

Report from a IAG Summer School

The International School of Theoretical Geodesy on "Boundary Value Problems and the Modeling of the Earth's Gravity Field in View of the One Centimeter Geoid" has been held in Como from May 27 to June 7, 1996.

We had 66 participants from 16 countries.

The contents of the lectures and the Seminars are here after described in a nice report due to R. Klees.

The content of the lectures has been in some instances very hard, yet I believe that the participation of the students has been high both in quality and interest.

We hope that this school, like past editions, will have an impact in future preparation of the many young geodesists that were there.

A special thank is due to Politecnico di Milano, CNR, IAG, IUGG, for the economical support to the direction of Villa Olmo and the Como Community for the warm reception.

A Lecture Notes Volume will be issued in the Springer Series in Spring '97.

Fernando Sansò

**Introduction of lectures by International Summer School of Theoretical Geodesy
(Como, May 27 - June 7, 1996)
R. Klees**

The lectures given at the International Summer School of Theoretical Geodesy on "Boundary Value Problems and the Modelling of the Earth's Gravity Field in View of the One Centimeter Geoid" can be divided in four groups: mathematical foundations (Martensen & Ritter, Biroli, Rozanov), mathematical geodesy (Heck, Sansò), numerics (Sünkel, Tscherning), and geodetic applications (Rapp, Schwarz, Schrama, Rummel). The lectures have been supplemented by seminars. They may also be assigned to the four categories mentioned before: mathematical foundations (Sacerdote, Martensen), mathematical geodesy (Holota, Grafarend), numerics (Klees, Lehmann), and geodetic applications (Keller, Sneeuw & van Gelderen, Schuyer). In the closing session a list of questions has been compiled in order to point out the most challenging open problems to be solved in future, potentially by the students of the Summer School. The lectures and seminars could only address some of them:

- why do we need boundary value problems at all?
- what is the surface of the Earth?
- is the time of free boundary value problems over?
- how to optimally combine spherical harmonic models with terrestrial data?
- are high degree spherical harmonic expansions needed?
- how to assess the limits of classical geodetic solutions when accuracy and resolution will increase?
- is geophysical information conceptually essential to set up our problems?
- does the classical geodetic approach converge to the boundary element approach and vice versa?
- how to solve the datum problem?
- how to solve mixed boundary value problems efficiently?
- what is an appropriate mathematical representation of sea surface topography and gravity field?
- what are mean anomalies on a rough surface and are there alternatives to working with mean anomalies?
- which theoretical improvements are needed in the estimation of high degree geopotential models and in the determination of geoidal undulations and height anomalies from spherical harmonic models?

1st group: Mathematical Foundations

Lectures:

E. Martensen, W. Ritter: Potential Theory

The series of mathematical lectures is opened by E. Martensen and W. Ritter with their lecture on "Potential Theory".

The lecture is divided into two parts. The first one, given by E. Martensen, focuses on classical potential theory in the space of continuous and Hölder continuous functions. It consists of three chapters: the first chapter deals with representation formulas and contains the complex and real versions of Cauchy's integral formula, and their generalizations to three dimensions for scalar and vector fields. The second chapter focuses on limit relations and integral equations. Starting with scalar fields, the concept of source distributions on closed surfaces is introduced, the jump relations of the potentials are given and the mapping properties and the spectrum of the corresponding boundary integral operators are discussed. The multiplicity of the eigenvalues ± 1 are determined in an elegant way making use of the (second) Betti numbers. The chapter finishes with some extensions to vector fields. The third chapter is devoted to boundary value problems of potential theory and integral equation methods. The (scalar and vector) Dirichlet, Neumann, and Robin boundary value problems for Poisson's and Laplace's equation are introduced and corresponding integral equations are derived making use of the so-called null field method. The second part is given by W. Ritter and aims at providing some mathematical background of boundary element methods, i.e. numerical techniques for solving integral equations making use of trial functions with finite support on the boundary. After discussing the pros and cons of boundary element methods, two discretization methods for boundary integral equations, the (exact) collocation method and the Galerkin method, are explained and some problems related to practical implementations discussed. Most emphasis is put on the definition of Sobolev spaces which is the class of functions normally used in boundary element methods because they provide convergence results and error estimates. Besides, the concept of strong ellipticity of boundary integral operators is introduced, a property that integral operators must have to construct an error and convergence analysis. Finally, the lemma of Céa is discussed which for strongly elliptic integral operators provides the convergence and error estimates of Galerkin boundary element methods.

Y. Rozanov: Random Harmonic Fields

These lectures are motivated by the need of studying various error propagation to the solutions of different B.V.P. (Boundary Value Problems) of geodetic interest, as well as by the new branch of overdetermined B.V.P.'s, because in these cases it is meaningful to ask oneself: what is the part of the solution corresponding to the boundary noise?

In mathematical literature, random fields as solutions of B.V.P.'s are characterized by a suitable, weak formulation, which directly suggests the use of generalized stochastic processes.

This concept however is classically related to linear stochastic functionals defined on spaces of very smooth functions, like the space D of L. Schwarz; the problem is to extend these functionals to much larger spaces in which D is densely embedded and try to characterize the (narrower) space to which the solution has then to belong.

Roughly speaking, for the Dirichlet problem, the reasoning sounds as follows: since a function $\mu \in H^{2,2}(\Omega)$ has a Laplacian in $L^2(\Omega)$ and boundary values in $H^{3/2}(S)$, these can be seen respectively as functionals on $H^{2,2}(\Omega)$ and $H^{3/2}(S)$; as a matter of fact these functionals can be constructed as limits of sequences of functions $\varphi \in D(\Omega)$; therefore if we provide a source (Δu) and a trace, $u|_S$, such that they are linear stochastic functionals continuous on $L^2(S)$ and $H^{3/2}(S)$ respectively, it is defined a linear stochastic functionals u , continuous $H^{2,2}(\Omega)$ which is the generalized solution of our problem.

The class of such solution is defined by Rozanov as a vector $\underline{H}^{2,2}(\Omega)$ space, characterized by mapping $H^{2,2}(\Omega)$ distributions onto random variables with finite variance.

M. Biroli: Free Boundary Problems

The lecture of M. Biroli deals with techniques for solving free boundary value problems that are used in mathematics. Whereas the solution of free geodetic boundary value problems is nowadays based on implicit function theorems, M. Biroli describes frequently used methods based on the calculus of variations which allow to transform the free boundary value problem into a usual minimization problem, thus avoiding the use of an implicit function theorem. Although these methods are not directly applicable to geodetic boundary value problems, they provide more insight into the whole complexity of free boundary value problems. After specifying the conditions for existence of minimum points of a functional making use of the concept of lower semicontinuity, four examples are given to explain the different methods: the equilibrium displacement of a membrane in presence of an

obstacle, the so-called dam problem which deals with finding the steady flow of water in a dam, the image segmentation problem, and finally a shape optimization problem, for Dirichlet problems. First, each example is described; then a mathematical formulation of the physical problem is given, and finally the problem is transformed into a variational problem (i.e. the problem of minimizing a certain functional on a given set), providing the solution of the free boundary value problem. The question of regularity of the free boundary is left out of consideration.

Seminars:

F. Sacerdote: Stochastic Boundary Value Problem Theory: An Elementary Example

The seminar of F. Sacerdote aims at giving an application of (a part of) the theory developed by Rozanov in his lecture. A Dirichlet problem for a random harmonic field on an open interval is taken as example. First the nonstochastic case is treated and solved in Sobolev spaces. Then, the stochastic case is considered and a rigorous mathematical meaning of the boundary condition and the solution of the problem is given.

E. Martensen: Analogy Between Vectorial and Scalar Formulations of Boundary Value Problems

In the seminar E. Martensen points out the analogies between vectorial and scalar formulations of boundary value problems in potential theory. As a first example the dualism concerning electrostatic and magnetic fields in the context of the Neumann problem is presented, and the properties of the respective eigenvalues are discussed, extending former work of Poincaré and Stakhov. For a further example, where the domain consists of a solid circle torus, the physical properties behind the eigenvalues are identified. Finally the transformation into integral equations is demonstrated by means of the vector Dirichlet problem for a three-axial ellipsoid and a cylinder, magnetized along its axis, which serve as boundary surfaces.

2nd group: Mathematical Geodesy

Lectures:

B. Heck: Formulation and Linearization of Boundary Value Problems: From Observables to a Mathematical Model

The lecture by B. Heck aims at showing the long way from the theory of geodetic boundary value problems to the approximations that underly all practical solutions today, producing a whole hierarchy of approximated problems. Therefore, it may be seen as the core lecture of the Summer School. For each of the three classical formulations of geodetic boundary value problems, namely the scalar and vector Molodensky problem, and the fixed gravimetric boundary value problem, the boundary conditions related to different levels of approximation are derived and thoroughly discussed. First, the boundary conditions in the classical geodetic boundary value problems are derived, and the practical relevance is discussed by comparing them with the real data situation. Then, the boundary conditions are linearized using a reference gravity field and a reference surface, respectively. Thereafter, the linearization errors are evaluated in terms of gravity anomalies and potential quantities, and the differences between the three formulations of the geodetic boundary value problem w.r.t. the impact of nonlinear effects are shown. Finally, starting from the specification of the linearized boundary conditions in several coordinate systems, the corresponding boundary conditions in spherical approximation are given. The fixed gravimetric BVP is taken as example to estimate numerically the influence of the spherical approximation and the constant radius approximation. Finally, the solvability of the first degree harmonic coefficients is discussed, based on the solution of the three basic boundary value problems in nonspherical form in terms of spherical harmonics.

F. Sansò: The Hierarchy of Geodetic Boundary Value Problems

F. Sansò gives in total three lectures which cover various topics of the theory of geodetic boundary value problems. The first lecture on "Wiener measures and the continuous formulation of observation equations" is related to the problem of replacing the original discrete problem of gravity field determination by a continuous problem in a stochastic setting without introducing a nonacceptable bias in the solution of the discrete problem. This leads to the concept of Wiener measures as generalization of the concept of discrete white noise to a continuous white noise. Based on that he introduces

the concept of a continuum of noisy measurements as the limit of a large dense data set of measurement points. Then, the question when a switch from continuous to discrete and vice versa is possible, is investigated. A Nyquist type relation is derived stating that the transition may be possible if the discretization error is in some sense "much smaller" than the noise in the data. Only then it is possible to formulate the problem of gravity field determination based on discrete (stochastic) observations in terms of boundary value problems based on a continuum of (stochastic) observations. In a second lecture he discusses the definition of the boundary surface in geodetic boundary value problems and the related problems. He shows that the used boundary is always a (smooth) approximation to the Earth's topography which is located partially inside and partially outside the masses. Therefore, the well known remove restore technique makes it possible to formulate the problem of gravity field determination in terms of boundary value problems. Finally some numerical estimates are presented about the relation between resolution and accuracy of a digital terrain model on the one hand and resolution and accuracy of gravity on the other hand to achieve a 1 cm geoid.

The last lecture is devoted to the analysis of the vector Molodensky problem. The analysis uses a Legendre transform, an approach which is better known in geodesy as the "gravity space approach" worked out by F. Sansò in the seventies. First, the necessary mathematical background such as Legendre transform, Hölder spaces, Schauder estimates for the Dirichlet problem, and local solvability of functional equations are provided. Then, the Legendre transform is applied to the vector Molodensky problem for a non-rotating earth transforming the original BVP in several steps to a fixed boundary value problem for a second order nonlinear partial differential operator. Finally, existence, uniqueness, and regularity of the solution of the transformed BVP are discussed.

Seminars:

P. Holota: Variational Methods for Geodetic Boundary Value Problems

This seminar gives an introduction to variational methods and their application to geodetic boundary value problems. P. Holota starts with a presentation of the functional-analytic foundations of the weak solution concept, concentrating on the construction of a weighted Sobolev space for an unbounded solution domain and estimates for equivalent norms and for traces of functions on a Lipschitz boundary surface. Then he adopts the

formulation of the linear Molodensky problem and other geodetic boundary value problems such that the weak solution concept can be applied. With respect to the question of solvability the Lax-Milgram theorem, associated with the ellipticity of bilinear forms, is the basic tool. The ellipticity conditions are interpreted in a geometric way. Finally, numerical solutions in the framework of Galerkin approximations are discussed.

E.W. Grafarend: Three Seminars on Geodetic Boundary Value Problems

The first seminar of E.W. Grafarend is devoted to the determination of the geoid as a (partly) internal level surface. This problem is formulated in the framework of a Cauchy problem for the domain enclosed by the Earth's surface. Special consideration is given to the potential field produced by the topographical and compensation masses, which is calculated by using a slicing technique. A procedure for the determination of the geoidal heights with respect to a given reference surface (sphere, ellipsoid of revolution) is outlined. This procedure is based on (1) Topo-Moho reduction of the potential data given on the Earth's surface, (2) downward continuation by solving an inverse Dirichlet problem making use of Tikhonov regularization, and (3) restoring the effects of topographical-isostatic masses. In the second seminar, this theory is applied to the calculation of the geoid in South-Western Germany from given geopotential numbers. Details about the applied procedure and about the geopotential data and topographic models are presented. The third seminar provides some type of "tool-box" for the representation of scalar, vector, and tensor fields by spherical and ellipsoidal harmonics which are generally used in gravity field analysis. Starting from a description of spherical (polar) and elliptical coordinates in several parameterizations and from the Helmholtz decomposition theorem for differential forms, fields of type scalar, vector and second order (Hesse) tensor are presented. It is shown that vector-valued spheroidal functions are composed of radial, spheroidal and toroidal components, while tensor-valued functions can be decomposed into normal/tangential, antisymmetric/symmetric and trace/trace-free (deviatoric) parts. The decomposition is illustrated by the aid of the zero, first and second derivatives of the gravitational potential in terms of spherical and ellipsoidal harmonics.

3rd group: Numerics

Lectures:

H. Sünkel: Classical Solutions of GBVP and Their Implementation

This lecture provides a review of the classical analytical approach into the solution of the GBVP, characterized by series expansions with respect to a "small" parameter; as such, it mainly summarizes the work by H. Moritz and the "Graz School". Furthermore, emphasis is given to the numerical implementation of the resulting solution formulae which have the form of spherical integrals. In the first part of the lecture the Stokes problem is treated, aiming at the determination of the geoid as an equipotential surface situated partly inside the earth's surface. After having applied the classical topographic-isostatic and free-air corrections as well as the standard approximations, the geoidal height can be described by the Stokes integral. The Stokes operator is represented in several forms, based on spherical and planar approximation; the effect of the error induced by spherical approximation is counteracted by ellipsoidal corrections. The (spherical/planar) Stokes integral is evaluated using fast spectral analysis procedures such as one- and two-dimensional Fast Fourier and Fast Hartley Transforms. The second part of the lecture concentrates on the analytical solution of Molodensky's problem in terms of series of spherical integrals. Main emphasis is given to the solution by analytical continuation of the boundary data as well as the evaluation of the integrals by FFT procedures. Finally practical examples and experiences are presented; for the area of Austria the differences between geoidal undulations and height anomalies are reported to amount up to 0.5 m which is about 10% of the geoidal height range in this area.

C.C. Tscherning: Topographic Effects in Gravity Field Modelling for BVP

In methodological respect the lecture by C.C. Tscherning, co-authored by R. Forsberg, is also related to the classical analytical solution approach. Considering geodetic boundary value problems as a subset in the class of gravity modelling approaches, different concepts of terrain reductions are elaborated, including the conventional and the remove-restore technique. The final aim behind terrain reductions in Stokes' as well as in Molodensky's approach is seen in smoothing the residual gravity field, facilitating interpolation, prediction, or the use of collocation. Since the magnitude of the terrain reduction is much larger than the non-level correction (free-air

reduction), in general, it is emphasized to use methods which do not require up- or downward movement of the observation points, simultaneously providing the smoothest results. Furthermore he recommends to use density information in terms of density anomalies or isostatic models such as the Airy-Heiskanen compensation model or Helmert's second condensation approach. As an alternative, or in addition, residual terrain modelling (RTM) can be applied, based on a smooth height reference surface. The problem of harmonic continuation from the true topography to the height reference surface is not yet satisfactorily solved in theoretical respect. Another part of the lecture is devoted to the use of terrain reductions in the context of Molodensky's theory. The paper is concluded by a summary of strategies for calculation of the terrain effects, including prism-integration and FFT.

Seminars:

R. Klees: Topics on Boundary Element Methods

The seminar given by R. Klees covers both theoretical and practical aspects of boundary element methods in the context of high resolution gravity field determination. Starting from the mathematical model derived by B. Heck in his lecture, several representation formulas for harmonic functions in an unbounded domain are discussed. Their jump relations provide the boundary integral equations which form the starting point of any boundary element method. Then, the construction of the approximating subspaces where the solution is looked for is discussed and the approximation properties in Sobolev norms of some subspaces relevant for geodetic applications are given. Thereafter, different discretization principles are investigated where most of the time is spent to the Galerkin method, the only discretization method relevant to geodetic problems. The core of the seminar are fast numerical algorithms for Galerkin boundary element methods which are essential for geodetic applications to restrict the numerical costs. Two algorithms are discussed in detail: the panel clustering which exploits the smoothness of the kernel of the integral operator by using certain Taylor series expansions for most of the entries of the equation matrix, and biorthogonal compactly supported wavelets. Both provide sparse linear systems with $O(N \cdot \log^s N)$ nonzero elements where N denotes the number of unknowns and s some positive real number. Thereafter, some of the problems of numerical integration of singular surface integrals are analyzed. They arise when evaluating the presentation formula to get pointwise estimates of the gravity potential and any functionals on the boundary surface. Finally, a detailed state of the art error analysis of Galerkin boundary element methods

is given. The three most important types of errors, namely the discretization error, the error of numerical integration, and the error of boundary surface approximation are discussed. Some estimates are given for geodetic boundary value problems making use of the approximation properties of the approximating subspaces.

R. Lehmann: Solving Geodetic Boundary Value Problems with Parallel Computers

This seminar focuses on the use of a special class of parallel computers, the so-called Multiple Instruction Multiple Data Stream (MIMD) computers for gravity field determination. The first part of the seminar deals with a review of modern supercomputer architectures like Single Instruction Multiple Data Stream (SIMD) computers, and MIMD computers with shared, distributed or virtually shared memory. Then some parallel computing concepts for MIMD computers are discussed, for instance interconnection networks, problem decomposition, interprocessor communication, granularity, and load management. Finally, several applications are presented, and analyzed from a parallel computing point of view: the problem of parallel numerical integration which arises for instance when computing terrain corrections, the application of Galerkin boundary element methods to the linearized fixed gravimetric boundary value problem, and some standard problems of numerical linear algebra.

4th group: Geodetic Applications

Lectures:

R. H. Rapp: Global Models for the 1 cm Geoid. Present Status and Near Term Prospects

The lecture given by R. Rapp can be seen as an application of the theory of boundary value problems to the determination of global gravity field models. It mainly focuses on the practical problems related to high resolution gravity field determination in terms of spherical harmonics. First the hierarchy of potential coefficient estimation procedures is outlined: the estimation of a satellite potential coefficient model, a low degree combination with altimeter data and surface gravity anomalies, and two procedures that yield high degree gravity field coefficients, the quadrature approach and the block diagonal procedure. Then, a review of past and current geopotential models is given with special emphasis on the GSFC/DMA joint gravity and geoid

improvement project. For the latter, the different types of data are discussed and an evaluation of some preliminary potential coefficient models is given. Finally, some theoretical challenges related to the procedures applied in practice for high resolution global gravity field recovery in view of the 1 cm geoid are discussed.

K.P. Schwarz: An Introduction to Airborne Gravimetry and Its Boundary Value Problems

In the series of lectures presented by K.P. Schwarz (co-author: Zuofa Li) three aspects of airborne gravimetry and gradiometry are elucidated. In the first part the system concepts of airborne gravimetry and gradiometry are explained. The fundamental problems in all approaches to airborne gravimetry are related to the stabilization of a direction in space under motion, and the separation between gravitational and non-gravitational accelerations. After discussing potential solutions to these problems, scalar and vector gravimetry and gravity gradiometry are identified as three basic approaches to the measurement of gravity from a moving platform. The central section of the lecture covers the modelling of the observables and estimation methods. The equations for modelling inertial data are reviewed, and an error model for airborne gravimetry is developed. In particular, the effects of attitude errors and insufficient synchronization between GPS and INS on the derived gravity values are studied, and the dominating role of vibration effects is emphasized. In order to minimize the effects of these and other error sources suitable estimation strategies have been designed. Presently the iterative use of time domain and spectral methods is the most promising way to extract the gravity signal from the noisy data. In the third part several variants of boundary value problems related to airborne gravimetry and gradiometry are formulated. The first group uses airborne data only and includes the Scalar Airborne Gravimetric BVP, the Vector Airborne Gravimetric BVP and the Airborne Gradiometry BVP as special cases, considering continuous or gridded data on a plane at constant flight level. The proposed solution concept makes use of Taylor expansions and Fourier transforms as well as a variational principle. In a second set of BVP airborne data are combined with ground data, stabilizing the downward continuation from flight level to ground level. Finally the solution concept for gridded data is generalized to multiresolution data sets. The lecture is concluded by a presentation of practical results obtained in Greenland and Switzerland and by a discussion of future plans related to airborne gravimetry.

E.J.O. Schrama: Satellite Altimetry, Ocean Dynamics, and the Marine Geoid

This lecture can be seen in some sense as complementary to the lecture given by C.C. Tscherning on the treatment and problems of land topography in the context of the geodetic boundary value problem. However, the time component makes the ocean topography much more challenging although the data coverage is mostly considerably better w.r.t. homogeneity and denseness than on land. The first part of the lecture deals with satellite altimetry as a geodetic measurement technique and is thought for those who are not familiar with satellite altimetry. The principles of altimetric measurements are explained, the corrections that have to be applied to fully exploit the capability of the radar sensor are discussed and the problem of precise orbit determination is briefly outlined. The main part of the lecture focuses on tides and ocean dynamics. The lecture on tides aims at giving an introduction to that subject. It covers such relevant topics like tide generating potential, Earth tides, and Love numbers. Methods for determining ocean tides from altimeter measurements are explained and the problem of load tides is briefly discussed. Finally, a review of recent developments that have dramatically improved the accuracy of deep ocean tide models is given. The last part of the lecture is related to ocean dynamics. It aims at discussing that part of the ocean topography that remains when the tidal effect, the inverse barometer effect, and wind waves have been subtracted: the mean dynamic topography as stationary part and the ocean variability as temporal part. Starting with the equation of motion the concept of a mean dynamic topography is introduced and a physical method for measuring it is explained. Thereafter, comparisons are shown between that method and the results obtained by satellite altimetry. The lecture is finished with some examples showing the temporal variations in the dynamic topography as observed by the French/American Topex/Poseidon altimeter satellite.

R. Rummel: Spaceborne Gradiometry

The last lecture in the group of geodetic applications of boundary value problems is devoted to gravity field determination from spaceborne gradiometry. The course presented by R. Rummel starts with an introduction into the principles of Satellite Gravity Gradiometry. Although the basic concepts of gradiometry are very simple, the technical realization has to struggle with problems like drag effects, calibration and orientation of the devices, and consideration of temporal variations of gravitating masses such as propellant. Due to the construction of gradiometers the error spectrum contains a white noise part as well as a frequency-dependent component. In

the second part of the lecture the origin and role of geodetic boundary value problems is explained. Originally the failure of Bruns' polyhedron approach gave rise to the boundary value approach; in the space age the role of the GBVP has been newly defined, e.g. due to the concept of GPS levelling. Satellite gradiometry, providing the second order derivatives of the gravitational potential at satellite height, is embedded into the broader context of gravity sensors and gravity quantities in general and their respective information content. In the third section of the lecture this context is offered by the so-called Meissl scheme, visualized by a "pocket guide" of Physical Geodesy. The structure of the relations based on various self-adjoint (rotational invariant or isotropic) operators is discussed in detail, and extensions to non-self-adjoint operators related to gravity gradiometry observables are pointed out. In the last block of his lectures R. Rummel addresses to spectral analysis of satellite gradiometry which can be considered as an inverse problem of the GBVP, since the disturbing potential on the surface of the earth (or equally well the harmonic coefficients) has to be determined from data given in the exterior. After discussing the signal content of satellite gradiometry and the construction of a noise model (band-limited white noise) several solution concepts are presented, each aiming at a stabilization of the estimation process which proves necessary due to measurement noise propagation. Restricting to a linear functional model the concepts of singular value decomposition, least squares estimation, least squares collocation or regularization, and biased estimation are applied to gravity gradiometry. The lecture is concluded by a synthetic numerical example for spectral estimation based on satellite gradiometry data.

Seminars:

W. Keller: Application of Boundary Value Techniques to Satellite Gradiometry

The seminar given by W. Keller focuses on the gravity field determination using gravity gradients at satellite altitude and terrestrial gravity data. The solution strategy is based on the continuous spacewise approach and the problem is formulated as an overdetermined boundary value problem with noisy data. It is assumed that the data taken along the satellite orbit have been reduced to some boundary surface which in some sense fits the orbit. To set up the functional model, Sobolev spaces on the sphere are used as function spaces and the connection between the error free data and the unknown solution is described by pseudodifferential operators on spheres.

Then, the definition of suitable continuous stochastic models describing the behaviour of the measurement noise, is discussed. He introduces the concept of Hilbert space valued random variables to set up this stochastic model. Thereafter, the solution of the overdetermined boundary value problem is pointed out. First he shows that a simple generalization of the usual least squares principle to infinite dimensional spaces is not possible. As alternative, a generalization of the concept of the best linear unbiased estimation (BLUE) to infinite dimensional spaces is developed and applied to the overdetermined boundary value problem. In addition, a simple a priori error estimate is derived based on the homomorphy of the algebra of pseudodifferential operators with the algebra of their symbols. It can be used in order to investigate the influence of certain mission parameters on the solution. Finally, the theory is applied to the ARISTOTELES and STEP satellite gravity gradiometry missions and some results of a simulation study are presented.

N. Sneeuw & M. van Gelderen: The Polar Gap

The seminar by N. Sneeuw and M. van Gelderen focusses on the effect of nonpolar orbits on the derivation of spherical harmonic coefficients from satellite gradiometry. Based on simulations for a one axis gradiometer (STEP satellite) and a three axis gradiometer (Gravity Explorer Mission), they show what the effect of data (polar) gaps on the estimation of potential coefficients and geoid heights are. Because the problem is ill-posed, regularisation is necessary even though it introduces biases into the potential coefficients. As a result, the polar gap induces a band of low order coefficients which are strongly disturbed and cannot be determined from a combination of gradiometric and GPS data. This could be explained by the support of the Legendre functions in the polar areas and by the condition numbers of the systems of normal equations. If propagated to the space domain (the geoid or the gravity field) the damage caused by the non-global coverage remains limited to an area not much larger than the polar gaps, notwithstanding the global support of the spherical harmonic base functions. Moreover it is demonstrated that the least-squares estimation might contain considerable aliasing errors.

M. Schuyer: European Capabilities and Prospects for a Spaceborne Gravimetry Mission

M. Schuyer presents the goals and the planned mission features of a potential future ESA satellite mission, the Gravity field and Ocean Circulation Earth

explorer (GOCE). This mission candidate will have strong benefits to Geodesy, satellite orbit modelling, Solid Earth Geophysics and Geodynamics, and Oceanography (ocean circulation). In technical respect, the GOCE proposal consists of a spacecraft carrying a three-axis gradiometer and a GPS receiver, and flying at 260-270 km altitude on a sun-synchronous dawn-dusk orbit. After a mission length of 8 months the accuracy and resolution requirements (1 mgal in gravity, 2 cm in geoidal height, up to degree and order 180) will be achieved. The gradiometry component is designed to measure the diagonal terms of the tensor of second order gravitational potential derivatives with high accuracy, the non-diagonal terms with less accuracy. The drag-free control of the space vehicle is discussed in detail.

**PC-based software for gridding and validating randomly spaced data :
A consumer's review of SurferTM and TransformTM**

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ABSTRACT

With the increase in the processing speed and lower cost of PC's, data processing that used to be restricted to workstations can now be performed on a personal computer. Two PC-based software packages for gridding random XYZ data are compared. While they do not allow interactive editing of the random data that automatically updates the grid, they produce not only gridded data but also plots of the data suitable for inclusion in reports.

INTRODUCTION

There are numerous applications (scientific disciplines) that collect randomly spaced data, and then require the data to be converted into a regular grid for analysis. In the broad field of Geomatics, the gridding of random spot heights is necessary, as the software used to calculate surface areas and volumes, produce contour maps, create raster images; or visualise data by a smooth surface, often require gridded data.

As personal computers (PC) are becoming smaller and more powerful, relying solely on a workstation for all data validation and processing is no longer necessary. For a small fraction of the cost of a workstation, and with appropriate software, the data validation and manipulation through to the final report can be performed on a PC.

This paper will summarise the features of two WINDOWS-based software packages, Surfer and Transform, both of which are not prevalent in the field of Geomatics. These are not the only software packages capable of gridding randomly spaced data and then displaying the grid as either contours or surfaces, but are examples of software that are not strictly Engineering Design software. Included are simple examples of the graphical output capabilities from both software packages. The examples will be based on the gridding of irregularly spaced gravity data. However, the compared software can process any irregular data that has x, y and z values. Even if randomly spaced data is not required in a gridded form, the process of gridding the data can be useful in validating data.

DATA AND HARDWARE

The comparison of the software was restricted to gravity data that consisted of four columns of data for each point; Latitude, Longitude, Free-Air Gravity Anomaly and Height. The first data set consisted of 930 data points, selected from an area in the lower half of the North Island of New Zealand. A second data set containing 65000 points, that included and surrounded the 930 data points was also tested. The 65000 point data set covered an area from Latitude S39.00° to S42.40°, and Longitude E173.60° to E176.88°.

The testing was performed on a 486DX2, 66 MHz PC running DOS version 6.20 and Windows version 3.1. The PC had 16 Mbytes of random access memory (RAM) and Windows running in enhanced mode with 36 Mbytes of virtual memory.

GRIDDING METHOD

To compare the gridding performance of the two software packages, it was decided that the comparison should be based on a common and processing-intensive method. The Kriging method was chosen because it is one of the most processing-intensive gridding methods and generally produces better estimates (Isaaks and Srivastava, 1989).

There are variations of the Kriging method. The Ordinary Kriging with a Linear variogram model was used as it is a base model from which the variations are developed. By using Ordinary Kriging with a Linear variogram model it was assumed that both software packages have implemented equivalent formulae.

Isaaks and Srivastava (1989) describe Ordinary Kriging as a method associated with the acronym B.L.U.E. for "best linear unbiased estimator". They continue by stating that "Ordinary Kriging is 'linear' because its estimates are weighted linear combinations of the available data; it is 'unbiased' since it tries to have the mean residual or error equal to zero; it is 'best' because it aims at minimising the variance of the errors." For further details of Kriging the reader is referred to Isaaks and Srivastava (1989) and Journel (1989). For the remainder of this paper, the method of Ordinary Kriging with a Linear variogram model will be referred to as Linear Kriging.

TRANSFORM BY SPYGLASS

The version of Transform tested was v3.0, dated 8 January 1994. Transform occupied 2.3 Mbytes of hard disk space when installed with examples, and used 1.5 Mbytes of memory when first started. The total memory required to run depends upon the size of the data file being analysed.

Unfortunately Transform does not provide an on-line help facility when running the software. The software is quite simple to follow, with the manual being needed only for those finer details about a particular option.

A nice feature of Transform is the ability to view the data before loading the file into memory. However the approximate format of the file (ie. ASCII or Binary, Columns or Matrix) needs to be known. Unfortunately it is not possible to work with random data directly in Transform. The data first has to be entered into a matrix before it can be viewed.

There are a number of techniques that can be used to get the random data into an initial matrix for Transform. These include NO FILL, which places a raw data value in the matrix if the coordinates agree with the matrix cell coordinates; and various interpolation methods that fill the entire matrix with values by using methods such as Linear Interpolation, Kernel Smoothing and Kriging.

Using Linear Kriging and setting the other options to be as close as possible to those used in Surfer, the 930 points took 6^m30^s to generate the full matrix (100x100). However, if the matrix was created first by using the NO FILL option and then Linear Kriging run on the matrix, it took 5^m55^s. For the file consisting of 65000 points, when Linear Kriging was attempted using the random data as input, to generate a 364x379 matrix, the processing appeared to be approximately one third of the way through after 3 days, before the processing had to be terminated. When a 364x379 matrix was created using NO FILL, then Linear Kriging performed on the matrix, the processing took 23^h45^m.

A feature not documented in the manuals is that Transform has a limit on Kriging. If the ratio of neighbouring cell locations in either the x or y direction is close to 1.0001 then Kriging will encounter difficulties and may fail (Pierce, Spyglass Technical Support, personal communication, 1995). This is the most likely reason why Kriging of the 65000 point file had difficulties. To avoid this Kriging limit the 65000 point file was gridded by Linear Kriging into a 83 x 86 matrix that took 6^m10^s to process. The processing was done by first creating a matrix (83x86) using the NO FILL option and then filling the matrix by performing Linear Kriging on the random point matrix. The ratio of neighbouring cells was higher than 1.0002.

Once the data has been entered into a matrix, it is then possible to visualise the data using options such as a contour map, surface plot, vector plot or raster image. If a point appears to be in error by clicking the cursor on the point in the image window, the point will be highlighted in the matrix window (refer to Figure 1). The Image generation function in Transform has 21 built-in colour tables which by varying can highlight features in the data that are less obvious with other colour tables. Transform also allows personalised colour tables to be created, which makes this feature useful for interpreting remotely sensed images.

Transform allows for cross-sections (slices) in the row or column directions to be easily generated. Unfortunately there did not appear to be any simple way of generating a cross-section along any chosen line. Spyglass does have another software package called PLOT. PLOT is designed to produce cross-sections and is compatible with Transform.

There is a comprehensive list of Macro functions available within Transform that can be used to edit columns of data, otherwise only single cell values can be changed manually.

Images can be saved in a number of formats including a TIFF image, a Metafile or a Bitmap file. The matrix can be saved to a number of proprietary HDF binary formats but there is only one ASCII format and it is in a matrix table form.

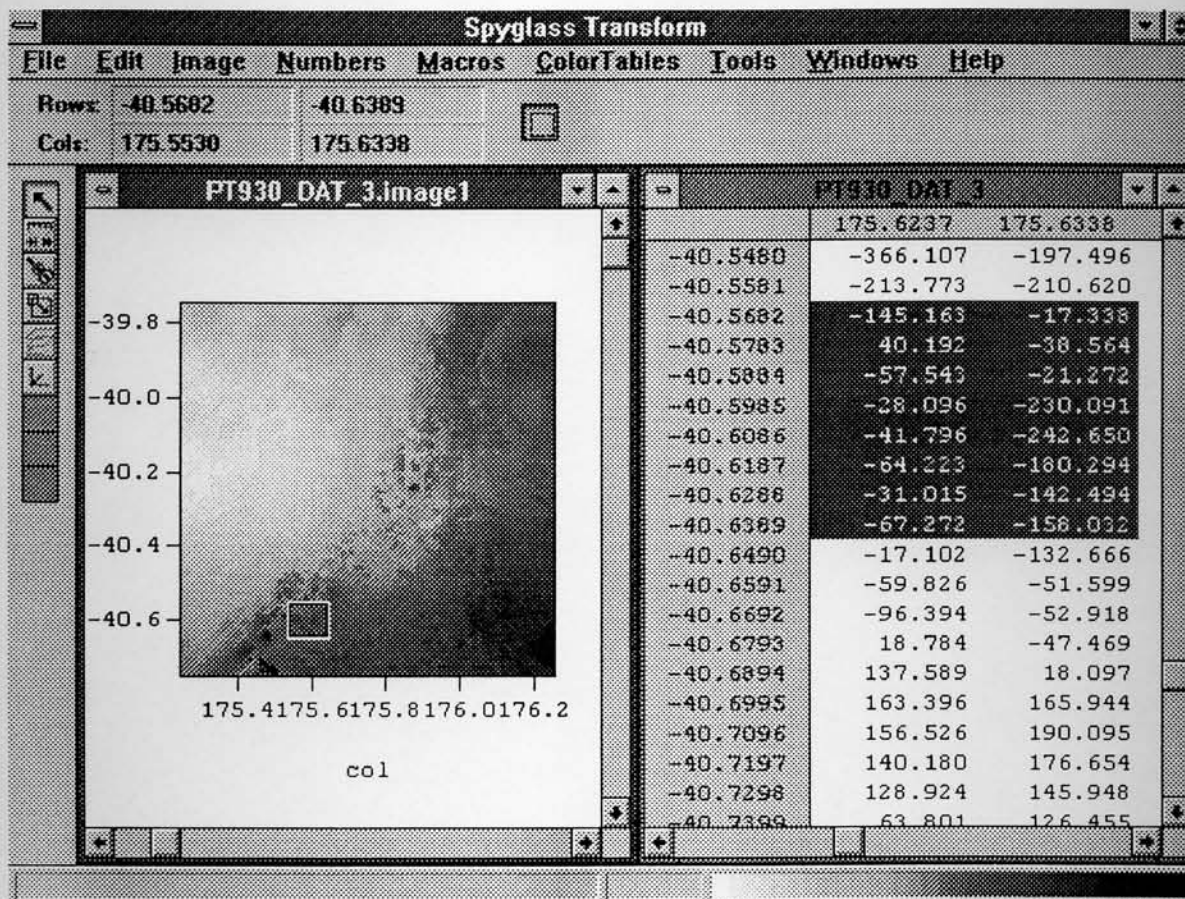


Figure 1 : Screen dump from Transform showing in the left window, a raster image of the matrix data shown in the right window. Note the data is highlighted in both screens when a cell or cells are selected.

SURFER BY GOLDEN SOFTWARE

The version of Surfer tested was v5.02, dated 16 October 1994. Surfer occupies 4.4 Mbytes of hard disk space when installed with examples and uses 0.2 Mbytes of memory when first started. The total memory required to run Surfer will depend upon the size of the data file being analysed.

When Surfer is first started there are a number of features that can be used while the data is still random. These include plotting the random points with a basemap and loading the random data into a worksheet (spreadsheet) to manipulate the data before gridding. This feature is valuable because it gives an overview of the data distribution and the location of any erroneous data. With the random data in a worksheet, mathematical formula can be applied to either a single cell or an entire column or row of cells. The worksheet was able to perform a calculation on all 65000 points with the UNDO feature disabled. This compared to Microsoft Excel version 5.0 which was only able to work with approximately 16000 points.

While working with Surfer there is detailed on-line help available from within most features. However, when gridding data the help is disabled, so being aware of the various options before entering the gridding option is necessary.

Surfer has an undocumented limit of "single" precision for the X and Y values, yielding 6 to 7 significant figures. Thus Longitude values in the example of 175 degrees are limited in precision to 0.0001 degrees (Bresnahan, Golden Software Support, personal communication, 1995), which is approximately 10 metres on the ground.

As with Transform, Surfer has a number of different methods of gridding the data. These include Inverse Distance to a Power, Shepard's Method, Triangulation with Linear Interpolation and Kriging.

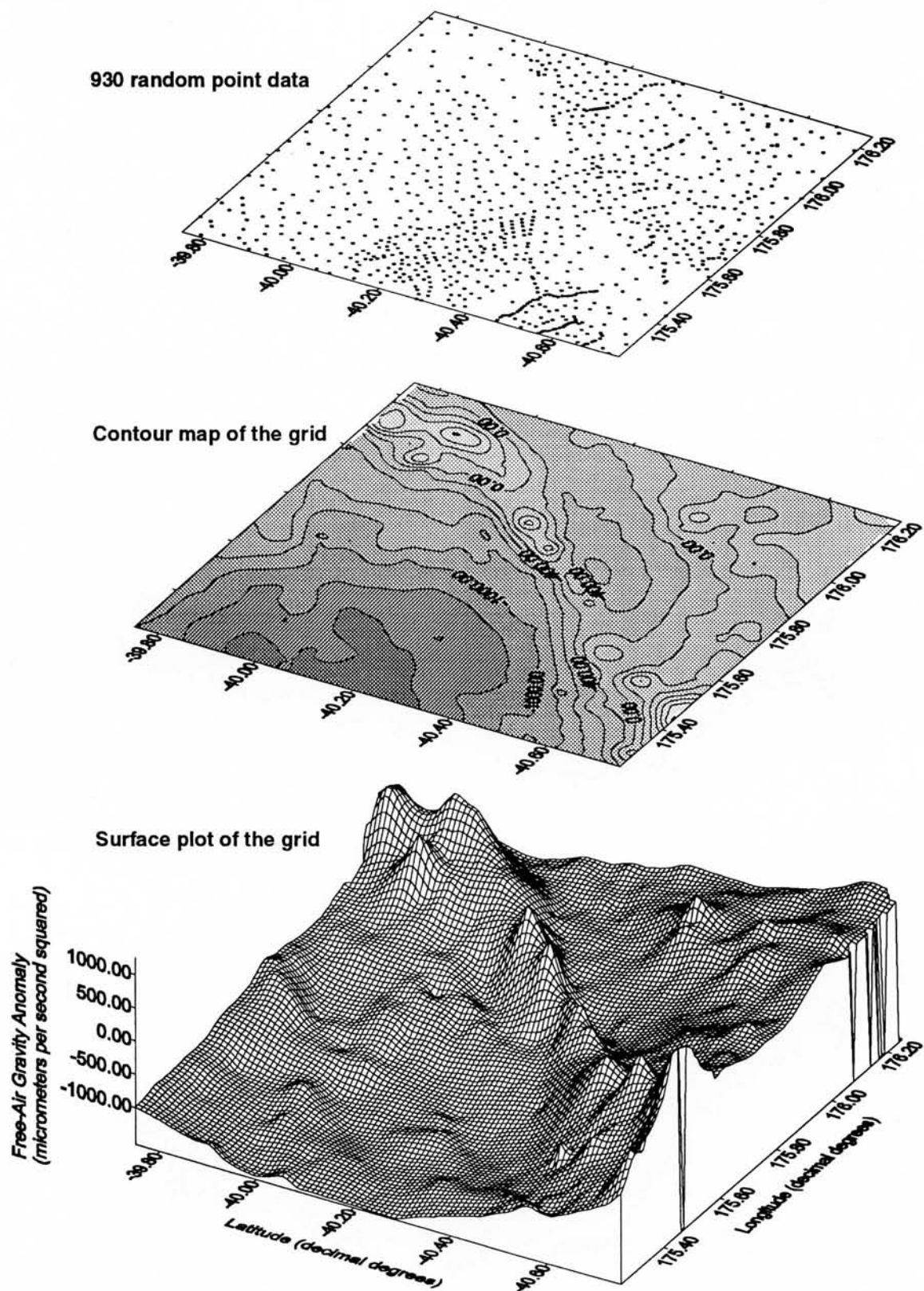


Figure 2: Stacking and Overlay presentation created in Surfer of the data used in Option A, Table 2. Note the points on the edge of the grid that were not calculated due to the lack of data at the edge.

Surfer was able to grid the 930 data point file using the Linear Kriging into a 100x100 matrix, and with the options set as close to those used in Transform, took 2^m00^s. When the 65000 point file was gridded into a 364x379 matrix using Linear Kriging, it took 1^h45^m00^s. The 65000 point file was also gridded into a 83 x 86 matrix, this took 5^m55^s, using Linear Kriging to compare results when the Transform grid was created with the neighbouring cell location ratio of higher than 1.0002.

Surfer, as with Transform, can then produce a contour map or a surface plot of the gridded data. Surfer cannot easily produce the quality raster images or vector plots that Transform can generate. However, Surfer can easily overlay and stack different contour maps and surfaces with the random data points (refer to Figure 2). Residuals can also be calculated between surfaces and between a surface and random points. This residual feature is useful for determining Z-values for new random points, and for checking how well the gridded data represents the random data. Surfer can import base maps in a range of formats, which include DXF, GIF and JPG.

Surfer allows cross-sections (slices) to be generated along any combination of lines and curves, although first a Surfer Blanking File containing the line and curve coordinates needs to be created. The slice function then produces a file that is intended for input into another package by Golden Software called Grapher, however the output file from Surfer is easily interpreted and could be input into other graphing software.

The Matrix data from Surfer can be output in proprietary binary formats or as two ASCII formats, one being in a matrix and the other XYZ. Diagrams which have been created in Surfer can be saved as a Windows Picture (WMF), Windows Clipboard (CLP) or AutoCAD DXF files.

SUMMARY OF LINEAR KRIGING PROCESSING TIMES

From the previous two sections, Transform by Spyglass and Surfer by Golden Software, the processing times of the various options are summarised in Table 1.

Table 1 : Summary of Linear Kriging processing times.

Number of Points	Matirx (grid) size	Data format	Surfer	Transform
930	100 x 100	xyz	2m 00s	6m 30s
		matrix	N/A	5m 55s
65000	364 x 379	xyz	1h 45m	1/3 done in 3 days
		matrix	N/A	23h 45m
65000	83 x 86	xyz	5m 55s	failed
		matrix	N/A	6m 10s

N/A : Not Applicable, is used when Surfer was to use a matrix of the data to perform the Linear Kriging. Surfer worked from the XYZ random data, whereas Transform had an option to Fill a matrix from random XYZ data, before Kriging. **Failed** : indicates the covariance matrix inversion failed in Transform during Linear Kriging.

COMPARISON OF SURFACES WITH INPUT DATA

To be able to determine which software package was producing gridded data that best represented the input random data, the matrices from Transform were imported into Surfer. For both the Surfer and Transform matrices, the residual function in Surfer was used to calculate the difference, at each random point, between the matrix z-value and the z-value in the random point file. Table 2 shows the results of trialing different sized matrices, using both the Linear Kriging and Linear Interpolation, on the 930 and 65000 point data sets.

From Table 2, option A, it is clear that when the x or y neighbouring grid location ratio is below 1.0001, as for this example, then Transform finishes Linear Kriging but the results, especially the average and standard deviation, show that a poor grid was produced.

Option B and C of Table 2 both use the same grid created from the 65000 point data set using Linear Kriging but the residuals were calculated at the 65000 (option B) or 930 (option C) points. In both cases the result from Surfer was significantly better than from Transform. As the 930 point data set is in the middle of the 65000 points, option C was performed to compare results without any edge effects.

Linear Interpolation was used instead of Linear Kriging to generate the grids in option D, Table 2. Both software packages produced similar results that were approximately as reliable as the Surfer result from option C, which used Linear Kriging. Surfer uses Triangulation with Linear Interpolation whereas Transform uses Linear

Interpolation. As a consequence of forming the triangles before Linear Interpolation, Surfer took 8^m15^s to process the data compared with 00^m05^s for Transform.

Table 2 : Statistics of the residuals between different grids and the random data used to create the grid.

Option	Software	Number of input & residual points	Sum	Average	Standard Deviation	Minimum (μms^{-2})	Maximum (μms^{-2})
Definition		n_i / n_r	Σz	$\frac{\Sigma z}{n_r}$	$\sqrt{\frac{\Sigma z^2 - \frac{(\Sigma z)^2}{n_r}}{n_r - 1}}$	min z	max z
A	Transform	930 / 930	69377.7	74.6	489.9	-1281.7	2297.4
	Surfer	930 / 927	5.4	0.01	19.9	-140.5	400.2
B	Transform (M)	65000 / 64819	-1870036.8	-28.9	101.3	-1021.4	1359.4
	Surfer	65000 / 64819	-455423.3	-7.0	54.7	-928.1	518.5
C	Transform (M)	65000 / 930	-11997.5	-12.9	141.4	-377.1	912.2
	Surfer	65000 / 930	-4083.4	-4.4	57.0	-191.1	518.5
D	Transform	65000 / 930	4381.3	4.7	69.0	-225.2	822.3
	Surfer	65000 / 930	-5066.0	-5.4	58.3	-189.5	572.4
E	Transform	Grid ratio below 1.0001					
	Surfer	65000 / 930	-347.0	-0.4	18.9	-183.9	346.8
F	Transform	65000 / 930	330.6	0.4	46.1	-189.1	535.3
	Surfer	65000 / 930	-774.6	-0.8	19.3	-148.0	380.1

Note: In the software column if Transform has a 'M' in brackets, then the Linear Kriging was performed on a matrix of random data not directly from the xyz input file. **Options :** A : 100x100 matrix created from, and residuals calculated at, the 930 point data set using Linear Kriging; B : 83x86 matrix created from, and residuals calculated at, the 65000 point data set using Linear Kriging; C : 83x86 matrix created from 65000 points and residuals calculated at the 930 point data set using Linear Kriging; D : 83x86 matrix created from 65000 points and residuals calculated at the 930 point data set using Linear Interpolation (Transform) or Triangulation with Linear Interpolation (Surfer); E : 364x379 matrix created from 65000 points and residuals calculated at the 930 point data set using Linear Kriging; F : 364x379 matrix created from 65000 points and residuals calculated at the 930 point data set using Linear Interpolation (Transform) or Triangulation with Linear Interpolation (Surfer).

Options E and F from Table 2 are a repeat of the options C and D except that the matrix size was increased to produce a denser grid (364x379). The use of a denser grid improved the results significantly. As was the case for the smaller matrix (83x86) there is no significant difference between a grid produced by Surfer using Linear Kriging and either software package using Linear Interpolation. Based on this test the marginal improvement in the grid produced in Surfer by Linear Kriging (1^h45^m00^s) rather than Triangulation with Linear Interpolation (8^m15^s) is of small benefit when the large difference in processing time is considered.

As can be seen by comparing option A and E of Table 2, the denser the grid the greater the reliability of the grid representing the random data. However, this does not necessarily mean that an interpolated value for a new random point will be predicted any more reliably.

For Australian gravity, Barlow (1977) reported the following estimates of errors (s.d.):

Observed gravity : 0.3 mGal

Elevation : 4-6m which is equivalent to 1.2 - 1.9 mGal Free-air anomaly.

Combining these errors and converting to μms^{-2} gives an error of 22 μms^{-2} for free-air anomalies. Assuming that the gravity data for New Zealand has similar sized errors to the Australian data, then Standard Deviation values in Table 2 of approximately 20 μms^{-2} are realistic even without considering the errors from the interpolation.

CONCLUSIONS

As with most software packages there are features in one package that would be nice to have in another package. Surfer and Transform are no exception. Both packages retail for approximately the same price and met the main requirements; that of being capable of gridding random data and displaying the results in a format suitable for a report.

Neither package allows interactive editing of a random data point, that then updates the grid automatically. The erroneous point needs editing, then the data re-gridded, to obtain a new grid.

If the raw data is random, Surfer can grid the data by the Linear Kriging method quicker than Transform. This becomes especially true for data sets containing tens of thousands of points. Surfer's ability to plot and manipulate the random data in a worksheet, allows it to offer easier data preprocessing features than Transform's macro functions.

For applications where the original (input) data is available in a grid format, Transform has features which allow a wider selection of final presentation formats than does Surfer (for example; Vector plots and Raster Images).

Both packages are relatively simple to use if familiar with Windows based software. As Surfer has an edge over Transform in preprocessing the data and Transform has a wider selection of final presentation formats, the choice of which software would be a more suitable application will depend on whether the raw data is random or gridded.

ACKNOWLEDGMENTS

Mr Guido Zelenka, who kindly provided a copy of Transform for evaluation. Mary Pierce at Spyglass for helping solve the initial problems I had running Transform. Professor Bill Kearsley for providing the resources and encouragement to evaluate the software. Lastly, Matt Higgins for insisting that I tested the special features of both software packages, discussing ideas with while writing up this paper and ensuring the Hot-Metal colour table is used at every opportunity.

SOFTWARE SUPPLIERS

Transform was provided by Mr G. Zelenka from Metronix Pty Ltd, 62 East Arterial Road, St. Ives, NSW 2075, Australia, phone 61-2-449 4620. Transform is a trademark of Spyglass, Inc., who should be able to provide full details of Transform and their suppliers through the following contact:

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Savoy, Illinois 61874
USA
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email: info@spyglass.com

Surfer was obtained from Hearne Scientific Software, Level 6, 552 Lonsdale St, Melbourne 3000, Australia, phone 61-3-602 5088. Surfer is a trademark of Golden Software, Inc., who should be able to provide full details of Surfer and their suppliers through the following contact:

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METHOD BOWRING FOR THE COMPUTATION OF LATITUDE φ FROM RECTANGULAR COORDINATES (X,Y,Z)

G.L. Strang van Hees

Summary.

In geodesy the transformation of rectangular coordinates (x,y,z) into ellipsoidal coordinates (φ, λ, h) is very important. Most people solve the latitude by iteration. I noticed that many geodesists are not familiar with the method Bowring published already in 1976. He gives a more precise formula without iteration. The aim of this article is to pay attention to this remarkable method and give some geometrical insight in the method.

Introduction.

The transformation from ellipsoidal coordinates (φ, λ, h) to (x,y,z) is given by the following formulas

$$\begin{aligned} x &= (N + h) \cos \varphi \cos \lambda \\ y &= (N + h) \cos \varphi \sin \lambda \\ z &= (N - e^2 N + h) \sin \varphi \end{aligned} \quad (1)$$

with

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}, \quad e^2 = \frac{a^2 - b^2}{a^2} \quad (2)$$

The inverse computation from (x,y,z) to (φ, λ, h) is complicated for φ . If φ is known, λ and h can be computed simply. In this paper the rather unknown solution of *Bowring* will be described, which is very convenient for practical computation.

Computation of φ .

From (1) follows:

$$\tan \varphi = \frac{z + e^2 N \sin \varphi}{r}, \quad \text{with } r = \sqrt{x^2 + y^2} \quad (3)$$

The usual method is to compute φ in an iterative way. ($N \cdot \sin \varphi$) in the right-hand side of (3) is computed with an approximate value of φ . The computation of φ with (3) can be illustrated with figure 1. φ' is the latitude of Q which is an approximate value for the unknown latitude φ of P.

For Q is:

$N(\varphi')$	the distance QN
$e^2 N$	the distance LN
$e^2 N \sin \varphi'$	the distance ON.

The latitude computed with (3) is the angle of the line PN with the horizontal axis. After 4 iterations the latitude approaches φ of P with a precision of $0''.0001$.

Another method is to solve φ from (3) explicitly. This gives a fourth degree equation in $\tan \varphi$. The solution is possible, but rather complicated (Heck, Lapaine).

Method Bowring.

Bowring published already in 1976 a simple and practical exact solution of the problem. Bowring found a very clever method to eliminate the iteration. He does not compute point N, but point M in figure 1. M is the cross-point of the normals to the ellipsoid in P and Q. It is the center of the radius of curvature of the ellipsoid. QM is the radius of curvature. If the coordinates of Q are known, then the coordinates of M can be computed. The angle of the line PM with the horizontal axis is the latitude φ of point P.

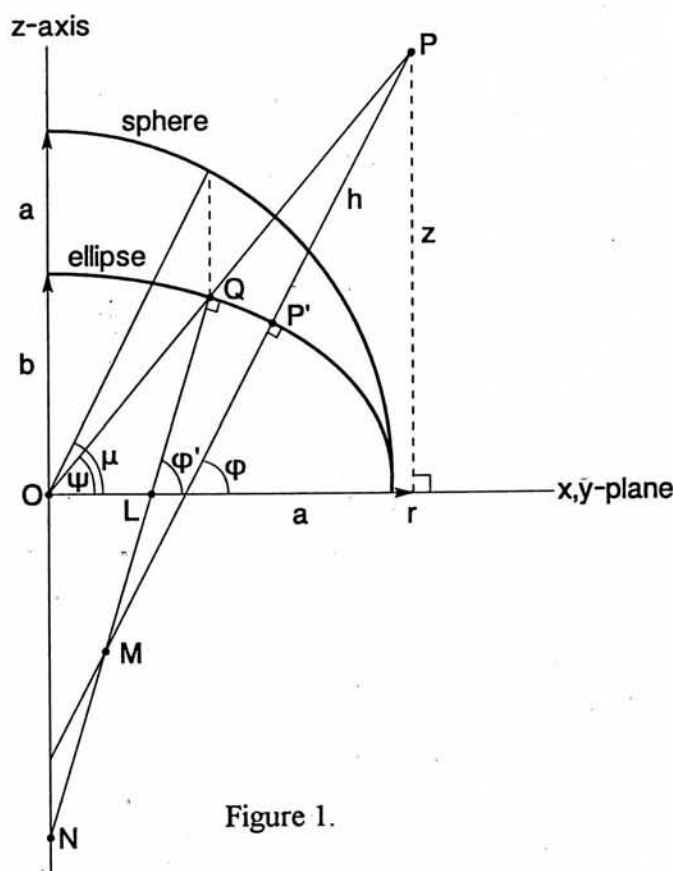


Figure 1.

The method Bowring can be explained as follows by figure 1.

The horizontal axis is: $r = \sqrt{x^2 + y^2}$

The equation of the ellipse is:

$$r = a \cos \mu, \quad z = b \sin \mu \quad (4)$$

with μ the reduced latitude.

The geocentric latitude of P and Q is:

$$\tan \psi = \frac{z}{r} \quad (5)$$

The reduced latitude of Q:

$$\tan \mu = \frac{a}{b} \tan \psi = \frac{a \cdot z}{b \cdot r} \quad (6)$$

The ellipsoidal latitude of Q:

$$\tan \varphi' = \frac{a}{b} \tan \mu = \frac{a^2 z}{b^2 r} \quad (7)$$

If the coordinates of Q are given, then we can compute the coordinates of M:

$$r_M = e^2 a \left(\frac{r_Q}{a} \right)^3, \quad z_M = -e'^2 b \left(\frac{z_Q}{b} \right)^3 \quad (8)$$

with $e^2 = \frac{a^2 - b^2}{a^2}$ and $e'^2 = \frac{a^2 - b^2}{b^2}$

see for example: Bartsch: Mathematische Formeln.

(8) can also be derived from the distance QM, which is the radius of curvature of the ellipse:

$$QM = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi')^{3/2}}$$

The coordinates of Q follow from (4):

$$r_Q = a \cos \mu, \quad z_Q = b \sin \mu \quad (9)$$

and μ follows from (6).

From (8) and (9) follow:

$$r_M = e^2 a \cos^3 \mu, \quad z_M = -e'^2 b \sin^3 \mu$$

The latitude φ follows from:

$$\tan \varphi = \frac{z_P - z_M}{r_P - r_M} = \frac{z + e'^2 b \sin^3 \mu}{r - e^2 a \cos^3 \mu} \quad (10)$$

Summary:

$$\tan \mu = \frac{a \cdot z}{b \cdot r} \quad (11a)$$

$$\tan \varphi = \frac{z + e'^2 b \sin^3 \mu}{r - e^2 a \cos^3 \mu} \quad (11b)$$

Remark.

If we differentiate the formula of Bowring (11b), we get:

$$\frac{d\varphi}{d\mu} = 0 \quad (12)$$

This is the power of the formula of Bowring. If we change μ a little bit, this has no influence on φ . In other words: formula (11b) converges very fast, so we don't need to iterate.

This method shows that the solution of formulas (1) is found by a geometric consideration of figure 1. It is not possible to find this solution with a mathematical computer program. This illustrates that in modern mathematics we should not forget to pay attention to geometry.

Precision.

The method Bowring is not complete exact. If h is big, in the order of satellite height, 1000 to 10000 km, then the points P' and Q are not very close and the normals do not intersect exactly in the center of curvature. The error made by this approximation is, however, very small.

height in km, $\varphi = 45^\circ$	error in φ in arcsec
0	0".000 000
100	0".000 003
400	0".000 06
1 000	0" 000 2
2 500	0".000 7
5 000	0".001 3
10 000	0".001 7
50 000	0".001 0
100 000	0".000 6
500 000	0".000 2
1 000 000	0".000 1
∞	0".000 0

0".0001 corresponds with 3 mm on the earth surface!

Thus, the formula is exact for points close to the earth surface.

Determination of height.

After the latitude φ has been computed the height h can be determined. We have the choice between three formulas:

$$h = \frac{r}{\cos \varphi} - N \quad (13)$$

$$h = \frac{z}{\sin \varphi} - N(1 - e^2) \quad (14)$$

$$h = r \cdot \cos \varphi + z \cdot \sin \varphi - a\sqrt{1 - e^2 \sin^2 \varphi} \quad (15)$$

with

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$$

Formulas (13) and (14) follow directly from (1), the third formula follows from:

$$(13) \cdot \cos^2 \varphi + (14) \cdot \sin^2 \varphi.$$

This formula has first been derived by Bowring (1985). Although all three formulas are correct, they are not equivalent concerning the influence of the rounding error in latitude φ .

If we differentiate the three formulas to φ we get:

$$\text{from (13): } \frac{dh}{d\varphi} = (N+h)\tan \varphi + \dots(e^2 N) \quad (16)$$

$$\text{from (14): } \frac{dh}{d\varphi} = (N+h) \frac{-1}{\tan \varphi} + \dots(e^2 N) \quad (17)$$

$$\text{from (15): } \frac{dh}{d\varphi} = 0 \quad (18)$$

Formula (18) can be checked as follows. Differentiate (15):

$$\frac{dh}{d\varphi} = -r \sin \varphi + z \cos \varphi + e^2 N \sin \varphi \cos \varphi$$

Substitute (1) for r and z :

$$\frac{dh}{d\varphi} = -(N+h)\sin \varphi \cos \varphi + (N+h - e^2 N)\sin \varphi \cos \varphi + e^2 N \sin \varphi \cos \varphi = 0$$

The second derivate is:

$$\frac{d^2 h}{d\varphi^2} = -(a+h) + \dots (e^2 N)$$

The error in h caused by a rounding error in φ , $\Delta\varphi$, is:

$$\Delta h = \frac{dh}{d\varphi} \cdot \Delta\varphi + \frac{1}{2} \frac{d^2 h}{d\varphi^2} \Delta\varphi^2 + \dots \quad (\Delta\varphi \text{ in radians})$$

This means that if we use formula (13) or (14) to compute h , a small rounding error in φ gives an error in h , corresponding to (16) and (17). Near the equator is $\tan \varphi = 0$ and (14) cannot be used and near the pole is $\tan \varphi = \infty$ and (13) can not be used. In between, both formulas give errors depending on $\tan \varphi$. On the other hand formula (15) is much better because a small error in φ gives a neglectable error in h . Therefore formula (15) is the best for all values of φ .

For example if the error in φ is 10^{-6} degree = $0",0036$, then the error in h is for a terrestrial point (small h) and for latitude $\varphi = 30^\circ, 45^\circ, 60^\circ$ respectively:

with formula (13) : error in h : 6, 11, 19 cm

with formula (14) : error in h : 19, 11, 6 cm

with formula (15) : error in h : 10^{-7} cm for all latitudes.

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**Towards a refined gravimetric geoid for Canada
1996-1999
S.D. Pagiatakis**

Preface

In 1996-1999, the Gravity Section of the Geodetic Survey Division (GSD) will develop and implement a strategy to determine an enhanced geoid model for Canada which, in combination with GPS, will provide an alternative and cost-effective method to geodetic levelling for orthometric height determination and for a suite of applications, such as mapping, geodetic positioning, geodynamics and geophysical exploration projects. In addition, an enhanced geoid model will contribute to the reconciliation of the different height systems in Canada and the further development and delivery of the Canadian Spatial Reference System (CSRS).

GSD recognizes that it is dealing with clients who need support and increased capability to position their markers at the centimetre level of accuracy. Although advances in geoid determination and refinement stem mainly from geodetic science, and despite the fact that major challenges for further improvements originate from GPS technology, geoid modelling has a considerable influence in all areas of earth, ocean and atmospheric sciences.

This three-year plan was discussed and accepted by the Geodetic Science and Technology Committee (GSTC) of the GSD, on the 4th of July 1996. It will be implemented and executed primarily by the Geodetic Survey Division, the University of New Brunswick and the University of Calgary, in cooperation with the provincial geodetic agencies and the international geodetic community. The present three-year plan has been prepared by the Geoid Group of the GSD, based on the discussion paper *The Canadian Gravimetric Geoid*, presented and discussed at the GSTC meeting (22 March, 1996) and it includes significant contributions from Prof. P. Vaníček (UNB) and Prof. M. Sideris (UofC).

1. Overview

One of the key goals of the Geodetic Survey Division is to develop a new high resolution geoid model to meet the needs and objectives of clients and to maintain and enhance the Canadian Spatial Reference System (CSRS), thus providing ties between GPS positioning and traditional survey networks. It is Geodetic's primary responsibility to ensure that CSRS is widely available to serve as the basis upon which all surveys in Canada rest.

A new, high resolution geoid for Canada requires the joint efforts and expertise of many agencies and institutions, especially during this period of limited resources. GSD will play a leadership role by supporting and contributing to the research and development on the geoid. GSD's leadership role will be in consolidating the close coöperation and support (contracts) of other research institutions in Canada. Geodetic Survey Division, University of New Brunswick and University of Calgary will collaborate during a three-year period to improve the infrastructure required (e.g. data sets, data base) for a high resolution geoid, while researching unsolved problems related to theoretical developments, data reduction and algorithm implementation. In addition, GSD will lead the overall progress of the geoid studies, while each of the three institutions will assume responsibility for well defined research objectives. Furthermore, GSD encourages the participation of other research institutions in this effort.

GSD will organize annual, three-day workshops, preferably during the first quarter of each calendar year, during which GSD personnel from the Geoid Group and the investigators from the two Universities will deliver technical presentations summarizing their activities and progress during the previous year, vis-a-vis their assumed responsibilities. The workshops will be open to provincial geodetic agencies, our American counterparts, as well as to graduate students, who are doing research on the geoid. The technical presentations will be scheduled during the first day of the workshop, whereas the remaining two days will be assigned to discussions and planning. The results presented at the workshop will be documented in brief reports and preferably, in technical papers published in refereed journals and conference proceedings.

In the next section, the main objectives, the milestones, the measures of success and the completion dates are presented and explained. Figure 1 illustrates the tasks involved and their inter-relations, while Table 1 shows the time frame of action. Tasks 2.3.2 through 2.3.7 will be mainly the responsibility of the two Universities; details will be included in an up-coming contract proposal prepared jointly by UNB and UofC.

2. Program Objectives, Milestones and Measures of Success

2.1. International Commitments

2.1.1. *Global Geopotential Models*: Fulfil commitments to international projects of the International Geoid Service (IGeS) by analyzing and testing the DMA/GSFC and other geopotential models (as they become available). (*Team Leader*: M. Véronneau. *Resource*: A. Mainville, M. Sideris).

- Milestones*:
- a) Technical report to be submitted to the Special Working Group (SWG) of the IGeS. (July, 1996),
 - b) Further tests may be conducted, if requested by the SWG and the model developers.

Measure of success: Recognition by the IGeS of the GSD contributions in the form of including the GSD report in M. Sideris's SWG final report and/or publishing it in the IGeS bulletin, if permission is given by NASA/DMA.

2.1.2. *Towards a North American Geoid:* The Canadian and American geoid models show disagreement in certain areas around the Canada-US border. It will be therefore premature to compute a preliminary North American geoid model (as indicated in the Canada-US Technical Memorandum of Agreement, of November 1994) before these discrepancies can be explained and resolved. It is anticipated that at the end of the present three-year plan, we will be in a position to explain the discrepancies and negotiate with the Americans a common realistic plan towards the determination of a North American geoid model. It is envisaged that the NA geoid model will be a combined product of both, the Canadian and the US models, after a reasonable agreement has been ensured at the border. This implies that an agreement on standards and methodology will be achieved between the two countries.

Milestone: Establishment of a Canada-US committee on the North American geoid to discuss and define common strategies for a unified North American geoid model (March, 1997).

Measures of success: a) Increased cooperation with US National Geodetic Survey,
b) Successful negotiation and fulfilment of MOUs.

2.2. Technical Issues

2.2.1. *Documentation of GSD95 Geoid Model:* The internal technical reports, related to GSD95 geoid model, will be used as a basis to prepare a unified report to describe the theory, assumptions, approximations and techniques used to produce GSD95, with input from internal and external reviewers [*Team Leader:* M. Véronneau. *Resource:* A. Mainville, S. Pagiatakis].

Milestones: a) Paper ready for internal review by September, 1996,
b) Paper to be submitted to a refereed journal (January, 1997),

Measure of success: Acceptance of paper for publication.

2.2.2. *Data Collection:* Efforts will increase to obtain detailed Digital Elevation Model (DEM) and Topographic Mass Density Data (TMDD) for the purpose of refining the terrain corrections and the calculation of orthometric heights [*Team Leader:* M. Véronneau. *Resource:* S. Pagiatakis, A. Mainville, B. Hearty, P. Kane, A. Goodacre, R. Gibb].

The involvement and potential contribution of the provincial geodetic agencies is significant to accomplish this task successfully. They can provide us with DEM data (when available) and this communication has started already [A. Kassam, BC]. It is anticipated that other provinces, such as New Brunswick [C. Carlin] and Alberta [C. Barnes] will make available detailed DEM data. In addition, the Centre for Topographic Information (Sherbrooke, Québec) can provide us with DEMs to fill void areas or improve the ETOPO5 model in central Québec [S. Plante].

There exists a TMDD file in the National Gravity Data Base (NGDB) that needs to be checked for suitability (March, 1997). GSC (Terrain Sciences Division) and the private sector [e.g. B. Jones] can contribute towards the collection and validation of a suitable TMDD model.

The regular exchange of gravity data between GSD and NGS is valuable. Other agencies, such as DMA and KMS (Denmark) can provide us with additional gravity data.

Satellite altimetry data collection will continue. Altimetric data will be analysed to determine the sea surface topography in Hudson Bay. The data will be archived and processed for improved marine gravity and geoid determination.

Milestones and measures of success:

- a) Available DEM and TMDD from the NGDB (March, 1998),
- b) Determination of SST in Hudson Bay using satellite altimetry data and regional marine tide models (March 1998),
- c) Increased cooperation with provinces and industry.

2.2.3. *Re-adjustment of the Levelling Network:* A re-adjustment of the junction points of the levelling network is required, using tide gauges, altimetric geoid undulations and sea surface topography (where available) as weighted constraints. This will provide us with a rigorous set of test points for the geoid and will facilitate the reconciliation project thus increased cooperation with the Geodetic Networks Section of GSD is essential. As the height information is essential for the assessment of the data (2.2.4) and the development of a new NGDB (2.2.5) a preliminary re-adjustment of the junction points for research purposes only will suffice [*Team leader:* A. Mainville. *Resource:* M. Bérubé, P. Vaníček].

Milestone: Adjusted junction points of the levelling network with weighted constraints (November, 1996).

Measures of success: a) Increased understanding of systematic effects in geodetic levelling,
b) Successful application of results to data assessment (2.2.4).

2.2.4. *Data Assessment and Integrity:* Data relevant to geoid model determination, that have been collected in the past, need to be checked for systematic errors. Such data include the elevations of terrestrial gravity measurements observed with altimeters. Furthermore, it is known that there exist biases and datum shifts in terrestrial (gravity and heights), marine and altimetry-derived gravity data. This is a very important issue and must be resolved before a new geoid model is calculated. Treatment of the biases and datum shifts must be performed in a rigorous manner (combined with the results obtained from the previous step 2.2.3) [*Team Leader:* S. Pagiatakis. *Resource:* M. Véronneau, R. Cooper, J. Halpenny].

Milestones and measures of success:

- a) Resolve problems with the elevations (altimetry-derived) of terrestrial gravity measurements (October, 1997),
- b) Resolve biases between shipborne gravity measurements and altimetry derived gravity data over the Atlantic Ocean (October, 1997),
- c) Improve gravity data set in the NGDB (December, 1997).

2.2.5. *National Gravity Data Base:* The NGDB was originally designed to meet the needs of both, geological and geodetic applications. The National Gravity Program (NGP), after its amalgamation with the Geodetic Survey Division, is undergoing changes and is becoming more focused on geodetic programs that require restructuring of the NGDB. Eventually, NGDB will be integrated with the Canadian Geodetic Information System (CGIS). The geoid project, being one of the major ones, requires a multitude of data sets that do not reside presently in the NGDB. This increases the possibility of data corruption, thus resulting in wrong geoid models. The aim of this task is to re-design and expand the NGDB to include all the data required for the reduction of gravity observations (e.g. DEM, TMDD) and the calculation of the geoid (e.g. mean Helmert anomalies). In this study, other types of data will also be considered (e.g. cryogenic data from CAGS).

This step involves the design of a prototype NGDB and relevant software for retrieval and storage of data. At a later stage, when step 2.3.2 is completed, the prototype will be expanded to include real data. Significant input and contributions are anticipated from our Gravity Surveys and Data Base group to complete this task. The study should be completed by December 1996 and a prototype should be implemented in the NGDB by March 1997. NGDB updates and maintenance will continue for the duration of this three-year plan [*Team Leader:* S. Pagiatakis. *Resource:* M. Véronneau, B. Hearty, P. Kane, P. Vaníček, M. Sideris, R. Gibb].

Milestones and measures of success:

- a) Design of a new NGDB (CGIS) to serve all geodetic requirements and tasks (December, 1996),
- b) Development and implementation of a prototype NGDB (March, 1997).

2.3. Theoretical Issues

2.3.1. *Synthetic Gravity Field:* For the purpose of testing rigorously the various numerical integration techniques, controlling the truncation errors, identifying kernel singularities and other numerical integration problems, a synthetic (simulated) global gravity data set (with DEM and TMDD) is needed. This task is fairly complicated; it involves the modelling of a reasonable topography and a continuous density distribution. Then, this combined topography-density model will be used to generate the gravity field in a geocentric coordinate system and then compute gravity anomalies on the geoid. International cooperation is needed here since it requires competent theoreticians to tackle it properly (e.g. University of Milan, Charles University). [*Leader:* F. Sansó (University of Milan). *Resource:* P. Vaníček, M. Sideris, D. Nagy, other international participation].

Milestones and measures of success:

- a) Synthetic DEM and TMDD data (December, 1997),
- b) Synthetic global gravity field (March, 1998).

2.3.2. *Data Reduction Algorithms:* Study and intercompare various methods of gravity averaging (e.g. mean Helmert anomalies) interpolation and prediction, as well as topographic reduction, condensation and continuation methods. The ellipsoidal and atmospheric corrections, as well as the flattening error should be well understood and implemented [*Team Leader:* P. Vaníček. *Resource:* M. Paul].

Milestones and measures of success:

- a) Rigorous integration of global GMs and observed gravity (long and short wavelengths),
- b) Propagation of errors in GM coefficients and in observed gravity to the geoid height estimation,
- c) Publication.

2.3.6. *Analysis and Assessment of Computational Methods:* Critical analysis of available geoid prediction methods and software packages. Discussion of open theoretical, technical and practical problems, such as truncation, kernel singularities, integration schemes for mean and point data [Team Leader: P. Vaníček. Resource: M. Véronneau, M. Sideris].

Milestones and measures of success:

- a) Adopt optimal methods, techniques and software for geoid determination (FY 1998-99),
- b) Publication.

2.3.7. *Heterogeneous Data Evaluation and Integration:* Investigation of the contribution of satellite altimetry and airborne gravimetry to gravity and geoid determination. Investigation of optimal combination techniques that make use of all types of available heterogeneous data and their errors, including deflections of the vertical as a data source yielding geoid improvement and proper incorporation of GPS, levelling and SST data into the geoid solution. This task will be carried out in the third year of studies, FY 1998-99 [Team Leader: M. Sideris. Resource: P. Vaníček, M. Véronneau, A. Mainville, M. Craymer].

Milestones and measures of success:

- a) Optimum use of heterogeneous data towards a high resolution geoid model for Canada,
- b) Plan for the determination of the Canadian Gravimetric Geoid CGG2000,
- c) Publication.

Other data correction methods, such as terrain corrections to gravity data and relevant software [e.g. TRITER] should be documented and standardised. Updates to NGDB should also be implemented [D. Nagy, B. Hearty].

Milestones and measures of success:

- a) Reduction algorithms (July, 1997),
- b) Software development, testing and standardization (October, 1997),
- c) Publication(s).

2.3.3. Gravity Data Accuracy and Distribution: The accuracy of the gravity data in Canada should be investigated to obtain realistic estimates of the random errors to be propagated in various gravity and geoid prediction methods. In addition, studies should be carried out to determine the required density of coverage of the gravity data for a high resolution geoid, for various regions of the country (mountainous, flat) and for different levels of accuracy of the geoid model. Presently, the gravity coverage in Canada can hardly support a 5'x5' geoid grid. This important issue is in effect a reorientation of the priorities of the gravity mapping program to meet the requirements for improved geoid determination. Other priorities, for example the needs of GSC, the exploration industry and USDMA will continue to be met. The above studies will facilitate the identification of weak gravity coverage in Canada [Team Leader: M. Sideris. Resource: P. Vaníček, S. Pagiatakis, R. Gibb].

Milestones and measures of success:

- a) Gravity data coverage and accuracy requirements for a high resolution geoid at the medium wavelengths (July, 1997),
- b) Identification of areas that need densification of gravity (October, 1997),
- c) Publication.

2.3.4. DEM and TMDD Accuracy and Distribution: Similar to 2.3.3 above, the accuracy and distribution of DEM should be studied, vis-a-vis the requirements for a high accuracy geoid at the short wavelengths. In addition, topographic mass density information can be assessed in a similar fashion. [Team Leader: P. Vaníček. Resource: D. Nagy, A. Goodacre, M. Véronneau, S. Pagiatakis].

Milestones and measures of success:

- a) DEM and TMDD coverage and accuracy requirements for a high resolution geoid at the short wavelengths (FY 97-98),
- b) Publication.

2.3.5. Global Geopotential Models and Long Wavelength Geoid: Study the error limitations imposed by the accuracy of the geopotential coefficients on the long wavelength geoid. Study the effect of the maximum degree of expansion of the GM in relation to the accuracy and distribution of gravity data available. The effect of using the same terrestrial gravity values to determine both, the underlying GM and the local/regional geoid (Stokes) has to be resolved. This task will be carried out in the second year of studies, FY 1997-98. [Team Leader: M. Sideris. Resource: P. Vaníček, M. Véronneau, S. Pagiatakis].

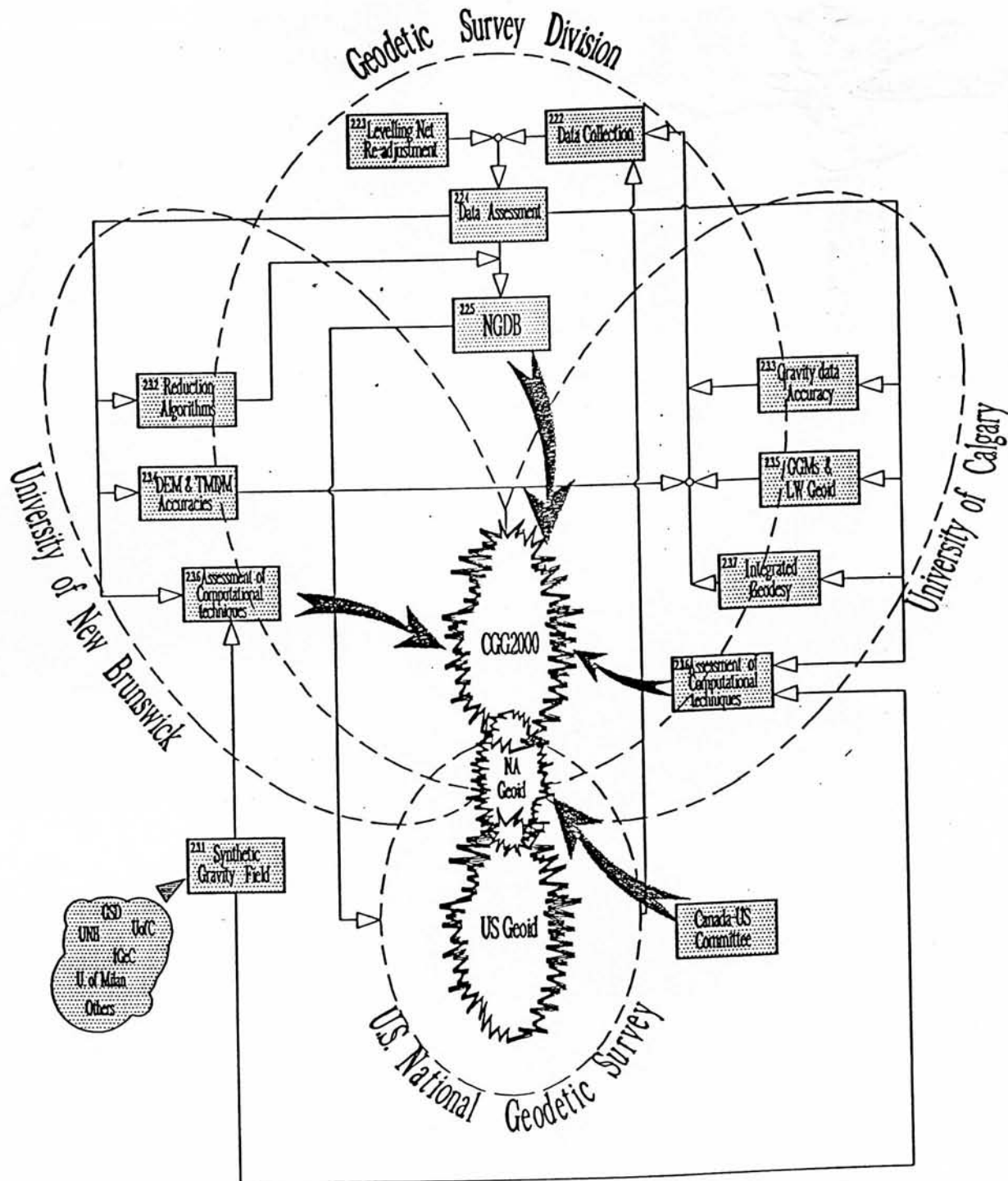
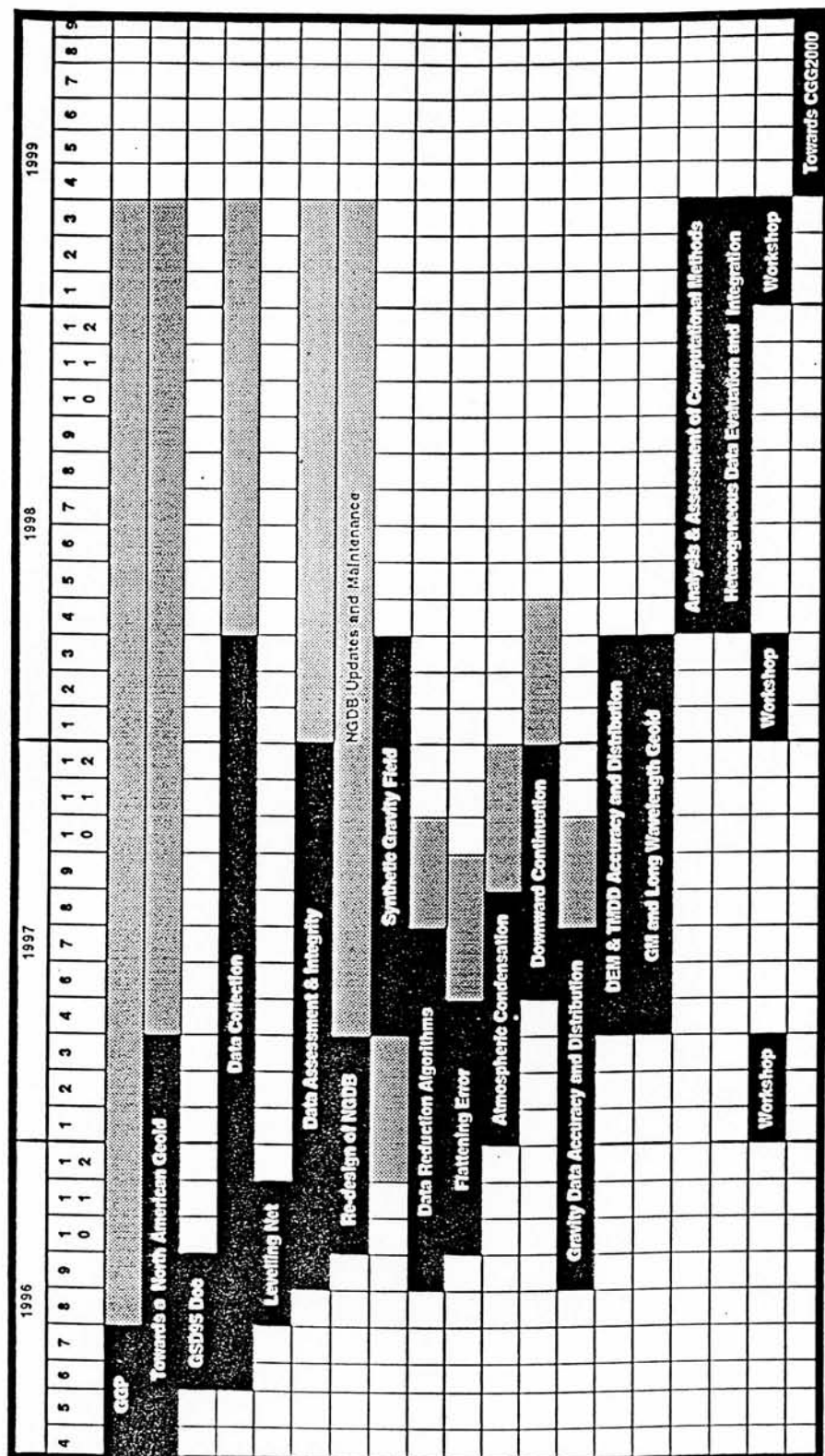


Fig. 1. Interrelationship among organizations and tasks



This table shows the course of action during the three-year period 1996-99. Dark areas show high activity, whereas gray areas indicate either preparation, or testing and maintenance (low activity).

The new Algerian quasi-geoid 96 computed using spherical FFT

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May 1996

ABSTRACT

The main objective of this paper is the use of FFT techniques for the efficient computation of the geoid and the quasi-geoid.

The evaluation of Stokes integral and Molodensky's series by numerical integration is very time consuming but now the burden has been removed and higher order terms can be evaluated in the spectral domain in planar approximation by the Fast Fourier Transform algorithm applied to gridded data.

The gradient solution (cf. Moritz 1980 pp.414) was computed as the simple Molodensky's solution and was compared to the geoid solution computed using the same technique.(cf. statistics table).

I- INTRODUCTION

Algeria has recently focused a part of the current research on the precise geoid determinations using different methods.

At present, demands on gravity determination have increased through the advent of the Global Positioning System, providing three dimensional relative positioning with cm accuracy even over longer distance namely 100 km.

Especially the combination of ellipsoidal heights from the now fully operational GPS with classical levelling data is one of the primary drivers for precise geoid calculations requiring cm accuracy over few km to a few 1000 km.

In order to obtain such a geoid, high resolution point or mean gravity field data and topographic information are needed as well as a good spherical harmonic reference model.

A geoid has been computed over an area covering Algeria using a gravity data file of 12000 gravity points, not homogeneously distributed and an elevation model of 15' x 15'.

A new geoid and a new quasi-geoid have been computed over the area $20 \leq \varphi \leq 37$ and $-7 \leq \lambda \leq 10$ covering Algeria, where φ is the latitude and λ is the longitude.

The same gravity data file was used in the computation, but a detailed digital terrain model was introduced.

In fact, a combination of a 1 km DTM with a 10 km DTM was used when computing the topographic effects on the geoid.

The former DTM did only cover the area $33 \leq \varphi \leq 38$ and $-3 \leq \lambda \leq 9$ and the later covers the area $15 \leq \varphi \leq 40$ and $-10 \leq \lambda \leq 15$.

The usual remove-restore method is used in order to smooth the gravity data which will be used to compute the residual geoid and quasi-geoid using spherical Fast Fourier Transform. (Forsberg and Tscherning 1994)

The comparison of the Algerian geoid with the GPS derived geoid heights is not possible due to the almost absence of GPS benchmarks in Algeria.

II- DATA DESCRIPTION

1- Gravity data

In the area $20 \leq \varphi \leq 37$ $-7 \leq \lambda \leq 10$ approximately 12000 free air gravity anomalies exist. The gaps were covered by predicted data using collocation. The data were made available by Bureau Gravimetrique International.

2- Elevation Model

The computation of the terrain effects on the geoid requires a detailed DTM model. For this purpose, two elevation grids of different resolution were used:

- 10 km DTM covering the area $15 \leq \varphi \leq 40$ $-10 \leq \lambda \leq 15$ originatives from ETOPO5

- 1 km DTM covering $33 \leq \varphi \leq 38$ $-3 \leq \lambda \leq 9$ subtracted from database.

In order to compute the reduced geoid and quasi-geoid in one run of the computer program TCFOUR we merged the two grids into one grid of $5' \times 5'$.

Nevertheless, the RTM effect computation was split up into two steps :

- RTM effect over the area $33 \leq \varphi \leq 38$ $-3 \leq \lambda \leq 9$ using 1km DTM.
- RTM effect over the area $20 \leq \varphi \leq 33$ $-7 \leq \lambda \leq 10$ using the merged grid.

The output files of the two steps were merged into one file.

3- Geopotential Model

The OSU91A1F complete to degree and order 360 was the reference field employed when subtracting its effect from free air gravity anomalies thus subtracting the long wavelength part of the earth's gravity field.

Therefore, the prediction based on the residual observation is of improved accuracy since we hereby have accounted for the main part of the gravity field outside algeria.

Furthermore, we used the FFT techniques the subtraction of the reference field effect may reduce the leakage error caused by the existence of long wavelength parts which cannot be resolved using only gravity data of limited extent.

III- QUASI-GEOID SOLUTION

The solution was performed according to the spectral methods (Schwarz & al 1990) and using the remove-restore procedure.

The long wavelength solution, ζ_r , was given by OSU91A1F complete to degree and order 360 and the short wavelength solution was modeled using the residual terrain model where the reference topography was constructed by a 10' x 10' moving average filter.

The field transformation from gravity anomalies to height anomalies was done using :

$$\zeta = \zeta_{ref} + \zeta_{red} + \zeta_{rm}$$

The residual terrain model effects with respect to a mean surface were computed using Fast Fourier Transform (TCFOUR: Forsberg 1989).

The residual gravity anomalies were gridded by least squares collocation onto 5' x 5' grid covering the area $20 \leq \phi \leq 37$ $-7 \leq \lambda \leq 10$ and this gives $204 \times 204 = 41616$ grid points and utilizing 5 closest points in each quadrant around the prediction points.

the residual height anomalies were calculated by :

$$\Delta g' = \Delta g - \Delta g_{ref} - \Delta g_{rm}$$

The residual height anomalies were obtained involving $\Delta g'$ and using the gradient solution described in Moritz 1980.

$$\zeta_{red} = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g' + G_1) S(\psi) d\sigma$$

where

$$G_1 = \frac{R^2}{2\pi} \iint_{\sigma} \frac{h - h_p}{l_o^3} \Delta g d\sigma$$

$S(\psi)$ is Stokes function

and Δg is the reduced gravity used in GEOFOUR program (Forsberg 1993).

VI- PRACTICAL RESULTS

The remove-restore procedure and FFT were used to get the quasi-geoid estimate in the following steps :

1/ Removal of the long wavelength of the gravity data using the global geopotential model OSU91A complete to the degree and order 360 :

$$\Delta g_{GMred} = \Delta g - \Delta g_{GM}$$

2/ Residual terrain effect computation and subtraction of such an effect to produce residual gravity values :

$$\Delta g_{icred} = \Delta g - \Delta g_{GM} - \Delta g_{rtm}$$

3/ Gridding Δg_{icred} to produce a 5' x 5' regular grid of Δg_{grid} over the area $20 \leq \phi \leq 38$ $-7 \leq \lambda \leq 10$

4/ Fast Fourier Transform computation of ζ_{fft} involving Δg_{grid} .

5/ Computation of height anomalies and gravity anomalies of the reference field ζ_{GM} and Δg_{GM}

6/ Computation of the topographic effect on the height anomalies ζ_{rtm}

7/ Restore of the model and the residual terrain model effects on height anomalies :

$$\zeta = \zeta_{GM} + \zeta_{FFT} + \zeta_{RTM}$$

8/ Computation of Geoid heights N according to :

$$N = N_{GM} + N_{FFT} + N_{RTM}$$

V- Comments :

The DTM file covers only the northern Algeria $33 \leq \varphi \leq 38$ $-3 \leq \lambda \leq 9$ with blank spaces in the area $33 \leq \varphi \leq 35$ $4 \leq \lambda \leq 9$. We decided to split up the gravity data into 2 parts : northern part and southern part.

We included the area of missing heights with the later.

The terrain correction was computed for northern Algeria involving 1km DTM and for southern Algeria involving 10 km DTM.

The final results were merged into one file, then the data were gridded.

The computer package used for all the steps listed above is GRAVSOF (Forsberg 94 and Tscherning 94).

	Δg (mgal)	Δg_{GM} (mgal)	ζ_{FFT} (cm)	ζ_{RTM} (m)	ζ (m)	N (m)	$\zeta-N$ (dm)
mean	3.72	0.38			36.63	36.64	0.88
σ	26.71	2.27			8.90	8.93	1.15
min	-82.60	-95.56	-1.58	1.00	23.79	23.74	-0.77
max	165.50	128.91	7.65	3.27	57.01	57.21	6.29

	$\Delta g - \Delta g_{GM}$ (mgal)	Δg_{tcred} (Southern (mgal) area)	Δg_{tcred} (Northern (mgal) area)
mean	0.38	-7.14	-1.97
σ	12.27	14.45	8.00
min	-95.56	-124.21	-66.52
max	128.91	56.24	39.71

Geoid and Quasi-geoid computation statistics

Acknowledgements

The work was supported by the Geophysical Department of the University of Copenhagen.

We gratefully acknowledge Professor C.C.Tscherning and Senior Scientist R.Forsberg for their assistance, help and advices

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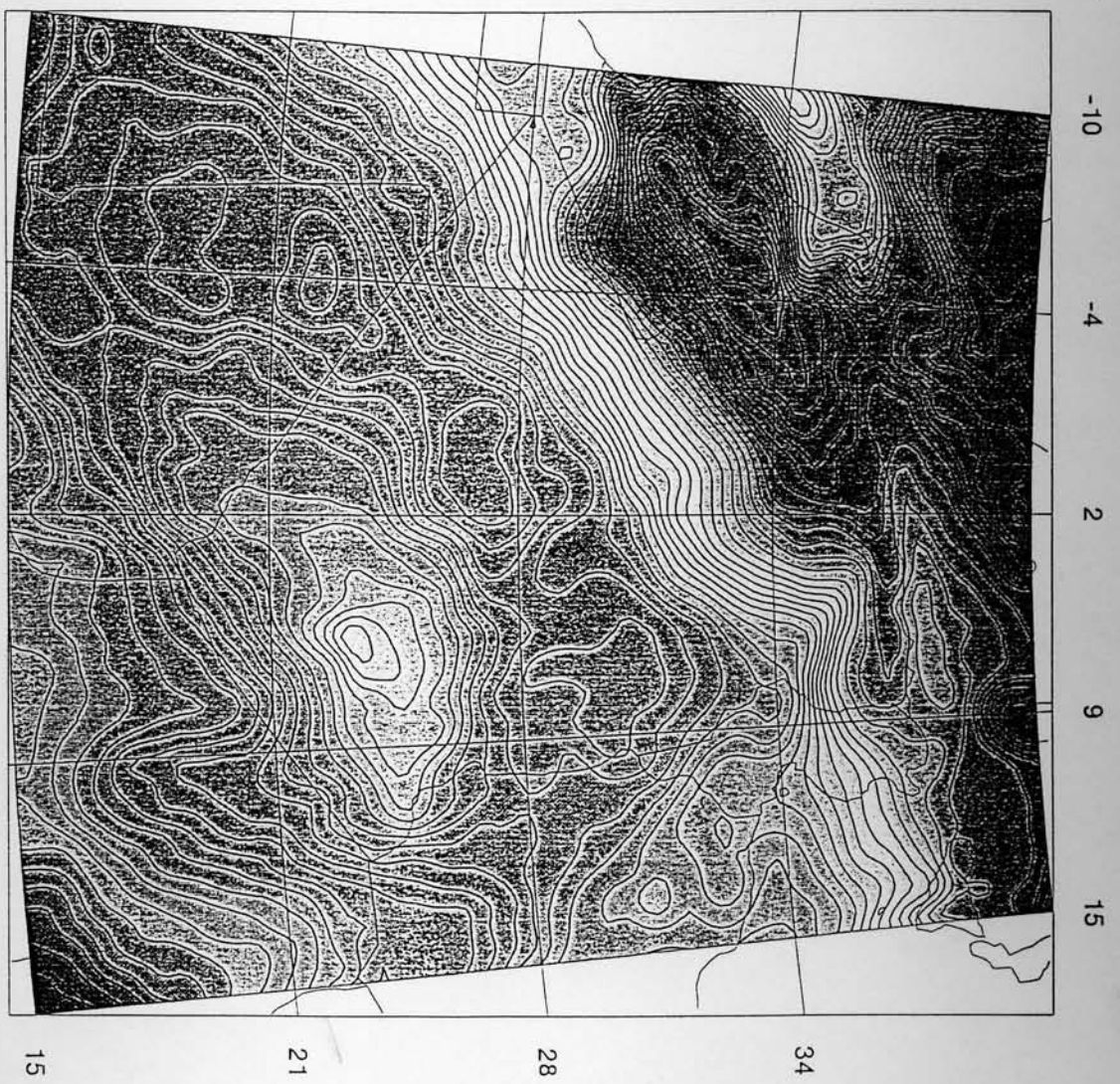
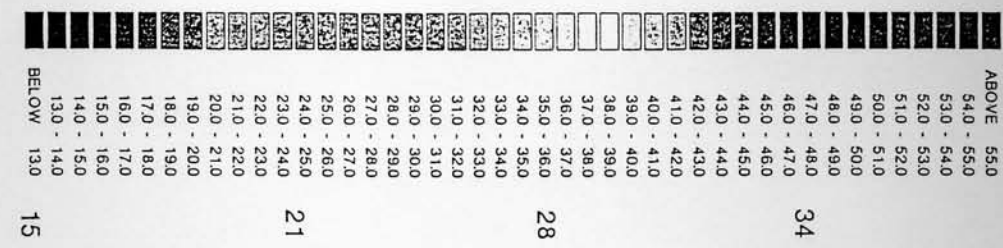
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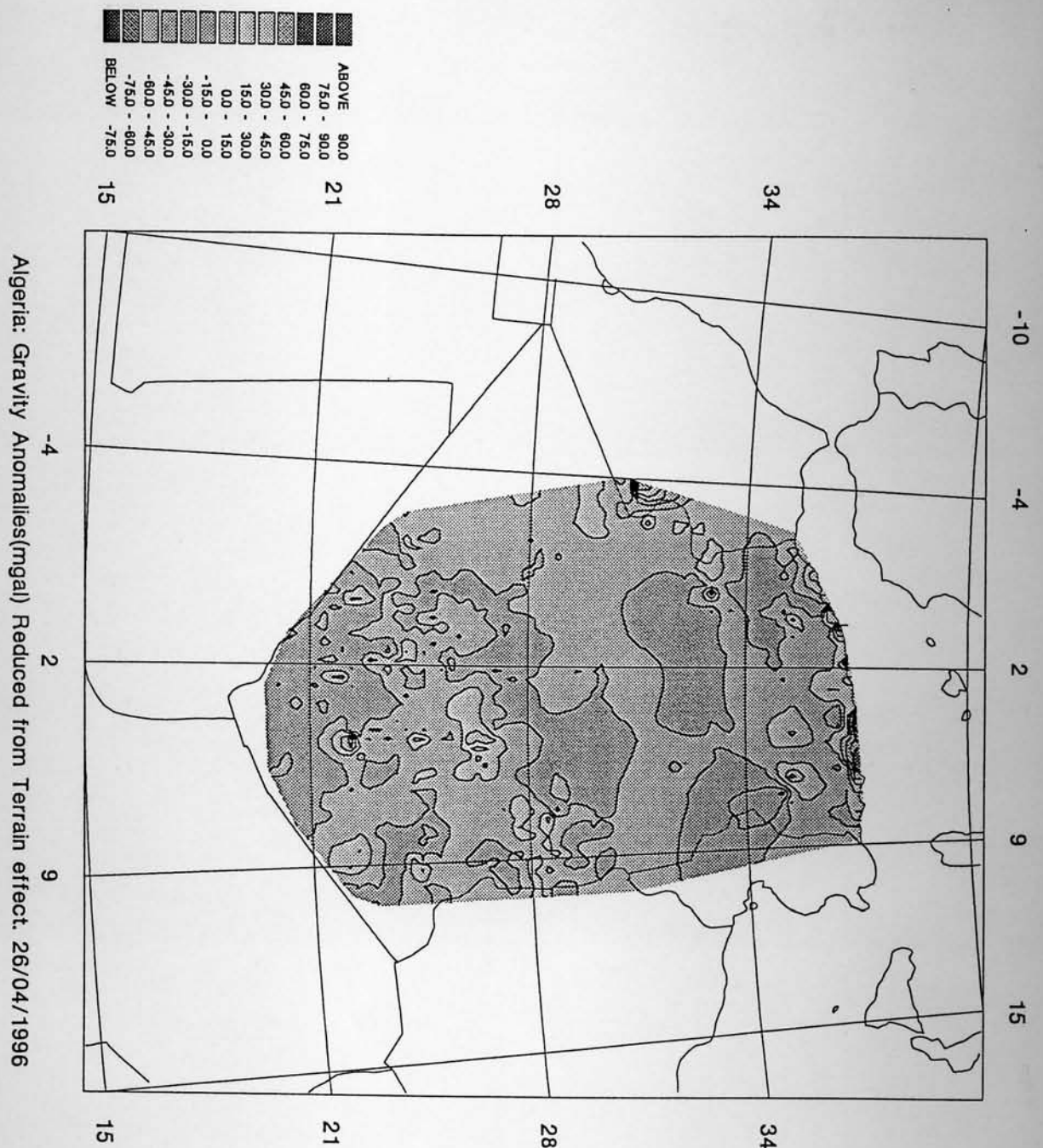
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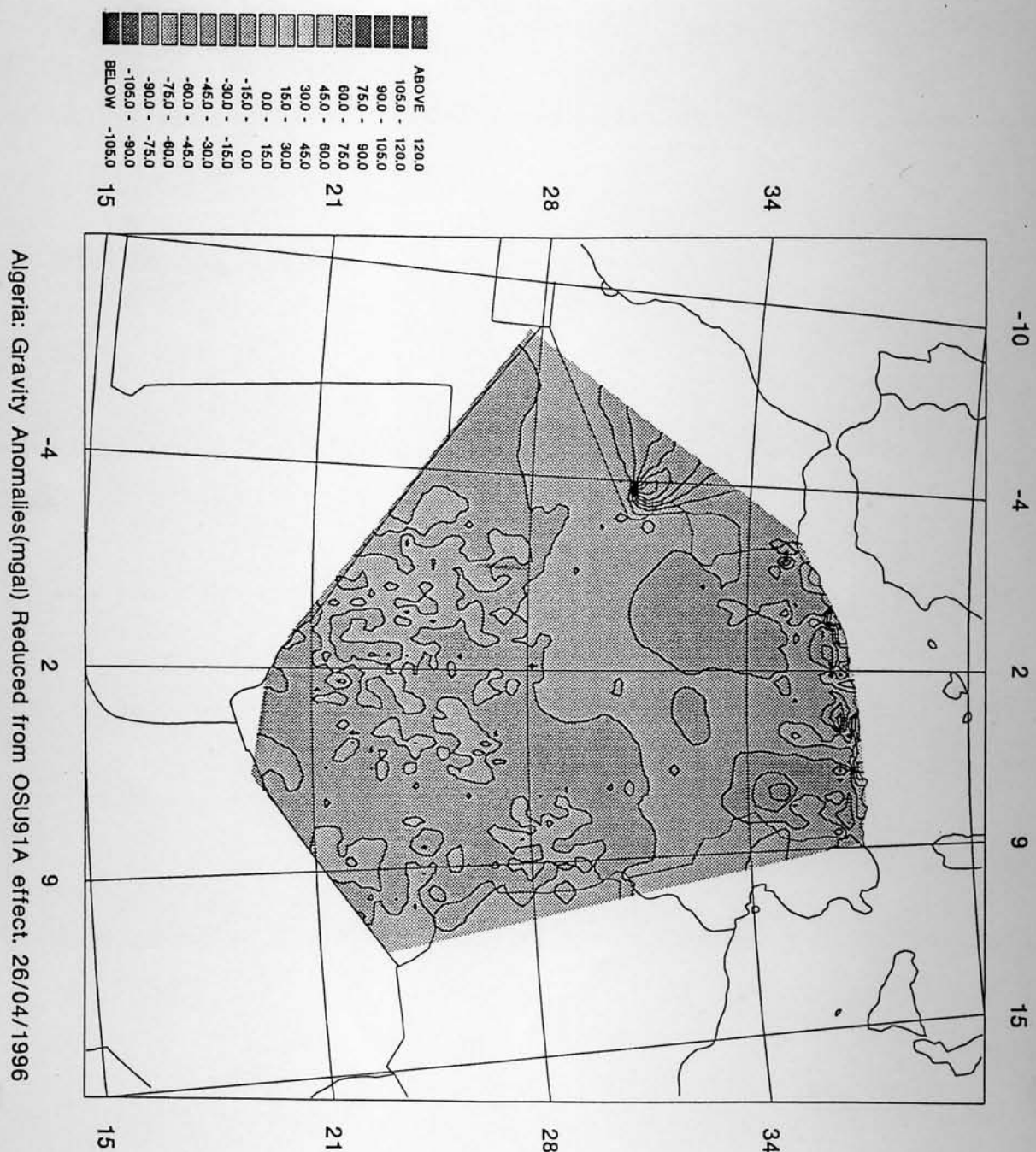
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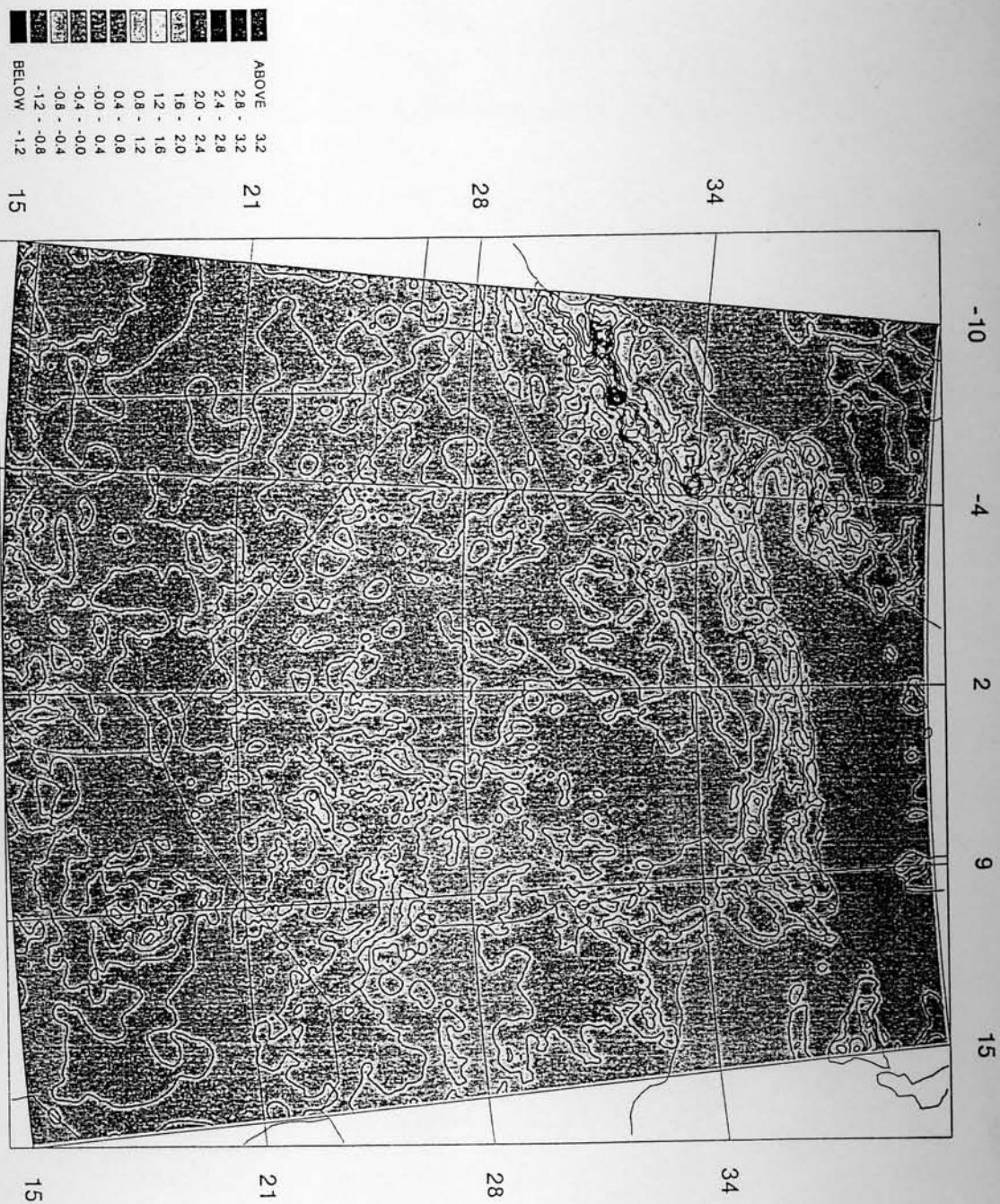


Algeria: OSU91A Geoid Heights(m) in a (15' x 15') grid 26/04/1996

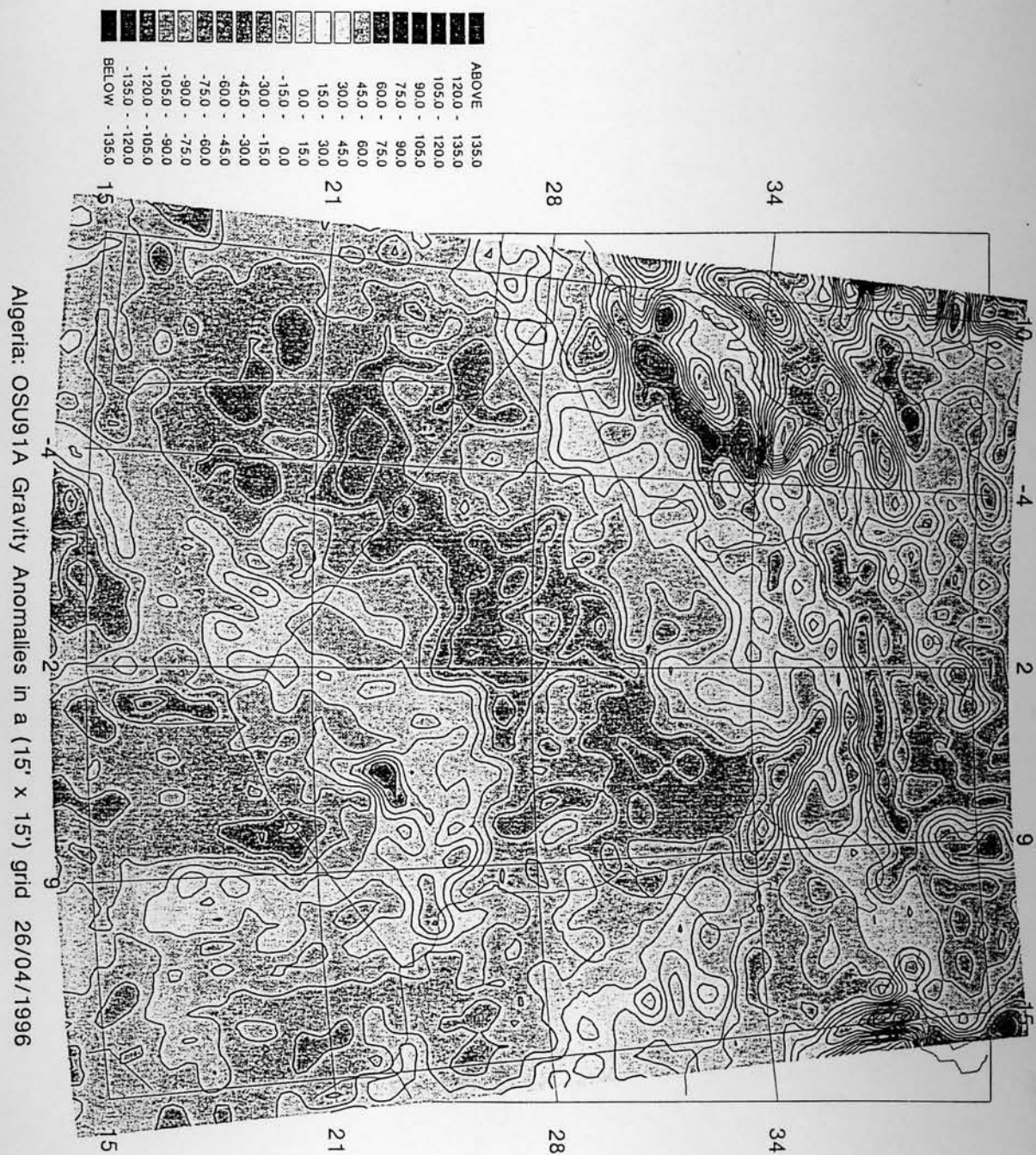


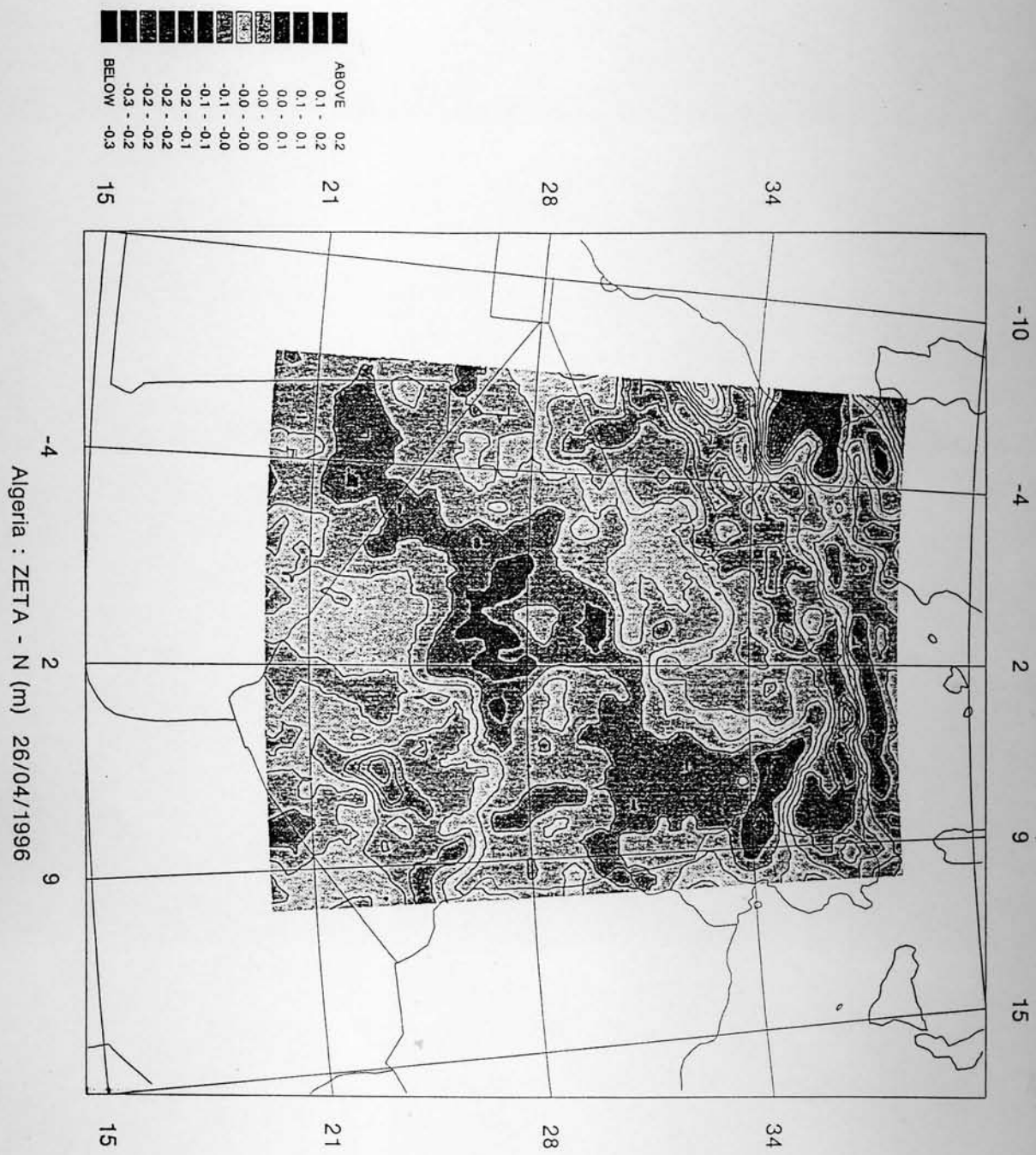
Algeria: Gravity Anomalies(mgal) Reduced from Terrain effect. 26/04/1996

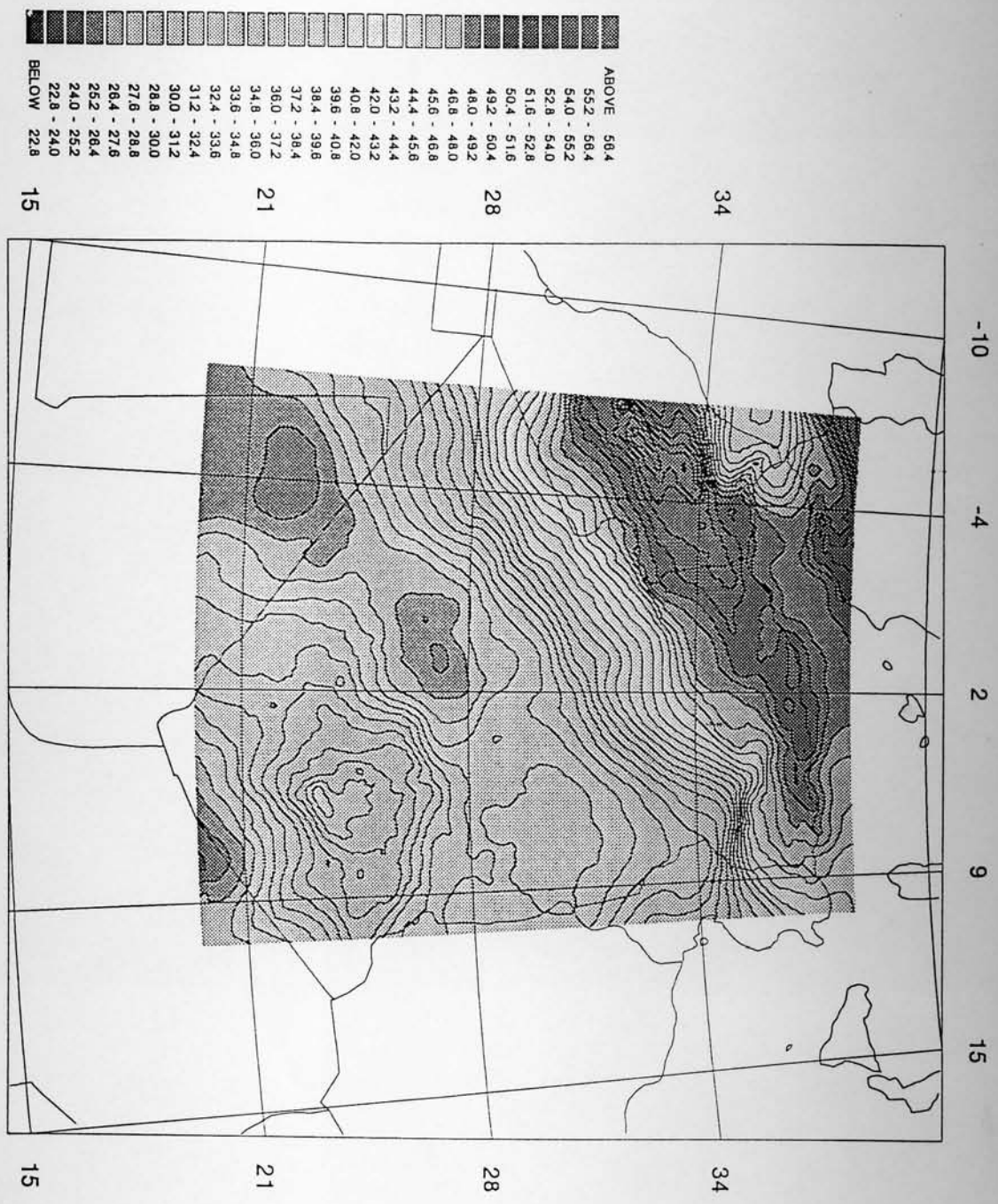




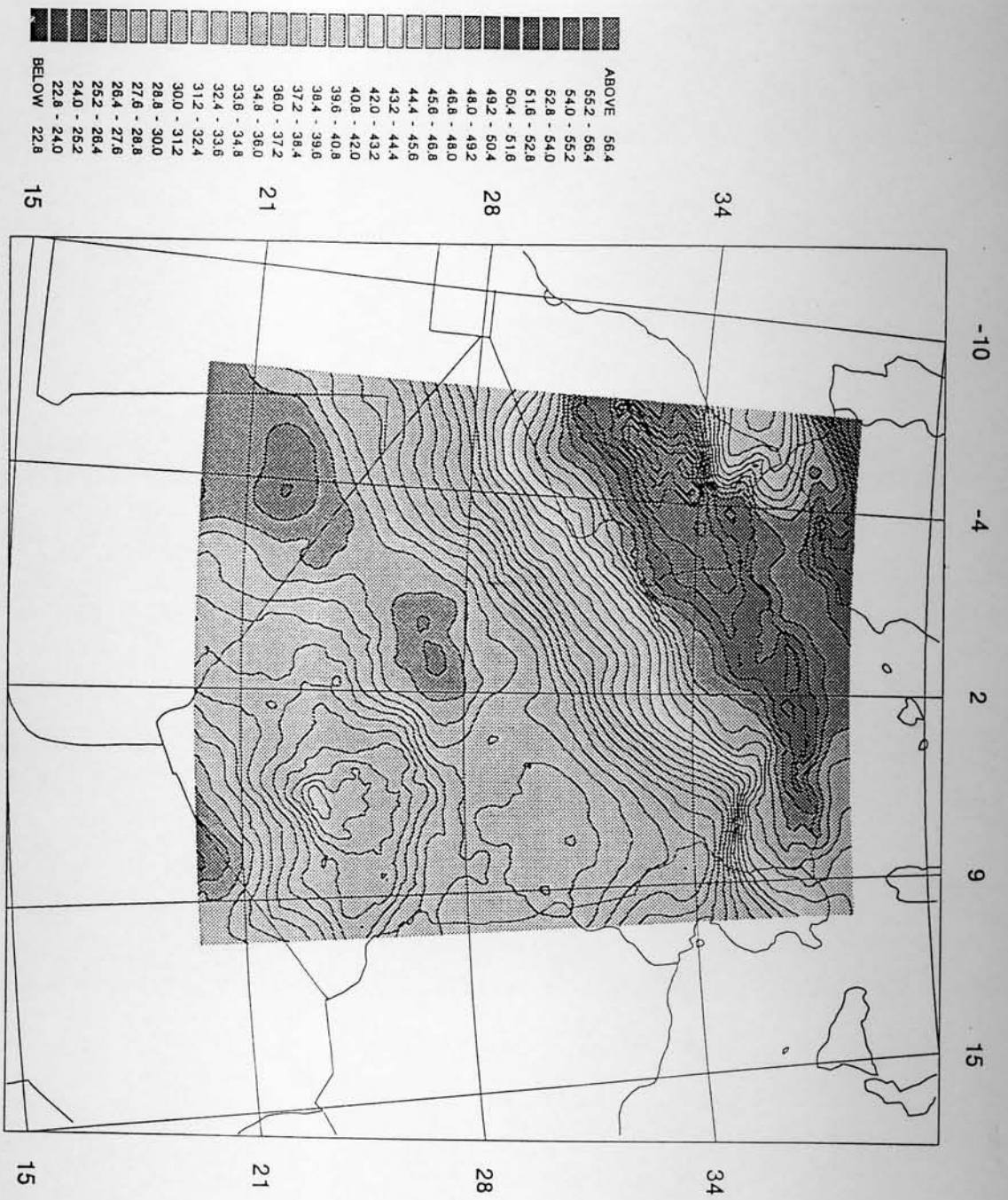
Algeria: RTM effect on Height Anomalies(dm)in a 5'x5' grid. 26/04/1996



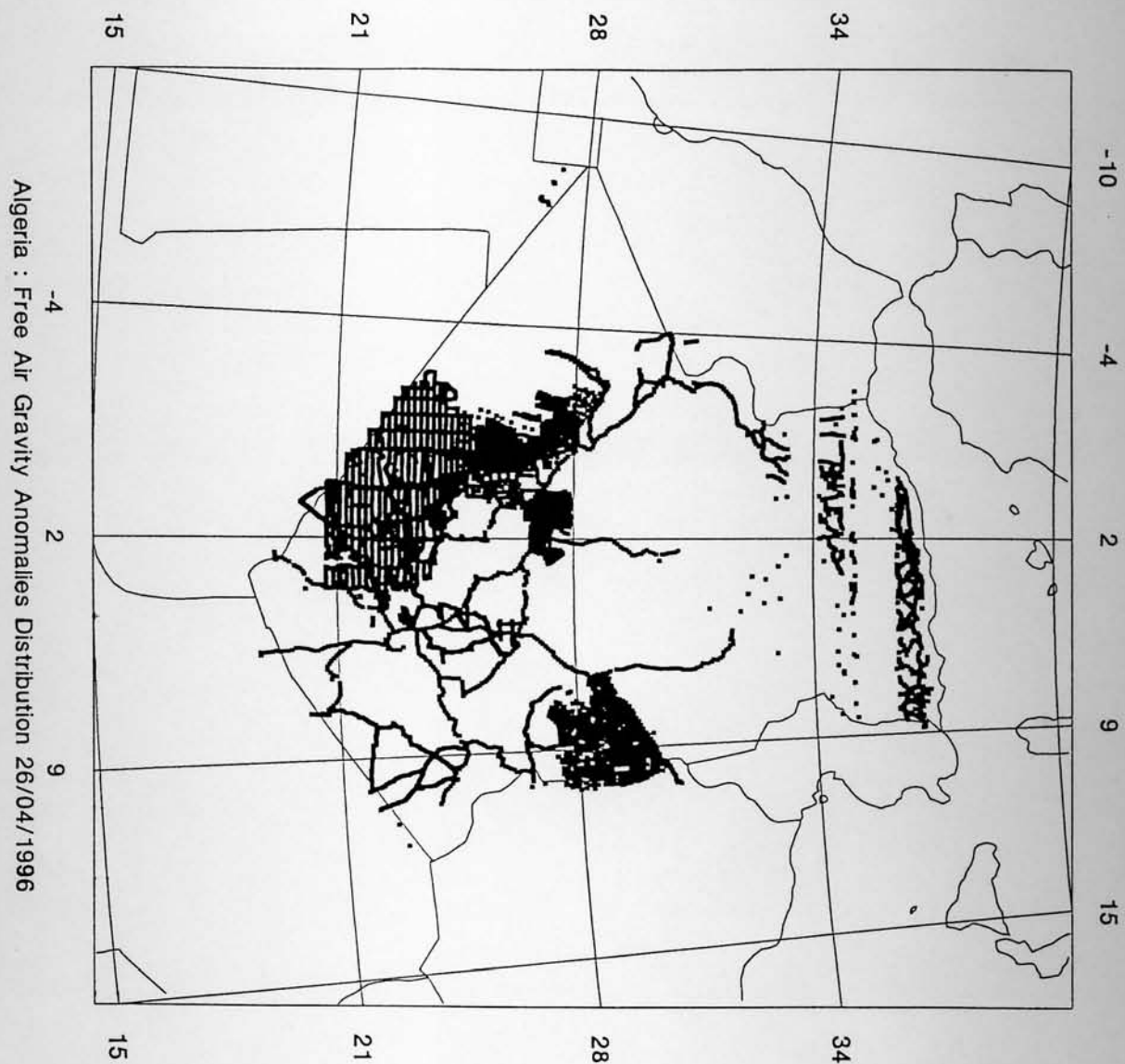




Algeria: FFT Solution for Height Anomalies(m). 26/04/1996



Algeria: FFT Solution for Geoid Heights(m). 26/04/1996



THE IGeS SPECIAL WORKING GROUP FOR THE EVALUATION OF THE NEW GSFC/DMA GEOPOTENTIAL MODELS

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ABSTRACT

This paper describes the activities of the Special Working Group (SWG) that was set up for testing the new GSFC/DMA geopotential models. The SWG is under the auspices of the International Geoid Service and reports its findings to the model development team through a world wide web home page maintained at the University of Calgary. Based on the test results reported by the SWG members, a final model has been selected and released to the public. It is of degree and order 360 and it is called EGM96. The SWG will conduct some further tests with EGM96 and all results will be published in an upcoming issue of the IGeS Bulletin.

1. INTRODUCTION

The U.S. Defense Mapping Agency (DMA) and the NASA Goddard Space Flight Centre (GSFC), based on an idea fostered by M. Kumar of DMA and D. Smith of NASA GSFC, have worked jointly towards developing, with the assistance of the Ohio State University and the U.S. Naval Surface Warfare Centre, an improved potential coefficient model complete to degree and order 360. The intent was to produce a global geoid with an absolute accuracy of ± 0.5 to ± 1.0 m. Besides this improved global geoid, the GSFC/DMA model will be used to support terrestrial and extraterrestrial scientific endeavors, to meet charting, mapping, navigation and orbit determination requirements, and to define a new world height system.

The new models benefit from the inclusion of new terrestrial and airborne gravity data, satellite altimetry data and proprietary DMA data. Due to different data selection, data weighting and estimation techniques, several different models were developed. The International Geoid Service (IGeS) agreed to provide leadership in evaluating these models. A Special Working Group (SWG) of international researchers from eighteen countries carried out extensive tests. Comparisons were made against existing geopotential models, and in terms of the models' use in predicting gravity anomalies, geoid undulations, satellite orbits, etc., using various marine, terrestrial, altimetric and satellite data sets. The findings were reported to NASA/DMA in order

to select the final model. Based on the SWG's contributions and on in-house tests by the development team, a final model, called EGM96, was selected and released to the public on September 1996.

2. SWG ESTABLISHMENT, MEETINGS AND MEMBERSHIP

A paper where the plans of NASA and DMA for a new, improved global model were discussed was presented at the meeting of the International Geoid Commission in Graz, in September of 1994 (Rapp and Nerem, 1994). A month later, during the International Geoid School for the Determination and Use of the Geoid in Milan, the idea of having an international group to test the new models was tossed around among the school teachers (R. Forsberg, R.H. Rapp, F. Sanso, M.G. Sideris and C.C. Tscherning). In January of 1995, I was officially given by IGeS the mandate to put together and chair an international evaluation group.

The SWG was fully established within a few months. A meeting between the SWG members and members of the model development team was held on July 13, 1995, during the IUGG General Assembly in Boulder. In this meeting, the evaluation procedures and future plans were discussed, including data sets, the area(s) and the types of the tests, any preliminary results of applying the proposed evaluation criteria to existing models, etc. Some SWG members also expressed interest in exchanging and sharing data sets. By the end of September, a home page was created at the University of Calgary for posting the results of the tests and any other relevant information as soon as it became available.

There are 19 regular members in the working group and 6 corresponding members, five of whom are from the model development team. Test results were also provided by a group of 16 contributing members, who worked either alone or in cooperation with the regular members. The list of members is given below.

Chair:

M.G. Sideris (Canada)

Members:

C. Merry	(South Africa)	A. Marchenko	(Ukraine)
G. Balmino	(France)	D. Milbert	(U.S.A.)
R. Barzaghi	(Italy)	Y. Neyman	(Russia)
D. Blitzkow	(Brazil)	J. Ries	(U.S.A.)

J.Y. Chen	(P.R. China)	J. Segawa	(Japan)
H. Denker	(Germany)	Z. Sima	(Czech Republic)
R. Forsberg	(Denmark)	H. Sünkel	(Austria)
A.H.W. Kearsley	(Australia)	C.C. Tscherning	(Denmark)
A. Mainville	(Canada)	I.N. Tziavos	(Greece)
C. Boucher	(France)		

Corresponding members:

S.C. Kenyon	(U.S.A)	R.S. Nerem	(U.S.A.)
N.K. Pavlis	(U.S.A)	R.H. Rapp	(U.S.A.)
R.D. Salman	(U.S.A)	F. Sansó	(Italy)

Contributing members:

D. Arabelos	(Greece)	J. Kirby	(Australia)
M. Bursa	(Czech Republic)	J. Li	(P.R. China)
D. Chao	(P.R. China)	J. Ning	(P.R. China)
L. Duhem	(France)	M. Pearse	(New Zealand)
W. Featherstone	(Australia)	D.A. Smith	(U.S.A.)
Z. Jiang	(France)	M. Veronneau	(Canada)
R. Biancale	(France)	V. Byvshev	(Russia)
Y. Kuroishi	(Japan)	N. Kuehtreiber	(Austria)

3. TEST MODELS AND EVALUATION SCHEME

The models provided to the SWG for testing are listed below. All are of degree and order 360, however, the EGM models contain proprietary DMA and other data that had not been available before.

- EGM-X01
- EGM-X02
- EGM-X03
- EGM-X04
- EGM-X05
- JGM3_OSU91A

- OSU91A

The first one, EGM-X01, was a preliminary model provided after many delays in December 1995 for the purpose of setting up software and test procedures. The major models to be evaluated were the following four, i.e. EGM-X02 to EGM-X05, and were made available to the SWG in April 1996. OSU91A and the hybrid model JGM_OSU91A were provided as the standard models against which the new models should be compared.

The list of data sets that was proposed to be used to compare and evaluate the GSFC/DMA models is as follows:

- GPS-leveling campaigns over large areas, such as the long traverses measured on leveling benchmarks in Europe, U.S.A., Canada and Australia;
- currently available regional geoid solutions, such as the European geoid, the Canadian geoid, the American geoid, the Mediterranean geoid, etc.;
- astronomical deflections of the vertical;
- satellite altimetry data;
- ground gradiometric data sets;
- gravity data not included in the models, including absolute gravity measurements;
- satellite tracking data; and
- airborne gravity data.

4. INTERNATIONAL TESTS AND THE NEW EGM96 MODEL

As it has been stated before, the four models EGM-X02 to EGM-X05 were provided to the SWG in April 1996. They were, and still are, accessible via anonymous ftp from *ftp.ensu.ucalgary.ca* in the */pub/SWG* directory. Because NASA/DMA requested that all tests should have been completed by the middle of July 1996, it was decided to scrap the idea of submitting a written report in favour of placing the results on an Internet server as they became available for immediate access by all interested parties. The URL address of the SWG's home page is <http://www.ucalgary.ca/~sideris/SWG/SWG.html>.

Since all reports submitted by the SWG members are available via the Internet, no results will be reported herein. Interested colleagues are encouraged to visit the SWG's home page to read the individual reports. In general, it can be said that the new models outperformed OSU91A, JGM3_OSU91A and every other global model they were tested against. This was evident not only in the standard geoid undulation tests but also in tests involving satellite orbits, satellite altimetry and airborne gravity. As expected, the most significant improvements over the older

models were obtained in areas where data were not previously available, such as Greenland, South America, China, Russia, the Arctic, part of the oceans, etc.

A summary of the results was presented at the Joint Meeting of the International Gravity Commission and the International Geoid Commission in Tokyo, Sept. 25 to October 5, 1996 (Sideris, 1996). In the same meeting, details on the development of these and the final EGM96 model were presented in papers by Kenyon and Pavlis (1996), Lemoine et al. (1996) and Pavlis et al. (1996). Right before the Tokyo meeting, this final EGM96 model was selected and made available for free distribution. It can also be obtained from the ftp and URL addresses mentioned above.

5. FUTURE WORK FOR THE SPECIAL WORKING GROUP

After the release of EGM96, it was decided to continue the work of the SWG with further tests that would include the final model. In addition, all test results will be published in their entirety in the next issue of the IGeS Bulletin, early 1997, which has been dedicated to the publication of the reports of the SWG members. More specifically, the following items have been identified for completion:

- Test EGM96 against the other available geopotential models.
- Revise individual reports to include test results from EGM96.
- Rewrite individual reports in journal paper format for publication in an upcoming dedicated issue of the IGeS Bulletin (February or March 1997).
- Update the SWG's home page with the new test results and access to the new model.

6. SUMMARY AND ACKNOWLEDGEMENTS

After two years of continuous involvement, the work of the SWG is near completion. We were all glad to see the new models outperform the older ones in virtually all types of tests and gratified that our work helped NASA and DMA select a final model for public release. After updating the test results with the inclusion of EGM96, the SWG's work will come to an end with the publication of the evaluation reports in the upcoming issue of the IGeS Bulletin. This was a joint effort, made possible only through the international cooperation and voluntary involvement of colleagues from around the world.

I wish to thank all regular and contributing members of the SWG for their interest and hard work on testing the models and providing the results. NASA, DMA, Nikos Pavlis and all other

members of the model development team are thank for making the models available and providing the necessary information. Fernando Sansó and IGeS are thanked for providing the vehicle for these tests. Last but not least, many thanks are due to Dick Rapp for acting as the bridge between the evaluation and the development groups, for his initiatives, and for his invaluable contributions to global gravity field modelling in general.

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Toward the Physical Interpretation of the Geoid in the Pannonian Basin Using 3-D Model of the Lithosphere¹

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A lot of efforts has been made to build up the initial version of a 3D crustal model for the Pannonian basin in Hungary to support the investigation of the regional gravity field. This model contains geometrical and physical parameters of the main structural elements of the crust arranged into a system of joining right rectangular prisms the optimized number of which is approximately 10.000 in the recent resolution of the model. It starts at the mean sea level, describes the geometry and density distribution of thick sediments above the crystalline basement, involves a density contrast surface inside the crust and extends down to the material of the upper mantle beneath the Mohorovičić discontinuity.

Based on this model, a synthetic reproduction of the geoid in the Pannonian basin was attempted with unexpected success. By spectral combination of the global geoid computed from OSU89B global geopotential model with the potential surfaces generated by the main structural elements of the crust a so called "crustal geoid" was determined. It was compared point by point to the HGQ91B national gravimetric quasigeoid solution and a surprising agreement — bias: +0.08 m, S.D.: ± 0.22 m — was detected.

Introduction

Possessing a 3-D finite element model of the crust based on data which are produced by different kinds of geological-geophysical exploration, basically in-

¹The research outlined in the paper is supported by National Scientific Research Foundation, Contract No. F014284

dependent from the Earth's gravity field, a new dimension opens for the quantitative interpretation of the gravity data and all the gravity related quantities (deflection of vertical, geoid undulations, gravity gradients) measured on the ground.

For 3-D forward gravity field computations, a density distribution model of the investigated area must be available. Because of basin evolution processes (McKenzie, 1978) the crust of the Pannonian basin has a strongly layered structure showing generally sharp boundaries between rocks having different lithological features. Therefore the model of the density distribution may be built up in two steps. In the first step the geometry of the layered structure should be determined to provide a geometrical frame for physical parameters in 3-D space. This frame can be realized using finite elements in a suitable form (e.g. right rectangular prisms) for computations. Then volume density values have to be applied in the model to generate masses the gravitational effect of which can be computed using rigorous or approximate formulae.

In Hungary a regional model of the lithosphere based on a system of right rectangular prisms is being developed continuously (Kalmár et al., 1995) because more and more geological and geophysical data concerning the crustal structure of the Pannonian basin are available. This is a consequence of significant efforts that have been recently made in the field of the evaluation of old and also new data and their organization into data bases by different institutions (Kovácsvölgyi, 1994; Sárhidai, 1994).

The regional model has been already used successfully e.g. to interpret the unusually moderate Bouguer gravity anomaly depression of the Little Hungarian Plain sub-basin (Northwest Hungary) inconsistent with the extreme structure of the sediments and the basement topography (Papp and Kalmár, 1995). It seems, that this "inconsistent behavior" of the gravity field has general validity for the deep (6–7 km) sub-basins of the Pannonian basin and therefore extensive investigations based on local 2-D and 3-D density models has been started recently as well (Nemesi and Stomfai, 1994, Kovácsvölgyi, 1994).

There are also some efforts (Geiger et al., 1990) e.g. to involve crustal pa-

rameters or models of significant density anomalies in the regularization of the gravity field in order to help the determination of the geoid. Even the application of digital terrain models, that is taking into account directly the gravitational effect of topographical masses in the process of geoid determination applying the so called remove-restore technics has become a standard procedure in the last years.

The purpose of the present study is to show that a realistic composition of a local geoid using the combination of a global geopotential model and a 3-D crustal model is possible and this result suggests a new deal in the evaluation of local geoid solutions based on surface gravity data. Furthermore the process of composition helps the geophysical interpretation of regional/local features of the geoid so it increases the scientific significance of the determination and use of the geoid.

Overview of the crustal structure in the Pannonian basin

Since this small piece of the whole surface of the globe "suffers" from intensive geoscientific exploration relative to its extent therefore the amount of textbooks, conference proceedings, papers and reports which concern the details of the structure of the Pannonian basin from different aspects is enormous. It is not possible to refer all of them but a general overview was given recently in the field of basin analysis by Royden and Horváth (1991) and by Cloetingh et al. (1993).

To give a rough idea about the main features and structure of the basin a short description is given in the following.

The surface topography of the Pannonian basin is smooth and flat (Table 1) relative to the surrounding areas. It is located on the boundary between the African and Eurasian plates inside the arc of Eastern-Alps, Carpathians and Dinarides which are chains of high mountains with rugged topography (Fig. 1). This relatively smooth terrain has a moderate Bouguer gravity anomaly field (Table 2) indicating a seemingly normal and uninteresting crustal structure.

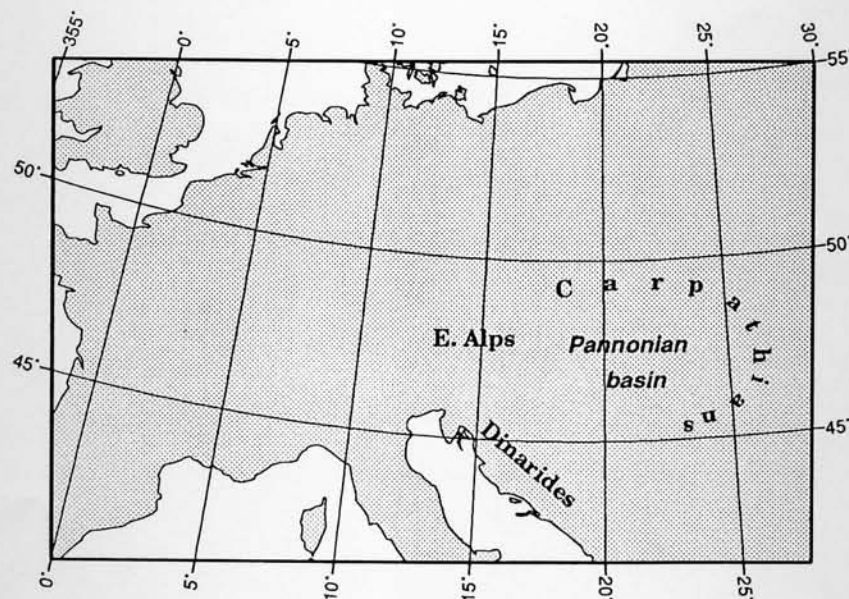


Figure 1. The Pannonian basin and its surroundings.

Large part of the basin is covered by Neogene-Quaternary sediments and there are only a few places inside the basin where the crystalline basement rocks outcrop. The soil on the surface mainly consists of clay, sand and sandstone. However, the results of borehole drillings and (deep) seismic soundings show that the structure of the crust has extreme characteristics.

Table I. Statistics of topographical elevations in Hungary derived from the 500 by 500 m Digital Terrain Model of Hungary

No. of data	min. [m]	max. [m]	mean [m]	S.D. [m]
463169	+68	+983	+155	±91

The mean thickness of the sediments is almost 2 km and it lies between 0 and 8 km (Papp and Kalmár, 1995). The borehole measurements show significant compaction of the sedimentary rocks and a typical density-to-depth function is given in Fig. 2. The weighted average density contrast is -460 kg/m^3 so the presence of the sediments represents a significant lack of mass relative to the 2750 kg/m^3 average density (Bielik, 1991) of the upper crust.

Table II. Statistics of Bouguer gravity anomalies derived from the data base of Eötvös Loránd Geophysical Institute of the Geological Survey of Hungary using 2670 kg/m^3 volume density in Bouguer reduction

No. of data	min. [mgal]	max. [mgal]	mean [mgal]	S.D. [mgal]
147974	-35.4	+29.6	-2.1	± 10.0

Having a look at the map of the Mohorovičić discontinuity beneath Central Europe (Fig. 3) it is obvious that the material of the upper mantle is in extremely elevated position beneath the basin. The average thickness of the crust beneath the continents is approximately 33 km whereas beneath the Pannonian basin it is only 26 km. Concerning the supposed $+500 \text{ kg/m}^3$ density contrast the mantle material in elevated position represents a mass surplus.

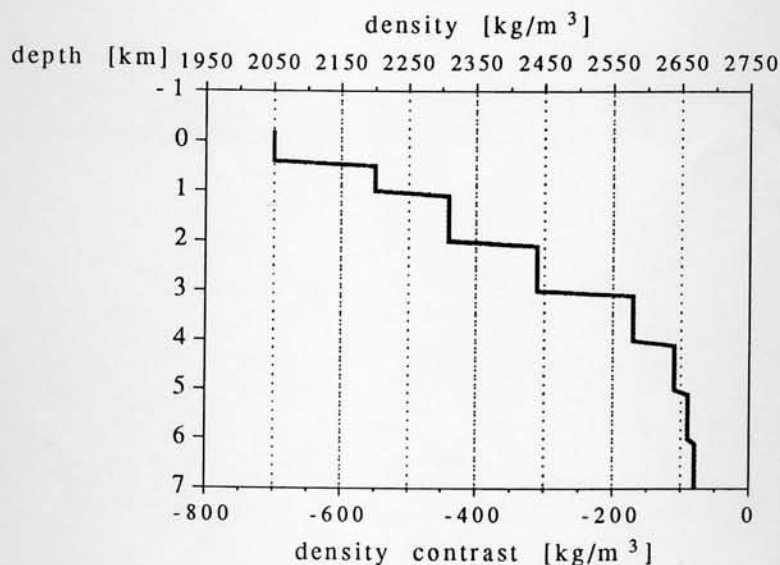


Figure 2. Bielik's density model for the Pannonian basin.

Unfortunately there is only a few data about the density distribution inside the crust but a density interface in the depth range of 10–15 km is indicated in some of the deep seismic sounding records (Posgay et al., 1986). At some places the geometry of this interface seems to be parallel to the geometry of Moho

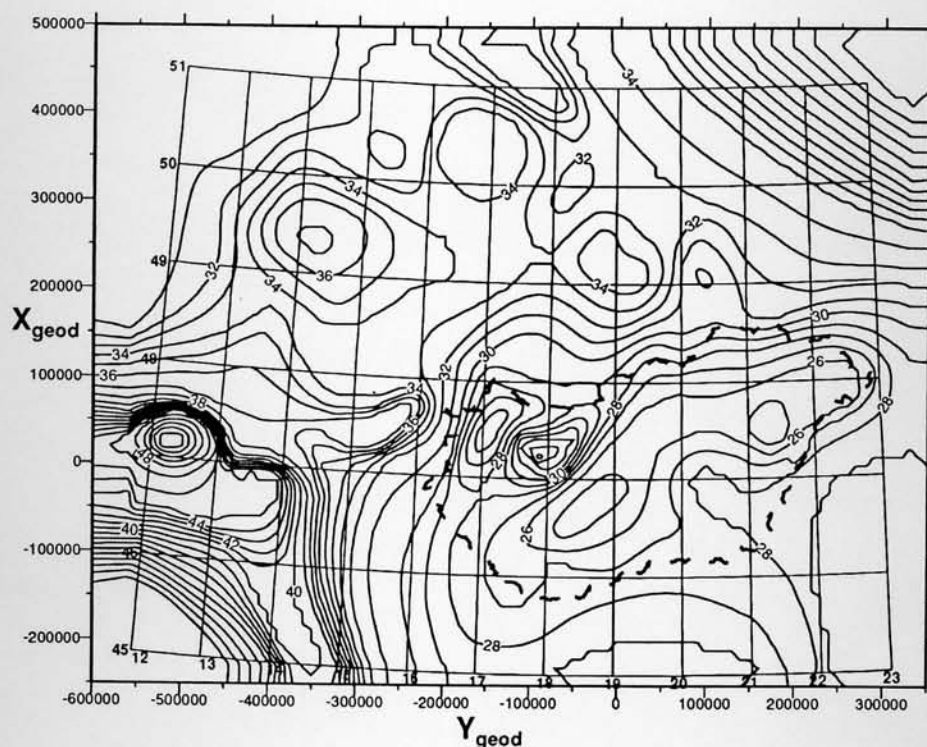


Figure 3. Contour line map of the Moho reconstructed from its DTM. Contour interval is 1 km. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Dashed line represents the state border of Hungary.

and probably the density jump of approximately $+300 \text{ kg/m}^3$ is above all responsible for the systematic discrepancies between Bouguer gravity anomalies and those gravity anomalies which can be computed from the combined model of the sediments and the Moho (Kovácsvölgyi, 1994; Papp and Kalmár, 1995).

These extreme conditions inside the lithosphere are resulted in by basin evolution processes which are connected to dynamical interaction of tectonic plates. The thinning of the crust and the mantle intrusion is a consequence of the horizontal extension of the basin while the cooling of hot mantle material in elevated position caused the subsidence of the basement rocks (Sclater et al., 1980).

In spite of these facts the moderate Bouguer gravity field indicates balanced overall conditions of the mass distribution and it is interesting to see how

this moderate field even the smooth geoid is being built up from individually extreme sources.

3-D model of the lithosphere in the Pannonian basin

The description of the first version of the model is given by Kalmár et al. (1995). Since it has been continuously updated, the present version has to be described. It has the following elements and characteristics.

The prism system modelling the density distribution inside the sediments consists of only 7713 joining prisms due to an optimization method (Kalmár et al., 1995). An elementary approach may result in approx. 90000 prisms on the same area using 2×2 km grid spacing. The model covers 141536 km^2 (Fig. 4). The vertical change of density is accounted for according to Table 3.

Table III. Bielík's density model for the sediments of Pannonian basin

depth interval		density	density contrast*
[m]		[kg/m ³]	[kg/m ³]
0	500	2050	-700
500	1000	2200	-550
1000	2000	2310	-440
2000	3000	2440	-310
3000	4000	2580	-170
4000	5000	2640	-110
5000	6000	2660	-90
6000	7000	2670	-80

The model of the mantle material beneath the Mohorovičić discontinuity covers a larger area than in the previous version. It consists of 1821 prisms on a square of $950 \times 750 \text{ km}$ (Fig. 3). Different maps (Lillie et al., 1994; Horváth, 1993) were used to extend the initial model but obviously because of the discrepancies between the maps applied the solution is not perfect. A constant

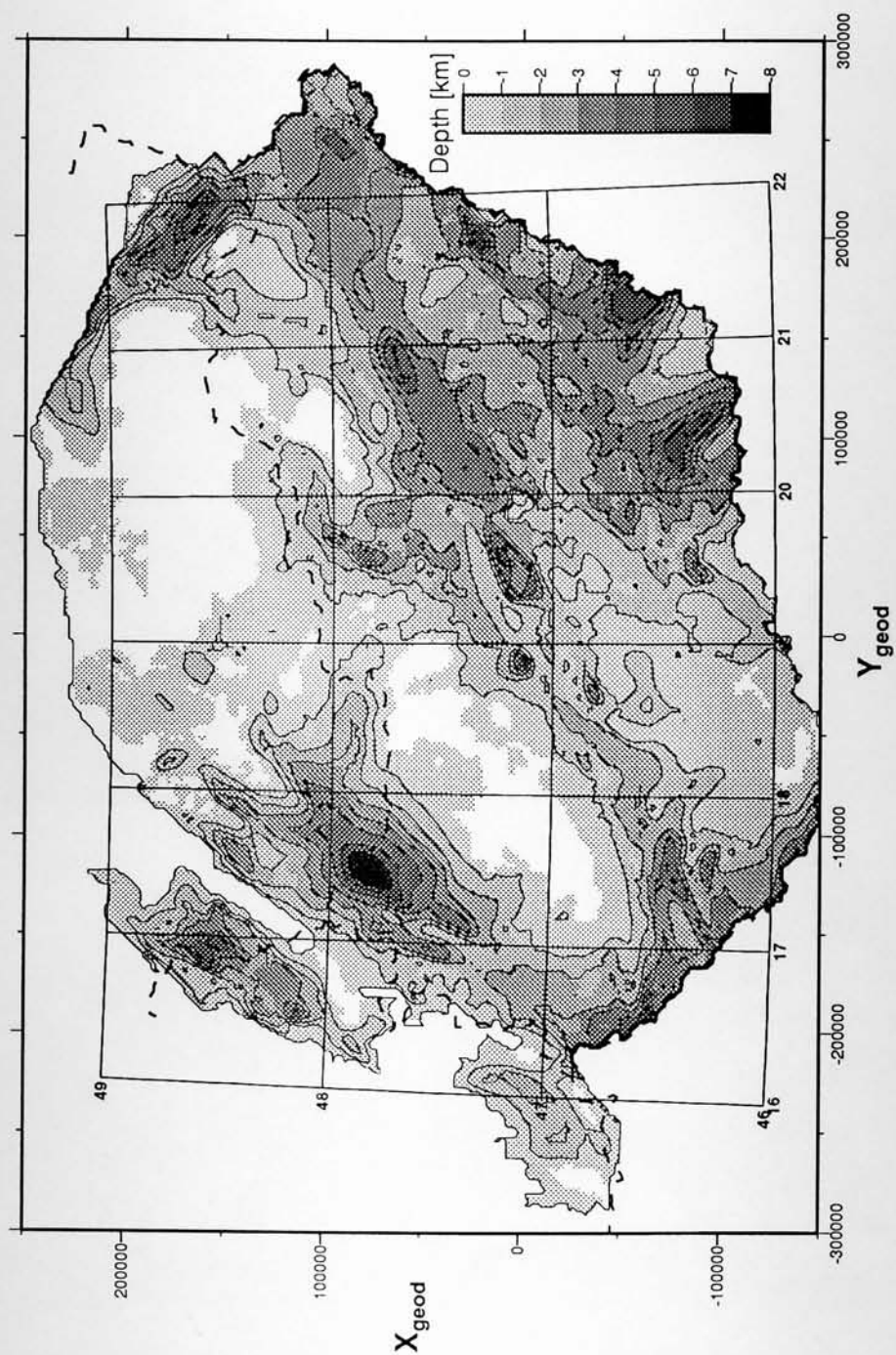


Figure 4. Contour line map of the pre-Tertiary basement combined with grey scale coding. Contour interval is 1 km. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees.

density jump of $+500 \text{ kg/m}^3$ was supposed on the Moho which results in 3250 kg/m^3 density for the upper mantle.

The most fictive and unreliable element of the lithosphere model is the model of the lower crust. As it was mentioned previously there are some seismic indications of a velocity jump inside the crust between depths of 10–15 km. Therefore an intuitive model was developed using the constrains that (1), the density jump must be located approximately in the depth range above (2), the density jump is $+300 \text{ kg/m}^3$ on the interface and (3), the gravitational effect of the model should be equal to the long wavelength discrepancies between the observed Bouguer gravity anomalies and the model anomalies obtained from the combined effect of the sediments and the Moho. Such way a rough system of prisms from 78 elements (Fig. 5) was built up the gravitational effect of which approximates well the Bouguer residuals in the wavelength range above 50 km with a transition band of 50 km ($\lambda_{\text{cutoff}} = 50 \text{ km}$; $\lambda_{\text{pass}} = 100 \text{ km}$).

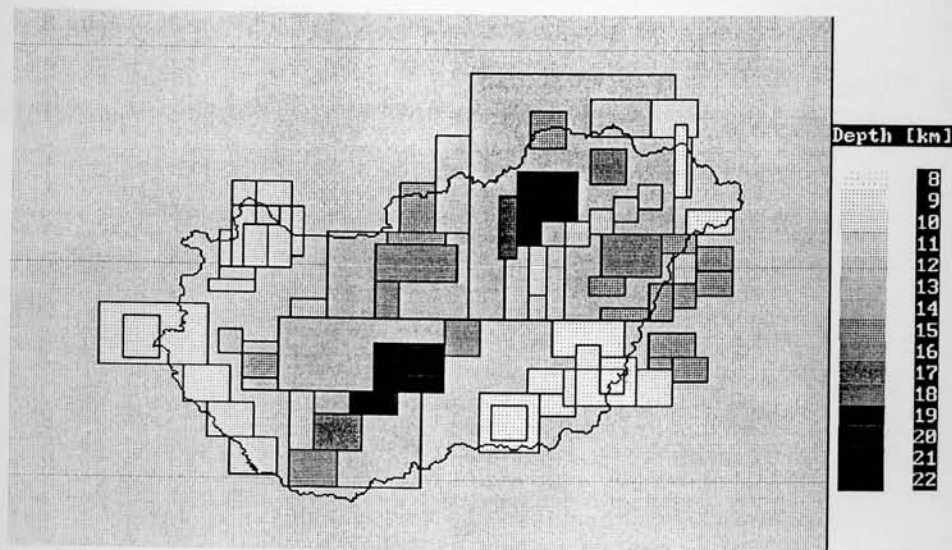


Figure 5. Horizontal arrangement of the prism system modelling the lower crust. Gray scale codes locate the depth of the interface between the upper and the lower mantle. The irregular line represents the state border of Hungary.

This model was used to reproduce short wavelength features of the Hungarian geoid solution HGQ91B (Fig. 6) which was derived by the application of

Meissl-Ostach truncation method of global geopotential coefficients (Kenyeres, 1992).

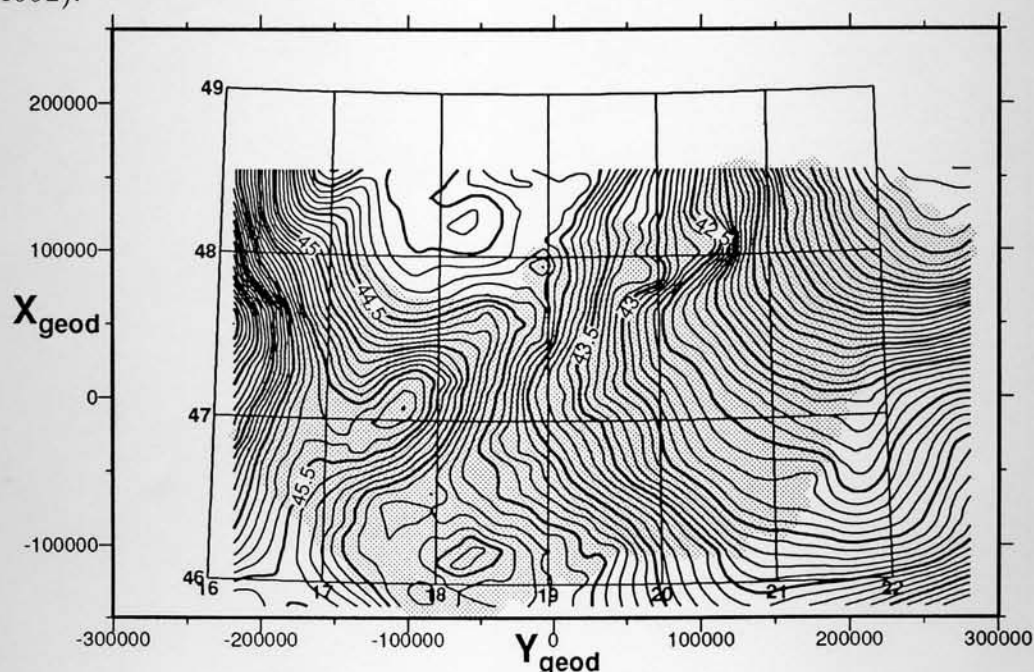


Figure 6. Contour line map of the HGQ91B geoid. Shaded area represents the territory of Hungary. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees.

Composition of the “crustal geoid”

Both the gravity field and the potential of the lithosphere model were computed on 10 by 10 km grids at $z=0$ m level, that is at the top plane of the model using the algorithm by Nagy (1988). The reference/normal potential U_{ref} was assumed to be zero in the horizontal extension of the investigated area therefore the disturbing potential T in Bruns' formula (Eq. 1) was replaced by the total potential V_{model} generated by the density distribution model to obtain its geometrical representation.

$$N = \frac{T}{\gamma} = \frac{V_{\text{model}}}{\gamma} \quad (1)$$

where N - the undulation of the potential surface relative to $U_{\text{ref}} = 0$
 γ - the normal gravity

Thus the non-spherical terms ($J_2, J_4, \dots, J_{2n}$) of the reference potential were neglected what is a satisfactory approximation concerning the small area of the Pannonian basin. This approximation results in a "planar geoid" with an R.M.S. of ± 2.936 m and causes a strong tilt in north-south direction ($\frac{\partial N}{\partial x} = -1.6314 \pm 0.0010$ m/km and $\frac{\partial N}{\partial y} = -0.0114 \pm 0.0008$ m/km) and a bias ($N_o = -2046.78 \pm 0.14$ m) of the geoid on the area investigated if it is used in global geoid computation (that is the coefficients of the reference potential are not subtracted from the corresponding coefficients of the global potential). Therefore the long wavelength or systematic features of the geoid surface cannot be explained by this 3-D model computation.

Since the model of the lithosphere is limited in space therefore a global geopotential model OSU89B was used to provide long wavelength information for the composition. However, a method had to be found to carry out the spectral merging of potential surfaces generated by different sources to avoid spectral distortions.

Assuming the isotropy of the potential fields the radial power spectrum (a 1D average of the 2D spectrum) of the OSU and the HGQ91B geoids were computed by FFT for this purpose using windowing and zero padding technics by the application of GMT utilities (Wessel and Smith, 1991) (Fig. 7 top). On the one hand it is obvious that the OSU89B solution does not contain wavelength information below 100 km (Rapp and Pavlis, 1990) and probably the coefficients having wavelength close to this λ_{Nyquist} are distorted by spectral folding (Papp, 1993). On the other hand the reliability of the long wavelength components in the power spectrum of HGQ91B geoid solution and also in the spectral contribution of the potential surface generated by the lithosphere model is low because of their limited horizontal extension. Therefore a transition band had to be determined in the two spectra from the lower limit λ_{min} of which the spectrum of HGQ91B geoid is valid alone down to $\lambda = 0$ and from the upper limit λ_{max} of which the spectrum of the OSU geoid is valid alone up to $\lambda = \infty$. From Fig. 7 (bottom) a realistic transition band can be identified above 200 km. Below the wavelength of 200 km the power in the spectrum of OSU89B geoid is systematically greater than the power in the spectrum of

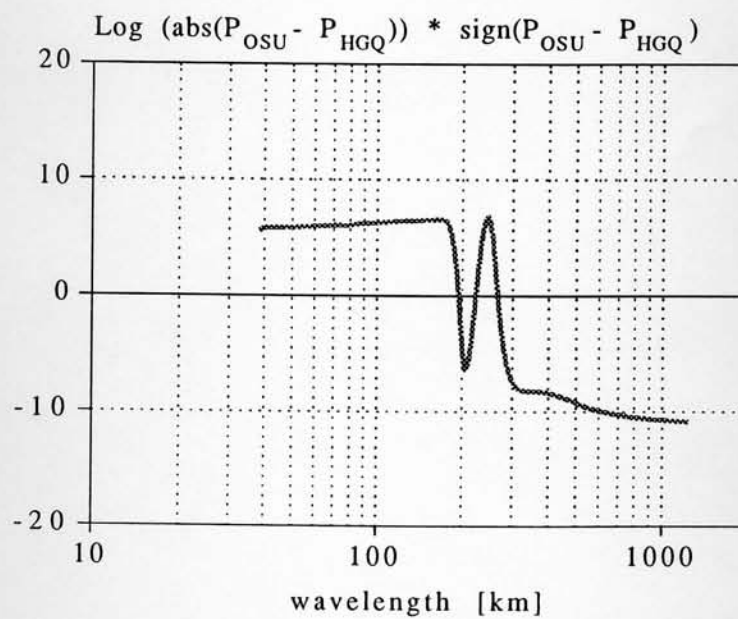
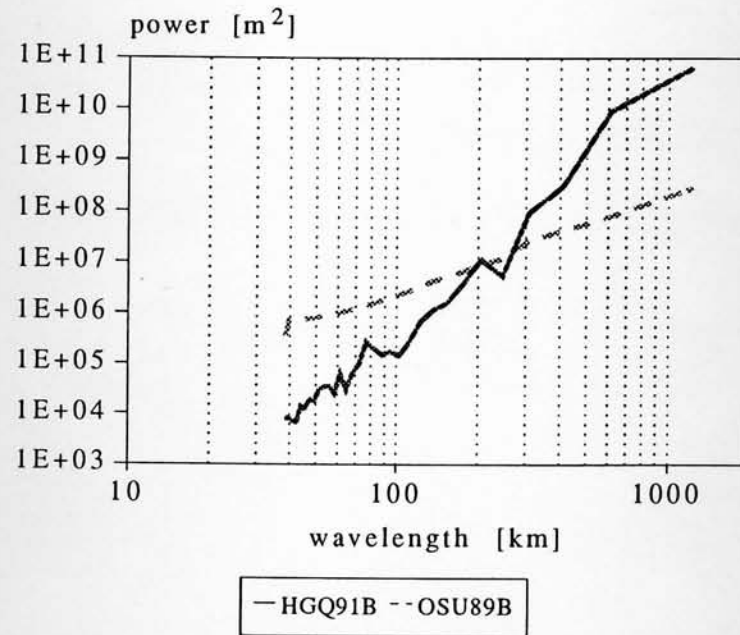


Figure 7. top) Radial power spectra of HGQ91B and OSU89B geoids. bottom) Difference of the two spectra.

HGQ91B. Above 300 km wavelength an opposite situation can be observed whereas between 200 km and 300 km the sign of the difference differs from wavelength to wavelength. Therefore the transition band was defined between 200 km and 300 km. It means that a low-pass filter with $\lambda_{\text{cutoff}} = 200$ km and $\lambda_{\text{pass}} = 300$ km was applied to filter out high frequency power from the OSU geoid and a high-pass filter with $\lambda_{\text{cutoff}} = 300$ km and $\lambda_{\text{pass}} = 200$ km was applied to cancel low frequency power from the potential surfaces generated by the components of the crustal model. In the transition band cosine tapering was used to insure gradual change of the power to decrease Gibbs' phenomena (Brigham, 1974). The transfer function of the low-pass filter is seen in Fig. 8. Before filtering of potential surfaces produced by the components of the lithosphere model the least-squares planes of the surfaces were removed also by using GMT utilities.

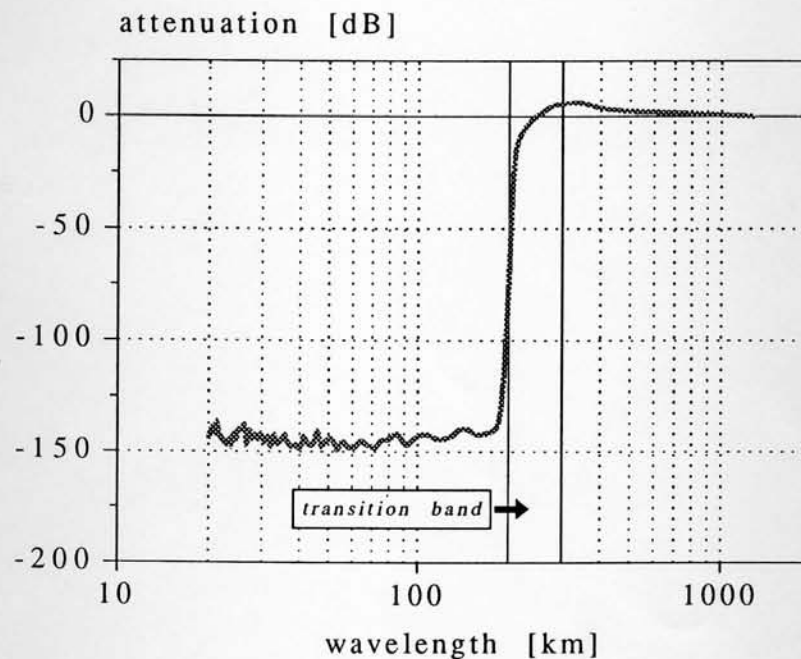


Figure 8. Transfer function of the low-pass filter used in computations.

From Figs 9 to 12 one can see the modifications of the potential surfaces caused by the filtering procedure. It is very interesting to see how the am-

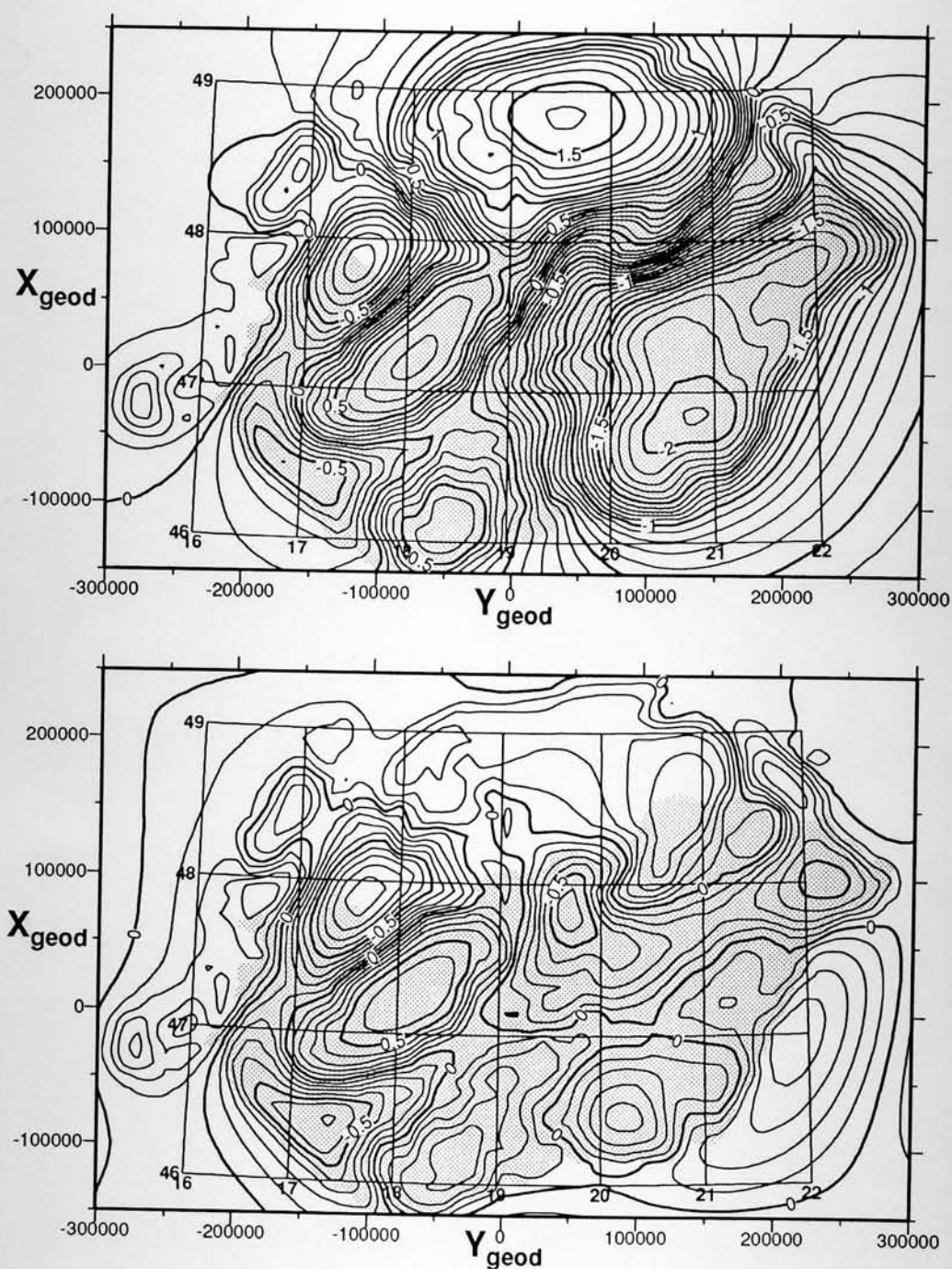


Figure 9. top) Undulation contribution of the Neogene-Quaternary sediments. bottom) Its short wavelength undulation contribution. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

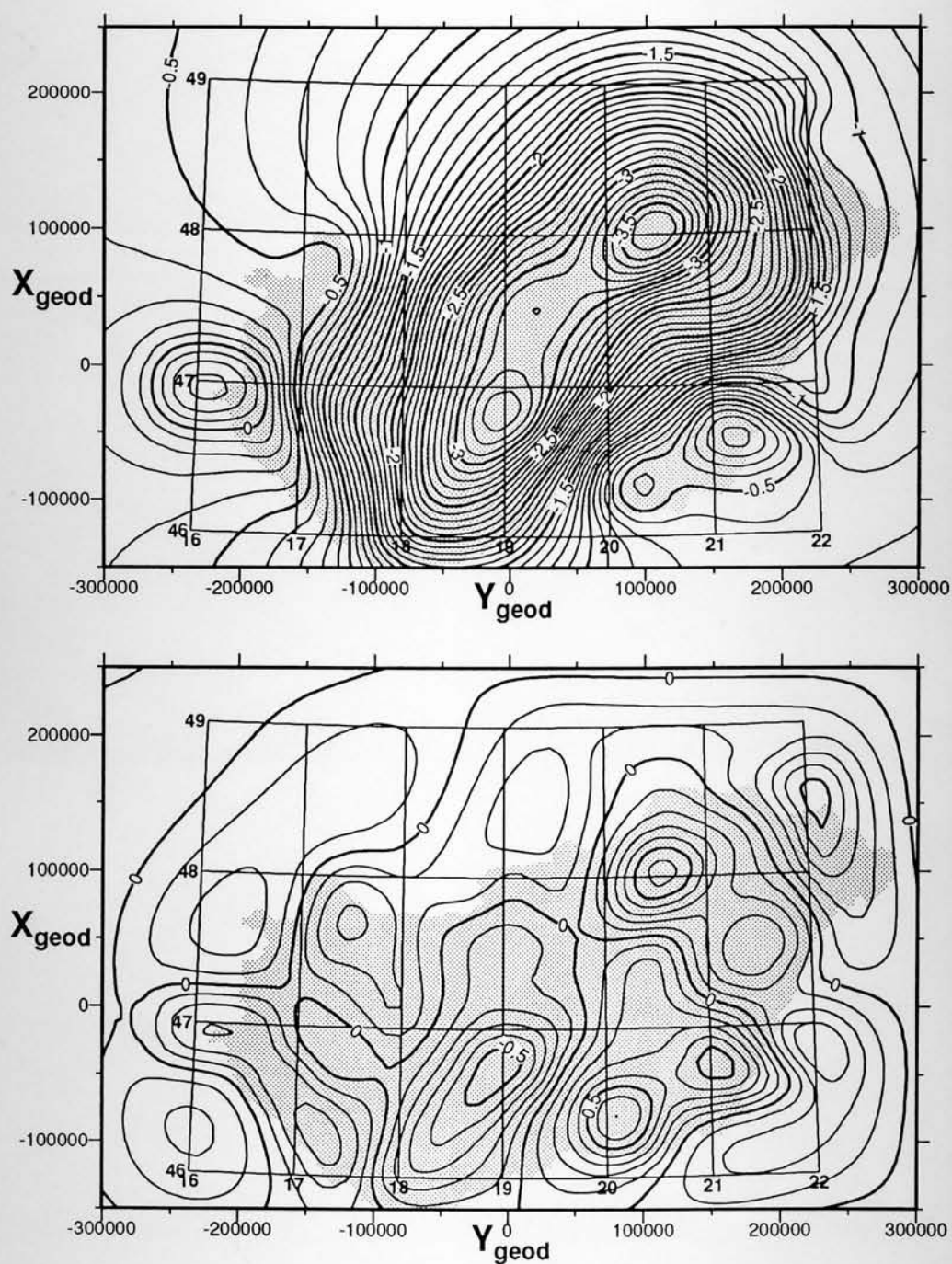


Figure 10. top) Undulation contribution of the lower crust. bottom) Its short wavelength undulation contribution. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

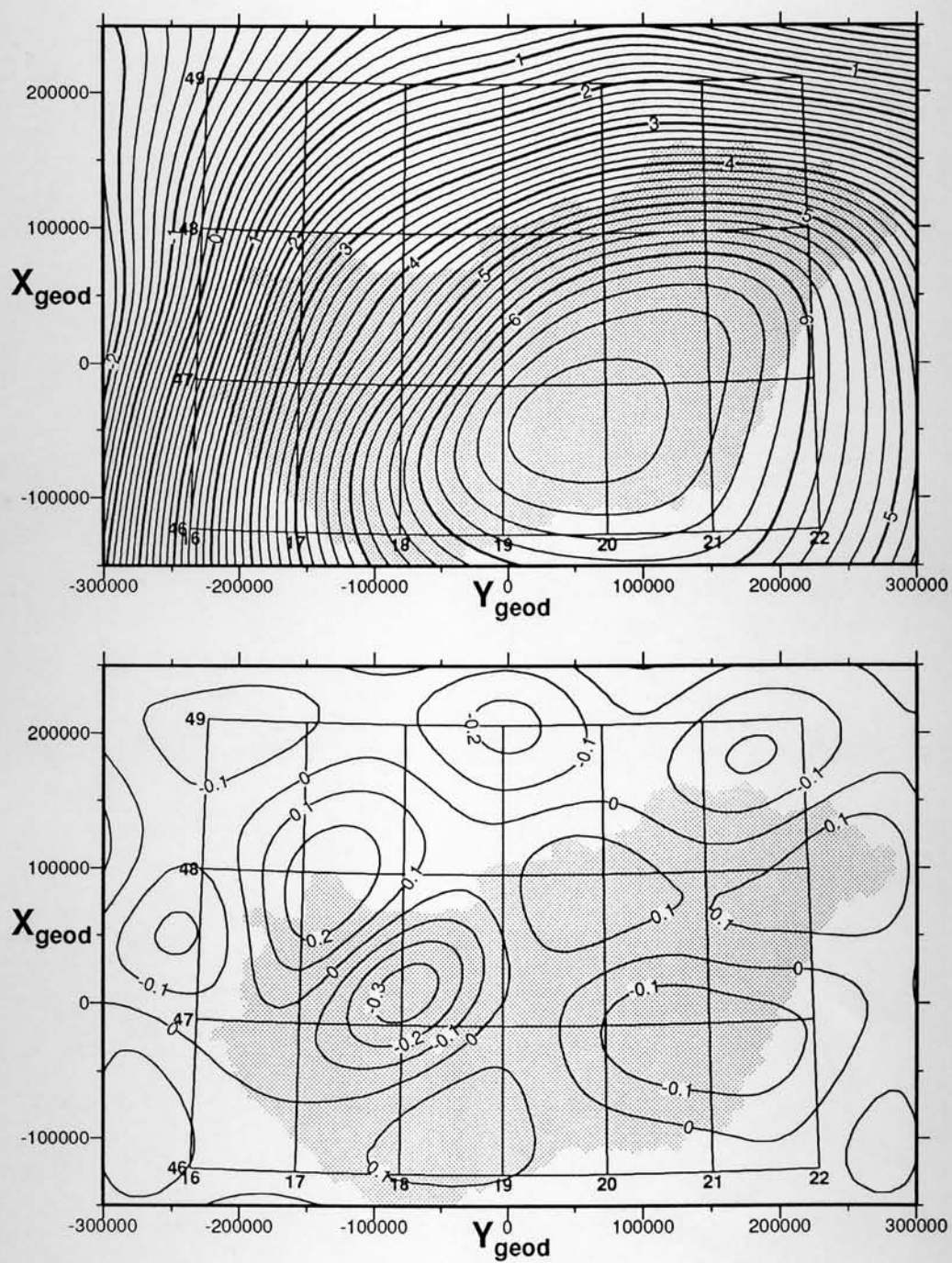


Figure 11. top) Undulation contribution of the upper mantle. bottom) Its short wavelength undulation contribution. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

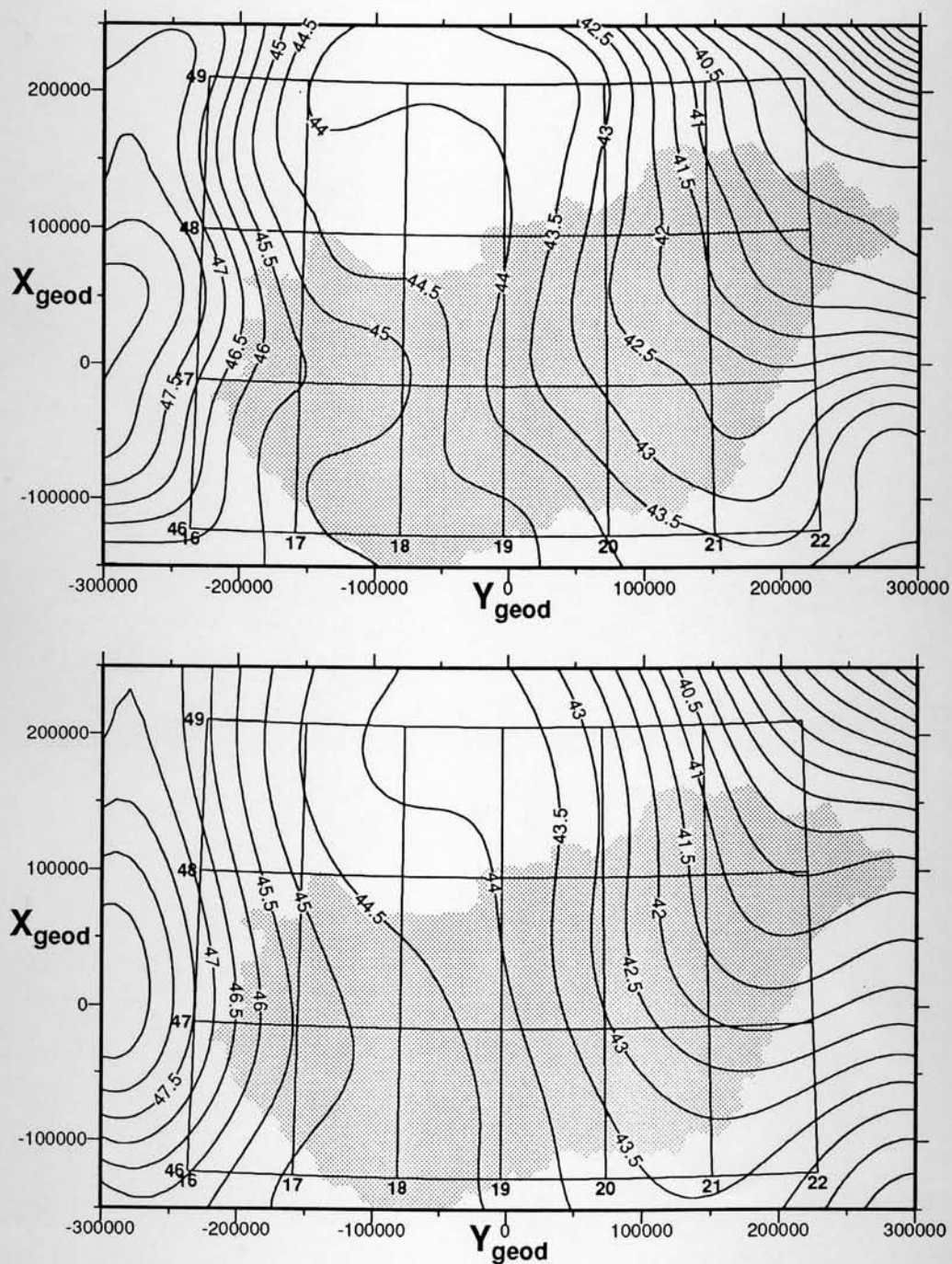


Figure 12. top) OSU89B geoid ($n,m=360$). bottom) Its low-pass filtered picture. Contour interval is 0.5 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

plitude and the horizontal gradients of short wavelength undulations decrease by the source depth that is by the depth of density inhomogeneity generating undulation of potential surface. It is obvious from this investigation that the relative contributions of lithospheric sources to the short wavelength undulations ($\lambda < 300$ km) of the geoid are — one by one — significant and 1–1.5 m differences of geoid heights are produced. However, the total sum of these potential surfaces (Fig. 13) shows a decrease of undulations (the maximum height difference is about 1 m) indicating that concerning the gravity field the evolution of the Pannonian basin is at a stage of balanced conditions.

Summing up the filtered potential surfaces and the OSU89B geoid the “crustal geoid” was obtained (Fig. 14).

Comparison of HGQ91B geoid with the “crustal geoid”

From a theoretical point of view the two solutions cannot be the same because HGQ91B geoid was produced using the combination of OSU89B coefficients and surface Free-air gravity anomalies which obviously contain the effect of topographical masses, whereas the “crustal geoid” may be — by definition — a “Bouguer geoid” in a classical sense consisting of undulations generated by density inhomogeneities being only below the Mean Sea Level. For statistical comparison the two grids were subtracted point by point by Eq. (2)

$$\Delta N = N_{\text{HGQ91B}} - N_{\text{“crustal geoid”}} . \quad (2)$$

Fig. 16 shows the areal distribution of residual undulations whereas its histogram can be seen in Fig. 15. The histogram was made up from grid values computed by Eq. (2) located inside the state border of Hungary to diminish edge effects caused by the prism model being finite. Obviously the reliability of HGQ91B undulations also is questionable outside the state border because of data availability problems. In order to indicate that the application of the density model of the lithosphere really results in improved sets of differences

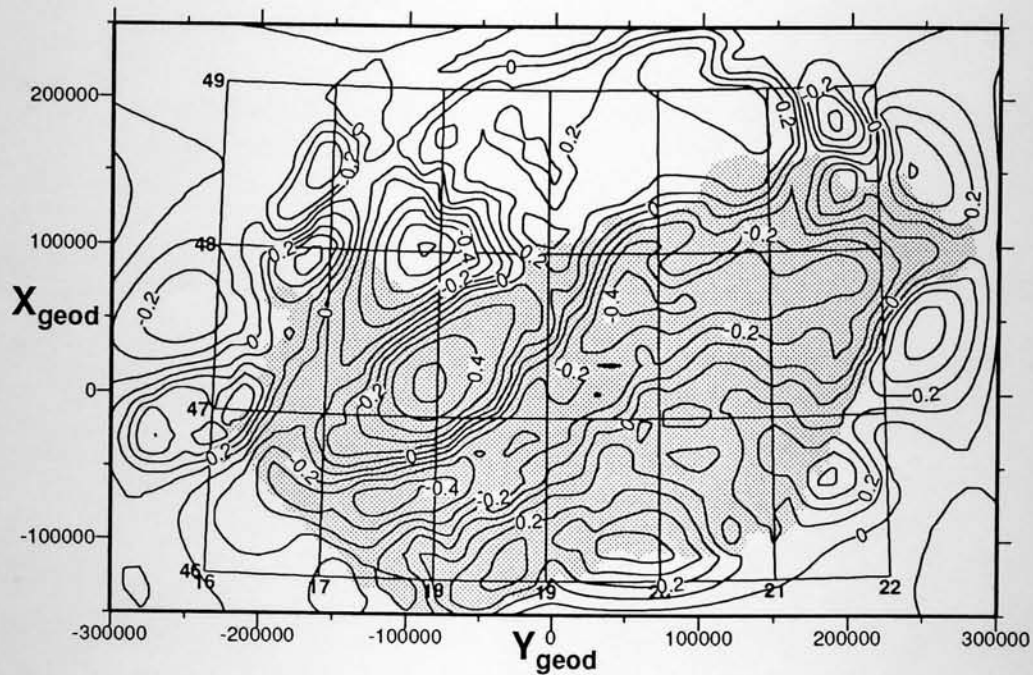


Figure 13. Short wavelength undulation contribution of the whole lithosphere model. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

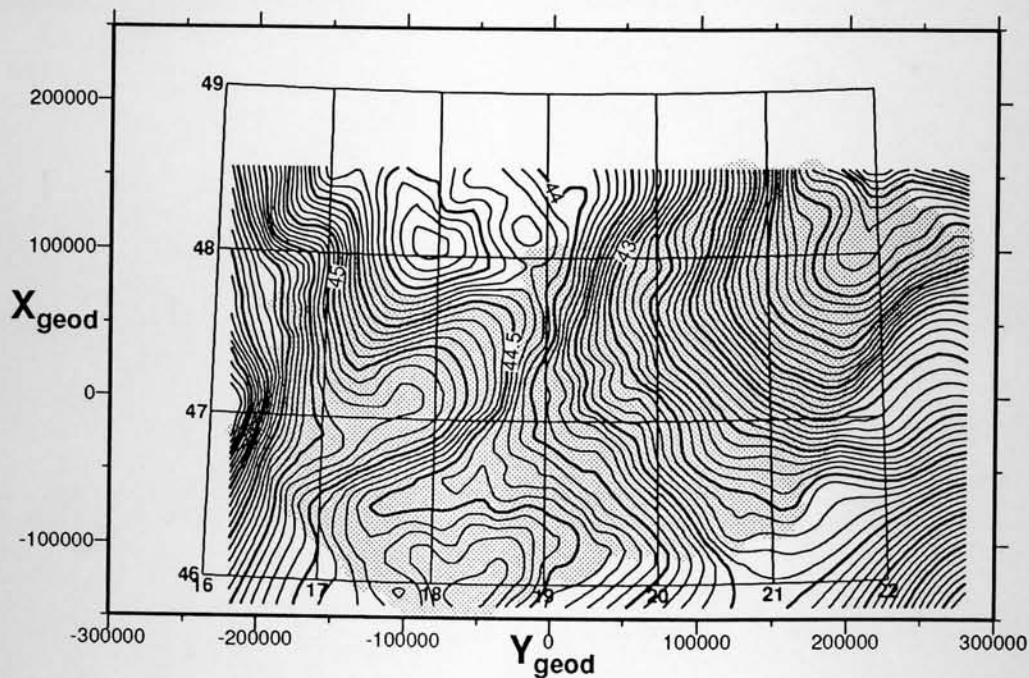


Figure 14. Contour line map of the "crustal geoid". Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

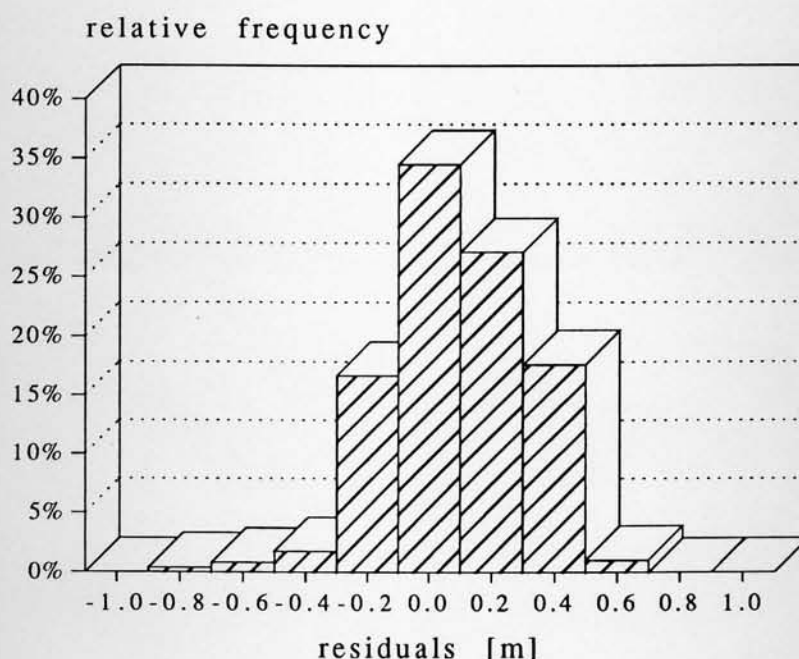


Figure 15. Histogram of residual geoid undulations.

between HGQ91B geoid and different global geoid solutions were derived as well (Table 4).

The crustal geoid generally approximates well the HGQ91B geoid solution. Obviously the smallest residuals what means the best approximation were produced by the comparison of HGQ91B and OSU89B ($n,m=360$) geoid undulations (data set *D* in Table 4) since this OSU model was generated from a data set which contains — for the first time in the history of OSU models — 509 Free-air gravity anomalies from the terrain of Hungary (Ádám, 1992). Data set *B* and *C* indicates that with respect to the statistics, filtering results in almost the same output as what a global geopotential solution which does not involve terrestrial gravity data from Hungary can produce. It also indicates that the selection of filter parameters is suitable for filtering out local effects in geoid undulations.

The constant bias between the two geoids is small, only +8 cm in the sense of Eq. (2) and very close to the mean values of the other data sets B,C,D. We

Table IV. Statistical parameters of different sets of geoid undulation residuals.
A: HGQ91B – “crustal geoid” *B*: HGQ91B – filtered OSU89B geoid ($n,m=360$)
C: HGQ91B – OSU81 geoid ($n,m=180$) *D*: HGQ91B – OSU89B ($n,m=360$)

Data set	No. of data	min. [m]	max. [m]	mean [m]	S.D. [m]
<i>A</i>	959	−0.80	+0.69	+0.08	±0.22
<i>B</i>	959	−1.16	+1.14	+0.05	±0.49
<i>C</i>	959	−1.21	+1.51	+0.04	±0.59
<i>D</i>	959	−0.20	+0.62	+0.08	±0.13

assume that it also indicates the appropriate choice of filter parameters mainly the λ_{cutoff} for the high-pass filter of potential surfaces generated by the density distribution inside the lithosphere. However, the influence of filtering on the statistics of residuals is not yet investigated in details at all and a proper filter design should be applied to eliminate distortions caused by digital filters. Also different taper types with different filter parameters should be applied to the data to monitor the effect of filtering.

Concerning the areal distribution of the residuals (Fig. 16) positive local extrema of the discrepancies can be found at places where the surface topography is relatively high (Fig. 17). It also proves the fact that the crustal model does not contain topographical information at the moment. However, there are also indications of possible model deficiencies on the area of the Great Hungarian Plain where both systematically positive and negative local extrema can be seen. It is almost obvious that this cannot be the effect of the unmodelled topography since it is “absolutely” plain here.

Conclusions

The results of model computations based on a 3-D finite element density model of the lithosphere in the Pannonian basin have shown that the reconstruction of geoid undulations with wavelengths shorter than 300 km is possible and a careful spectral merging of a global geopotential model like OSU89B and

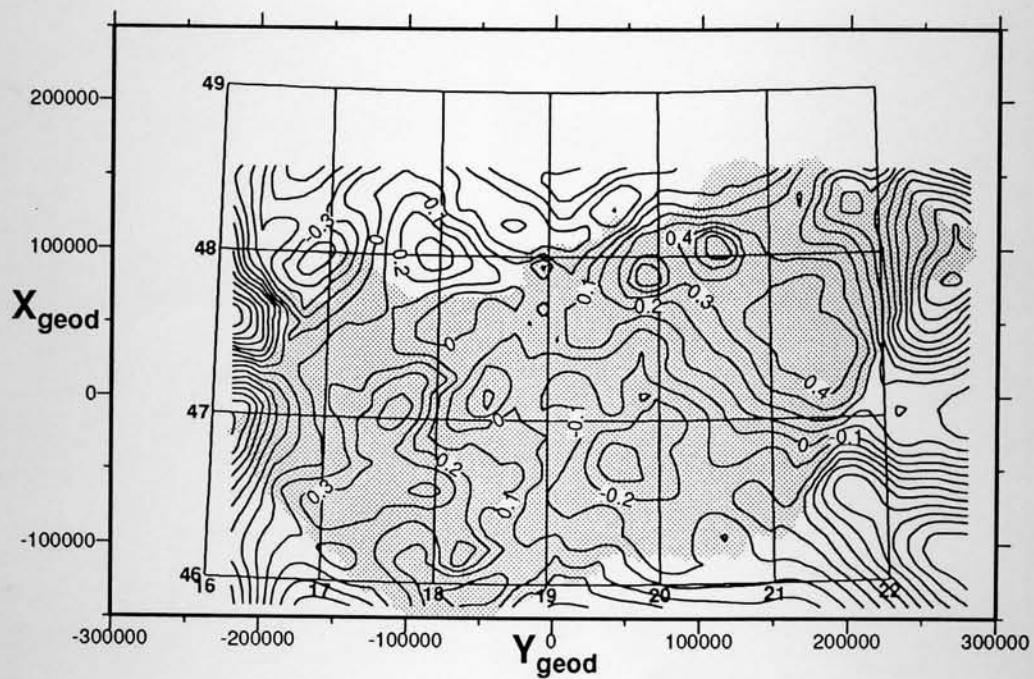


Figure 16. Areal distribution of geoid undulation residuals. Contour interval is 0.1 m. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Shaded area locates the territory of Hungary

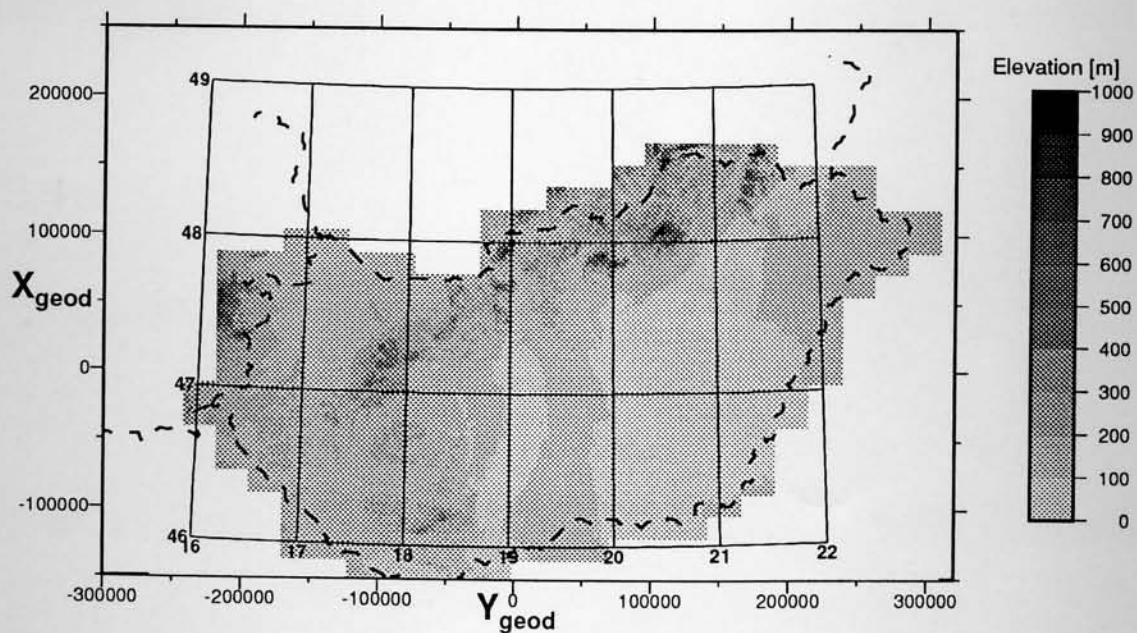


Figure 17. Gray scale map of the surface topography. Plane coordinates are in meters. Ellipsoidal coordinates are in degrees. Dashed line represents the state borders between Austria, Hungary and Slovakia.

of the potential contribution produced by the density model may result in reliable approximation of a local quasi-geoid solution at a decimeter level. To achieve this accuracy (1) a detailed 3-D model of the lithosphere described in this paper should be available and (2) a proper filtering of data, that is of the potential surfaces to be merged is necessary to separate spectral bands where the spectral contribution of different data sources is possibly unique. An empirical procedure was given to determine the spectral band and the transition band for high-pass and low-pass filtering. Because of the "bulking" effect of forward gravity field modelling based on finite element density models defined in Cartesian coordinate systems the long wavelength anomalies or undulations which are close to even larger than the dimensions of the area investigated cannot be interpreted. This is also a reason why filtering has to be applied. However, there is a possibility at least to estimate the exactness of spectral tailoring if the mean value of the residuals determined by Eq. (2) is considered. A small bias may indicate that appropriate filter parameters were used in the filtering before the combination of different potential surfaces.

The model computations also served a natural way to start the physical interpretation of the geoid undulations in the Pannonian basin. Excluding the effect of topography what is still unknown at the moment it can be stated that the contribution of lithospheric sources located not deeper than 30–35 km to the gravity potential of the Earth is significant in the wavelength range below 300 km. The greatest effect (1.5 m undulation) was induced by the sediments that is the topography of the crystalline basement. However, because of the gravitational equilibrium state of the lithosphere on the investigated area the effect of the Moho and of crustal density inhomogeneities compensate it partly producing a smooth gravity potential field.

Acknowledgements

The authors are grateful to the Military Mapping for providing statistics of the 500 by 500 m DTM of Hungary and also to Mr. Zoltán Páncsics, Eötvös Loránd Geophysical Institute for computing the statistics of Bouguer gravity anomalies de-

rived from the data base of Geological Survey of Hungary. They wish to thank Mrs. Fleischhacker for the careful editing and processing the text of the paper.

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ELEVATION DETERMINATION USING GPS AND GEOID93: GETTING THE TOPOGRAPHY OUT OF GRAVIMETRY

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ABSTRACT

One of the largest sources of error in gravimetric surveys is that associated with terrain effects. High-resolution gravity surveys, recently done by the Phillips Laboratory, have revealed deficiencies in existing digital elevation data. Recent advances in GPS and geoid models have allowed for the development of precise terrain models. A small-scale terrain data model was developed using topographic maps, GPS, and a new geoid model, GEOID93. The model was compared against known elevations to verify its precision. These data were then used in terrain correction of relative gravimetric data to greatly improve the results.

INTRODUCTION

The Phillips Laboratory (PL) Geophysics Directorate has been doing precise, high-resolution gravimetry for over 20 years. One of the most important aspects of this work is the correction for terrain. Terrain variations, if improperly modeled, could easily alias the gravity signature. The difficulty lies in obtaining precise terrain models that can be used to correct gravimetric data. Clearly, doing a high-precision survey while using inaccurate terrain data is counterproductive. In the past, terrain corrections have been done by dividing the survey area into sectors and then obtaining mean elevations for a given sector from contour maps. Data bases of digital terrain exist but are not very detailed and their precision is at best on the order of a few meters.

Large scale topographic maps covering the continental United States are readily available and provide a wealth of terrain information. The U. S. Geological Survey (USGS) produces 1 : 24000 scale, 7.5×7.5 arc-min, topographic maps. These map elevations, purported to be precise to half a contour interval, can be digitized and grided to provide an initial terrain data base. These data can then be supplemented with Global Positioning System (GPS) observations to produce detailed terrain models that can be used for terrain corrections. Two recent developments have facilitated the use of GPS in elevation determination: The first is a fully operational GPS constellation. The second is the development of precise grided data bases (e.g. GEOID93 within the continental U.S.) that enables GPS derived elevations to be converted to topographic elevations. We will discuss how a combination of map digitization, GPS, and GEOID93 can be used to produce precise terrain models, and then to employ these models in computing terrain effects.

POSITION AND ELEVATION DATUMS

In order to use USGS maps with GPS data we must first establish a common reference system. USGS maps display the latitude and longitude in the North American Datum of 1927 (NAD27) or the North American Datum of 1983 (NAD83). USGS topographic elevations or orthometric heights, H , are referenced to mean sea level (MSL). MSL is represented by the vertical datums: the Northern Geodetic Vertical Datum of 1929 (NGVD29) or the North American Vertical Datum of 1988 (NAVD88). Terrain corrected gravity anomalies, however, are referenced to the geoid. MSL is an approximation to the geoid but recent studies have shown that the two surfaces can differ by as much as 2 m in some areas of the world (Rapp 1994). In computing relative elevations, however, this difference can be ignored, and for the purposes of this paper we will assume the geoid and MSL are one in the same.

GPS latitudes, longitudes, and elevations are referenced to the World Geodetic System of 1984 (WGS84). This is currently the most widely known ellipsoid and is defined by two main parameters, the semimajor axis, $a = 6378137 \pm 2$ m, and the flattening, $f^{-1} = 298.257223563$. Since NAD27 and WGS84 are both mathematically defined surfaces, multiple regression equations can be used in the transformation of latitudes and longitudes from NAD27 to WGS84 and vice versa with no loss of precision. The problem lies in the incongruity between the ellipsoid and the geoid. The difference between the geoid and the ellipsoid, known as the geoid undulation, N , is the quantity that must be determined if GPS derived elevations, known as ellipsoidal heights, h , are to

be converted to orthometric heights. Given the geoid undulation, orthometric heights can be easily computed from ellipsoidal heights as:

$$H = h - N. \quad (1)$$

The determination of orthometric heights using conventional surveying techniques can be done to a high degree of precision (~0.005 m). Computing orthometric heights of commensurate precision using Eq. 1 requires both precise ellipsoidal heights and geoid undulations. Consequently, the ability to determine orthometric height using GPS is a function of the accuracy of GPS and the precision of the geoid undulations.

THE STATUS OF GPS

In early 1994 the GPS constellation achieved Initial Operational Capability (IOC) and is today fully operational. The constellation presently consists of 24 satellites (21 satellites and 3 spares) in six orbital planes. All satellites are Block II/IIA satellites; starting in 1996 these will be replaced by the newer Block IIR satellites. The advent of IOC for GPS, has made vertical positioning much less labor intensive. Elevation determination for a large set of points, once done by running time-consuming level lines, can now be accomplished in a matter of days with GPS to better than centimeter accuracy. The only inconvenience related to GPS is in its restriction to civilian users.

The Department of Defense restricts the use of GPS with Selective Availability (SA) and Anti-Spoofing (AS). Each GPS satellite transmits a preliminary range estimate known as the pseudorange, ρ . In the SA environment (enabled at the start of IOC), the pseudorange is intentionally perturbed by a discrete amount, $d\rho$. This has the effect of limiting GPS accuracies to about 100 m when GPS is used for point positioning. In differential GPS, however, the affects of SA can largely be removed by employing double differencing procedures during post processing. This can easily be shown by examining the GPS observation equation:

$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} - d_{trop}. \quad (2)$$

For the purposes of the SA discussion we will only focus on one part of Eq. 2 which is the orbital error component given by

$$d\rho = d\rho_n + d\rho_{sa}$$

where

$d\rho_n$ = nominal orbital error component

$d\rho_{sa}$ = orbital error component due to SA.

In solving Eq. 2 using receiver-satellite double differences, the orbital errors, $d\rho$, which include the affects of SA, are subtracted out and we are left with the remaining unknowns which can be determined from observations:

$$\nabla\Delta\Phi = \nabla\Delta\rho + \lambda(\nabla\Delta N) - \nabla\Delta d_{ion} - \nabla\Delta d_{trop}. \quad (3)$$

In Eq. 3, Δ represents differences between one satellite and multiple receivers; and ∇ represents differences between one receiver and multiple satellites.

The other restriction, AS, only affects P-code receivers. AS is a procedure whereby the P-code is encrypted into what is known as the Y-code. This has the affect of denying the more precise P-code (an order of magnitude more precise than the C/A-code) to civilian users. Since most GPS receivers are C/A-code receivers, however, they are unaffected by AS. In addition, a new Z-code GPS receiver has recently been developed. The Z-code is a decoded version of the Y-code and possesses essentially the same precision as the P-code.

UNDULATION INTERPOLATION VERSUS GEOID93

Early geoid undulation models, used mainly for global gravity modeling, possess insufficient resolution for precise geodetic applications. Furthermore, their relative precision was no better than a meter. The National Geodetic Survey (NGS) with assistance from Ohio State University (OSU) has recently developed a new geoid model for the continental U.S. known as GEOID93 (Milbert 1992). GEOID93 is a grided undulation file (approximately 5 km spacing) with a relative precision of better than two parts per million. The development of GEOID93 makes GPS an even more powerful tool for surveying. Given the extremely precise ellipsoidal heights determined by GPS, and a comprehensive geoid undulation model, orthometric heights can be readily determined.

Orthometric heights are determined from ellipsoidal heights in one of two ways: relative undulation interpolation and direct elevation computation using a geoid model. We will refer to these two procedures as methods 1 and 2 respectively. Method 1 requires at least second order benchmarks to be located in the perimeter of the survey area to be incorporated into the GPS survey. Given that both the orthometric height and the ellipsoidal height will be known at the benchmarks (upon completion of the GPS survey), the geoid undulation can be computed from Eq. 1. At that point, a surface (e.g. a plane) can be fit to the relative geoid undulations between the benchmarks. The resultant plane can be used to interpolate relative undulations to the interior of the survey region and to compute orthometric heights for the remaining survey points. This is done by using least-squares to minimize the following equation:

$$\sum_{i=1}^P [(N_0 - N_i) - x(\lambda_0 - \lambda_i) - y(\varphi_0 - \varphi_i) - z]^2 = 0 \quad (4)$$

where

$N_0 - N_i$ = relative undulation

$\lambda_0 - \lambda_i$ = longitude distance

$\varphi_0 - \varphi_i$ = latitude distance

x, y, z = plane coefficients

P = number of benchmarks.

Here the relative undulation is the undulation difference between a reference benchmark, N_0 , and the remaining benchmarks. Similarly the longitude and latitude distances are between the same reference benchmark and the remaining benchmarks. A minimum of three benchmarks allows for the solution of Eq. 4 and results in the determination of the three coefficients that define the plane.

A general rule-of-thumb is that interpolated heights should not be used if a precise geoid is available. Method 2 computes orthometric heights through the use of GEOID93 and ellipsoidal heights derived from GPS observations. If the terrain is reasonably flat, the geoid (in most cases) tends to be smooth and both methods are comparable. In mountainous terrain, however, the geoid contains high frequency variations rendering the planar interpolation less precise. In these instances a precise geoid model (if one is available) such as GEOID93 is the more attractive method.

A TEST CASE

PL conducted a GPS survey in November 1991 in Inverness, MS, (Romaides et al., 1994). In 1991 there were far fewer GPS satellites in orbit than the present time, and SA had not yet been enabled. Due to the limited satellite coverage at the time, this test provided a lower limit on the achievable accuracies in using GPS and GEOID93 for elevation determination. The survey, observed using two Ashtech M-XII and four LD-XII C/A code, dual-frequency receivers, consists of 102 points and approximately 500 baselines. Eight of the survey points are benchmarks with a known orthometric height and three are triangulation stations with known latitude and longitude. Baseline lengths range from 0.15 to 25 km with a mean length of about 3 km. All observations were processed in a least-squares network adjustment. In the adjustment two of the eight benchmarks and one of the three trigonometric stations were held fixed; the others were allowed to float within the constraints of the adjustment. The results of the adjustment are a root-mean-square (rms) error of 0.02 m in the latitude and longitude and 0.03 m in the ellipsoidal height.

The two methods (described above) were used to compute orthometric heights. First, using Eq. 4, relative geoid undulations in the survey area can be computed. Five of the eight benchmarks occupy the perimeter of the survey area with the remaining three in the interior (Figure 1). Since this area is very flat the relative geoid undulations can be adequately modeled with a plane. The resulting equation,

$$N_0 - N_p = [(6.8 \times 10^{-7})(\lambda_0 - \lambda_p)] + [(3.9 \times 10^{-6})(\phi_0 - \phi_p)] - 1.65 \times 10^{-2}, \quad (5)$$

defines the relative geoid undulation within the survey area. We can now recast Eq. 1 as a difference between a point of known elevation and an unknown point as follows:

$$H_p = H_0 + (N_0 - N_p) - (h_0 - h_p). \quad (6)$$

Given the known orthometric height of a benchmark, H_0 , Eq. 6 can be used to determine the elevation of the unknown point, H_p . The ellipsoidal heights, h_0 and h_p , are determined by the GPS survey and given a known latitude and longitude, the relative undulations are computed from Eq.

5. In this way the undulations are interpolated to the interior of the survey region to compute orthometric heights. The results of the five benchmarks used in this exercise are displayed in Table 1. The observed and the predicted undulations are in meters relative to MSN0; the residuals are the

Table 1. Results of plane fit to relative undulations.

Station	Observed Undulation	Predicted Undulation	Residual
MSN0	0.0000	-0.0228	0.0228
MSN3	-0.1150	-0.1135	-0.0015
MSN4	-0.0480	-0.0383	-0.0097
MSN7	0.0010	0.0114	-0.0104
MSNC	-0.0850	-0.0838	-0.0012

observed minus predicted elevations and possess an rms of 0.012 m. In Table 2 the orthometric heights for all the benchmark elevations computed using Eqs. 5 and 6 are displayed along with the true elevations. The results of method 1 indicate good agreement between the true heights and predicted heights, and with the exception of MSKA all the differences are 0.03 m or less.

Table 2. Orthometric heights computed using undulation interpolation versus true heights.

Station	True Height	Predicted Height	Difference
MSN0	36.125	36.148	-0.023
MSN3	35.382	35.380	0.002
MSN4	37.244	37.234	0.010
MSN7	38.405	38.395	0.010
MSNA	33.966	33.996	-0.030
MSNB	36.344	36.325	0.019
MSNC	33.909	33.908	0.001
MSKA	35.644	35.580	0.064

If a comprehensive geoid model that encompasses the survey area is available, method 2 can be applied. Such is the case in Inverness where the geoid undulations are obtained from GEOID93 and displayed in Figure 2. Orthometric heights for all the points in Table 2 were then computed directly using Eq. 1 and are shown in Table 3. The true-minus-model results were obtained by differencing the computed orthometric heights using method 2 with the true heights given in Table 2.

Table 3. Orthometric heights computed from GPS and GEOID93 versus true heights.

Station	Geoid Undulation	Ellipsoidal Height	Orthometric Height	True Minus Model
MSN0	-26.645	9.486	36.131	-0.006
MSN3	-26.748	8.626	35.374	0.006
MSN4	-26.743	10.540	37.283	-0.039
MSN7	-26.595	11.769	38.364	0.041
MSNA	-26.744	7.251	33.995	-0.029
MSNB	-26.647	9.658	36.305	0.039
MSNC	-26.728	7.185	33.913	-0.004
MSKA	-26.721	8.864	35.585	0.059

In this region the two methods are indeed comparable; the rms of the true minus predicted elevations is 0.028 m for method 1 and 0.034 m for method 2. In a sense this is not quite a fair comparison since in the least-squares plane fit of method 1, five of the benchmarks are forced to fit the undulations whereas in method 2 there are no such a priori constraints. The results demonstrate two things: First, although this region of the country is flat, the ability of using GPS and GEOID93 in the determination of orthometric heights is clearly evident. Second, if a precise geoid model is not available, undulation interpolation can be used if sufficient care is taken.

BUILDING A TERRAIN MODEL

Currently, the most comprehensive digital terrain model resides at the Defense Mapping Agency (DMA). DMA possesses a grided data base of digital terrain elevation data (DTED) at a 3 arc-sec spacing; this results in an approximately 100×100 m grid at the equator. The accuracy of the DTED data in most areas is no better than a few meters and in some areas no better than tens of meters. The shortcomings of the DTED data became abundantly clear during a previous gravity survey in Clayton, NC. Figure 3 shows the topography in a 4×4 km region digitized from a USGS map; figure 4 shows the DTED data over that same area. Examination of the two figures clearly shows that the DTED data looks nothing like the true terrain. Consequently, any terrain corrections done using these data would be in error. For applications where 2 mGal precision is required, these data are sufficient; for precision gravimetric work (~ 0.02 mGal), however, they are largely inadequate ($1 \text{ mGal} = 10^{-6} \text{ cm s}^{-2}$). As a result, we must resort to other methods to obtain terrain information.

Armed with USGS maps, GPS, and GEOID93 we can now proceed with the development of a grided terrain model. Using the two detailed surveys that PL has done in Clayton, NC, and Inverness, MS, we first evaluate the precision of the USGS maps. Examination of the two previous PL survey areas reveals that the elevations of the USGS maps are indeed precise to half a contour interval. In Clayton, the maps are contoured at 3.05 m (10 ft) intervals. We compared 77 point elevations and found an rms difference between the elevations of our data (precise to 0.02 m)

and those of USGS to be 1.12 m. In Inverness, maps are contoured at 1.52 m (5 ft); a comparison using 128 positioned points (precise to 0.03 m) reveals an rms difference of 0.85 m. Both results show good agreement between the USGS maps and survey points.

The next phase is to digitize the maps to create a preliminary terrain model. For this example we will focus on the area of Inverness, MS because the digitization was done in greater detail than that of Clayton, NC. We digitized the elevations in a 20×20 km region. In areas of little detail there were fewer points and conversely areas of high detail contained a greater density of points. Once this time-consuming task was completed, a computer program was used to connect all the digitized points forming optimal triangles with one elevation value at each vertex. A 50 m grid was then superposed onto the area. Fitting a plane to the three triangle vertices allowed us to interpolate the elevations onto the grid points thus creating a data file at 50 m spacing. The results of this exercise are shown in Figure 5 which depicts a contour plot of the digitized data over the region. A visual comparison with the actual USGS map indicates excellent agreement in the general terrain features. The DTED data for the same area is shown in Figure 6. Although these data are superior to the DTED in Clayton, there are still significant differences. We compared the DTED data to the digitized USGS data and found a mean elevation difference of 0.68 m. Once again, it is clear that for coarse survey applications the DTED data are acceptable but are quite inadequate for precise work.

The USGS maps, however, also contain errors as newer topographic features do not appear on the maps. In fact this was just the case in Clayton and Inverness. A closer examination of the elevation comparison revealed that most of the larger differences (2 - 4 m) between the actual data and the map elevation are a direct result of newer features that are absent on the maps. The next step is to correct the digitized elevations using the GPS survey points. By using the elevation differences between the GPS points and the USGS elevations, a new grided data file was created at the same spacing as the elevation file. We then used this file to correct the original data creating a new data file. This procedure was only performed once, but it is obvious that some form of an iterative process can be used. Ideally one would obtain a grided data file from digitized topography and collect some GPS data. These data would then be used to correct the preliminary data base. Next, additional GPS points can be collected and compared with the updated terrain data base. If the resultant precision is sufficient, the procedure can be stopped; if not, additional GPS data can be collected until such time as the terrain data meet the requirements.

Our GPS survey was done in the static mode at a time when the GPS constellation was incomplete. GPS is now a fully operational positioning system. Using the much faster kinematic or pseudo-kinematic GPS surveying and GEOID93, elevations for a large number of points can be determined in a very short period of time. These points can then be used to supplement and correct the elevations digitized off topographic maps to create a grided terrain data base of high precision. Furthermore, we digitized the USGS maps manually; another means to expedite the procedure is the willingness of USGS to supply grided data files with point elevations on CD-ROM.

TERRAIN CORRECTIONS

The high-frequency portion of the gravitational signal is dominated by the terrain whereas the long-wavelength component is due to mass distributions inside the Earth. Figure 7 depicts the power spectral density of both the terrain-implied and geology-implied gravity disturbances in a

relatively flat region of Oklahoma. The figure clearly shows that for wavelengths shorter than 3.5 km, the terrain-implied disturbance overwhelms the geology-implied disturbance. Consequently, detailed gravimetry will always require corrections for terrain.

For the purposes of this paper we will focus on the numerical summation of right rectangular prisms to correct for terrain. If terrain data, however, are in the form of a regular grid, the much faster Fast-Fourier Transform (FFT) can be used for terrain correction. Formulae to compute the gravitational effect of a right rectangular prism from an exterior point are given by Forsberg and Tcherning, 1981. Using these equations the grided data set can be used to compute terrain-corrected Bouguer anomalies, Δg_b , as follows:

$$\Delta g_b = \Delta g_f - 2\pi\rho G(h - \bar{h}) + \Delta g_t \quad (7)$$

where Δg_f is the free-air anomaly, ρ is the mean crustal density (2670 kg m^{-3}), G is the gravitational constant, h is the elevation, $\bar{h}$ is the mean elevation of the terrain in the area, and Δg_t is the terrain correction. In Eq. 7 both the terrain and Bouguer corrections are done with respect to the same datum plane which is the mean of the elevations in the survey area, $\bar{h}$. In this way the Bouguer slab serves as the first approximation to the removal of terrain effects. The terrain correction is then added to the correction for the Bouguer slab as mass excesses are removed and mass deficiencies are added to obtain the final result.

Previous experiments performed by PL demonstrate the need for precise terrain corrections in a gravity survey (Romaides et al., 1991). Depending on the density of the soil, both the Bouguer and terrain corrections are approximately 0.1 mGal for every 1 m of topography. If the aim of a gravity survey is a 0.02 mGal rms error or better, it would take only a 0.2 m error in the terrain to equal the entire error budget. It has been previously shown that the DTED data in Clayton cannot be used for precise terrain corrections. In Inverness, the DTED data do resemble the USGS data but there are still differences. For example, the mean of the USGS and the DTED data differ by $\Delta h = 0.68 \text{ m}$. If the density of the soil is assumed to be $\rho = 2670 \text{ kg m}^{-3}$, this elevation error will result in a mean terrain correction error of $2\pi\rho G\Delta h$ or 0.076 mGal.

The importance of precise terrain corrections can be shown by examining the results of the Clayton land gravity survey. The aim of the survey was an rms gravity observation error of no more than 0.02 mGal. Upon completion of the data collection the gravity data were corrected for terrain using two grids; a 5 m grid was used for a region where the data are closely spaced (40 points in 0.04 km^2) and a 33 m grid where the data are more sparse (79 points in 225 km^2). For the dense region, the rms terrain correction to the Bouguer anomaly is 0.093 mGal and for the sparse region it is 0.077 mGal. Had the DMA DTED been used or had no terrain corrections been performed, the resultant error would be far greater than the allowed 0.02 mGal.

SUMMARY AND FUTURE PROSPECTS

We have seen the importance of removing the effects of terrain in precision gravimetry. For localized survey applications we now have very powerful tools to construct terrain data bases. GPS and GEOID93 have eliminated one of the largest sources of error in precision gravimetric

surveys and have paved the way to improved terrain corrections. The procedure for obtaining a terrain data base can briefly be outlined in a series of five steps:

1. Digitize USGS topographic maps and construct a preliminary terrain grid.
2. Collect a uniform sampling of GPS data and compute orthometric heights.
3. Compare GPS points with map elevations and correct terrain grid accordingly.
4. Collect additional GPS data and re-evaluate updated grid.
5. Return to step 2 if updated terrain grid is unsatisfactory.

Clearly removing the terrain effects over the entire surface area of survey would not be possible; to do so would require an infinite number of GPS measurements. Depending on the project requirements, however, it is possible to reduce the terrain-induced errors by following the procedure outlined above. It is important that all steps be performed. Our Clayton survey showed that simply digitizing maps or using existing data is insufficient. The test case using GEOID93 was done in a flat region of Inverness but can be used for the entire U.S. including the mountainous regions of the west. Recent work has led to more precise geoid computations, thus even in the western mountains GEOID93 has been shown to be precise to better than 0.09 m (Rapp 1994).

In addition to GEOID93 (which has been recently updated to G9501) there are many areas of the world with precise geoids (e.g. EGG95 in Europe, UC93 in Canada, and SCGSA in South America). Ideally it would be very desirable to extend this procedure to the rest of the world. PL is currently supporting research by OSU, DMA, and NASA that will eventually lead to a World Height System which will link the many existing and inconsistent vertical datums and provide mathematical techniques to transform from one datum to another (Balasubramania 1994). Furthermore, a new global geoid undulation model (WHS96) is currently under development that will enable orthometric heights to be computed from GPS heights all over the world. In the coming years we may finally see the end of height errors as a major source of concern in geodesy and geophysics.

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FIGURE CAPTIONS

Figure 1. The distribution of survey points along with the existing benchmarks used in the GPS survey. The dots represent points of unknown elevation and the boxes are the benchmarks.

Figure 2. Contour plot of the geoid undulations in the survey area as derived from GEOID93; contour interval is 0.02 m.

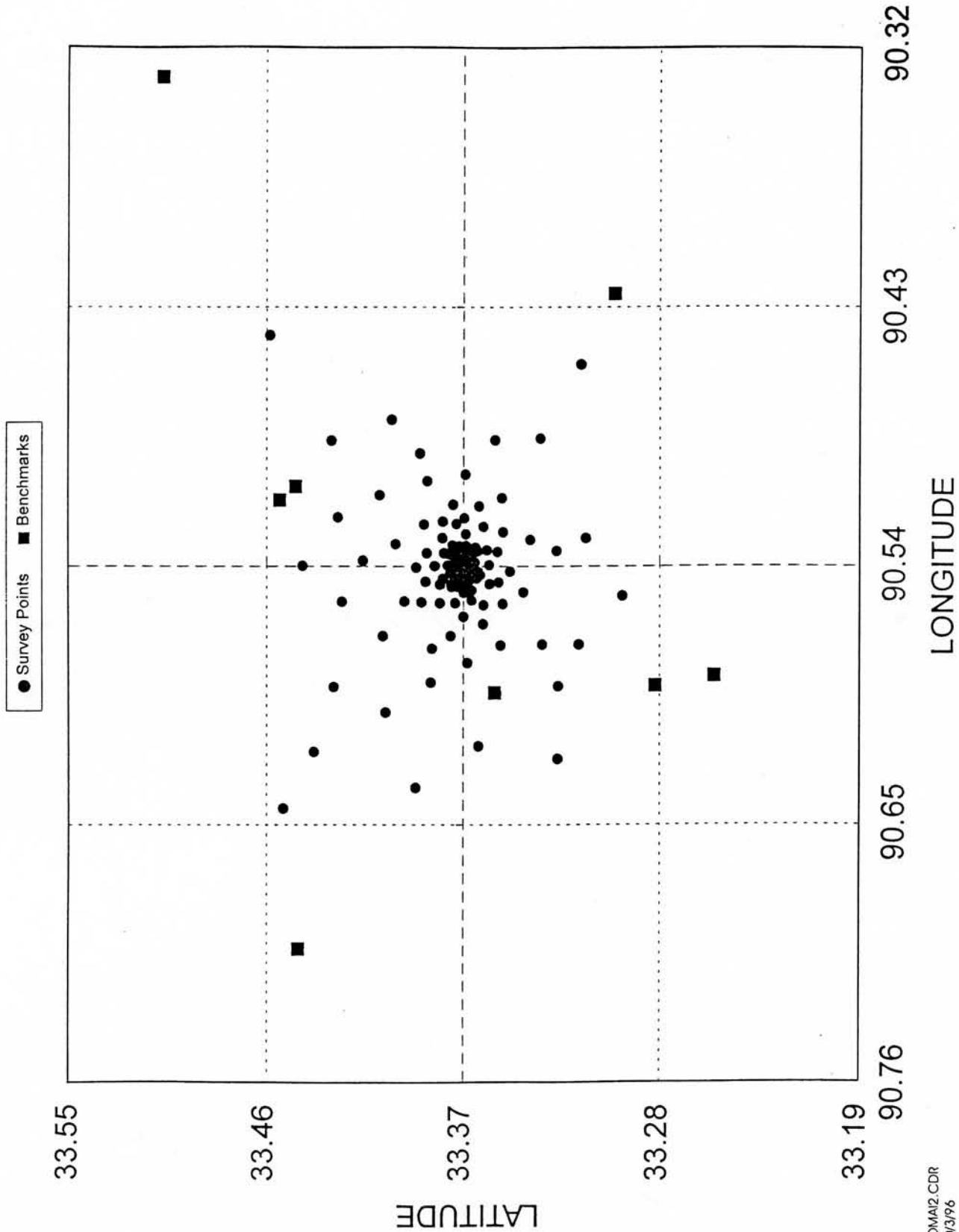
Figure 3. Contour plot of the elevations in the Clayton, NC survey area digitized from the USGS topographic maps; contour interval is 2.5 m.

Figure 4. Contour plot of the elevations in the Clayton, NC survey area as represented by the DMA DTED data; contour interval is 2.5 m.

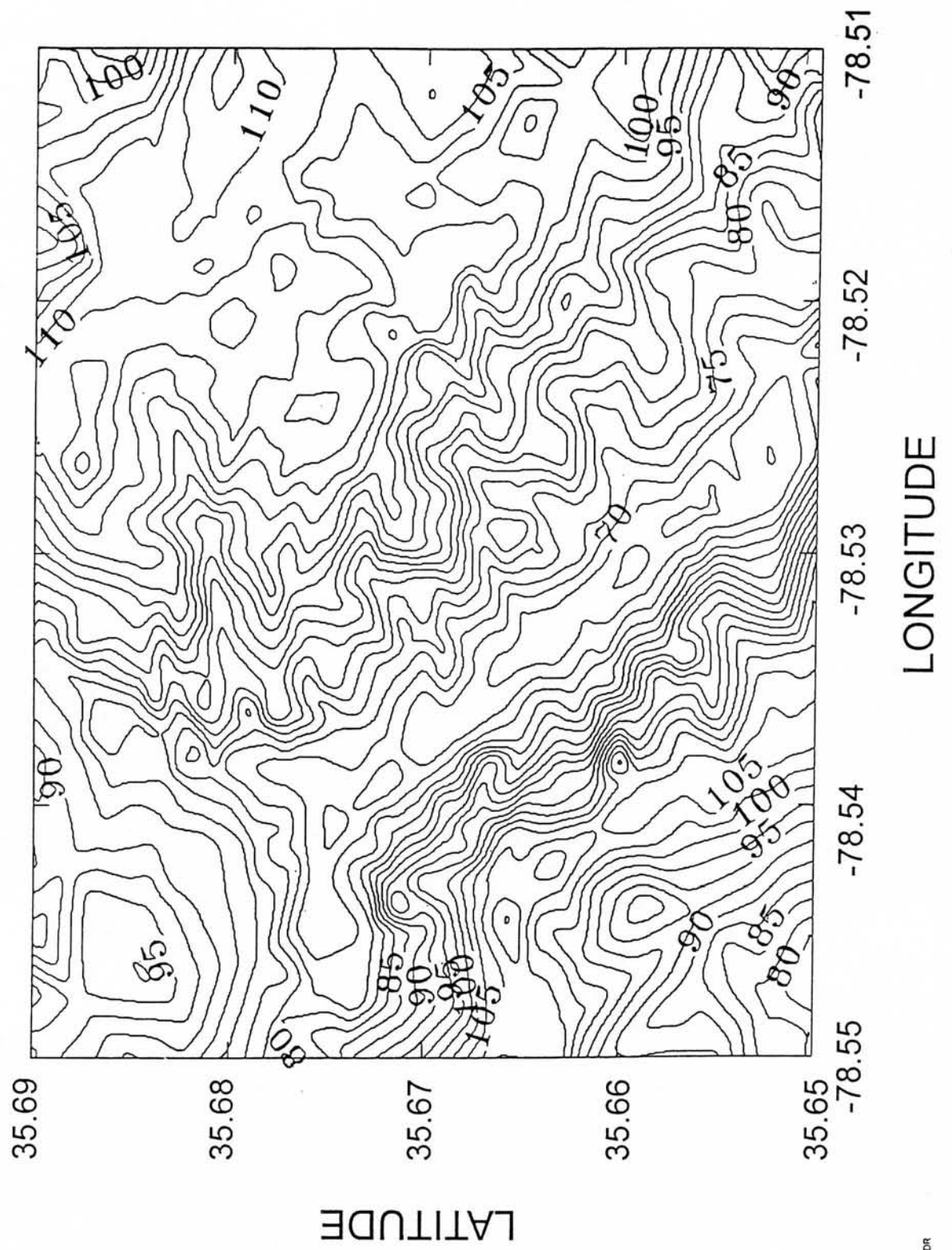
Figure 5. Contour plot of the elevations in the Inverness, MS survey area digitized from the USGS topographic maps; contour interval is 1 m. The dots show the locations of the PL data and the triangles show the existing DMA data.

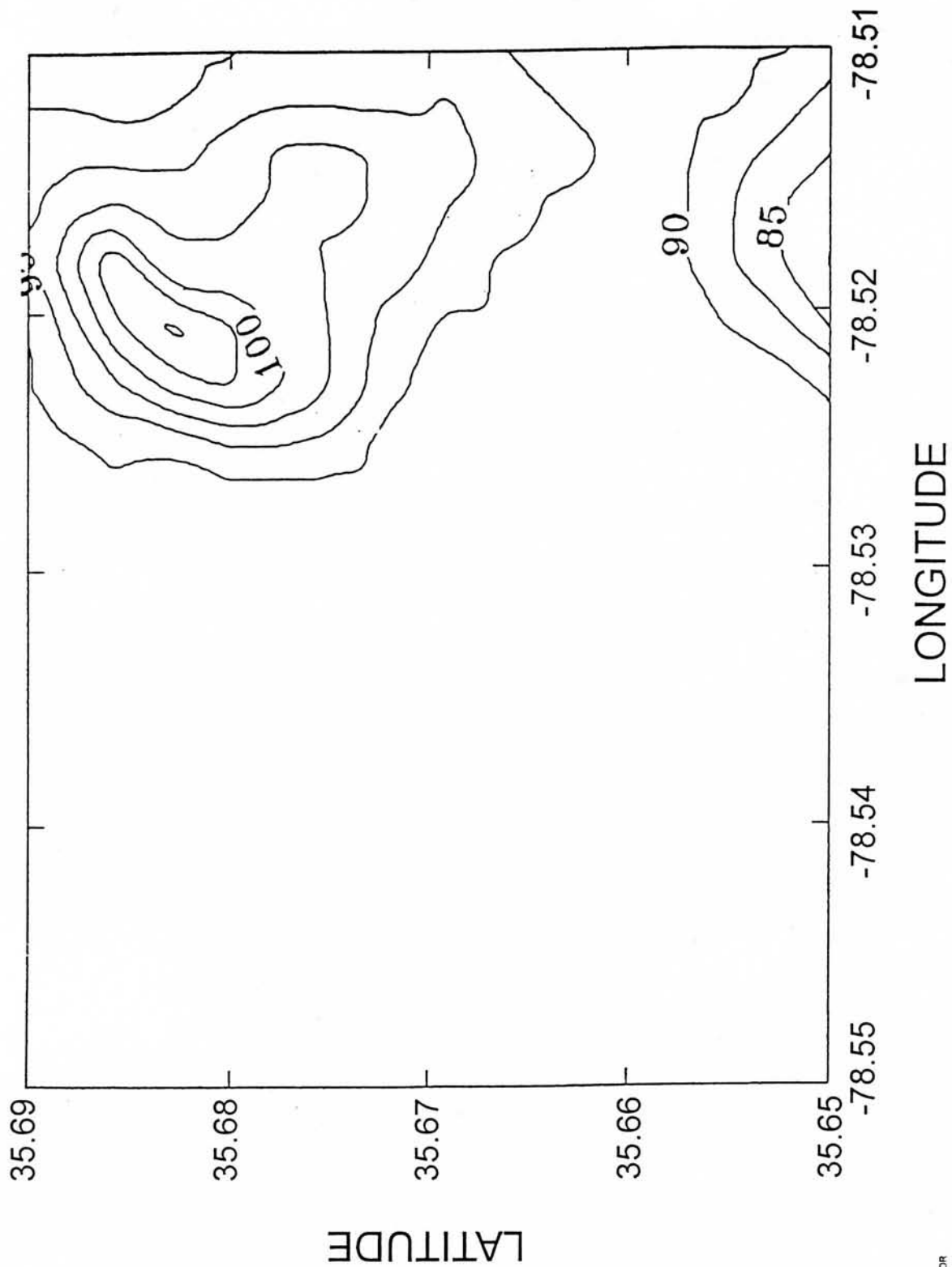
Figure 6. Contour plot of the elevations in the Inverness, MS survey area as represented by the DMA DTED data; contour interval is 1 m. The dots show the locations of the PL data and the triangles show the existing DMA data.

Figure 7. Two dimensional power spectral density of the terrain implied and geology implied gravity disturbances in a flat region of Oklahoma.

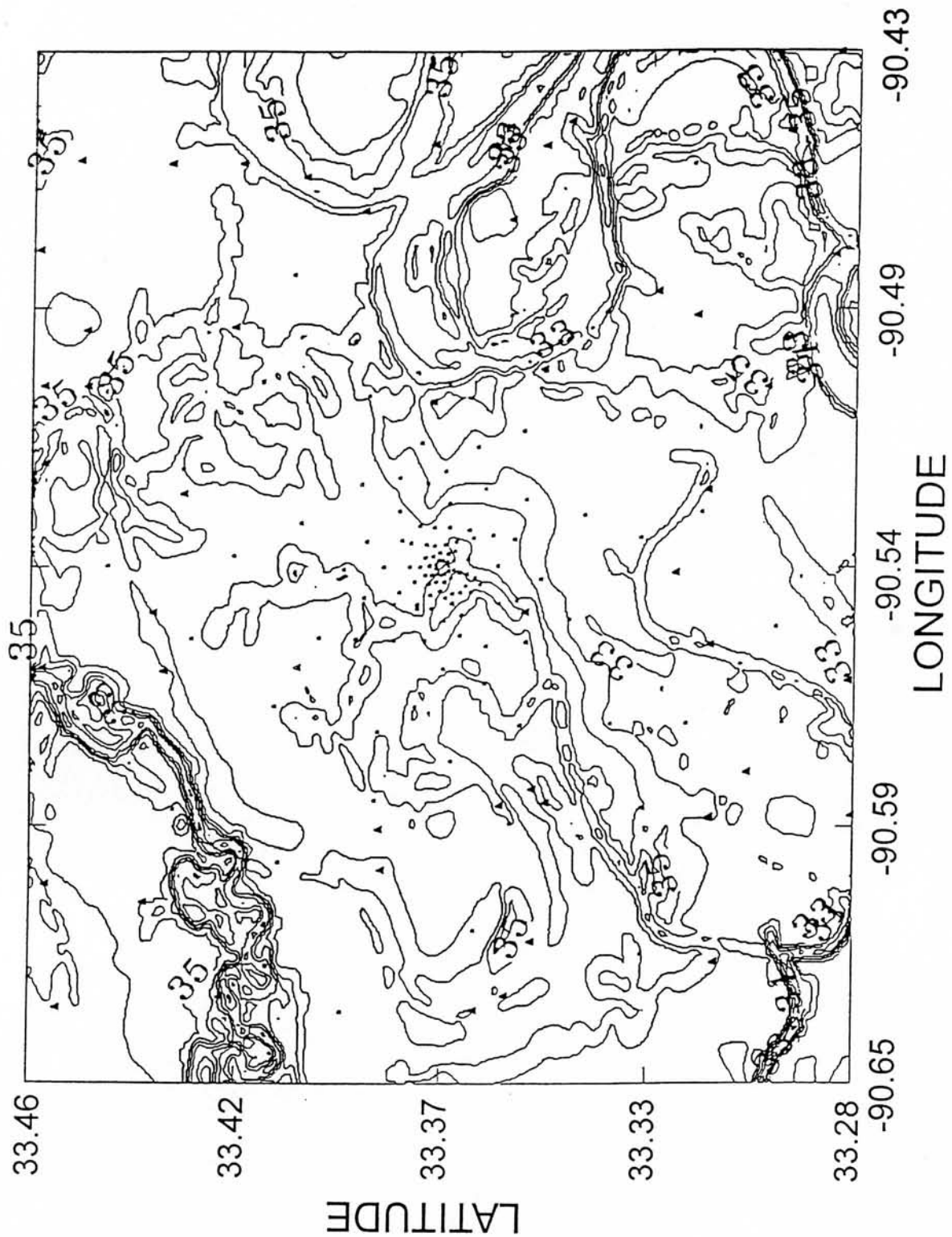


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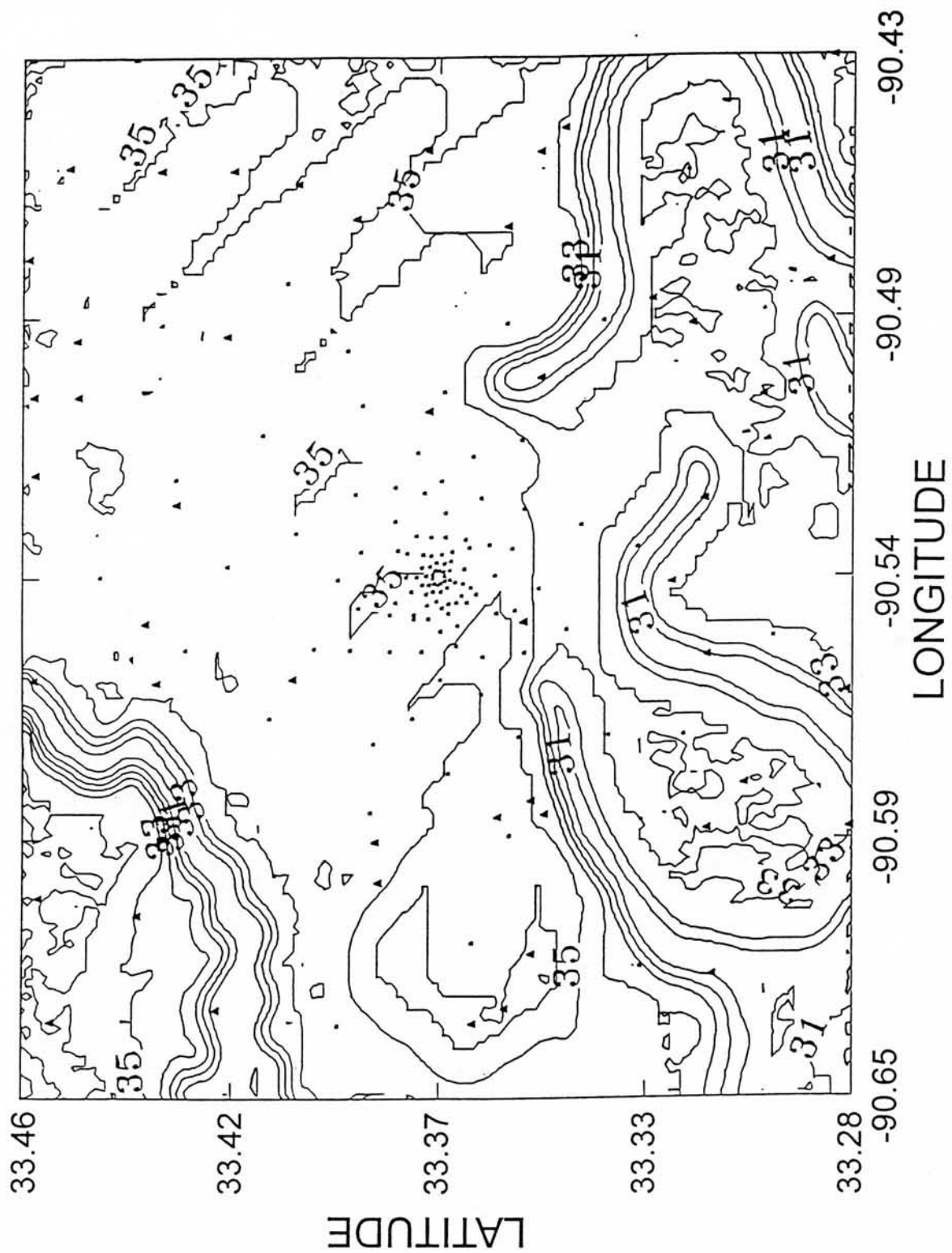




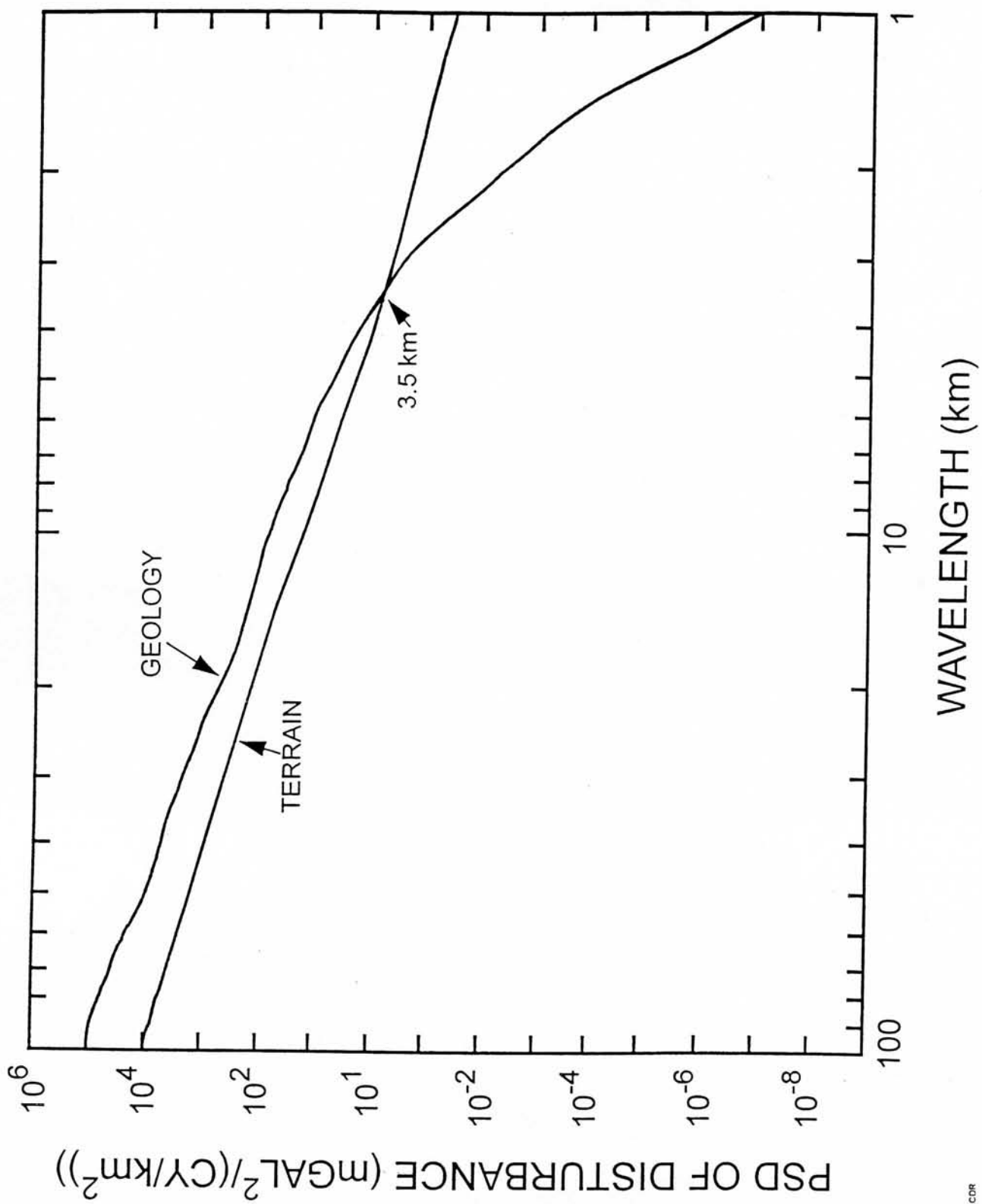
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The Polar Gap

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1. Introduction

For geodetic purposes the ideal satellite orbit would be polar, simply because in that case the pattern of groundtracks covers the entire earth. For several reasons, especially engineering ones, a sun-synchronous orbit is favourable, though. The inclination for low earth orbiters would be around 97° . This will leave caps around the poles without data coverage, denoted as *polar gaps*.

In the literature, the polar gap problem has hardly been studied, presumably because only polar orbits were assumed for geodetic missions. A further reason might be, that most simulation models employ spherical harmonics as base functions. Since these are global functions, they are awkward for taking into account local data gaps. Moreover, simulation results used to be presented in terms of error degree variances, which do not reveal the polar gap effects on the full spherical harmonic (SH hereinafter) error spectrum. The first attempts of such a description in the 2-D spectral domain were made in the course of investigations of the STEP gradiometry mission, cf. (Belikov et al. 1994) and (Sneeuw 1994).

Although the polar gaps are relatively small, about 7° radius for sun-synchronous orbits, they do influence gravity field recovery. Considered as a spherical harmonic analysis problem, it is the task to derive the SH spectrum from a function, multiplied by a rectangular window. In Fourier terms this causes leakage. In analogy the polar gap causes leakage of the SH spectrum.

Several strategies are possible to find the potential coefficients from the (incomplete) data. The problem can be formulated in the time domain (see section 4.) or in the space domain. Although they are essentially identical, they each have their peculiarities (Rummel et al. 1993). Then we can either use a quadrature approach to solve for the potential coefficients, which implies that the inverse relation is available, or minimize some error (e.g., least-squares). The latter is often preferred as it has, obviously, minimal propagated variances. The next problem is that the system of equations can be unstable. If it needs to be regularized, bias will be introduced, cf. (Xu 1992).

Finally, we only try to determine the SH coefficients up to a certain degree. Thus aliasing will play a role as well. This aliasing is not only due to undersampling, as it is defined classically. In general it is signal that will be estimated on the wrong frequencies during the inversion.

2. Distortion of the Spherical Harmonic Spectrum

The polar gap causes distortion of the SH spectrum. In order to see which part of the spectrum is affected, a number of error simulations have been performed with increasing polar gap radius $\theta_0 = |\pi/2 - I|$. The second radial gradient T_{zz} was simulated with white noise assumption, which should give a homogeneous SH error spectrum, that only depends on the degree l . The simulation results are displayed in fig. 2.1 with growing polar gap. Note that only the spectral domain centered around the zonal coefficients is displayed.

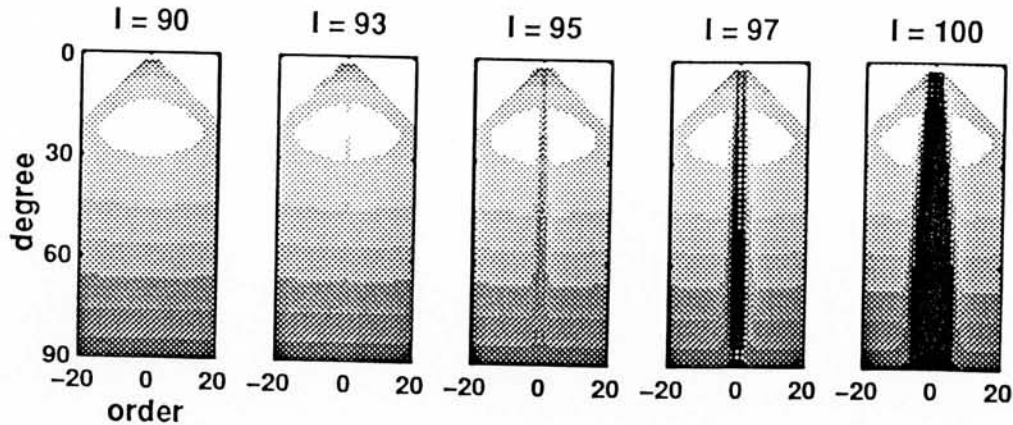


Figure 2.1. Polar gap wedge from least-squares error analysis of T_{zz} tensor component under increasing gap radius.

Without going into details about computation methods, see (Rummel et al. 1993), about meaning of the gray values and about actual results, the figures make clear that only a polar orbit assures a homogeneous error spectrum (for the T_{zz} -observable, to be precise). At 93° inclination, or $\theta_0 = 3^\circ$, the zonals show slightly reduced performance already. At $I = 95^\circ$ the domain of affected coefficients shows up as a narrow wedge. This *polar gap wedge* grows wider for increasing gap radius.

So the low order part of the SH spectrum is seen to be affected by the polar gap. This fact would not be clear if only the degree error variances would be given. They are horizontal averages, loosely speaking, in the figures 2.1. The effect of a polar gap would be invisible, though present. Thus special care must be taken, when representing the results from a non-polar satellite mission by degree error measures. Such a 1-D error representation is not representative for the full σ_{lm} -spectrum anymore.

The wedge shape is explained partly by analyzing the definition, e.g. (Heiskanen and Moritz 1967), of the associated Legendre function:

$$P_{lm}(x) = (1 - x^2)^{m/2} \frac{d^m P_l(x)}{dx^m}$$

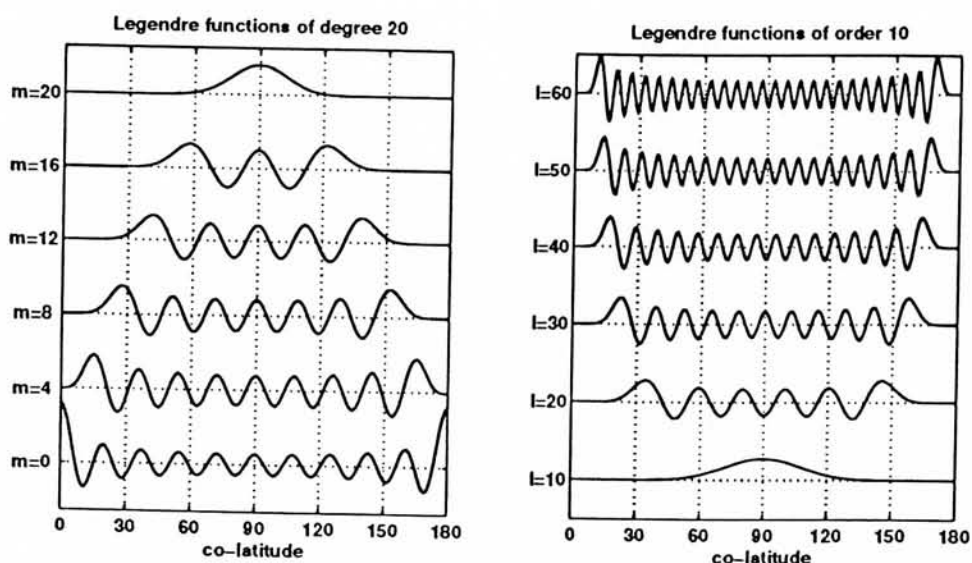


Figure 2.2. Squeezing effects for changing degree or order.

The Legendre polynomial P_l is of degree l , and has l roots on $x \in (-1, 1)$. The derivative $d^m P_l(x)/dx^m$ is a polynomial of degree $l - m$, having $l - m$ roots. The latter polynomial gets multiplied by the function $(1 - x^2)^{m/2}$. Substituting $x = \cos \theta$, the factor becomes $\sin^m \theta$, which will give more and more weight to the area around the equator for increasing order m . The left hand figure in fig. 2.2 show this effect for $P_{20,m}(\cos \theta)$. The only Legendre functions – and thus SH-coefficient – that contribute to the function at and near the poles are the zonal and low-order ones. Vice versa it must then be concluded that the only coefficients, likely to be affected by missing data around the poles, are the near-zonals. Besides the order m , the degree l also plays a role, though less pronounced. For fixed order and increasing degree, the support of the Legendre functions grows, cf. the right hand side of fig. 2.2, displaying $P_{l,10}(\cos \theta)$.

3. A Simple Approach

From a simplified point of view, the polar gap problem is that of estimating the SH spectrum of a function $f(\theta, \lambda)$ on the sphere, multiplied by the rectangular window $h(\theta, \lambda)$, which equals zero in the polar gaps and 1 for $\theta \in [\theta_0, \pi - \theta_0]$. If the potential coefficients are estimated by means of a quadrature approach from potential observations on the the earth's surface, the estimator reads:

$$C_{lm} = \frac{1}{4\pi} \int_{\sigma} f(\sigma) Y_{lm}^*(\sigma) d\sigma$$

in which $Y_{lm}(\sigma)$ are the fully normalized surface spherical harmonics:

$$Y_{lm}(\sigma) = Y_{lm}(\theta, \lambda) = e^{im\lambda} N_{l,|m|} P_{l,|m|}(\cos \theta).$$

Here, N_{lm} denotes the normalization factor. Due to the window function, the above quadrature is now replaced by, cf. (v. Gelderen & Koop 1996):

$$\hat{C}_{lm} = \frac{1}{4\pi} \int_{\sigma} f(\sigma) h(\sigma) Y_{lm}^*(\sigma) d\sigma = \frac{1}{4\pi} \int_{\sigma-\sigma_0} f(\sigma) Y_{lm}^*(\sigma) d\sigma. \quad (3.1)$$

Here σ_0 stands for both polar gaps. The relative error in the coefficients reads $\epsilon_{lm} = (\hat{C}_{lm} - C_{lm})/C_{lm}$, whose error measure:

$$M\{\epsilon_{lm}^2\} \approx \frac{1}{4\pi} \int_{\sigma_0} Y_{lm}^*(\sigma) Y_{lm}(\sigma) d\sigma \quad (3.2)$$

can be considered the *loss of power* of the spherical harmonic base functions in the polar gaps σ_0 . The error ϵ_{lm} represents a signal error alone. No stochastics have been included yet.

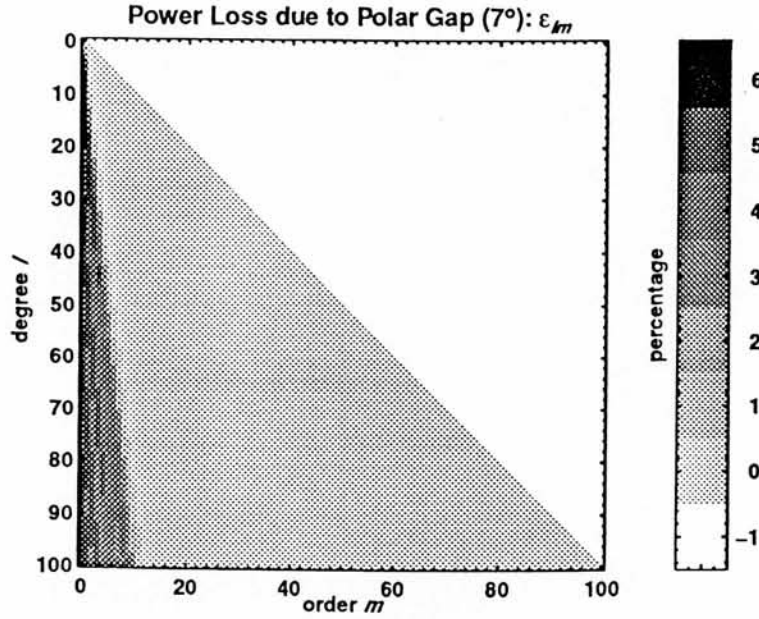


Figure 3.1. Relative error ϵ_{lm} in percent.

The power loss is displayed for a 7° polar gap in fig. 3.1, revealing a wedge shape indeed, delimited rather sharply. In (v. Gelderen & Koop 1996) the following rule of thumb is derived for the maximum order $m_{\max}$ (at a specific degree) that delimits the polar gap wedge:

$$m_{\max} = |\pi/2 - l| \cdot l = \theta_0 \cdot l, \quad \theta_0 \text{ in radians.} \quad (3.3)$$

Remark 3.1. Inspection of fig. 2.1 shows that the rule of thumb does not strictly apply to those wedges. They are in fact narrower. An explanation in terms of power loss, based on the quadrature approach, is apparently too simplistic for more realistic calculations. As a rule of thumb it performs well enough, though.

4. A Time-wise Approach: Lumped Coefficients

The polar gap problem is inherent to any type of satellite-based observations of functionals of the geopotential, e.g. specific gradiometer components or GPS derived orbit perturbations. Each functional has its own specific transfer and therefore its own gravity field recovery characteristics. Consequently the way that the polar gap problem interferes with the coefficient recovery must depend on observable type. E.g. the cross-track gradient tensor component yields a gravity field that is weaker in the lower orders than one from radial gradiometry. So a polar gap cannot inflict much harm there (Sneeuw 1994).

In order to separate the behaviour of the basic linear system from other particularities (observable type, orbit height), we will investigate now a simplified situation. Suppose the earth were a sphere and a satellite would revolve around it at zero height. The observable would be the geopotential directly, instead of its functionals. The along-orbit time-series of this observable can be expressed as the following Fourier series (Wagner & Klosko 1977):

$$V(t) = \sum_{m=-L}^L \sum_{k=-L}^L A_{mk} e^{i\psi_{mk}(t)} \quad (4.1)$$

whose coefficients A_{mk} are the so-called *lumped coefficients*. The angular argument $\psi_{mk} = k(\omega + M) + m(\Omega - \theta)$ is composed of the argument of latitude $\omega + M$ (the in-orbit angle, counted from the ascending node) and the longitude of the ascending node $\Omega - \theta$. The spectral lines of the signal $V(t)$ are thus defined by ψ_{mk} . The lumped coefficients are a linear combination of the potential coefficients:

$$A_{mk} = \frac{GM}{R_E} \sum_l F_{lmk}(I) C_{lm}. \quad (4.2)$$

Here the functions $F_{lmk}(I)$ are complex *inclination functions*. Every functional can similarly be expressed as a lumped coefficient series, based on eq. (4.2). Examples of equations for gradiometer components and of orbit perturbations can be found in (Sneeuw 1994).

Now each order m gives a separate linear system, yielding a block system. Apart from the constant GM/R_E the linear systems consist only of the inclination functions $F_{lmk}(I)$. They build matrices F , again for each order m separately. The lumped coefficients of this order are stored in a vector a , and

the potential coefficients in c . Now the linear system (4.2) simply becomes $a = Fc$. The extent to which this linear system can be solved, solely depends on the conditioning of the matrix F .

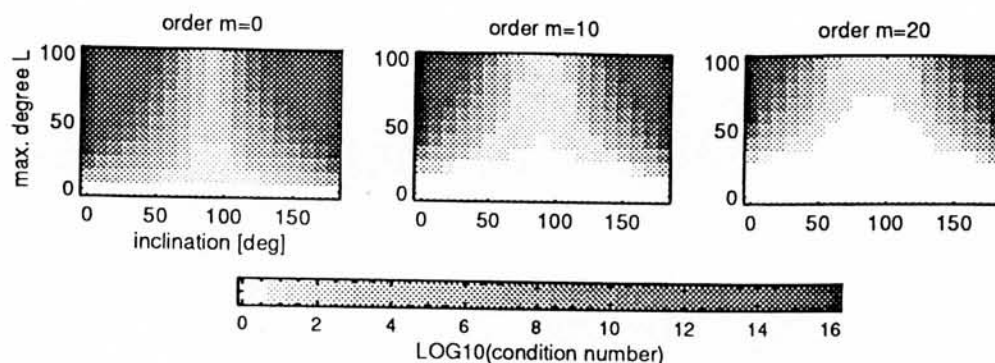


Figure 4.1. Conditioning of inclination function matrices, depending on inclination I , maximum degree L and order m .

Figure 4.1 displays the condition numbers of several such F -matrices, i.e. the ratio of the maximum and minimum singular values, depending on inclination, order and maximum degree. To be precise, the logarithm of the condition numbers has been rendered. This is a measure for the number of valid digits that can be lost during inversion of $a = Fc$, due to round-off and/or measurement errors (Strang 1986). Thus the gray value white (logarithm equals zero) represents the optimal situation; black denotes that F cannot be inverted.

With the inclination changing from 0° towards 180° , the conditioning moves from black to white (at $I = 90^\circ$) to black again. A polar orbit is optimal for inverting the linear system. Thus a good ground-track coverage, i.e. sampling of the earth, is translated in numerical terms as stability of the inclination function matrices F . The speed by which the conditioning changes with inclination depends on the order m . The smaller the order, the narrower the white band of good invertibility around $I = 90^\circ$.

The maximum degree L determines the size of the matrix F , which has size $(2L+1) \times (L-m+1)$, since $-L \leq k \leq L$ and $m \leq l \leq L$. Figure 4.1 shows that for smaller L , conditioning gets better, which is a general behaviour: the smaller the matrix, the better its conditioning. A 1×1 matrix has condition number 1. This fact will be of importance to processing GPS-type observables, which typically are included up till degree $L = 50$ or $L = 75$ at most. As can be seen from the plot, even for the case $m = 0$ the F -matrices for $L = 50$ are still stable when the orbit is not strictly polar. Thus, loosely speaking, GPS observables can always be inverted (for reasonable inclination).

5. A Least-Squares Formulation

A more thorough analysis of the polar gap problem must also include a bias consideration. Figure 4.1 explains that for low orders the matrices F become unstable. Their inversion will require regularization by means of adding a priori knowledge of the unknown potential coefficients, usually in the form of a signal power spectrum, such as Kaula's rule. Xu (1992) argues that this type of regularization gives rise to biased estimation of the unknowns.

Another aspect to be treated is aliasing. Due to limitation of the SH domain, only a limited number of coefficients C_{lm} will be estimated. The following general model is able to treat these aspects:

$$y = [A_1 \ A_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (5.1)$$

in which y denotes the vector of observations, x_1 the vector of coefficients to be solved, i.e. up to degree L , and x_2 the unresolved coefficients, in principle up to infinity. The vector x_2 (and thus the matrix A_2) have been added for analysis of the aliasing phenomenon. The design matrices A_1 , A_2 may come from a space-wise or from a time-wise formulation, which is irrelevant at this point. In the latter case they consist basically of the inclination functions, eq. (4.2), and of the specific transfer, due to the observable type. Least-squares inversion of the linear system (5.1) yields the estimate $\hat{x}_1 = By$, where the matrix B stands for:

$$B = (A_1^T P A_1)^{-1} A_1^T P \quad \text{in the unbiased case, or}$$

$$B = (A_1^T P A_1 + R)^{-1} A_1^T P \quad \text{with regularization matrix } R.$$

P denotes the weight matrix of the vector of observations y , which is the inverse of the covariance matrix Q_{yy} . Now three types of errors must be investigated in order to assess a realistic error for the polar gap:

1. A straightforward *noise propagation* gives the covariance matrix of the estimated unknowns:

$$Q_{x_1 x_1} = B Q_{yy} B^T = B P^{-1} B^T.$$

In the unbiased case, this reduces to the inverse of the normal matrix: $Q_{x_1 x_1} = (A_1^T P A_1)^{-1}$. In the biased case this will only approximately be true: $Q_{x_1 x_1} \approx (A_1^T P A_1 + R)^{-1}$. Also note that $(A_1^T P A_1 + R)^{-1}$ will be smaller than $(A_1^T P A_1)^{-1}$ (Xu 1992).

2. For the *bias* the following expression holds: $x_1 - \hat{x}_1 = (I - B A_1) x_1$. Without regularization and with full invertibility of the normal matrix (polar orbit or global coverage), one has $B A_1 = I$. This expresses unbiasedness. Since the unknowns are unknown indeed, the above expression cannot be evaluated. In a mean square sense one would have:

$$M\{(x_1 - \hat{x}_1)^2\} = (I - B A_1) C_{xx} (I - B A_1)^T.$$

The matrix C_{xx} represents a variance model, e.g. from Kaula's rule or from an actual gravity field model (OSU91a). The sum of propagated measurement noise $Q_{x_1 x_1}$ and bias variance is called MSE (mean square error), cf. (Xu 1992).

3. Considering the full linear model (5.1), where A_2 is not a zero matrix, part of the signal in vector x_2 will map onto the estimate $\hat{x}_1$. This *aliasing* is expressed by the second term in:

$$x_1 - \hat{x}_1 = (I - BA_1)x_1 + BA_2x_2.$$

The latter part has the following error measure: $BA_2C_{xx}A_2B$. Again, for the evaluation of it, one can insert a signal model of SH coefficients for the matrix C_{xx} .

In fig. 5.1 the three effects are displayed separately.

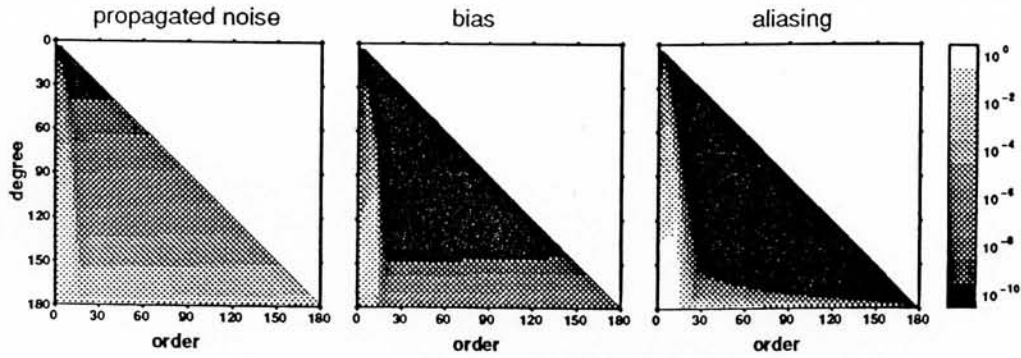


Figure 5.1. The three errors (an example with V_{zx} observed).

6. Error Propagation onto the Sphere

The polar gaps distort the low-order SH coefficients in the polar gap wedge. Although spherical harmonics are functions with *global* support, the polar gaps themselves are well-localized functions. This paradoxical situation will now further be investigated by a formal covariance propagation. The *full* covariance block matrix will be propagated onto geoid errors on the sphere. In this case the propagation result can be shown to be longitude-independent on the sphere. Moreover the propagated variance will be symmetric in the equator. It suffices therefore to represent the propagated geoid errors as a function of co-latitude θ only, on the interval $\theta \in [0; \pi/2]$.

The propagation results are separated in non-regularized (left hand side) and regularized (right hand side) simulations, cf. figure 6.1. Simulation #1 corresponds to a polar orbit (left picture in fig. 2.1) and #2 to a non-regularized simulation with $I = 97^\circ$. The corresponding simulations with

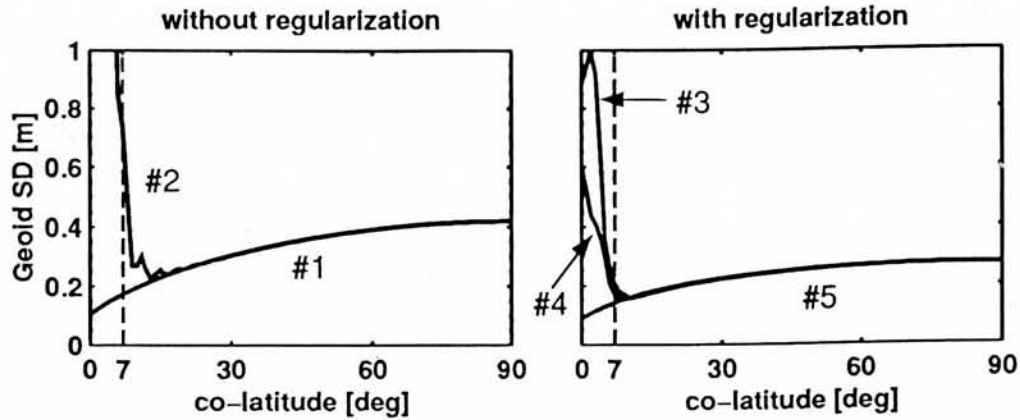


Figure 6.1. Propagated geoid standard deviations.

regularization (by Kaula's rule) are #5 and #3 respectively. For simulation #4 GPS-type observables are included. First compare the geoid standard deviations of the two polar simulations. It is seen that $\sigma_N^{\#5}(\theta)$ remains below $\sigma_N^{\#1}(\theta)$. This is an artefact, due to the regularization. Since the curves come from error propagation of the inverted normal matrices $Q_{x_1 x_1}$, it is basically a comparison of $(A_1^T P A_1 + R)^{-1}$ vs. $(A_1^T P A_1)^{-1}$, see item 1. above.

The full polar gap effect gets visible in the unregularized #2. The $\sigma_N^{\#2}(\theta)$ curve stretches far beyond the plot margin of 1 m (to around 10 m). At the edge of the polar gap however, at $\theta = 7^\circ$, the curve falls down towards the reference curve of simulation #1. From $\theta \approx 20^\circ$ onwards, both geoid error curves are equal. The lack of data within the polar gap does hardly propagate to the area outside of it. Only a rather small area around shows increased errors. Outside the gap, and the surrounding areas, the errors remain well-behaved, at the level of a polar orbit simulation. So even though the SH error spectrum, representing errors in the coefficients of global functions, the errors as a spatial function are confined to the gaps.

The error curve $\sigma_N^{\#3}(\theta)$ shows the influence of regularization. The maximum standard deviation within the gap is reduced to 1 m. At the same time, the radius of the transition zone at the polar gap edge is reduced. The curve conforms to the $\sigma_N^{\#5}(\theta)$ -curve directly beyond $\theta = 7^\circ$, whereas $\sigma_N^{\#3}(\theta)$ fluctuates till about $\theta = 20^\circ$. The GPS-like simulation has the same effect. The geoid standard deviation within the gap is even further reduced.

7. Conclusions

The effects of a slightly non-polar orbit on gravity field recovery, both in the spectral and in the spatial domain, are summarized by the following conclusions:

- In the SH *spectral* domain the low order coefficients are affected, which can be concluded from both a space-wise and a time-wise view. The spectral band of these coefficients has a wedge shape, whose upper limit in m -direction is represented by the rule of thumb $m_{\max} = |\theta_0|l$, (v. Gelderen & Koop 1996), which follows from a quadrature. For least-squares inversions the gap is even smaller than that, approximately by a factor 2.
- This wedge can be understood in numerical terms by ill-conditioned normal matrices, which are based on inclination function matrices F . The fact that the smaller the matrix (through smaller L), the better its condition, explains why GPS-type observations hardly suffer from the polar gap.
- Analytically, the wedge shape is explained by the shape of Legendre functions, which quickly tend towards zero at the higher latitudes when the order m increases. Thus the polar gap can only interfere with the low order Legendre functions. To a lesser extent, the degree l plays a similar role.
- The polar gap has relatively little influence on observation types which have reduced performance at the lower orders anyway, e.g. V_{yy} or V_{yz} .
- Irrespective of the global nature of spherical harmonics, the polar gap error in the *spatial* domain is delimited to the polar gap itself and a small surrounding transition area. Outside this area, the error characteristic is the same as for polar simulations. Thus regional gravity field recovery methods do not suffer from a polar gap.
- Regularization has a dual effect. It shrinks the transition area outside the gaps, and it reduces the error within the gaps, cf. fig. 6.1.

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Collocation with finite covariance functions

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Abstract

Preliminary prediction results using Least Squares Collocation (LSC) with finite covariance functions showed satisfactory agreement with corresponding results obtained with full covariance functions in the case of differentiation (e.g. upward continuation). But in the case of integration (e.g. computation of gravity and geoid from gradients) the results when using finite covariance functions are not so encouraging. This calls for a better modelling of the finite covariance functions. The computational savings which might be gained in finite covariance functions shows that LSC in the future could be used globally. Iterative solutions based on solutions where finite covariance functions are used may be the way to go.

1. Introduction

The Least Squares Collocation (LSC) is a very flexible method in gravity field modelling, which permits the use of data of different kinds and with different noise-characteristics. It also permits the computation of drift parameters and of error estimates of both computed values and parameters. These possibilities have been used extensively in earlier studies (see e.g. Arabelos and Tscherning, 1996a).

The method has however the drawback that one needs to solve a system of normal equations involving as many unknowns as the number of data and parameters. The matrices of normal equations are full matrices because the covariance functions even for very long distances are different from zero due to the harmonicity of the covariance function. This gives a heavy computation burden for simulations as well as for actual computations tasks to be solved in practice. The reason permitting the use of finite instead of full covariance functions is that after a certain distance the auto- and cross-correlation between the quantities related to the gravity field becomes very weak. Taking this into account the corresponding covariance values could be approximated by zeros. Other methods for gravity field modelling use base functions, like finite elements, which results in the use of sparse matrices. The same would be in the case if we in LSC gave up the requirements that our base functions are harmonic.

The computational savings which might be gained if finite covariance functions could be used would be large, (see e.g. Schuh, 1994). If we used a grid with $N \times N$ values, the full matrices would contain $N \times N \times (N \times N - 1)/2$ elements, i.e. of the order of N raised to the power of 4. If finite covariances are used, -and the observations are ordered in a reasonable manner, the matrices would contain non-zero elements of the order on N raised to the power 3. The computational savings would be even larger.

This fact made Sansò and Schuh (1987) propose to use the so-called finite covariance functions, i.e. covariance functions with values equal to zero for distances larger than a given radius. We have here studied their use for gravity field modelling and it seems that they may be useful in solving some tasks. The consequence of using finite covariance functions is obviously that we disregard long-distance correlations. Consequently for differentiation we could expect good results (gravity gradients from gravity) while for integration (geoid and gravity from gravity gradients) we could expect biased

results. This is also what we have found and what is new here is the magnitude of the errors we commit using these functions.

For the second- order derivatives of the anomalous potential T with respect to z of a local (x, y, z) coordinate frame (East, North, Up) we use the notation: $\partial^2 T / \partial z^2 = T_{zz}$.

2. Finite covariance functions modelling

The combination of ground and satellite data may be done using several different methods. The optimal one (in a very specific sense, see Moritz, 1980) is LSC. The cost of this “optimality” is as noted above that systems of equations with a dimension as large as the number of observations have to be solved. The equations to be solved have associate matrices, which are full, since the base functions (generally) used are harmonic. This is a consequence of that the anomalous gravity potential T is a harmonic function, and that the only function which is both zero in an open set and harmonic is the function which is identically zero.

One may however use more general functions, like finite elements. But then the observation that the Laplace equation applied to T is harmonic in every point must be added to the observations. One may also use functions which are good approximations to harmonic functions, or in the case of LSC, good approximations to the analytic covariance functions. This was suggested by Sansò and Schuh (1987), who suggested various kinds of covariance functions which were zero for distances larger than a certain distance.

Various methods for obtaining a finite covariance function have been suggested. One method is to multiply a “real” covariance function with one being zero at a given distance. However the functions suggested by Sansò and Schuh all give very strong changes of the original covariance functions.

But if one inspects the finite covariance functions suggested for use, they all look (for their first part) like any other covariance function. We therefore decided to try to use one of the most reasonable suggested. It has the following very simple expression (Sansò and Schuh, 1987, eq. 30):

$$\text{cov}(d) = \frac{1}{3}R^6\pi - \frac{1}{2}R^4d^2\pi + \frac{1}{3}\left(R^4d + \frac{4}{3}R^2d^3 - \frac{1}{12}d^5\right)\sqrt{R - \frac{d^2}{4}} + \left(R^4d^2 - \frac{2}{3}R^6\right)\sin\left(\frac{d}{2R}\right), \quad (1)$$

$\text{cov}(d) = 0$ for $d \geq 2R$,

with R a constant and d the variable distance.

The values to be selected in order to fix the covariance function is the quantity R and a scale-factor so the functions agree with the original one for $d = 0$. R may be selected so that we get the best approximation to the original function but we chose simply to fix R so that we had the same correlation distance d_1 for the finite and the original,

$$\text{cov}(0) 0.5 = \text{cov}(d_1) \quad (2)$$

Consequently, a number of simple subroutines have been written, finding the correlation distance d_1 from the original function and subsequently the value of R for the finite covariance function. These subroutines have been included in the program GEOCOL, version 11. Other routines were changed, so that they for a specific covariance function would use the finite one instead of the original.

3. Data

In order to study the performance of LSC with finite covariance functions in different cases, various experiments were done using different types of data. The following data sets were used:

1. Marine gravity data

From the GEODAS -Marine Geological and Geophysical Data from NGDC CD-ROM, published by NOAA, a number of 29,760 point gravity values have been extracted covering the area bounded by ($70^\circ \leq \phi \leq 83^\circ$, $-10^\circ \leq \lambda \leq 20^\circ$). These gravity values are coming from different Institutions and different surveys. For the main part of these data there is the information that the Gravity System is Potsdam and the reference system is International formula 1930. We assume that also in the cases that this information is missing, the gravity system and the reference gravity field are the same (Potsdam/Int.Form. 1930). However this assumption could introduce compatibility problem when merging all data sources. No information concerning the accuracy of the data is available. The statistics of this data set after the transformation to IGSN71/GRS80 and subtraction of the OSU91 is shown in Table 1.

Table 1. Statistics of the gravity data distributed in an $11^\circ \times 30^\circ$ area in the Arctic zone.
Unit is mGal.

	Mean	Std	Min.	Max.
original	34.99	24.94	-63.81	162.88
original -OSU91 (360)	-2.80	22.22	-86.20	133.63

From this data set a new data set was created by selecting point data closest to nodes of a $10' \times 10'$ grid. 3,710 point values were selected in this way having the distribution showed in Fig. 1. The gravity field from this data set after the subtraction of the OSU91A component is plotted in Figure 2.

In Central Mediterranean we have used the gravity data described in numerous earlier studies (see, e.g., Arabelos and Vermeer, 1996c). The test area in this case is bounded by ($33^\circ \leq \phi \leq 37^\circ$, $16^\circ \leq \lambda \leq 20^\circ$).

2. Airborne data:

A set of 5,292 point free-air gravity anomalies covering part of Greenland ($52.25^\circ \leq \phi \leq 73.68^\circ$, $-58.32^\circ \leq \lambda \leq -20.25^\circ$) is available from (Forsberg and Brozena, 1992). The measuring height was about 4 km and the distribution of the data is shown in Figure 3. These values are mean values along the plain tracks with a sampling rate corresponding to about 18 arcmin. From these values the contribution of an upgraded version of OSU91 Geopotential model (OSU91aC) to degree 180 was subtracted. In Table 2 the statistics of the original and the reduced data is shown.

Table 2. Statistics of the original and reduced airborne gravity anomalies in Greenland.
Unit is mGal.

	No.	Mean	Std	Min.	Max.
original	5292	25.108	35.338	-103.440	137.880
original - OSU91aC	5292	-2.348	27.851	-108.690	115.220

The free air gravity field at measuring altitude is shown in Figure 4.

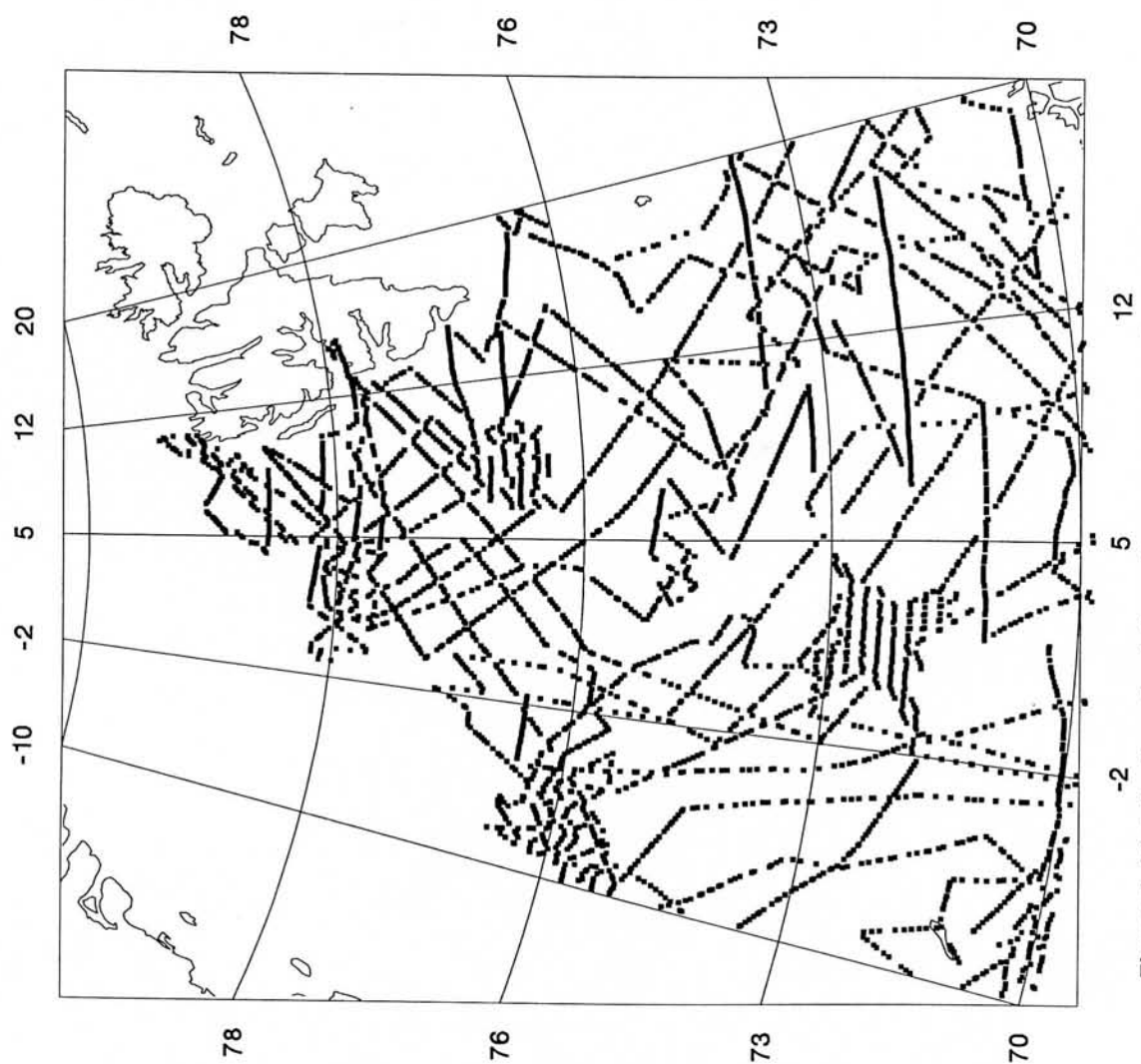


Figure 1: Distribution of point gravity measurements in the Arctic zone.

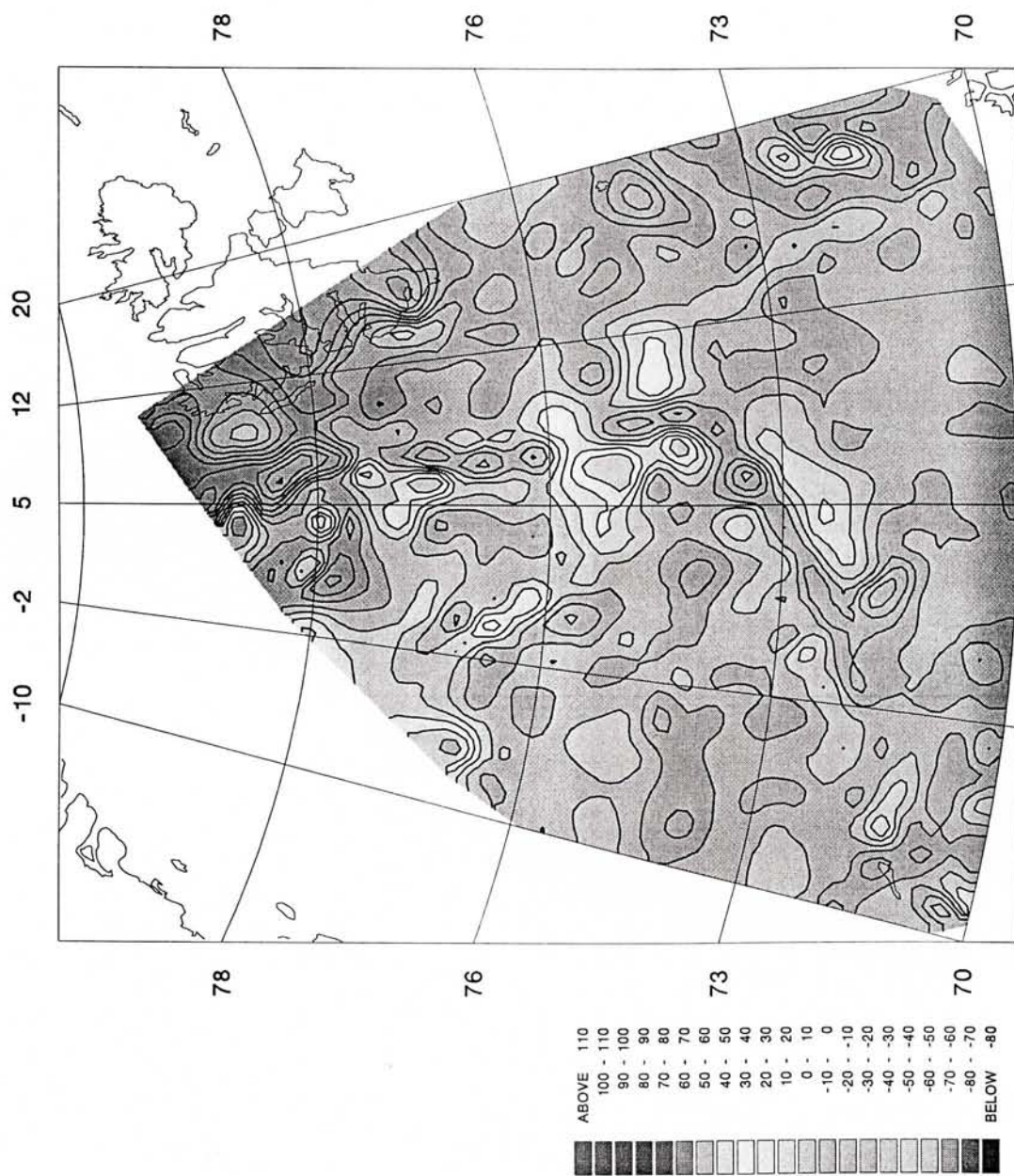
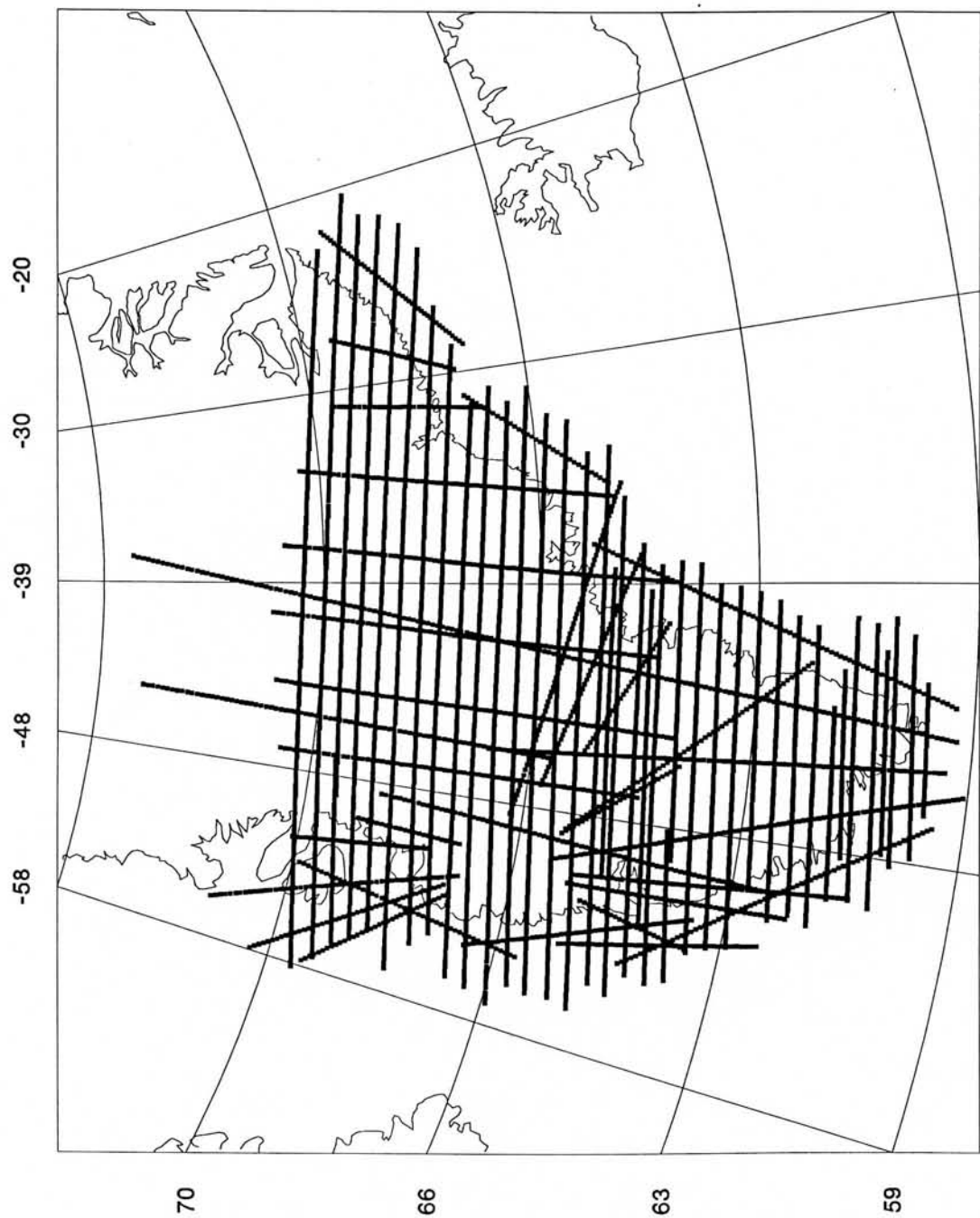


Figure 2: Free air anomalies - OSU91A in the Arctic zone



-48 -39 -30
Figure 3: Distribution of airborne gravity data over Greenland.

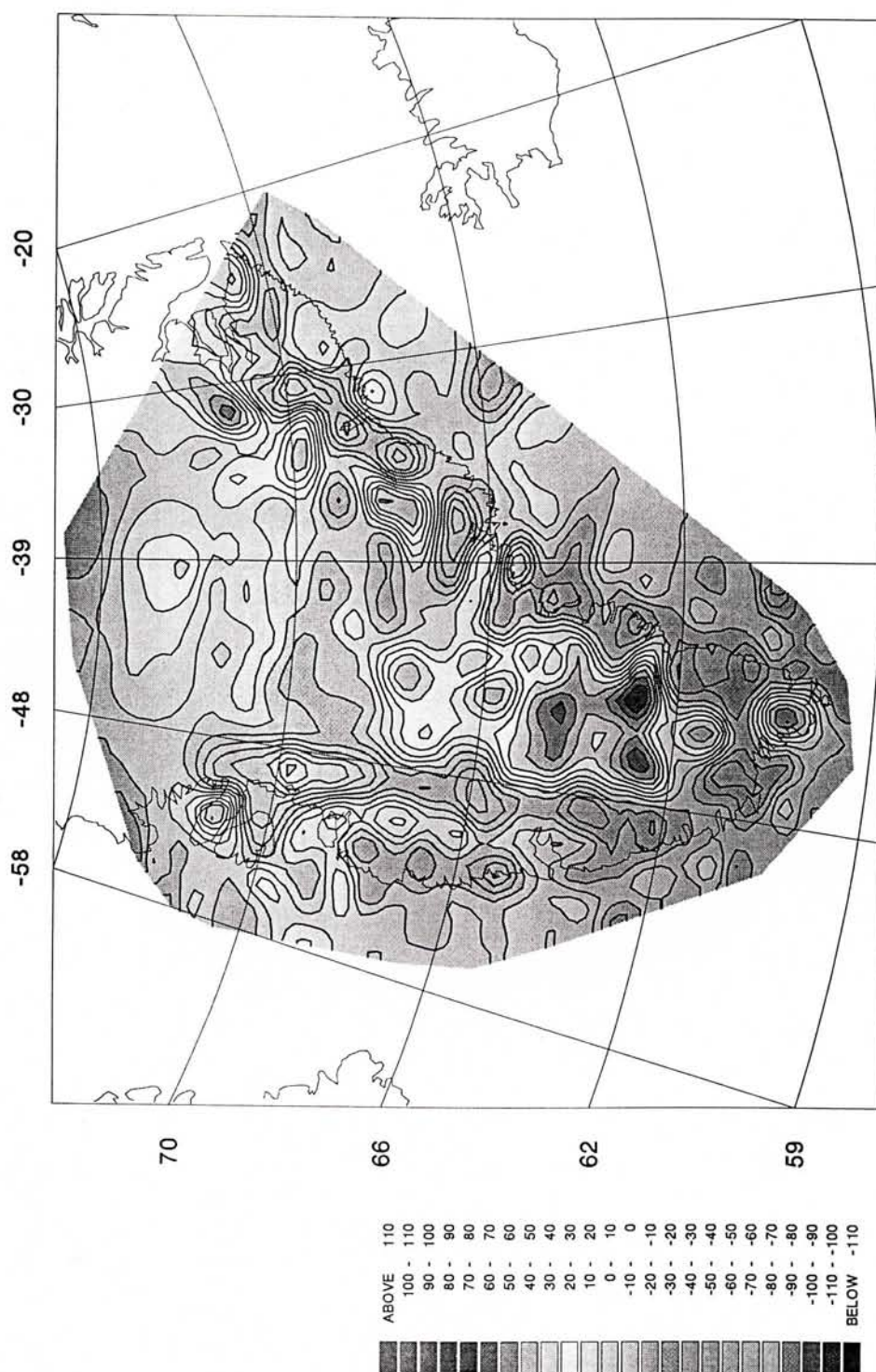


Figure 4: Airborne free air gravity field at 4 km altitude in Greenland (mGal).

3. Satellite altimeter data

A set of common adjusted ERS1/ERM, ERS1/GM and TOPEX data have been used to create a $5' \times 5'$ sea surface height grid in Central Mediterranean. The area bounded by $(33^\circ \leq \phi \leq 37^\circ, 16^\circ \leq \lambda \leq 20^\circ)$ is covered by this grid. The editing and adjustment procedure of these data is described in (Arabelos and Vermeer, 1996c).

4. Experiments with finite covariance functions

With the software described in section 2 and the data described in section 3, several experiments were carried out. First we computed T_{zz} at 200 km altitude from point surface gravity anomalies in the Arctic zone using full (traditional) and finite covariance functions. In Fig. 5. the model (full) covariance function of free air gravity anomalies at sea level reduced to OSU91A, the corresponding finite as well as their differences are shown. The results of the prediction are shown in Table 3 for both, (a) full and (b) finite covariance functions. The results in terms of the mean value and standard deviation of differences (a)-(b) were encouraging enough to continue the finite covariance investigations.

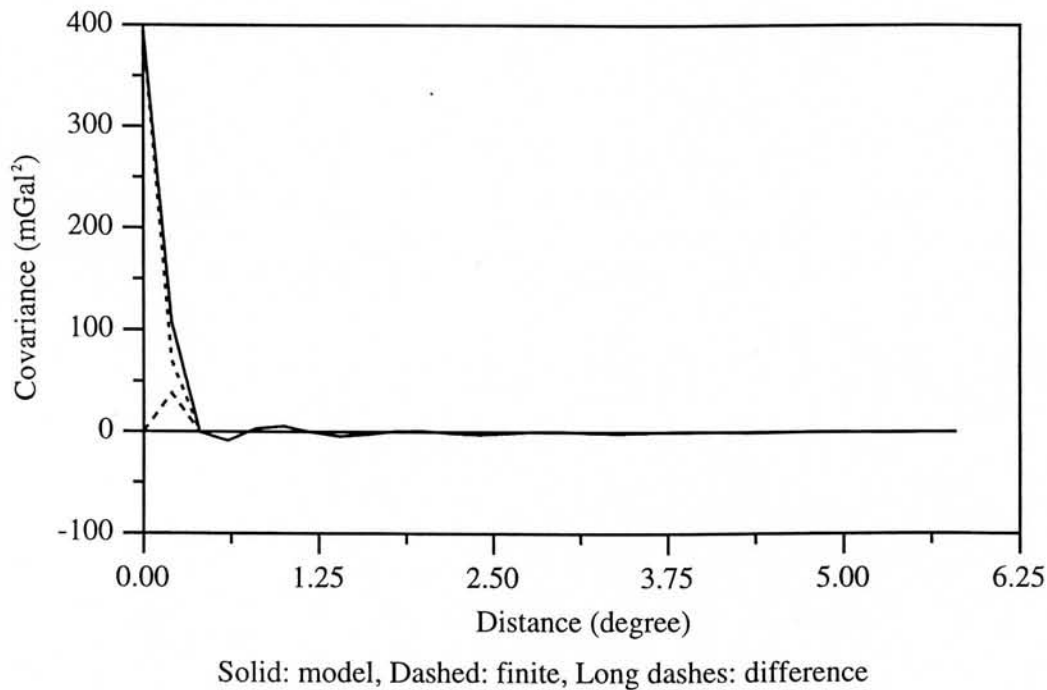


Fig. 5. Covariance functions of gravity anomalies in the Arctic zone.

Table 3. Statistics of 496 T_{zz} values predicted from point gravity anomalies in the Arctic zone. (a) full and (b) finite covariance functions were used. Unit is EU.

	Min.	Max.	Mean value	Std
(a)	-0.0108	0.1459	0.0743	0.0361
(b)	-0.0410	0.1438	0.0575	0.0430
(a) - (b)	0.0012	0.0321	0.0167	0.0078

The differences (a) - (b) are shown in Fig. 6. These differences are small but not negligible.

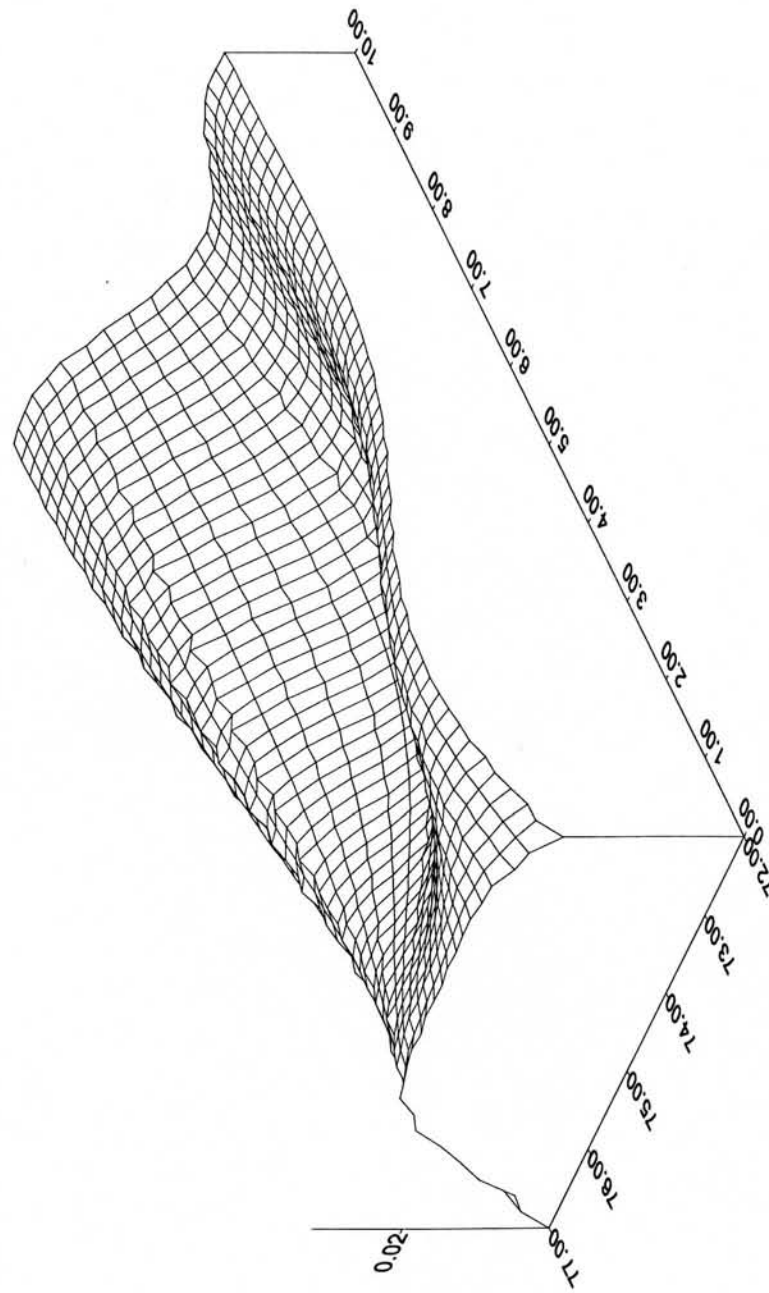


Fig. 6. Differences between T_{zz} at 200 km predicted from point gravity data (Arctic zone), using
(a) full and (b) finite covariances. Unit is EU.

Subsequently, one further upward continuation experiment using (a) full and (b) finite covariance functions was executed using the airborne data described in section 3. T_z field at 400 km altitude was predicted from airborne gravity anomalies in Greenland. The corresponding covariance functions (model, finite and their differences) of gravity anomalies at zero level, reduced to OSU91C are shown in Fig. 7. The results of this experiment from (a), (b) and (a)-(b) are shown in Table 4. The differences (a) - (b) are shown in Fig. 8.

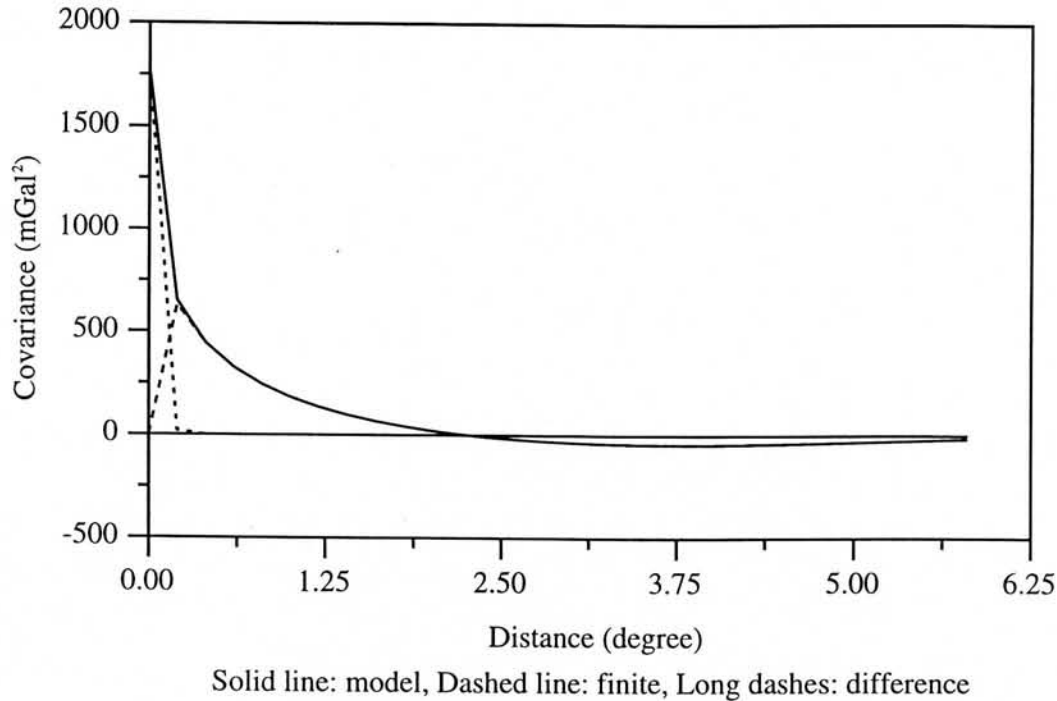


Fig. 7. Covariance functions of gravity anomalies in Greenland.

Table 4. Statistics of T_z values (248) predicted from airborne gravity anomalies in Greenland.
(a) full and (b) finite covariance functions were used. Unit is mGal.

	Min.	Max.	Mean value	Std
(a)	-0.2400	0.1700	-0.0178	0.1149
(b)	-0.5800	0.6600	0.1384	0.3338
(a) - (b)	-0.5000	0.3400	-0.1562	0.2214

The results of this upward continuation experiment in terms of the statistics of Table 4 are not so encouraging as in the previous experiment. Moreover, there is an -almost constant- difference between the collocation error estimates: In (a) the error estimate is 0.02 mGal larger than in (b).

We also conducted a downward continuation experiment. Gravity and geoid values were predicted in Greenland at ground level from simulated T_{zz} at 200 km altitude. These simulated values were predicted from the airborne gravity data described in section 3, using LSC with full covariance functions (see Arabelos and Tscherning, 1996b). The 496 T_{zz} values were distributed on a $20' \times 20'$ grid and considered to have an accuracy equal to 0.01 EU (Arabelos and Tscherning, 1996). The area covered by the T_{zz} grid (part of Greenland) is bounded by ($64^\circ \leq \phi \leq 69^\circ$, $-50^\circ \leq \lambda \leq -40^\circ$). The T_{zz} field is shown in Fig. 9. In Fig. 10 the model (solid line) and the finite (dashed line) covariance functions of the T_{zz} values at 200 km altitude are shown. The differences between model and finite covariance functions are depicted in the same figure (10).

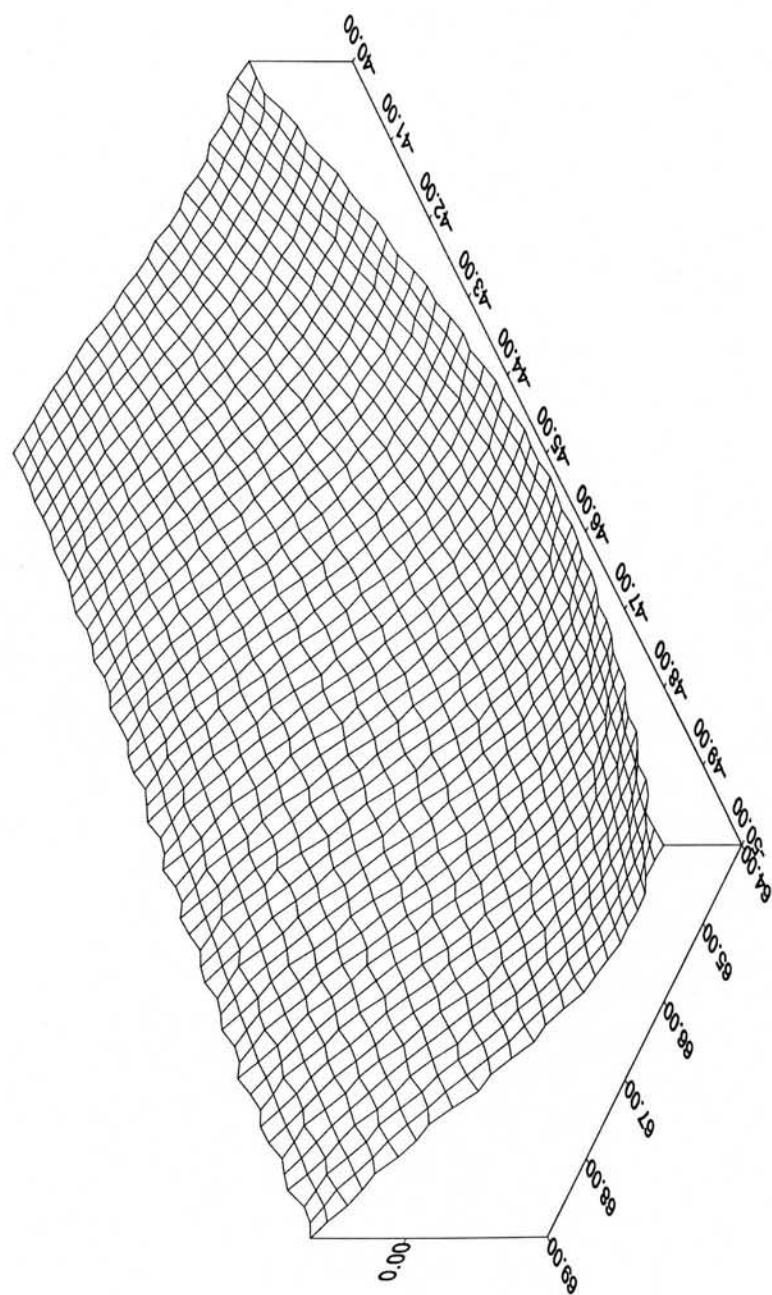
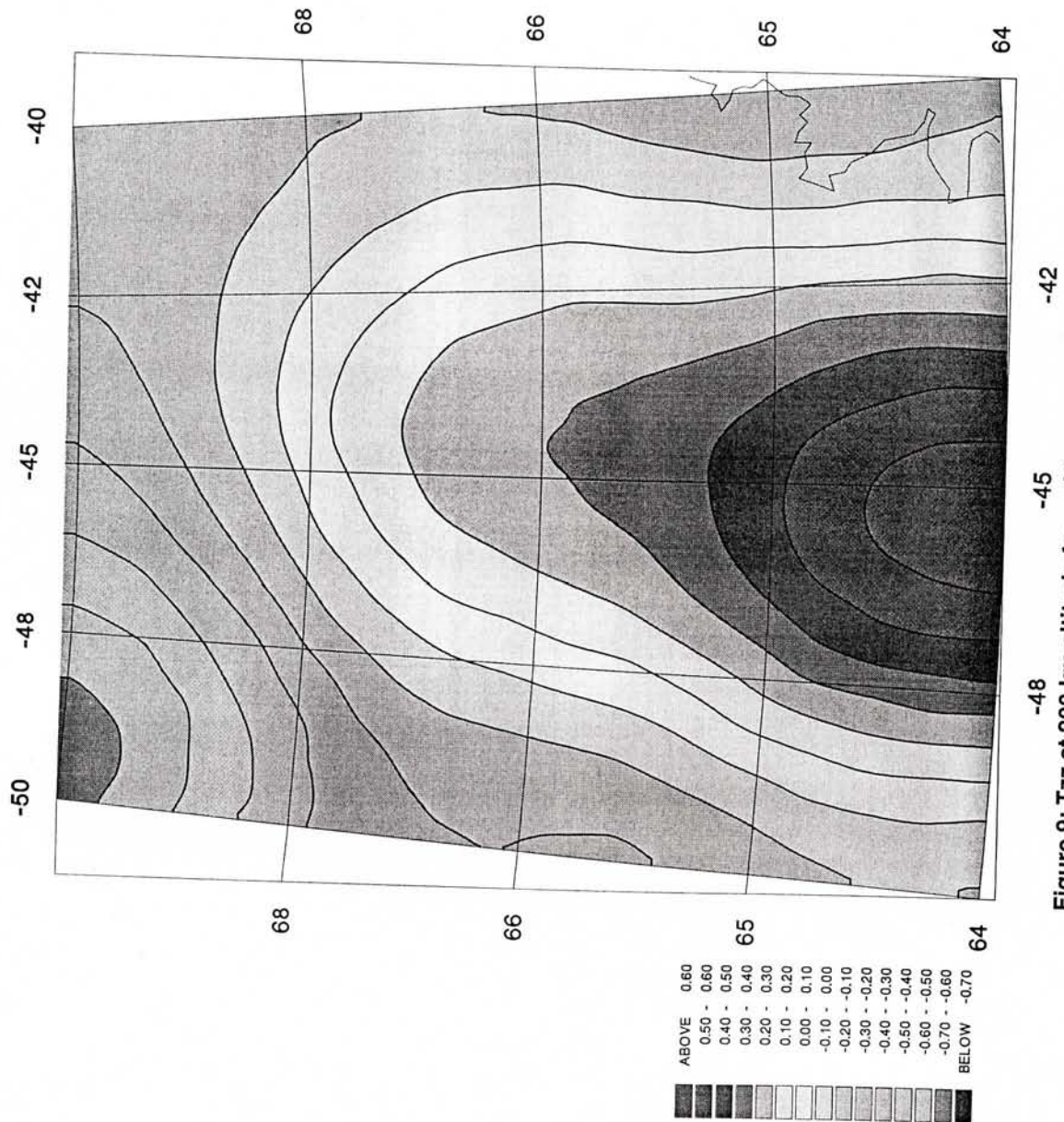


Fig.8. Differences (a) - (b) between T_z at 400 km predicted from airborne gravity anomalies (mean values along tracks) in Greenland, Full covariance functions have been used in (a), finite covariance functions in (b). Unit is mGal.



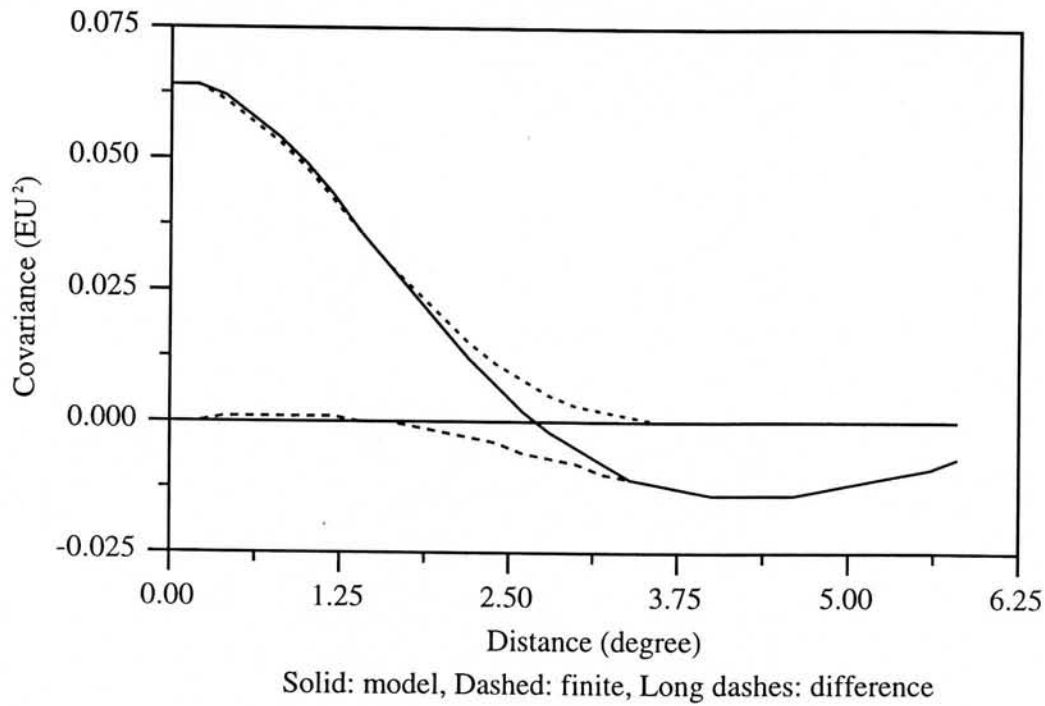


Fig. 10. Model and finite covariance functions of T_{zz} at 200 km altitude, as well as their differences.

Free air gravity anomalies and geoid heights were predicted on a $10' \times 10'$ grid covering the area ($65^\circ \leq \phi \leq 68^\circ$, $-48^\circ \leq \lambda \leq -42^\circ$) (703 values). Using finite covariances 12,675 covariances were set to zero. The collocation error estimation in the case of gravity prediction using full covariance functions was equal to about 21 mGal. The corresponding quantity in the case of finite covariance functions was 2 mGal higher (equal to about 23 mGal). There are large discrepancies between the corresponding quantities predicted using (a) full and (b) finite covariance functions as it is shown in Tables 5 and 6.

The differences between the predicted gravity anomalies using full and finite covariance functions are shown in Fig. 11, while the corresponding differences for the geoid are shown in Fig. 12.

Table 5. Statistics of the results of gravity prediction at ground level from simulated T_{zz} at 200 km altitude in Greenland. (a) full and (b) finite covariance functions were used. Unit is mGal.

	Min.	Max.	Mean value	Std
(a)	-34.130	58.960	17.046	15.924
(b)	-26.440	53.380	18.308	13.553
(a) - (b)	-27.180	15.790	-1.262	9.974

Table 6. Statistics of the results of geoid prediction from simulated T_{zz} at 200 km altitude in Greenland. (a) full and (b) finite covariance functions were used. Unit is m.

	Min.	Max.	Mean value	Std
(a)	-3.40	4.79	1.40	1.43
(b)	-3.30	5.99	2.01	1.88
(a) - (b)	-1.92	0.46	-0.61	0.68

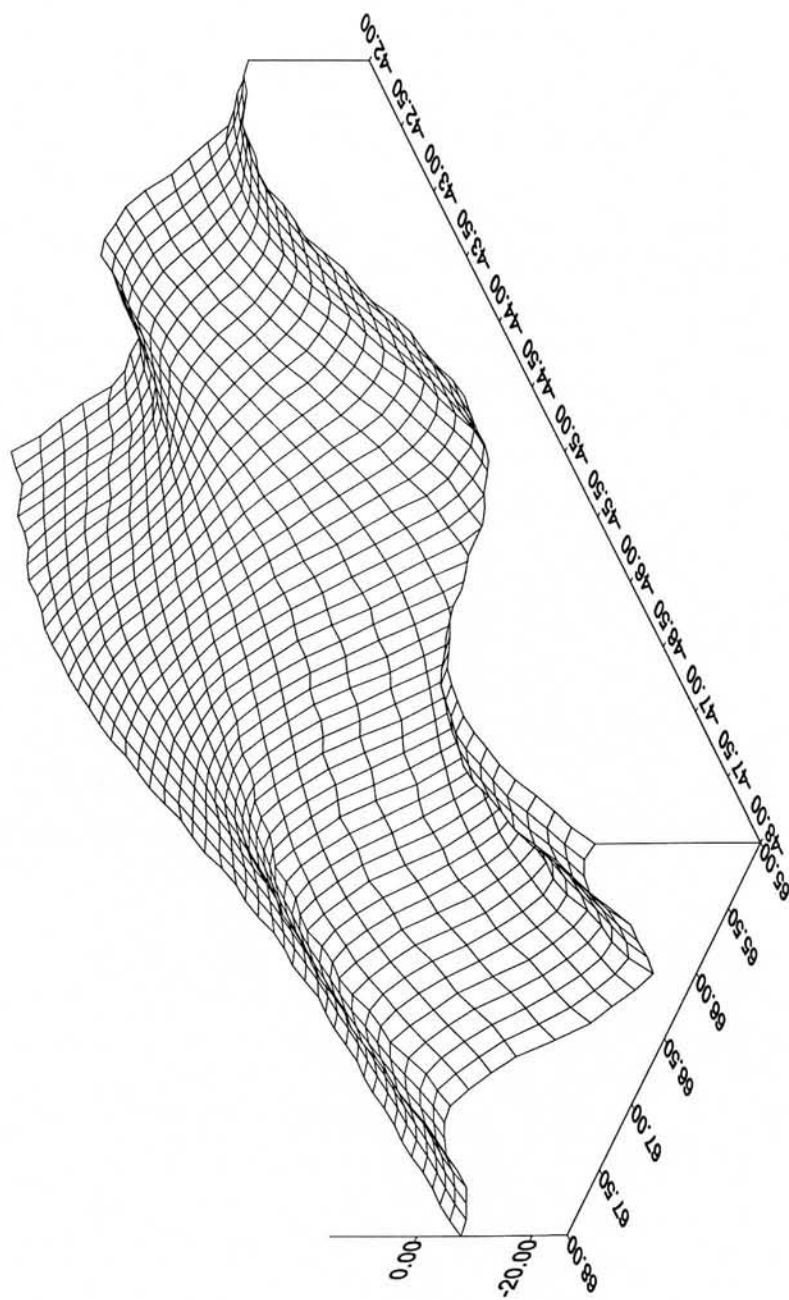


Fig. 11. Differences (a) - (b) between surface free air gravity anomalies in Greenland predicted from T_{zz} at 200 km altitude using (a) full and (b) finite covariance functions. Unit is mGal.

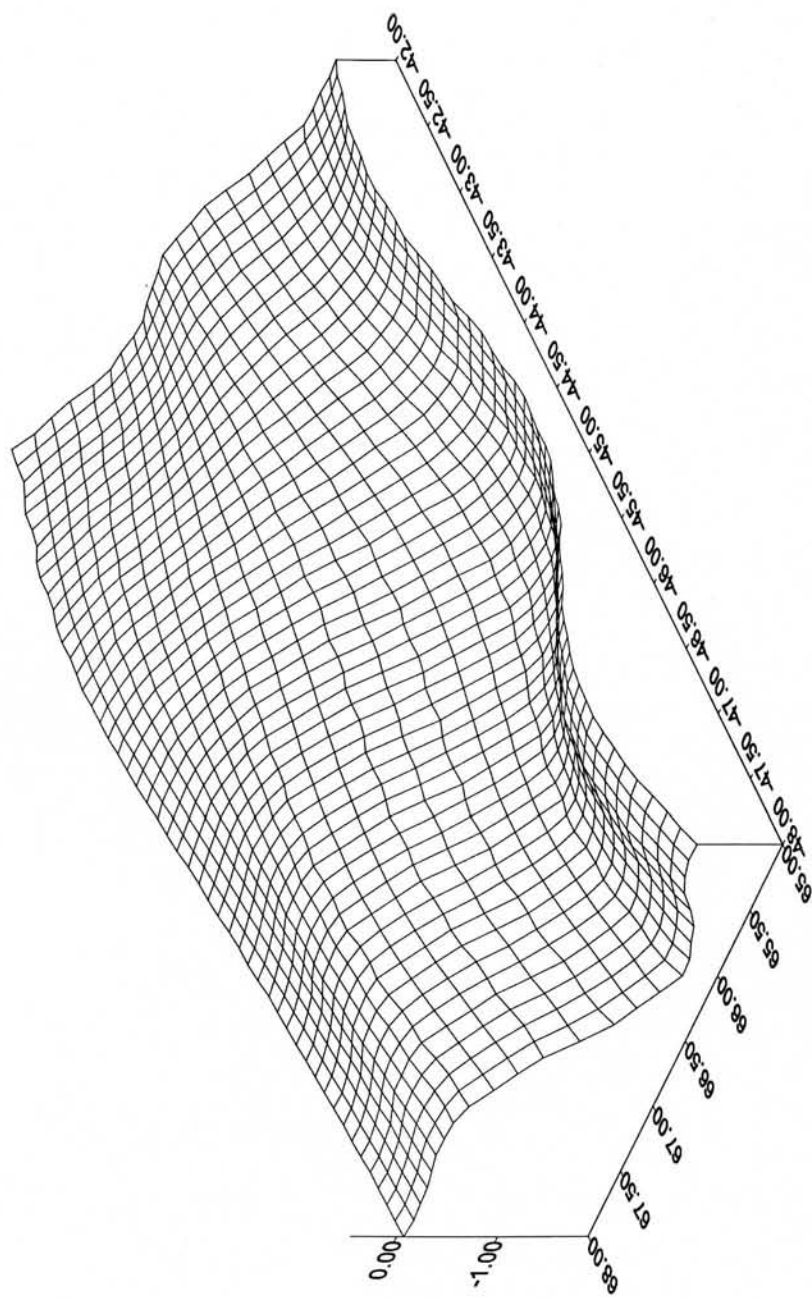


Fig. 12. Differences (a) - (b) between geoid heights in Greenland predicted from Tzz at 200 km using
 (a) full and (b) finite covariance functions. Unit is m.

Finally we performed experiments in the Central Mediterranean using sea gravity and satellite altimeter data. The experiments concern the prediction of gravity from altimetry and vice-versa, using full and finite covariance functions. The results of these predictions are compared with observed data in order to have a realistic estimation of the effect of cutting the covariance function in such a way as it was described in section 2. We started with the prediction of geoid heights from the gravity data for the Central Mediterranean, using full as well as finite covariance functions. The prediction was made on a $5' \times 5'$ grid (1,369 points) covering the inner $3^\circ \times 3^\circ$ area bounded by $(33.5^\circ \leq \phi \leq 36.5^\circ, 16.5^\circ \leq \lambda \leq 19.5^\circ)$. The accuracy adopted for the gravity data was 5 mGal in both cases (full and finite covariance functions). In Fig. 13 the model and the finite covariance functions of gravity anomalies are shown. It should be noted that the collocation error estimation was varying between 0.12 and 0.13 m for the prediction with full covariances and between 0.11 and 0.14 m for the prediction with finite covariances.

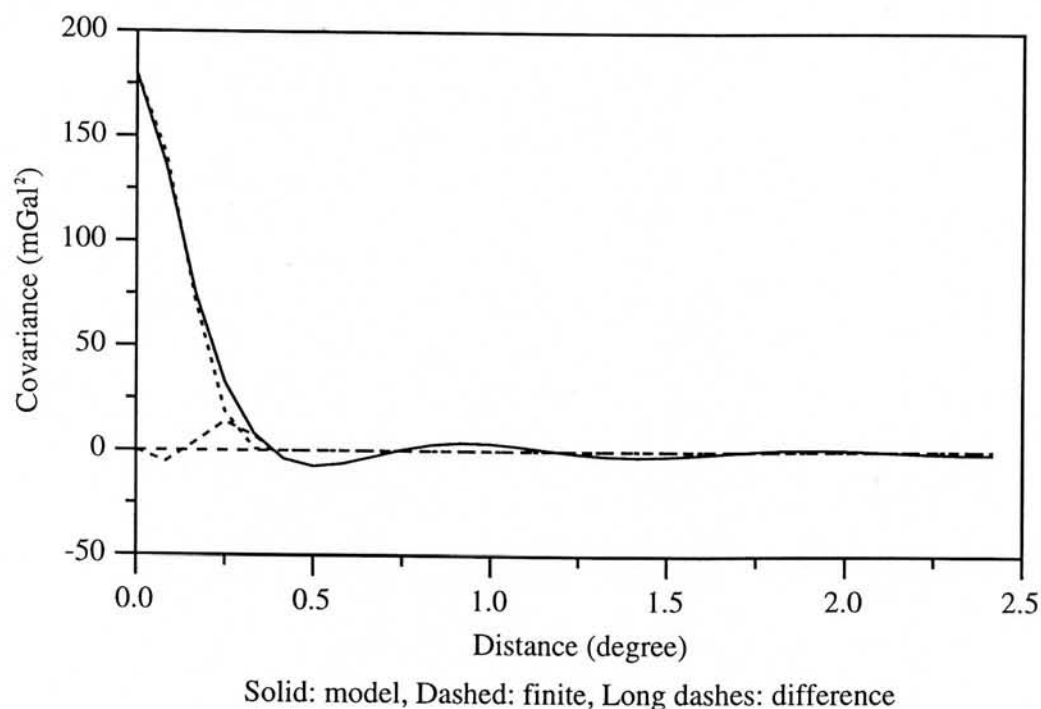


Figure 13. Covariance functions of gravity anomalies in the Central Mediterranean

The statistics of the comparison between predicted geoid heights and satellite sea surface heights is shown in Table 7. From Table 7 it is shown that the results of the prediction in terms of the mean value and standard deviation of the differences between predicted geoid heights and satellite sea surface heights using full and finite covariance functions are very close although we have to do with an integration procedure.

Table 7. Statistics of the differences between geoid heights predicted from sea gravity data and satellite sea surface heights, using full and finite covariance functions. Unit is (m).

	Observations	Full covariance functions		Finite covariance function	
		Predictions	Difference	Predictions	Difference
Mean value	0.20	0.30	-0.10	0.26	-0.06
Std	0.30	0.28	0.10	0.31	0.11

The opposite way, i.e. the prediction of gravity from altimetry using the same covariance functions as in the previous experiment was not an encouraging experience. We started adopting a realistic error estimation for the satellite altimeter grid equal to 0.1 m. The comparison of the predicted using full covariance functions gravity anomalies with ground truth at 1,369 points gave the very good agreement reported in earlier studies (e.g., Arabelos and Vermeer, 1996c). The standard deviation of the differences was equal to 4.87 mGal. The collocation error estimation in this case was equal to about 7 mGal. Using finite covariance functions with the same accuracy assumption for the satellite altimeter data the results of the prediction were strongly biased. Supposing that the problem was due to high frequency features of the altimeter data we tried to apply a damping procedure (similar to that used by FFT techniques) by filtering the data. In the case of LSC this could be done by increasing the error estimation of the data. After some experiments with different accuracy assumptions for the altimeter data we produced the results given in Table 8.

Table 8. Change of the prediction results by adopting different accuracy values of the altimeter data. Unit is mGal.

Error adopted for the altimeter data	Collocation error estimation	Std of the predicted gravity	Std of the differences (observed-predicted)
0.1	500	73.15	65.74
0.4	40	19.36	14.17
0.6	9	10.76	8.65
0.7	10	8.59	8.23
1.0	12	5.32	8.88

The case giving the best results corresponds to an accuracy equal to 0.7 m which is about 7 times higher than the actual accuracy of the altimeter data. In Table 9 the results of the prediction using full (with accuracy assumption for the altimeter data equal to 0.1 m) and finite (with accuracy assumption for the altimeter data equal to 0.7 m) covariance functions are summarized. From Table 9 it is shown that even after the filtering of the high frequencies, the use of the finite covariance function gives no satisfactory results. On the other hand the number of covariances set to zero was 1,845,780. The computational saving gained in this case might would be extremely large. Obviously the reason is the considerable discrepancies between full and finite covariance functions (see Fig. 14).

Table 9. Statistics of the differences between gravity anomalies predicted from satellite SSH, using full and finite covariance functions. 0.7 m error adopted for SSH. Unit is mGal.

	Observations	Full covariance functions		Finite covariance function	
		Predictions	Difference	Predictions	Difference
Mean value	2.40	1.95	0.44	2.52	-0.11
Std	11.79	10.35	4.87	8.59	8.23

The results - good or bad - of these preliminary experiments are according to expectations. Cutting off the tails of the covariance functions, we disregard long-wavelength correlations. Consequently the finite covariance functions will work well when we differentiate (e.g. in the case of upward continuation), but not so well when we integrate (e.g. in the case of computation of gravity and geoid at ground level from gravity gradients at satellite altitude).

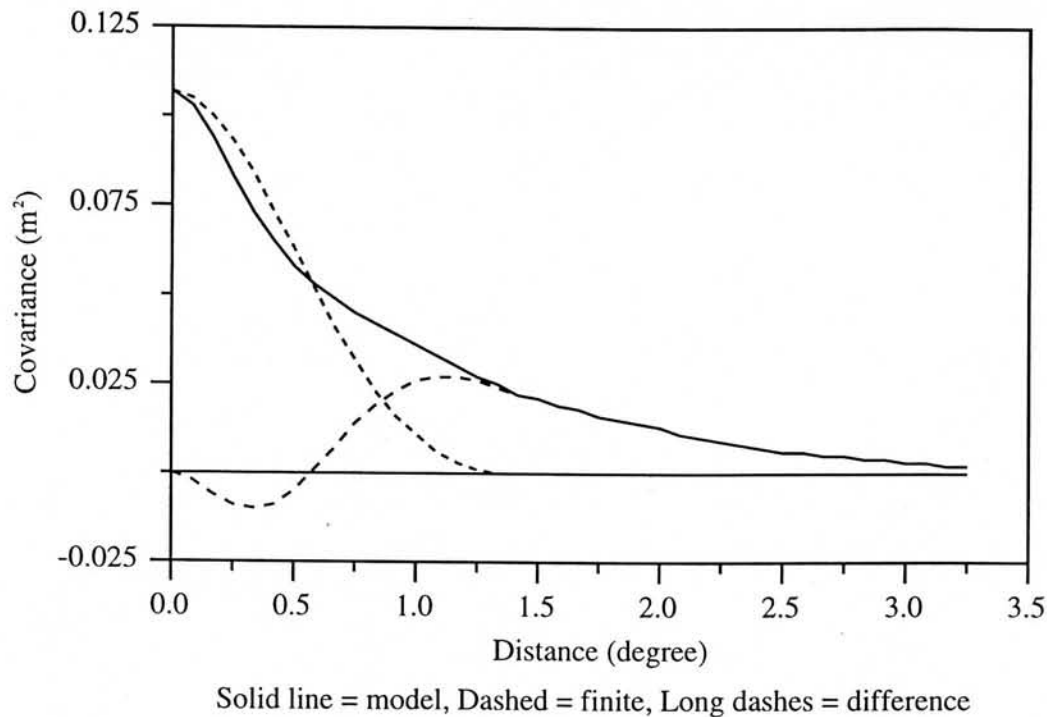


Fig. 14. Covariance functions of satellite altimeter data in Central Mediterranean

Considering all the previously reported results it is obvious that further investigation needed for a better modelling of the finite covariance functions. However, in the case of a good agreement between finite and full covariance functions, finite covariance functions work properly also when we integrate.

3. Conclusion

The first experiments with finite covariance functions were encouraging in the sense that here is a research track to be followed. We only used the most crude finite covariance functions, where there exist several alternatives which it would be interesting to study in the future. Also finite covariance functions which have a consistent height dependence should be tried (Schuh, 1989). The preliminary results confirms the intuitive expectations one might have to the consequence of using finite covariance functions, namely that long wavelength information is lost, or could be damaged.

If methods like LSC in the future should be used globally, one has to find ways to reduce the requirements on both storage and computer time. Iterative solutions based on solutions where finite covariance functions are used may be the way to go.

Acknowledgements. Thanks to R. Forsberg (National Survey & Cadastre, Denmark) for providing gravity data. Part of this paper has been carried out in the framework of a project supported by ESA/ESTEC contract JP/95-4-137/MS/NR.

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The problem of polar caps in a gradiometric mission

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1 Introduction

In the analysis of the data coming from a gradiometric mission, the polar caps problem is defined as the study of the influence of the missing observations in polar caps on the final estimate of the coefficients of the harmonic expansion of the anomalous potential T .

This problem is in principle a pure "conditioning problem". In fact from the theoretical point of view, giving the values of a harmonic function on any portion of a sphere all embedded in the harmonicity domain, univocally determines the harmonic function itself in its harmonicity domain (cfr. Sacerdote -Sansó', 1990). Naturally in practice even missing a small portion of data kills the B.V.P. approach; this however becomes again feasible when, as done in all other methods, the signal is assumed to be band limited, i.e. in case of a harmonic function, this is truncated at a maximum degree L . The real problem therefore was to see what is the influence of polar caps on the accuracy of the estimated coefficients.

The least squares approach seems to be able to control the error in the geoid at least when we stay away from polar caps, as shown in the simulation presented in this paper.

2 Simulation and error analysis

The simulation has been performed with the following characteristics:

- the $\frac{\partial^2 T}{\partial r^2}$ has been simulated at the centers of a regular grid of $0^\circ.5 \times 0^\circ.5$ ranging from $6^\circ.5$ to $173^\circ.5$, using for T the OSU91A model from degree 13 to degree 180, complete for all orders;
- the altitude of the orbit of the satellite was $H = 290$ km with an inclination equal to $96^\circ.5$;
- the observation rate of 1 measurement every 3 s for a 6 months span provides a number of 5184000 observations;
- the measurement noise was taken at the level of $10^{-12} s^{-2} (= 10^{-3} E.U.)$: correspondingly, a noise of $10^{-3} E.U. / \sqrt{N_\theta}$ (N_θ = number of observations in the

block, at colatitude ϑ) has been directly simulated at the centre of each grid block.

The set of pure observations $\left\{ \frac{\partial^2 T}{\partial r^2}(\varphi_i, \lambda_k) \right\}$ has then been used into our least squares processor to verify whether it could supply the correct coefficients. Afterwards, we have computed the mean square root of the relative error of the reconstructed coefficients (numerical error):

$$\mathcal{E}(\ell) = \sqrt{\frac{\sum_{m=-\ell}^{\ell} (T_{\ell m} - \hat{T}_{\ell m})^2}{\sum_{m=-\ell}^{\ell} T_{\ell m}^2}} \quad (1)$$

where $T_{\ell m}$ are the OSU91A coefficients and $\hat{T}_{\ell m}$ are the estimated coefficients.

The result is displayed in Fig. 1: $\mathcal{E}(\ell)$ is definitely below the 10^{-6} level, i.e. it is practically equal to zero.

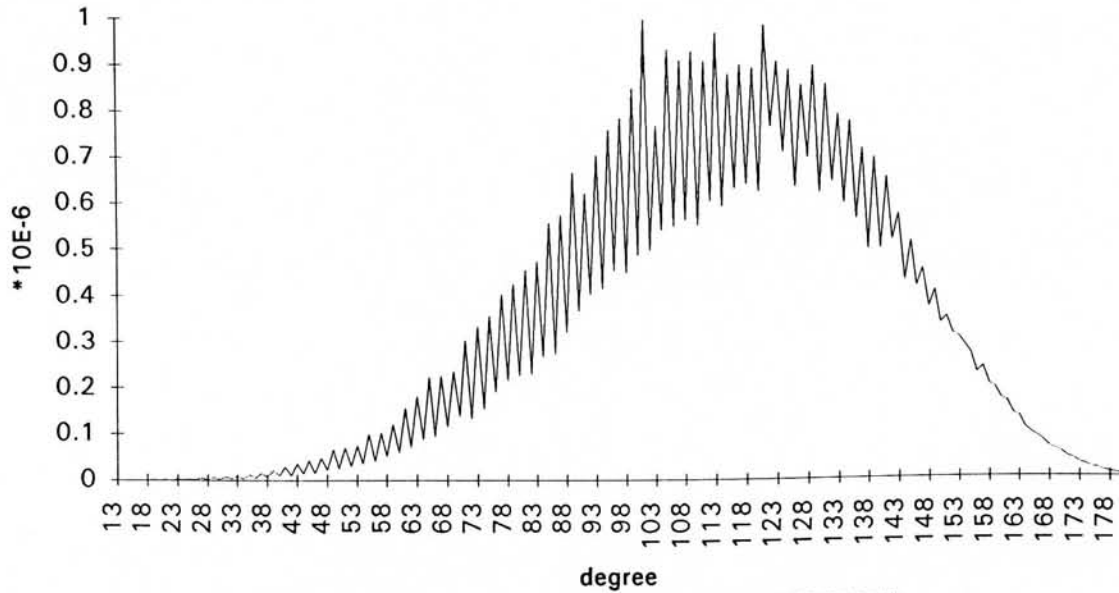


Figure 1: Numerical error per degree.

Then the analysis has been repeated for the measurement errors, which give rise to errors in the coefficients equal to $\delta T_{\ell m}$.

First we have computed the usual signal degree variances and noise degree variances (as propagated to the coefficients). The square root of the ratio between the two

$$\frac{S}{N}(\ell) = \sqrt{\frac{\sum_{m=-\ell}^{\ell} \hat{T}_{\ell m}^2}{\sum_{m=-\ell}^{\ell} \delta T_{\ell m}^2}} ; \quad (2)$$

is plotted in Fig. 2 (lower curve).

This signal to noise ratio is indeed too low in the range $90 \leq \ell \leq 150$; however this is basically due to the unduly large values of the errors on the low order coefficients. In fact repeating the same computation, skipping the orders below $\ell = 5$, gives the higher curve which is quite acceptable.

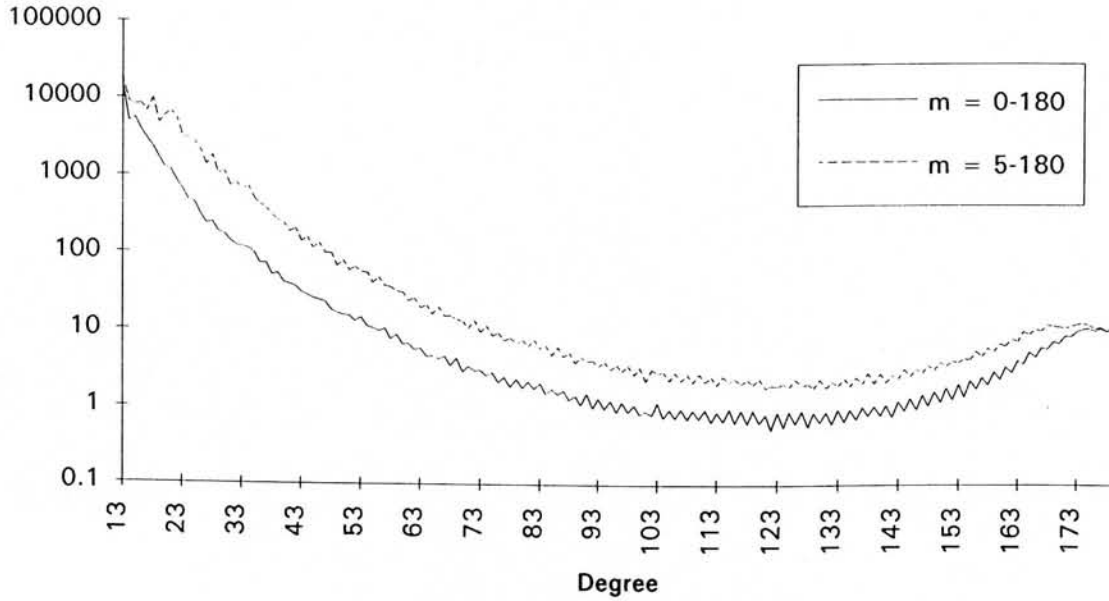


Figure 2: Signal to noise ratio per degree.

This pattern of the problem has suggested to compute “order variances” for both the signal, $\hat{T}_{\ell m}$, and the “noise coefficients”, $\delta T_{\ell m}$; the representation of the corresponding square root of the ratio

$$\frac{S}{N}(m) = \sqrt{\frac{\sum_{\ell=m}^L \hat{T}_{\ell m}^2}{\sum_{\ell=m}^L \delta T_{\ell m}^2}} ; \quad (3)$$

is given in Fig. 3 and Fig. 4, where we see that our guess was basically correct.

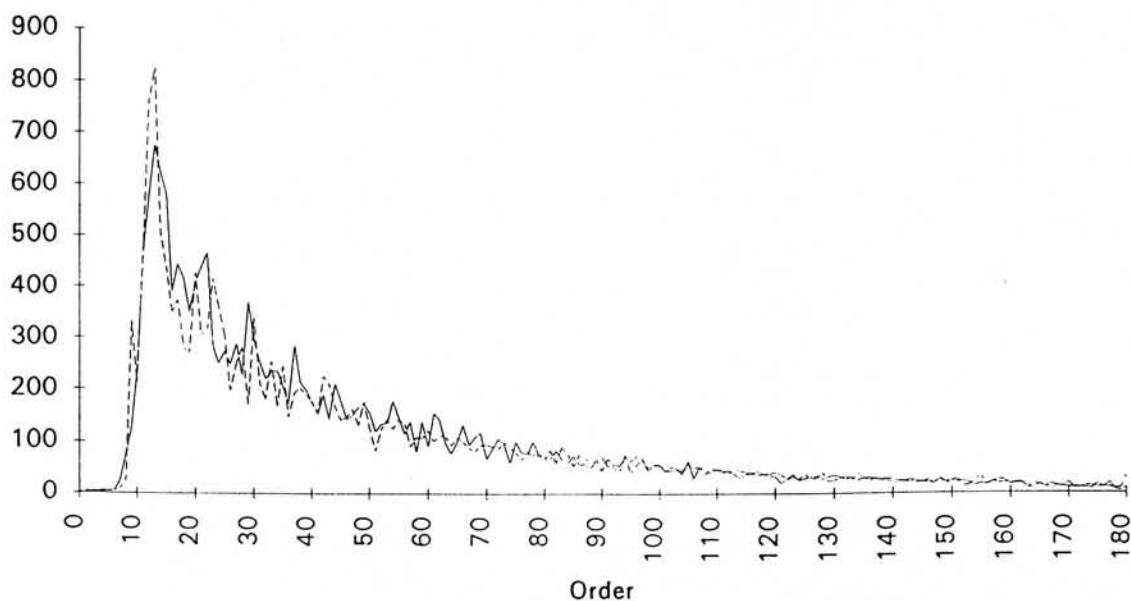


Figure 3: Average signal to noise ratio per order: the solid line is for $m \geq 0$ and dashed line is for $m < 0$.

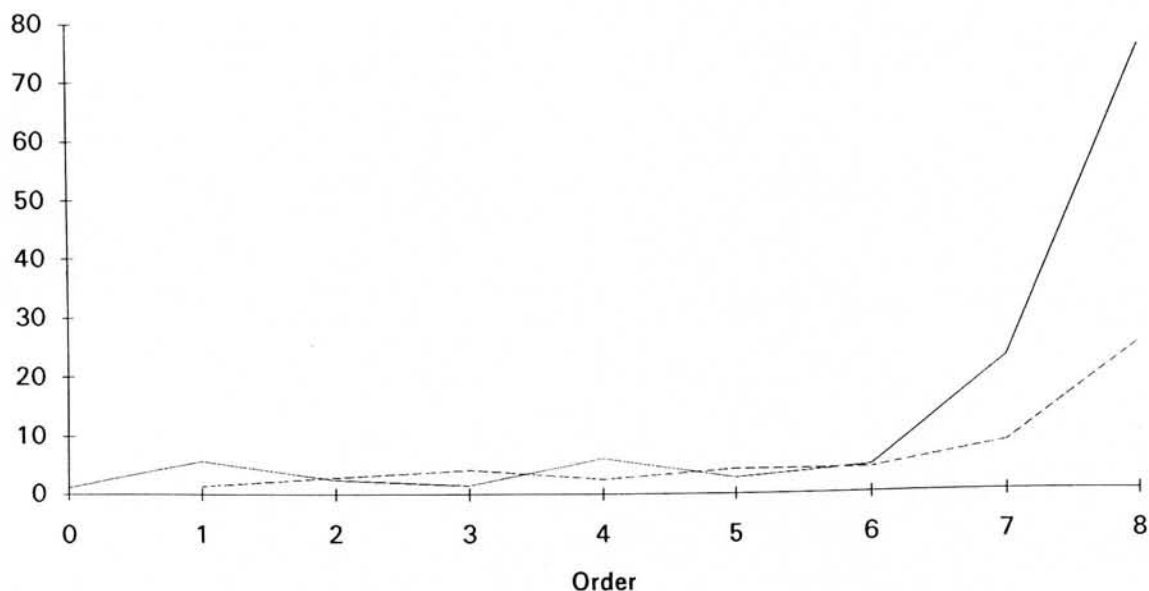


Figure 4: Average signal to noise ratio order by order (first 8 orders).

Yet this type of analysis is not really able to completely catch the pattern of the error implicit in this data treatment; in fact we suspected that as a matter of fact the large errors in the low order coefficients could build a “very wrong” reconstruction of the signal at latitudes where no data is given, i.e. in polar caps.

Following an idea of R. Rummel, we were therefore interested in seeing what was the behaviour of the error in terms e.g. of geoid at latitudes far from polar caps. In order to represent this error as a function of latitude, its average value and r.m.s. were computed fixing ϑ every 2° , from $\vartheta = 10^\circ$ to $\vartheta = 170^\circ$. The results are shown

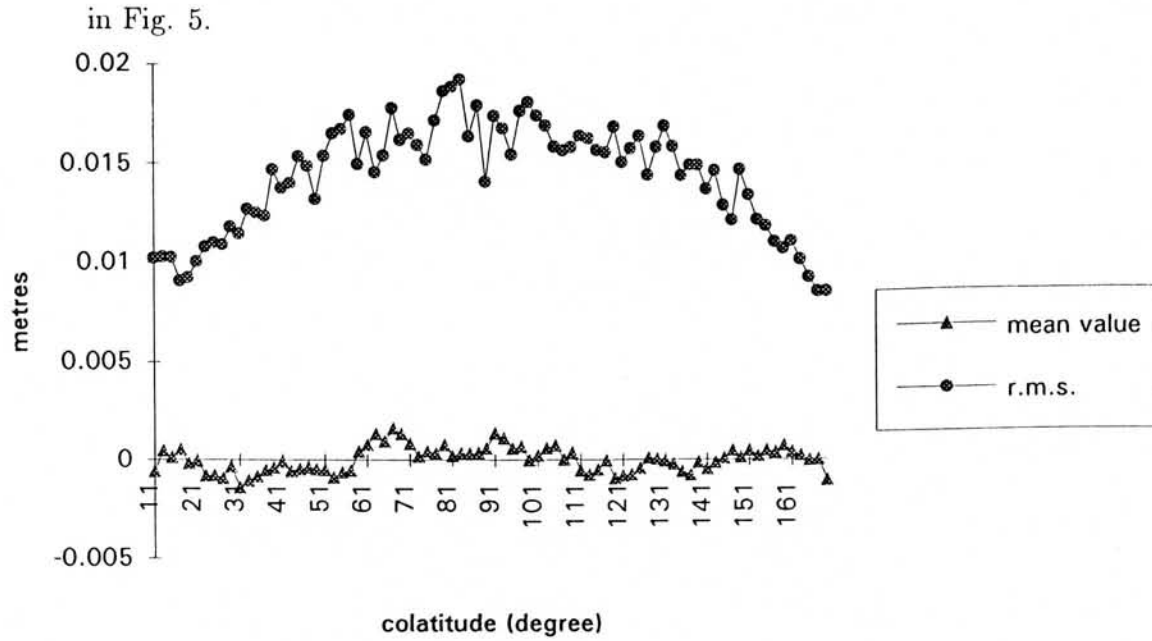


Figure 5: Geoid estimation error away from polar caps.

Just to have the feeling how fast this result degrades when we approach the polar caps, one can look at Fig. 6, where also latitudes inside the caps are displayed; going deeper into polar caps gives even worse results.

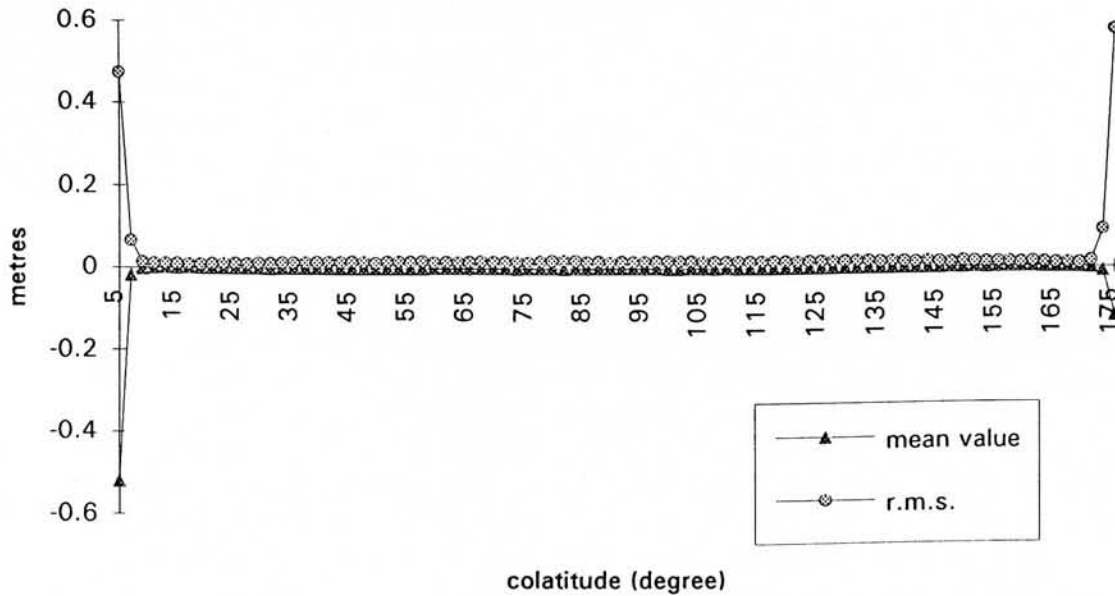


Figure 6: Geoid estimation error from $\vartheta = 5^\circ$ to $\vartheta = 175^\circ$.

As for the error in the reconstruction of the field of gravity anomalies, we have used the same approach, computing averages and standard deviations per parallel belts at different latitudes with a 2° lag. The result is presented in Fig. 7 for colatitudes $10^\circ \leq \vartheta \leq 170^\circ$ and, as one can see, the error lies in a small range about 1 mGal.

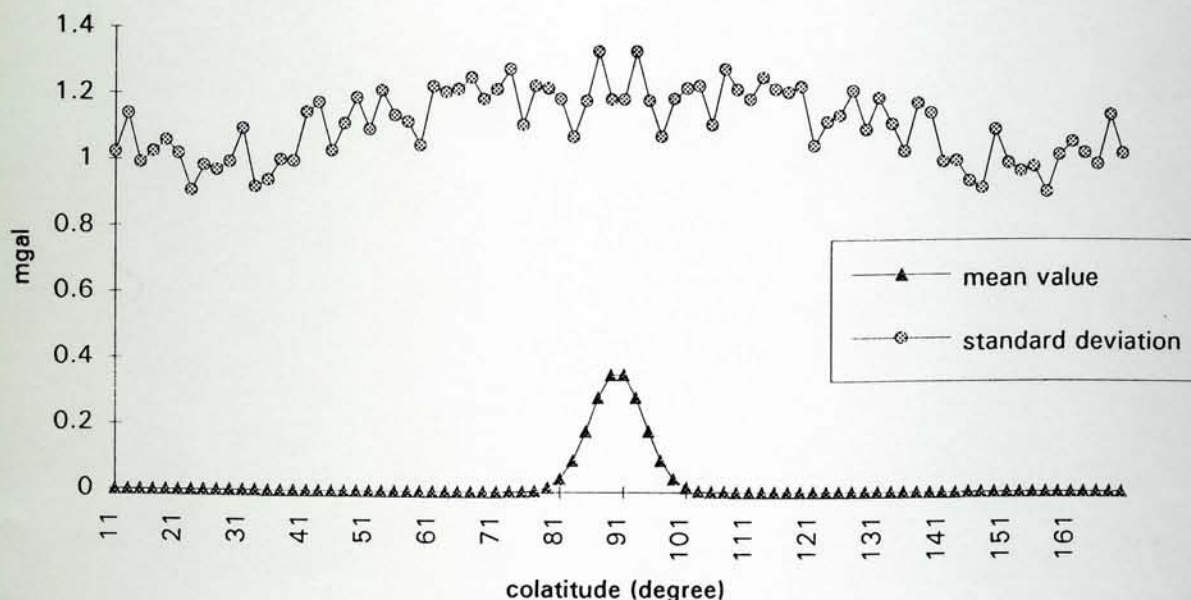


Figure 7: Gravity anomalies error.

Concluding, we can say that most of the coefficients of the anomalous potential are retrieved by a least squares approach to a reasonable degree of accuracy; only for small orders ($m \leq 5$) the results are not satisfactory. It has been recognized that these bad results are related to the lack of data in polar caps.

Now we want to study if it is possible to repair only the bad coefficients. This will be described in the following paragraph.

3 Analysis of the estimation error for the order $m = 0$ only

The idea here is to concentrate on the orders that are harmful in the estimation of the potential coefficients, particularly $m = 0$. Let us recall that in the series expansions

$$T(\vartheta, \lambda) = \frac{\mu}{R} \sum_{\ell=L_0}^L \sum_{m=-\ell}^{\ell} T_{\ell m} \left(\frac{R}{r} \right)^{\ell+1} Y_{\ell m}(\vartheta, \lambda) \quad (4)$$

$$\frac{\partial^2 T}{\partial r^2}(\vartheta, \lambda) = T_{rr}(\vartheta, \lambda) = \frac{\mu}{R^3} \sum_{\ell, m} (\ell+1)(\ell+2) T_{\ell m} \left(\frac{R}{r} \right)^{\ell+3} Y_{\ell m}(\vartheta, \lambda) \quad (5)$$

the $T_{\ell m}$ coefficients for each order m can be isolated by taking the Fourier transform along parallels, namely

$$\begin{aligned}
\bar{u}_m(\vartheta_k) &= \frac{\mu}{R^3} \sum_{\ell=m}^L (\ell+1)(\ell+2) T_{\ell m} \left(\frac{R}{r} \right)^{\ell+3} \bar{P}_{\ell m}(\vartheta_k) = \\
&= \frac{\pi}{N} \sum_{i=1}^{2N} T_{rr}(\vartheta_k, \lambda_i) \left\{ \begin{array}{c} \cos m \lambda_i \\ \sin m \lambda_i \end{array} \right\}
\end{aligned} \tag{6}$$

The point is that the relation (6) is available only for values ϑ_k outside the polar caps. In particular for $m = 0$ we have

$$\bar{u}_0(\vartheta_k) = \sum_{\ell=0}^L a_{\ell} T_{\ell 0} \bar{P}_{\ell 0}(\cos \vartheta_k) \tag{7}$$

where

$$\begin{aligned}
a_{\ell} &= \frac{\mu}{R^3} (\ell+1)(\ell+2) \left(\frac{R}{r} \right)^{\ell+3} \\
\bar{P}_{\ell 0}(\cos \vartheta_k) &= \sqrt{2(2\ell+1)} P_{\ell}(\cos \vartheta_k)
\end{aligned} \tag{8}$$

while the quantity $\bar{u}_0(\vartheta_k)$ can be computed from the observed values of $T_{rr}(\vartheta_k, \lambda_i)$:

$$\bar{u}_0(\vartheta_k) = \frac{\pi}{N} \sum_{i=1}^{2N} T_{rr}(\vartheta_k, \lambda_i) \tag{9}$$

The behaviour of $\bar{u}_0(\vartheta_k)$ including polar caps computed by (7), using the OSU91A coefficients, is represented in Fig. 8, where the limits of polar caps have also been evidenced.

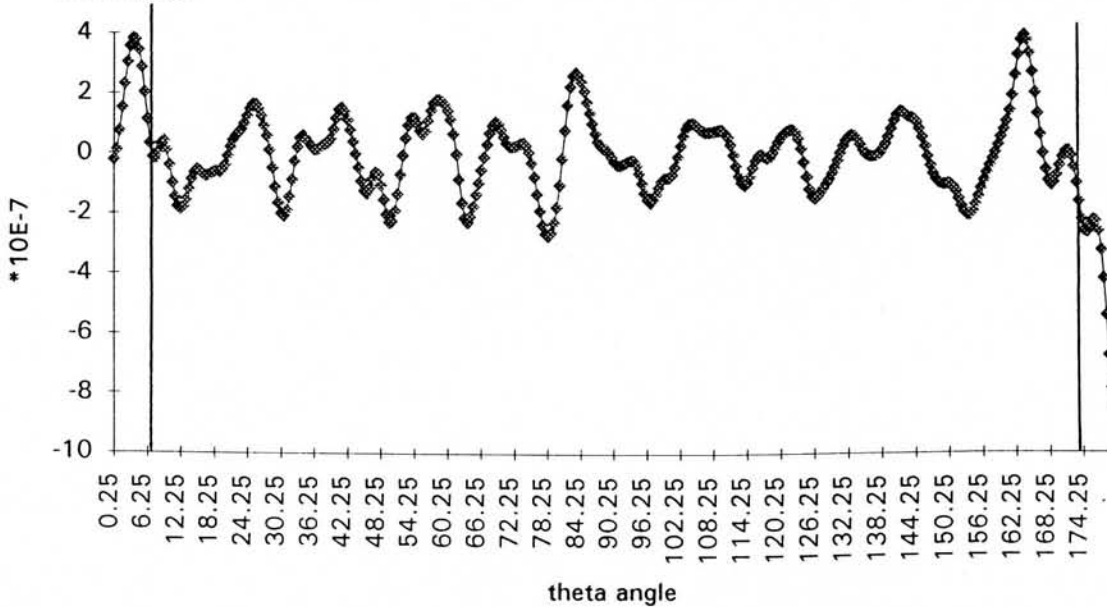


Figure 8: $\bar{u}_0(\vartheta_k)$.

As one can see these values display a quite peculiar behaviour; this is probably due to the lack of real data on polar caps in the mentioned model, implying that many errors have been swept in these areas through aliasing.

Yet one could think of using the known values of $\bar{u}_0(\vartheta_k)$ computed from (9) to set up a prediction of $\hat{u}_0(\vartheta_k)$, the unknown values on the caps. This has been done by 1-D collocation with the covariance function (empirical and theoretical model) shown in Fig. 9.

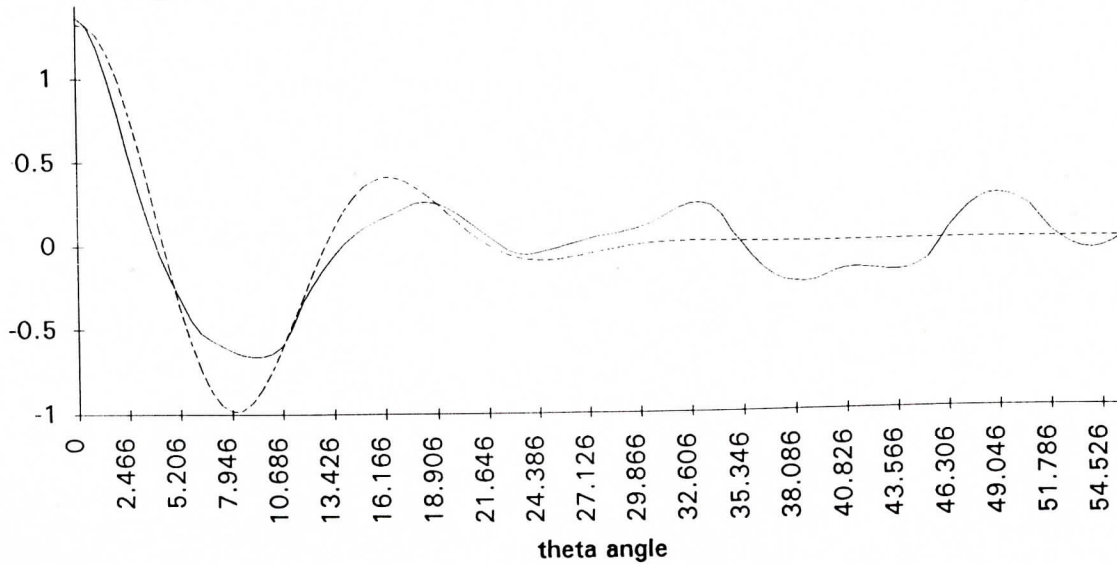


Figure 9: Covariance function for the 1-D collocation: solid line = empirical covariance function; dashed line = theoretical covariance function.

The results of the prediction are shown in Fig. 10 and Fig. 11 where the north pole displays a very high estimation error while the south pole is slightly better.

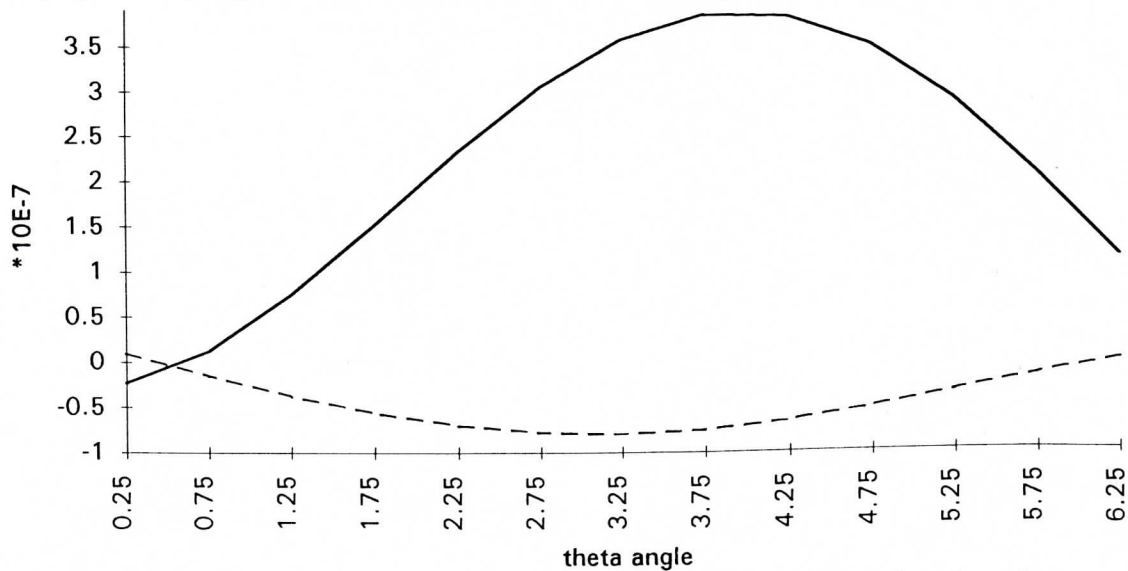


Figure 10: Prediction by 1-D collocation in the north cap (the solid line is for values computed from OSU91A and dashed line is for predicted values).

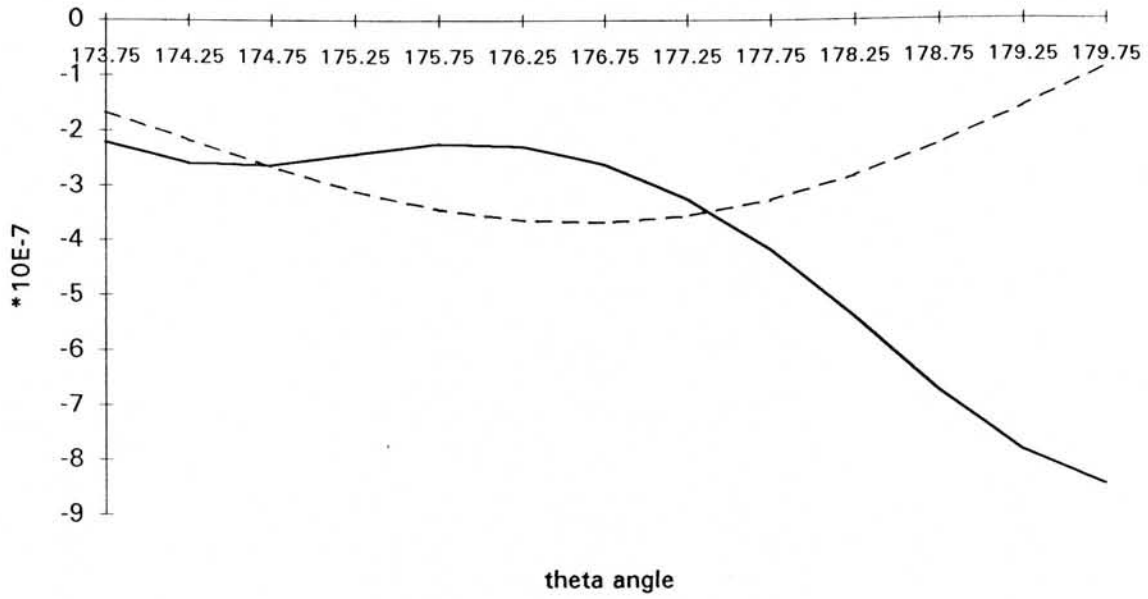


Figure 11: Prediction by 1-D collocation in the south cap (the solid line is for values computed from OSU91A and dashed line is for predicted values).

Both the values $\bar{u}_0(\vartheta_k)$ computed from (7) up to the poles and the same values computed in the measurement area together with the predicted values on the caps $\hat{u}_0(\vartheta_k)$, have then been analyzed by discrete quadrature formulas. The two corresponding sets of coefficients $T_{\ell 0}^q$ and $\hat{T}_{\ell 0}^q$ (where q stems for quadrature) have been compared with the original OSU91A values $T_{\ell 0}^{osu}$. The absolute differences per degree are displayed in Fig. 12 and Fig. 13 respectively.

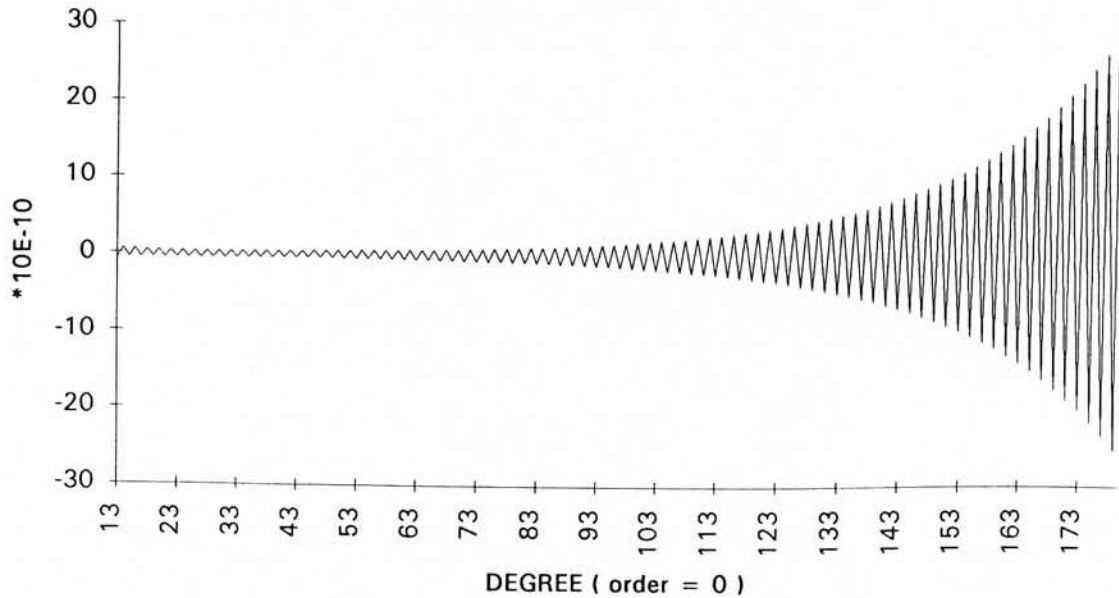


Figure 12: Absolute differences $(T_{\ell 0}^q - T_{\ell 0}^{osu})$.

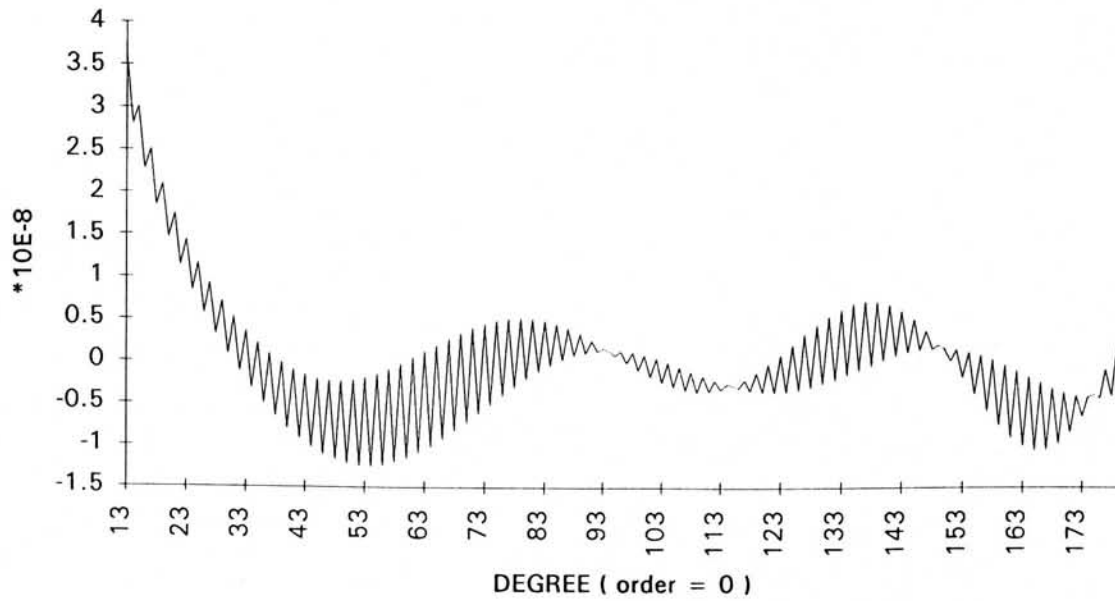


Figure 13: Absolute differences ($\hat{T}_{\ell 0}^q - T_{\ell 0}^{osu}$).

Although it is clear that the second case is worse than the first, no one of the two results is acceptable, showing an intrinsic weakness of the quadrature method.

For the sake of comparison, also $\hat{T}_{\ell 0}^{\ell s}$ coefficients have been computed by means of a least squares procedure, analyzing the set of observations with no data on the polar caps. Afterwards the absolute differences ($\hat{T}_{\ell 0}^{\ell s} - T_{\ell 0}^{osu}$) were computed.

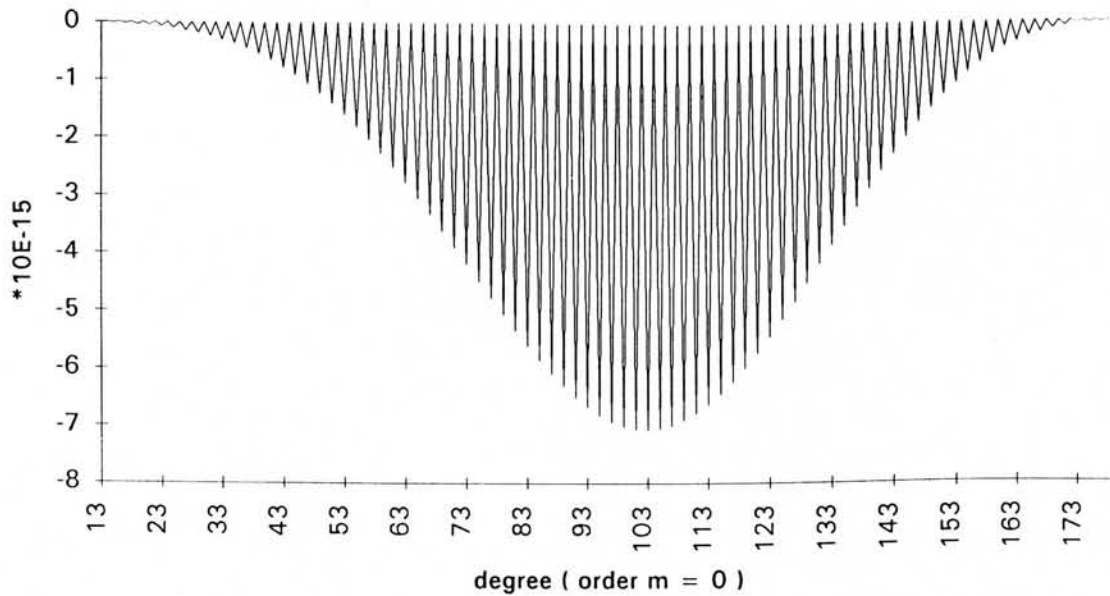


Figure 14: Absolute differences ($\hat{T}_{\ell 0}^{\ell s} - T_{\ell 0}^{osu}$).

Already at this point we can immediately see that the least squares solution is superior by many orders of magnitude to the quadrature solution, even if data are missing on polar caps.

Also a comparison between Fig. 12 and Fig. 14 shows that the least squares errors are five orders of magnitude lower than the pure numerical and discretization error due to the quadrature formulas.

At this point the result already synthetically presented in Fig. 1 has been confirmed, showing that a least squares procedure is able to control the pure numerical error.

We were also interested in reconstructing the error in terms of geoid at ground level generated by these errors in the coefficients, i.e. we have computed:

$$\delta N_0(\vartheta_k) = R \sum_{\ell=13}^{180} (T_{\ell 0}^q - T_{\ell 0}^{osu}) Y_{\ell 0}(\vartheta_k) \quad (10)$$

$$\delta N_0^q(\vartheta_k) = R \sum_{\ell=13}^{180} (\hat{T}_{\ell 0}^q - T_{\ell 0}^{osu}) Y_{\ell 0}(\vartheta_k) \quad (11)$$

$$\delta N_0^{ls}(\vartheta_k) = R \sum_{\ell=13}^{180} (\hat{T}_{\ell 0}^{ls} - T_{\ell 0}^{osu}) Y_{\ell 0}(\vartheta_k) \quad (12)$$

The results are shown in Fig. 15a, Fig. 16a and Fig. 17a respectively for the whole sphere, while only the central values of these plots are evidenced in Fig. 15b, 16b and 17b.

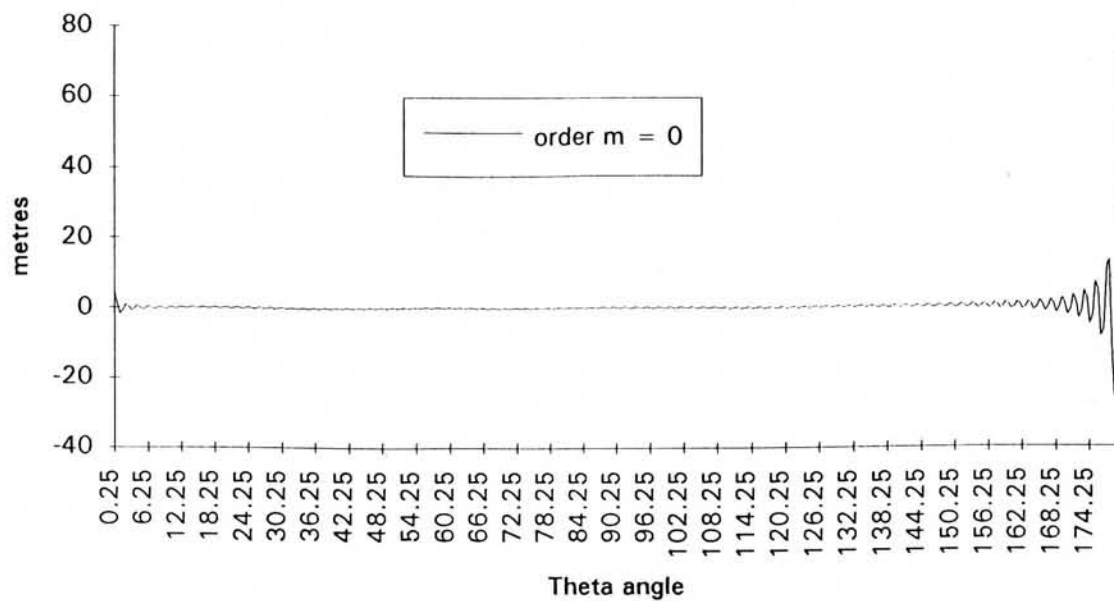


Figure 15a: Geoid estimation error with $\bar{u}_0(\vartheta_k)$ computed from (7) and analysis by discrete quadrature formulas.

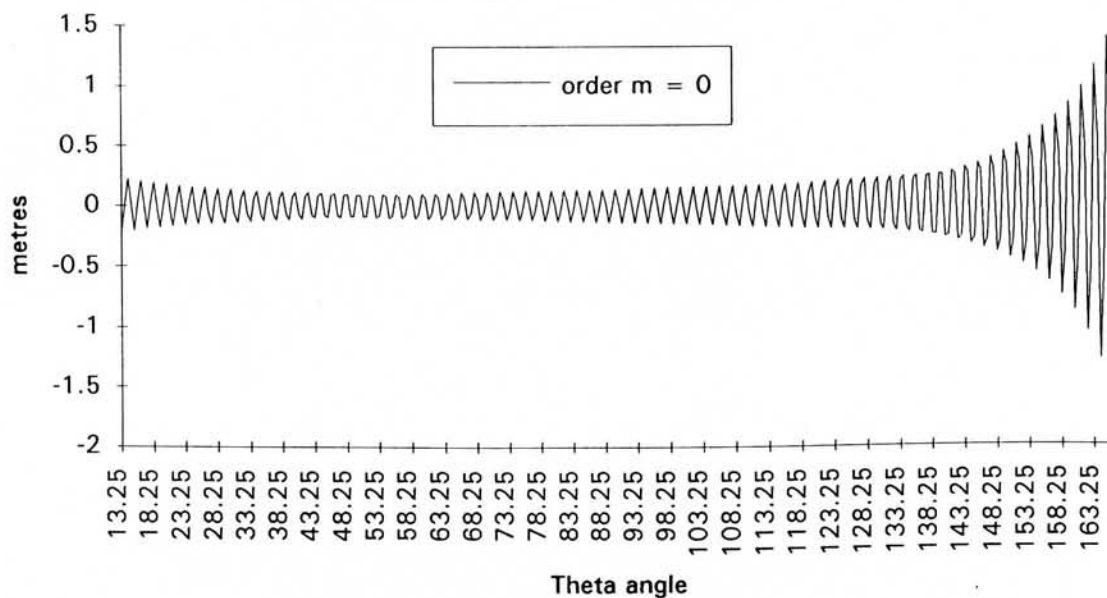


Figure 15b: Central values of Fig. 15a.

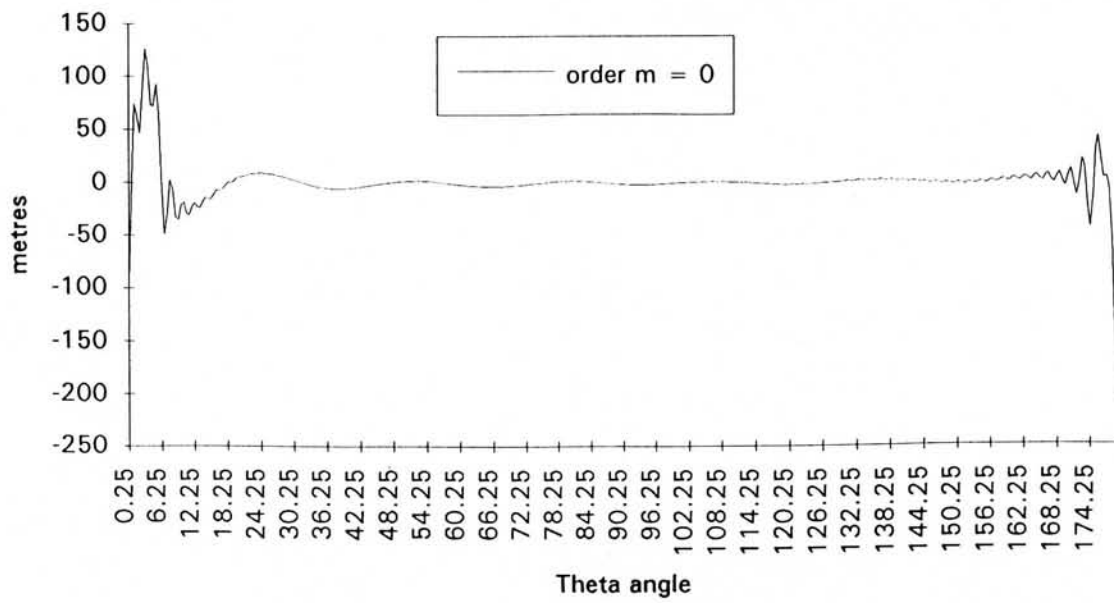


Figure 16a: Geoid estimation error with collocation prediction and analysis by discrete quadrature formulas.

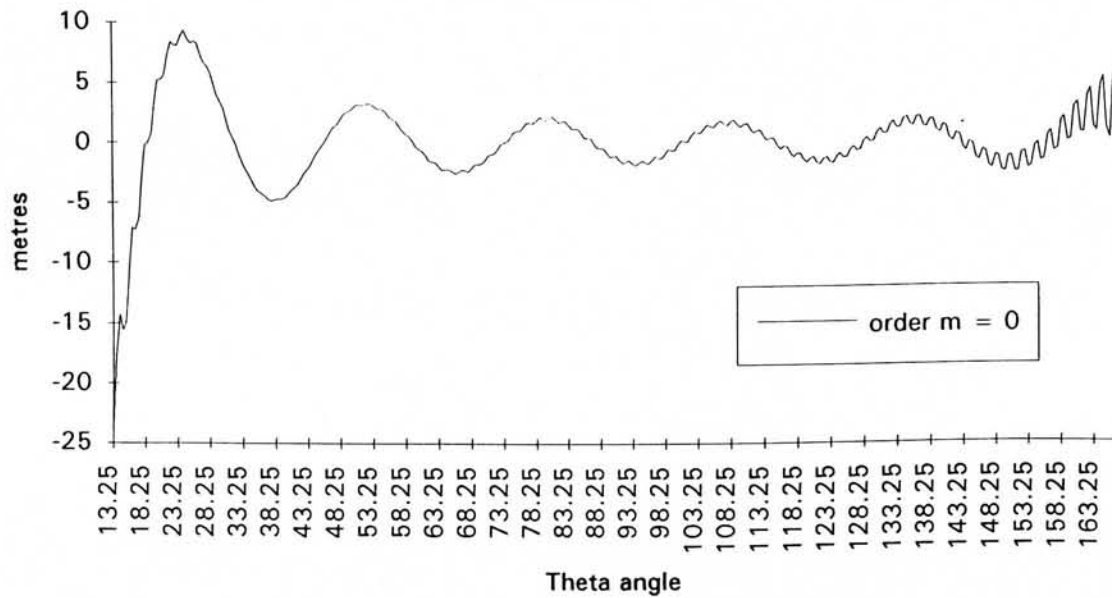


Figure 16b: Central values of Fig. 16a.

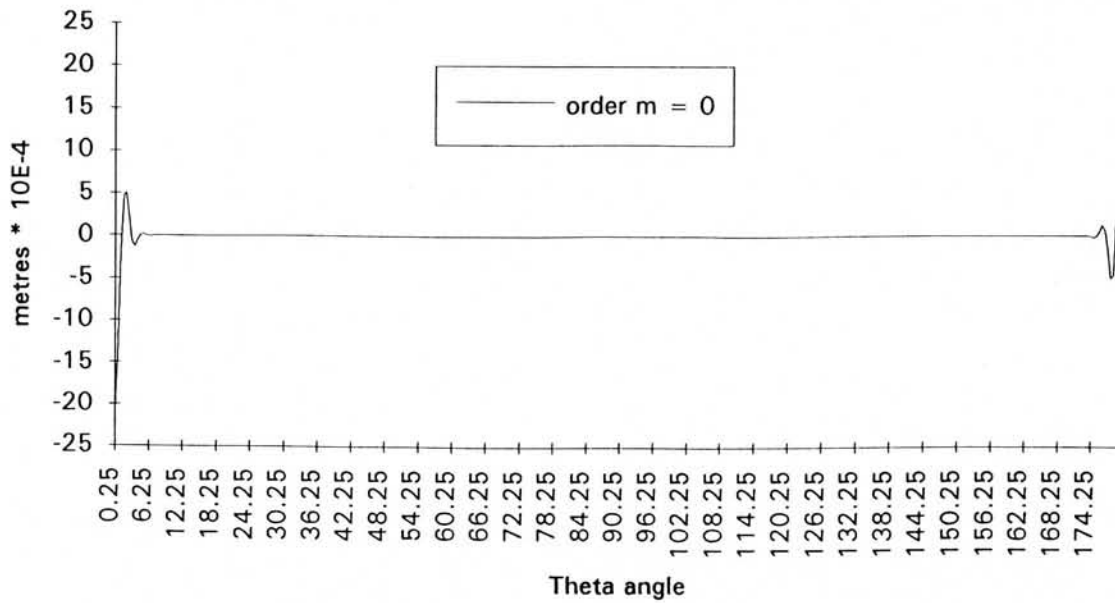


Figure 17a: Geoid estimation error with l.s. solution.

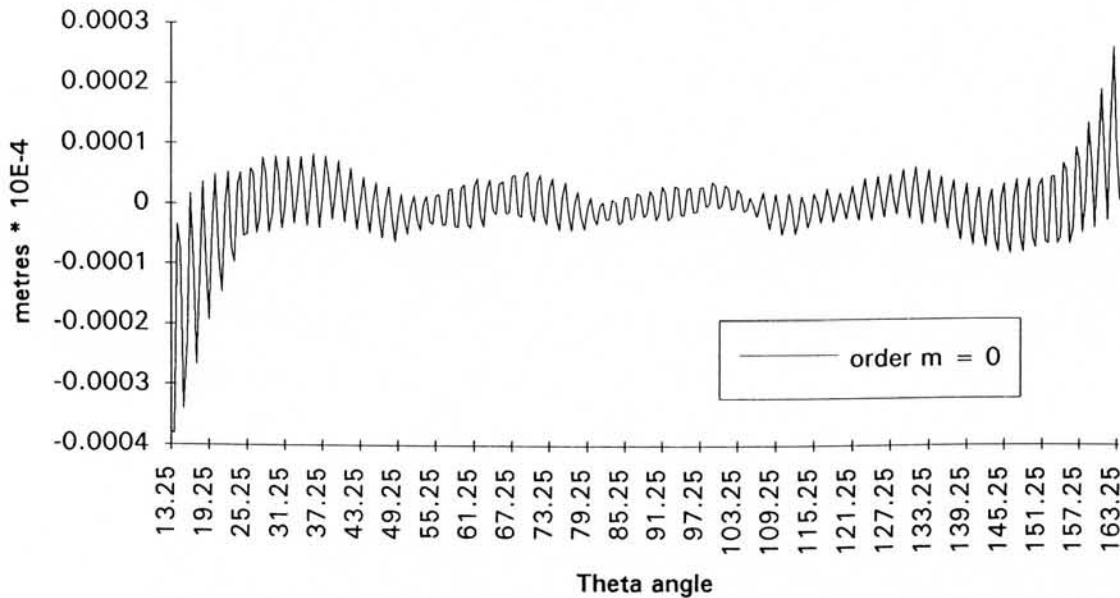


Figure 17b: Central values of Fig. 17a.

As one can see in all cases the errors close to the north and south poles become large, while in the central part the range is some 10 centimetres r.m.s. for the coefficients $T_{\ell 0}^q$, some metres for the coefficients $\hat{T}_{\ell 0}^q$ and becomes practically zero for the coefficients $\hat{T}_{\ell 0}^{\ell s}$.

In order to complete the analysis of the behaviour of the least squares solution for the order $m = 0$, we have investigated how the measurement error is propagated to the geoid estimation error, using the equation:

$$\delta N_0^{err}(\vartheta_k) = R \sum_{\ell=13}^{180} \delta \hat{T}_{\ell 0}^{ls} Y_{\ell 0}(\vartheta_k) \quad (13)$$

where $\delta \hat{T}_{\ell 0}^{ls}$ represents the error in the $\ell 0$ coefficient. The result are shown in Fig. 18a and Fig. 18b.

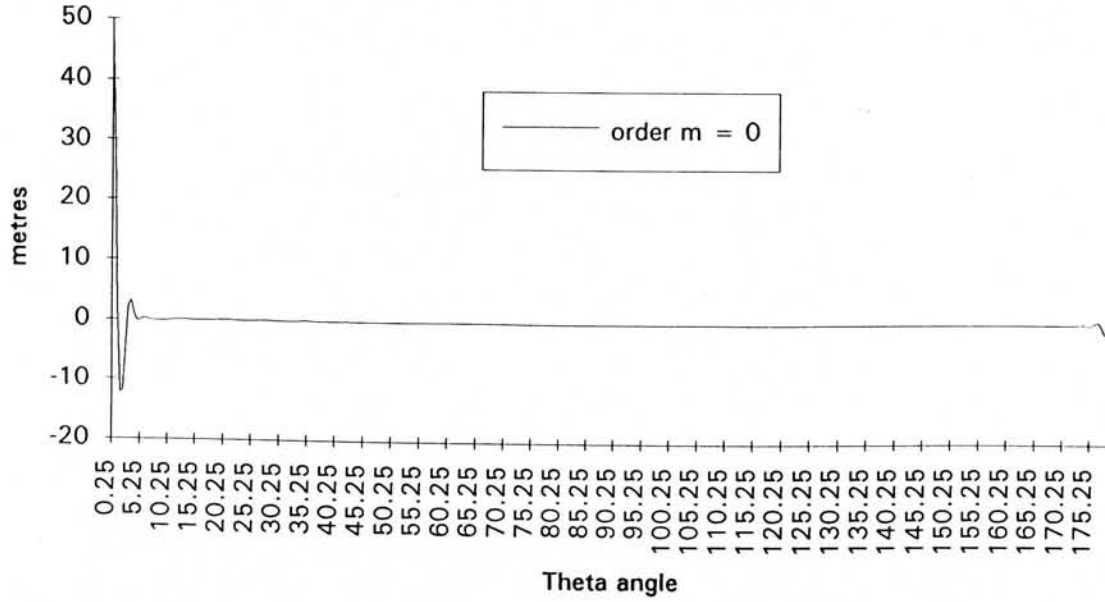


Figure 18a: Geoid estimation error due to the measurement noise.

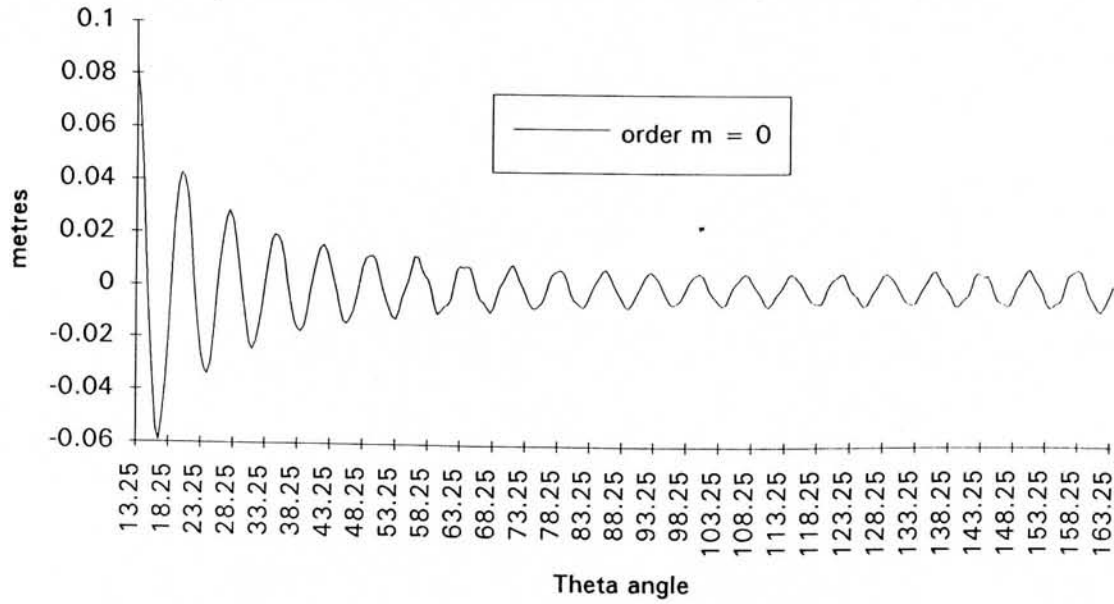


Figure 18b: Central values of Fig. 18a.

4 Conclusion

The effect of polar caps on the estimation of harmonic coefficients of the gravity potential has been studied, when data are taken from a simulated gradiometric mission.

In principle a least squares approach should be able to retrieve exactly the model coefficients used in the simulation (see Fig. 1), but when measurement noise is added a worse performance of least squares estimates is obtained (see Fig. 2). However most of the estimation error giving a low signal to noise ratio, is due to low order coefficients (see Fig. 3, Fig. 4).

An attempt to remedy to this situation has been done by applying a one dimensional prediction of collocation type to the values $\sum_{j=0}^{720} T_{rr}(\vartheta_k, \lambda_j)$, $6,5^\circ \leq \vartheta \leq 173,5^\circ$, which had to be extrapolated into the polar caps to derive a better estimate of the $T_{\ell,0}$ ($\ell \leq 180$) coefficients. The attempt has failed (see Fig. 10, Fig. 11), in our opinion also because the global model used in the simulation has little information in the polar areas.

In any way, although the low order coefficients are unavoidable badly estimated, the effect of this error on the geoid reconstruction at the ground level and far from polar caps is still rather good, as shown in Fig. 5 and Fig. 18b. This proves that already at this level the oceanographic target of a gradiometric mission can be fully hit.

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THE USAGE OF STOKES IN THE POSSESSIVE

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INTRODUCTION

Over the years, the work of George Gabriel Stokes (1819-1903) has been referred to in several different, and usually incorrect, ways. Specifically, there is considerable variation present in physical-geodetic literature for the genitive or possessive citation of Stokes.

We hope to use the International Geoid Service as a medium to encourage consistency in the citation of Stokes (1849), and others whose name ends in s, by geodesists.

GRAMMATICAL RULES FOR THE POSSESSIVE

1. The Singular

Use 's for the possessive when it refers to a single person, animal, organisation, time or place (Murphy 1987, p.158). Respective examples are: William's car, the cat's tail, government's address, yesterday's news, and Canada's climate.

A fact that is not widely adhered to is that this rule extends to the singular noun which ends in an s, especially in British English (Swan 1980, p.261). Examples of this usage include Charles's friend and Burns's poems (Strunk and White 1979, p.1). Therefore, this should be applied to the citation of Stokes.

2. The Plural

The possessive is indicated by an apostrophe alone, only when the noun plural ends in an s. For example, two surveyors' measurements, my parents' house, and seven days' work. An exception holds for the irregular plural, where the plural does not end in an s, such as in children's future (Swan 1980, p.261).

3. The Definite Article

Neither use the possessive apostrophe nor *s* if the noun is preceded by the definite article. In this case, the possessive phrase is converted to a descriptive phrase. For instance, Newton's hypothesis becomes the Newton hypothesis or the hypothesis of Newton.

THE SPECIFIC CASE OF STOKES

The word Stokes is a singular noun and thus follows rule 1 above, even though it ends in an *s*. However, there is some quite considerable variation throughout the literature and it is common to see the possessive written as Stokes'. This is grammatically incorrect, as this type of punctuation only applies to the plural, which Stokes is not.

The correct usage in the possessive is Stokes's. Examples of the possessive include: Stokes's assumptions, Stokes's theorem, Stokes's formula, and of similar relevance to physical geodesists is Bruns's formula. Alternatively, the definite article may be used to change the possessive phrase to a descriptive phase, specifically, the Stokes formula or the formula of Stokes.

There is one exception to this rule of modern English usage. This is the case of using the possessive for older, foreign and classical (ancient Roman and Greek) names (Swan 1980, p.261). Examples are: Eratosthenes' arc, Pythagoras' theorem and Copernicus' hypothesis.

SUMMARY

This discussion is intended to generate some uniformity in the possessive usage of Stokes amongst geodesists. The author should remember that the final choice of grammatical style is made by the editor. However, if everyone is to adhere to this scheme, the often thankless task of the editor will be made slightly simpler. Finally, we would like to thank Ms. Wendlynn Wells for bringing this inconsistency to our attention.

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