_2 and R can be solved. We get:

$$R = \frac{h_2 - p h_1}{p - 1} \quad \text{with} \quad p = \left(\frac{t_2}{t_1} \right)^{\frac{3}{2}} \quad (8)$$

In practice one should take into account the eccentricity of the satellite orbit, the flattening of the earth and the position of the laser stations. One gets a number of equations with many unknowns from which the radius of the earth, the flattening and the position of the stations can be solved by least squares adjustment. Finally the mass of the earth can be solved from the third Kepler law.

APPENDIX.

Refraction.

Here follows an explanation on the computation of the refraction correction. Refraction is the curvature of a ray of light caused by the atmosphere. According to the law of Frenet the ray of light going from A to B does not follow the shortest way (straight line), but the fastest way. The velocity of light through the air depends on the density ρ of the air. Less dense air gives a faster speed, in vacuum is the speed of light, c , maximum. Close to the earth in the air more dense than high above, so the speed is faster in the higher air layers. A horizontal light ray from A to B will tend to go a little bit above the straight line, so the light ray bends with the earth curvature.

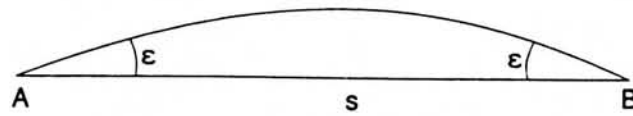


Figure 6.

We can consider the orbit of the light ray as a circle bow with curvature $k = \frac{1}{r}$, with r the radius of the circle. The small angle ϵ follows from the geometry of a circle

$$\epsilon = \frac{1}{2} k s \quad (9)$$

The distance s to the skyline is according to fig. 2:

$$s = R \tan \alpha = R \cdot \alpha \quad (\alpha \text{ is the dip of the skyline}).$$

$$\text{So: } \epsilon = \frac{1}{2} k R \alpha \quad (10)$$

The curvature k is dependent on the change of speed of light with height:

$$= \frac{1}{c} \frac{dv}{dh} \quad (11)$$

The speed v is dependent on the density of the air:

$$v = c(1 - a\rho) \quad (a \text{ is a constant})$$

$$\text{or: } \frac{c - v}{c} = a\rho$$

differentiate:

$$dv = -c a d\rho$$

$$\text{or } \frac{dv}{c - v} = - \frac{d\rho}{\rho} \quad (12)$$

Following the general gas law of Boyle-Guy Lussac:

$$\frac{\rho T}{P} = \text{constant} \quad (13)$$

ρ = density, T = temperature, P = air pressure.

Differentiation of (13) gives:

$$\begin{aligned} \frac{d\rho}{\rho} + \frac{dT}{T} - \frac{dP}{P} &= 0 \\ \text{or: } - \frac{1}{\rho} \frac{d\rho}{dh} &= \frac{1}{T} \frac{dT}{dh} - \frac{1}{P} \frac{dP}{dh} \end{aligned} \quad (14)$$

Substitute (12) and (14) in (11):

$$k = \frac{c - v}{c} \left(\frac{1}{T} \frac{dT}{dh} - \frac{1}{P} \frac{dP}{dh} \right) \quad (15)$$

The value for $\frac{c-v}{v}$ equals about 0.000 270. We insert some approximate values for a standard atmosphere:

$$T = 20^{\circ} \text{C} = 293 \text{ Kelvin},$$

$$P = 1013 \text{ mbar},$$

$$\frac{dT}{dh} = -10 \text{ Kelvin per km height},$$

$$\frac{dP}{dh} = -120 \text{ mbar per km height}.$$

The curvature of a light ray is:

$$k = 0,000\ 270 \left(\left(\frac{-10}{293} \right) - \left(\frac{-120}{1013} \right) \right) = 0,000\ 022 \text{ (km}^{-1}\text{)} \quad (16)$$

If we insert k and $R = 6400 \text{ km}$ in (10):

$$\epsilon = 0,07 \quad \alpha = 7\% \text{ of } \alpha \quad (17)$$

This correction must be added to the measured dip of the skyline in order to find the straight line dip α .

Refraction has also an other aspect. Because the speed of light is dependent on the density of the atmosphere, a correction must be applied to electronic distance measurements and to distance measurements to satellites like GPS. The uncertainty in temperature, air pressure and humidity of the atmosphere is the most important source of errors in geodetic measurements. A german professor in geodesy said once:

Der Liebe Herr Gott hat die Refraktion den geodäten geschenkt damit viele geodäsie Professoren Ihr täglich Brot verdienen können.

Or: Our dear Father God has given the Refraction to the geodesists so that many Professors can earn their daily bread.

On the numerical evaluation of Stokes' integral

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1 Introduction

In geoid computations from gravity data the Stokes' integral formula is used

$$N(P) = \frac{R}{4\pi\gamma} \int_{\lambda} \int_{\varphi} St(\psi_{PQ}) \Delta g(Q) \cos \varphi d\varphi d\lambda, \quad (1)$$

where R is the mean earth's radius, γ mean normal gravity, St is Stokes' function, ψ a spherical distance, Δg are free air anomalies and P is the geoid computation point and Q the free air anomaly points.

In practice the continuous gravity function is usually replaced by a block signal. Mean block values are computed from 'equiangular' areas with constant sidelengths $\Delta\varphi$ and $\Delta\lambda$, for which a constant value $\overline{\Delta g}_i$ is assumed. So Stokes' integral has to be rewritten as (Heiskanen & Moritz, 1967, p.118)

$$N(P) = \frac{R}{4\pi\gamma} \sum_{i=1}^I \overline{\Delta g}_i \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} St(\psi_{PQ}) \cos \varphi d\lambda d\varphi = \sum_{i=1}^I w_i^{St} \overline{\Delta g}_i. \quad (2)$$

Here Q indicates the midpoint of the block area i .

The size of the block areas must be chosen such that the block signal describes the real gravity signal sufficiently well. The error caused by using the block signal instead of the real gravity signal is called discretization error. Usually mean block values of $5 \times 5 \text{ km}^2$ will be good enough for geoid computations, especially when topography effects have been taken away in hilly or mountainous areas.

This paper will focus on the computation of the Stokes' weights w_i^{St} , the integrated Stokes' function over the block area i

$$w_i^{St} = \frac{R}{4\pi\gamma} \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} St(\psi_{PQ}) \cos \varphi d\varphi d\lambda. \quad (3)$$

2 Computation of Stokes' weights w_i^{St} : numerical approach

The Stokes' function is (Heiskanen and Moritz, 1967, p.94)

$$St(\psi) = \frac{1}{\sin(\psi/2)} - 6 \sin(\psi/2) + 1 - 5 \cos \psi - 3 \cos \psi \ln \left(\sin(\psi/2) + \sin^2(\psi/2) \right) . \quad (4)$$

Since it is not possible (at least I didn't succeed) to solve the integration (3) of (4) analytically, the straightforward approach is to do the integration numerically. For any block area with respect to the computation point one starts the computation with 1 step, then increase it to 2^2 , then 4^2 , and so on, until the change of result is less than ε . This maximum error ε can be chosen by the user and depends on the required accuracy. For local geoid computations using residual gravity anomalies (after removing a global geopotential reference model) usually $\varepsilon = 10^{-4}$ will be sufficient.

Another way to find the necessary number of integration steps is by approximating Stokes' function by $2/\psi$ and applying flat surface approximation. Since the term $\frac{1}{\sin(\psi/2)} \approx \frac{2}{\psi}$ is the dominant term in Stokes' function for short distances (say $< 2^\circ$), this function can be used to find the required number of integration steps. In flat surface approximation the analytical solution of $\int_x \int_y \frac{2}{\psi} dy dx$, with $\psi = \sqrt{x^2 + y^2}$, is known, namely

$$\int_{x_0}^{x_1} \int_{y_0}^{y_1} \frac{2}{\sqrt{x^2 + y^2}} dy dx = 2 \left[x \ln(y + \sqrt{x^2 + y^2}) + y \ln(x + \sqrt{x^2 + y^2}) \right] \Big|_{y_0}^{y_1} \Big|_{x_0}^{x_1} . \quad (5)$$

Using this analytical solution, one can find the smallest number of integration steps necessary to allow a maximum error ε .

From tests it was found that the number of integration steps depends on the ratio of the distance between computation point P and midpoint of the mean block area Q , and the blocksize Δ , $\frac{\psi_{PQ}}{\Delta}$. For instance, if blocks of 10 km sidelength are used and the midpoint of a block is 30 km from the computation point P , then 8^2 integration steps must be used. And if blocks of 5 km sidelength are used and the midpoint of a block is 15 km from the computation point P , then also 8^2 integration steps must be used. However, if blocks of 5 km sidelength are used and the midpoint of a block is 30 km from the computation point P , then 4^2 integration steps can be used.

In table 1 the minimum number of integration steps is given for relative distances $\frac{\psi_{PQ}}{\Delta}$, to compute the Stokes' weights with some chosen maximum error.

maximum error ε	number of integration steps N^2 per block												
	4096^2	2048^2	1024^2	512^2	256^2	128^2	64^2	32^2	16^2	8^2	4^2	2^2	1
$5 \cdot 10^{-3}$						0					1	2	3
$1 \cdot 10^{-3}$				0						1	2	$3\frac{1}{2}$	7
$5 \cdot 10^{-4}$			0						1	$1\frac{1}{2}$	$2\frac{1}{2}$	5	9
$1 \cdot 10^{-4}$	0							1	$1\frac{1}{2}$	3	6	10	20
$5 \cdot 10^{-5}$							1	$1\frac{1}{2}$	2	4	7	14	28

Tabel 1: Number of integration steps from a relative distance $\frac{\psi_{PQ}}{\Delta}$, to make a maximum error ε in the Stokes' weight w_i^{St} . P is the geoid computation point, point Q is the midpoint of block area i .

So, if the computation point is in the midpoint of a gravity block, and a maximum error of 10^{-3} is allowed, 512^2 integration steps must be used. If the computation point is in the middle of the first neighbour block ($\psi_{PQ} = \Delta$) 8^2 integration steps should be used for a maximum error of 10^{-3} and 32^2 integration steps should be used for a maximum error of 10^{-4} .

The number of integration steps in table 1 was found both by the first method using Stokes' function and by the second method using the analytical flat surface integral solution of $2/\sqrt{x^2 + y^2}$.

3 Computation of Stokes' weights w_i^{St} : 'analytical' approach

As is shown in the previous section the number of numerical integration steps to compute the Stokes' weight w_i^{St} is very large for neighbouring blocks close by. Therefore another approach is presented in this section. The Stokes' function is split in several terms.

$$\begin{aligned}
 w_i^{St} = & \frac{R}{4\pi\gamma} \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} \frac{1}{\sin(\psi/2)} \cos \varphi \, d\varphi \, d\lambda \\
 & - \frac{R}{4\pi\gamma} \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} 3 \cos \psi \ln (\sin(\psi/2) + \sin^2(\psi/2)) \cos \varphi \, d\varphi \, d\lambda \\
 & - \frac{R}{4\pi\gamma} \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} (6 \sin(\psi/2) - 1 + 5 \cos \psi) \cos \varphi \, d\varphi \, d\lambda
 \end{aligned} \tag{6}$$

The second and third terms can be computed by numerical integration in only a few integration steps (1 or 2²). Only the first term needs more attention.

For small distances ψ one could write $\frac{1}{\sin(\psi/2)} \approx 2/\psi$. It is also allowed to apply some kind of flat surface approximation for this small distances. If we use

$$x = (\lambda_Q - \lambda_P) \cos \varphi_Q \quad \text{and} \quad y = (\varphi_Q - \varphi_P) \tag{7}$$

then $2/\psi$ can be replaced by $2/\sqrt{x^2 + y^2}$ and the analytical solution of this function is known, see previous paragraph. A sufficient approximation for small distances of the first term of the integrated Stokes' function (6) is found. Figure 1 shows the used coordinates in the analytical solution.

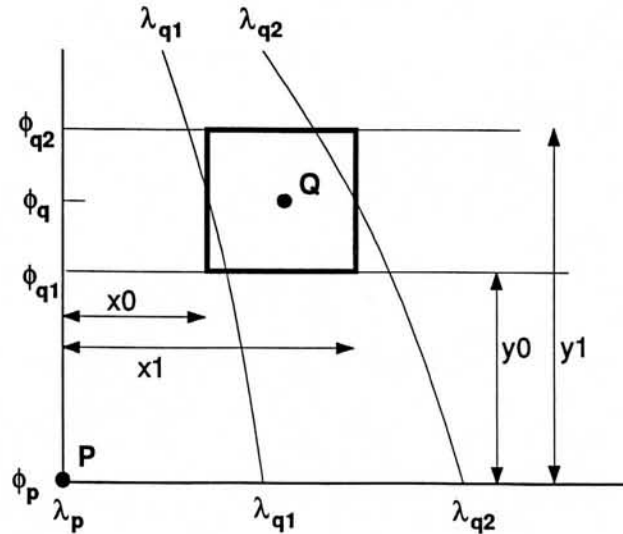


Figure 1: Coordinates in flat surface approximation.

The second term of the integrated Stokes' function can also be computed by an analytical solution if flat surface approximation is applied and the $\sin^2(\psi/2)$ -term within the $\ln(\cdot)$ -term is neglected. However, the numerical solution can be computed in only 2^2 integration steps, which is more convenient.

All the other terms of the integrated Stokes' function in the third term of (6) can be computed in one numerical integration step.

The final computation of the integrated Stokes' function than reads

$$\begin{aligned}
w_i^{St} &= \frac{R}{4\pi\gamma} 2 \int_{\lambda=\lambda_Q-\frac{\Delta\lambda}{2}}^{\lambda_Q+\frac{\Delta\lambda}{2}} \int_{\varphi=\varphi_Q-\frac{\Delta\varphi}{2}}^{\varphi_Q+\frac{\Delta\varphi}{2}} \frac{1}{\sqrt{x^2+y^2}} dx dy \\
&\quad - \frac{R}{4\pi\gamma} 3 \cos \psi \sum_{i=1}^2 \sum_{j=1}^2 \ln \left(\sin(\psi_{ij}/2) + \sin^2(\psi_{ij}/2) \right) \frac{\Delta x}{2} \frac{\Delta y}{2} \\
&\quad + \frac{R}{4\pi\gamma} (1 - 5 \cos \psi - 6 \sin(\psi/2)) \Delta x \Delta y \\
&= \frac{R}{4\pi\gamma} 2 \left[x \ln(y + \sqrt{x^2+y^2}) + y \ln(x + \sqrt{x^2+y^2}) \right. \\
&\quad \left. \Big|_{y=\varphi_Q-\frac{\Delta\varphi}{2}-\varphi_P}^{\varphi_Q+\frac{\Delta\varphi}{2}-\varphi_P} \Big|_{x=(\lambda_Q-\frac{\Delta\lambda}{2}-\lambda_P)\cos\varphi_Q}^{(\lambda_Q+\frac{\Delta\lambda}{2}-\lambda_P)\cos\varphi_Q} \right] \\
&\quad - \frac{R}{4\pi\gamma} 3 \cos \psi \sum_{i=1}^2 \sum_{j=1}^2 \ln \left(\sin(\psi_{ij}/2) + \sin^2(\psi_{ij}/2) \right) \frac{\Delta x}{2} \frac{\Delta y}{2} \\
&\quad + \frac{R}{4\pi\gamma} (1 - 5 \cos \psi - 6 \sin(\psi/2)) \Delta x \Delta y ,
\end{aligned} \tag{8}$$

where x and y are in radians, and $\Delta x = \Delta\lambda \cos \varphi_Q$ and $\Delta y = \Delta\varphi$.

For distances smaller than 50-100 km the error in this formula is smaller than 10^{-4} .

Neglecting all terms but the first one, as is sometimes done in flat surface (FFT) computations, is not sufficient. This introduces errors of 1-8% for distances of 10-100 km.

4 Final approach

The final total approach is to use the 'analytical' approach for distances upto 50 km, while for larger distances the numerical approach is used with maximum 2^2 to 8^2 numerical integration steps. In this way Stokes' weights with relative errors smaller than 10^{-4} are used without large computational efforts.

5 Concluding remarks

In this paper a description is given of the computation of Stokes' weights with high precision and in an efficient way.

By using only one integration step for all block areas large errors can be obtained (upto 20%).

The computation of the Stokes' weights as described in section 4 is implemented in the Stokes' numerical integration program **stokes70** by the author. This program also has the possibility to use 1D-FFT techniques for very fast evaluations (see Haagmans et al., 1993). The program and its manual can be obtained from the author or from the International Geoid service in Milan.

Acknowledgment: I am grateful to Khafid for starting a discussion about this topic.

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DEVELOPMENT, STATE OF THE ART AND PROBLEMS AT LARGE SCALE GEOID DETERMINATIONS

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1. The Geoid: Concept and Fundamentals

The geoid is the equipotential surface of the earth's gravity field approximating mean sea level in an optimum way, and extended under the continents. This modern definition can be traced back to *Carl Friedrich Gauss* (1828) when introducing a "geometrical" surface of the earth in this sense while the name "geoid" was only introduced in 1872 by Listing, cf. *Moritz* (1977). A more specified description was given by *Helmert* (1880), who clearly distinguished between the ideal surface of the oceans as part of that distinguished level surface "geoid" ("mathematical" surface of the earth), and the real "physical" earth's surface.

The definition of the geoid given above contains several *aspects* important for geosciences, surveying and mapping, and engineering:

- about 2/3 of the earth's surface (oceans) approximately coincide with the geoid, the rest (continents) can be easily referred to it by leveling and gravity measurements. The geoid thus is a natural reference for describing the heights of the topography on land (continental topography) as well as on sea (sea surface topography): *geodetic aspect* (height reference surface),
- the earth model "geoid" as a geometrical representation of the earth's gravity field contains information about the density distribution, primarily in the deeper interior of the earth's body: *geophysical aspect*,
- being an equilibrium surface of the earth's gravity field only, the geoid is an ideal reference for modeling the "stationary" sea surface, generated by ocean currents and other sources: *oceanographic aspect*,
- classical geodetic measurements get their orientation in space by relating them to the direction of the plumb line, they thus refer approximately to the geoid or (distances) are easily reduced to it: *surveying aspect*,
- long-term tide gauge observations and nowadays also satellite altimetry missions deliver mean sea level information, which by definition is needed for deriving the geoid: *realization aspect*.

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Obviously, the geoid definition includes several *problems*:

- mean sea level is not an equipotential surface of the earth's gravity field, due to quasistationary effects of oceanographic, meteorological and water budget type, (deviations < 2 m). The reduction of the sea surface related observations to the geoid, and the "optimum approximation" postulated in the geoid definition thus create a *definition problem*,
- the existence of the "sea surface topography" results in different height reference surfaces if different tide gauges have been used for deriving mean sea level, and produces height network distortions if several tide gauges have been included into one network: *height datum problem*,
- within the continents the geoid extends into the earth, consequently a density model of the topography (hypothesis!) is needed for this downward continuation. In addition, discontinuities of the geoid curvature occur inside the topographical masses, which theoretically prevent a single analytical representation: *theoretical problems*,
- the geoid being an equipotential surface of the gravity field participates in variations of the earth's body with time, with main contributions (≤ 1 mm/a) coming from large-scale and long-term variations: *epoch problem*.

The development of *theory and modeling* strategies shall be characterized here by a few names, without any attempt to obtain completeness:

- *Stokes* (1849) derived a surface integral for calculating geoid heights from gravity anomalies derived from surface gravity data after proper reduction to the geoid,
- *Helmert* (1884) through the line integral of "astronomical leveling" provided the transformation from observed deflections of the vertical derived from astronomical observations, to geoid height differences,
- *Jeffreys* (1931) performed detailed investigations about the use of free air anomalies at the calculation of the geoid,
- *Molodenski* (1945) introduced the concept of the "quasigeoid" which avoids the problems related to the downward continuation into the solid earth,
- *Krarup* (1969) and *Moritz* (1972) by developing and investigating "least squares collocation" provided a powerful tool to combine heterogeneous and unevenly distributed gravity field data to an optimum solution, taking the statistical field behaviour and the data errors into account.

2. Early Results

We now shortly describe some earlier attempts and results of local, regional and global geoid calculations. By simple *modeling* of the continental masses taken as isostatically uncompensated, *Helmert* (1884) found a geoid rise on the continents up to more than 400 m, and similar depressions on the oceans. He also stated that in reality the effect of the continents is more or less compensated. Later he assumed an uncomplete compensation, resulting in a corresponding geoid rise resp. depression of 27 m only, holding geoid heights of more than ± 100 m as very unlikely (*Helmert* 1910). In the same paper he calculated the average geoid height from Stokes' formula, based on the rather poor knowledge about the gravity anomalies, available at that time. The result was a mean geoid height of about ± 50 m, and maximum values of 100 m. These early estimates already gave the right order of magnitude for the geoid heights.

The *astrogeodetic geoid determination* prevailed until the 1960's, due to the local and regional availability of astrogeodetic deflections of the vertical. The "Mitteleuropäische Gradmessung" included the studies of the earth curvature anomalies already into the first research program (1864), and the "International Association of Geodesy" since 1886 published regularly the observed deflections of the vertical. A detailed survey of this type was started in 1865 in the Harz mountains, Germany, under the direction of J.J. Baeyer, and after thorough theoretical studies by Helmert, concluded by a geoid calculation presented by *Galle* (1914). The geoid in this area (dimensions ~ 60 km x 100 km, maximum elevation ~ 1000 m) could be obtained with a relative accuracy of ± 0.05 to 0.1 m over distances of 15 to 100 km, as shown by comparisons with more recent astrogeodetic and gravimetric calculations (r.m.s. discr. ± 0.06 m and ± 0.08 m resp., *Torge* 1993). With the improvement of the national horizontal control networks and their combination to regional systems, the practical need for vertical deflections (corresponding to geoid height differences) increased. *Wolf* (1949) presented a geoid calculation for the *Central European Net*, and the European geoid then was carefully investigated and improved by IAG research groups until the 1980's, with the "*Bomford*" geoid (first version by *Bomford* 1954, last version by *Levallois* and *Monge* 1978) as outstanding result. These regional determinations were based on some 100 resp. some 1000 vertical deflection points, and had a relative accuracy of $\pm 1 \dots 3$ m, depending on station distances and topography.

The *gravimetric geoid calculation* presupposes a global gravity coverage, a centralized data collection, and a common gravity standard. IAG again involved itself early in this field, by encouraging gravity measurements on land and sea, providing a gravity standard with the "Potsdam Gravity System" (1909), and later the "International Gravity Standardization Net 1971", and by collecting and making available global gravity data (Bureau Gravimetrique International since 1951). *Hirvonen* (1934) the first time used Stokes' formula for a rough estimation (more than 4000 gravity values, 18 % global coverage), resulting in an r.m.s. geoid height of ± 50 m. A first attempt to an harmonic expansion (up to degree 8) was made by *Zhongolovich* (1952), and gave maximum geoid values of -160 m. A systematic collection and reduction of gravity data then started at the IAG Isostatic Institute in Helsinki (since 1936), and was continued at the Ohio State University in Columbus, Ohio. *Tanni* (1952) had more than 13000 gravity values available, which reduced the r.m.s. geoid height to ± 26 m, and gave a relative accuracy of $\pm$ few m for Central Europe. The famous "Columbus-Geoid" (*Heiskanen* 1957) then represents

a certain end of the attempts to calculate a global "pure-gravimetric" geoid by Stokes' formula.

A new method for determining the (long-wave) geoid structures was available since the end of the 1950's by modeling the gravitational field in the orbit analysis of *artificial satellites*. The spherical harmonic expansion could now be employed efficiently and soon delivered significant results. *O'Keefe* (1958) and *King Hele* and *Merson* (1958) derived the second degree harmonic ("dynamical flattening"), and *Lundquist* and *Veis* (1966) with the SAO Standard Earth I presented a (8,8) spherical harmonics model.

Satellite derived gravity field information including satellite altimetry (since 1975) rapidly increased since that time. The global coverage with terrestrial gravity data also improved steadily since the 1970's, including now also airborne data (since the late 1980's). Finally, digital terrain data are available now globally and regionally, delivering most of the high frequency part of the gravity field. This led to the development and application of combination techniques for the geoid calculation, which will be discussed later.

3. Recent Requirements

In the following we concentrate on regional geoid determinations, i.e. on areal dimensions of some 100 km to some 1000 km: Here demands on the accuracy and the related spatial resolution of the relative geoid (geoid differences) have increased remarkably in the last decade, and may be described as follows (e.g. *Torge* 1990):

- $\pm 0.1 \dots 1$ m for the reduction of terrestrial distance measurements to the ellipsoid as reference surface for the calculation of positions. This demand is fulfilled in many regions of the world, by astrogeodetic or gravimetric methods,
- $\pm 0.005 \dots 0.01 \dots 0.05$ m/1 ... 10 ... some 1000 km for the mutual transformation and combination of heights derived from GPS (Global Positioning System) and geometrical leveling, and for eventually substituting this time consuming technique through GPS-leveling. This "cm"-accuracy level has not been reached, except in a few local test areas. However, due to the rapidly increasing use of GPS heighting, which already now gives the same accuracy as leveling over some 10 to 100 km (see Fig. 1), there is an urgent need to provide the "cm"-geoid to geodesists, surveyors and other engineers employing heights,
- ± 0.01 m for describing the sea surface topography in coastal zones and border seas. This demand is also not fulfilled, and renders more difficulties through the fact, that terrestrial techniques as gravimetry and astronomy face special problems in coastal waters,
- ± 0.001 m for geoid application in vertical crustal movements research. This high demand may be necessary only in regions with higher movement rates, as tectonically active zones, regions of isostatic land uplift, and areas affected by man-made variations (mining, oil and natural gas exploitation, large scale engineering projects).

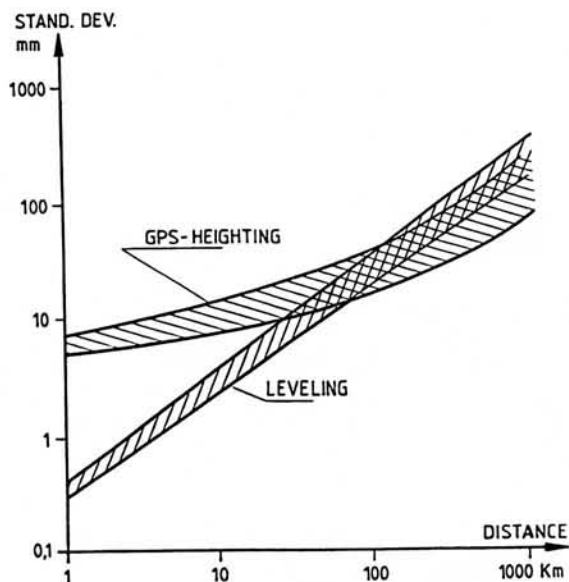


Fig. 1: Accuracy of GPS-heighting and leveling

In order to estimate the spatial resolution and the accuracy of the gravity field data to be employed for these solutions, we may use available information about the *statistical behaviour* of the *gravity field*. If we take the global spectral decomposition, as provided e.g. by the anomaly degree variance model derived by *Tscherning and Rapp* (1974), we recognize that the omission errors resulting from neglected high frequency field parts at a resolution (corresponding to an average station distance) of a few km are less than 1 cm for geoid heights, a few $10 \mu\text{ms}^{-2}$ for gravity anomalies, and 1" for vertical deflections respectively. This also gives an idea about the necessary observation accuracy of the gravity field data which can be used. As the omission errors and the observation errors add in the calculations, the average distance of the data points should not exceed the limit given above.

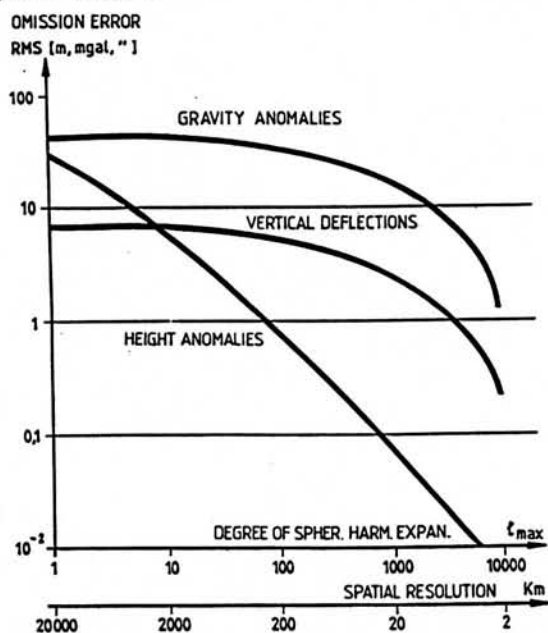


Fig. 2: Omission errors for different gravity field parameters as a function of the maximum degree l_{max} of a spherical harmonic expansion, model *Tscherning and Rapp* (1974).

4. Data Sources

We now summarize the gravity field data available for geoid calculations. *Observed* gravity field quantities include:

- global "*satellite-only*" *models*. They deliver the long-wave part of the gravity field, in the form of spherical harmonic expansions up to degree and order (36,36) or (50,50). Recent examples are the Goddard GEM T2 (*Marsh et al.* 1990) and the GRIM 4-S3 models (*Schwintzer et al.* 1993). It has to be considered that the coefficients of these developments are significant (at the 1σ -level) until about degree 30 at the most, corresponding to wavelengths of about 12° spherical distance or 1300 km. The errors of these long-wave geoid parts are about ± 1 m,
- global data sets of *mean free air gravity anomalies*, for $1^\circ \times 1^\circ$ (about 75 % global coverage) and $30' \times 30'$ (about 30 % coverage) compartments, as collected at the Ohio State University (*Kim and Rapp* 1990). The errors of these mean anomalies are $\pm 50 \dots 200 \mu\text{m/s}^2$ on the average,
- global data sets (oceans only) of *mean altimetric geoid heights*, for $1^\circ \times 1^\circ$ and $30' \times 30'$ compartments and with accuracies of $\pm 0.5 \dots 1$ m,
- regional data sets of *mean (free air) gravity anomalies* for block sizes of $5' \times 5'$ (in Europe $6' \times 10'$) or $15' \times 15'$, corresponding to smallest wavelengths of about 60 km and 20 km respectively. Average accuracies are $\pm 50 \dots 100 \mu\text{m/s}^2$ (e.g. *Torge et al.* 1984),
- global and regional data sets of *point (free air) gravity anomalies*, with point distances partly as close as 2 km, and accuracies of $\pm 2 \dots 20 \mu\text{m/s}^2$. A global data base is managed by the Bureau Gravimetrique International (BGI), including now more than 4.5 million point values,
- point (foot-print) *altimetric geoid heights* from different altimetry missions, with an along track resolution of about 10 km, and track distances varying between some km and 100 km. Accuracies (after crossing point adjustment) have improved from about ± 0.5 m (GEOS-3) to ± 0.1 m and better (TOPEX-POSEIDON, ERS-1), and gridded values are usually provided as final results,
- regional data sets (continents only) of *astrogeodetic vertical deflections*, with station distances of 10 km (only few areas) to 50 or 100 km, and accuracies of $\pm 0.5 \dots 1''$,
- regional *point geoid heights* (continents only) as differences between *Doppler* resp. *GPS* (ellipsoidal) *heights* and *levelled heights*. While the absolute accuracy of these results is not better than $\pm 1 \dots 2$ m, the relative (distance-dependant) accuracy is $\pm 0.2 \dots 0.5$ m/100 ... 1000 km (Doppler), and $\pm 0.01 \dots 0.1$ m / 1 ... 1000 km.

For later gravity field modeling, important *source information* also is available nowadays:

- global *topographic* or *topographic-isostatic models*, as spherical harmonic expansion up to degree and order 360 (Sünkel 1985, Pavlis and Rapp 1990),
- *global digital terrain models* of block size 5' x 5', as ETOPO 5,
- *regional digital terrain models* with resolutions varying from 30 m x 30 m to 1 km x 1 km and more.

Global data sets are generally collected at global data bases, and available for scientific research. Regional high-resolution data are stored at national agencies, and not always freely available. For dedicated projects those data have been released as the example of the IAG project "European Geoid" demonstrates. The IfE Hannover operating as the computing center for this project now has more than 1.5 million point gravity values, and more than 600 million digital terrain data (550 million from Germany) in its data base (e.g. Denker and Torge 1993).

One remark with respect to the *reference systems* used at the gravity field data sets may be useful. Obviously, all field quantities have to be referred to the same references. At global and regional calculations, these should be the geocentric coordinate system realized by the International Earth Rotation Service (IERS), which is now increasingly adopted also for continental and national control systems. Gravity values have to be given in the International Gravity Standardization System 1971 (IGSN 71). The earth standard model should be the Geodetic Reference System 1980 (GRS 80), the four definition parameters of this system practically coincide with the World Geodetic System 1984 (WGS 84) used for the Global Positioning System. Finally a unique height reference (geoid approximation by mean sea level) must be used for large-scale geoid calculations.

5. Modeling Strategies

For modeling the earth's gravity field - in our case the geoid as one geometric field quantity - different strategies are available (cf. also section 1). They are all based on linear relations between the different anomalous gravity field quantities (for further details see textbooks as Heiskanen and Moritz 1967 or Torge 1991):

- the *spherical harmonic expansion* allows a global parametrization and a simple combination of different "observed" gravity field quantities. It presupposes a reasonable good global data coverage, which today limits the degree of expansion to 360. Recent developments combine "satellite-only" (50,50) models and 1° x 1° and 30' x 30' mean anomalies derived from gravimetry and altimetry (OSU 91 A, Rapp et al. 1991; OGE 12, Gruber and Bosch, 1993). By regional fitting of global models to higher-resolution data, "tailored" models approximate the geoid in that area (Bašić et al. 1989),

- for regional and local applications high resolution (few km) and corresponding high accuracy (see section 3) are demanded. This can be achieved by *integral formulas* (Stokes' or Molodenski's formula, astronomic leveling), which in principle can transform *one* kind of gravity field data only to geoid or quasigeoid heights. Manifold combination strategies have been developed in order to combine heterogeneous data sets, taking also the observation errors into account. The results of these calculation are provided in gridded form, the grid dimensions correspond to the data resolution and accuracy,
- an elegant technique for combining different data sets is *least squares collocation*, with the still existing disadvantage of computing problems at very large data sets. This eventually forces to subdivide the area into smaller computation blocks, with resulting problems at the blocks' unification. Again the results generally are presented in a gridded form.

Concentrating now on *regional geoid calculations*, different *steps* may be distinguished, according to the state of data collection:

- the first step simply consists of using one of the available *global (360, 360) spherical harmonic models*, corresponding to an 0.5° field resolution. The model coefficients today are significant (1σ -limit) only until degree 250 approximately, and the average geoid accuracy is 0.3 to 0.5 m on the oceans and $\pm 1 \dots 1.5$ m on land, the omission error amounting to ± 0.2 m. It must be mentioned, that in regions without or with insufficient gravity coverage, the geoid error may reach 3 to 4 m, see also section 6,
- at a second step a *global model* (either a "satellite-only" or a (360, 360)-model) is combined with higher resolution *mean gravity anomalies* (e.g. $5' \times 5'$) and eventually also with *mean altimetric geoid heights*, to derive a geoid with corresponding resolution. As more recent examples show (see section 6) a relative accuracy of $\pm 0.3 \dots 1$ m/100 ... 1000 km may achieved, while the absolute accuracy naturally cannot exceed $\pm 1 \dots 1.5$ m. A certain accuracy improvement may be possible by including topographic information of a higher resolution than the gravity field,
- at a third step, one may proceed to a more precise geoid, by combination of a (360, 360) *global model* with high-resolution data sets, as *point gravity anomalies*, *altimetric geoid heights*, or (locally) *deflections of the vertical*. *Digital terrain models* are now used in order to deliver information about the high-frequency parts of the gravity field. At this stage separate solutions for either gravity anomalies (+ eventually altimetry) or vertical deflections (+ eventually altimetry) may be useful, for independent comparisons and later combination. Another independent check is provided by comparisons with Doppler or GPS/leveling geoid heights. The relative accuracy of these solutions is estimated to $\pm 0.01 \dots 0.1$ m/ 1 ... 1000 km,
- the most advanced fourth step finally should *combine* all gravity field information available, in an *optimum manner*. Correspondingly these combinations will include a high-resolution global model, point and if necessary also mean gravity anoma-

lies, altimetric geoid heights (with properly reduced sea surface topography), deflections of the vertical, GPS/leveling geoid heights and topographic (evtl. topographic-isostatic) data. From experiments carried out among others at IfE Hannover (see section 6), a relative accuracy of $\pm 0.005 \dots 0.01 \dots 0.05 \text{ m} / 1 \dots 10 \dots \text{some } 1000 \text{ km}$ may be expected.

For the combination of a global model with regional data, the "*remove-restore-technique*" has proved to be an efficient strategie (*Denker et al. 1986*). Here the gravity field information is splitted up into three different parts:

- the long wave part up to about 200 km wavelength is taken from a global or regionally tailored geopotential (spherical harmonic) model;
- the medium wave part (200 km to 5 ... 20 km wavelength) is taken from terrestrial point gravity field data, as gravity anomalies and astrogeodetic vertical deflections;
- the short wave part is derived from a high resolution digital terrain model.

Removing the long wave and the short wave part from the point data (which corresponds to a spectral filtering) leads to a residual gravity field data set, on which gravity field transformation algorithms for deriving the geoid (or quasigeoid) are applied. As shown, among others, by *Denker (1988)* and *Bašić (1989)*, integral formulas and least squares collocation give comparable results at this transformation. By restitution of the long and short wavelength gravity field part to this residual geoid solution, the final geoid is derived. Simulation studies revealed that by applying this technique, the data collection area in integral formulas and collocation may be strongly reduced, leading to more economic calculations.

6. Regional Geoid Case Studies

We now shortly describe a few case studies of large scale (quasi) geoid determinations, carried out at Institut für Erdmessung (IFE), University of Hannover:

- European gravimetric quasigeoid EGG1 as example of continental dimensions, representing the second step modeling strategy introduced in the preceding section,
- European gravimetric quasigeoid EGG2, as a representative of the third step strategy,
- gravimetric quasigeoid of Venezuela, as a regional example of the second step,
- gravimetric quasigeoid of Germany, representing the third step on a regional scale.

Here one remark about the introduction of the *quasigeoid* is appropriate. At IFE Hannover the quasigeoid calculation is preferred in order to avoid the problems which

occur at the "inverse" problem of the geoid determination. Only gravity field data observed at the earth's surface and the exterior enter into the calculations, and no assumptions about the mass distribution in the interior have to be made. As normal heights have been introduced in many countries with the same arguments, this strategy also is in agreement with practical considerations. If orthometric heights are used at the definition of a national height system, the transformation from the quasigeoid to the geoid can easily be performed with the density model of topography applied for the calculation of the orthometric heights.

The calculation of the *European gravimetric quasigeoid* EGG1 (Torge et al. 1982) is characterized by the following features:

- establishment of a *gravity data base* ($1^\circ \times 1^\circ$ and $6' \times 10'$ mean anomalies) with careful analysis and screening of the data (Torge et al. 1984)
- *statistical analysis* of the data, and derivation of signal and error *covariance functions*, for prediction and error propagation. Due to independent overlapping gravity surveys especially in marine areas a distance dependent error correlation could be derived (Weber and Wenzel 1982),
- least squares *spectral combination* with closed *integral formulas* (Wenzel 1982), using a satellite-only model (GEM9), $1^\circ \times 1^\circ$ anomalies (integration radius 20°), and $6' \times 10'$ anomalies (integration radius 3°), including straightforward error estimation,
- *quasigeoid heights* in a $12' \times 20'$ grid, with an absolute error estimate of ± 0.9 m, and relative errors of $\pm 0.3 \dots 1.1$ m/100...1000 km,
- by including about 5000 deflections of the vertical, EGG1 was extended to the *astrogravimetric quasigeoid* EAGG1 (Brennecke et al. 1983), where the astrogeodetic data improved the gravimetric solution mainly in areas with lacking or insufficient gravity data.

In the 1980's new gravity field data sets of high quality became available. This fact triggered the calculation of an improved quasigeoid for Europe, with the establishment (1990) of an IAG subcommission for the European geoid, which asked IFE Hannover to serve as a computing center (Torge and Denker 1991). Main working steps and (preliminary) results of the *European gravimetric quasigeoid* EGG2 are (Denker and Torge 1992, 1993, Torge 1992):

- extension of the IFE *gravity data base* by including point (and partly high resolution mean) gravity anomalies (spatial resolution between 1 km and 10 km) and terrain data (block size between 30 m and 10 km),
- *validation* and *merging* of the data (including transformation to a geocentric system), *gridding* of gravity ($1' \times 1.5'$) and terrain ($7.5'' \times 7.5''$ and multiples of this) data,

- calculation of *quasigeoid heights* employing the *remove-restore technique*, with the global model OSU 91 A and 30' x 30' RTM (residual terrain model) reductions, and planar FFT evaluation of *Stokes' formula*,
- the result are *quasigeoid heights* in a 1' x 1.5' grid, with an *error estimation* by comparison with GPS/leveling geoid heights (r.m.s. discrepancies including errors of GPS and leveling) of a few cm over some 10 to some 100 km, better than 0.1 m over 1000 km, and a few dm over some 1000 km, after bias and tilt reduction. Long-wave errors (resulting in a tilt) are supposed to stem from the global model and/or systematic effects of sea gravimetry.

The *comparison* with *GPS/leveling quasigeoid heights* has proved to be an efficient method for the accuracy estimation of gravimetric or combined solutions. In Europe, the NS-GPS traverse is available, among other surveys, for this purpose (Torge et al. 1989). Fig. 3 gives the results of these comparisons with the solutions EGG1 (gravimetric mean 6' x 10' anomalies), EAGG1 (EGG1 + vertical deflections), EGG2 (preliminary state of 1993, point gravity values + terrain data).

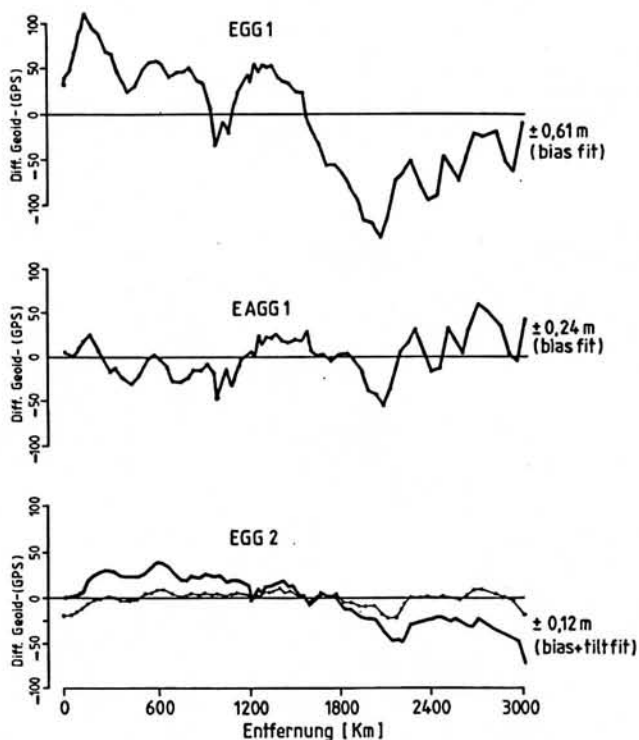


Fig. 3: Comparison of GPS/leveling quasigeoid heights with the results of gravity modeling for Europe, along the central and northern part of the European GPS traverse, bias resp. bias + tilt fit.

The (preliminary) *gravimetric quasigeoid calculation* for *Venezuela* was intended to demonstrate how a global model can be improved by regional data, to the "m to dm" accuracy level (Torge and Behrend 1992). The solution is characterized by

- establishment of a *gravity and terrain data base*, with point anomalies compressed to 5' x 5' mean values, and a 5' x 5' digital terrain model,
- *quasigeoid solution* after reduction of a global model (OSU 91 A) and the terrain part, by applying *Stokes' formula* in plane approximation using the *FFT method*,
- the result are *quasigeoid heights* in a 5' x 5' grid, with an error estimate on the 1 m accuracy level or better depending on the availability and quality of the terrestrial data. This is confirmed by a comparison with 4 "Doppler" geoid heights, with an r.m.s. discrepancy of ± 1.2 m. Table 1 gives some statistics for this solution showing that the residual gravity part adds (r.m.s.) ± 2 m to the global model part, and the terrain part still contributes with ± 0.3 m (r.m.s.).

Table 1: Statistics for the (preliminary) quasigeoid solution (Nov. 1992) for Venezuela

height anomaly	mean value (m)	min. value (m)	max. value (m)	r.m.s. value (m)
model part (OSU 91 A)	- 10.7	- 48.4	+ 23.6	-
residual gravity part	0.4	- 4.2	+ 9.2	± 1.9
RTM part	0.0	- 2.1	+ 3.3	± 0.3
total	- 10.7	- 48.9	+ 22.7	-

Finally, we mention the *quasigeoid* determination for the western part of *Germany* (Denker 1989):

- this gravimetric solution is based on *point gravity data* with an average spacing of 2 to 5 km, 6' x 10' *mean anomalies* for the outer zones, and a 30" x 50" digital *terrain model*,
- the *computation* was carried out using the remove-restore technique, with *Stokes' integral* formula (plane approximation) evaluated by FFT. Residual gravity anomalies were obtained by subtracting the (360, 360) tailored (for Europe) model IFE 88 E1 (Bašić 1989) and RTM effects (6' x 10' moving average filter for the reference topography), and gridded in a 60" x 100" grid,
- the resulting *quasigeoid heights* are provided in a 60" x 100" grid, with a bilinear interpolation algorithm. The *accuracy* was estimated by comparison with different GPS/leveling control nets or profiles, leading to r.m.s. discrepancies of $\pm 0.01...0.05$ m/15...800 km (Denker 1991). Table 2 gives the field statistics with r.m.s. residual gravity and residual terrain part contributions of ± 0.18 m and ± 0.02 m, respectively.

Table 2: Statistics for the 1989 quasigeoid solution for western Germany

height anomaly	mean value (m)	min. value (m)	max. value (m)	r.m.s. value (m)
model part (IFE 88 E1)	45.91	39.20	50.47	-
residual gravity part	- 0.01	- 0.49	+ 1.17	± 0.18
RTM part	0	- 0.14	+ 0.17	± 0.02
total	45.89	39.11	51.11	-

7. Problem Areas and Trends

From theoretical and practical considerations as well as from the results of more recent large-scale quasigeoid and geoid determinations, we can identify a number of problem areas and trends in this field. Without any attempt for completeness we specify some of those topics:

Data collection

- *Space missions* running, scheduled or under discussion for this decade and later include satellite altimetry (TOPEX/POSEIDON, ERS-1, ERS-2), satellite-to-satellite tracking (high-low-mode) and gravity gradiometry. They could not only improve the long-wave part of the gravity field but also extend the spectral resolution from over degree and order (100,100) to (360, 360). In marine areas, altimetry will provide a 10' and better spatial resolution, and further progress may be possible also in coastal zones and land areas,
- *gravity data* can now rapidly be collected by airborne gravimetry as recent large-scale surveys (Greenland) have shown, and further progress can be expected with respect to accuracy and resolution (few km). There is still a lack of gravity data measurements in coastal zones, where neither land nor sea gravimetry can be operated. We face the continuing problems of the release of (mainly) land gravity data for scientific investigations due to economic and military restrictions, here flexible solutions should be sought for (release of observed data, gridded data, mean values),
- the number of point *GPS/leveling geoid heights* increases rapidly, through continental and national GPS surveys, as well as through dedicated GPS networks and profiles. Those surveys should be optimized also with respect to heighting and well connected to the leveling system (bench marks, tide gauges). The adequate storage of this information and its release to the scientific community is not yet sufficiently solved,

- as regards *terrain data*, an improved global set may come from future space missions, while regional DTM's will be established (and improved) for more and more areas. Again we face the problem of releasing those data sets for geoid calculations which may be as serious as the release of gravity data. More attempts should be made to also include *density* (surface and subsurface) into the DTM's.

Data screening and reductions

- Data *screening* is a tedious but extremely important part at establishing a high-level data base, as systematic and gross errors of the data affect the geoid or quasi-geoid in an unpredictable manner. We especially mention the problems at *sea gravimetry* (cross-over discrepancies and bias, stemming from different error sources) and at digital *terrain data* (gross errors),
- data *reductions* should follow adopted standards but at least be homogeneous and clearly documented at one large-scale (quasi) geoid calculation. As examples we mention the atmospheric, tidal and sea surface topography reductions, which in many cases have been differently (if at all) modelled at different data sets. SST reduction (satellite altimetry, gravimetry) at border seas is still a problem if the cm-accuracy level is strived for,
- the *datum* problem has to be mentioned in this connection too. All data sets have to refer to the same and (generally) internationally recommended reference systems. Different *vertical* datums and height systems seriously affect gravimetry and leveling (realization problem of a global height reference surface!) while the problem of different *gravimetric* datums should in general be sufficiently solved. Different *horizontal* datums produce systematic effects at combining data sets from different regions but (more serious) also enter as errors if heterogeneous data based on different datums are combined (e.g. gravimetry and topography).

Spectral analysis of the gravity field

Although an enormous work has been done in spectral analysis and statistical gravity field description, still questions are open, as

- the application of *global* spectral *models* to *regional* calculations (scaling),
- the determination and use of *empirical signal covariance* functions (random errors),
- the inclusion of *GPS/leveling geoid heights* into the spectral investigations,
- the role and handling of *terrain* data (including the density of topography) and *crustal models*.

Error models

A priori error models (including possibly also error covariances) are important in order to perform a straightforward error propagation at the (quasi) geoid calculation. The models can be later improved by the comparison of independent data sets and by an analysis of the adjustment residuals. These investigations should also include spectral analyses of the errors. We still have problems at

- *global models* with long-wave errors, uncertainties of higher order terms, and too optimistic accuracy estimates,
- *satellite altimetry* with bias problems, large-scale errors and SST reduction errors,
- *gravimetry* with error correlation, prediction errors, and error models for sea and airborne gravimetric techniques,
- *deflections of the vertical* with longitude errors and (in many cases) too optimistic accuracy estimates,
- *GPS/leveling* with an appropriate error model, taking the main error sources into account, and finally leading to a distance dependent (?) model including correct covariances.

Theoretical problems

There are a number of theoretical problems which are not sufficiently solved. They practically cover all areas mentioned, but some of them shall be listed here separately:

- the internal geodetic *boundary value problem* (downward continuation into the solid earth, cogeoid, indirect effect),
- the *geoid* versus *quasigeoid* question,
- the *overdetermined* mixed geodetic boundary value problem and its correct handling with integral formulas,
- *optimization* procedures for obtaining a certain (quasi)geoid result (accuracy, resolution) with heterogeneous data sets (different gravity field quantities with different spectral behaviour), at a certain gravity field structure,
- investigations of all steps of a (quasi)geoid determinations with respect to a *1 cm-*, *5 mm-*, and *1 mm-accuracy* level..

Field transformation procedures

Although formulas for field transformation are available and applied since long time, a number of problems are not yet or only unsufficiently solved:

- at applying the *remove-restore technique* the spectral combination taking the error model into account,
- the optimum combination of *different data sets*,
- detailed investigations about the *efficiency* (accuracy and spectral part) of different data sets, especially with respect to GPS/leveling (quasi)geoid heights,
- investigation about the efficiency and comparison of *integral formulas* and *least squares collocation*,
- *error propagation* through the transformation formulas and realistic *error models* for the results.

"Sub-cm"-geoid

Although the "cm"-geoid has not yet been reached (except in a few local test areas), theoretical and numerical work should already start for an eventual "sub-cm"-geoid. The following questions then arise:

- is there a *need* for a "sub-cm"- or even "mm"-geoid (precise heighting)?
- which parts of the *theory* and of *field transformation* procedures of the geodetic boundary value problem have to be improved in order to guarantee the "mm"?
- which demands have to be put on the gravity field *data*, in order to reach the "mm" accuracy?
- how has the *time component* of the gravity field to be handled at approaching the "mm"-accuracy level? At which level is the "*environmental*" noise, which prevents a further accuracy increase?

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THE GEOID AND THE STEADY CIRCULATION PATTERN IN THE MEDITERRANEAN SEA

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1. Introduction

The GEOMED project (Brovelli and Sansò, 1992) led to a new estimate of the geoid in the Mediterranean area. Three estimates of the geoid undulation were computed in the western, the central and the eastern Mediterranean Sea (Barzaghi et al, 1992, 1993, Arabelos et al. 1994) using gravity data which were collected and validated by the GEOMED research team (Sevilla, 1993).

The gravimetric geoids were calculated via remove-restore technique and fast collocation which allows to estimate the undulation N , for each area, in a single step.

By subtracting the estimated geoids from altimetric data, properly preprocessed and crossover adjusted, an estimate of the Sea Surface Topography (SST) has been also separately derived in the three areas previously mentioned.

A further step, to which this paper relates, has been the estimation of a global solution, covering the whole Mediterranean, which has been obtained by merging the three partial estimates and two new solutions, computed to minimize the edge effects in the patching procedure.

Furthermore, a new SST has been derived using such a global estimate of N and the entire data set of altimetric data covering the Mediterranean Sea. This global SST seems to be physically meaningful and the steady circulation pattern based on it is in good agreement with direct marine observations, at least in the western and in the central part of the Mediterranean.

2. The geoid in the Mediterranean

As mentioned before, three local geoid estimates were derived in the western (W), central (C) and eastern (E) part of the Mediterranean Sea in the framework of the GEOMED project. The geoid, in each area, was computed on a regular geographical grid with $\Delta\varphi = \Delta\lambda = 5'$; the boundaries of the three local estimates and the statistics of N in these areas are summarized in Table 1.

Area	Boundaries	N of points	Average (m)	St. dev. (m)
W	$35^{\circ} \leq \varphi \leq 43^{\circ}$ $-5^{\circ} \leq \lambda \leq 10^{\circ}$	17557	47.99	3.40
C	$30^{\circ} \leq \varphi \leq 47^{\circ}$ $8^{\circ} \leq \lambda \leq 20^{\circ}$	29725	38.82	7.03
E	$30^{\circ} \leq \varphi \leq 41.5^{\circ}$ $17^{\circ} \leq \lambda \leq 36^{\circ}$	31831	25.78	10.65

Table 1 - Geoid statistics (the three original estimates)

The statistics show quite evidently a well known property of the geoid in the Mediterranean, i.e. its increasing roughness moving eastward.

Furthermore, the three areas were defined in such a way that overlapping zones (of about two degrees in longitude) exist between adjacent solutions, so to allow the merging of the local solutions into a global geoid estimate.

This was the initial idea to get a unique geoid over the whole Mediterranean Sea; however, the projected overlappings turned out to be insufficient in patching the three solutions. Infact, relevant differences in the mean of adjacent solutions were revealed, but it was not clear, being the overlappings between the areas quite small, if these relates to edge effects or to biases in the estimates.

To have a deeper insight on this problem and to devise a reliable procedure to get a global geoid over the Mediterranean from the local solutions, it was decided to compute two new geoid estimates centered on the edges of the three solutions previously obtained. The position of the five computation areas covering the Mediterranean is presented in Fig. 1 and the statistics of the two last geoid estimates are collected in Table 2: W/C is the area superimposed to the W and the C areas, while C/E is the one straddling the C and the E zones (estimates have been computed on a regular geographical grid with $\Delta\varphi = \Delta\lambda = 5'$).

Area	Boundaries	N of points	Average (m)	St. dev. (m)
W/C	$35^{\circ} \leq \varphi \leq 45^{\circ}$ $5^{\circ} \leq \lambda \leq 15^{\circ}$	14641	44.19	3.17
C/E	$30^{\circ} \leq \varphi \leq 47^{\circ}$ $8^{\circ} \leq \lambda \leq 20^{\circ}$	14641	28.67	7.26

Table 2 - Geoid statistics (the two new estimates)

The first analysis that we performed was the one concerning possible biases which can be evidenced more clearly with the aid of two new solutions. We considered two geoid sections over two parallels: the first one involves the W, the W/C and the C estimates and the reference parallel is $\varphi = 40^\circ$ (see Fig. 2); the second one displays the geoid values belonging to the C, the C/E and the E estimates along the $\varphi = 35^\circ$ parallel (see Fig. 3). As one can clearly see, the two sections show that biases are present in the computed geoid undulations which, however, have a remarkable consistency at high frequencies (apart from expected edge distortions). Furthermore, the statistical comparisons between C – W/C and C – C/E in the overlapping areas proved that these estimates are nearly unbiased (see also Fig. 2 and Fig. 3) since the mean of the differences are of -0.02 m and of -0.24 m respectively, while the situation is worse if we compare the W estimate with the W/C and the C/E estimate with E.

These are infact the two critical cases which have unacceptable biases, the mean of the differences being of 0.72 m and 1.10 m respectively.

There are several possible explanations of such a behaviour. First of all, the W solution was based on the OSU89B global geopotential model while the remaining four solutions were computed using the OSU91A model; this can account for low frequency discrepancies between the W and the W/C geoid estimates. Secondly, different DTMs were used to compute the RTC in these two areas and this can introduce further distortions. Finally, the sharp bias in the difference between the C/E and the E geoid estimates could be related, in the opinion of the authors, to serious data problems in the E zone. The east northern Africa and the eastern boundary have an insufficient data coverage and on Turkey, geoid inverted gravity anomalies were used (Tscherning, 1993) (we remark that this lack of data reflects not only on the GEOMED geoid estimates but also on the OSU91A model which can have low frequency distortions exactly in this part of the Mediterranean).

Considering these results, we decided to remove the biases taking as reference level the C geoid estimate, which is at least in good agreement with its two adjacent solutions.

So, strictly speaking, we obtained in the end a relative geoid, referred to the geoid computed in the central part.

Then, to merge the five "relative" local solutions, we used a simple weighted mean of the values belonging to them. In the area

$$30^\circ \leq \varphi \leq 47^\circ$$

$$-5^\circ \leq \lambda \leq 36^\circ$$

which includes the five local geoid estimates, we considered a grid of $5' \times 5'$; in each grid knot P_i , the value of $N(P_i)$ was computed as

$$N(P_i) = \frac{\sum_j^K p_{ij} N^j(P_i)}{\sum_j^K p_{ij}} \quad K = 1, 2, \dots, 5$$

where:

$N^j(P_i)$ = local geoid estimate in the point P_i belonging to the j -th area

$$p_{ij} = \frac{d_{\min}^i(j)}{\text{diag}(j)}$$

$d_{\min}^i(j)$ = minimum distance from point P_i to the edges of the j -th area

$\text{diag}(j)$ = diagonal of the j -th area

In this way, edge effects are minimized since the estimator gives less weight to values which are closer to the border. Furthermore, the OSU91A geoid undulation was used to assign a value to grid knots inside the selected area but outside the union of the five local estimates. The global geoid that we got in such a way is plotted in Fig. 4, while in Fig. 5 the standard deviation of the computed weighted mean is drawn. As it expected, major troubles are present in the eastern part where data problems and geoid roughness led to less reliable estimates.

The statistics of the geoid for the whole area are

$n = 102465$ (72409 coming from the weighted mean and 30056 from OSU91A model)

Average = 37.19 m

St. dev. = 11.33 m

3. The stationary Sea Surface Topography and the steady circulation pattern in the Mediterranean Sea.

In the Mediterranean Sea, a set of stacked and adjusted SEASAT, GEOSAT and ERS1 data has been made available by dr. P. Knudsen in the framework of the GEOMED project (see Fig. 6).

Using such a data base and the global geoid estimate that we got, we derived a stationary Sea Surface Topography for the whole Mediterranean.

We subtracted from the altimetric values the geoid, interpolated from the global grid on the altimetric points, and subsequently we estimated and removed from the residuals the free surface of the cross-over adjustment (Schrama, 1989).

At the end of this process, we obtained the SST which is plotted in Fig. 7; the statistics of the SST are the following

$n = 32217$

Average = -0.07 m

St. dev. = 0.50 m

The computed SST seems to have quite a high st.dev. which is again connected to the eastern part of the Mediterranean Sea, where both geoid and altimetry may have distortions (we recall that the global adjustment of altimetric data has been performed using the OSU91A geoid undulation which, as previously pointed out, could have troubles in this area). Nevertheless, the western and the central pattern of the SST seems to be physically consistent and so, reasonable currents can be derived.

To compute the steady circulation flows, we applied the equation describing the local geostrophic approximation, in Cartesian coordinates, of the Navier-Stokes equations (Wunsch, 1993)

$$\begin{cases} u = -\frac{\gamma}{f} \frac{\partial \zeta}{\partial y} \\ v = \frac{\gamma}{f} \frac{\partial \zeta}{\partial x} \end{cases} \quad \begin{array}{l} u = \text{eastward velocity component} \\ v = \text{northward velocity component} \end{array}$$

with

ζ = stationary Sea Surface Topography

γ = normal gravity

$f = 2 \omega \sin \varphi$ (ω = Earth mean angular rotation velocity)

$x = R (\lambda - \lambda_0) \cos \varphi$ $y = R \varphi$ (R = mean Earth radius)

The result of such a computation is plotted in Fig. 8. The circulation pattern in the western and in the central Mediterranean is meaningful; in particular, in the Tyrrhenian Sea its feature is in agreement with ship measurements of currents performed by the Italian Navy. While moving eastward, the circulation pattern becomes more and more confused and the agreement with flow maps derived from the oceanographers with independent data (Malanotte, 1994) is quite poor. The comparison in this area shows that the main structures of the flow field have been evidenced, but relevant distortions in the absolute values and in their positions are present.

However, in our opinion, this is quite a satisfactory result if we take into account the problems that we had in estimating the geoid in this area and considering how rough are here the geoid and the SST.

4. Conclusions

This final step of the GEOMED project concluded this first attempt of computing a reliable geoid for the Mediterranean area. As it has been pointed out in this and in the related

papers, there are still serious problems in the geoid estimate that we finally computed. They are mainly related to the lack of data exactly in the area (the eastern part) where the structure of the geoid would require a denser coverage of good gravity data.

Although this is only an intermediate solution (in the next two years a new GEOMED project will recompute the Mediterranean geoid with an improved gravity data base), it can give good quality information, at least in the western and in the central area.

The first by-product that we got from this geoid and from altimetric data, i.e. the computation of the stationary Sea Surface Topography and the implied current flows, gave interesting results which can be used, together with ship measurements, to compute integrated flows maps.

In the near future, we plan to use such a geoid for geophysical investigation, e.g. to invert it for estimating the Moho depth in the Mediterranean area.

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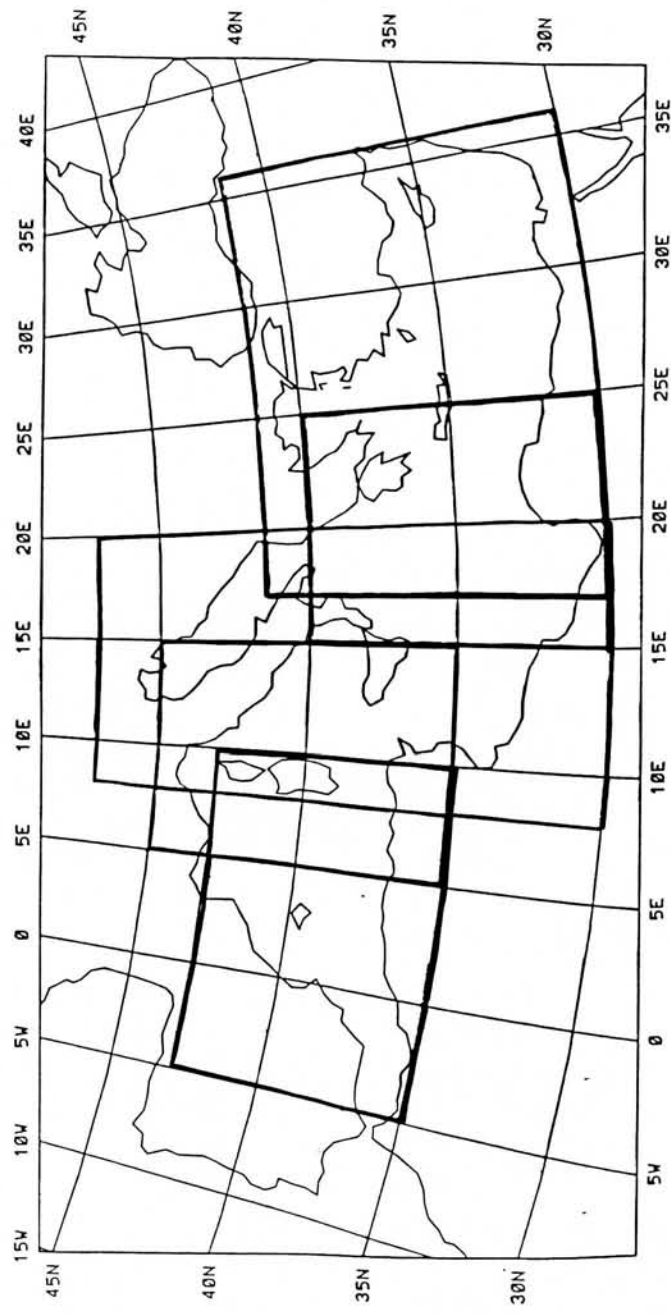


Fig. 1 - The five computation areas

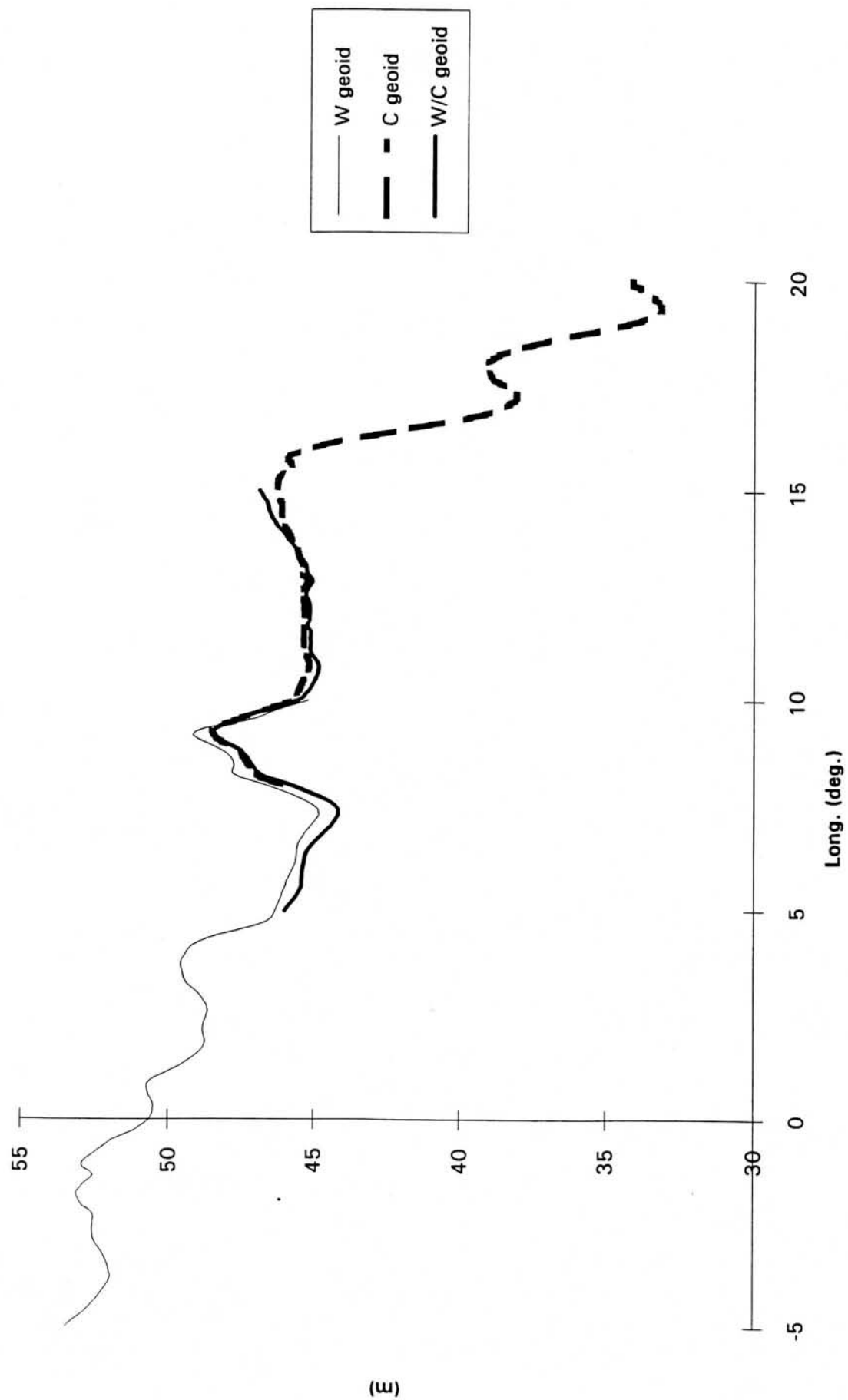


Fig. 2 - The W, W/C and C geoid estimates along $\phi = 40^\circ$

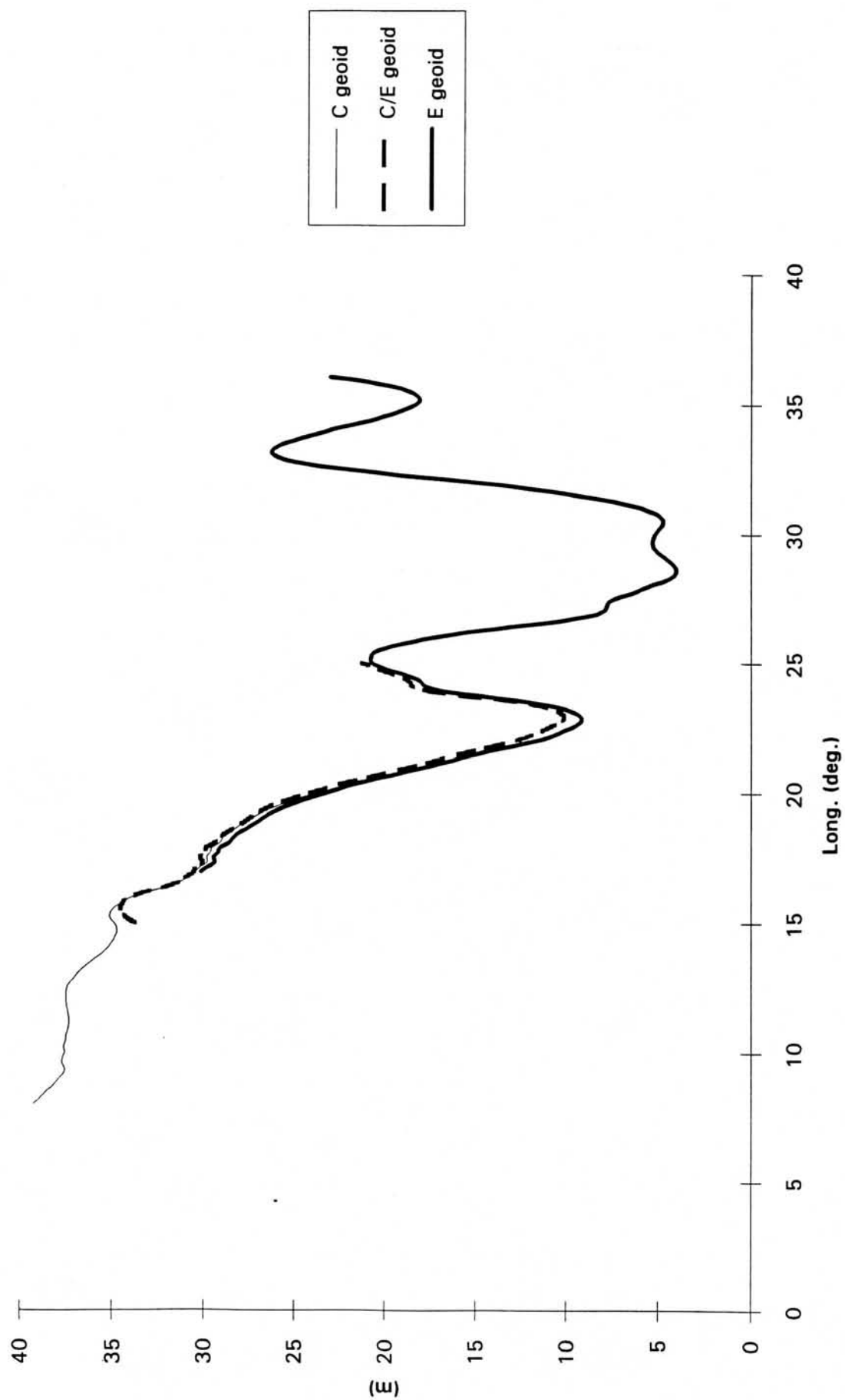


Fig. 3 - The C, C/E and E geoid estimates along $\varphi = 35^\circ$

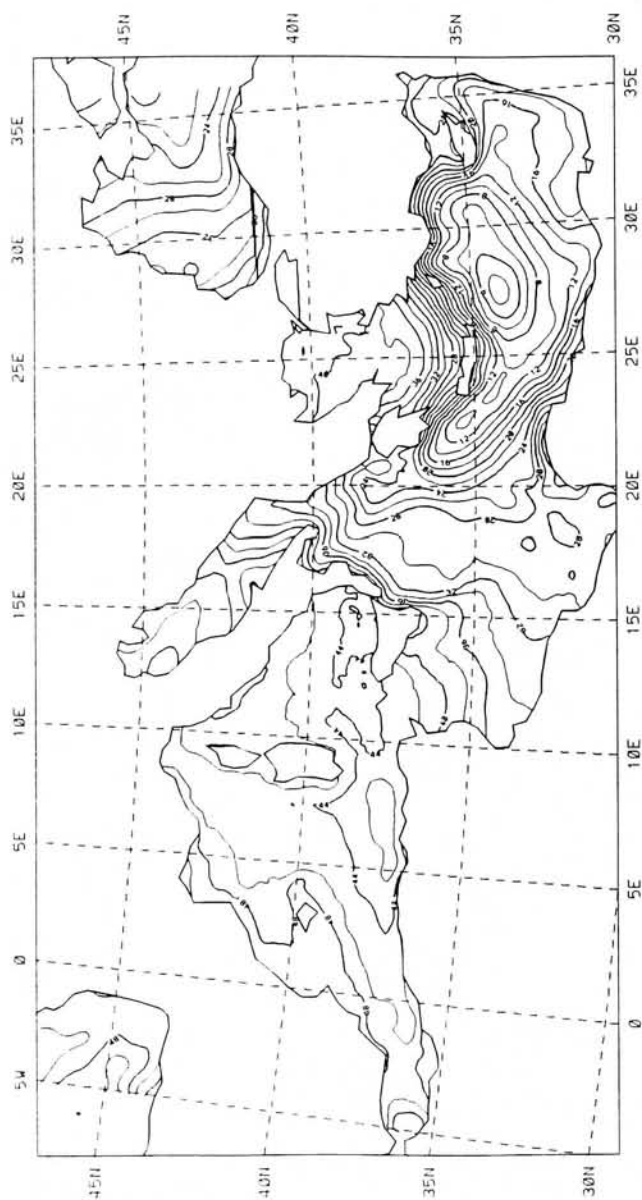


Fig. 4 - The geoid in the Mediterranean Sea (m)

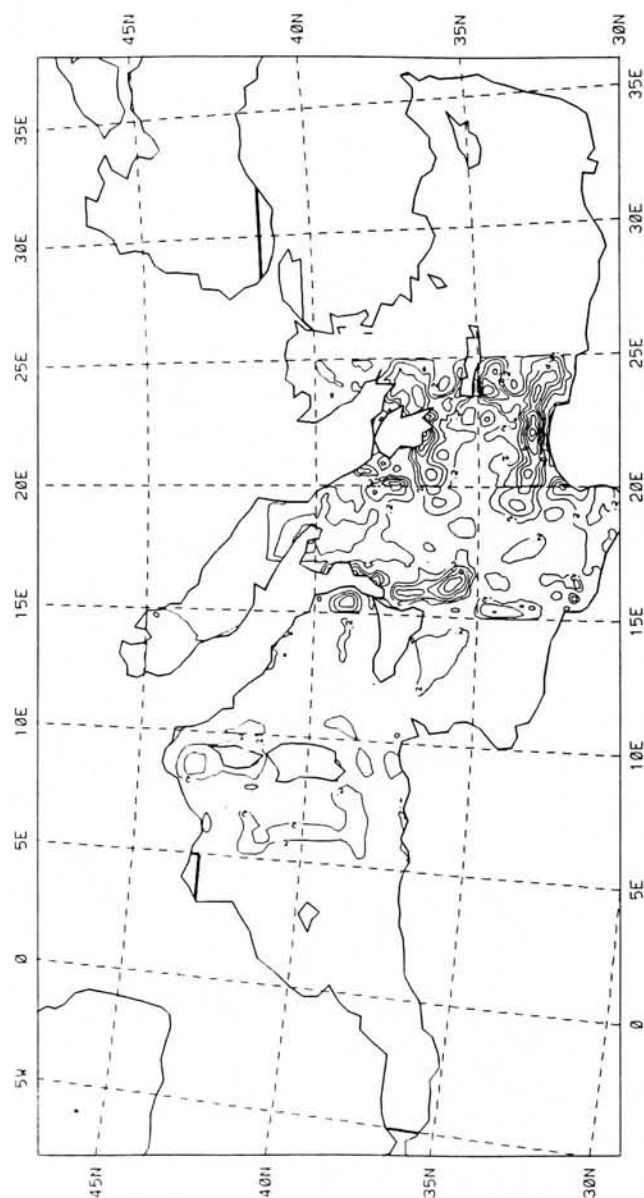


Fig. 5 - The st.dev. of the weighted mean operator (m)

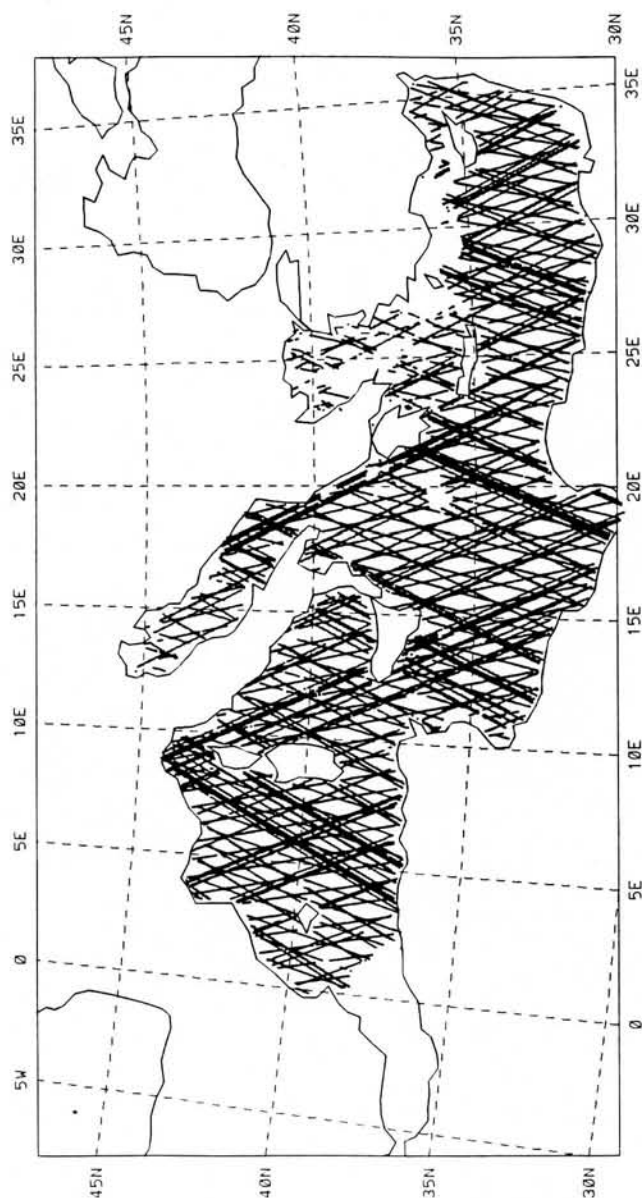


Fig. 6 - Altimetric track pattern

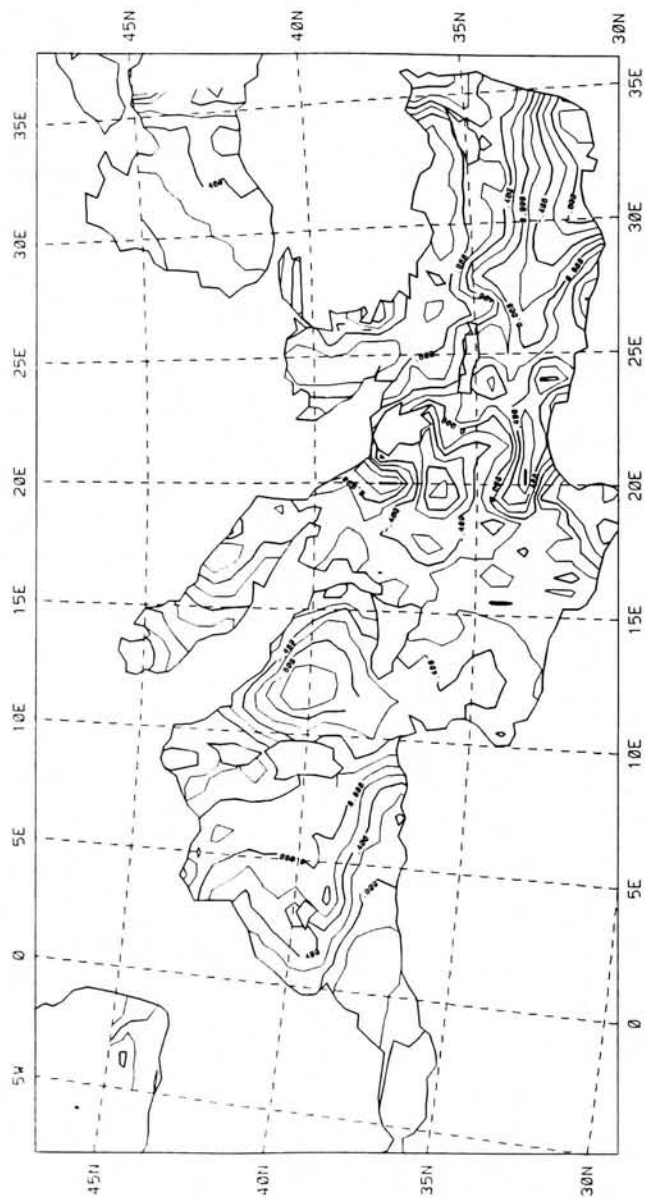


Fig. 7 - The estimated stationary SST (m)

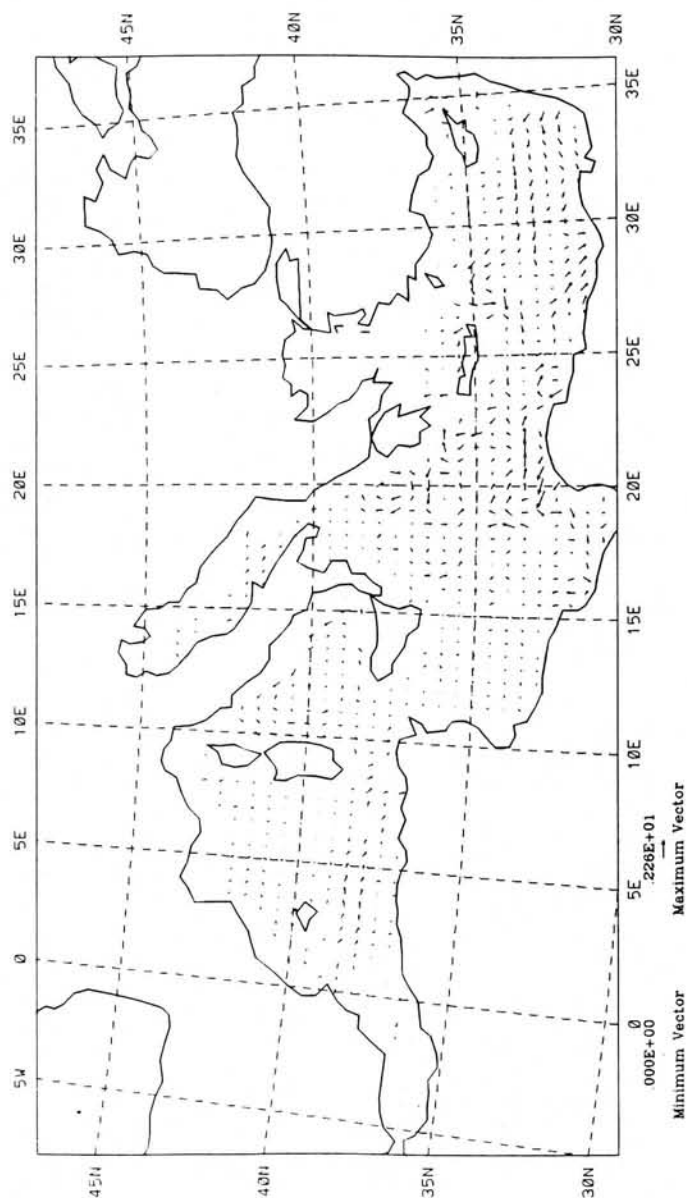


Fig. 8 - The steady circulation pattern in the Mediterranean Sea (m/s)

AN ATTEMPT FOR A GRAVIMETRIC GEOID IN SOUTH AMERICA

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ABSTRACT

South American Gravity Project (SAGP), a project undertaken and coordinated by the Geophysical Exploration Technology (GETECH) - University of Leeds, has been responsible for the collection and compilation of gravity data as well as topographic data in South and Central America. The project compiled 2,297,373 gravity points on land, marine and airborne. All gravity values have been adjusted to IGSN71 by using "Latin American Gravity Standardization Net 1977" (SILAG 77) and "Rede Gravimétrica Fundamental Brasileira" established by Observatório Nacional in Brazil. The anomalies are referred to WGS-84. The data were acquired from the International Gravity Bureau (BGI), Toulouse, France, from Defense Mapping Agency, Saint Louis, from oil companies and many national academic and non-academic organizations in different countries. Topographic and bathymetric data were used to generate a topographic model in a 3' grid and from that a terrain correction grid of 5' has also been derived. Nevertheless, several data gaps still exist and new measurements are needed to accomplish a homogeneous coverage. In Brazil a specific project called Anglo-Brazilian Gravity Project (ABGP) has been designed to infill some of these gaps in the north and west parts of the country. A new effort is now being undertaken by Escola Politécnica - University of São Paulo (EPUSP) and GETECH to estimate mean gravity anomaly values and to use them for geoid computations in South America. This paper is intended to describe the processing carried out with all the available informations from SAGP and ABGP in order to derive the best Helmert mean gravity anomaly value of 30' x 30'. A resume of the main theoretical topics related to the geoid determination will be summarized. A geoid model for Brazil will be presented.

1. INTRODUCTION

Geophysical Exploration Technology (GETECH) - University of Leeds has been involved in the compilation of gravity and geomagnetic data from many different parts of the world since 1986. One of its projects was the South American Gravity Project (SAGP) which collected gravity data for South and Central America and its margins delimited by the latitudes of 60° S and 25° N and longitudes 25° W and 100° W. Topographic and bathymetric data have been collected in the same area and a digital terrain model generated. Nevertheless, several data gaps still exist and new measurements are needed to accomplish a homogeneous coverage. In Brazil a specific project called Anglo-Brazilian Gravity Project (ABGP) has been designed to infill some of these gaps in the north and west parts of the country. In the first three years of the ABGP (1991-1994) several thousands of new gravity stations have already been established. ABGP has been benefited from an agreement between Fundação Instituto Brasileiro de Geografia e Estatística (IBGE - DEGED) and EPUSP, but also had collaborations from other organizations like Observatório Nacional (ON) [SOUSA, et al., 1993] and Companhia de Pesquisas de Recursos Minerais (CPRM). Data have been delivered to the project by these organizations as well as by IAG-USP [SHUKOWSKY et al., 1991]

Taking advantage of the SAGP and ABGP gravity data base, a new effort is now being undertaken by GETECH and Escola Politécnica - University of São Paulo (EPUSP), through the EPUSP and GETECH agreement, to estimate mean gravity anomalies in block sizes of 30' x 30' as a first step to develop a geoid model for South America. The results of this effort is expected to be used by the Sub-Commission for the Geoid in South America for a geoid model for the continent and for local geoid model in each country.

2. THE GRAVITY FIELD

In a convenient cartesian coordinate system X,Y,Z, the gravitational potential of the earth in a point P(x,y,z) may be expressed by: [MORITZ, 1980]

$$V(P) = V(x,y,z) = G \iiint_V \frac{\rho(Q)}{l} dv_Q \quad (1)$$

dv_Q is the volume element centered in a point Q with mass density $\rho(Q)$ and l the distance between P and Q. G is the Newtonian gravitational constant:

$$G = 6,672 \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

The integral is extended over the whole earth which includes the solid and liquid parts. The atmosphere in general is not included in the integral and its effect is taken into account separately through out appropriate corrections. For practical purposes, the earth is considered rigid and temporal variations like earth tides and polar motion are also considered separately. Other temporal variations are sometimes taken into account with specific corrections when necessary. [GROTEN, 1984]

The potential due to the centrifugal force is given by:

$$V_c = \frac{1}{2} \omega^2 (x^2 + y^2) \quad (2)$$

ω being the angular velocity of the earth's rotation. The gravity potential W is the sum of the gravitational potential and the centrifugal potential:

$$W(x, y, z) = V(x, y, z) + \frac{1}{2} \omega^2 (x^2 + y^2) \quad (3)$$

The potential V is related to the gravitational field and the potential W to the gravity field. So, the gravitational vector $\mathbf{f}$ is the gradient of V :

$$\mathbf{f} = \text{grad } V = [V_x \ V_y \ V_z]^T \quad (4)$$

and the gravity vector $\mathbf{g}$ the gradient of W :

$$\mathbf{g} = \text{grad } W = [W_x \ W_y \ W_z]^T \quad (5)$$

It is important to mention the (second order) gravitational gradient tensor V_{ij} :

$$V_{ij} = \begin{bmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{bmatrix} \quad (6)$$

whose elements are second order partial derivatives of V . The gravity gradient tensor is defined in a similar way:

$$W_{ij} = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix} \quad (7)$$

The traces of the matrices (6) e (7) represent the Laplacian of V and W respectively:

$$\nabla^2 V = V_{xx} + V_{yy} + V_{zz} \quad (8)$$

and

$$\nabla^2 W = W_{xx} + W_{yy} + W_{zz} \quad (9)$$

Outside of the attracting masses V satisfies the Laplace's equation:

$$\nabla^2 V = 0 \quad (10)$$

and in the interior of the masses the Poisson's equation:

$$\nabla^2 V = -4\pi G\rho \quad (11)$$

Similar relations hold for the gravity potential:

$$\nabla^2 W = 2\omega^2 \quad (12)$$

outside the masses, and

$$\nabla^2 W = -4\pi G\rho + 2\omega^2 \quad (13)$$

inside.

Due to the fact that V satisfies equation (10) is called a HARMONIC FUNCTION. Because of this it can be represented in a series of spherical harmonic functions (see 22).

The norm of the gravity vector

$$g = ||\mathbf{g}|| \quad (14)$$

is of extreme importance because is a quantity which can be easily observed directly or indirectly. The direction of $\mathbf{g}$ is defined by the versor $\mathbf{n}$ with the following components:

$$\mathbf{n} = \begin{bmatrix} \cos\phi_A \cos\lambda_A \\ \cos\phi_A \sin\lambda_A \\ \sin\phi_A \end{bmatrix} \quad (15)$$

where ϕ_A, λ_A are astronomic coordinates. In (14) the direction of $\mathbf{n}$ is upward, so:

$$\mathbf{g} = -g\mathbf{n} \quad (16)$$

Associated with the gravity field are surfaces with a constant potential, EQUIPOTENTIAL SURFACES. The equipotential surface which coincides with the undisturbed mean sea level is called geoidal surface. It is the limit of a geometric form called GEOID.

3. NORMAL EARTH AND ANOMALOUS FIELD

An ellipsoid of revolution having the same angular velocity and the same mass of the earth, the potential U_0 on the ellipsoid surface equals the potential W_0 on the geoid, with the center coincident with the center of mass of the earth is called

NORMAL EARTH. Related to the normal earth is the gravity potential U and the normal gravity vector:

$$\gamma = \text{grad } U \quad (17)$$

with

$$U = U_g + \frac{1}{2} \omega^2 (x^2 + y^2) \quad (18)$$

U_g is the gravitational potential of the normal earth.

The gravity potential U is a good approximation to the gravity potential W . The difference in a point P

$$T(P) = W(P) - U(P) \quad (19)$$

is relatively small and called anomalous potential or disturbing potential. The gravity anomaly vector is defined as:

$$\Delta g(P) = g(P) - \gamma(Q) = [\gamma_\eta \gamma_\xi \Delta g]^T \quad (20)$$

The third component is simply called gravity anomaly. It is related to the anomalous potential through the following relation:

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{R} T \quad (21)$$

which is the spherical approximation of the fundamental equation of physical geodesy. It is used as a boundary condition to the solution of T which is similar to the third boundary value problem of potential theory (Hilbert problem) since the boundary is convenient chosen. One possibility is to choose the geoidal surface and in this case we are in the Stokes theory. If the option is the physical surface of the earth, the solution is based on the modern Molodenskii theory. Two approximations have been used in the solution of one or the other problem: the series of spherical harmonic functions or the integral equations. Certainly the more convenient is a combination of the two possibilities after a convenient separation of components. A solution that has been very much used in geodesy uses the Hilbert space with reproducing kernel called least squares collocation [MORITZ, 1980]. Necessarily explores the separation of the two components. On the other hand, as the integrals involved are always convolution integrals the fast Fourier transform (FFT) can be used with some advantages [SCHWARZ et al., 1990].

4. SPHERICAL HARMONIC FUNCTIONS AND THE STOKES INTEGRAL

The solution of the Laplace equation (10) can be represented in a series of the following form:

$$V(r, \theta, \lambda) = \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^n (A_{nm} Y_{nm}^c + B_{nm} Y_{nm}^s) \quad (22)$$

$$Y_{nm}^c = P_{nm}(\cos \theta) \cos m\lambda$$

$$Y_{nm}^s = P_{nm}(\cos \theta) \sin m\lambda$$

where $P_{nm}(\cos \theta)$ represents the eigenfunctions of the Legendre associated equation and n, m the eigenvalues. Y_{nm}^c, Y_{nm}^s can be viewed as eigenfunctions of the Laplace equation on the sphere; they are called (surface) spherical harmonic functions [VANÍČEK and KRAKIWSKY, 1986]. If coefficients without unit are used, (22) takes the form:

$$V(r, \theta, \lambda) = \frac{GM}{r} \left\{ 1 - \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\bar{J}_{nm} \bar{Y}_{nm}^c + \bar{K}_{nm} \bar{Y}_{nm}^s) \right\} \quad (23)$$

The relations between the coefficients A_{nm}, B_{nm} with potential units, J_{nm}, K_{nm} without units, and the full normalized coefficients $\bar{J}_{nm}, \bar{K}_{nm}$ are: [VANÍČEK and KRAKIWSKY, 1986]

$$A_{nm} = \frac{GM}{a} J_{nm} \quad n \neq 0$$

$$B_{nm} = \frac{GM}{a} K_{nm}$$

$$\bar{J}_{n0} = \frac{1}{\sqrt{(2n+1)}} J_{n0}$$

$$\bar{J}_{nm} = \sqrt{\frac{(n-m)!}{2(2n+1)(n-m)!}} J_{nm}$$

$$\bar{K}_{nm} = \sqrt{\frac{(n-m)!}{2(2n+1)(n-m)!}} K_{nm}$$

The gravitational potential of the normal earth can be represented in a similar way:

$$U_g(r, \theta) = \frac{GM}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{2n} \bar{J}_{2n}^N \bar{P}_{2n}(\cos \theta) \right] \quad (24)$$

where the form $2n$ for the sub-index and the exponent means an even number and the index N refer to the normal earth in this case.

Assuming that the centrifugal force related to the potentials U and W are exactly equal, the anomalous potential satisfies the Laplace equation and can be represented in a series as:

$$T(r, \theta, \lambda) = -\frac{GM}{r} \left\{ 1 - \sum_{n=2}^{\infty} \sum_{m=0}^n [\bar{J}'_{nm} \bar{Y}_{nm}^c + \bar{K}'_{nm} \bar{Y}_{nm}^s] \right\} \quad (25)$$

$$\bar{J}'_{nm} = \bar{J}_{nm} - \bar{J}_{nm}^N$$

$$\bar{K}'_{nm} = \bar{K}_{nm} - \bar{K}_{nm}^N$$

The expression (25) is a consequence of (19) assuming some simplifications. In general the interest is on values on the surface of the earth approximating by a sphere of radius R ($r = R$). On the other hand, it has been adopted a scale factor $a = R$ so that the radial function equals to unit. It is assumed that both the earth and the ellipsoid have the same quantity of mass and that the gravity potential on the geoid equals the normal gravity potential on the ellipsoid. In these conditions there will not be zero order terms in (25). Finally, if the centre of the ellipsoid coincides with the centre of mass of the earth the first order term will be null.

The gravity anomaly and the geoidal height, as components of the anomalous potential, accomplish the following relations on the geoid: [HEISKANEN and MORITZ, 1967]

$$\Delta g = \frac{1}{R} \sum_{n=0}^{\infty} (n-1) T_n(\theta, \lambda) \quad (26)$$

$$N(\theta, \lambda) = \frac{T(\theta, \lambda)}{\gamma} \quad (27)$$

The anomalous potential can be obtained from (25) if values of the coefficients at least up to a certain degree and order 1 are known. Another possibility is to use the Stokes integral:

$$T(\theta, \lambda) = \frac{R}{4\pi} \iint \Delta g S(\psi) d\sigma \quad (28)$$

A form of representation of the geoidal height in terms of spherical harmonic functions is: [BLITZKOW, 1986]

$$N(\theta, \lambda) = -R \sum_{n=2}^{\infty} \sum_{m=0}^n (\bar{J}'_{nm} \bar{Y}_{nm}^c + \bar{K}'_{nm} \bar{Y}_{nm}^s) \quad (29)$$

which is a result of (25) and (27) assuming $\gamma = GM / R^2$. The terms of zero and first order were considered null.

5. THE SPECTRAL COMPONENTS

The determination of geopotential models and their improvements in the last few years are responsible for the decomposition of the elements of the anomalous potential, in particular the geoidal height, in two different spectral components: one of

long wavelength and another of short wavelength. The first one is derived directly from the model. The second is computed using a convenient modification of the Stokes integral. When least squares collocation is used the geopotential model is important to eliminate the systematic part which allows to consider the short wavelength component as a random variable. Recently, the facilities to obtain digital terrain models (DTM) in a grid and in this way to use interpolation procedures, can be a good reason to use FFT technique.

6. THE MODIFIED STOKES INTEGRAL

The expression (29) can be split out in the following form according to: [BLITZKOW, 1986]

$$N(\theta, \lambda) = -R \sum_{n=2}^l \sum_{m=0}^n (\bar{J}_{nm} \bar{Y}_{nm}^c + \bar{K}_{nm} \bar{Y}_{nm}^s) - R \sum_{n=l+1}^{\infty} \sum_{m=0}^n (\bar{J}_{nm} \bar{Y}_{nm}^c + \bar{K}_{nm} \bar{Y}_{nm}^s) \quad (30)$$

or briefly:

$$N(\theta, \lambda) = N_1(\theta, \lambda) + \delta N_1(\theta, \lambda) \quad (31)$$

which means to separate the geoidal height in two different spectral components, one of long wavelength and another of short wavelength. The first component N_1 is obtained easily from a geopotential model. If gravity data is available in the region around the computation point the second component can be estimated using a convenient modification of the Stokes integral. Without details which can be found in [BLITZKOW et al., 1991] the mathematical expressions are:

$$\delta N_1(\theta, \lambda) = \frac{R}{4\pi\gamma} \iint \delta \Delta g_l \delta S_l^m(\psi) \sin \psi d\psi d\alpha \quad (32)$$

where:

$$\delta \Delta g_l = \Delta g - \sum_{n=0}^l \sum_{m=0}^n (C_{nm} Y_{nm}^c + D_{nm} Y_{nm}^s) \quad (33)$$

which means to subtract the long wavelength component from the observed gravity anomaly, and

$$\delta S_l^m(\psi) = \delta S_l(\psi) - \overline{\delta S_l(\psi)} \quad (34)$$

with

$$\delta S_l(\psi) = S(\psi) - S_l(\psi) \quad (35)$$

$$S_l(\psi) = \sum_{n=2}^l \frac{2n+1}{n-1} P_n(\psi) \quad (36)$$

$$\overline{\delta S_l(\psi)} = \sum_{i=0}^l \frac{2i+1}{2} t_i P_i(\cos \psi) \quad (37)$$

for the determination of t_i coefficients see [VANÍČEK et al., 1987] and [BLITZKOW et al., 1991].

Some remarks are important at this point related to the modified Stokes integral. In the past due to the lack of gravity data all over the world the integration was limited to a spherical distance ψ_0 . The fact that the area outside of ψ_0 was not considered was responsible by a truncation error studied extensively in the bibliography, e. g. [HEISKANEN & MORITZ, 1967] and led to the Molodenskii coefficients. The idea was to choose ψ_0 as big as possible in order to minimize the truncation error. With geopotential models available the choice of ψ_0 has another principle. It is a function of the degree and order of the model used. A model of high degree ($l \geq 50$) means that the integration can be performed in a shorter distance from the computation point ($\psi_0 \leq 4^\circ$). However, model of high degree are always derived from satellite observations and surface gravity data. These kind of data haven't a homogeneous distribution in most cases. The consequence is that the geopotential models doesn't have very precise high order coefficients. In this way, in general is more convenient to use a model derived from satellite observations only which means basically degree and order 36 and in this case to extend the integration up to $\psi_0 = 5^\circ$.

7. GETECH DATA BASE

At the end of SAGP 2,297,373 land, marine and airborne gravity points were collected in South and Central America [GREEN & FAIRHEAD, 1991]. All gravity values have been adjusted to IGSN71 by using "Latin American Gravity Standardization Net 1977" (SILAG 77) and "Rede Gravimétrica Fundamental Brasileira" established by Observatório Nacional in Brazil [FAIRHEAD et al., 1991]. The data have been acquired from the International Gravity Bureau (BGI), Toulouse, France, from Defense Mapping Agency, Saint Louis, USA, from oil companies and many national academic and non-academic organizations in different countries. The final gravity distribution is shown in fig.1. The analysis carried out on the gravity data to check against inconsistencies and different origins are explained in [GREEN & FAIRHEAD, 1991].

Besides the gravity stations, a 3' x 3' grid of topography and bathymetry was produced in order that terrain corrections could be applied. Four types of elevation data have been used to construct the grid:

1. Worldwide topographic grid (ETOPO5).
2. Values picked from topographic maps.
3. Heights at gravity stations.
4. Shoreline location.

A detailed description of these sources and the way they have been used can be found in [GREEN & FAIRHEAD, 1991] and fig. 2 shows the spatial distribution of the different types of elevation.

8. DATA PROCESSING

8.1 SAGP PROCESSING

After the validation process was completed at the time of SAGP, terrain correction, Free Air and Bouguer anomalies were computed. The derived gravity anomalies are referred to WGS-84 through the theoretical formula:

$$\gamma_{84} = 978032.67714 \frac{(1 + 0.00193185138639 \sin^2 \phi)}{\sqrt{1 - 0.00669437999013 \sin^2 \phi}} \text{ mGal} \quad (38)$$

The Free Air correction has been applied using the following empirical equation:

$$FAC = 0.3083293357 + 0.0004397732 \cos^2 \phi) h + 7.2125 \times 10^{-8} h^2$$

which is a function of the latitude ϕ and the orthometric height h .

The sea bottom points have been treated according to the following expression: [BGI, 1991]

$$FAC = (2k\rho_w^s - \Gamma) D_1 \text{ mGal}$$

where D_1 is the depth of water, $\rho_w^s = 1027 \text{ kg x m}^{-3}$, $\Gamma = 0.3086 \text{ mGal/m}$, $k = 2\pi G$ with G the gravitational constant. For the Bouguer correction an infinite slab expression has been used with a density of 2.67 g/cc for land stations and 1.17 g/cc for marine points. A curvature correction has also been applied. For a detailed description refer to [GREEN & FAIRHEAD, 1991].

Terrain corrections were calculated for all land and marine gravity points in South America taking into consideration only the "outer zone" (from 5 km to 166.7 km). To estimate this correction the 3' digital terrain model described before has been used. For more details about the terrain correction and informations on the range of this correction in particular in the Andes, refer again to [GREEN & FAIRHEAD, 1991].

Because the expression (1) includes the mass of the atmosphere, a correction has been applied to the observed gravity values to be consistent according to the following equation:

$$\delta g_A = 0.87 e^{-0.116 \times H^{1.07}} \text{ mGal}$$

where H is the elevation in kilometers.

8.2 PRESENT PROCESSING

The onshore data base available at GETECH (which includes coordinates, heights and anomalies computed according to the aforementioned description, and a source code) have been sorted in longitude by blocks of $1^\circ \times 1^\circ$ and sorted in latitude in each block. The arithmetic mean has been used to estimate a mean anomaly for each block of $5' \times 5'$ if at least two points existed in the cell. When a mean value was computed, the coordinates of the centre of the block was derived as a position for the mean. If just one point was available in one $5'$ block, the anomaly value with the existing coordinates were maintained. As a result the final file with mean values of $5' \times 5'$ grid is not in a complete regular grid, but some coordinates are shifted from the centre. The estimated mean values for the whole area covered by SAGP are shown in figures 3 and 4, south and north hemisphere respectively. The SAGP data file does not include any information on the accuracy of the different values available in the record for each station. So, an estimation of accuracy of the mean gravity value has been computed using the following slightly modified version of the empirical formula presented in [KIM and RAPP, 1990]:

$$\vartheta = \text{MAX}\{2, \text{INT}(20 / \sqrt{n} + 0.05 + |\Delta g| + 0.5)\}$$

which is a function of the number of points 'n' and the module of the gravity anomaly Δg , between other constants.

Once the mean value was estimated the difference between the mean and individual values has been calculated and recorded in a specific file if the absolute value exceeds 200% of the mean. This procedure has been used to look for any big error still present in the original file of gravity anomalies. Despite carrying out a careful validation process in SAGP, a few new errors have been found and corrected. Cells of $1^\circ \times 1^\circ$ with suspected problems has been analysed using NDVS (New Data Validation Software) developed at GETECH. It provides with facilities to look for the anomalies in a 3-D and in this way find out some inconsistencies.

After completing the estimation of mean values an interpolation process was carried out: the objective being to estimate a value for as many empty blocks of $5'$ as possible by taking advantage of the existing values around them. Therefore a mean value has been assigned to every empty block when at least three values were available either to the north, south, east or west of the empty cell. In this process an arithmetic mean has been used again. Figures 5 and 6 show the distribution of the interpolated blocks, south and north respectively. From this complete file which includes estimated and interpolated values, a $30' \times 30'$ grid mean values have been derived using again the arithmetic mean. In the whole process it has been derived mean values for the height, for the Free Air anomaly and for the Bouguer anomaly. Due to the smoothness of the last one, it is very well known that the mean value of this anomaly is more representative than the two other quantities, in particular when the distribution of the point values in the cell is poor.

In a further step, the 3' topographic grid mentioned above has been used to estimate mean heights for blocks of 30' x 30'. As the data are in a 3' grid, for each cell a total of 11 x 11 values occur considering the borders. The fact that the values of the borders are used for two different cells creates some correlation that has not been taken into account in the mean estimation at this time. The mean heights derived can be used to restore the mean Free Air anomaly in the blocks of 30' x 30'.

9. RESULTS AND CONCLUSION

The final distribution of mean gravity values of 30' x 30' for the area related to SAGP is shown in fig. 7. In order to use these data for the South America a validation experiment has been done in Brazil.

In a first step 399 Doppler and GPS points on the levelling network have been used to derive the satellite geoidal height N_S . The cartesian coordinates related to WGS-84 have been referred to ITRF90 using the following transformation parameters: [McCARTHY, 1992; p.18]

$$\begin{aligned} T_1 &= -0.006 \text{ m} & \alpha_1 &= -0.01830 \text{ ''} \\ T_2 &= 0.517 \text{ m} & \alpha_2 &= 0.0003 \text{ ''} \\ T_3 &= 0.223 \text{ m} & \alpha_3 &= 0.0070 \text{ ''} \\ K &= 0.011 \text{ PPM} \end{aligned}$$

The idea is to have a geoid referred to a geocentric/ITRF90 oriented ellipsoid defined by:

$$\begin{aligned} a &= 6,378,137 \text{ m} \\ e^2 &= 0.00669437999 \end{aligned}$$

In order that a gravimetric geoid (derived from gravity data and/or geopotential model) be referred to WGS-84 ellipsoid it is necessary to estimate a zero order undulation N_0 due to the fact that the best fit ellipsoid to the geoid is not (or may not be) the WGS-84 ellipsoid. To do that the satellite undulations have been compared in a first step with GEMT2. The term N_0 is computed according to: [BLITZKOW & SÁ, 1983]

$$N_0 = \frac{\sum_{i=1}^n (N_{Si} - N_{Gi})}{n} \quad (20)$$

The result of the comparison with 399 points has been $N_0 = -1.37 \text{ m}$.

The mean gravity anomaly of 30' blocksize are mean Faye anomaly. In other words, they are Free Air anomaly corrected of the terrain correction. The referred anomaly is a good approximation to the Helmert second condensation method. [BLITZKOW et al., 1994].

The mean gravity anomalies, after remove the long wavelength component according to (33), have been used in the modified Stokes integral (32). Finally the spheroidal undulation derived from GEMT2 has been added. This is the " remove-restore " technique. A new comparison has been done with the satellite undulations. The differences are shown in the Fig. 8. The new zero order undulation estimated is $N_0 = -1.56$ m. The RMS difference is 2.1 m.

Using both the satellite and gravity derived undulations the geoid (quasi-geoid) map (Fig. 9) for Brazil has been computed and presented. The satellite undulations are referred to WGS-84/ITRF90 according to the transformation parameters mentioned before. To the gravity/GEMT2 undulations the zero order term of -1.56 m has been added. A similar geoid map for South America is in preparation.

It is difficult to estimate the error of the geoidal heights now presented. It depends on many different conditions: quality and distribution of the gravity data, quality and distribution of the Doppler and GPS points, the accuracy of the levelling network, etc. The RMS difference obtained from the comparison of satellite and gravimetric undulations is certainly a good measure. Actually the relative error should be better. It is expected a relative error better than 1 cm/km.

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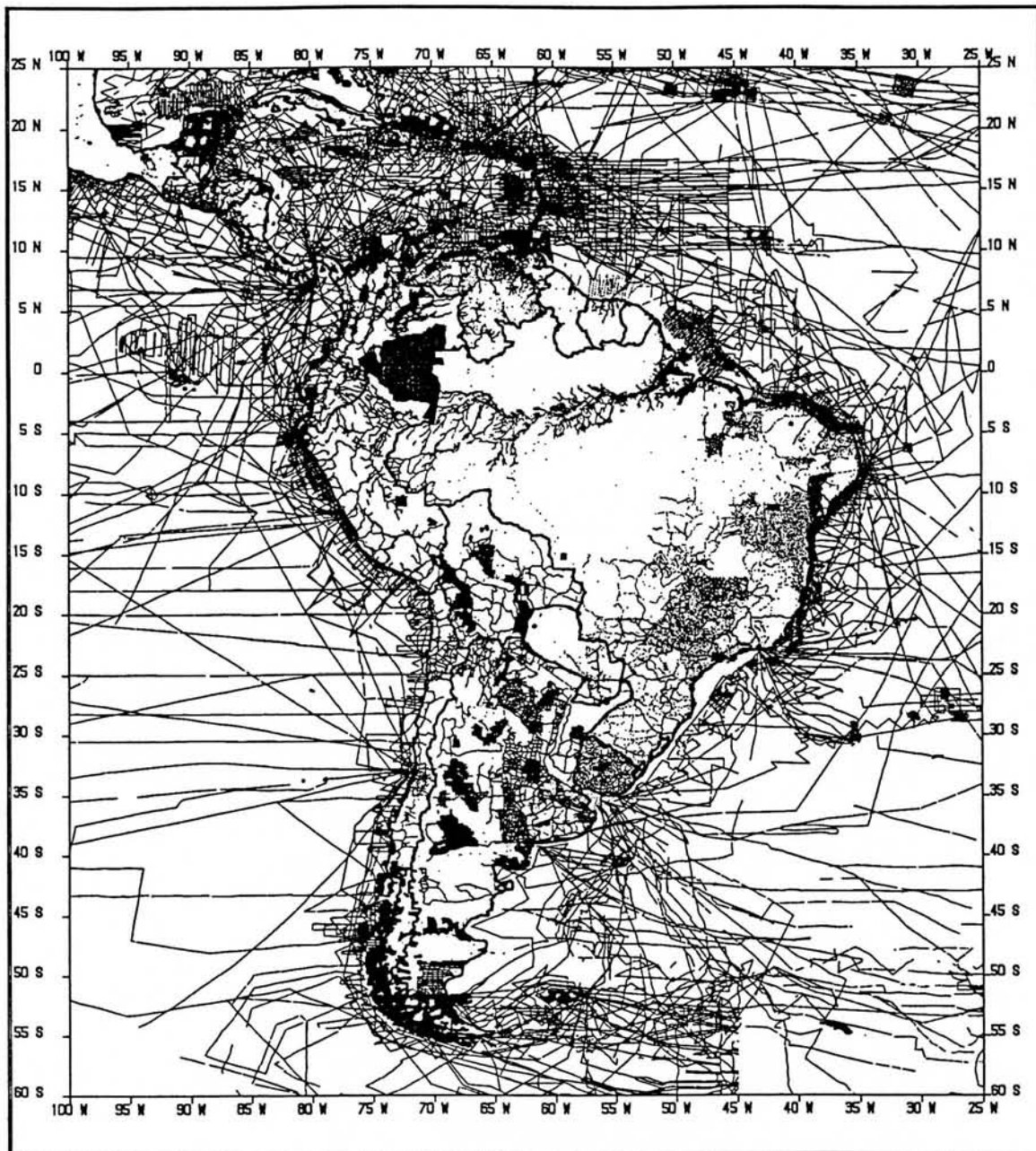


Fig. 1 - Final gravity distribution for SAGP.



Fig. 2 - Contribution of ETOPO5 (white), map derived grid (light grey) and station heights (dark grey) to the final SAGP topographic grid.

Estimated mean values of 5' grid.

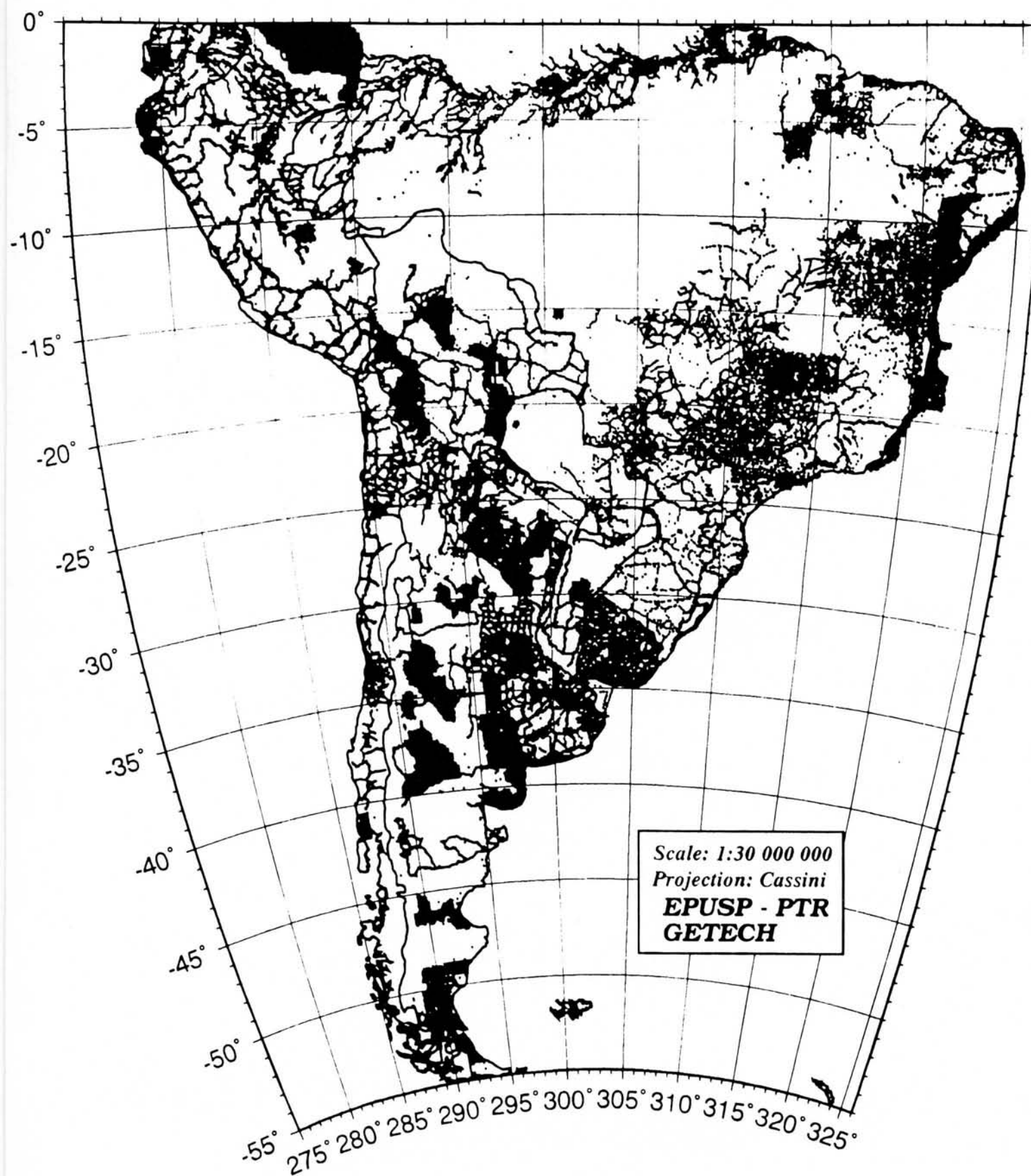


Fig. 3 - Estimated mean values (45 413 points).

Estimated mean values of 5' grid.

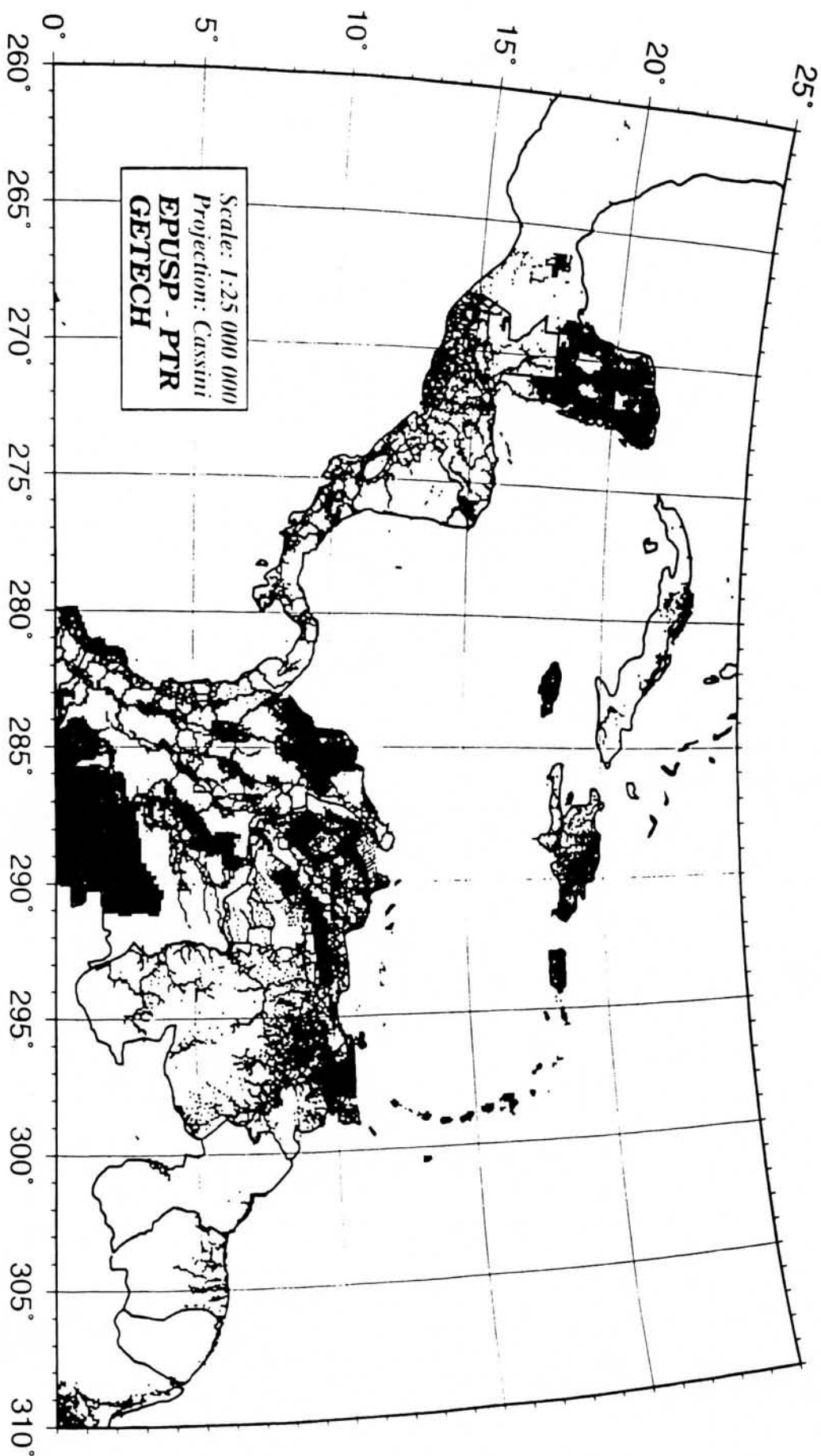


Fig. 4 - Estimated mean values (18 243 points).

Interpolated mean values of 5' grid.

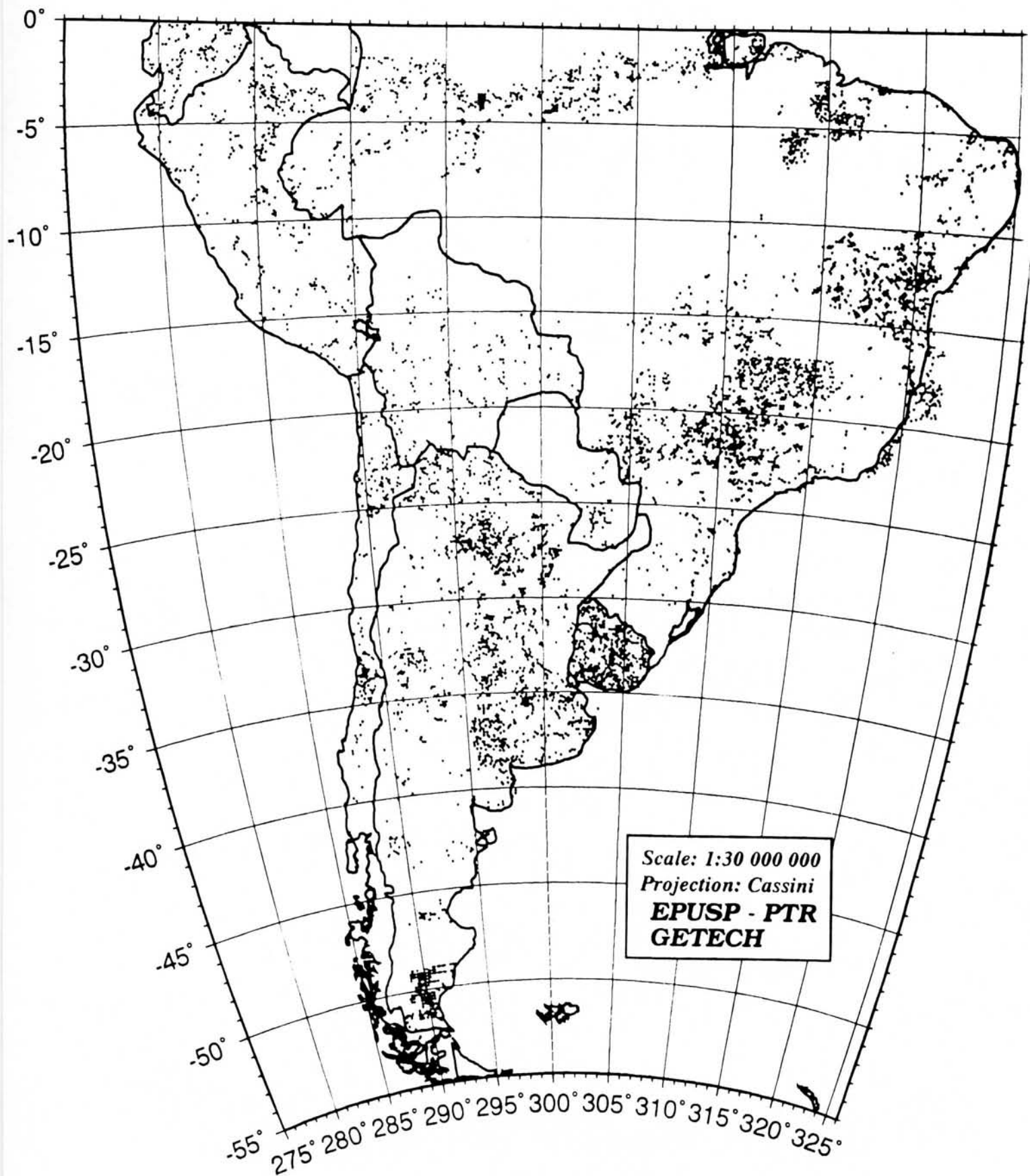


Fig. 5 - Interpolated mean values (5 517 points).

Interpolated mean values of 5' grid.

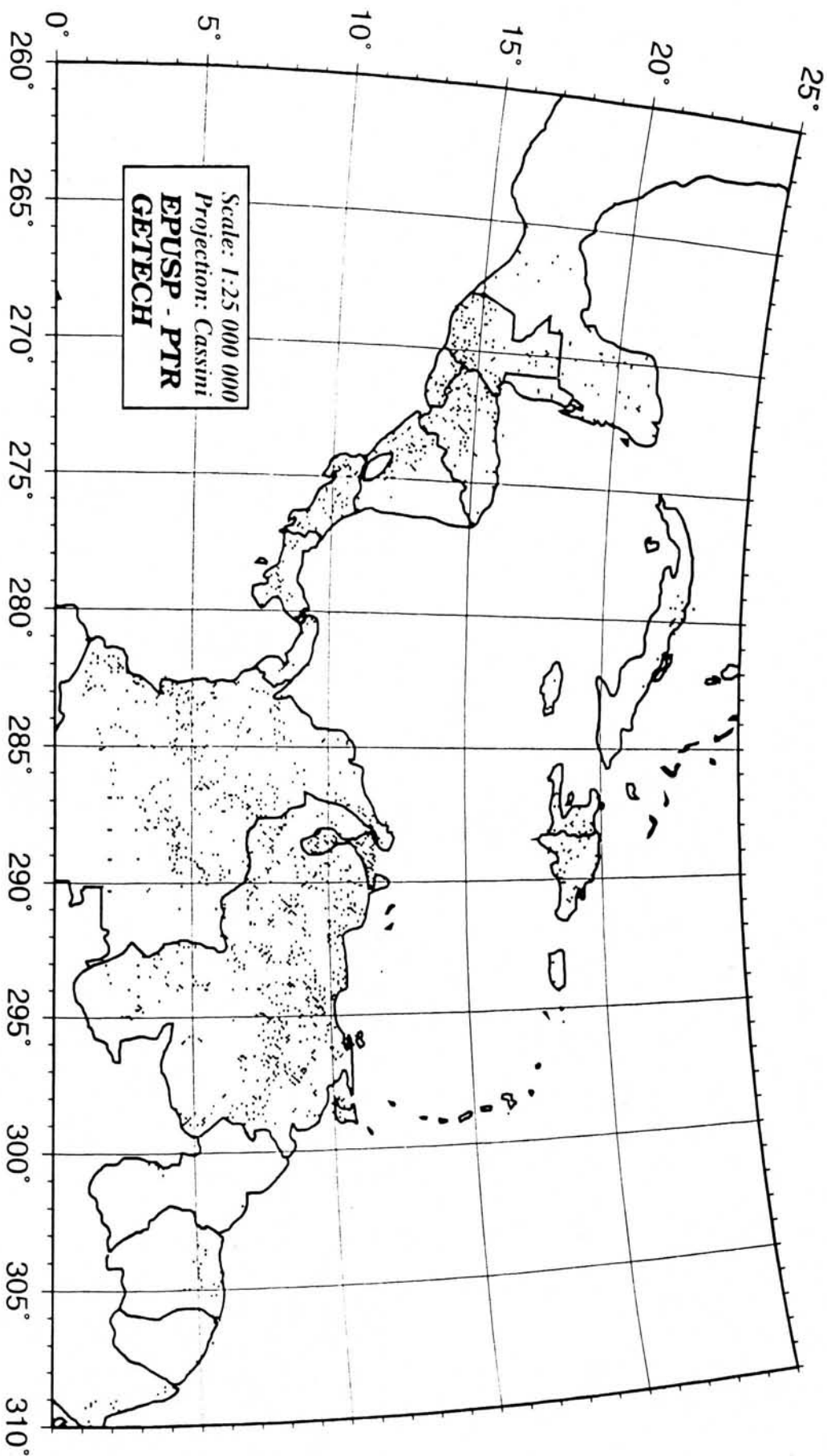


Fig. 6 - Interpolated mean values (1542 points).

Estimated mean values of 30' grid.

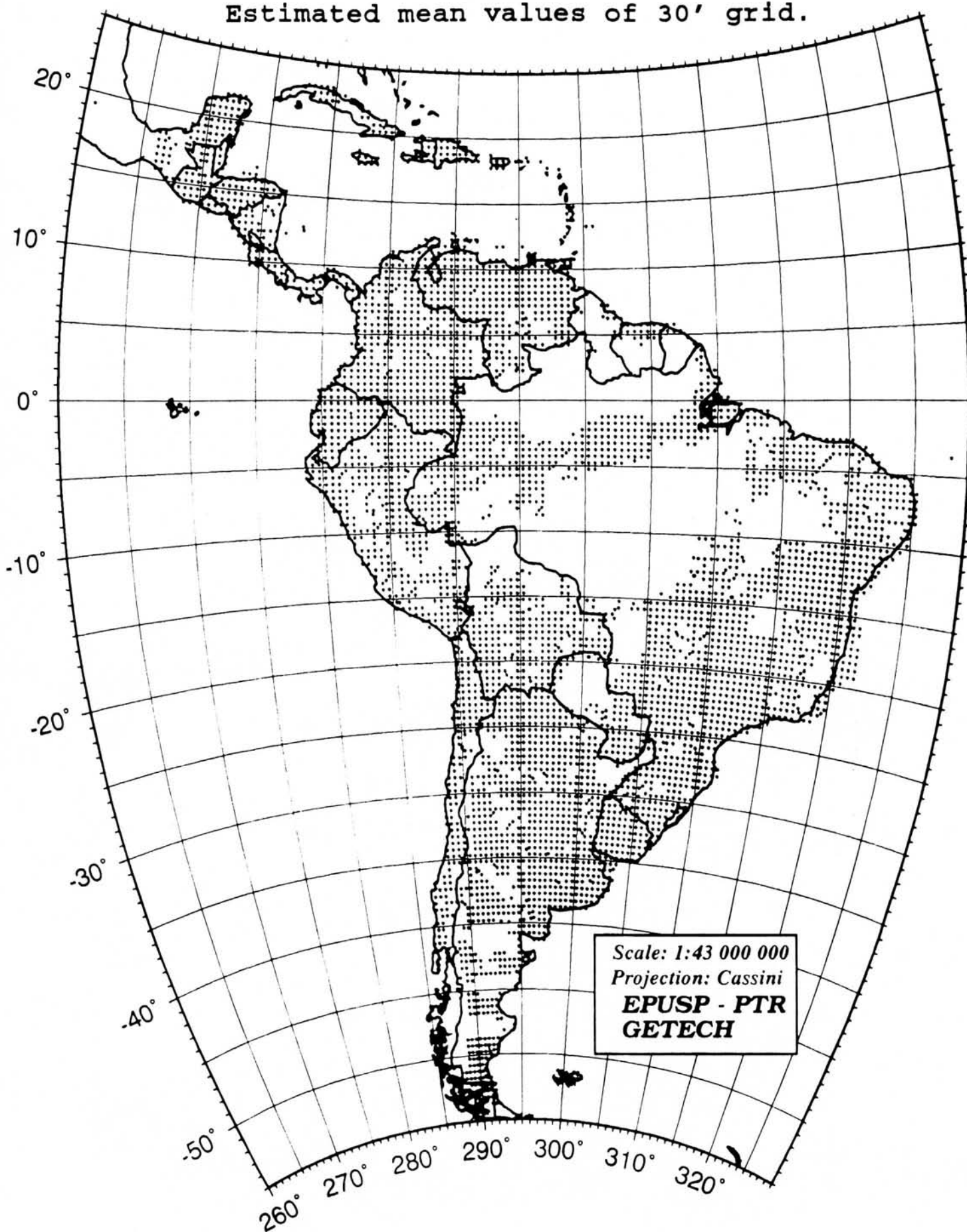


Fig. 7 - Final distribution of the 30' grid.

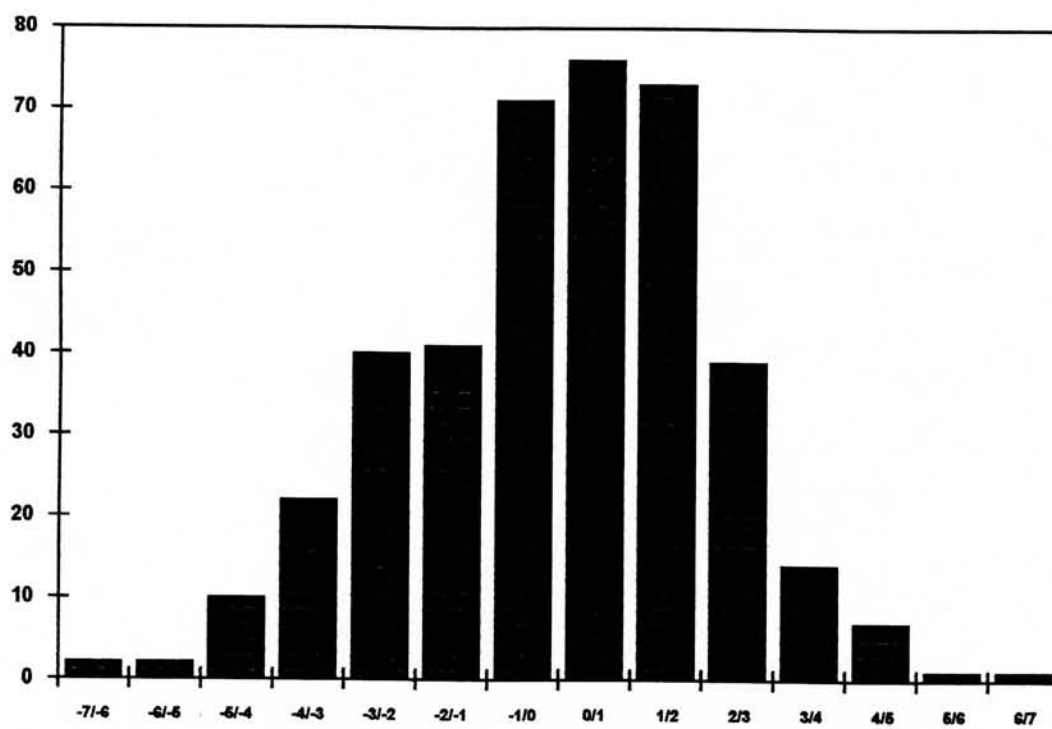


Fig. 8 - Satellite and gravimetric undulation differences.

Geoidal map of Brazil

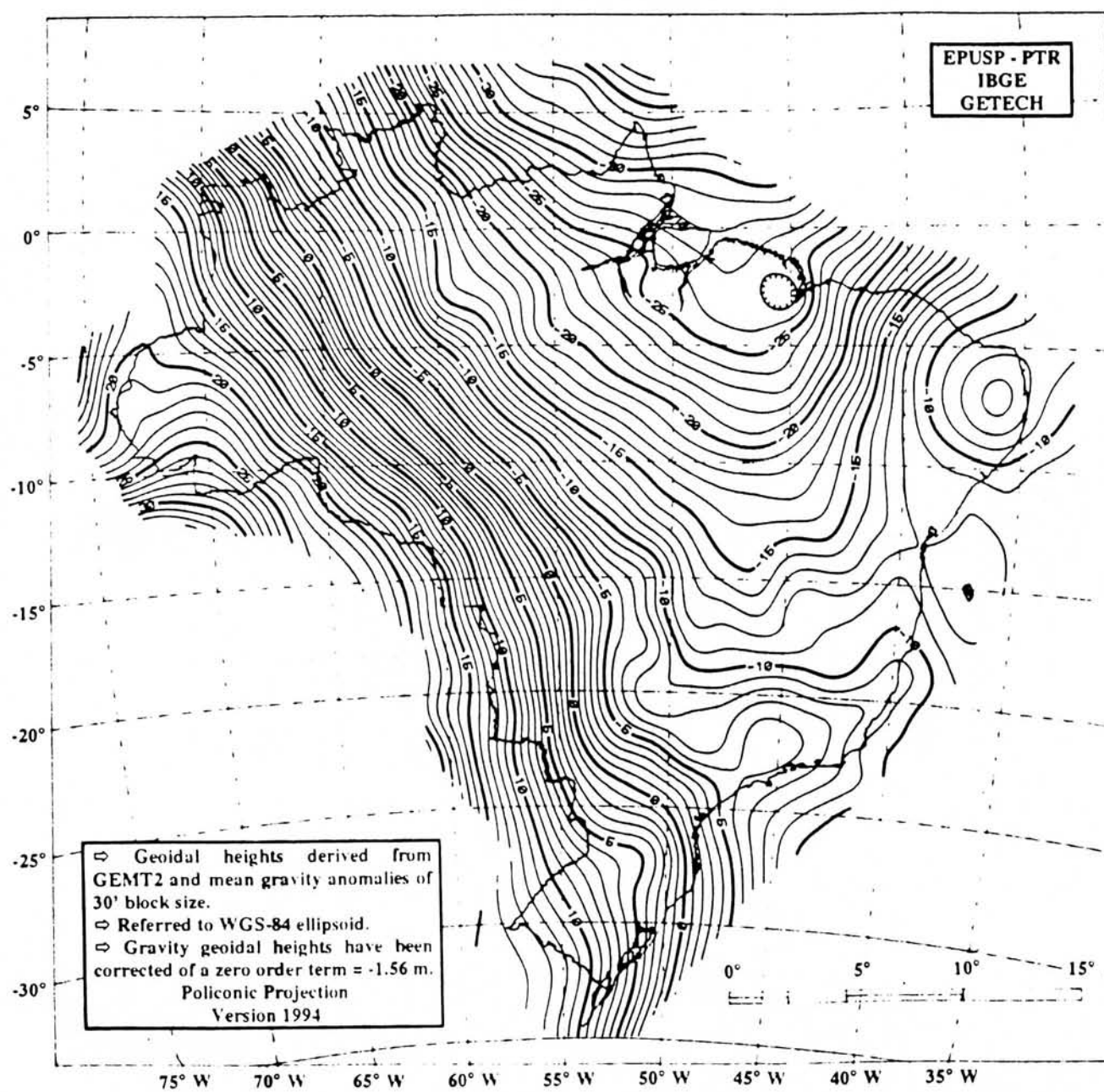


Fig 9 - Geoidal map for Brazil.

The Regional Model of High-Degree Gravitational Field and the
Potential Coefficient Model of the Qinghai-Tibet Region

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Abstract: This paper deals with the method for calculating the regional high degree potential model by using regional gravity data. The initial model used here is OSU91A1F given by Rapp, in which a number of systematical corrections are included and the formulas are developed based on the ellipsoidal coordinate and ellipsoidal harmonics. By the derived formulas, the terrestrial gravity anomalies in the Qinghai-Tibet area are used to obtain a regional potential coefficient model QZ93G (completed to degree 360). The mean anomalies from the QZ93G model have a RMS discrepancy of $\pm 11.8 \times 10^{-5} \text{ ms}^{-2}$ with the terrestrial anomalies. The new model represents a substantial improvement over the OSU91A1F model.

Key words: regional geopotential model, Residual gravity anomaly field, Ellipsoidal correction, Qinghai-Tibet region.

1. Introduction

With the rapid development of satellite technique, and rich in global gravity data, the gravitational potential models of the earth can be developed in higher degree. Beginning in the 1986, a number of advances have taken place in the computation of more accurate high-degree model, especially in Ohio State University, such as OSU86F, OSU89A-B and recently OSU91A1F, which completes to degree 360 (Hsu, 1991. Rapp, 1991).

As well known to all, the high-degree model of earth may reflect

the fine structure of gravity field with a high resolution, It may play a important role in space sciences, determination of geoid, research for tectonics and plate movement etc.. However, a global model is less of accuracy to represent the region geopotential field. To get a precise regional gravity model, which has been of great interest to geoscientists. People usually uses the tailored method, this corresponding approach has been taken, G.Weber and H.Zomorrodian in 1988 (Weber, et al.,1988), and T.Basic in 1989 (Basic, 1989), in which they used the formulas in frame of spherical system to determinate the regional model. By 1991, H.T.Hsu gave the formulas including more systematic corrections of "tailored model" following Rapp's idea, i.e. and using ellipsoidal coordinates and expansions. In the following we deal with the modeling principles, and derive the fundamental formulas for the computation of regional high-degree geopotential model by means of regional gravity data. Using these derivations and taking OSU91A1F as the initial model, a regional geopotential model QZ93G (complete to degree 360) suitable to the Qinghai-Tibet region in China is developed. Finally, some results concerning the model accuracy are discussed.

2. Modeling Principle of Regional Field

We take a global geopotential model as the initial model and revise the corresponding coefficients in such a way, that the residual field of gravity anomalies formed from the initial model and regional observed data turns to minimum. Then a regional geopotential model can be determined.

Over the years the modeling theories of the spherical approximate B.V.P. (boundary value problem) was replaced gradually by the strict B.V.P.. For this reason, the formulas for revising coefficients in this paper included the following systematic corrections: atmospheric correction, second-order vertical gradient of the normal gravity and ellipsoidal corrections. Moreover the analytical continuation for anomalies were downward to ellipsoidal reference surface, and the formulas are developed employing system of ellipsoidal coordinate and ellipsoidal harmonics, as pointed out by Rapp (Rapp, Pavlis 1990).

2.1 The relationship between residual anomalies and revising coefficients The gravity disturbing potential is referred with the real surface of the earth in Molodensky's theories and the spherical harmonic representation of disturbing potential T at a point (r, ϑ, λ) can be expanded as spherical harmonics,

$$T(r, \vartheta, \lambda) = \frac{GM}{r} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=-n}^n \bar{C}_{nm}^s \bar{Y}_{n|m|}(\vartheta, \lambda) \quad (1)$$

$$\bar{Y}_{nm}(\vartheta, \lambda) = \begin{cases} \cos m\lambda \\ \sin m\lambda \end{cases} \bar{P}_{n|m|}(\cos \vartheta) \quad \begin{matrix} \text{if } m \geq 0 \\ \text{if } m < 0 \end{matrix}$$

where r is the geocentric distance, ϑ is the geocentric colatitude, and λ the longitude. GM is the multiplication of geocentric gravitational constant. G and earth mass M , a semimajor axis of an adopted mean-Earth ellipsoid. $\bar{C}_{nm}^s$ is the fully normalized spherical coefficients, $\bar{P}_{n|m|}$ the fully normalized associated Legendre functions. We used the subscripts n, m to denote the order and degree in harmonics expansions. SI. (international system of unit) was used in this paper.

The surface free-air gravity anomaly Δg is defined as the difference of the actual gravity acceleration g_p at a point P of the surface the normal gravity acceleration γ_Q at the corresponding telluroid point Q , i.e.,

$$\Delta g = |g_p| - |\gamma_Q| \quad (2)$$

The Molodensky's boundary condition connected the gravity anomaly and the disturbing potential takes the following form

$$\Delta g = -\left(\frac{\partial T}{\partial r}\right)_Q - \frac{2}{r} T_Q + (\varepsilon_h + \varepsilon_r + \varepsilon_p)_Q, \quad (3)$$

where $\varepsilon_h, \varepsilon_r, \varepsilon_p$ are the terms for taking into account of ellipsoidal corrections. Let $\Delta g^c(r, \vartheta, \lambda)$ denote the gravity anomaly after application of all systematic corrections, it satisfies

$$\Delta g^c(r, \vartheta, \lambda) = -\left(\frac{\partial T}{\partial r}\right)_Q - \frac{2}{r} T_Q \quad (4)$$

Substituting (1) into (4), we have

$$\Delta g^c(r, \vartheta, \lambda) = \frac{GM}{r^2} \sum_{n=2}^{\infty} (n-1) \left(\frac{a}{r}\right)^n \sum_{m=-n}^n \bar{C}_{nm}^s \bar{Y}_{n|m|}(\vartheta, \lambda) \quad (5)$$

Now the gravity anomalies on earth surface can be calculated from the geopotential coefficients $\bar{C}_{nm}^s$ by using the expression

$$\Delta g = \Delta g^c(r, \vartheta, \lambda) + (\varepsilon_h + \varepsilon_r + \varepsilon_p) - \delta g_A - \delta g_h \quad (6)$$

In (6) δg_A and δg_h are the atmospheric correction and

correction due to the neglecting of second-order vertical gradient of the normal gravity, respectively.

The gravity anomaly $\Delta g^c(r, \vartheta, \lambda)$ is not a harmonic function, but $r\Delta g^c(r, \vartheta, \lambda)$ is, in the space outside the sources of the disturbing potential. According to Rapp and Pavlis, the fundamental mathematical model to compute the geopotential coefficients $\bar{C}_{nm}^s$ can be obtained in the following,

$$\bar{C}_{nm}^s = \frac{1}{4\pi a \gamma} \sum_{i=0}^{N-1} r_i^e \sum_{k=0}^{s'} \frac{L_{nmk}}{\bar{S}_{n-2k,m}(\frac{b}{E})} \frac{I\bar{P}_{n-2k,m}^i}{(n-2k-1)q_{n-2k}^i} \sum_{j=0}^{2N-1} \Delta g_{ij}^e \left\{ \begin{matrix} IC \\ IS \end{matrix} \right\}_m^j \quad \begin{matrix} m \geq 0 \\ m < 0 \end{matrix} \quad (7)$$

where

$$\gamma = \frac{GM}{a^2}, \quad I\bar{P}_{nm}^i = \int_{\delta_i}^{\delta_{i+1}} \bar{P}_{n|m|}(\cos \delta) \sin \delta \, d\delta, \quad (8)$$

$$\left\{ \begin{matrix} IC \\ IS \end{matrix} \right\}_m^j = \int_{\lambda_j}^{\lambda_{j+1}} \left\{ \begin{matrix} \cos m\lambda \\ \sin m\lambda \end{matrix} \right\} d\lambda.$$

In (7) δ represents the reduced colatitude, λ the longitude. b the semiminor radius of the ellipsoid. $E=ae$, where a is the semimajor axis and e is the first eccentricity of the reference ellipsoid. The renormalization functions $\bar{S}_{nm}$ are related to the associated Legendre functions of the second kind. S' is the greatest integer less than or equal to $\frac{1}{2}(n-|m|)$, L_{nmk} is defined by Gleason (Gleason, 1988), r_i^e is the radius of the point on ellipsoid. q_n is the smoothing factors, Δg_{ij}^e the average value over the block on the ellipsoid. We use the subscripts i, j to designate a block with colatitude δ_i and longitude λ_j , respectively, and the superscripts e to designate a value on the ellipsoid. we have

$$\Delta g_{ij}^e = \Delta g_{ij}^c + (\delta g_A)_{ij} + (\delta g_h^2)_{ij} - (IEh + IEr + IEp)_{ij} + (g_1)_{ij}, \quad (9)$$

and

$$\Delta g_{ijM}^e = (\Delta g^c)_{ij} + (g_1)_{ij} \quad (10)$$

In (10), Δg_{ijM}^e is the average value over the block on the ellipsoid from (5). IEh, IEr, IEp are ellipsoidal corrections over the block on the ellipsoid. $(g_1)_{ij}$ means the correction of analytical downward continuation (Wang, 1988)

For a certain investigated region, it is easy to obtain the differences between the gravity anomaly calculated values from initial geopotential model and those measured practically:

$$\delta \Delta g_{ij} = \Delta g_{ij}^e - \Delta g_{ijM}^e$$

$$\begin{aligned}
&= \Delta \bar{g}_{ij} + (\delta \bar{g}_A)_{ij} + (\delta \bar{g}_h^2)_{ij} - (IE_h + IE_r + IE_p)_{ij} \\
&- \frac{GM}{r_{ij}^2} \sum_{n=2}^{360} (n-1) \left(\frac{a}{r_{ij}} \right)^n \sum_{m=-n}^n \bar{C}_{nm}^s \bar{Y}_{nm}(\vartheta, \lambda)
\end{aligned} \quad (11)$$

we call it regional residual anomalies. The existence of residual field comes mainly from two factors. One is ineffective the initial global model which is a model on the average, the other is the inaccurate and incomplete data set used for the solution, in particular, lack of terrestrial gravity sources in the local region. Therefore we can use the modified model and meet the need of the application in individual local area.

For this purpose, the whole global is divided into two parts, local part Σ_1 and external part Σ_2 , and only the effect of local part Σ_1 is taken into account, while the residual field in Σ_2 is assumed to be zero. Based on (7) the corrections of geopotential coefficients caused by regional residual field can be written as,

$$\Delta \bar{C}_{nm}^R = \frac{1}{4\pi a \gamma} \sum_{i=1}^T r_i^e \sum_{k=0}^{S'} \frac{L_{nmk}}{\bar{S}_{n-2k,m} \left(\frac{b}{E} \right)} \frac{I \bar{P}_{n-2k,m}^t}{(n-2k-1) q_{n-2k}^t} \delta \Delta \bar{g}_t \left\{ \begin{matrix} IC \\ IS \end{matrix} \right\}_m^t \quad \begin{matrix} m \geq 0 \\ m < 0 \end{matrix} \quad (12)$$

where T is the total amount of the blocks in local area Σ_1 . Thus, the model coefficients of regional gravity field become

$$\bar{C}_{nm}^{SR} = \bar{C}_{nm}^S + \Delta \bar{C}_{nm}^R \quad (13)$$

Since the lower-degree set of geopotential coefficients obtained from satellite solutions have enough accuracy at present, nearly all of the high-degree global model of gravity field take advantage of these satellite results to a great extent, for example, in case of OSU91A1F model, solution of GEMT3 is used. Moreover, to avoid the distortion of gravity field, we should not change these lower-degree coefficients, one method is to introduce the weight functions, we have

$$W_n = \begin{cases} 0 & n < N_{min} \\ (n - N_{min}) / (N_{max} - N_{min}) & N_{min} \leq n \leq N_{max} \\ 1 & N_{max} < n \end{cases} \quad (14)$$

where N_{min} is the lowest degree, that should be kept in original model and N_{max} is the starting degree, that should be fully corrected. In this case, formula (13) reduces to

$$\bar{C}_{nm}^{SR} = \bar{C}_{nm}^S + \Delta \bar{C}_{nm}^R \cdot W_n \quad (15)$$

The weight function (14) can be looked upon as a high-pass filter, and be chosen arbitrarily. However, the effects of

weight-function on calculating coefficients and physical meaning should be taken into account

2.2 Accuracy Analysis

It can be proved that if the orders higher than e^2 are neglected, the error of the modified coefficients will be

$$\sigma_R^2(n, m) = \sigma_s^2(n, m) + \varepsilon(n, m) \quad (16)$$

where $\sigma_s^2(n, m)$ is the accuracy of the initial model coefficients.

Let

$$U^2(n, m) = \frac{1}{[4\pi\gamma(n-1)\bar{S}_{nm}(\frac{b}{E})]^2},$$

$$V^2(n, m, t) = \left(\frac{r_t^e}{a}\right)^2 \left(\frac{1}{q_t^n}\right)^2 (IP_{nm}^t)^2 \cdot \left\{ \frac{IC^2}{IS^2} \right\}_m^t \quad \begin{matrix} m \geq 0 \\ m < 0 \end{matrix}$$

then

$$\varepsilon(n, m) = U^2(n, m) \sum_{t=1}^T V^2(n, m, t) [(W_n^2 - 2W_n) \sigma^2(\Delta g_{Mt}^e) + W_n^2 \sigma^2(\Delta g_t^e)] + O(e^2)$$

$$= \begin{cases} 0 \\ U^2(n, m) \sum_{t=1}^T V^2(n, m, t) W_n^2 [\sigma^2(\Delta g_t^e) + (1 - \frac{2}{W_n}) \sigma^2(\Delta g_{Mt}^e)] + O(e^2) \\ U^2(n, m) \sum_{t=1}^T V^2(n, m, t) [\sigma^2(\Delta g_t^e) - \sigma^2(\Delta g_{Mt}^e)] \end{cases}$$

$n < N_{min}$

$N_{min} \leq n \leq N_{max} \quad (17)$

$N_{max} < n$

where, $\sigma^2(\Delta g_{Mt}^e)$ and $\sigma^2(\Delta g_t^e)$ separately represent gravity anomaly error by computation from initial model and that by observation after the correction of downward continuation to ellipsoid, $O(e^2)$ is the truncation error.

It is evident the new model accuracy are dependent upon the choose of weight function and the accuracy of observed anomaly from (16) and (17), i.e., if degree n below N_{min} , the new model accuracy is the same as the initial model, and if degree n above N_{min} , obviously, due to the weight function W_n less or equal to one, and $\sigma^2(\Delta g_t^e) < \sigma^2(\Delta g_{Mt}^e)$, so that $\sigma_R^2(n, m) < \sigma_s^2(n, m)$. It means that the accuracy of modifying model is better than the original one due to the supplement of the local observed data.

3. Numerical Results in the Qinghai-Tibet region

In this work we update the terrestrial anomaly data in the Qinghai-Tibet region starting from a 5'x5' mean grid file and observational data. All of the original data are taken to form the mean values of 30'x30' grids (related to GRS80 system).

To the model calculation, the first step is to compute gravity anomalies in the investigated area by OSU91A1F model, and constitute the residual anomaly field (Fig.2). Then, the corrected coefficients is calculated according to the above-developed formulas. Finally, a regional model QZ93G (complete to degree 360) is be obtained.

We compare the anomalies computed by model OSU91A1f and QZ93G with the observed anomalies in the size of 30'x30' grid, to check the validity of this regional model QZ93G the digital results indicate that the RMSE (root mean square error) decreases from the original $\pm 35.67 \times 10^{-5} \text{ mS}^{-2}$ (OSU91A1F model) to $\pm 11.83 \times 10^{-5} \text{ mS}^{-2}$ (QZ93G model), the accuracy increase by three times, and corresponding range of error decreases from $442.5 \times 10^{-5} \text{ mS}^{-2}$ to $95.6 \times 10^{-5} \text{ mS}^{-2}$, as listed in table 1.

Tab.1 Accuracy comparison between the computed anomaly by model and the observed anomaly (unit: 10^{-5} mS^{-2})

model	OSU91A1F	QZ93G
mean error	+ 4.25	+4.13
RMSE	± 35.67	± 11.83
minimum derivation	-219.4	-45.1
maximum derivation	+223.5	+50.5
range of derivation	442.5	95.6

The residuals are shown in Fig.2 and Fig.3. Obviously, the errors in OSU91A1F are overwhelming large than in QZ93G within the area of Qinghai-Tibet. Although the OSU91A1F model included more accurate satellite model and global terrestrial data, and used the global digital terrain model as well as the topographic-isostatic potential model to fill in gravity anomaly data for China, it has still larger residual anomaly undulation by OSU91A1F model in Qinghai-Tibet region (Fig.2). This is due to the complex

topography and internal structure, and incomplete isostatic compensation in this region. In contrast to this, the residual anomaly in QZ93G model is smaller and smoother (Fig.3).

Figure 1 gives information of absolute error distribution about modified model QZ93G in the region. The maximum is $50.5 \times 10^{-5} \text{ mS}^{-2}$ (only one). The absolute error between 40 and $50 \times 10^{-5} \text{ mS}^{-2}$ is a total of nine, account for 0.38%, error between 20 and $30 \times 10^{-5} \text{ mS}^{-2}$ is a total of 38, account for 1.58%, and 85.58% of the error are distributed between -10 and $+20 \times 10^{-5} \text{ mS}^{-2}$. This situation accords with normal distribution rule of error. Besides, from Fig.1 we can find a smaller systematic error, it should be related to the lower-degree coefficients in model.

In figure 4, we compared the anomaly degree variances of modified and initial model with from Kaula's rule. It can be seen that the degree variance differences between modified and initial model are quite small and the trends are both in good agreement with Kaula's rule. Moreover, the degree variance values of new model is little large than values for initial model, as seen in Fig.5. This could be attributed to the enhancement of the information in Qinghai-Tibet.

Figure 6, 7, and 8 are contour-lines chart of anomaly, obtained by observation, by model QZ93G and OSU91A1F in the size of $30' \times 30'$ grid in Qinghai-Tibet region, respectively. It can be seen that Fig.7 is so much similar to Fig.6 that it can describe the detailed anomaly situation in region, but Fig.8 is on the contrary.

4. Some Remarks

a. At present, the research for the geopotential model has developed to Molodensky's method from Stocks method, and a number of advances have taken place in the computation more accurate high-degree models as a result of improved theory, more accurate and complete satellite models, and extended terrestrial data. In this paper, we have derived the quintessence from these latest solutions, and developed modified formulas to calculate the regional model. The formulas, including all systematic corrections, i.e. atmospheric correction, ellipsoidal corrections,

second-order vertical gradient correction of normal gravity, and analytical downward continuation, have been referred to the ellipsoidal coordinates and ellipsoidal harmonics. These systematic corrections should be taken into account in computing a high accurate and high-degree gravity potential model. In our work, the systematic corrections in the Qinghai-Tibet region gave a undulation between -0.9 and $+2.6 \times 10^{-5} \text{ mS}^{-2}$.

b. Based on the modified formulas connected to Rapp's development theory, it is allowed to determine a regional high-degree geopotential model by combining the global model with the regional gravity sources. The new model keep the advantage of original global model, and contains more local information and represents the regional field in more detail.

The advantages of proposed approach are effective, flexible and simple. It is easily to change the fitting area according to the collected data, to follow the latest model and to improve the validity of high-degree model in a particular region.

c. On the basis of the comparison and analysis in section 3, the regional geopotential model QZ93G (complete to degree 360) by combing the OPS91A1F model with the terrestrial data represents a substantial improvement over the OSU91A1F model in Qinghai-Tibet region. It can be said that the QZ93G model has good quality in respect of representing the fine structure of gravity field in the region external accuracy is checked to be $\pm 11.8 \times 10^{-5} \text{ ms}^{-2}$.

d. We have dealt with the improved "tailored method" for calculating the regional high-degree potential model by means of theory and computation, respectively. Obviously, the accuracy of the model is directly related to the accuracy of the used regional data. To obtain a high accurate regional model, we should employ more accurate data. Besides, the weight function (Eq.14), has a direct effect on the spectrum structure of modifying model, should be a high pass filter. Its determination depends on the accuracy of collected data. The lower-degree parts of satellite model, corresponding to a long-wavelength parts in spatial domain, should be kept so that the modified model in harmony with the initial

global model, as a whole.

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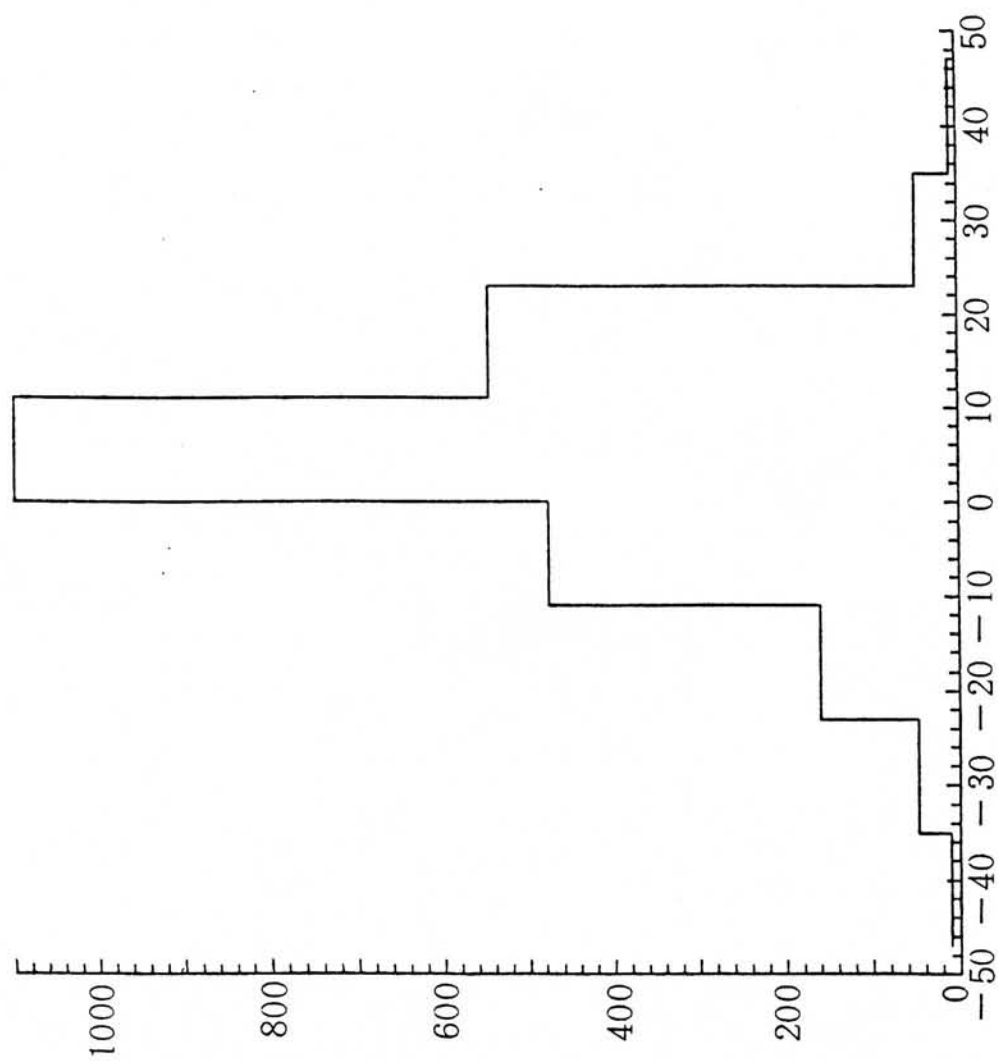


Fig. 1 Error distribution of QZ93G model.

图1
误差分布

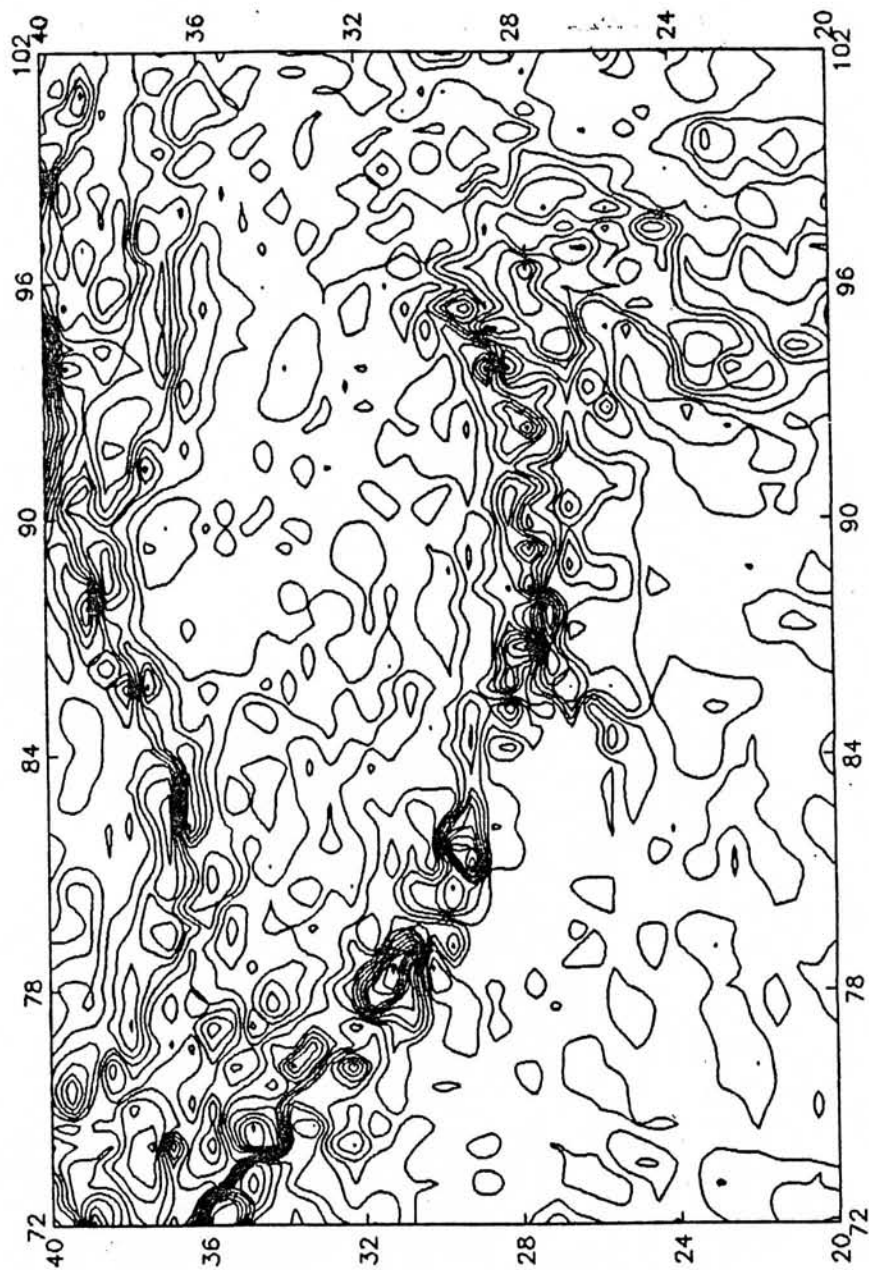


Fig. 2 Difference between observed anomalies and OSU91A1f model
(c.i.= 40 mGal).

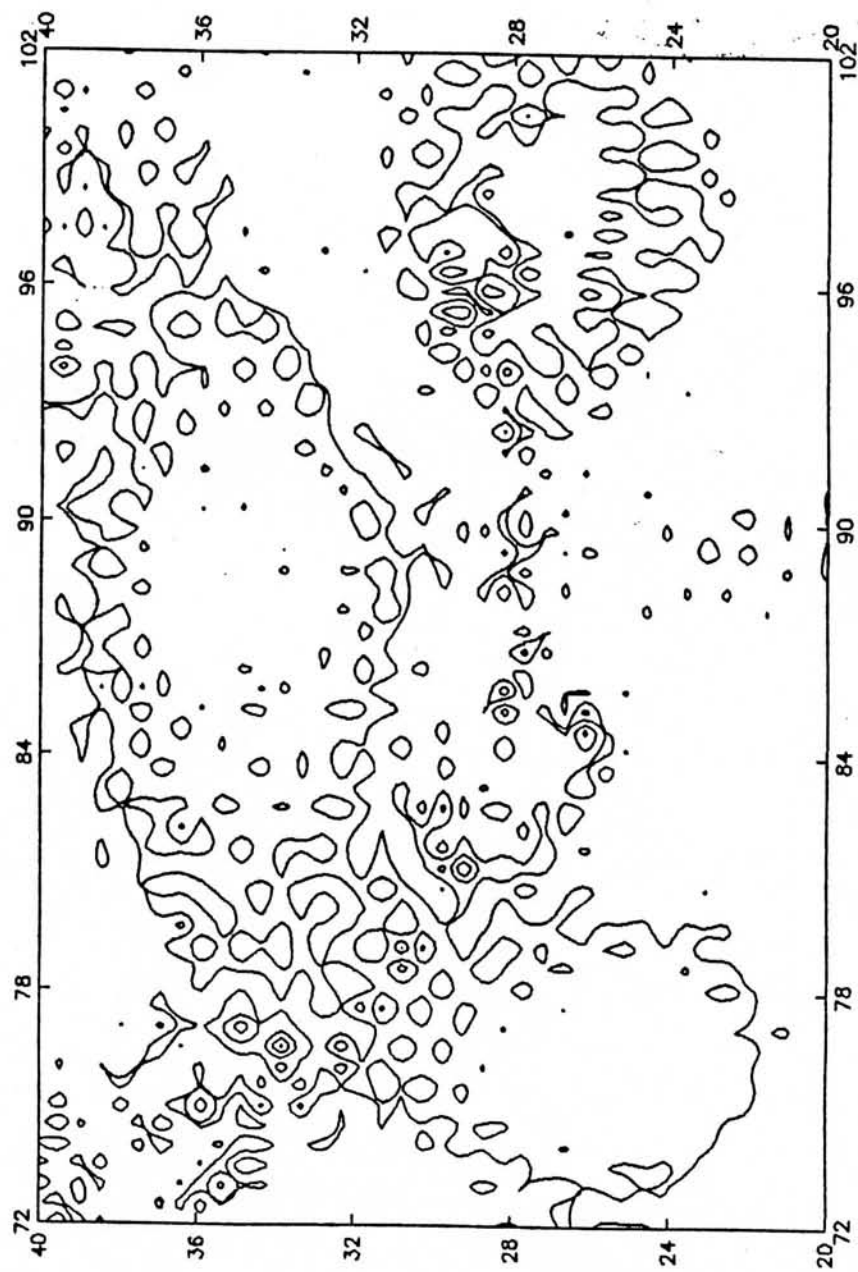


Fig. 3 Difference between observed anomalies and QZ93G model
(c.i. = 20 mGal).

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陸洋

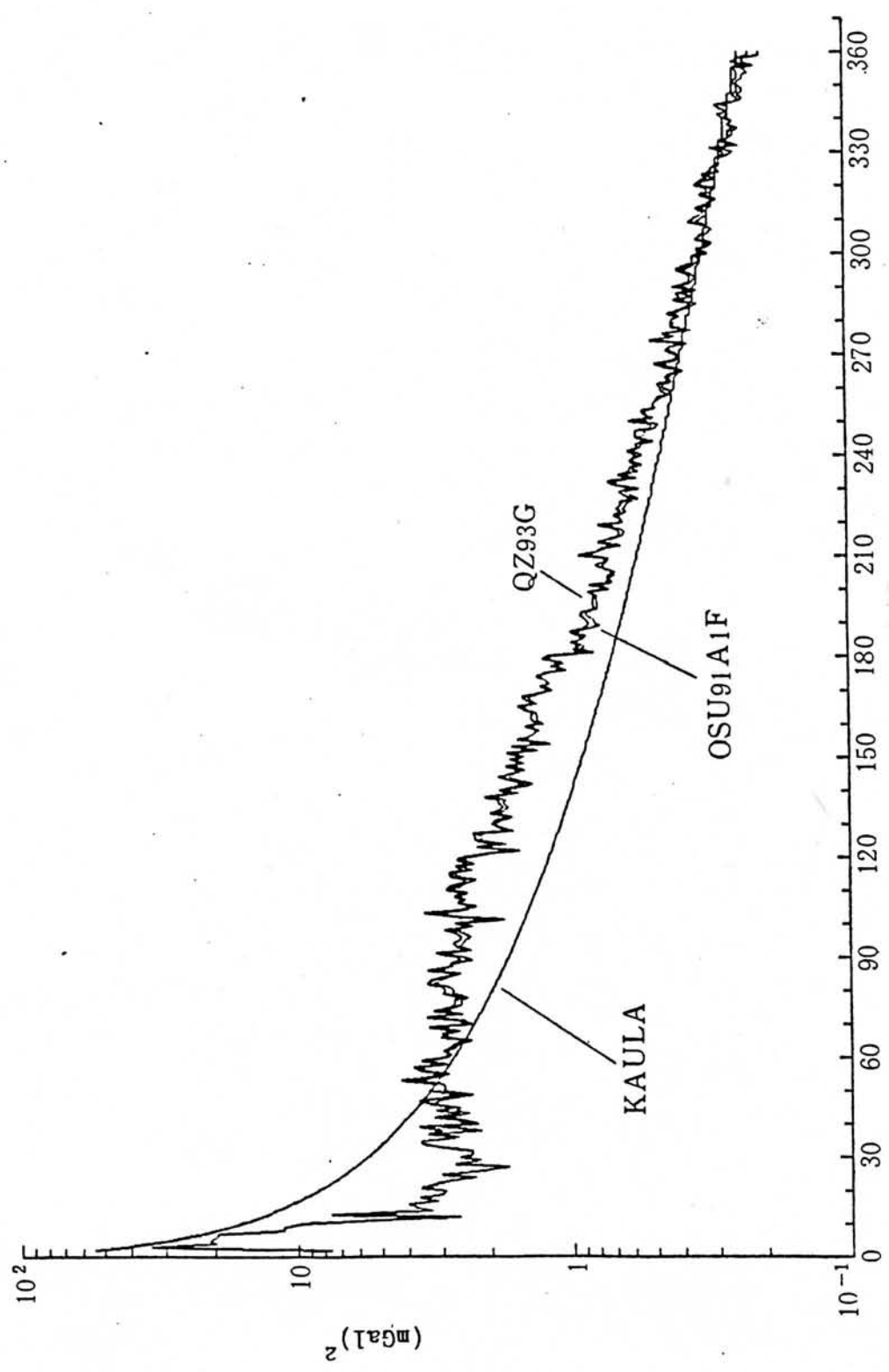


Fig. 4 Degree variance of potential models.

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陸洋

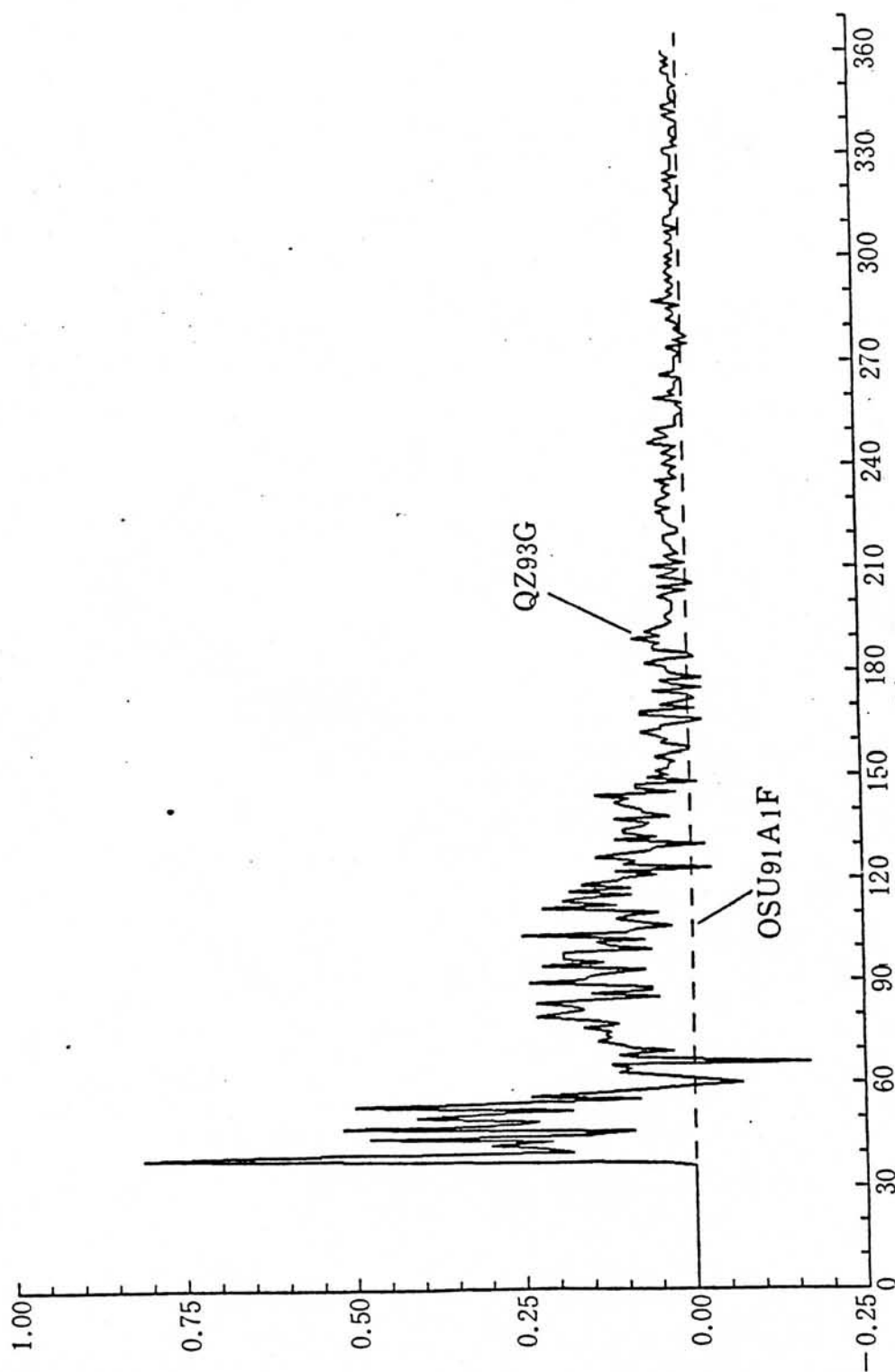


Fig. 5 Difference of degree variance between QZ93G and OSU91A1F.

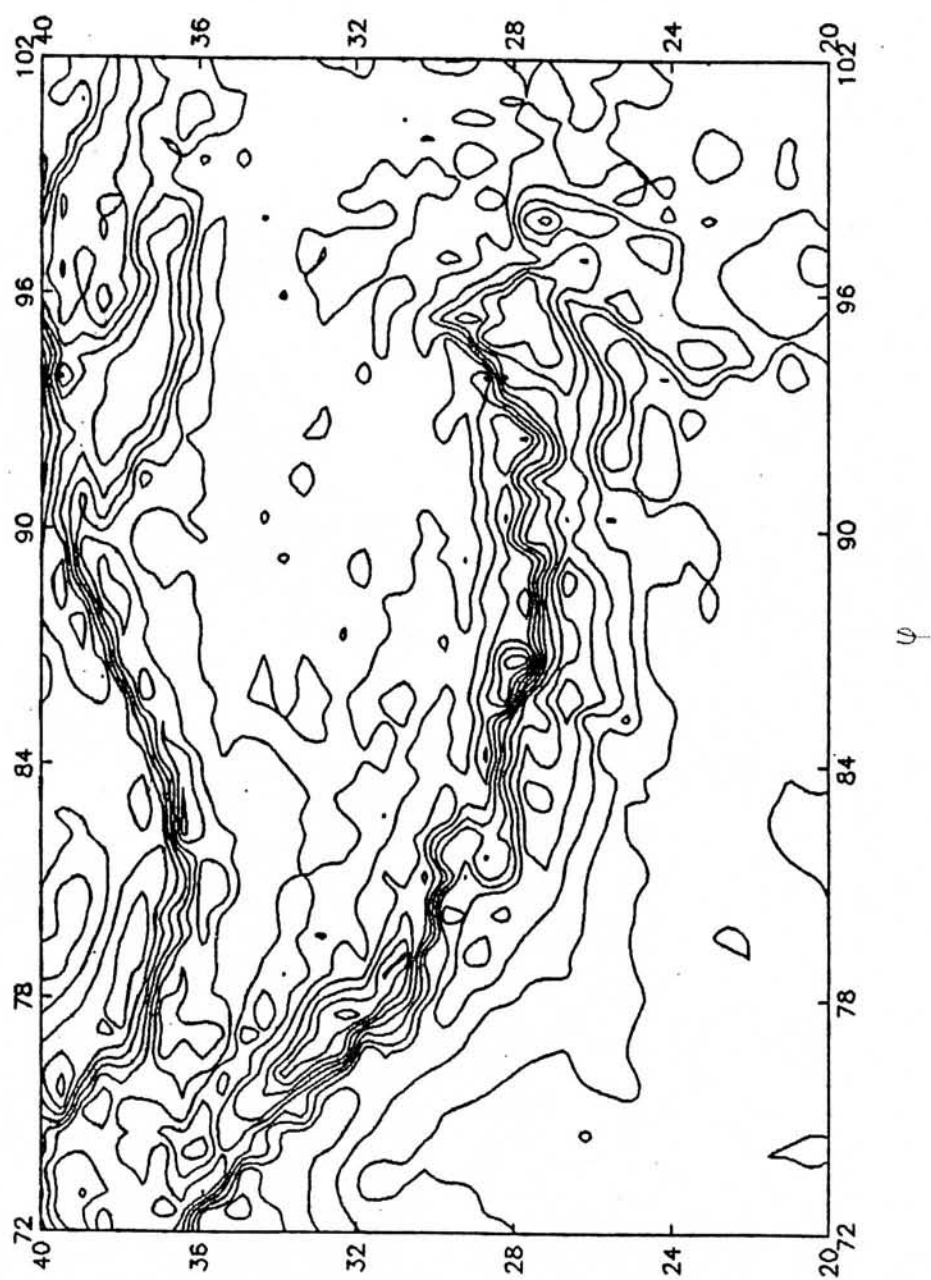


Fig. 6 Observed anomalies in Qinghai-Tibet of China(c.i.=40 mGal)

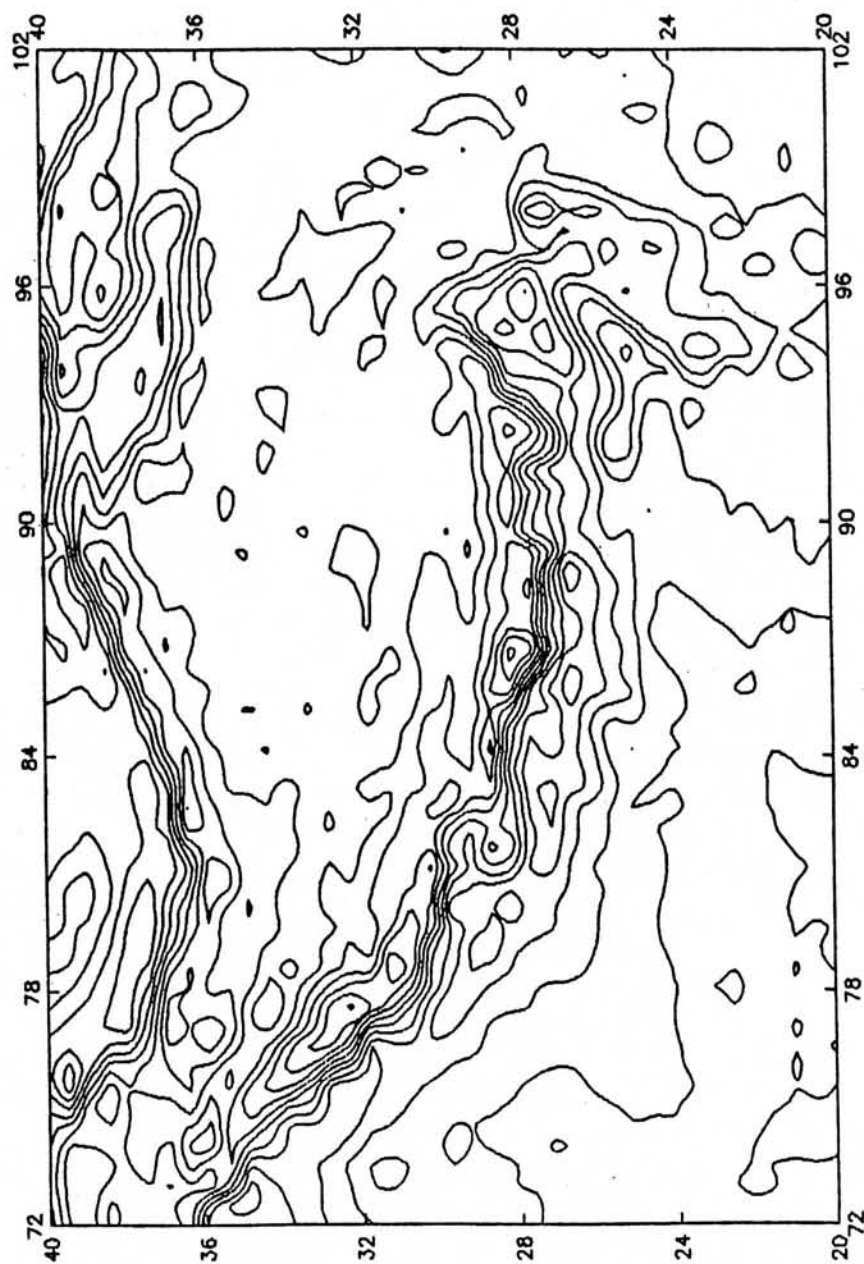


Fig. 7 Anomalies by QZ93G model in Qinghai-Tibet (c.i.=40 mGal).

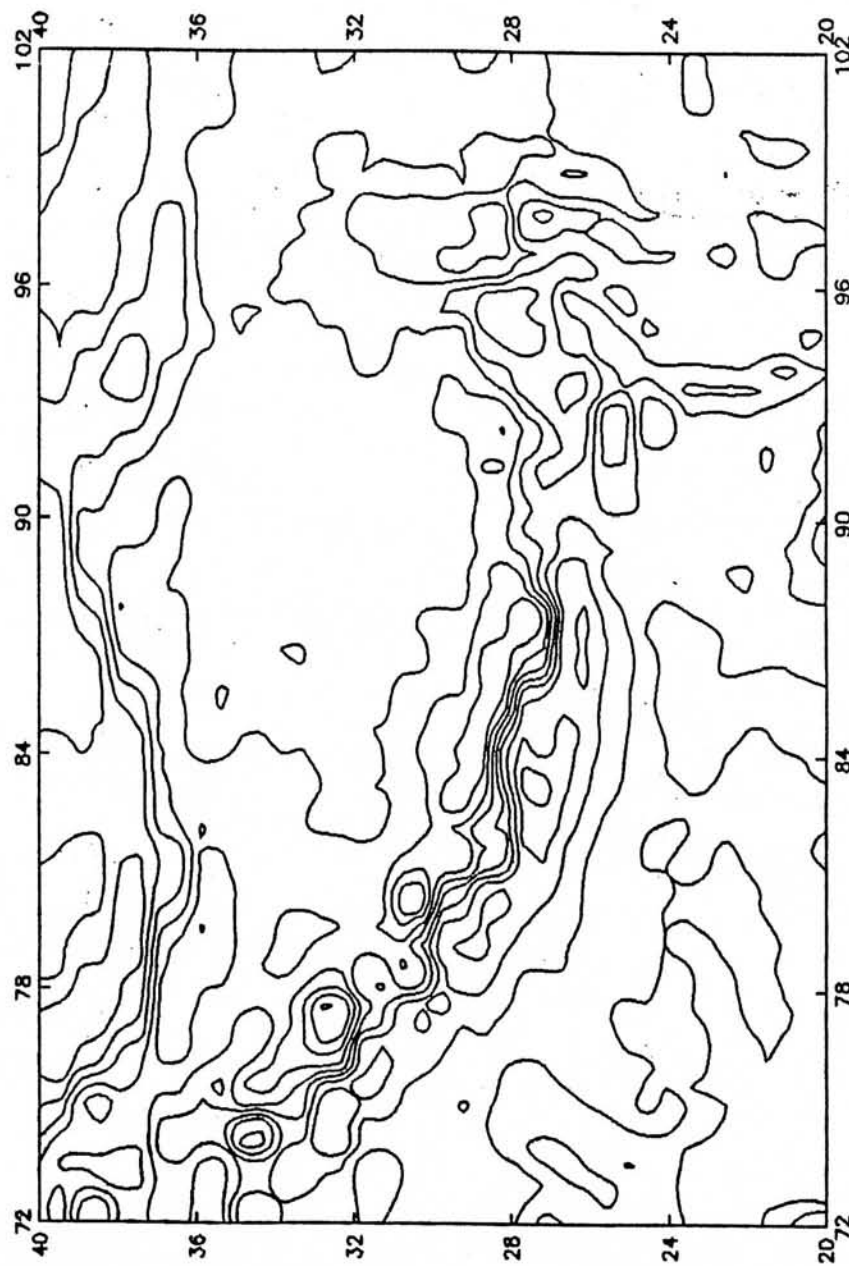


Fig. 8 Anomalies by OSU91A1F model in Qinghai-Tibet (c.i.=40 mGal)