

FOREWORD

At the joint Symposium of the International Gravity Commission and the International Geoid Commission of IAG (Section III) in Graz (September 1994) a joint project between GSFC/NIMA(DMA) and NASA has been announced to determine a new global model of the anomalous gravity field of the earth, including an input data set much larger than in the past for both primary gravimetric quantities in areas like Russia, China, etc. and altimetry-derived gravimetric quantities, as well as subsidiary data necessary for the scope.

The approach to the processing of these data for the gravity model determination, has been fixed by scientists working in the above organizations in cooperation with the Ohio State University group; in particular the project has been described by R. Rapp and S. Nerem in the proceedings of the Graz Symposium.

At the IUGG General Assembly in Boulder (July 1995) a meeting has been run by GSFC/NIMA-NASA under the auspices of the International Geoid Service (IAG-Sect. III) to establish a working group inside IGeS with the purpose of evaluating several different solutions that were on the meantime under computation with slightly different strategies; namely the idea was to provide parameters for a choice of the "best" model by comparing it with already determined local geoids, stationary sea surface solutions, GPS levelling data sets, etc.

The work has then been performed by the established working group, chaired by Michael Sideris (University of Calgary).

The actual special publication of the IGeS Bulletin is a comprehensive report of the work done by many scientists engaged in the problem of geoid computation.

I would like to express my sincere thanks to all the actors of this project which I think will remain for long as a positive example of fair cooperation between different organizations like Governmental Institutions, Universities, Research Center, etc.

First of all thanks to NASA and NIMA who have agreed to cooperate with an ad-hoc international structure set up in the framework of IAG.

Also thanks are due to the Goddard Space Flight Center (NASA) for supporting economically this publication.

Thanks also to Michael Sideris for the fine and fruitful organization of the work.

Thanks to all the participants to the project and to the authors of the paper presented in this number.

I express my wishes that this cooperation will last longer.

Fernando Sansò

COMPARISON OF RECENT GEOPOTENTIAL MODELS WITH SATELLITE DATA

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The following evaluation of the joint NASA/NIMA test (X02-05) and final (EGM-96) geopotential models is a comparison of the quality of the orbit determination for various geodetic satellites, as measured by the root-mean-square (RMS) of the tracking data residuals. The absolute magnitude of these fits are highly dependent on the arc lengths and the set of parameters estimated, so comparisons with the JGM series of models (Nerem et al., 1994; Tapley et al., 1996), the European GRIM4-C4 model (Schwintzer et al., 1997), and the DGM-E04 model (Scharroo et al., 1997), are included to provide a context to evaluate the results. The test orbits used are not sensitive to the very high degrees provided in the test fields, so these tests used only the coefficients up to degree and order 70 (the X04 field was identical to X03 for that portion and was not tested). The arc lengths and estimated parameters are shown for each case. Except for GPS-MET and Topex/Poseidon, the tests are based on satellite laser ranging (SLR) tracking, since this is the only precise tracking data type available for most of these geodetic quality satellites.

The orbit tests in Table 1 were performed with and without the estimation of one-cycle-per-revolution (1-cpr) empirical accelerations. These empirical accelerations are generally employed to remove, to a large degree, the effect of surface force modeling errors, but they also have the result of removing the secular and long-period orbit errors due to the even and odd zonals. Consequently, the tests without 1-cprs constitute tests where the effect of the zonal errors are still included, whereas the tests with 1-cpr parameters evaluate the gravity model without the dominant effect of the zonal errors included.

Field	Ajisai	Starlette	Stella	GFZ-1	Ajisai*	Starlette*	Stella*	GFZ-1*
JGM-1	5.3	4.8	47.1	70.7	3.4	4.1	24.1	40.4
JGM-2	5.3	4.8	52.2	70.3	3.3	4.1	21.7	40.6
JGM-3	5.0	4.8	9.4	68.2	3.2	4.1	6.3	39.6
DGM-E04	5.1	4.9	9.0	73.2	3.3	4.2	6.0	43.5
GRIM4-C4	5.6	7.8	12.6	140.5	3.4	6.6	7.1	76.7
X02	5.1	4.7	8.9	61.7	3.3	4.0	7.9	48.8
X03	5.3	4.7	8.8	68.8	3.2	3.9	7.4	52.2
X05	5.3	4.7	8.9	68.3	3.2	4.0	7.4	51.9
EGM-96	5.1	4.7	8.8	56.6	3.3	4.0	6.2	45.4

Table 1. Gravity Field Evaluation using Geodetic Satellite Orbit Fits to Laser Range Data

Notes:

- 1) All units are cm
- 2) 6-day arcs used for Ajisai, Starlette and Stella; 3-day arcs for GFZ-1
- 3) Daily C_d estimated for Ajisai, Starlette and Stella; twice-daily C_d estimated for GFZ-1
- 4) One set 1-cpr empirical accelerations (along-track and cross-track) also estimated where indicated by *

The orbit tests for the GFZ-1 satellite are included but should be interpreted with caution. The SLR tracking of GFZ-1 is very sparse, and the drag modeling errors are considerable. The value of the residual RMS can be almost any value, depending on the number of drag parameters adjusted or the sparseness of the tracking. The arcs and parameterization chosen here represent an attempt to avoid over-parameterizing drag so that some distinction between gravity models is obtained. Generally, the models perform comparably, with the exception of the GRIM4-C4 model. The EGM-96 model incorporated GFZ-1 tracking data, but extending it to include terms to degree and order 99 provided only minor improvement.

Some additional orbit tests are summarized in Table 2. There is little difference in the Lageos-1 and Lageos-2 fits, indicating that all the models are performing well at the long wavelengths to which these high altitude satellites are sensitive. To detect the small residual gravity model error effects on Lageos-1 and Lageos-2 requires significantly more elaborate tests, such as those performed in the evaluation of the JGM-3 model (Tapley et al., 1996). The orbit fits for the ERS-1 and GPS-MET satellites provide some differentiation between the models in spite of the heavy parameterization being used. Only the NASA/NIMA models incorporated GPS-MET data, yet JGM-3 and DGM-E04 perform slightly better. Because the DGM-E04 model is essentially the JGM-3 model slightly tuned by incorporating ERS-1 altimeter crossover data, it is not surprising that it performs similarly to JGM-3, except for ERS-1.

Field	Lageos-1	Lageos-2	ERS-1	GPS-MET
JGM-1	2.6	2.6	12.9	4.1
JGM-2	2.8	2.7	10.6	3.0
JGM-3	2.5	2.4	6.2	2.8
DGM-E04	2.5	2.4	5.3	2.6
GRIM4-C4	2.6	2.5	7.0	3.9
X02	2.6	2.5	6.9	3.2
X03	2.6	2.5	6.6	3.2
X05	2.6	2.5	6.6	2.9
EGM-96	2.6	2.5	6.0	3.1

Table 2. Additional Evaluations Using SLR or GPS Tracking Data

Notes:

- 1) All units are cm for Lageos-1, Lageos-2 and ERS-1 laser range and GPS-MET double-difference phase
- 2) 3-day arcs used for Lageos-1, Lageos-2 and ERS-1; 1-day arcs for GPS-MET
- 3) Only initial conditions estimated for Lageos-1 and Lageos-2
- 4) One set of empirical 1-cpr accelerations and twice-daily C_d estimated for ERS-1
- 5) One set of empirical 1-cpr accelerations and a C_d every 2 orbit revolutions estimated for GPS-MET

The altimeter satellite Topex/Poseidon (T/P) provides a particularly important test because of the high accuracy requirements of the mission and because there are a number of tracking data types available for the evaluation. In addition, a high precision orbit from JPL based on GPS tracking and the reduced-dynamics strategy was available for the test arc to provide an independent orbit accuracy assessment.

Field	SLR	DORIS	Altimeter Crossovers	Radial	Along-track	Cross-track
JGM-1	3.7	0.56	9.2	4.0	8.6	8.3
JGM-2	2.4	0.55	8.8	2.5	7.2	7.3
JGM-3	2.0	0.55	8.5	1.4	4.3	5.3
DGM-E04	2.0	0.55	8.5	1.5	4.4	5.3
GRIM4-C4	2.4	0.55	8.9	2.4	6.9	6.6
X02	2.1	0.55	8.5	1.9	5.2	5.9
X03	2.1	0.55	8.6	1.9	5.2	5.9
X05	2.1	0.55	8.6	1.9	5.2	6.0
EGM-96	2.0	0.55	8.5	1.8	5.1	5.5

Table 3. Evaluation using Data Fits and Orbit Comparisons for Topex/Poseidon

Notes:

- 1) All units are cm except mm/sec for DORIS doppler data
- 2) Original GDR tide model used for altimeter crossovers
- 3) 10-day arc for T/P Cycle 45; daily 1-cpr and 8-hour along-track constant empirical accelerations estimated
- 4) Orbit comparisons (radial, along-track and cross-track differences) relative to JPL reduced-dynamics GPS orbit

Conclusions

These tests evaluate the quality of the NASA/NIMA models in the long and medium wavelength components. Although they do not provide a quantitative assessment of the geoid accuracy, they do provide one measure of the overall quality of the gravity model. In this regard, the EGM-96 model appears to compare well with the other current models. There appears to be little difference between the various NASA/NIMA models, although the final model performs as well or better than the test models in nearly all cases. It would appear that the attempt to increase the accuracy of the geoid model has not been detrimental to the quality of the satellite orbits.

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Evaluation of Preliminary Models of the Geopotential in the United States

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Introduction

As members of the Special Working Group of the International Geoid Service, we have evaluated the preliminary geopotential models X01 through X05; computed by the joint NASA-GSFC/DMA project (Rapp and Nerem 1994). Our focus has been the behavior of these models in the conterminous United States. In this effort, our strategy has been to compare geoid models derived from geopotential coefficients against GPS ellipsoid heights located on optically-leveled benchmarks. This comparison is done in two ways. In one approach, the GPS benchmarks are compared against the geoid values obtained from what is essentially direct evaluation of the preliminary models. This approach suffers from the omission or truncation error inherent in an $n=360$ model computation. In the other approach, the point gravity and digital terrain in and around the United States are used to compute high resolution geoid models in remove, compute, and restore procedures about the various preliminary models. While this approach alleviates the omission error problem, it is seen to remove portions of the geopotential model commission error.

In this report, we begin by outlining a few important points in the computation of gravimetric quantities from geopotential coefficients. Next, we review the features of the GPS benchmark data set that was used in evaluation of the preliminary models. We discuss the computation procedure used to obtain the high resolution geoid models, and display results with the GPS benchmark comparison. Then, we compare the geopotential models directly against the GPS benchmarks, without the removal of omission error. We close the report with conclusions and recommendations.

Geoid Height Computation

It stands to reason that if one chooses to evaluate geopotential coefficients by means of GPS on leveled benchmarks, then one must use the utmost rigor in computation of the geoid heights from the coefficients. Towards this goal, we have developed formulations to support the use of geopotential coefficients computed with an arbitrary set of normal ellipsoid constants, and to support the computation of various gravimetric quantities relative to a different, arbitrary set of normal ellipsoid constants (which we take as GRS80). In addition, it must be recognized that the theory used to establish the geopotential coefficients is rooted in a solution of Laplace's equation. However, Laplace's equation only holds in a vacuum. The geoid over land is typically interior to the Earth's topography, where Poisson's equation would operate.

We begin by considering the evaluation of gravimetric quantities from spherical harmonic coefficients with arbitrary values of GM and a . The total external gravitational potential of the Earth can be expressed as:

$$V_1(r, \theta, \lambda) = \frac{GM_1}{r} \sum_{n=0}^{\infty} \left(\frac{a_1}{r} \right)^n \sum_{m=-n}^n \bar{C}_{nm} \bar{Y}_{nm}(\theta, \lambda)$$

where:

r, θ, λ = geocentric radial distance, colatitude, longitude

GM_1, a_1 = scale factors associated with the C_{nm} values

$\bar{C}_{nm}$ = fully normalized coefficients of the total field

$\bar{Y}_{nm}$ = fully normalized Legendre functions

And, the normal external gravitational potential of the Earth can be expressed as:

$$V_2(r, \theta, \lambda) = \frac{GM_2}{r} \sum_{n=0}^{\infty} \left(\frac{a_2}{r} \right)^n \sum_{m=-n}^n \bar{D}_{nm} \bar{Y}_{nm}(\theta, \lambda)$$

where:

GM_2 = Gravitation-Mass constant of the normal field

a_2 = semi-major axis of the ellipsoid and which is, of itself, a "spheropotential" surface, with constant value of normal gravity potential, U_0 .

$\bar{D}_{nm}$ = fully normalized coefficients of the normal field
(=0 if n is not positive even, or m is not zero)

The total external gravity potential of the Earth is:

$$W(r, \theta, \lambda) = V_1(r, \theta, \lambda) + \Phi(r, \theta)$$

and the normal external gravity potential of the Earth is:

$$U(r, \theta, \lambda) = V_2(r, \theta, \lambda) + \Phi(r, \theta)$$

where $\Phi(r, \theta)$ is the centrifugal potential.

We make the following assumptions:

1) r, θ, λ are the same in both equations, that is, the origins of the two fields coincide, as do their three Cartesian co-ordinate axes.

2) the value of normal gravity potential on the ellipsoid (U_0), is equivalent, numerically, to the value of total gravity potential on the geoid (W_0)

3) The total and normal centrifugal potentials are equivalent, thus the disturbing potential, will be: $T = W - U = V_1 - V_2$

Upon evaluating the disturbing potential, we pay careful attention to the differing values of GM_1 versus GM_2 and a_1 versus a_2 . We start by rewriting the equation for V_1 as (and truncating the series at the given high value of 360):

$$V_1(r, \theta, \lambda) = \frac{GM_2}{r} \sum_{n=0}^{360} \left(\frac{a_2}{r} \right)^n \sum_{m=-n}^n \left(\frac{GM_1}{GM_2} \right) \left(\frac{a_1}{a_2} \right)^n \bar{C}_{nm} \bar{Y}_{nm}(\theta, \lambda)$$

where the scaled coefficients will be expressed as:

$$\bar{E}_{nm} = (GM_1/GM_2) * (a_1/a_2)^n C_{nm} = \text{scale}(n) C_{nm}$$

Thus:

$$V_1(r, \theta, \lambda) = \frac{GM_2}{r} \sum_{n=0}^{\infty} \left(\frac{a_2}{r} \right)^n \sum_{m=-n}^n \bar{E}_{nm} \bar{Y}_{nm}(\theta, \lambda)$$

Now, with a common GM_2 and a_2 value "out front", we can now write the formula of the disturbing potential as:

$$T(r, \theta, \lambda) = \frac{GM_2}{r} \sum_{n=0}^{\infty} \left(\frac{a_2}{r} \right)^n \sum_{m=-n}^n [\bar{E}_{nm} - \bar{D}_{nm}] \bar{Y}_{nm}(\theta, \lambda)$$

Of primary importance here is the fact that the degree zero term is non-zero if $GM_1 \neq GM_2$. In the case of GM_1 for OSU91A (Rapp et al. 1991) versus GM_2 of GRS80 (Moritz 1992), this effect is on the order of 1 meter, in geoid undulation.

Of secondary importance is the scaling of the coefficients. The scale factors are on the order of 10^{-4} , and this effect has been shown in some of our tests to have an RMS of 0.3 mm, with min/max values of -0.3 mm and +0.5 mm for the geoid. While any one effect of sub-mm level will not adversely effect our results, we feel that the number of such approximations that currently exist in the theory need to be removed to prevent the possibility of their combined effects being noticeable at the 1 cm level. It should be noted that this effect of properly scaling

the coefficients is not encompassed in a simple geometric transformation, but represents a fundamental correction to the way that a spherical harmonic expansion is evaluated.

In general remarks, we are also careful about the difference between $\partial/\partial r$ and $\partial/\partial h$. The difference between the two derivatives will have small (about 5 ppm), but calculable effects, on quantities of interest (gravity anomalies, upward derivatives of the height anomaly, etc...). And, wherever possible, exact (closed) formulas or iterations are performed (for example, computing the flattening from J_2 , ω , GM , and a), rather than using any approximate formulas. Considering the speed with which iterations can be performed, we felt it was best to remove as many approximate formulas as was reasonably possible.

Next, we consider the issue that geopotential coefficients do not accurately represent the behavior of the actual geopotential interior to the Earth's surface. This problem was raised in Rapp (1992), and amplified in Rapp (1996). Basically, one must evaluate the geopotential on or above the Earth's surface, and then develop the geoid undulation from that computation. It is convenient, therefore, to first compute height anomalies, ζ , and then convert them to geoid undulation, N , by means of Eq. (8-10) of Heiskanen and Moritz (1967)

$$N = \zeta + \frac{\Delta g_B}{\gamma} h$$

where:

Δg_B simple (non-terrain corrected) Bouguer anomaly

A popular choice in geopotential coefficient evaluation is the Clenshaw summation approach described in Tscherning, et al (1983). This implementation, however, must reform certain sums whenever the geocentric radius, r , varies. To aid in computational efficiency, Rapp (1996) computes the height anomaly on the ellipsoid, ζ_o , and then upward continues to the surface by the orthometric height, H ,

$$\zeta_p = \zeta_o + \partial\zeta/\partial r H$$

In an enhancement to this approach, we perform the upward continuation by

$$\zeta_p = \zeta_o + \partial\zeta/\partial h h$$

where, after suppressing a negligible θ dependence (0.1 ppm),

$$\partial\zeta/\partial h = \partial(T/\gamma)/\partial h = [(1/\gamma) \partial T/\partial r \partial r/\partial h - (T/\gamma^2) \partial\gamma/\partial h]$$

and where ellipsoidal height, h , is established by H and a preliminary geoid height, N . Note that the second term, which contains the normal gravity gradient, $\partial\gamma/\partial h$, can be of the same magnitude as the first term.

As additional confirmation that some type of “upward/downward” procedure is needed to establish accurate geoid heights interior to the Earth’s surface, we compare the geoid heights computed from 2497 GPS benchmarks to the geoid heights computed from the X01 model. In Figure 1a, the X01 coefficients were directly evaluated at a geocentric radius, r_N , which is continually re-evaluated to conform to the geoid surface. In Figure 1b, our enhanced “upward/downward” procedure is followed. Both figures incorporate the GM and a variations discussed earlier. Comparison of the plots demonstrate clear height dependent error in Figure 1a when one attempts to directly compute the geoid undulations interior to the Earth’s surface.

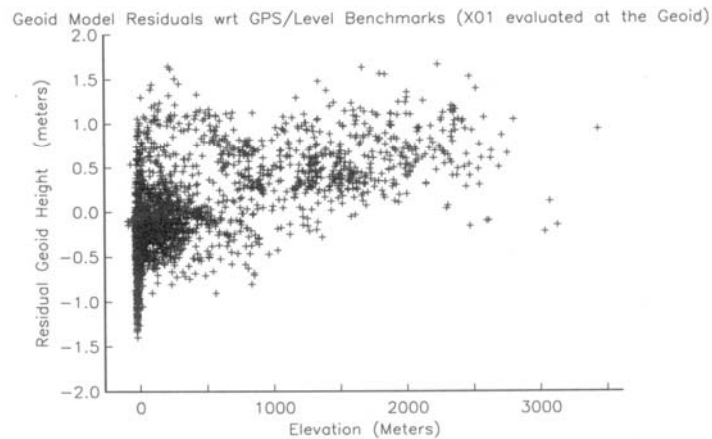


Figure 1a: Geoid model residuals to GPS/benchmark geoid heights: X01 model evaluated at the geoid surface.

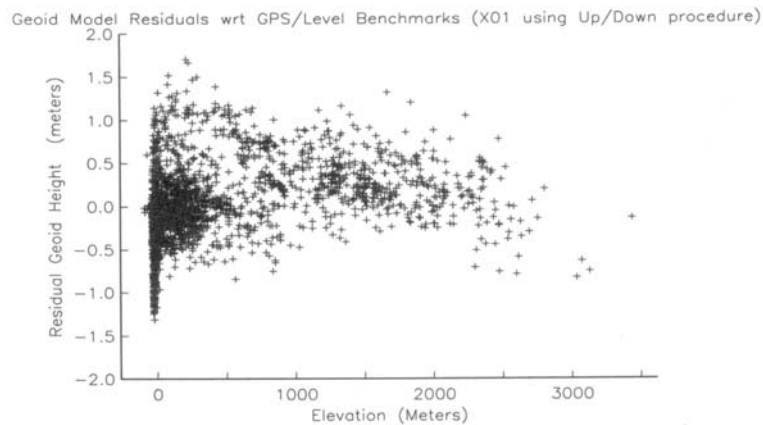


Figure 1b: Geoid model residuals to GPS/benchmark geoid heights: X01 evaluated using the Upward/Downward procedure.

The GPS Benchmark Data Set

NGS is engaged in a project to establish a high accuracy Federal Base Network (FBN), and an associated Cooperative Base Network (CBN), through nationwide measurement of GPS baselines to 1 ppm accuracy or better. In the course of this survey effort, many of those FBN/CBN points are established on NAVD 88 optically leveled benchmarks. Figure 2 displays the locations of 2497 points that are leveled benchmarks with NAVD 88 Helmert orthometric heights, and which have GPS measured ellipsoidal heights in the NAD 83 (86) reference system as of March 1996. The irregular distribution illustrates the state-by-state approach to the surveying, and the different levels of state participation.

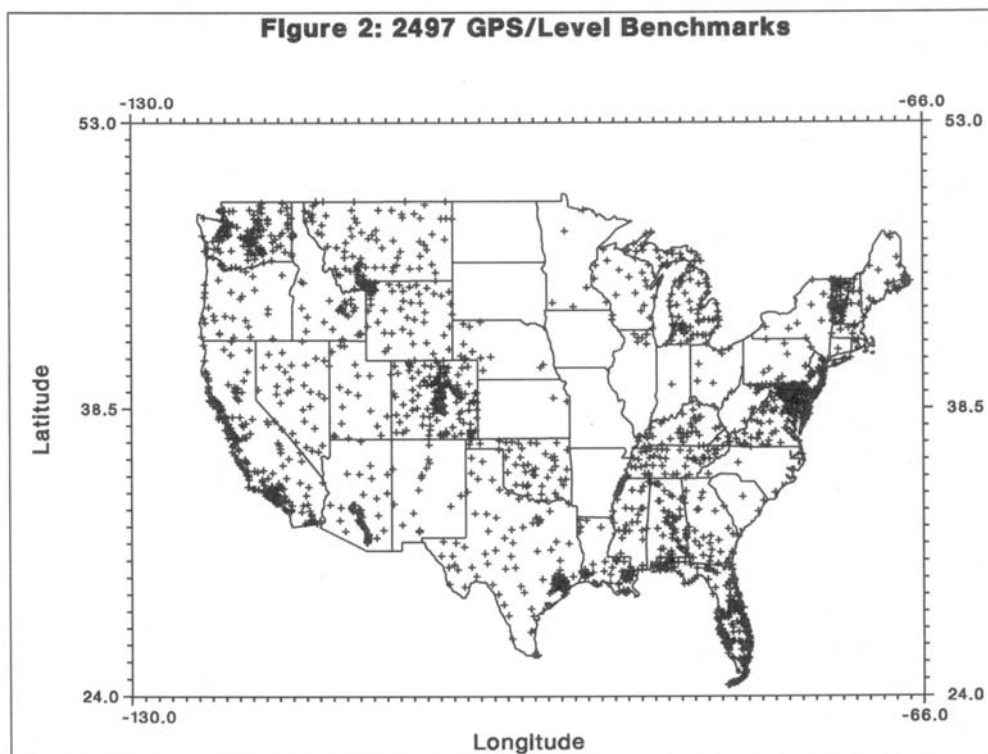


Figure 2: 2497 Leveled Benchmarks with NAVD 88 Helmert Orthometric Heights and GPS Ellipsoidal Heights in the ITRF93(1995.0) Reference System.

For the purposes of the preliminary model evaluation, the NAD 83 (86) coordinates are converted into the ITRF93(1995.0) reference frame. The transformation was performed using parameters established by Steve Frakes, National Geodetic Survey. These parameters are summarized in Table 1. In this paper, all ellipsoid heights are expressed relative to the GRS80 normal ellipsoid in the ITRF93(1995.0) frame.

Table 1 -- Transformation from NAD 83 (86) to ITRF93(1995.0)			
ΔX	-0.9769	± 0.0166	m
ΔY	+1.9392	± 0.0137	m
ΔZ	+0.5461	± 0.0141	m
ω_x	-0.0264	± 0.0006	arc sec
ω_y	-0.0101	± 0.0005	arc sec
ω_z	-0.0103	± 0.0004	arc sec
scale	-0.0068	± 0.0017	ppm

The NAVD 88 datum is expressed in Helmert orthometric heights, and was computed in 1991. The network contains over 1 million kilometers (km) of leveling at precisions ranging from 0.7 to 3.0 mm/ $\sqrt{\text{km}}$, and incorporates corrections for rod scale, temperature, level collimation, astronomic, refraction, and magnetic effects (Zilkoski et al. 1992). The NAVD 88 datum was realized by a single datum point, Father Point/Rimouski, in Quebec, Canada. It must be mentioned that there is no guarantee that the NAVD 88 datum coincides with a given normal potential, U_0 . For example, Rapp (1996) found a mean offset for the NAVD 88 datum of -34 cm relative to a U_0 established by $a=6378136.59$. For additional details on the GPS benchmark data set, please refer to Milbert (1995).

High Resolution Geoid Computation and Evaluation

The computation of the high resolution geoid models is based on the use of the Fast Fourier Transform (FFT) to evaluate Stokes equation. As described in Schwarz et al. (1990), a bandwidth-limited signal is needed for input to the FFT convolution. This mandates the use of a remove, compute, and restore procedure; where gravity anomalies from a global geopotential model are subtracted from gravity anomaly data, followed by convolution of the residual anomalies into residual geoid height, and then followed by restoration of geoid heights from that same geopotential model. Such an approach was used in the GEOID90 and GEOID93 computations for the United States (Milbert 1991, Milbert and Schultz 1993).

For the test model evaluation, a standard grid of terrain corrected, Bouguer anomalies was generated. Now, any gridding data set is subject to aliasing in the presence of high-frequency signal. To remove as much predictable, high-frequency content as possible, gridding is performed on terrain corrected, Bouguer anomalies, Δg_{TB} . For anomalies on land

$$\Delta g_{TB} = g + \frac{2\gamma_a}{a} [1 + f + m + (-3f + \frac{5}{2}m) \sin^2 \phi] H - 3 \frac{\gamma_a}{a^2} H^2 - \gamma + A + C - 0.1119H$$

where

$$A = 0.8658 - 9.727 \cdot 10^{-5} H + 3.482 \cdot 10^{-9} H^2$$

and

g	surface gravity, tide corrected, IGSN71 system (mgals)
H	orthometric height, NAVD 88 datum (meters)
γ	normal gravity on ellipsoid, GRS80 (Somigliana's formula) (mgals)
A	atmospheric correction (Wichiencharoen 1982) (mgals)
C	terrain correction from 30" elevation grid (mgals)
ϕ	geodetic latitude, NAD 83 (86) datum
a	semi-major axis, GRS80 (6378137 meters)
γ_a	normal gravity at equator, GRS80 (978032.67715 mgals)
f	ellipsoid flattening, GRS80 (0.00335281068118)
m	0.00344978600308 (GRS80)
ρ	density of topographic masses (2.67 gm/cm ³)
G	gravitational constant
R	mean radius of the earth
ψ	spherical distance

The gridding algorithm uses a method of continuous curvature splines in tension (Smith and Wessel 1990) with tension parameter $T_B = 0.75$. The method is one which honors the data and does not display large oscillations in areas without data coverage. The standard Bouguer grid is a 3' by 3' regular grid extending from 24°N to 53°N and 230°E to 294°E (66°W to 130°W), and contains 581 rows and 1281 columns. All anomalies are prefiltered by computing mean value and mean location of the anomalies in 3' x 3' cells centered over the regular 3' latitude and longitude intersections. This prefiltering step is recommended by Smith and Wessel to reduce spatial aliasing effects prior to gridding.

Since we are using Stokes theory to compute the geoid, we require gravity anomalies on the geoid surface, rather than at the Earth's surface. We adopt Helmert's second method of condensation to regularize the problem and include the indirect effect with Grushinsky's formula

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g_{TB} + 0.1119H) S(\psi) d\sigma - \frac{\pi G \rho H^2}{\gamma}$$

where Stokes function is

$$S(\psi) = \frac{1}{\sin \frac{\psi}{2}} - 4 - 6 \sin \frac{\psi}{2} + 10 \sin^2 \frac{\psi}{2} - (3 - 6 \sin^2 \frac{\psi}{2}) \ln(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2})$$

This setup is solved in a remove-compute-restore procedure using the FFT formulation of Strang van Hees (1990),

$$\Delta g_+ = (\Delta g_{TB} + 0.1119H) - \Delta g'_{360}$$

$$N_+ = \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} F^{-1}[F(\Delta g_+ \cos\phi_Q) F(S(\phi_P - \phi_Q, \lambda_P - \lambda_Q))]$$

$$N = N_+ + N'_{360}$$

where

Δg_+	high-frequency part of gravity anomalies
$\Delta g'_{360}$	geopotential model gravity anomalies at geoid surface
$\Delta\phi, \Delta\lambda$	grid spacing
F, F^{-1}	direct and inverse, two-dimensional, Fourier transforms
N_+	high-frequency part of geoid undulations
N'_{360}	geopotential model geoid undulations at geoid surface

and, where Stokes function, S , is evaluated with a mean latitude, ϕ_m , and the approximation,

$$\sin^{1/2}\psi \doteq [\sin^2 1/2(\phi_P - \phi_Q) + \sin^2 1/2(\lambda_P - \lambda_Q) (\cos^2 \phi_m - \sin^2 1/2(\phi_P - \phi_Q))]^{1/2}$$

The input grid, Δg_+ , (580 rows, 1280 columns) had 50% zero padding on all four edges to eliminate the effect of cyclic convolution (Gleason 1990). No tapering of Δg_+ was performed, since the long wavelength part has already been removed. The FFT subroutine has an option which exploits the Hermitian symmetry resulting from real valued grids. Thus, doubled computation speed and storage efficiency was obtained without resorting to Hartley transforms.

An important issue must be mentioned at this point. The geopotential model geoid undulations, N'_{360} , are evaluated at the geoid surface. From the results of Figures 1a and 1b, it is seen that these geoid heights are biased. However, the Stokes procedure described above will remove that height dependent error if the gravity anomalies, $\Delta g'_{360}$, are also evaluated in a fashion consistent with the geoid undulations. Since it is not convenient to apply some form of “upward/downward” procedure to the gravity anomalies, we choose to evaluate both geoid and anomalies at the geoid surface. The key point is that we are consistent in our choice of surface for coefficient evaluation.

As a formal demonstration, consider Stokes equation written in a remove, compute, and restore procedure:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\psi) d\sigma = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g - \Delta g'_{360}) S(\psi) d\sigma + \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g'_{360} S(\psi) d\sigma$$

The term on the right formulates the δN , which must be restored, as a function of the model gravity anomalies, $\Delta g'_{360}$, that were removed. Expanding Stokes function in Legendre polynomials and the gravity anomalies in terms of the harmonic coefficients yields

$$\delta N = \frac{R}{4\pi\gamma} \iint_{\sigma} \left[\frac{GM}{r} \sum_{n=2}^{360} \left(\frac{a}{r} \right)^n \left(\frac{n-1}{r} \right) \sum_{m=-n}^n \bar{F}_{nm} \bar{Y}_{nm}(\theta, \lambda) \right] \left[\sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos \psi) \right] d\sigma$$

where $\bar{F}_{nm} = \bar{E}_{nm} - \bar{D}_{nm}$. Perform the multiplication, exchange the order of the sums and integrals, under a standard assumption of convergence, combine terms, and invoke the orthogonality relationships of Legendre polynomials. After algebra, the equation above will simplify into

$$\delta N = \frac{GM}{r\gamma} \sum_{n=2}^{360} \left(\frac{a}{r} \right)^n \sum_{m=-n}^n \bar{F}_{nm} \bar{Y}_{nm}(\theta, \lambda)$$

if the leading R before the integral is taken to be r . The equation above is equivalent to N'_{360} . Other than the $R=r$ requirement of the lead term, the development did not invoke any specific surface mandated by a remove, compute, and restore procedure. The development did show the consistent N'_{360} and $\Delta g'_{360}$ expressions.

The high resolution geoid models are derived from four data sources: point gravity measurements, satellite altimetry-derived gravity anomalies, digital terrain, and geopotential coefficients. A few remarks are appropriate. About 1.8 million gravity points, both ship and terrestrial, went into the gridding. These data were a combination of NGS-held data and quality controlled data from the Defense Mapping Agency. The satellite altimetry-derived gravity anomalies were computed by Sandwell and Smith (1996). These were included in gravity void regions where depths exceeded 500 meters and which were 200 km or more from land. The digital terrain data came primarily from the 30" point topography database, TOPO30, distributed by the National Geophysical Data Center (Row and Kozleski 1991). The 30" elevation set was used to compute both terrain corrections, and 3' x 3' mean elevations. And, of course, the preliminary geopotential models, X01 through X05, were used as the coefficient sources. We denote the resultant high resolution models as 9620 through 9624, respectively.

We explore the general properties of the solutions by computing geoid height residuals, e , in the sense of

$$e = \text{high-res. geoid height} - (\text{ITRF93(1995.0) ellipsoid height} - \text{NAVD 88 orthometric height}).$$

Tilted plane models are fit to the 2497 residuals and are summarized in Table 2.

Model	Offset (cm.)	Tilt (ppm)	RMS about plane (cm)	Azimuth (deg)
9620	-4.58	0.04	35.07	293
9621	-5.90	0.06	36.14	317
9622	-2.07	0.05	36.14	319
9623	-2.05	0.06	36.56	321
9624	-2.05	0.05	36.52	321

Table 2. Tilted plane fits to 2497 high resolution geoid model residuals .

These results begin to indicate model differences. The average offsets suggest that X01 and X02 used differing data and/or procedures from X03, X04, and X05. The small magnitude of the offsets must be considered in the context of the comparisons. The GRS80 normal ellipsoid is used to express the ellipsoidal heights, the normal gravity, and the high resolution geoid models. Thus, the small offsets are not inconsistent with the -34 cm found by Rapp (1996).

Next, following the approach of Milbert (1995), smoothed residual surfaces are computed by means of collocation for all 5 high resolution models. Empirical covariance functions were fit to the covariance statistics of the detrended geoid residuals. The fits were made using a simple function of the form

$$C = C_0 e^{(-d^2/L^2)}$$

where

- d = the spherical distance between points (km)
- L = characteristic length (km)
- C_0 = function variance (m^2)

The function fits were virtually identical, with only minor variations of $L = 485$ km and $C_0 = (0.31)^2 m^2$. As was seen in Milbert (1995), the residual error is sizable and correlated over a long length scale. The detrended residual error, $\hat{s}$, is predicted on a 30' x 30' grid using least-squares collocation with noise (Moritz 1980, p.102-106). The prediction formula is

$$\hat{s} = C_{st} (C_{tt} + C_{nn})^{-1}$$

where

- C_{tt} = signal covariance between observations
- C_{st} = signal covariance between predicted signal and observations
- C_{nn} = covariance of random measuring errors, taken as diagonal and constant: $C_{nn} = s_o^2 I$

Before the grids are computed, the prediction process is iterated to establish a value of s_o^2 consistent with the residual misfit about the predictions. It was found the RMS of residuals from the prediction step matched the assigned noise when $s_o^2 = (5.6)^2 cm^2$ for the $n=2497$ points. The trends reported in Table 2 were restored to the detrended signal grids, resulting in the final corrector surface grids (adjusted in sign to provide an additive correction).

Figure 3 portrays a trend surface that, when added to the 9622 geoid model, will directly relate ITRF93(1995.0) ellipsoid heights and NAVD 88 Helmert orthometric heights. In viewing this figure it must be recalled that the GPS benchmarks used to develop this grid all lie within the U.S. borders; and that highs or lows in the oceans or in other countries are extrapolations, and are not reliable. In the interests of concision, the trend surfaces for the other 4 models are not displayed, since they appear almost identical.

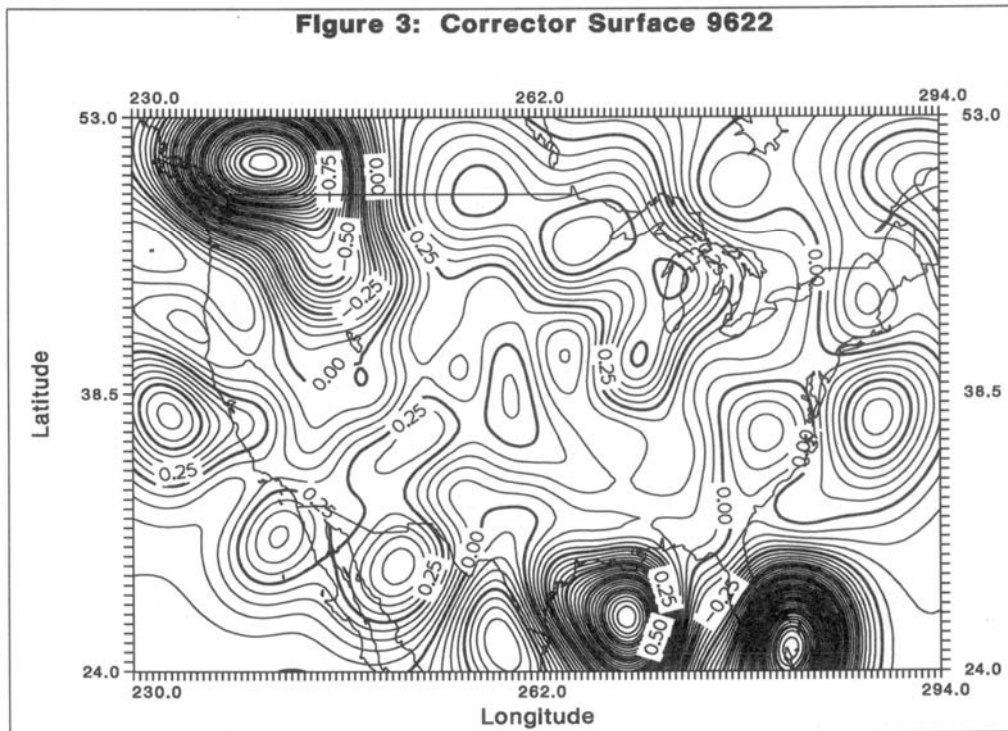


Figure 3: Trend Surface to Relate High-Resolution Model 9622 to GPS Benchmarks (Contour Interval = 0.05 m).

As part of the evaluation process, we computed all possible combinations of the residual trend surface differences, 10 in all. These are reproduced in Figures 4a through 4j. Inspection of the combinations indicate that the 9623 and 9624 models were virtually identical, and that 9622 was very close to the 9623 model. (One must discount features in the GPS benchmark void areas.) It is also seen that model 9621 shows some departures from 9622, and that 9620 is the most different of all the models. These results confirm the similarities of the 9622, 9623, and 9624 models, and also suggest that both 9620 and 9621 are distinct from them and are distinct from one another.

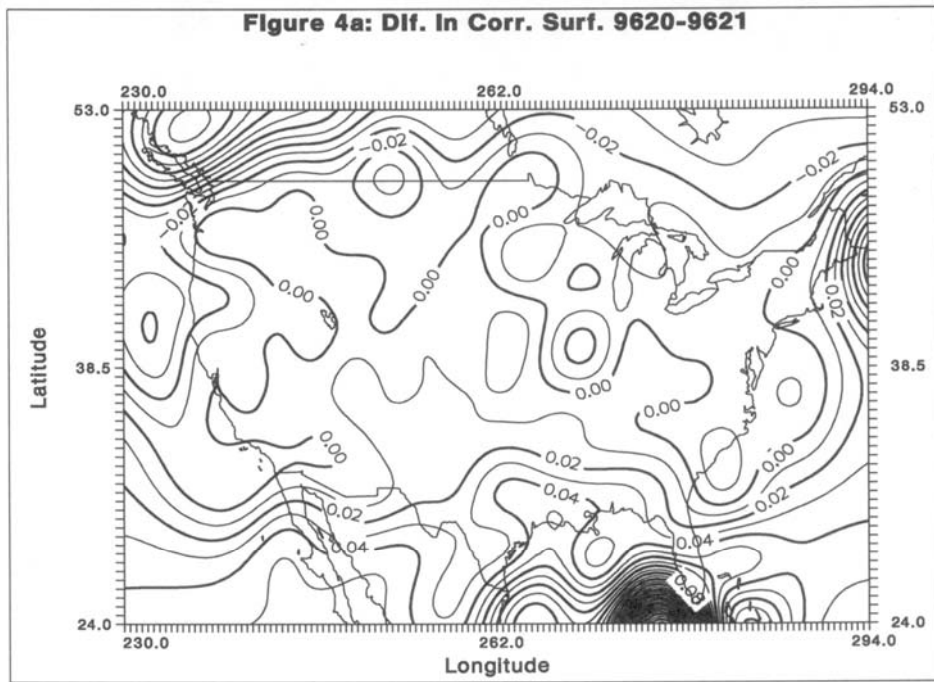


Figure 4a: Differences between Trend Surfaces 9620 and 9621 (Contour Interval = 0.01 m)

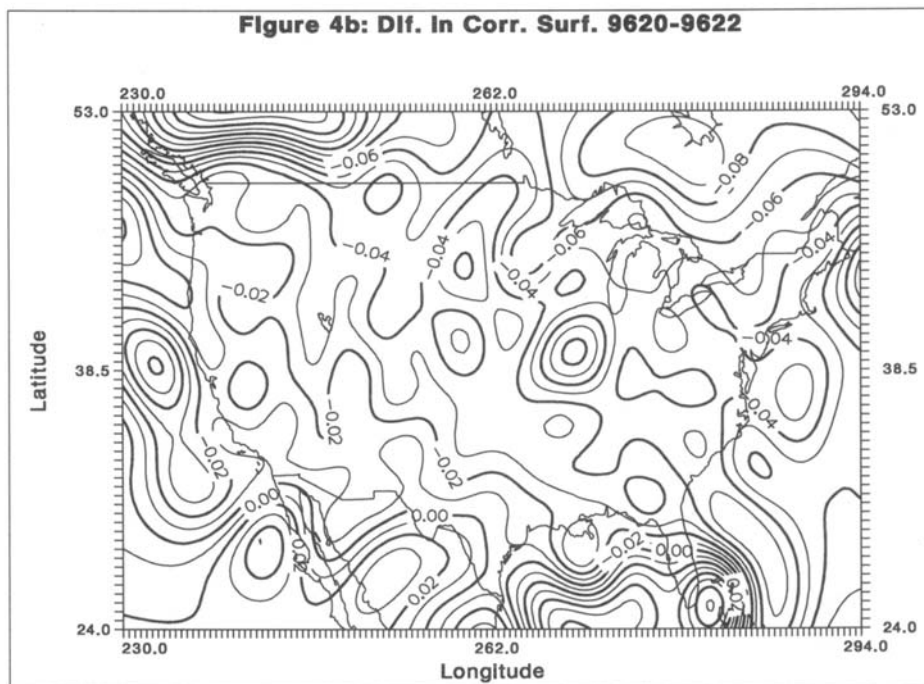


Figure 4b: Differences between Trend Surfaces 9620 and 9622 (Contour Interval = 0.01 m)

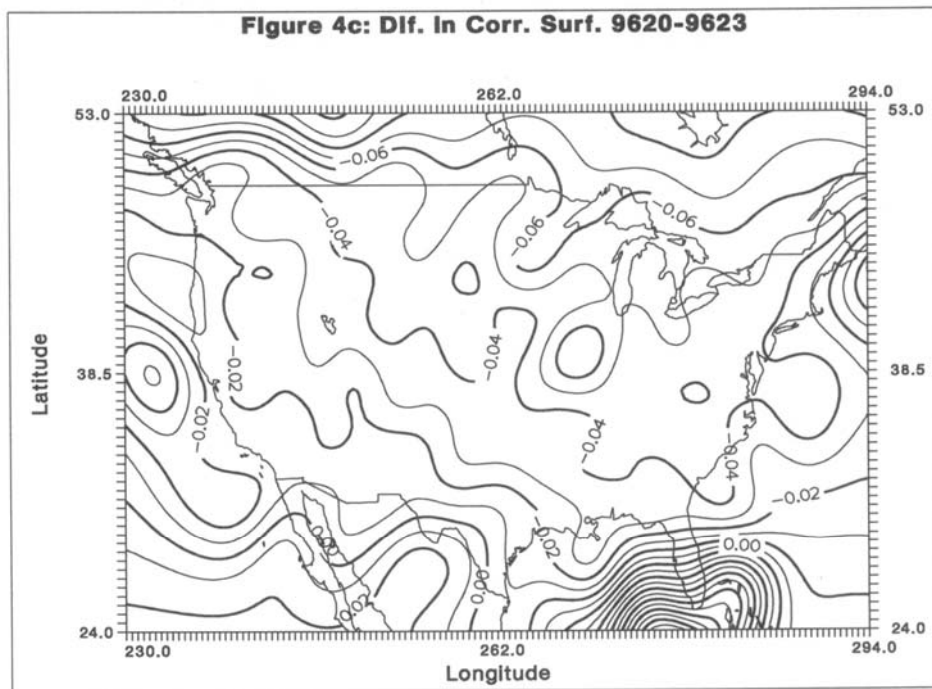


Figure 4c: Differences between Trend Surfaces 9620 and 9623 (Contour Interval = 0.01 m)

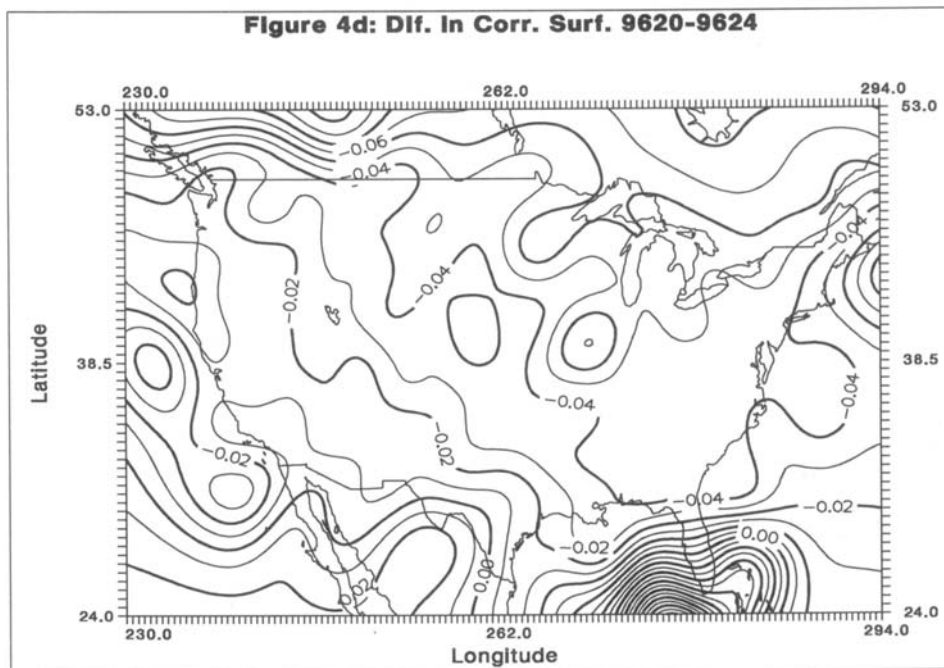


Figure 4d: Differences between Trend Surfaces 9620 and 9624 (Contour Interval = 0.01 m)

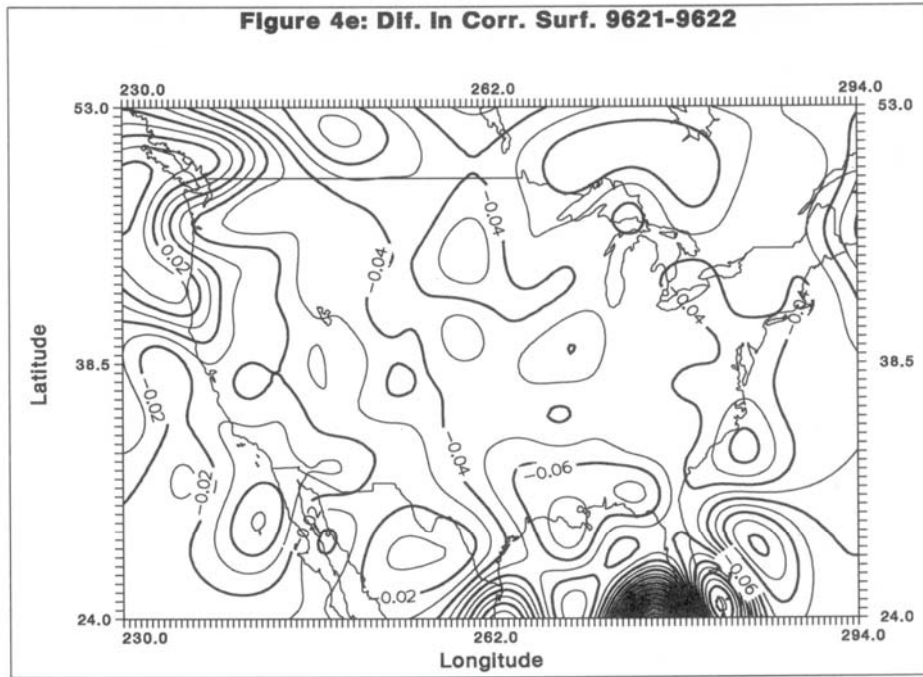


Figure 4e: Differences between Trend Surfaces 9621 and 9622 (Contour Interval = 0.01 m)

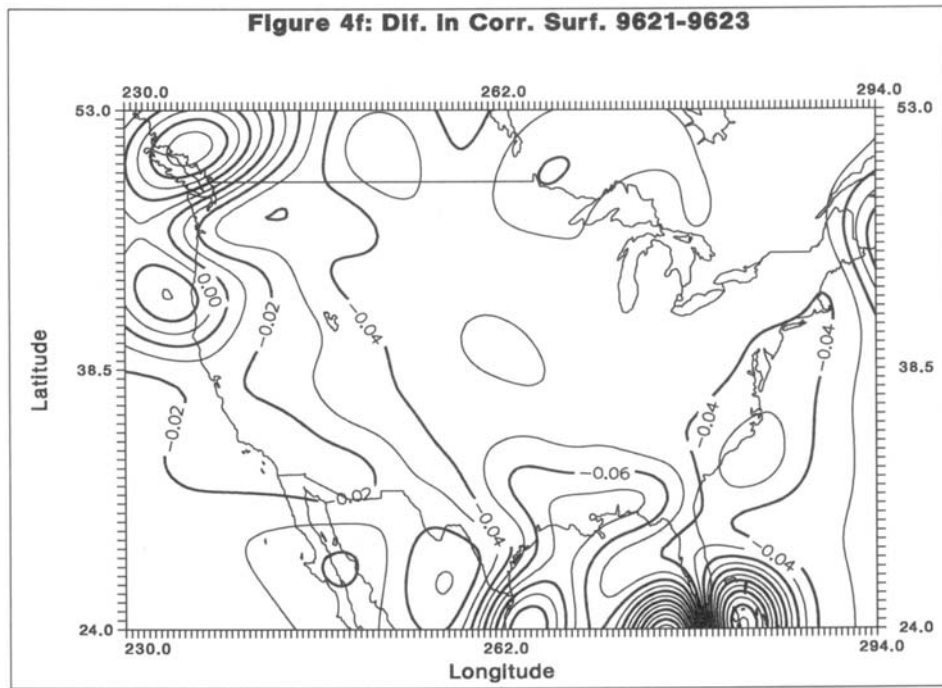


Figure 4f: Differences between Trend Surfaces 9621 and 9623 (Contour Interval = 0.01 m)

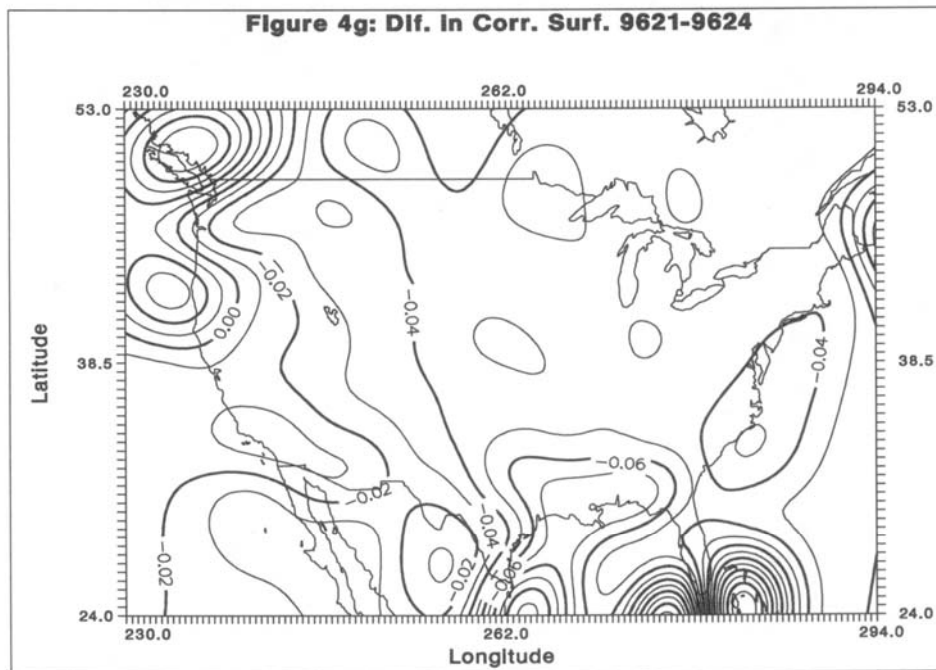


Figure 4g: Differences between Trend Surfaces 9621 and 9624 (Contour Interval = 0.01 m)

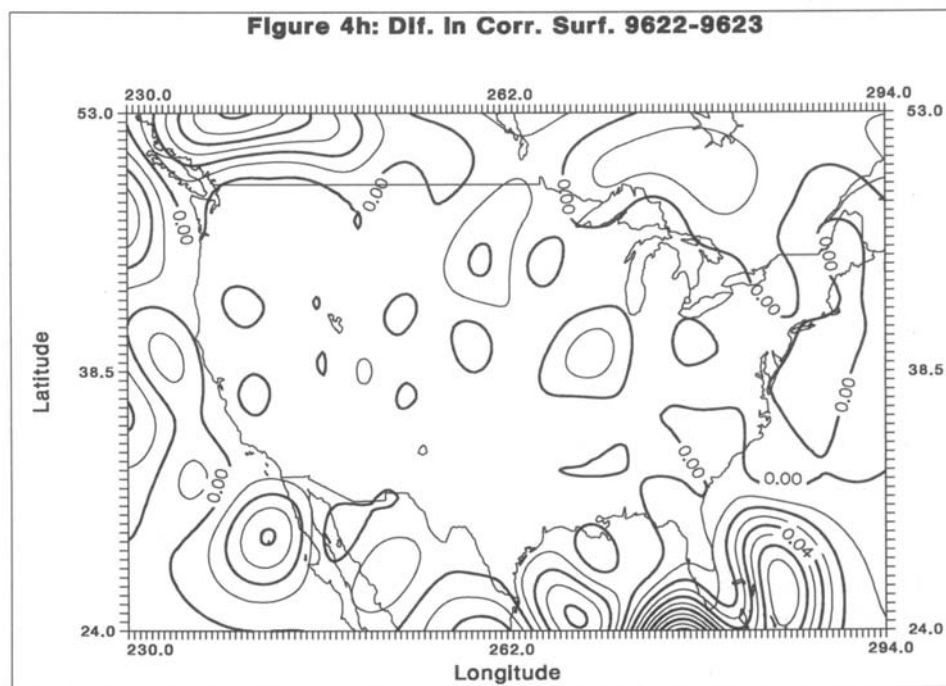


Figure 4h: Differences between Trend Surfaces 9622 and 9623 (Contour Interval = 0.01 m)

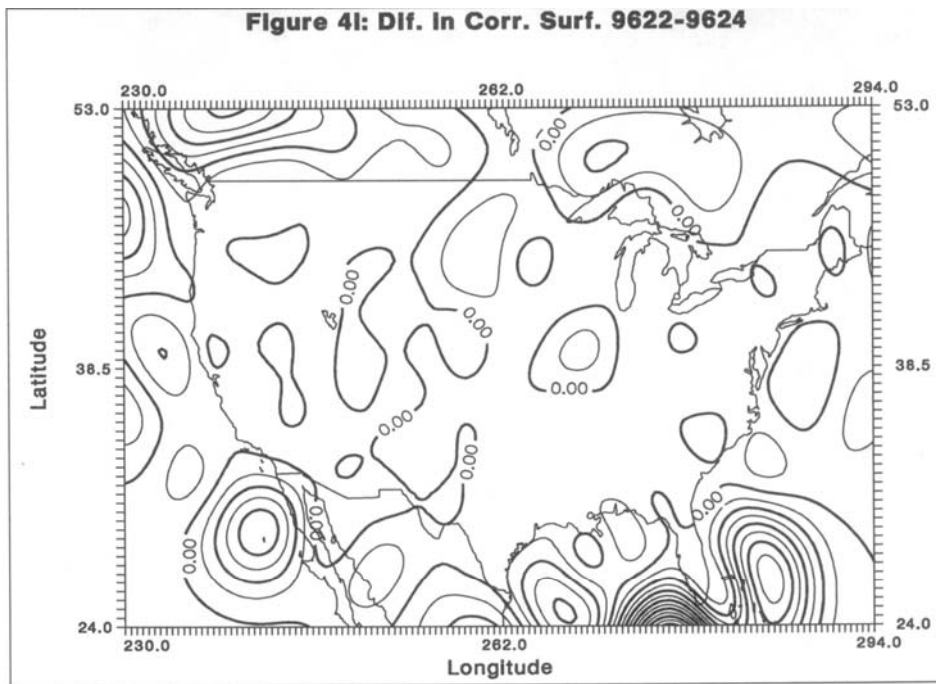


Figure 4i: Differences between Trend Surfaces 9622 and 9624 (Contour Interval = 0.01 m)

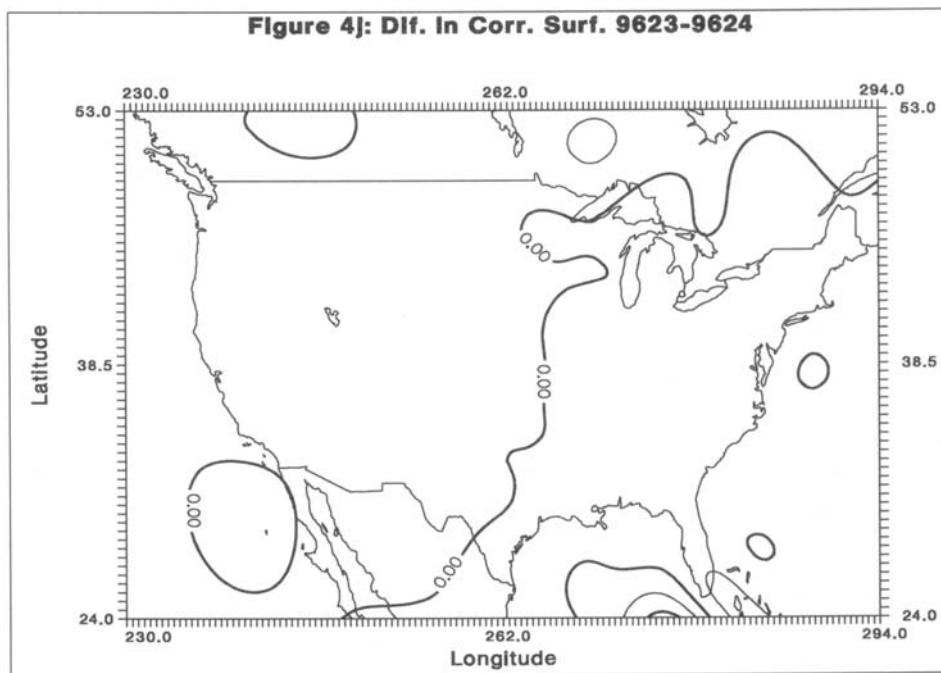


Figure 4j: Differences between Trend Surfaces 9623 and 9624 (Contour Interval = 0.01 m)

At this point it is helpful to review particular features seen in the trend surfaces. A tilt is evident in Florida, and it is virtually the same in all models. This effect has been observed with both the GEOID90 and GEOID93 models, as well as with numerous test models. Given the great care taken by the geopotential model computation team in the intercomparisons of marine gravity and altimetry derived gravity, and given the very flat topography of Florida, it is unlikely that the geoid models are the source of this feature. Further, since the GPS network is anchored to the Richmond VLBI site at the tip of Florida, it is also unlikely that the ellipsoid heights are biased in that region. The leveling is suspect. On the other hand, the tilt along the Gulf Coast is probably gravimetric in origin; and will be discussed later in this section.

A North-South tilt is also evident in Washington State. The source of this feature seems to be the terrain corrections computed from digital terrain data in southern British Columbia. We are working with the Geodetic Survey of Canada to resolve this problem. It is thought, however, that the mean gravity anomalies used to compute the preliminary geopotential models are accurate. The tilt is seen to blend into an elongated feature over the northern Rockies. This raises some concern over possible height-dependent error.

As a different view of the models, we made scatter plots of the geoid height residuals relative to elevation. A sample for the 9621 model is given in Figure 5, since the scatter plots look similar. The dispersion of values near the zero elevation reflect the issues in Florida and the Gulf Coast noted earlier. No obvious height dependence is evident in Figure 5, unlike the effect seen in Figure 1a. However, overlays of the the 5 different scatter plots do show small variations that could have an elevation dependence.

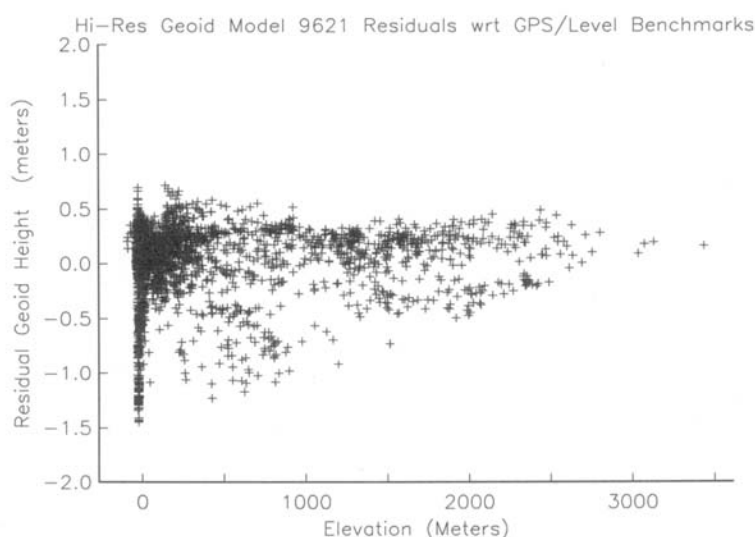


Figure 5: High Resolution Model 9621 Height Residuals to GPS Benchmark Geoid Heights

In pursuit of this possibility, an exploratory GPS benchmark data set was formed by withholding the points in Florida south of 30°N, and by withholding the points north of 48°N in Washington State. The average offset, and the RMS about the offset are then computed. In addition, the offset and RMS are computed where data are withheld below 1000 m, below 1500 meters, and below 2000 meters in elevation. The statistics are grouped by model, ordered by ascending elevation, and displayed in Table 3. In addition, the differences in average offset, and second differences are displayed.

9620 Model (X01)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.85	9.15		
	1000	448	23.99	6.58		
	1500	268	24.98	6.32	-0.26	
	2000	107	24.79	6.70	0.38	0.64
9621 Model (X02)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.59	9.82		
	1000	448	23.76	7.50		
	1500	268	24.71	7.15	-0.35	
	2000	107	24.51	7.57	0.42	0.77
9622 Model (X03)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.64	9.71		
	1000	448	23.77	7.07		
	1500	268	24.78	6.92	-0.15	
	2000	107	24.50	7.46	0.54	0.69
9623 Model (X04)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.66	9.92		
	1000	448	23.78	7.38		
	1500	268	24.77	7.26	-0.12	
	2000	107	24.47	7.83	0.57	0.69
9624 Model (X05)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.73	9.84		
	1000	448	23.83	6.97		
	1500	268	24.80	6.88	-0.09	
	2000	107	24.50	7.43	0.55	0.64

Table 3. Geoid residual statistics ordered by elevation cutoff tolerance.

Table 3 enables us to discern more about the computation of the preliminary models. The differences in the average offset between the 0 m elevation cutoff and the 1000 m cutoff are probably indicative of remaining systematic errors in the GPS benchmark or in the detail gravity data, rather than reflecting characteristics of the global models. We know that the average offset will contain NAVD 88 vertical datum error. We don't know what that error is, but we can expect that error to be constant with respect to elevation. Thus, we can expect a zero value for the *differences* in average offset. Height dependent effects will cause variations in the average offsets as the elevation cutoff is varied.

Looking at the offset difference column, we see that X03, X04, and X05 are distinct from X01 and X02. In fact, the close similarities suggest that X03, X04, and X05 are members of a family, perhaps controlled by some tuning parameter(s), such as anomaly weighting. And, it

seems that the models X03, X04, and X05 are provided in sequence; corresponding to monotonic changes in the hypothetical parameter.

At this stage it is seen that model 9624, computed from X05, displays the most uniform average offsets.

To close this section, a result was obtained that sets the stage for the next section of the report; where the models are directly compared against GPS benchmarks. Recall that in this section, the geopotential models are evaluated through high-resolution geoid models computed in a remove, compute, and restore procedure. One of the products of this procedure is the output grid from the FFT convolution, which we denoted, N_+ . This grid contains the high-frequency part of the geoid (truncation or omission error), but also contains information about lower-frequency commission error.

One must interpret an N_+ grid with caution. First, the grid, when added to the preliminary model geoid, N_{360} , will yield the co-geoid. One can think of the N_+ grid as being biased by the indirect effect, since Faye anomalies were used in its computation. Second, our model geoids, N_{360} , were biased, since they were evaluated interior to the Earth. Therefore, our N_+ grids will also contain that height-dependent bias. Despite these drawbacks, the N_+ grid certainly contains useful information at lower elevations.

Figure 6 portrays a color shaded relief image of the N_+ grid associated with the X02 model. Sixteen hues were allocated from the low of -1.5 m (magenta) to the high of +1.5 m (red). Sixteen levels of gray shading were added to give the effect of illumination from the East. The magnitudes of the grid actually ranged from -4.1 m to 2.8 m. Thus, values which exceed the ± 1.5 meter interval will “saturate” into the magenta or the red coloring. The grids from the other preliminary models look very similar.

Most noticeable is the large blue and magenta structure in the Rockies. Because of the bias issues discussed above, one can, at most, say that it hints at significant commission error in the Rockies. This issue will be explored in the next section.

Other features are interesting. The lows in Lake Superior may be due to our treatment of the elevations of the ship gravity data. The low in the Bermudas may also be related to our gravity data set, and will be investigated at a later stage. While the Bermudas feature does induce a tilt across Florida, it is localized to the south, and does not explain the complete amount of tilt we have seen in the past. In addition, some portion of the low seen in Washington State and southern British Columbia is due to currently unresolved problems with terrain corrections, discussed earlier.

On the other hand, the 0.5 m feature in the Central U.S., and the 1 m features in the Gulf of Mexico and off the Atlantic seaboard are less easily explained. These are regions of low elevations, and are rather broad. The data in the Gulf and near-shore Atlantic are ship gravity data. The altimeter-derived gravity anomalies from Sandwell and Smith (1996) were only used in the southern part of the Gulf and in the deep Atlantic. We suspect these particular features

may be due to preliminary model commission error; perhaps correlated with some sea surface (dynamic) topography effect in the ocean.

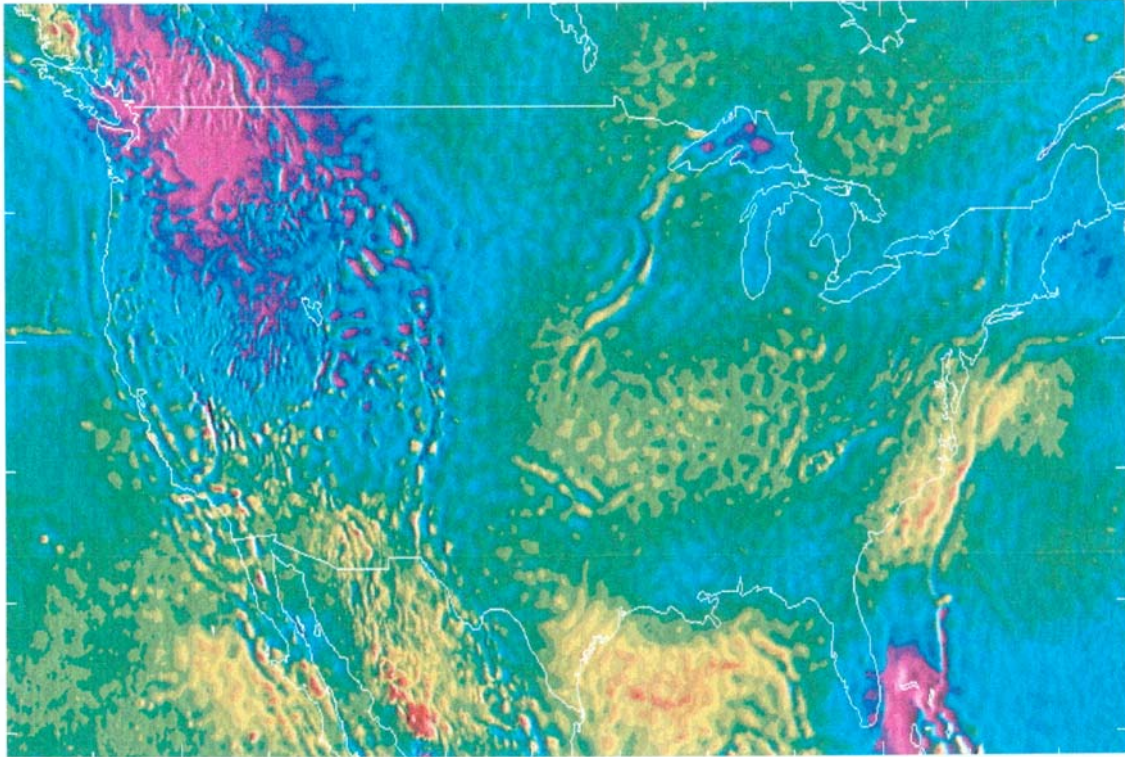


Figure 6: High Frequency Component of Geoid Height for High-Resolution Model 9621 Relative to Preliminary Geopotential Coefficient Model X02.

Low Resolution Geoid Computation and Evaluation

In this section, we compare geoid heights computed directly from the preliminary geopotential models against the GPS benchmark data set. While these statistics are very noisy, due to omission error, they can portray long wavelength commission error.

Five geoid grids at 3' x 3' spacing were computed from the X01 through X05 coefficients. All grids were computed with the “up/down” procedure, where the coefficients are evaluated at the ellipsoid, upward continued to the surface to form a height anomaly, and then converted “down” to a geoid height by means of Eq. (8-10) of Heiskanen and Moritz (1967). This last step is implemented as a 3' x 3' grid which has been low pass filtered to correspond to an $n=360$

resolution. We emphasize that these geoid grids are *not* subject to the height-dependent error illustrated in Figure 1a.

Tilted plane models are fit to 2497 residuals about the low resolution geoid grids, and are summarized in Table 4.

Model	Offset (cm.)	Tilt (ppm)	RMS about plane (cm)	Azimuth (deg)
X01	-2.16	0.40	26.52	338
X02	1.02	0.32	29.77	336
X03	0.07	0.35	26.22	334
X04	0.43	0.35	25.99	335
X05	0.76	0.35	26.10	335

Table 4. Tilted plane fits to 2497 low resolution geoid model residuals .

These results are remarkable, in that they show a significant tilt to the Northwest. This confirms the suspicion that the large blue and magenta feature seen in the Rockies in Figure 6 is at least partly due to preliminary geopotential model commission error. It should be recalled that this tilt was not evident when the GPS benchmark residuals to the high-resolution geoid models were computed in Table 2. It appears that the high-resolution models remove not only omission error, but also commission error.

To produce an image for evaluation, smoothed residual surfaces were computed by means of collocation for all 5 low resolution models. Empirical covariance functions were fit to the covariance statistics of the detrended geoid residuals. The function fits were virtually identical, with only minor variations of $L = 200$ km and $C_0 = (0.21)^2 \text{ m}^2$. The trends reported in Table 4 were restored to the detrended predicted signal grids, resulting in the final smoothed residual surfaces. The surfaces for the 5 models were virtually identical, so only the result for model X01 is displayed in Figure 7.

The complex structure seen in Figure 7 is a representation of the preliminary model omission error as sampled by the GPS benchmarks. These small features can be reduced with more stringent smoothing, and should be ignored. Similarly, features in the oceans (e.g. near Florida) or beyond the U.S. borders should also be ignored, since those represent extrapolations. Further, the presence or absence of structure in the central U.S. has no meaning, due to the absence of GPS benchmarks.

However, an offset in the northern Rockies, relative to the plains, is evident in Figure 7. Unfortunately, shortages of GPS benchmarks in the central U.S., in the Carolinas, and south of Houston, Texas, make it difficult to comment on other features discussed in Figure 6. It is noteworthy that the tilt in Florida is still evident; a result consistent with a hypothesized problem in the level network.

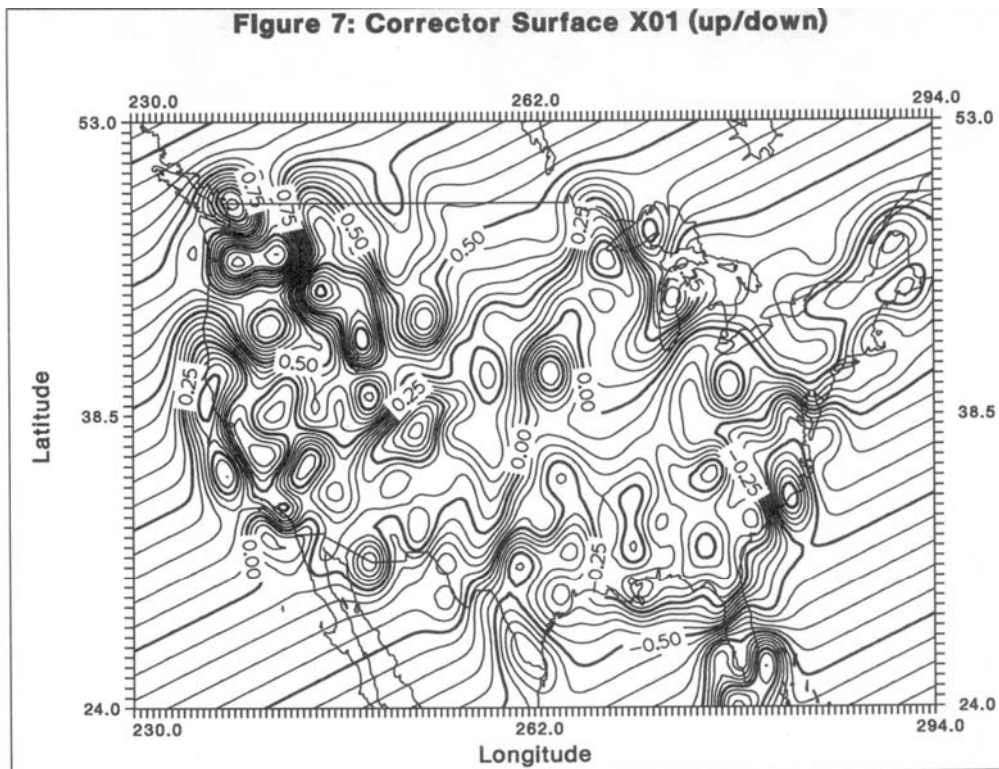


Figure 7: Trend Surface to Relate Low-Resolution Geoid Model X01 to GPS Benchmarks (Contour Interval = 0.05 m).

To further study the behavior of the preliminary models and the possibility of commission error in the Rockies, 505 GPS benchmarks in the region of 38° - 49° N and 230° - 255° E were extracted from the full data set. These points were then grouped into 500 m elevation cohorts, and statistics were computed. The statistics are grouped by model, ordered by ascending elevation, and displayed in Table 5. In addition, the differences in average offset, and, second and third differences are displayed. Only 15 GPS benchmarks are present in the "over 2500 m" cohort, and those statistics are not felt to be reliable.

X01 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	46.67	70.05			
	500-1000	97	27.69	69.22			
	1000-1500	93	28.78	49.63	-19.59		
	1500-2000	91	31.17	35.16	-14.47	5.12	
	2000-2500	71	34.38	18.02	-17.14	-2.67	-7.79
	Over 2500	15	34.92	-23.29			

	All	505	42.38	49.75			

X02 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	37.41	66.00			
	500-1000	97	26.80	62.36			
	1000-1500	93	29.39	49.42	-12.95		
	1500-2000	91	35.50	38.88	-10.54	2.41	
	2000-2500	71	30.62	23.28	-15.60	-5.06	-7.47
	Over 2500	15	37.65	-4.00			
	All	505	37.26	49.28			
X03 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	49.90	73.29			
	500-1000	97	30.50	69.54			
	1000-1500	93	27.14	45.97	23.57		
	1500-2000	91	32.09	36.37	9.59	13.98	
	2000-2500	71	32.07	19.11	-17.26	-7.67	-21.65
	Over 2500	15	36.65	-16.48			
	All	505	43.45	50.60			
X04 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	48.76	72.19			
	500-1000	97	31.32	69.65			
	1000-1500	93	28.26	44.36	-25.29		
	1500-2000	91	32.13	35.78	-8.58	16.70	
	2000-2500	71	31.75	20.01	-15.77	-7.19	-23.89
	Over 2500	15	36.04	-12.59			
	All	505	42.96	50.16			
X05 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	49.27	71.02			
	500-1000	97	31.76	69.20			
	1000-1500	93	28.49	42.93	-26.27		
	1500-2000	91	32.98	34.90	-8.03	18.24	
	2000-2500	71	29.93	18.54	-16.36	-8.33	-26.57
	Over 2500	15	35.05	-12.43			
	All	505	43.14	49.13			

Table 5. Geoid residual statistics ordered by 500 m elevation cohorts.

Table 5 supports many of the conclusions developed from analysis of Table 3. Models X03 through X05 seem to be members of a family, and seem ordered relative to some tunable parameter(s). Models X01 and X02 are distinct from that family, and from one another. Unlike the situation in Table 3, some interpretation can be applied to the average offset column. While we do not know the true value of the datum offset associated with NAVD 88 (in GRS80), the high-resolution geoid model results in Table 3 indicate that it is a value closer to 10 cm than to 40 cm. This leads us to prefer models with a smaller average offset, since we feel the offset is partly due to commission error. Models X02 and X05 look best. In general, model X05 has smaller offsets per cohort, but shows greater height dependence of those offsets when compared to X02. The smaller RMS about the average offsets of model X02 is also attractive.

Conclusions and Recommendations

Differencing the geoid grids computed from the preliminary geopotential model coefficients show the test models to be very similar. So it is clear that any of the test models represents a major advance over currently available geopotentials. With this said, we have a preference for models X05 and X02, with a very slight edge towards X05. This preference is based on criteria of low height dependence of geoid residuals, low average offsets of the residuals (for the case of the low-resolution models), and low RMS values. It is interesting that these criteria lead to the same model preferences irrespective of the high-resolution or the low-resolution approach.

Since it is likely that the final model will not be any one of the beta test models, it is worthwhile to state that an ideal model would combine characteristics from X02 and X05. However, all the preliminary models share the commission error seen in the region 38° - 49° N, 230° - 255° E. If this effect could be lessened, then the final model would be even better.

Acknowledgments

Work of this scope required the contributions of dozens of individuals. Virtually every employee of the National Geodetic Survey has assisted in this effort. The gravity data set, the NAVD 88 project, and the GPS state upgrade program were vital to this study. The Defense Mapping Agency (DMA) provided a major portion of the NGS land gravity data set, and was instrumental in creation of the various 3" and 30" elevation grids in existence. Dr. Walter Smith, NOAA, provided the altimeter-derived gravity anomalies.

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Evaluation of the EGM96 Model of the Geopotential in the United States

(Addendum to: Evaluation of Preliminary Models of the Geopotential in the United States)

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Introduction

As a final report for the Special Working Group of the International Geoid Service, we have evaluated the geopotential model EGM96. The tests are all similar to those performed on models EGM-X01 through EGM-X05, and comparisons between the final model and the preliminary models are made. The EGM96 model served as the base model in the remove-compute-restore procedure in the calculation of G96SSS, the recently released high-resolution model of the geoid for the United States (and the foundation of the GEOID96 product).

It must be noted that some of the data sets used in this evaluation are not the final data sets used in the computation of G96SSS. These older data sets were used in this evaluation strictly to maintain compatibility with the previous beta test report (Smith and Milbert, 1996).

Use of EGM96 Coefficients in Computing Geoid Heights

Between the time of the beta report and the testing of EGM96, some clarifications were made regarding the computation of geoid undulations from a spherical harmonic model of the Earth's external gravitational potential. Specifically, the treatment of the degree zero component of the geoid undulations was investigated. It was pointed out that a correction for the difference between the normal potential of GRS-80 and the true potential of the geoid (as best we know it), should be applied to geoid undulation computations, when using GRS-80 as the chosen reference field. However, in an effort to maintain compatibility with results in the beta model report, we will continue to neglect this correction in this report. However, it has been implemented in the computation of GEOID96. The effect of this term amounts to a constant bias of 41 cm in the geoid undulation computations.

Low Resolution Geoid Computation and Evaluation

Using the methods described in the previous section, combined with the height anomaly to geoid undulation correction (the "up/down" procedure) discussed in the beta report, geoid undulations were computed from EGM96, incorporating a 3'x3' ellipsoid height DTED for the conterminous United States, and a 3'x3' grid of simple Bouguer anomalies. This grid of geoid undulations (N_{EGM96}) was compared to the geoid undulations implied by 2497 GPS ellipsoidal heights on leveled benchmarks (N_{BM}). A tilted plane was fit through the residuals ($N_{\text{EGM96}} - N_{\text{BM}}$) in the same way as the beta models, and the results of that fit are shown below.

Model	Offset (cm.)	Tilt (ppm)	RMS about plane (cm)	Azimuth (deg)
X01	-2.16	0.40	26.52	338
X02	1.02	0.32	29.77	336
X03	0.07	0.35	26.22	334
X04	0.43	0.35	25.99	335
X05	0.76	0.35	26.10	335
EGM96	2.01	0.41	27.03	343

Table 1. Tilted plane fits to 2497 low resolution geoid model residuals .

These statistics show EGM96 is something of an outlier, relative to the five beta models, without being unreasonably different. In order to help illustrate the continental scale of the tilt, the following figures have been provided. Figure 1 shows a scatter plot of geoid undulation residuals relative to orthometric height. It is encouraging to see that there is little dependence upon elevation, though a small correlation could possibly be inferred. Figure 2 shows the scatter plot of geoid undulation residuals relative to latitude, and Figure 3 shows them relative to longitude. The combined message of these two plots is that a northerly and westerly (i.e. azimuth of 343) upward tilt is clearly occurring across the entire United States, and is not just being driven by any cluster of localized residuals. Please keep in mind that these statistics do not reflect the final data sets and theory used in G96SSS, but rather are being kept internally consistent with the report on the beta models.

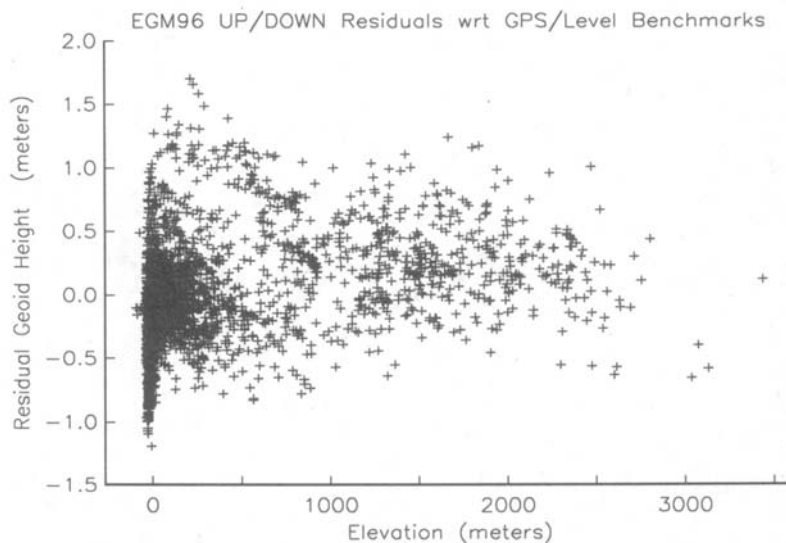


Figure 1: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to height

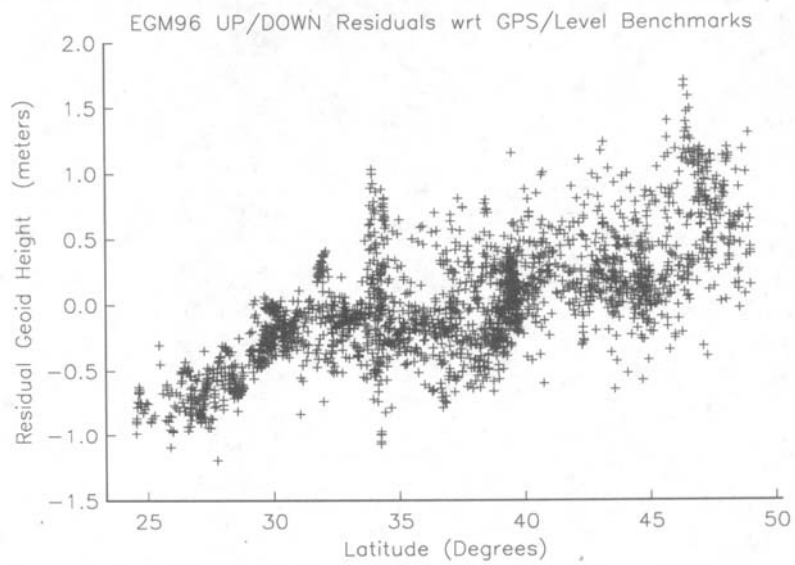


Figure 2: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to latitude

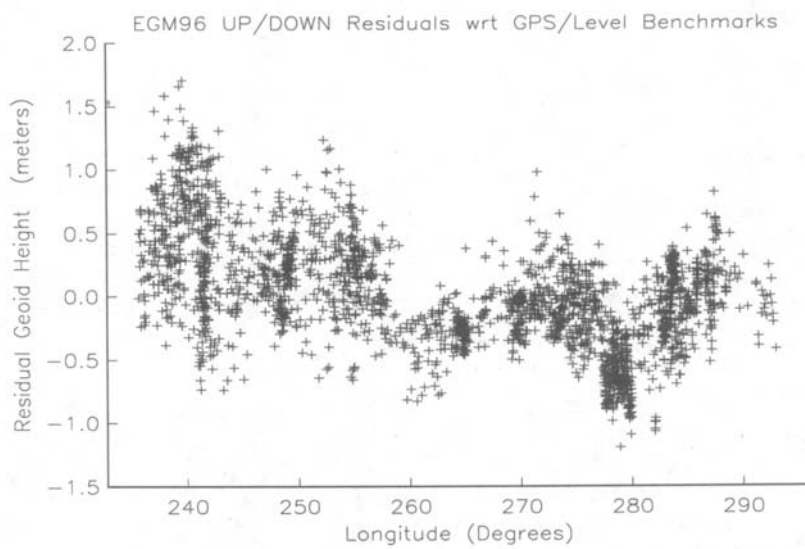


Figure 3: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to longitude

Although it is important to maintain consistency with the Smith and Milbert, 1996 (for the rest of this paper, called the “beta report”), it is also quite useful to see how EGM96 compares to our final, best gravimetric geoid model, G96SSS (Milbert and Smith, 1996). As such, Figure 4 shows the difference between the geoid undulations of G96SSS and the undulations of EGM96, corrected for the height anomaly to geoid undulation (Rapp, 1996). The color scale in Figure 4 ranges from -150 cm (magenta) to +150 cm (red), though the data actually range from -186 cm to +334 cm. In this figure only, we use the more recent gravity data sets and theory to show our final version of potential “commission error” which is inherent in EGM96. This figure is not completely consistent with Figure 6 of the beta report, but is very useful in showing the areas where surface data and EGM96 differ from one another.

The behavior of the geoid in the Pacific Northwest is of special interest due to its rugged terrain and difficulty in matching GPS benchmarks with the geoid in the area. As per the beta model test, a study of the EGM96 model in the area of 38° - 49° N and 230° - 255° E was made. In that area, 505 GPS benchmarks are found. To avoid redundancy with the beta model report, only the EGM-X05 results are repeated below, along with the EGM96 results. The statistics are ordered by ascending elevation groups. Additionally, first, second and third differences are presented.

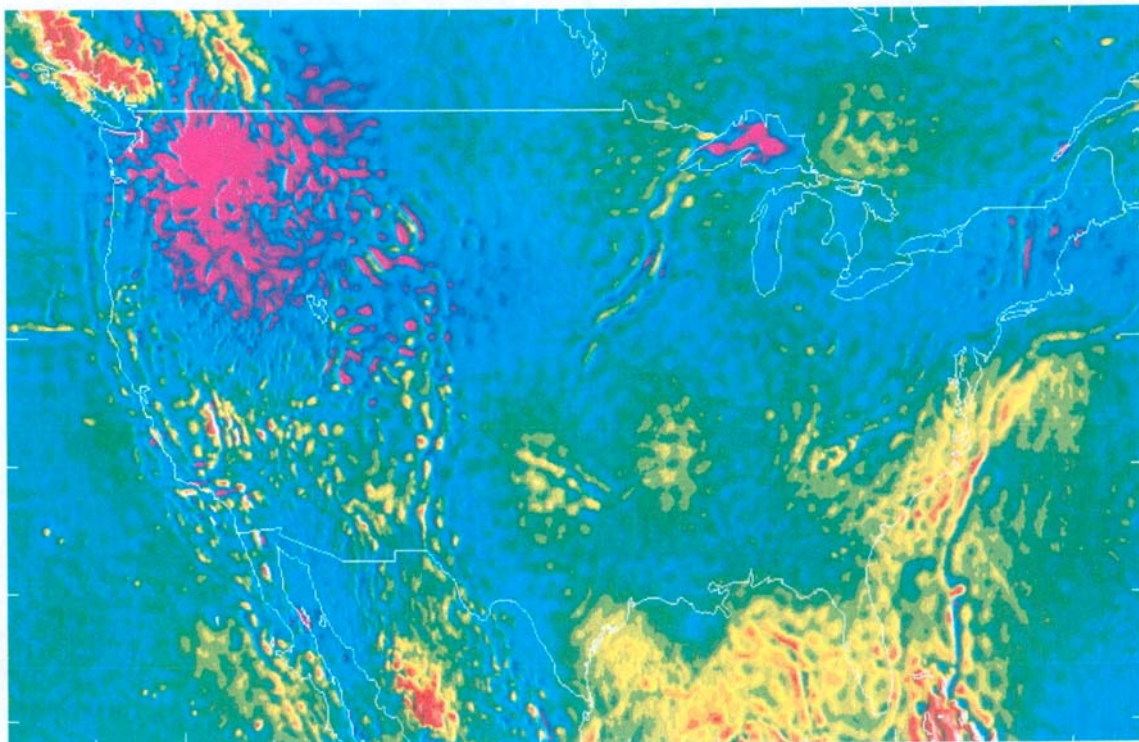


Figure 4: Geoid Undulation Residuals, G96SSS vs. EGM96

Table 2 shows that EGM96, as mentioned earlier, is something of an outlier relative to the five beta models (although, to be succinct we did not repeat the results of X01-X04). While its 2nd and 3rd differences have similarities to EGM-X05, the differences in offsets at various elevation groups show EGM96 to lie outside the three families (X01, X02 and X03-X05) identified in the beta model report. Table 2 corresponds with Table 5 in the beta model report. Results in Table 2, as well as the information contained in Figure 4, show that EGM96 also contains the elevation dependent commission error evident in the earlier beta models. Additional investigations with subsets of the GPS benchmark residuals (not reproduced here) show the elevation dependence is not due to GPS ellipsoid height scatter above 42 degrees latitude (discussed in the next section).

X05 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	49.27	71.02			
	500-1000	97	31.76	69.20			
	1000-1500	93	28.49	42.93	-26.27		
	1500-2000	91	32.98	34.90	-8.03	18.24	
	2000-2500	71	29.93	18.54	-16.36	-8.33	-26.57
	Over 2500	15	35.05	-12.43			
	All	505	43.14	49.13			
EGM96 Model	Cohort	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff	3rd-Diff
	0- 500	138	45.68	68.07			
	500-1000	97	31.45	66.24			
	1000-1500	93	29.16	44.42	-21.83		
	1500-2000	91	33.49	39.00	-5.42	16.41	
	2000-2500	71	28.69	25.18	-13.82	-8.41	-24.81
	Over 2500	15	38.22	-3.46			
	All	505	40.21	49.97			

Table 2. Geoid residual statistics ordered by 500 m elevation cohorts.

High Resolution Geoid Computation and Evaluation

The details surrounding the computation of high resolution geoid models are found in the beta model report. Using the identical procedures and data sets as the beta models, a new high resolution geoid model for the United States was produced, which we call model 9696 (Please note that this is *not* the same as G96SSS, as the methods and data sets used in the beta model report are somewhat inconsistent with the final data and procedures of G96SSS). Model 9696 is consistent with models 9620 - 9624 (see beta report), which are computed using EGM-X01 - EGM-X05 respectively.

In a way similar to the low resolution model test, a tilted plane was fit to the high resolution models, to find how additional gravity data can be used to remove commission error in the low resolution models. The results were encouraging, showing that most of the tilt which appears in the low resolution models is removable by computing a high resolution model. The statistics of the fitted planes are found in Table 3 below, which corresponds to Table 2 of the beta report.

Model	Offset (cm.)	Tilt (ppm)	RMS about plane (cm)	Azimuth (deg)
9620	-4.58	0.04	35.07	293
9621	-5.90	0.06	36.14	317
9622	-2.07	0.05	36.14	319
9623	-2.05	0.06	36.56	321
9624	-2.05	0.05	36.52	321
9696	-1.99	0.02	34.63	304

Table 3. Tilted plane fits to 2497 high resolution geoid model residuals .

Table 3 shows an interesting, and somewhat surprising result. Considering the larger magnitude of the tilt of EGM96, compared with the beta models, it is surprising that the final impact on the high resolution geoid undulation grid would be to show a slight improvement. As will be mentioned in a later section, the ability to remove commission error through high-resolution geoid computation does not include the ability to remove any form of very long wavelength (i.e. tilt) errors in the models. As such, we are pleased for the reduction in the magnitude of tilt of a high resolution geoid, due to the use of EGM96.

Care should be taken, however, in interpreting the small magnitude of the tilt in the high resolution models. Scatter plots of the residual geoid undulations show distinct problems in certain areas, which happen to have the property of canceling one another in their contribution to a continental tilt. These scatter plots are presented in the next 3 figures. Figure 5 shows the geoid residuals relative to orthometric height. As we expect, there is significantly less scatter than in Figure 1, and a dependence of geoid undulation residual upon height does not appear to be present. Figure 6 shows the scatter relative to latitude, and Figure 7 shows it relative to longitude. In all three figures, certain characteristic problems are noted. First, a low-elevation, south and east cluster of negative residuals is part of a difficulty in modeling the Florida geoid (a problem that has subsequently been corrected for G96SSS). Also, a mid/high elevation, north and west spread of residuals represents the difficulties in modeling the Pacific northwest (again, some of this spread has tightened up with G96SSS). These features happen to have canceling effects upon the tilt of the high-resolution model (i.e. a least-squares adjustment fits a flat line through Figure 6, even though the general pattern is curved).

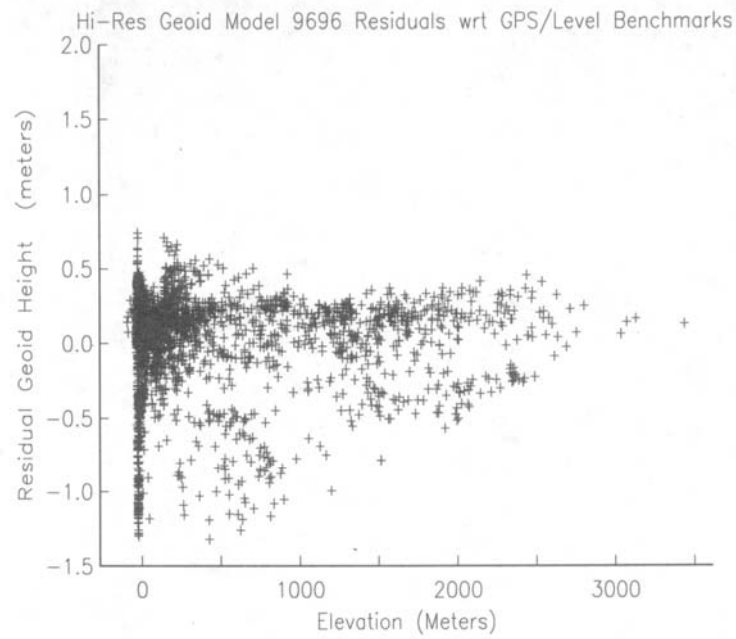


Figure 5: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to elevation

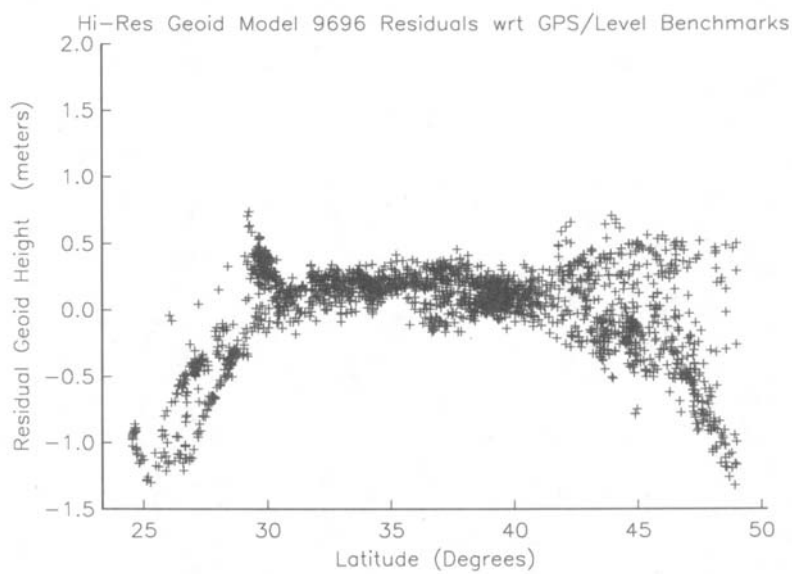


Figure 6: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to latitude

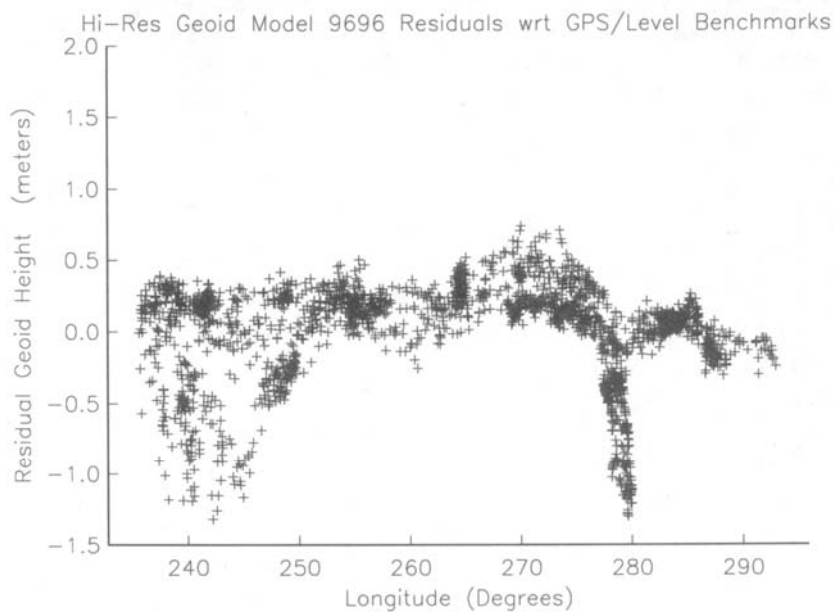


Figure 7: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to longitude

To better get an understanding of any potential elevation dependence that may not be immediately obvious in Figure 5, an “elevation cohort” test, identical with that of the beta model report, was conducted. In this test, geoid undulations residuals were grouped into elevation groups, where benchmarks near Florida (south of 30°) and Washington State (north of 48°) are withheld from the analysis. The results are presented in Table 4 below. Table 4 corresponds with Table 3 of the beta model report.

9624 Model (X05)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	24.73	9.84		
	1000	448	23.83	6.97		
	1500	268	24.80	6.88	-0.09	
	2000	107	24.50	7.43	0.55	0.64
9696 Model (EGM96)	Cutoff (m)	n	RMS (cm)	Offset (cm.)	Diff	2nd-Diff
	0	2190	25.15	8.52		
	1000	448	24.52	3.22		
	1500	268	25.43	3.16	-0.06	
	2000	107	25.17	3.58	0.42	0.36

Table 4. Geoid residual statistics ordered by elevation cutoff tolerance.

Table 4 enables us to discern more about the potential for elevation dependence in the geoid undulation residuals for the high resolution models. Previously, model 9624 (based on

EGM-X05) showed the most uniform average offsets (as seen in the magnitudes of 1st and 2nd differences). However, model 9696 (based on EGM96) shows even greater stability in the average offsets. On the other hand, a larger RMS and greater discrepancies between average offset of all GPS benchmarks and those above 1000 m shows a greater dependence on elevation, and thus could be considered an artifact of a tilt in the model.

Comparison of EGM96 with EGM-X05

Prior to the release of EGM96, our preference amongst the beta models was EGM-X05. This was the model used in the final preparations and tests for G96SSS/GEOID96, in anticipation of the final EGM96 release. As such, it was interesting to compare these two models with one another, and see what was the final effect of using EGM96 rather than EGM-X05 in high resolution geoid calculations. Geoid undulations were evaluated directly on the geoid (rather than using an up/down procedure) for both EGM-X05 and EGM96, with the differences (EGM96 minus EGM-X05) shown in Figure 8, covering the range of 24-53 degrees North in latitude, and 230-294 degrees East in longitude. This image, done in color to aid in clarity, shows large bowl shaped discrepancies, with a peak-to-peak magnitude of 95 cm (magenta values are -49 cm, red values are +46 cm). Reports from the EGM96 team indicate that the most significant change between EGM-X05 and EGM96 occurred in the satellite (i.e. degree 2 to 70) solution. This agrees with the most pronounced undulation differences, which seem to have a characteristic wavelength of about 8-10 degrees (around spherical harmonic degree 40).

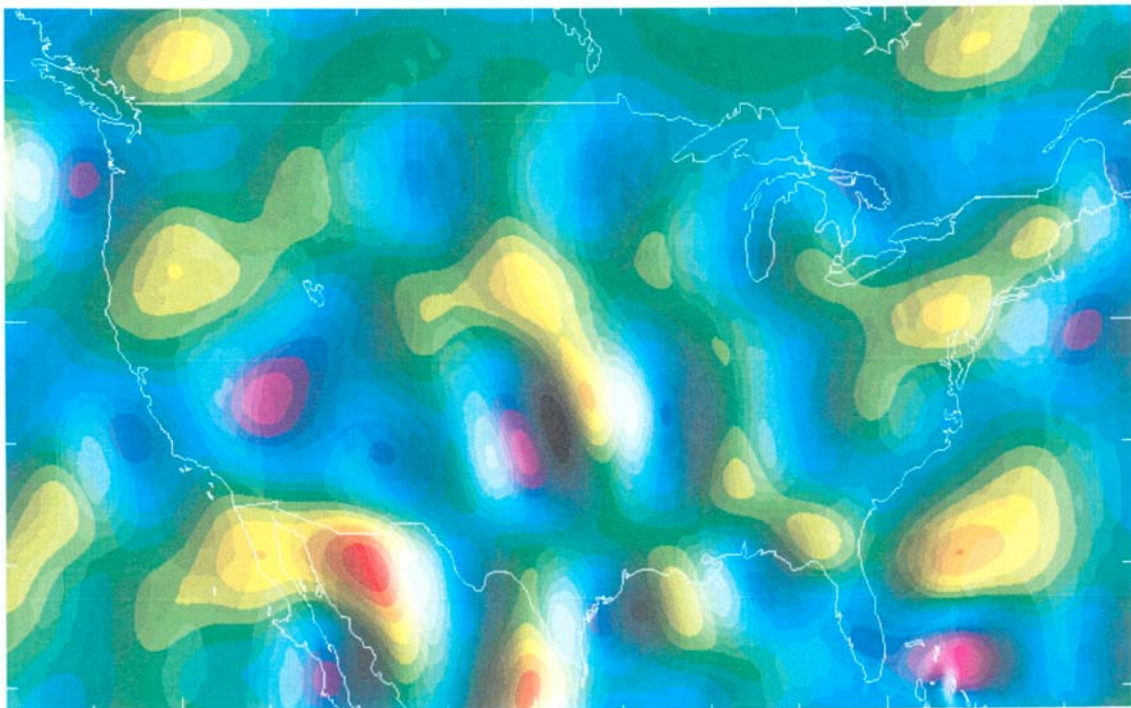


Figure 8: Geoid undulation differences, EGM96 minus EGM-X05

To confirm our visual findings, radially averaged power was computed for the region 25° - 50° N, 258° - 283° E on the differences seen in Figure 8. A plot of the power against the frequency is shown in Figure 9 on a log scale, and in Figure 10 on a log-log scale. The spike at the left of Figure 9 is indicative of additional power at a specific frequency band. To help isolate the frequency band, Figure 10 is plotted on a log-log scale. In this case, any deviation from a linear graph indicates a signal that is specific to one particular scale. In this case, we see the spike of Figure 9 translates into a large bump, deviating from the linear trend, around 0.1 - 0.2 cycles/degree, or a wavelength of about 5-10 degrees. This agrees nicely with the visual estimate obtained from Figure 8. A signal of this wavelength corresponds roughly to spherical harmonic degrees 30-50; which is within the spectral range of the satellite only, and combination solutions of EGM96 (to degree 70).

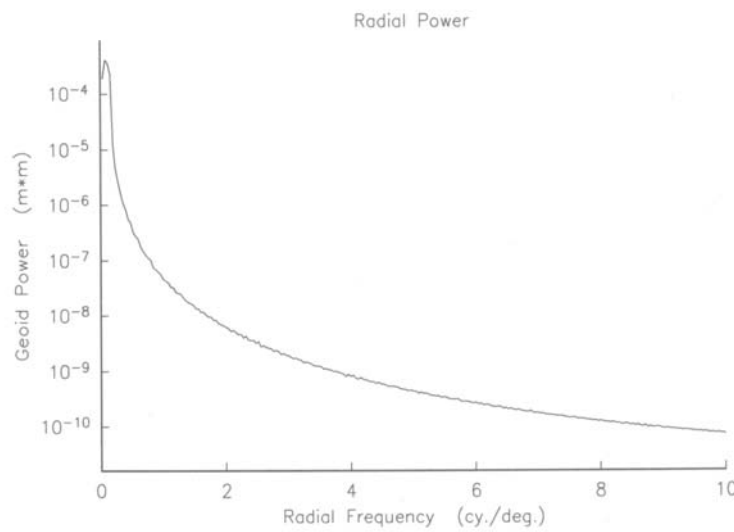


Figure 9: Radial power of EGM96 minus EGM-X05 undulation differences

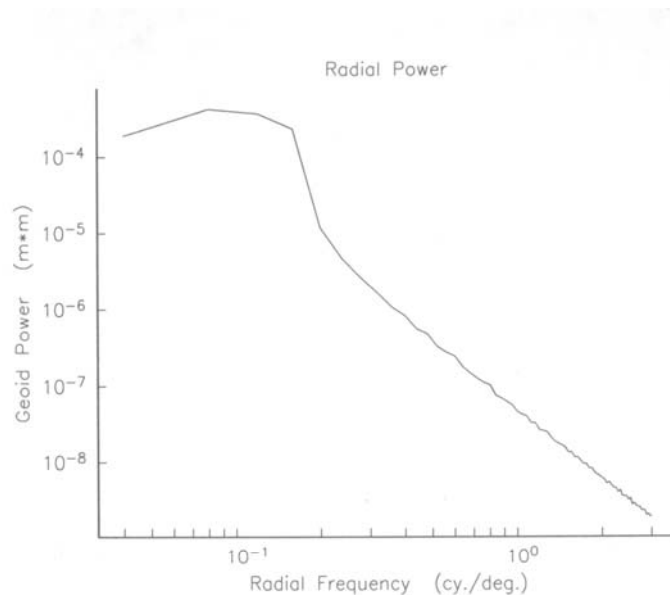


Figure 10: Radial power of EGM96 minus EGM-X05 undulation differences

In an effort to determine whether these large undulating differences were corrections, we investigated one of the largest differences between EGM-X05 and EGM96: the 1 ppm tilt across Oklahoma. As seen in Figure 8, this tilt connects a low (blue) in Texas to a high (red) in Kansas. Figure 11 shows a contour plot of the tilt, and the position of 16 GPS/level benchmarks in the area (indicated by “+”). Two tests against “ground truth” were made. In the first test, surface gravity data were gridded (see Smith and Milbert, 1996) and the anomalies implied by both EGM-X05 and EGM96 were removed. These two 2' x 2' residual gravity anomaly grids were run through the 1-D spherical FFT form of Stokes' integral to generate two 2' x 2' residual geoid undulation grids for the United States (with 8056 points in the study area). The size of the residual undulations is indicative of the fit between the surface gravity data and the two models (a comparison of the residual free-air anomalies could also be made, but we were interested in the fit of undulations at this point). Table 5 shows the results of this comparison.

Model name	Average residual undulation	RMS about the average
EGM-X05	13.1 cm	12.9 cm
EGM96	18.5 cm	16.6 cm

Table 5. Residual geoid undulations in Oklahoma

Table 5 shows a clear indication that for this area, the 1 ppm slope induced by changing from EGM-X05 to EGM96 is introducing a disagreement with the geoid implied by surface gravity. To further investigate this finding, a second test was made. In the second test, the geoid undulations implied by the 16 GPS/level benchmarks in Oklahoma were compared with the undulations computed solely from the n=360 models. The orthometric heights were originally

NAVD 88 values, but were corrected for a 43.4 cm bias in NAVD 88. As in the “low resolution geoid” section of this report, the undulations implied by the two models were corrected from surface height anomalies to geoid undulations. The undulations from the two models were subtracted from those implied by the GPS/level benchmarks, and the results of this comparison are shown in Table 6 below.

Model Name	Average difference	RMS about the average
EGM-X05	-9.1 cm	13.3 cm
EGM96	-17.5 cm	14.9 cm

Table 6. Residual geoid undulations on GPS/level benchmarks in Oklahoma

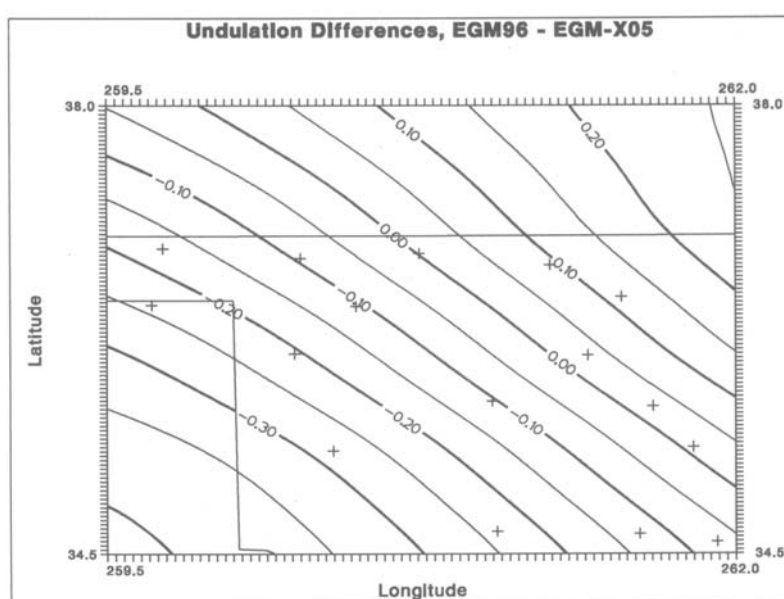


Figure 11: Undulation differences in Oklahoma, EGM96 minus EGM-X05, with GPS Benchmarks

Table 6 confirms the findings of Table 5, that the features introduced in Oklahoma by EGM96 are causing larger disagreements with surface data. It must be noted, however, that due to the semi-periodic nature of the differences between EGM96 and EGM-X05, a number of such slopes are induced across the United States, but time prevented us from studying them further. Oklahoma was chosen as an area with good gravity and GPS on benchmarks coverage, as well as coinciding with one of the larger differences between the models. Further studies should be done to determine the magnitude of the other differences in their respective regions.

As previously mentioned, much of the commission error in the spherical harmonic models can be removed by the use of the remove-compute-restore procedure in the creation of a high resolution model. However, any commission error with a wavelength longer than 29 degrees (i.e. spherical harmonic degrees 0 through 6 or 7) will not be repairable, due to the limited size (29 degrees north/south) of the grid which covers the United States. To exemplify this point, two high resolution geoids were produced using our most recent data sets, and procedures: one geoid based on EGM-X05, and one based on EGM96 (this second geoid is what we've termed G96SSS). The differences between the two geoids has been calculated, and a plot (minus a few degrees along the edges to reduce the visual impact of edge effects) of the differences is shown in Figure 12. Aside from the edge effects, the difference between the geoids is seen to be a very long wavelength structure across the United States. There is no indication of the large bowl-shaped differences seen in Figure 8. Thus, it seems clear that the structure of the geoid is not significantly dependent upon the shorter (29 degrees or smaller) wavelengths of the spherical harmonic model, but is rather controlled by the gravity data themselves. Any long wavelength (29 degrees or greater) errors, however, that may be in the model are not removable using this procedure.

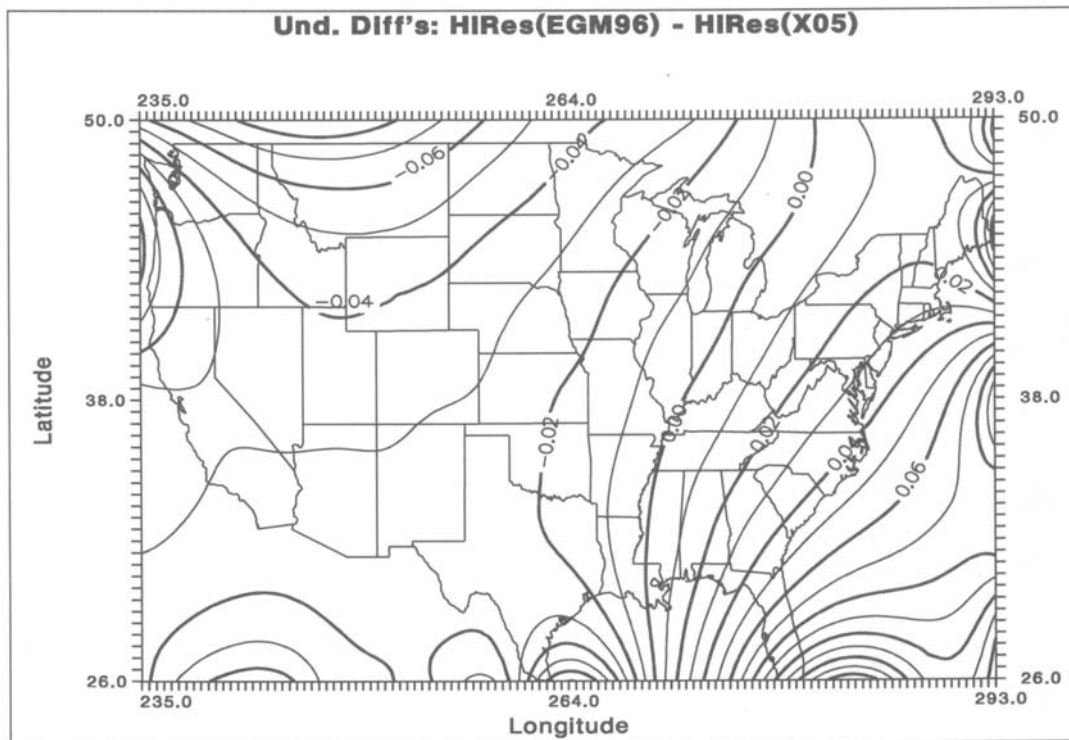


Figure 12: High-resolution undulation differences, CI = 1 cm

Conclusions

Upon first inspecting the differences between EGM96 and EGM-X05, it was surprising to see how large, and especially how semi-periodic, the differences were. These differences were primarily seen as bowl-shaped features approximately 1000 km wide, superimposed over a slight tilt. Overall, however, no significant change in the continental GPS/BM residual statistics occurred with EGM96, aside from the larger tilt. A regional study shows that one of the larger differences between EGM96 and EGM-X05 causes disagreement with surface data, but these regional features are removed in the remove-compute-restore procedures used in high-resolution geoid modeling. The importance of EGM96 became evident when it was used in the creation of model 9696, a high resolution model that was less tilted, and slightly more consistent at high elevations, relative to GPS benchmarks. This property, coupled with the known global improvements of EGM96, make us confident that EGM96 will serve as a firm foundation for the G96SSS and GEOID96 high-resolution geoid height models.

Acknowledgments

We would like to extend our sincere thanks and congratulations to all persons involved in the production and dissemination of EGM96. In addition, the coordinating efforts of Michael Sideris allowed an outstanding availability of global results from all members of the SWG. The efforts of numerous NGS employees in the creation and evaluation of the gravity, NAVD88 and GPS data were essential to this effort. The National Imagery and Mapping Agency (NIMA, formerly DMA) provided a major portion of the NGS land gravity data, and was instrumental in the creation of various 3" and 30" digital elevation data grids in use. Dr. Walter Smith, NOAA, provided the altimeter-derived gravity anomalies used in the high-resolution geoid computations.

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Evaluation of the NASA/NIMA Earth Geopotential Model (EGM96) over Canada

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Abstract

The NASA Goddard Space Flight Centre and the National Imagery and Mapping Agency jointly developed an improved Earth Geopotential Model (EGM) of degree and order 360. The EGM96 model was presented and made available at the International Symposium on Gravity, Geoid, and Marine Geodesy in Tokyo, October 1996. The Geodetic Survey Division evaluated the EGM96 geopotential model over Canada according to guidelines proposed by the International Geoid Service. This report consists of comparisons of the EGM96 model to the JGM3_OSU91A model, the high-resolution GSD95 geoid model for Canada, GPS-Levelling data, astro-geodetic deflections of the vertical, and TOPEX sea surface heights. The comparison indicates that over most regions of Canada there is no significant difference between EGM96 and JGM3_OSU91A. However, over the Western Cordillera, EGM96 is superior, but gives inconsistent results over Ungava Bay and Lake Superior.

1. Introduction

The NASA Goddard Space Flight Centre (GSFC) and the National Imagery and Mapping Agency (NIMA), with the assistance of the Ohio State University (OSU) and the U.S. Naval Surface Warfare Centre, jointly developed an improved spherical harmonic model of degree and order 360 representing the Earth gravitational potential (EGM96). The intent of this collaboration was the production of a global geoid model with an absolute accuracy of 0.5 to 1.0 metre. The EGM96 model will be the reference surface for the World Height System (WHS) and will be used to support terrestrial and extraterrestrial scientific endeavours (ocean circulations, charting, mapping, orbit determination requirements).

Prior to the development of EGM96, GSFC and NIMA developed four preliminary models in order to analyse different weighting schemes. The International Geoid Service (IGeS) agreed to provide leadership in the evaluation of these preliminary models. The president of the IGeS, Prof. Sansò, instructed Dr. Sideris to set up a Special Working Group (SWG) of international scientists who will evaluate the preliminary models under the auspices of the IGeS. The Geodetic Survey Division (GSD) evaluated all four models over Canada; GSD's report of this evaluation is available at: <http://www.geod.nrcan.gc.ca/~marc/>, and its findings are not included in this report because the preliminary models and EGM96 do not show any significant differences over Canada.

Prof. Sansò proposed guidelines to assist in the evaluation of the preliminary GSFC/NIMA models. The proposal encompassed comparisons of the models to GPS-levelling campaigns, regional geoid solutions, astro-geodetic deflections of the vertical, satellite altimetry data, ground gradiometric data sets, absolute gravity measurements and satellite tracking data. GSD applied most of these proposed guidelines to evaluate EGM96. In addition, the evaluation included a comparison of EGM96 to JGM3_OSU91A, which is a composite solution that consists of the harmonic coefficients of the JGM3 model (from degree 2 to 70) augmented by the coefficients of the OSU91A model (from degree 71 to 360).

In this paper, the evaluation of the EGM96 model is divided into 4 parts. First, the JGM3_OSU91A and

comparison of the three models mentioned above to GPS-levelling data at 925 sites across Canada. In section 3, astro-geodetic deflections of the vertical at 932 sites across Canada are compared with JGM3_OSU91A, EGM96 and GSD95. Finally, EGM96 is evaluated with respect to TOPEX observations over the Atlantic Ocean, Pacific Ocean and Hudson Bay. Each part briefly describes the data used in the comparison to EGM96 and highlights the large discrepancies. The Appendix includes Tables and Figures of the comparisons. However, this report will not discuss all the data contained in the Appendix.

2.1 The Canadian Geoid Model (GSD95)

The Geodetic Survey Division of Natural Resources Canada develops high-resolution geoid models for Canada which are distributed to GPS users, geophysicists and oceanographers. The latest model made available to the public was completed in June 1995. This model, GSD95 (Véronneau, 1997), is computed in three components using a remove-restore technique (Rapp and Rummel, 1975). The first component is determined from the OSU91A geopotential model (Rapp et al., 1991). The second component is the contribution of the local gravity measurements reduced by the global field (OSU91A). Finally, the third component is the primary indirect effect on the geoid (Heiskanen and Moritz, 1967). Therefore, the computational approach of GSD95 geoid height can be expressed as:

$$N \approx N_{gm} + \frac{R}{4\pi\gamma} \int \int_{\sigma_0} \Delta g_c^H S(\psi) d\sigma - \frac{\pi G \rho}{\gamma} H^2 \quad (1)$$

where the second term is evaluated using a multi-band 2D-FFT (Forsberg and Sideris, 1993). The integration zone extends from latitudes 41°N to 72°N and from longitudes 218°E to 314°E. The Helmert anomalies (Δg^H) are estimated in three steps: (1) apply gravity reductions in a planar approximation to the observed gravity values; (2) estimate anomalies at a 5'-spacing by a least-squares collocation method; and (3) add a Bouguer plate where the thickness is determined from a Digital Elevation Model (DEM). These three steps can be summarized as:

$$\Delta g_c^H = C(g_p - \gamma_q - D_\gamma - A_t + \delta g_a - 2\pi G \rho H - \Delta g_{osu91A}) + 2\pi G \rho H_{DEM} \quad (2)$$

where the C corresponds to the least-squares collocation operator. The other terms in equation (2) are: surface gravity measurement (g_p), normal gravity (γ_q), free-air reduction (D_γ), terrain correction (A_t), atmospheric correction (δg_a), height of gravity station (H), density (ρ), Newton's gravitational constant (G) and gravity anomaly (Δg). The estimated gravity anomalies are determined from 20 observations (5 measurements in each quadrant).

The geoid heights derived from JGM3_OSU91A and EGM96 are compared to those of the GSD95 geoid model. For the geopotential models, the geoid heights are computed using the following spherical harmonic expansion:

$$N \approx \frac{GM}{r\gamma} \sum_{n=2}^{360} \left(\frac{a}{r}\right)^n \sum_{m=0}^n \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right) P_{nm}(\sin \phi) \quad (3)$$

where γ is the normal gravity, r is the geocentric radial distance of the computation point projected on the ellipsoid, a is the semi-major axis, ϕ is the latitude and λ is the longitude. Finally, C_{nm} and S_{nm} are the har-

tion in the context of this report; however, geoid heights could be computed more accurately using equation (4) in Rapp (1992).

The differences between the GSD95 geoid model and the two geopotential models are summarized in Table 1; and they are depicted in Figures 1.1 and 1.2 in the Appendix. The EGM96 model is in closer agreement than the JGM3_OSU91A model with GSD95 in British Columbia, the Yukon Territory, Baffin Island and a small region next to Lake Superior. The former three areas have rough topography. However, the EGM96 model shows significant disagreement with the GSD95 model over Greenland, Ungava Bay, Foxe Basin and Lake Superior. The discrepancies over Greenland are due to the fact that no surface gravity measurements were available for the computation of GSD95; therefore, the model relies entirely on OSU91A. EGM96 model includes the airborne gravity surveys over Greenland. For Ungava Bay and Foxe Basin, the gravity measurements in GSD95 are obtained from Sandwell and Smith's global gravity grid (1994). Finally, the GSD95 geoid model uses only the surface gravity measurements over Lake Superior. The discrepancies over Ungava Bay and Lake Superior are tested against TOPEX data in Section 2.4.

Table 1: Comparison of the JGM3_OSU91A, the four preliminary GSFC/NIMA models and EGM96 to the Canadian high-resolution GSD95 geoid model.

Model	No of Points	Min. (m)	Max. (m)	Mean (m)	St. Dev. (m)
GSD95 - JGM3_O	428544	-10.07	8.60	0.11	0.84
GSD95 - EGM_X02	428544	-5.08	5.85	0.10	0.66
GSD95 - EGM_X03	428544	-5.83	6.06	0.10	0.65
GSD95 - EGM_X04	428544	-5.81	6.05	0.10	0.65
GSD95 - EGM_X05	428544	-5.91	6.15	0.10	0.65
GSD95 - EGM96	428544	-5.59	6.00	0.10	0.65

Most of the remaining differences between EGM96 and GSD95 result from to the higher resolution of the GSD95 geoid model. The additional gravity information in GSD95 permits improved definition of the Western Cordillera, mid-continent gravity high and Appalachian Mountains.

The comparison of the geoid models is a useful tool to identify significant discrepancies, but it does not correspond to the best evaluation because of the high correlation. In the following sections, the three models are tested against GPS-levelling data, astro-geodetic deflections and TOPEX data.

2.2 GPS on Levelling Lines

Since 1983, the GSD has conducted GPS campaigns across Canada for the purpose of improving the infrastructure for the national network, monitoring crustal deformation, calibrating instrumentation and validating geoid models. The GPS-levelling campaigns are carried out by collecting GPS measurements at selected bench marks, which are 30 km apart, for a period of approximately 4 hours. The early campaigns used single frequency receivers. However, throughout the series of GPS-levelling campaigns, there was a significant improvement in the technology of the GPS instruments and software.

The geographical coordinates (ϕ , λ , h) are determined from a national adjustment of all GPS observations and constrained to stations of the Canadian Active Control System (CACS) (Kouba and Popelar, 1994) and Canadian Base Network (CBN). The CACS and CBN are high accuracy networks. The precision of the

the epoch of observations. The geographical coordinates are expressed in ITRF92 reference system on an ellipsoid with $a = 6378136.3$ metres and an $1/f = 298.256415099$. The distribution of the 2204 GPS sites across Canada is shown on Figure 2.

The Canadian Geodetic Datum of 1928 (CGD28) (Canon, 1929), which is the official height system in Canada, is not compatible with the geoid because it is determined from normal gravity, but it is not a normal height system either. The CGD28 heights are expressed as "normal orthometric heights." Therefore, the orthometric heights used in this report are determined from a preliminary adjustment of the Canadian first-order levelling network known as Oct95 (Mainville and Bérubé, 1996). Its datum is realized by holding fixed a single point in Rimouski, Québec. The Oct95 adjustment indicates that the water level in the vicinity of Vancouver (Pacific Ocean) is higher than the water level in Halifax (Atlantic Ocean) by 88 cm. There are 925 GPS stations referenced to the Oct95 adjustment.

An indication of the precision of JGM3_OSU91A, EGM96 and GSD95 can be evaluated independently against geoid heights determined from GPS-levelling data. The discrepancies (ϵ) can be expressed as:

$$\epsilon = (h_{GPS} - H_{Oct95}) - N, \quad (4)$$

where h_{GPS} is the ellipsoidal height, H_{Oct95} is the orthometric height and N is the geoid height. The result of the comparisons at ten Canadian regions is summarized in Table 1.1 in the Appendix. The standard deviation of the discrepancies ϵ is given before and after fitting out the systematic biases and tilts by using a four-parameter transformation (Sideris, 1993), which absorbs long wavelength errors in GPS measurements, levelling data and geoid heights. The discrepancies for JGM3_OSU91A, EGM96 and GSD95 for nine of these sectors are depicted on Figures A.1 to A.9, B.1 to B.9 and C.1 to C.9, respectively. The first region encompasses all GPS sites with Oct95 orthometric heights. The other regions are sub-sets of the Canadian sites by province or territory with the exception of the Traverse which consists of GPS sites along the Trans-Canada Highway extending from Vancouver, British Columbia, to Halifax, Nova Scotia (≈ 6500 km).

The EGM96 geopotential model is superior to the JGM3_OSU91A model by a factor of two and three in British Columbia and the Yukon Territory, respectively. The standard deviation of the discrepancies ϵ have improved from 153 cm to 44 cm in the Yukon Territory and from 91 cm to 48 cm in British Columbia. However, the precision of GSD95 in the Western Cordillera is 17 cm. In Alberta, EGM96 indicates better results than JGM3_OSU91A for the Rocky Mountains area.

In Québec, GSD95 has a precision similar to the global models because of the poor DEM in central and northern Québec. The outliers, seen on Figure C.7, are located on the levelling line extending from James Bay to Labrador and are associated to a sector of poor DEM. An improved DEM is presently in production for Canada, based on the specifications for Digital Terrain Elevation Data (DTED). Thus, all maps, at a scale of 1:250000, are being scanned to produce a DEM at a 3 seconds of arc spacing. In addition, this new DEM will ameliorate the global geopotential model over Québec because EGM96 is also affected by the present DEM as shown in Figure B.7.

The discrepancies of EGM96 along the GPS Traverse on Figure B.9 indicates an east-west systematic error, which is probably due to the Oct95 adjustment. A recent levelling, north of Lake Superior, which is not yet incorporated into the national network, shows a height difference of 20 cm with the previous levelling. This levelling line is really important because it is one of the two lines connecting the western and eastern parts of the network. The other line was last levelled in the 1920s and its quality is doubtful. Thus, the new

wavelength agreement between GPS-levelling data and the EGM96 global model. A new and improved adjustment is presently in progress in order to replace the Oct95 adjustment.

A more significant systematic error can be seen when comparing the GSD95 geoid model against the GPS Traverse (Figure C.9). The error is compounded by problems in the long wavelength of GSD95. GSD is presently investigating the possibility of a bias in the shipboard gravity data over the Atlantic Ocean. A bias of -1 to -2 mGals is measured when the shipboard data are compared to Sandwell and Smith's global gravity grid (1996). The completion of the new DEM for Canada will allow the computation of terrain corrections throughout Ontario, Québec and the Maritimes. The absence of terrain corrections produces a systematic error in the GSD95 geoid model. These two corrections would increase the GSD95 geoid heights in an absolute sense in eastern Canada.

2.3 Deflections of the Vertical

The EGM96 model is also evaluated by using astronomical observations (Φ, Λ) taken from 1910 to 1974 at 932 stations across Canada. The distribution of the observations is shown on Figure 3 in the Appendix. A report by Vámosi (1977) gives some details about the reduction of the Canadian astronomical observations in connection with the readjustment of the North American Datum of 1983 (NAD83). He mentions that all data pertaining to geodetic astronomy have been reassessed and corrections have been derived and applied to reduce all existing measurements to a common reference system, where feasible or else re-observed. All star coordinates refer to FK4 catalogue and all astronomical data refer to the CIO pole. The astronomical measurements were given errors ranging from 0.3 to 0.5 second of arc, when used in the NAD83 readjustment (Strange, 1989). The geodetic latitudes (ϕ) and longitudes (λ) of the test points are in the ITRF93 reference system.

The two components of the deflections of the vertical can be determined from astronomical and geodetic coordinates. The north-south and east-west components can be expressed as (Heiskanen and Moritz, 1967):

$$\xi = \Phi - \phi \quad \text{and} \quad (5)$$

$$\eta = (\Lambda - \lambda) \cos(\phi) , \quad (6)$$

respectively.

The results of the comparison between GSFC/DMA, JGM3_OSU91A and GSD95 models and astro-geodetic deflections of the vertical are summarized in Tables 2.1 and 2.2 in the Appendix. Table 2.1 contains the results of the comparison to the north-south component while Table 2.2 contains results for the east-west component. The results are divided by province and territory and summarized for the entire country.

The results of comparisons using both components of the deflection of the vertical are as follows: (1) The EGM96 and JGM3_OSU91A models give similar results across Canada with the exception of British Columbia and the Yukon Territory, where the standard deviation of EGM96 is smaller by approximately one second of arc, and (2) GSD95 is in better agreement, than the global models, with the astro-geodetic deflections, especially in mountainous regions.

The difference of GSD95 with the north-south component of the deflections is 1.9 seconds of arc across the country. However, the discrepancy is as little as 0.8 second of arc for the 72 points in Saskatchewan and as large as 3.1 seconds of arc at the 200 points in British Columbia. On the other hand, the difference of

British Columbia are 1.2 and 5.4 seconds of arc, respectively. A disagreement of one second of arc corresponds to an error of approximately 5 mm/km on the slope of the geoid model.

The discrepancy of the gravimetric models to the east-west component is in the same range as the north-south component. However, a mean bias of 0.2 (EGM96) to 0.3 (GSD95) second of arc is measured at the 932 points across Canada. A bias of 0.65 second of arc was observed by Kouba (1979) in the astronomical longitude using space geodetic techniques. In addition, a nominal correction of 0.5 second of arc was applied to the astronomic longitude for the NAD83 adjustment (Strange, 1989).

2.4 TOPEX Data

The success of radar-altimeter missions has established satellite altimetry as an important technique to measure the sea surface height (SSH), which is the height of the ocean surface above the reference ellipsoid. Therefore, it offers a technique to assess and evaluate gravimetric geoid models over the oceans. The difference between the geoid height (N) and the SSH is the sea surface topography (SST), which can be expressed as:

$$SST = SSH - N \quad (7)$$

In this section, GSD95, JGM3_OSU91A and EGM96 models are compared to sea surface height obtained from TOPEX observations during the month of August, 1995 (Cycle 108). The selection of this cycle was made to assure the absence of ice over the oceans. The TOPEX altimeter ranges are corrected for five systematic effects: wet troposphere, dry troposphere, ionosphere, electromagnetic bias and inverse barometer. All these corrections are obtained from the PO.DAAC CD. In addition, the SSH's are corrected for the total tide effect. The solid Earth tide and the pole tide are also obtained from the PO.DAAC CD. However, the elastic ocean tide and radial load tide are determined using the model version 3.0 from the Center for Space Research at the University of Texas (Bettadpur and Eanes, 1994). The SSH's are referenced to an ellipsoid with a semimajor axis of 6378136.3 metres.

The results of the comparison between the geoid models (JGM3_OSU91A, EGM96 and GSD95) and the sea surface height of the Atlantic Ocean, Pacific Ocean and Hudson Bay are shown in Table 3.1 in the Appendix. Table 3.1 also includes statistics for sub-regions of the Atlantic Ocean: Ungava Bay, Davis Strait, Labrador Sea, sectors east and south of Newfoundland, and south of Nova Scotia. The Pacific Ocean has one sub-region and it is west of Vancouver Island and the state of Washington. The geoid models are also evaluated above the Great Lakes (Ontario, Erie, Huron, Michigan and Superior) and the three following lakes: Winnipeg, Great Slave and Great Bear. Finally, the SST determined from EGM96 and GSD95 for the Atlantic, Hudson Bay and Pacific is depicted on Figures D, E and F, respectively. These figures are produced after applying a low-pass filter to the SST in order to smooth the contour lines. The tracks of TOPEX data (cycle 108) are shown on Figure 4.

The depiction of the SST over the Atlantic Ocean on Figures D.2 (GSD95) shows well the cold Labrador current and the effect of the Hamilton Bank and Grand Banks of Newfoundland on the ocean topography. However, GSD95 has a bias in comparison to EGM96. The SST from GSD95 is higher than EGM96 by 75, 104, 55, 45 cm for the areas of Labrador Sea, east Newfoundland, south Newfoundland and south Nova Scotia, respectively. This bias could be corrected if GSD could demonstrate without any doubt the existence of a bias in the shipboard gravity data.

In the Ungava Bay, the SST derived from GSD95 geoid model has a smooth transition and increases to a

metres. The standard deviation of the SST at 156 measurements in the Ungava Bay is 1.13 metres for JGM3_OSU91A, 0.64 metre for EGM96 and 0.31 metre for GSD95. These large discrepancies in SST are coming directly from the differences between the geoid models (see section 2.1).

The SST in the Gulf of Maine for EGM96 and GSD95 is presently under investigation. However, in the case of GSD95, part of the problem is due to the absence of shipboard gravity data in the gulf.

In respect of Hudson Bay, the SST's on Figure E.1 and E.2 are quite similar. The mean SST for the Bay is 0.03 ± 0.22 metre and -0.05 ± 0.36 metre when derived from EGM96 and GSD95, respectively. These results are preliminary and consists of a first effort by GSD to determine its SST. For GSD95, the geoid model over Hudson Bay is entirely determined from shipboard gravity measurements.

Over the Pacific Ocean, the SST derived from GSD95 is smoother than the SST obtained using EGM96. For GSD95, the poor determination of the SST next to Alaska is caused by the fact that no surface gravity measurements are available. The gravity data are estimated from sparse gravity measurements over land and distant shipboard gravity measurements. The mean SST west of Washington State is 0.45 ± 0.16 metre for EGM96 and -0.26 ± 0.07 metre for GSD95. In the GSD95 computation, the American shipboard gravity data west of the states of Washington and Oregon are shifted by a bias of -3 mGals. The bias is measured by using Sandwell and Smith's global gravity grid (1994) and the overlapping Canadian shipboard gravity data. If the bias had not been applied to the American shipboard data, the mean SST obtained from GSD95 would have been approximately -1.0 metre.

The relation between the SST of the Pacific and the Atlantic oceans is useful for the analysis of systematic error in the geoid model and levelling data. On average, the Pacific Ocean near Vancouver is higher than the Atlantic Ocean near Halifax by approximately 25 cm according to EGM96 geoid model. However, GSD95 indicates that the Atlantic Ocean is higher than the Pacific by approximately 90 cm. This contradiction in water level might be explained by the possible bias in gravity measurements over the Atlantic Ocean and the omission of terrain corrections in eastern Canada.

Finally, it was previously mentioned that EGM96 and GSD95 differ significantly over Lake Superior. The discrepancies of GSD95 with 65 TOPEX measurements have a standard deviation of 4 cm; whereas, the standard deviation measures 30 cm when EGM96 is compared to the 65 TOPEX data. For the JGM3_OSU91A model, the standard deviation is 22 cm.

3. Conclusion

The EGM96, developed from a joint project between the NASA Goddard Space Flight Centre and the National Imagery and Mapping Agency, is a spherical harmonic model of degree and order 360 representing the Earth's geopotential. This model has shown an improvement over the JGM3_OSU91A model which was considered a state-of-the-art before the joint project. The EGM96 improvements are beneficial for the Western Cordillera of Alberta, British Columbia and the Yukon Territory. This is demonstrated by comparing it to the high-resolution GSD95 geoid model for Canada, GPS data on levelling lines and astro-geodetic deflections of the vertical. In addition, EGM96 is in better agreement than JGM3_OSU91A over Baffin Island and a small region west of Lake Superior.

However, the EGM96 geopotential model requires a more detailed analysis over Foxe Basin, Ungava Bay and Lake Superior. For these three regions, the comparisons to the GSD95 geoid model shows large discrepancies. In addition, the EGM96 model does not agree well with the TOPEX measurements from

cycle 108 over Ungava Bay and Lake Superior. On the other hand, GSD95 shows a good agreement with the same data.

An east-west tilt (approximately 1 metre over 6000 km) is present in GSD95. This slope can be due to the omission of terrain corrections in eastern Canada and a possible bias in the shipboard gravity data. The determination of the sea surface topography using GDS95 and TOPEX data indicates that the Pacific Ocean (Vancouver) is lower than the Atlantic Ocean (Halifax) by 90 cm. This same surface determined by EGM96 shows the Pacific higher than the Atlantic by 25 cm. This result is more compatible with the geodetic levelling from Halifax to Vancouver.

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Figure 1.1: Difference, in metres, between JGM3_OSU91A and GSD95 geoid models. The contourline interval is 50 cm.

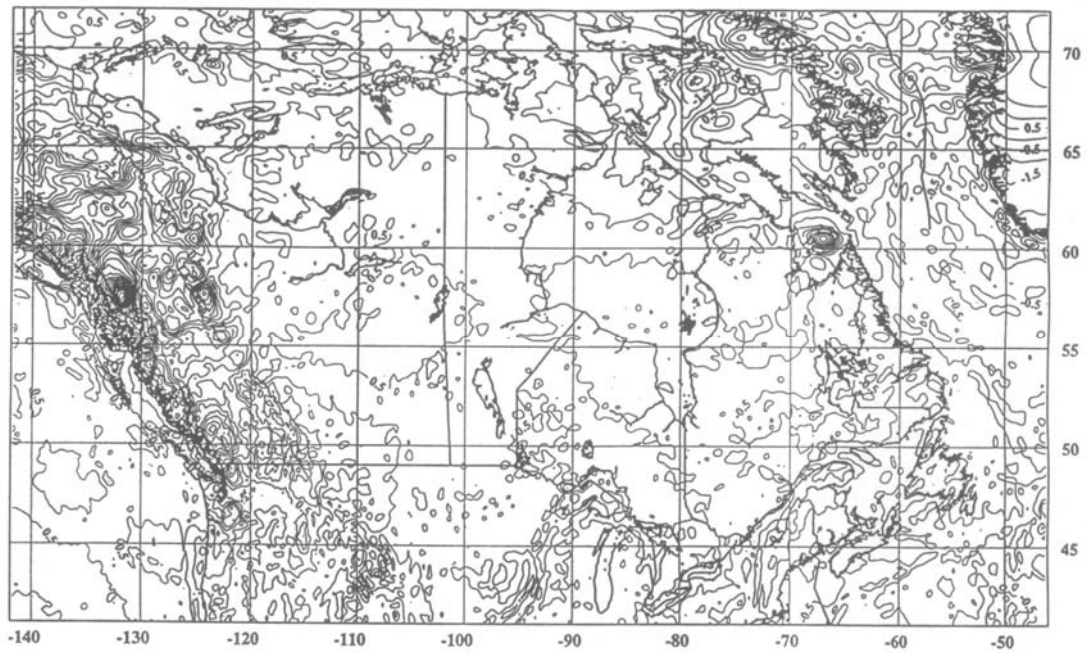


Figure 1.2: Difference, in metres, between EGM96 and GSD95 geoid models. The contourline interval is 50 cm.

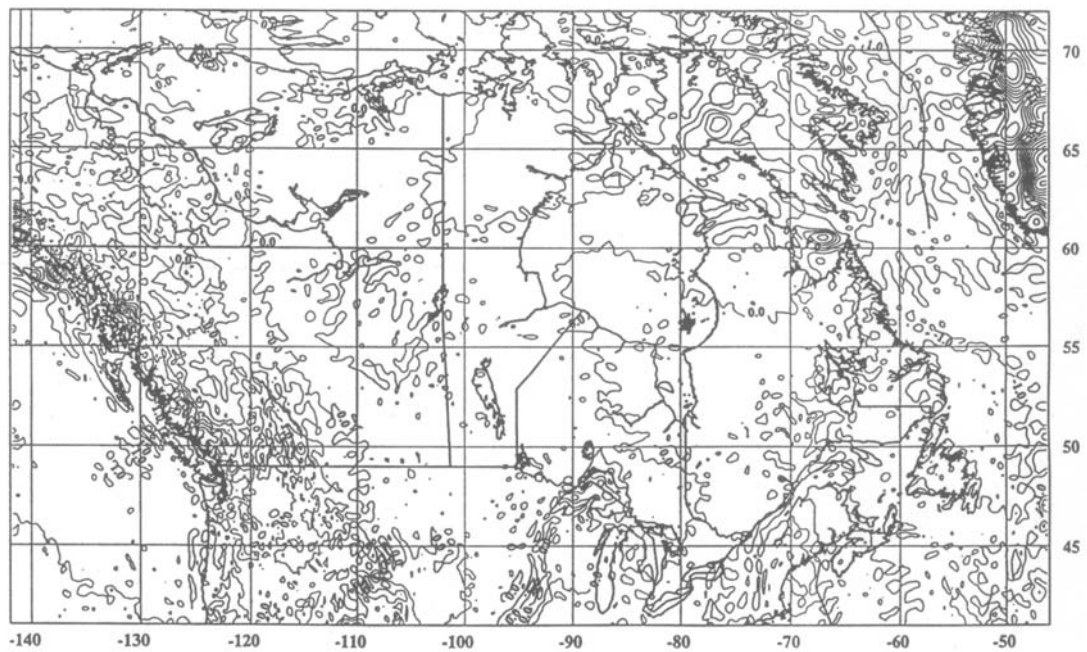


Figure 2: Distribution of the Canadian GPS sites within the area of GSD95 geoid model (not all these sites have orthometric heights).

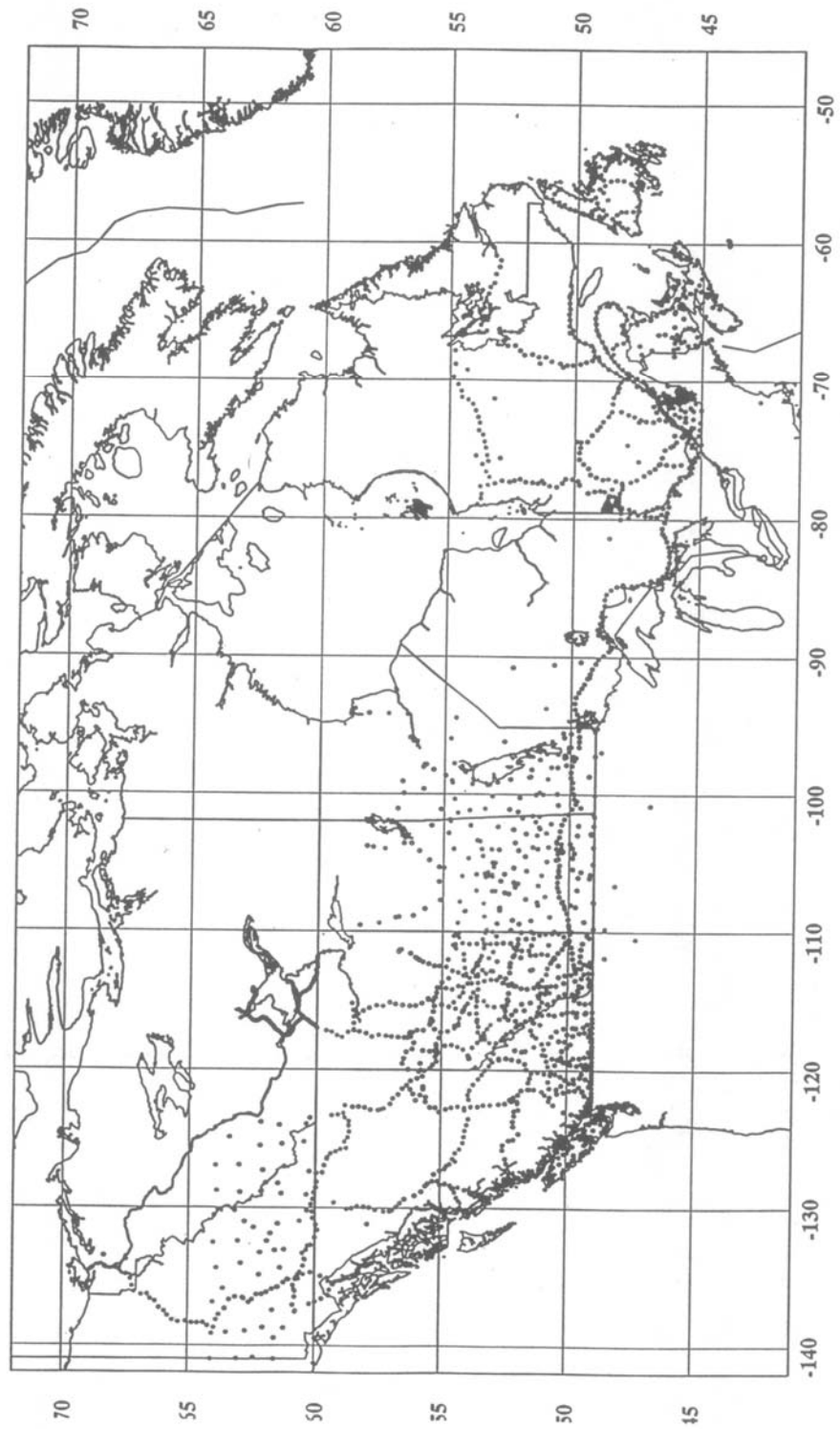
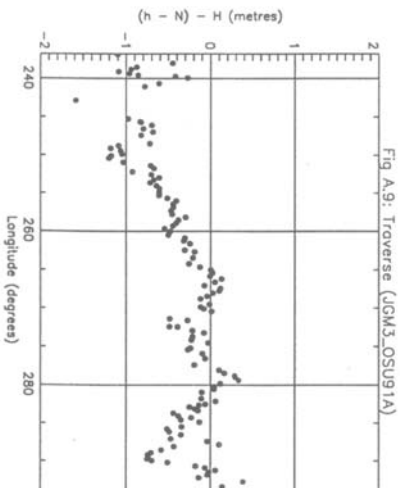
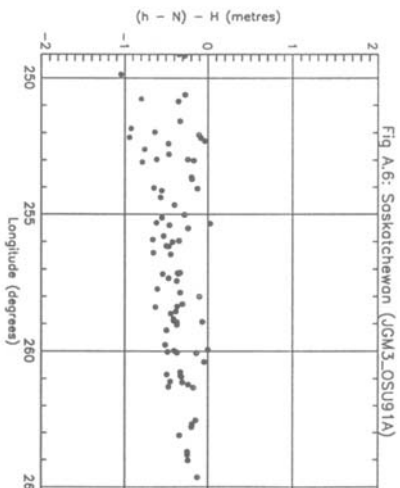
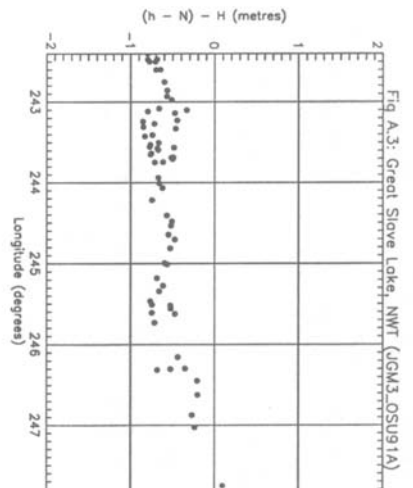
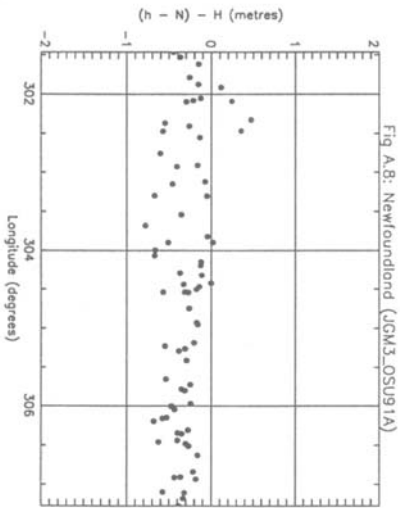
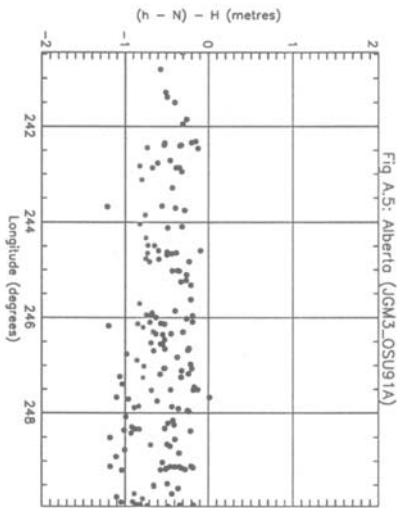
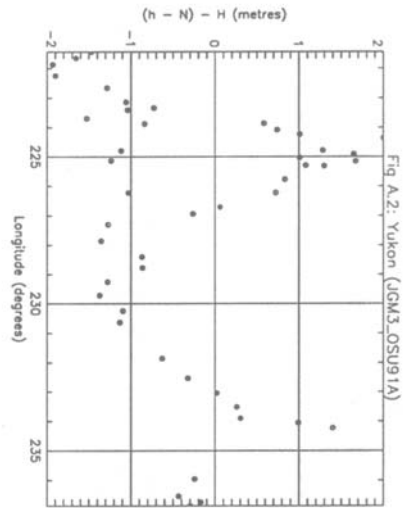
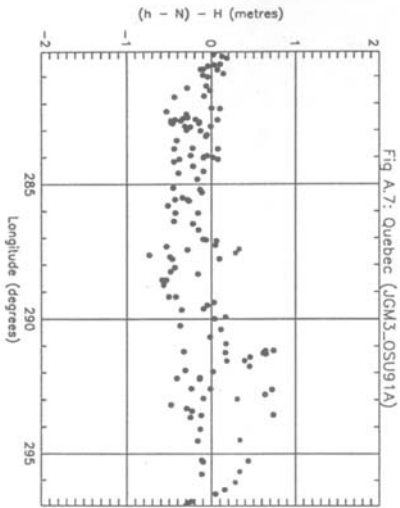
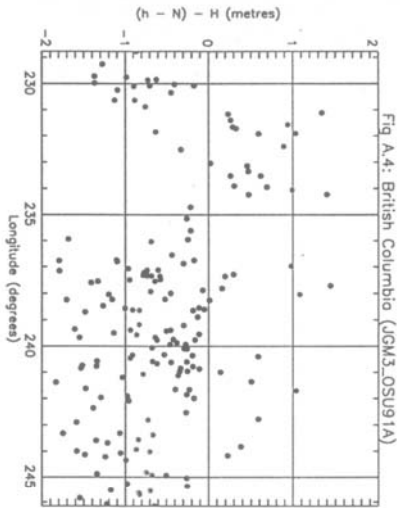
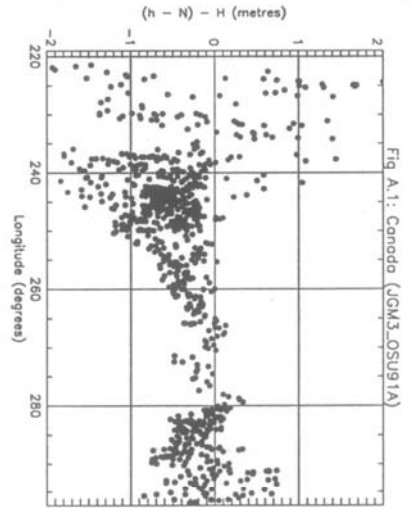


Table 1.1: Comparison of the JGM3_OSU91A, EGM96 and GSD95 geoid model to GPS-leveling data across Canada. The geometric and orthometric heights are referenced to ITRF92 and Oct95, respectively.

Region	No of Points	Model	Min. (m)	Max. (m)	Mean (m)	$\sigma_{(h-H-N)}^a$ (cm)
Canada	925	JGM3_O	-4.521	3.035	-0.403	65 (64)
		EGM96	-1.861	0.815	-0.442	37 (31)
		GSD95	-1.187	0.706	-0.466	42 (17)
Yukon	59	JGM3_O	-3.106	3.035	-0.337	153 (143)
		EGM96	-1.740	0.119	-0.869	44 (47)
		GSD95	-1.187	-0.515	-0.845	16 (12)
Great Slave Lake	68	JGM3_O	-0.853	0.097	-0.576	20 (12)
		EGM96	-0.752	0.175	-0.494	18 (11)
		GSD95	-1.004	-0.288	-0.727	12 (10)
British Columbia	182	JGM3_O	-4.521	2.686	-0.596	91 (88)
		EGM96	-1.861	0.569	-0.638	48 (46)
		GSD95	-1.142	-0.346	-0.683	18 (12)
Alberta	160	JGM3_O	-1.216	0.012	-0.548	27 (19)
		EGM96	-0.962	-0.159	-0.595	17 (12)
		GSD95	-0.919	-0.438	-0.692	9 (6)
Saskatchewan	88	JGM3_O	-1.048	0.024	-0.382	21 (12)
		EGM96	-0.867	-0.102	-0.366	13 (12)
		GSD95	-0.944	-0.355	-0.668	10 (6)
Québec	141	JGM3_O	-0.736	0.739	-0.107	30 (26)
		EGM96	-1.061	0.458	-0.159	29 (22)
		GSD95	-0.680	0.706	0.173	29 (16)
Newfoundland	76	JGM3_O	-0.780	0.473	-0.304	23 (19)
		EGM96	-0.578	0.466	-0.127	21 (18)
		GSD95	0.558	0.844	0.681	7 (6)
Traverse	148	JGM3_O	-1.587	0.678	-0.370	39 (27)
		EGM96	-0.962	0.815	-0.280	31 (28)
		GSD95	-0.813	0.487	-0.258	42 (14)
Maritimes	27	JGM3_O	-0.843	0.678	-0.229	41 (24)
		EGM96	-0.570	0.815	-0.099	30 (23)
		GSD95	0.246	0.487	0.364	7 (6)

a. The results in parentheses indicate the standard deviations after fitting out systematic biases and tilts by using a four-parameter transformation



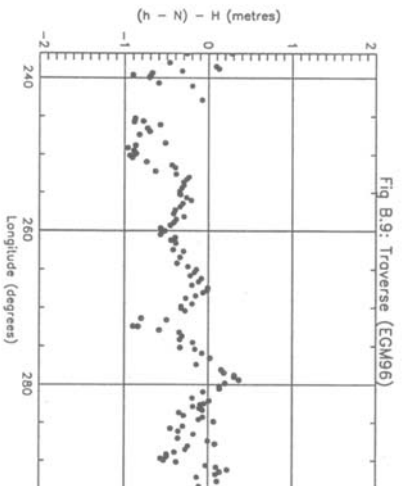
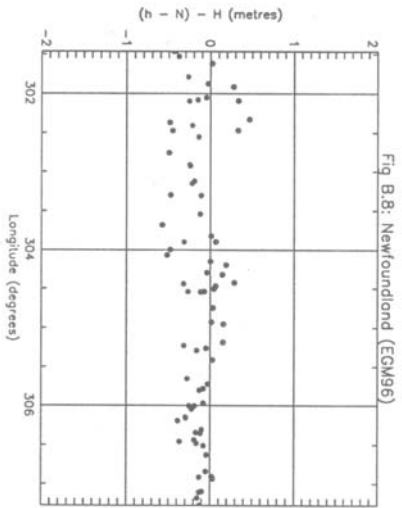
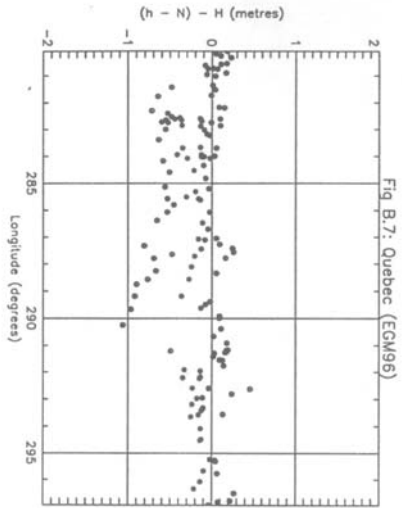
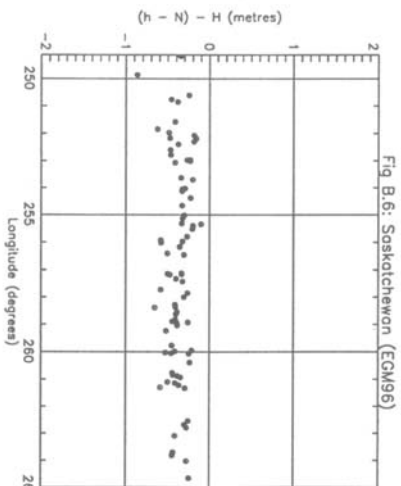
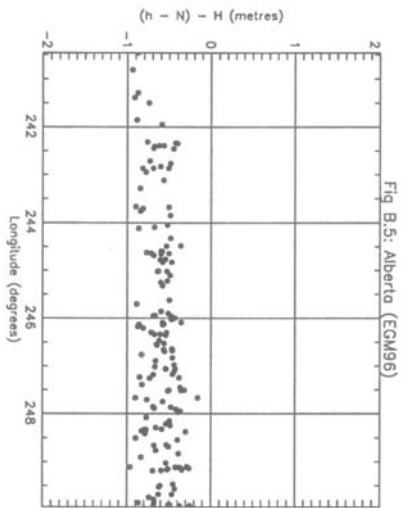
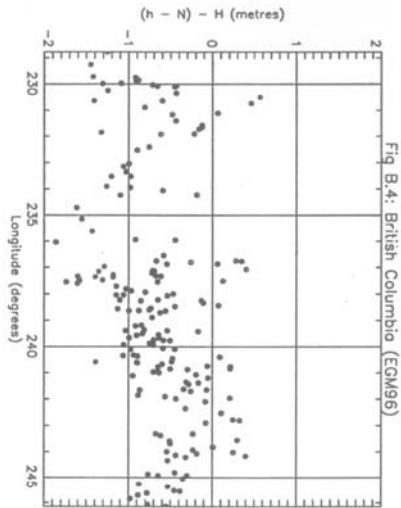
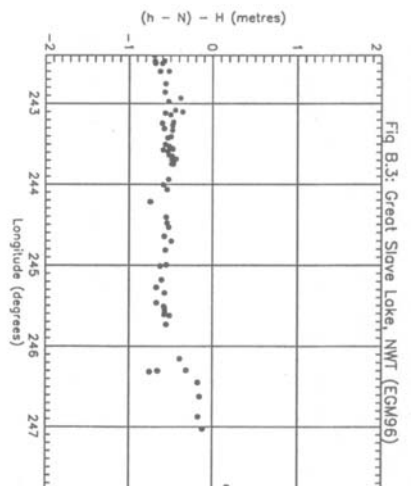
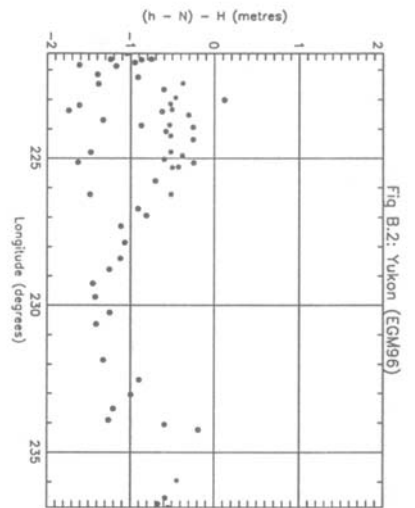
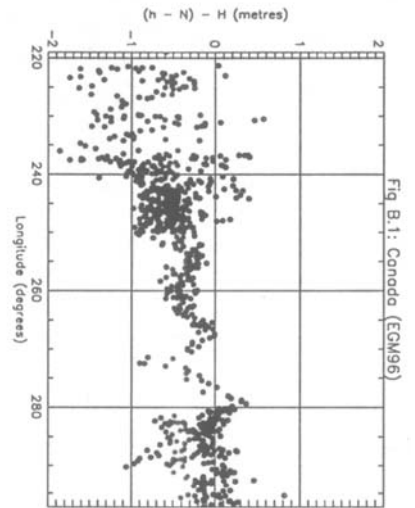


Fig C.1: Canada (GSD95)

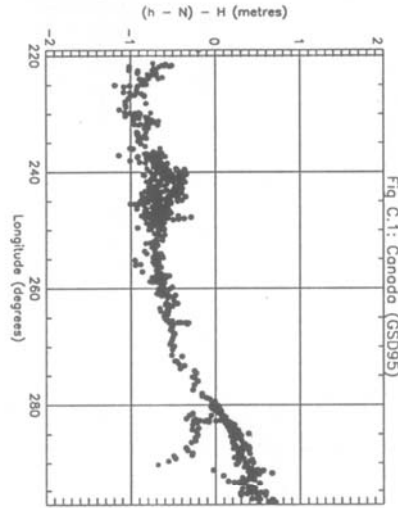


Fig C.2: Yukon (GSD95)

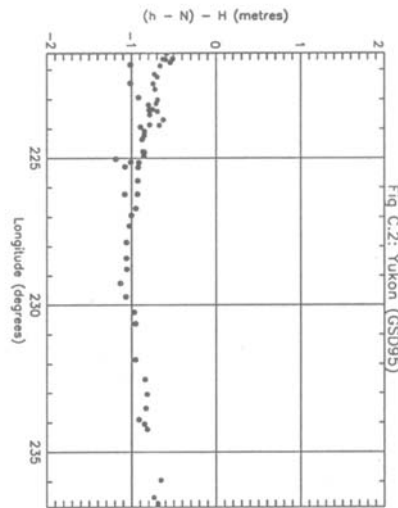


Fig C.3: Great Slave Lake, NWT (GSD95)

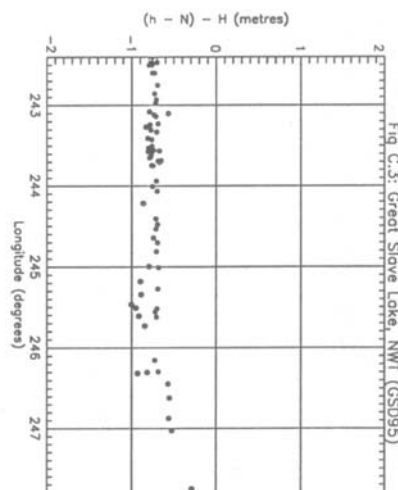


Fig C.4: British Columbia (GSD95)

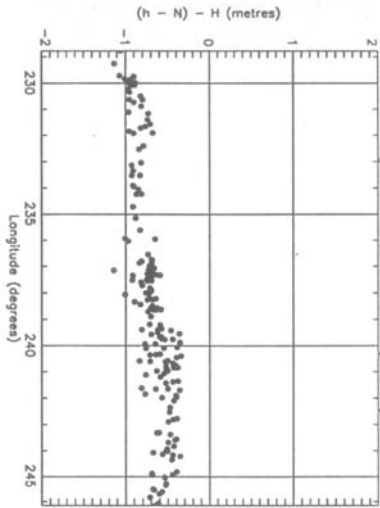


Fig C.5: Alberta (GSD95)

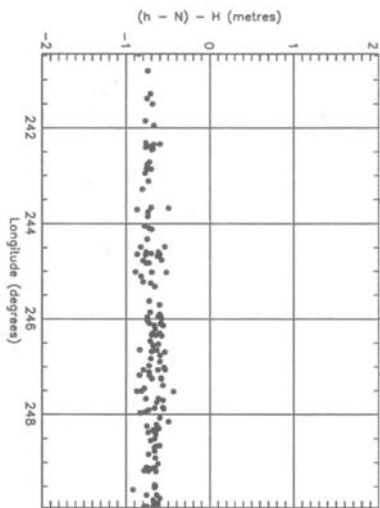


Fig C.6: Saskatchewan (GSD95)

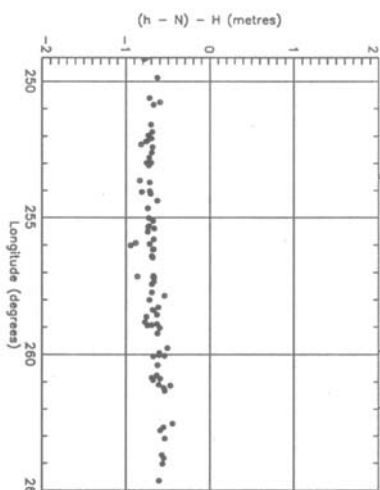


Fig C.7: Quebec (GSD95)

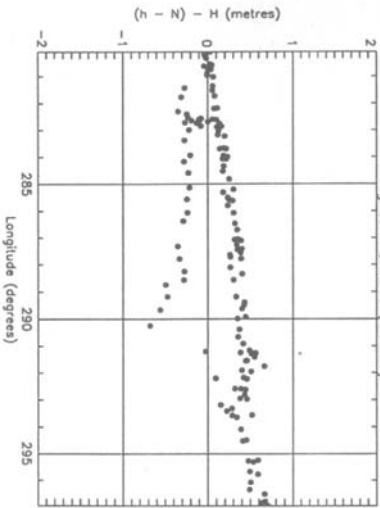


Fig C.8: Newfoundland (GSD95)

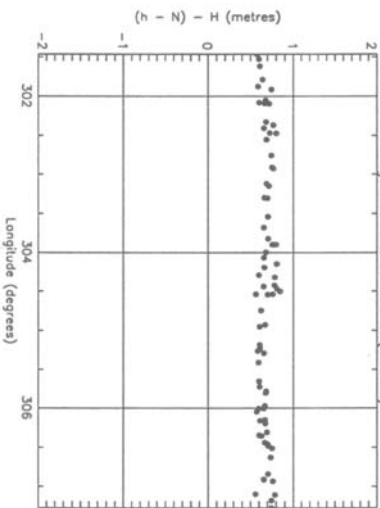


Fig C.9: Traverse (GSD95)

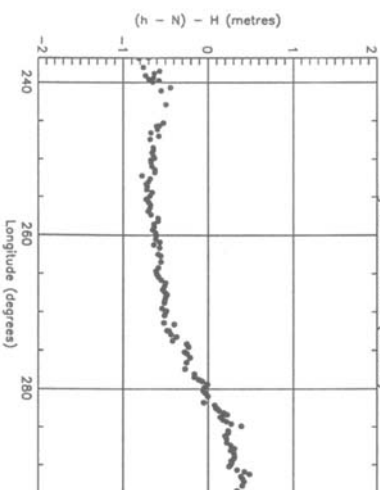


Figure 3: Distribution of the astro-geodetic deflections of the vertical within the area of GSD95 geoid model.

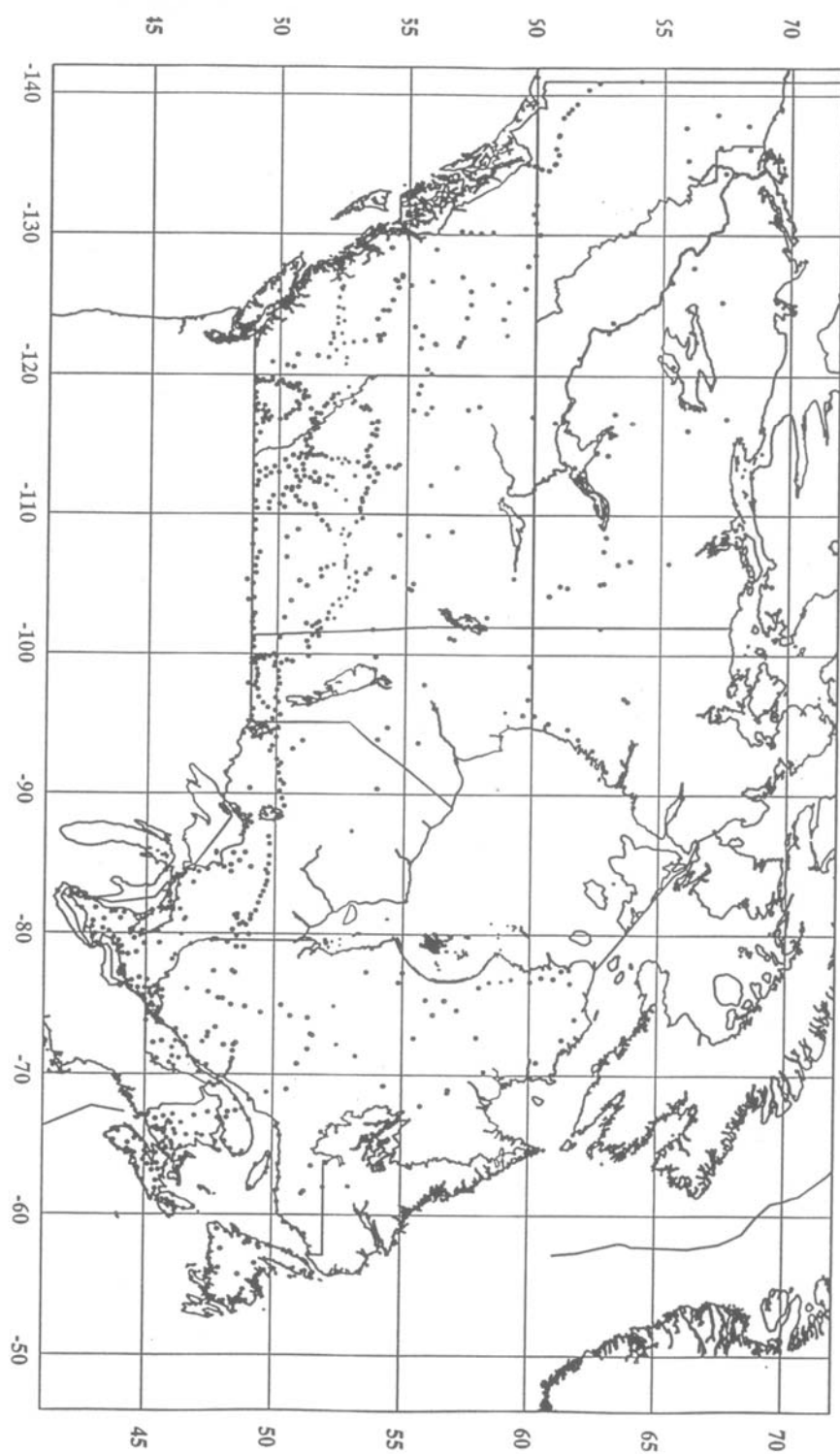


Table 2.1: Comparison of the JGM3_OSU91A, EGM96 and GSD95 geoid model to the north-south component of astro-geodetic deflections of the vertical across Canada. The geodetic coordinates deflections are in the ITRF93 system.

Region	No of Points	Model	Min. (arcsec)	Max. (arcsec)	Mean (arcsec)	St. Dev. (arcsec)
Newfoundland	52	JGM3_O	-9.35	7.44	-0.56	2.79
		EGM96	-9.42	7.30	-0.37	2.79
		GSD95	-6.05	5.64	-0.29	1.75
Maritime	74	JGM3_O	-4.35	8.43	0.16	2.09
		EGM96	-4.51	7.56	0.11	2.13
		GSD95	-2.41	3.83	0.27	1.10
Québec	127	JGM3_O	-6.87	10.78	-0.13	2.56
		EGM96	-6.10	8.93	-0.21	2.31
		GSD95	-4.03	3.33	-0.05	1.22
Ontario	147	JGM3_O	-4.81	5.83	0.16	1.60
		EGM96	-5.36	5.25	0.03	1.63
		GSD95	-2.77	2.60	0.01	0.90
Manitoba	66	JGM3_O	-4.40	4.24	0.13	1.51
		EGM96	-4.55	4.29	0.16	1.54
		GSD95	-3.22	2.71	0.27	1.06
Saskatchewan	72	JGM3_O	-2.18	5.12	0.30	1.20
		EGM96	-2.39	4.91	0.25	1.21
		GSD95	-1.45	2.82	0.08	0.78
Alberta	115	JGM3_O	10.44	7.33	0.40	2.64
		EGM96	-9.28	7.95	0.26	2.51
		GSD95	-7.00	10.29	0.16	1.78
British Columbia	200	JGM3_O	-21.56	12.62	-1.19	5.69
		EGM96	-21.83	14.79	-0.13	5.33
		GSD95	-11.79	9.42	-0.19	3.06
Yukon T.	23	JGM3_O	-9.73	11.75	-1.06	5.51
		EGM96	-8.03	8.13	-0.43	4.16
		GSD95	-3.27	9.86	1.42	2.91
Northwest T.	56	JGM3_O	-4.79	9.62	0.23	2.40
		EGM96	-4.78	9.71	0.32	2.36
		GSD95	-5.24	9.97	0.25	2.11
Canada	932	JGM3_O	-21.56	12.62	-0.20	3.41
		EGM96	-21.83	14.79	0.01	3.15
		GSD95	-11.79	10.29	0.05	1.92

Table 2.2: Comparison of the JGM3_OSU91A, EGM96 and GSD95 geoid model to the east-west component of astro-geodetic deflections of the vertical across Canada. The geodetic coordinates deflections are in the ITRF93 system.

Region	No of Points	Model	Min. (arcsec)	Max. (arcsec)	Mean (arcsec)	St. Dev. (arcsec)
Newfoundland	52	JGM3_O	-2.78	6.83	0.64	1.93
		EGM96	-2.91	6.38	0.56	1.88
		GSD95	-2.45	3.41	0.58	1.24
Maritime	74	JGM3_O	-7.83	4.10	0.59	1.84
		EGM96	-7.37	3.78	0.42	1.83
		GSD95	-2.89	5.36	0.60	1.03
Québec	127	JGM3_O	-9.66	6.56	0.22	2.48
		EGM96	-7.98	6.05	0.29	2.39
		GSD95	-5.93	5.54	0.37	1.55
Ontario	147	JGM3_O	-3.50	3.84	0.20	1.42
		EGM96	-3.49	4.13	0.22	1.51
		GSD95	-2.57	2.52	0.38	0.97
Manitoba	66	JGM3_O	-4.49	5.38	-0.02	1.64
		EGM96	-4.65	5.25	0.01	1.62
		GSD95	-2.46	6.13	0.20	1.35
Saskatchewan	72	JGM3_O	-3.26	4.40	0.38	1.45
		EGM96	-3.27	5.10	0.38	1.43
		GSD95	-1.91	5.82	0.56	1.09
Alberta	115	JGM3_O	-9.65	15.14	0.25	3.00
		EGM96	-9.96	13.92	0.26	2.89
		GSD95	-5.98	9.64	0.25	2.06
British Columbia	200	JGM3_O	-22.61	33.00	-0.58	7.83
		EGM96	-20.28	20.57	0.01	6.81
		GSD95	-12.80	11.31	0.03	3.47
Yukon T.	23	JGM3_O	-10.24	10.53	-1.44	5.25
		EGM96	-10.10	7.74	-0.44	4.31
		GSD95	-1.36	3.26	0.75	1.29
Northwest T.	56	JGM3_O	-8.63	4.00	0.17	2.25
		EGM96	-8.76	4.82	0.36	2.36
		GSD95	-5.74	3.07	-0.09	1.35
Canada	932	JGM3_O	-22.61	33.00	0.05	4.18
		EGM96	-20.28	20.57	0.21	3.71
		GSD95	-12.80	11.31	0.30	2.04

Figure 4: Tracks of TOPEX data (cycle 108).

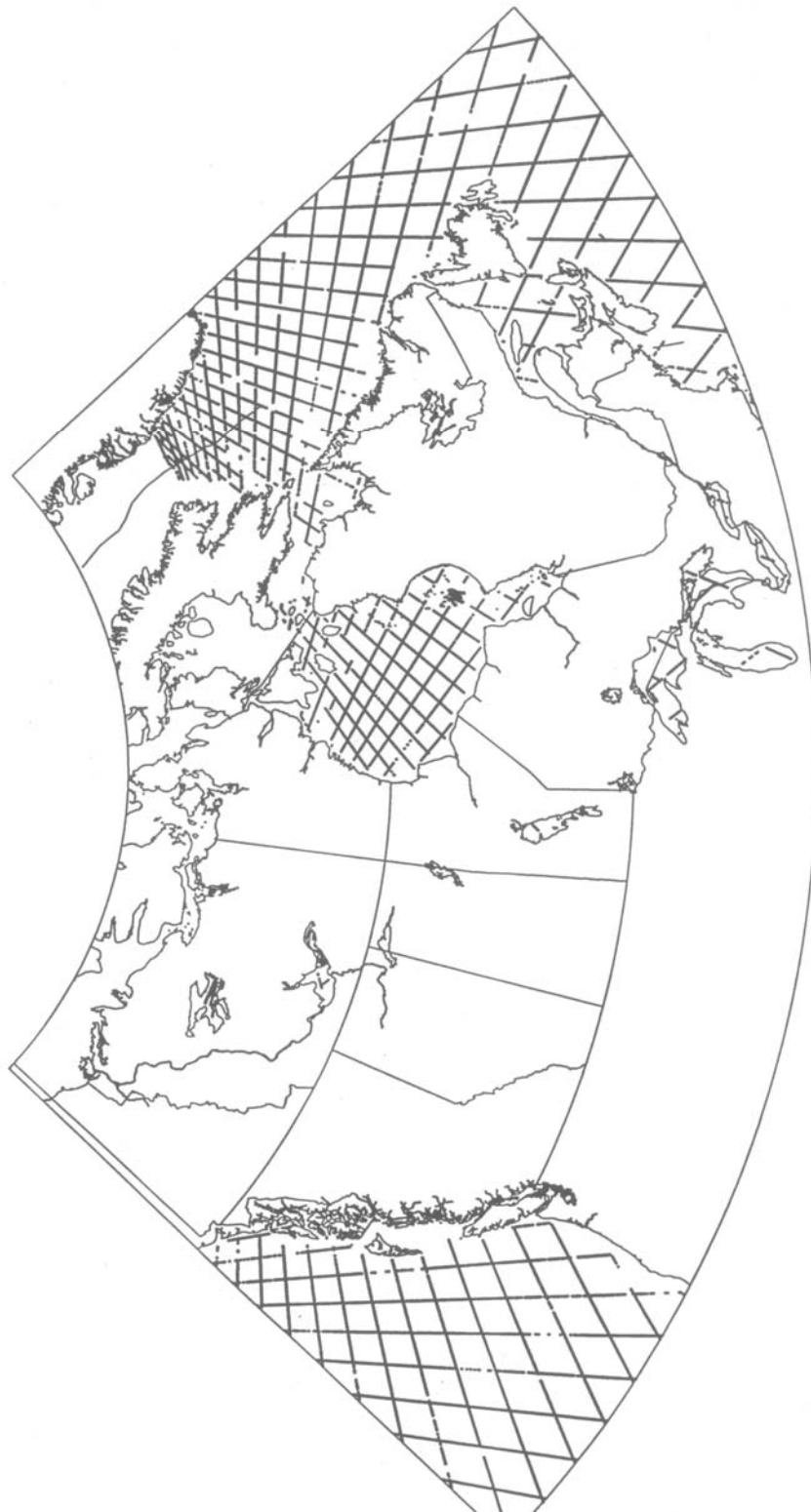


Table 3.1: Comparison of the JGM3_OSU91A, EGM96 and GSD95 geoid model to TOPEX sea surface height (cycle 108).

Region	Area N/S/W/E	No. Pts	Model	Min. (m)	Max. (m)	Mean (m)	St. Dev. (m)
Atlantic	60/41/218/236	5068	JGM3_O	-1.83	3.99	-0.39	0.47
			EGM96	-1.74	2.25	-0.27	0.37
			GSD95	-1.22	1.59	0.27	0.40
Pacific	65/41/288/314	3191	JGM3_O	-0.78	2.18	0.47	0.30
			EGM96	-0.63	1.66	0.45	0.20
			GSD95	-1.50	0.74	-0.22	0.27
Hudson Bay	64/41/264/284	1920	JGM3_O	-1.22	1.05	0.06	0.24
			EGM96	-0.78	0.85	0.03	0.22
			GSD95	-1.07	0.92	-0.05	0.36
Ungava Bay	62/58/288/296	156	JGM3_O	-0.92	3.99	0.13	1.13
			EGM96	-0.71	2.25	0.07	0.64
			GSD95	-1.00	0.49	-0.51	0.31
Davis Strait	66/60/296/314	1565	JGM3_O	-1.84	0.31	-0.79	0.28
			EGM96	-1.11	0.58	-0.26	0.24
			GSD95	-0.98	0.64	-0.29	0.29
Labrador Sea	60/52/296/314	1681	JGM3_O	-1.06	0.30	-0.50	0.21
			EGM96	-1.01	0.49	-0.55	0.19
			GSD95	-0.25	0.96	0.20	0.26
East Nfld	52/46/304/314	688	JGM3_O	-1.07	0.85	-0.40	0.26
			EGM96	-1.03	0.89	-0.41	0.23
			GSD95	0.05	0.93	0.63	0.15
South Nfld	46/41/300/314	948	JGM3_O	-0.65	0.80	0.09	0.22
			EGM96	-0.65	0.85	0.11	0.20
			GSD95	0.25	1.30	0.67	0.15
South N.S.	46/41/294/302	423	JGM3_O	-0.42	0.84	0.18	0.22
			EGM96	-0.41	1.01	0.20	0.19
			GSD95	0.28	1.13	0.65	0.14
St-Laurent G.	51/46/295/302	251	JGM3_O	-0.64	1.02	-0.16	0.29
			EGM96	-0.68	1.05	-0.02	0.30
			GSD95	-0.08	1.24	0.69	0.20
West Wash.	50/46/232/236	134	JGM3_O	-0.08	0.69	0.40	0.15
			EGM96	-0.13	0.80	0.45	0.16
			GSD95	-0.46	-0.07	-0.26	0.07

Table 3.2: Comparison of the JGM3_OSU91A, EGM96 and GSD95 geoid model to TOPEX sea surface height (cycle 108) over lakes.

Lake	Area N/S/W/E	No. Pts	Model	Min. (m)	Max. (m)	Mean (m)	St. Dev. (m)
Ontario	44/43/280/284	7	JGM3_O	74.15	74.28	74.21	0.04
			EGM96	74.37	74.68	74.46	0.10
			GSD95	75.03	75.14	75.10	0.03
Erie	43/41/276/281	16	JGM3_O	174.00	174.25	174.12	0.07
			EGM96	174.21	174.52	174.33	0.08
			GSD95	174.49	174.77	174.60	0.07
Huron	46/43/276/280	52	JGM3_O	176.52	176.89	176.71	0.10
			EGM96	176.25	176.85	176.53	0.17
			GSD95	176.50	176.77	176.66	0.07
Michigan	46/41/272/275	32	JGM3_O	175.48	176.63	176.33	0.35
			EGM96	175.62	176.53	176.26	0.24
			GSD95	175.98	176.89	176.54	0.27
Superior	49/46/267/276	65	JGM3_O	182.67	183.70	183.10	0.22
			EGM96	181.98	183.06	182.53	0.30
			GSD95	182.88	183.10	182.98	0.04
Winnipeg	55/50/260/264	7	JGM3_O	217.49	217.81	217.67	0.12
			EGM96	217.50	217.76	217.65	0.10
			GSD95	217.46	217.51	217.48	0.02
Great Slave	64/60/240/252	19	JGM3_O	156.10	156.49	156.29	0.11
			EGM96	155.94	156.35	156.16	0.12
			GSD95	155.99	156.19	156.09	0.06
Great Bear	68/64/232/244	107	JGM3_O	155.56	156.20	155.78	0.13
			EGM96	156.39	156.95	156.65	0.13
			GSD95	155.71	156.43	156.03	0.12

Figure D.1: Determination of the Sea Surface Topography (SST) over the Atlantic Ocean near the Canadian coast using TOPEX data (cycle 108) and the EGM96 geoid model.

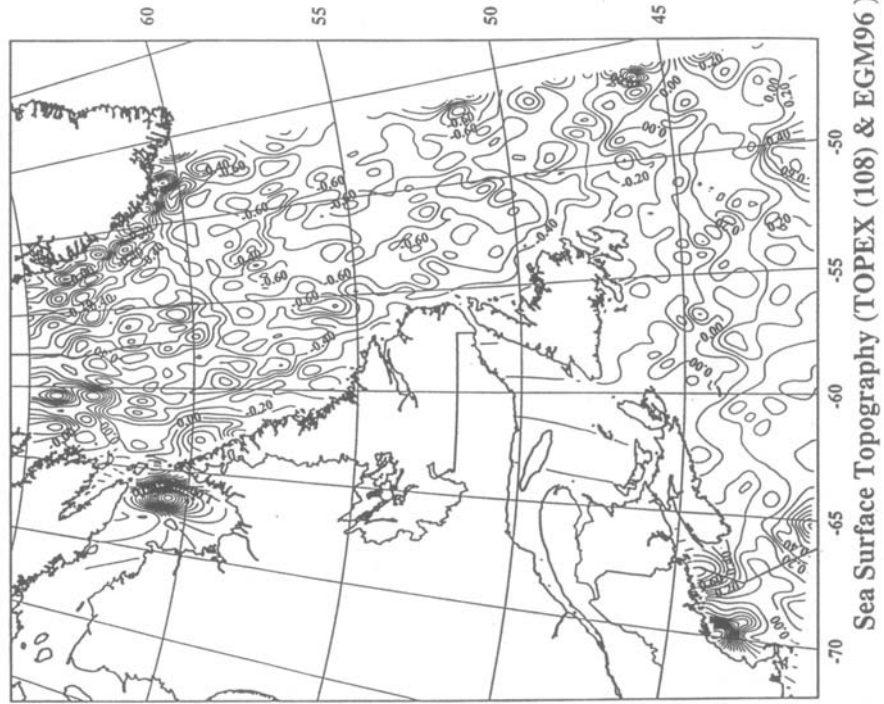


Figure D.2: Determination of the Sea Surface Topography (SST) over the Atlantic Ocean near the Canadian coast using TOPEX data (cycle 108) and the GSD95 geoid model.

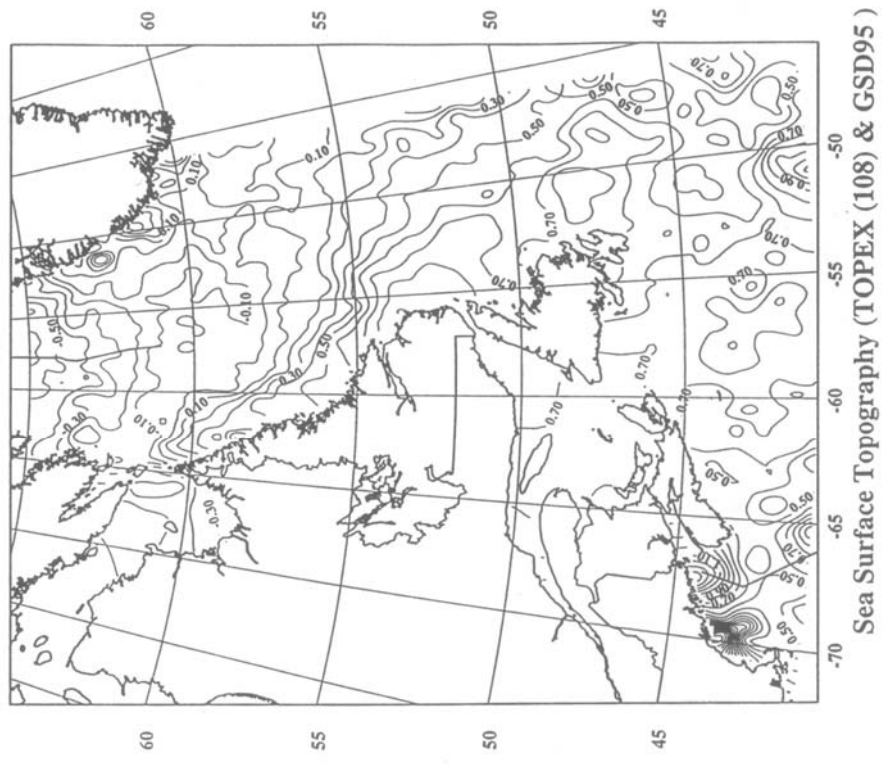


Figure E.1: Determination of the Sea Surface Topography (SST) over the Hudson Bay using TOPEX data (cycle 108) and the EGM96 geoid model.

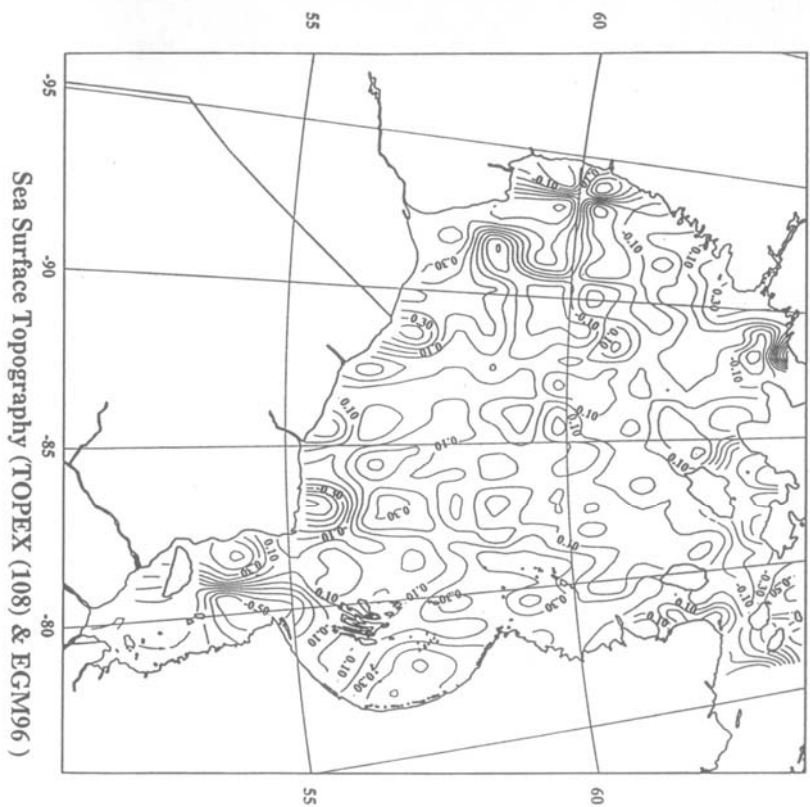


Figure E.2: Determination of the Sea Surface Topography (SST) over the Hudson Bay using TOPEX data (cycle 108) and the GSD95 geoid model.

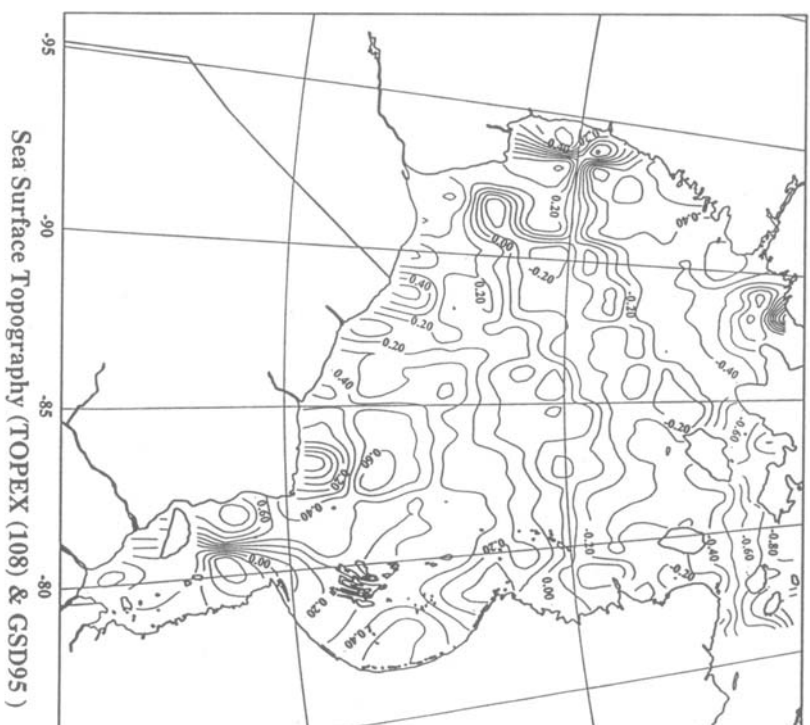


Figure F.1: Determination of the Sea Surface Topography (SST) over the Pacific Ocean near the Canadian coast using TOPEX data (cycle 108) and the EGM96 geoid model.

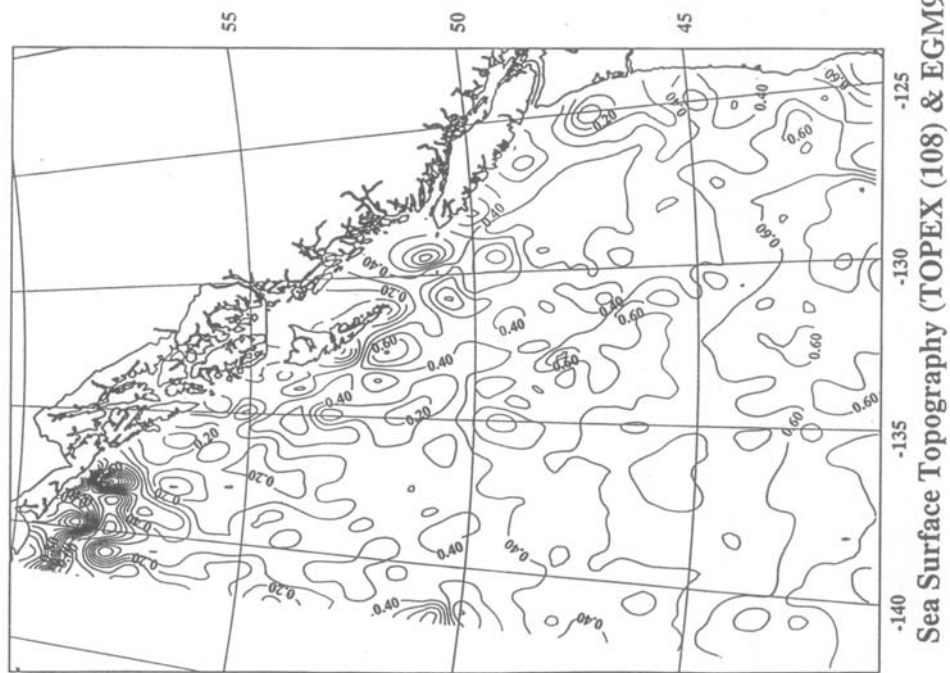
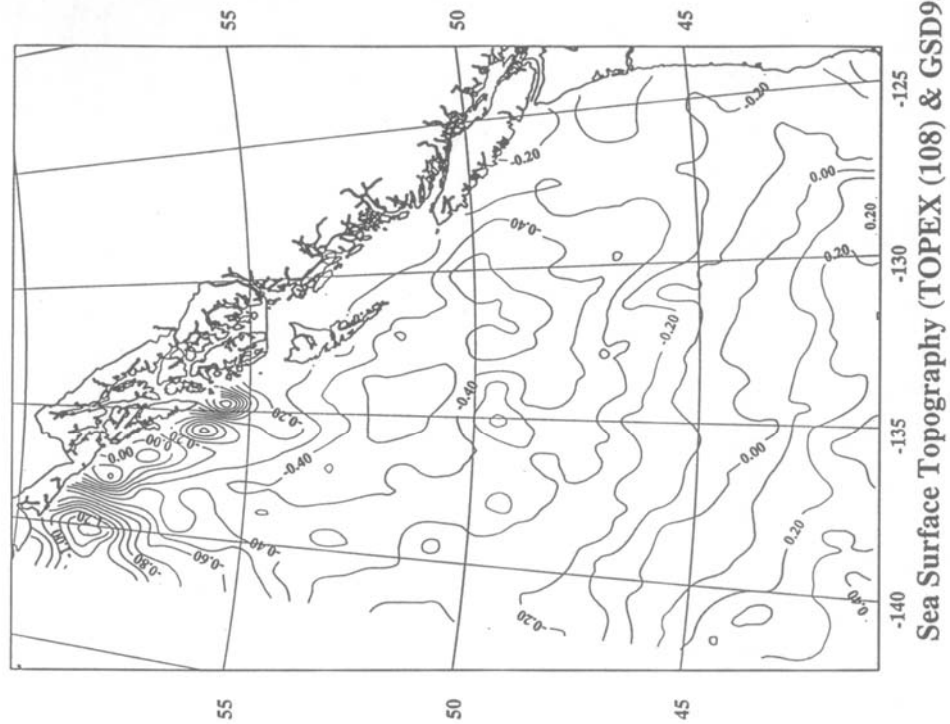


Figure F.1: Determination of the Sea Surface Topography (SST) over the Pacific Ocean near the Canadian coast using TOPEX data (cycle 108) and the GSD95 geoid model.



NASA(GSFC)/NIMA Model Evaluation

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ABSTRACT

The evaluation of the NASA/NIMA geopotential model has been carried out in South America using Doppler and GPS observations (DOP/GPS) on the levelling network as well as gravity anomalies. For the evaluation of the preliminary five models it has been used a total of 429 DOP/GPS points. The evaluation of the final model, EGM96, a total of 460 DOP/GPS points and 3831 updated 30' x 30' mean anomalies derived from point gravity were available in South America. Most of the gravity anomalies have been provided by Geophysical Exploration Technology (GETECH). The results presented show that the preliminary models are quite similar. The final model EGM96 is remarkable better than the previous JGM3-OSU91A in terms of the comparison with gravity anomalies.

1 - INTRODUCTION

The evaluation of the NASA(GSFC)/NIMA (National Agency and Space Administration (Goddard Space Flight Center)/National Imagery and Mapping Agency - former DMA) geopotential models has been carried out under the auspices of the Sub-Commission for the Geoid in South America (SCGSA) and with an important cooperation of Geophysical Exploration Technology (GETECH)-University of Leeds. Comparisons have been done with Doppler and GPS observations on the geometric levelling network in South America, from now on referred as DOP/GPS, and mean gravity anomalies. The cooperation of the different countries has been important to improve the data coverage released to NIMA for the computation of the model and recently for the evaluation. This one was carried out in two stages. First of all, after the five preliminary models have been released, a first comparison has been carried out using only DOP/GPS points. The comparison included also the JGM3-OSU91A model. The final model delivered and presented at GraGeoMar 96, in Japan, has been subjected recently to a specific analysis in South America.

2 - PRELIMINARY MODELS

After the five preliminary models, EGM-X01 to X05, have been released to SWG members for evaluation, an analysis has been carried out for South America. For this purpose a total of 429 DOP/GPS points have been used. They are distributed in Brazil (404) and in Venezuela (25). Most of the points in Brazil are derived from the old Doppler observations carried out at the Transit satellites. They have been selected only observations reduced with the precise ephemeris, at that time referred to NSWC-9Z2 (Naval Surface Weapons Center-9Z2). The coordinates have been transformed to WGS-84 (World

Geodetic Sistem-1984) using the following parameters: [BOUCHER and ALTAMINI, 1985]

$$TZ = 4,5 \text{ m} \quad (1a)$$

$$\alpha_3 = -0,814'' \quad (1b)$$

$$K = -0,6 \text{ PPM} \quad (1c)$$

The more recent GPS points are already referred to WGS-84. The model EGM-X02 which gave the minimum RMS difference has been checked also against ITRF90. For this purpose the WGS-84 coordinates have been corrected of the following parameters: [McCARTHY, 1992]

$$TX = -0,060 \text{ m} \quad (2a)$$

$$TY = 0,517 \text{ m} \quad (2b)$$

$$TZ = 0,223 \text{ m} \quad (2c)$$

$$\alpha_1 = -0,0183'' \quad (2d)$$

$$\alpha_2 = 0,0003'' \quad (2e)$$

$$\alpha_3 = 0,0070'' \quad (2f)$$

$$K = -0,6 \text{ PPM} \quad (2g)$$

The DOP/GPS data have certainly many uncertainties. The levelling network in Brazil is very large and not completely adjusted. We have many Doppler points in some long levelling lines across the jungle in Amazonas without control. In particular, one line extends from the South of Amazonas up to the Venezuela border, where a difference of 2 m exists between the networks of the two countries. In this way it is difficult to predict the accuracy of the data being used for comparison. The expected accuracy of the old Doppler observations was 1.5 m when precise ephemeris were used. A recent comparison between GPS results and the old Doppler in the center part of Brazil showed a difference of 0.5 m in each of the coordinates X,Y,Z. (Fonseca, 1996). Finally, it is important to point out that, according to Levitus model of the Sea Surface Topography (SST), the correction in the origin of the levelling network in Brazil, a tide gauge located at Imbituba, is close to zero. So, it is not expected that any large zero order term comes from SST.

A major work has been done in Brazil after 1991 to improve the gravity data coverage. After the creation of SCGSA an effort has also been addressed to other countries like Argentina, Uruguay, Chile, Colombia, etc. When the idea of the new NASA/NIMA model came up, the file of gravity data at NIMA has been updated as much as possible as an effort of SCGSA. But, new gravity data have been collected recently. On the other hand, the digital terrain model (DTM) has been significantly improved in the second semester of 1996 in Brazil and Argentina as a joint effort between SCGSA and GETECH. DTM is very helpful in the estimation of mean gravity anomalies. In this way, the present evaluation included more information, gravity and others, than those entered in the computation of the new model.

3 - RESULTS OF THE PRELIMINARY MODELS

The results related to the five preliminary models delivered by NASA/NIMA and the JGM3-OSU91A model, are presented in the table 1. It shows the mean difference value, which is a zero order term N_0 , and the RMS difference. This one has been obtained after N_0 was added to the undulations derived from the model. There is a good agreement between the models in terms of this comparison.

Model	N_0 (m)	RMS difference (m)
EGM-X01	-0.86	2.19
EGM-X02	-0.77	2.17
EGM-X03	-0.79	2.18
EGM-X04	-0.79	2.19
EGM-X05	-0.79	2.20
EGM-X02 (ITRF90)	-1.45	2.22
JGM3-OSU91A	-0.79	2.08

Table 1 - Mean values and RMS differences in terms of 'geoid height'.

The EGM-X02 is slightly better if the RMS value is considered. When referred to WGS-84 a zero order term of about -0.80 m indicates a semi-major axis for the best ellipsoid fitted to the model on the order of 6 378 136.2 m. When referred to ITRF90 the value of N_0 is about two times smaller (-1.45). In this case the semi-major axis of the best fitted ellipsoid would be on the order of 6 378 135.6 m. It is important to point out that the previous model JGM3-OSU91A shows an RMS difference better than the new models in terms of this comparison (table 1).

Figures 1 to 7 show the distribution of the differences after the respective zero order term has been added to the values derived from the models.

4 - RESULTS OF THE FINAL MODEL

The final model, EGM96, has been validated against a more complete set of data than the preliminary models. A total of 460 DOP/GPS points has been used with extra 31 points delivered by the University of La Plata, Argentina. A total of 3831 mean gravity anomalies in blocks of 30' x 30' has been estimated in South America and used for comparison too. For the derivation of the 30' mean, in a first step 5' x 5' mean values for free air and Bouguer anomalies have been derived using all the available point gravity in South America. Second, mean values for the height has also been obtained from the 3' DTM grid. The 30' mean free air anomalies used have been restored from mean Bouguer anomalies and mean heights. The activity related to the estimation of 5' mean values is part of the effort of SCGSA and has been possible due to the strong cooperation with Geophysical Exploration Technology (GETECH) - University of Leeds.

In terms of the DOP/GPS points the comparison is shown in the table 2. The figures 8 and 9 show the distribution of the differences.

Model	N_o (m)	RMS difference (m)
EGM96 (WGS-84)	-0.65	2.18
EGM96 (ITRF90)	-1.20	2.25

Table 2 - Mean values and RMS differences in terms of 'geoid height'.

In terms of mean anomalies the results are presented in the table 3 and the distribution of the differences in the figures 10 and 11, for JGM3-OSU91A and EGM96.

Model	Δg_o (mGal)	RMS difference (mGal)
JGM3-OAU91A (WGS-84)	-0.1	30.4
EGM96 (WGS-84)	-0.6	17.7

Table 3 - Mean values and RMS differences in terms of gravity anomaly.

5 - CONCLUSIONS

The five different models related to the effort NASA(GSFC)/NIMA gave very similar histograms related to the comparison with the 429 DOP/GPS points. The model EGM-X02 has a slight predominance of differences between 0 and 1 which is the reason for an RMS difference slightly smaller. When the reference is ITRF90 the differences in the interval 0 - 1 are marked greater. The old model JGM3-OSU91 has a slight predominance of positive values over negatives.

The comparison with 30' x 30' mean gravity anomalies has been carried out over the final model and JGM3-OSU91A. EGM96 is remarkably better as can be seen by the RMS difference (table 3) and by the histograms (figures 10 and 11).

Finally, it is important to emphasize that what has been done is a comparison of different quantities: the models and DOP/GPS points and gravity anomalies. It is certainly possible that problems exist with the DOP/GPS points in terms of the coordinates as well as in terms of the levelling. Also the gravity data are not well distributed and DTM may not have a very good resolution everywhere. So, the comparison has to be seen as a relative validation. We can certainly say that the new model in South America is better than any other previous model and should be used as a reference field from now on to any detailed computation of the geoid or other applications.

ACKNOWLEDGEMENT

The final calculations for this report has been prepared recently at GETECH - University of Leeds during one month stay. I am very much grateful to Prof. J. Derek Fairhead for his support to the gravity and to the geoid related activities in South America. I also acknowledge the support of FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) for funding my stay in Leeds.

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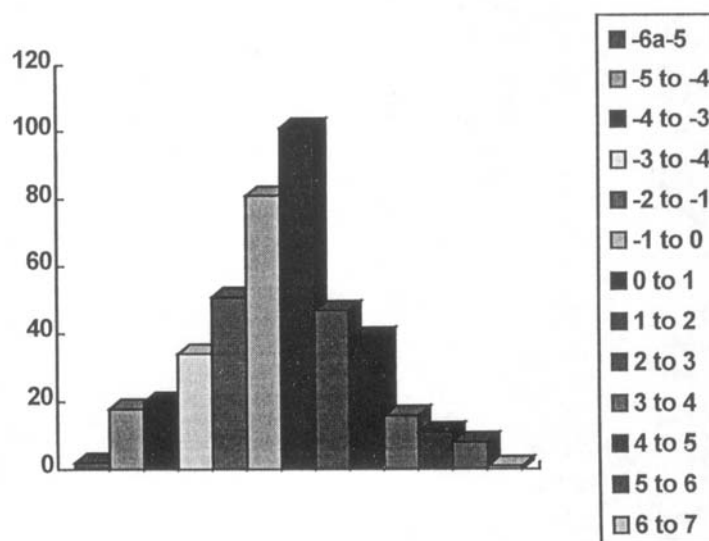


Fig. 1 - Histogram of the differences between EGM-X01 and DOP/GPS points (WGS-84).

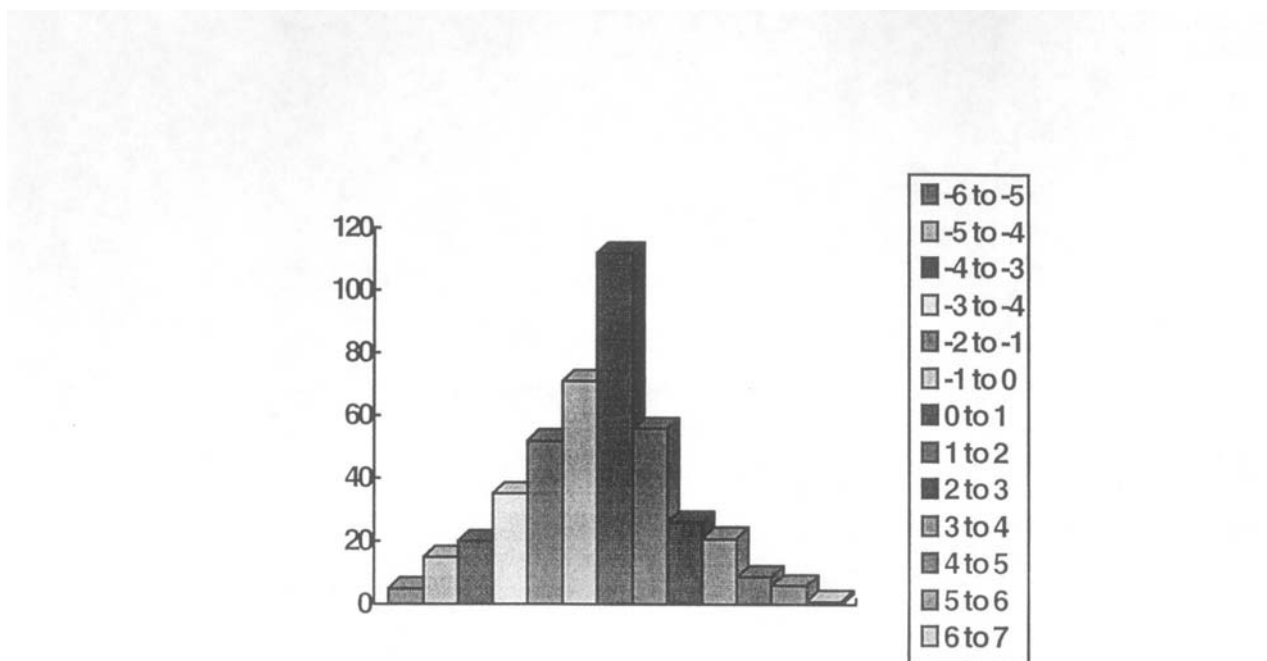


Fig. 2 - Histogram of the differences between EGM-X02 and DOP/GPS points (WGS-84).

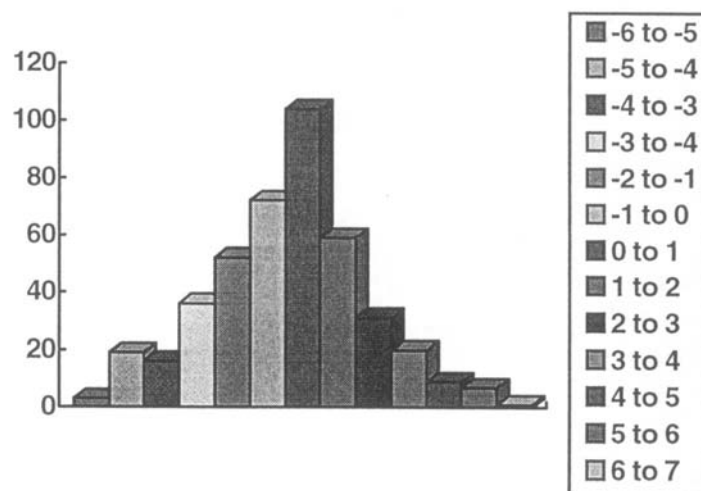


Fig. 3 - Histogram of the differences between EGM-X03 and DOP/GPS points (WGS-84).

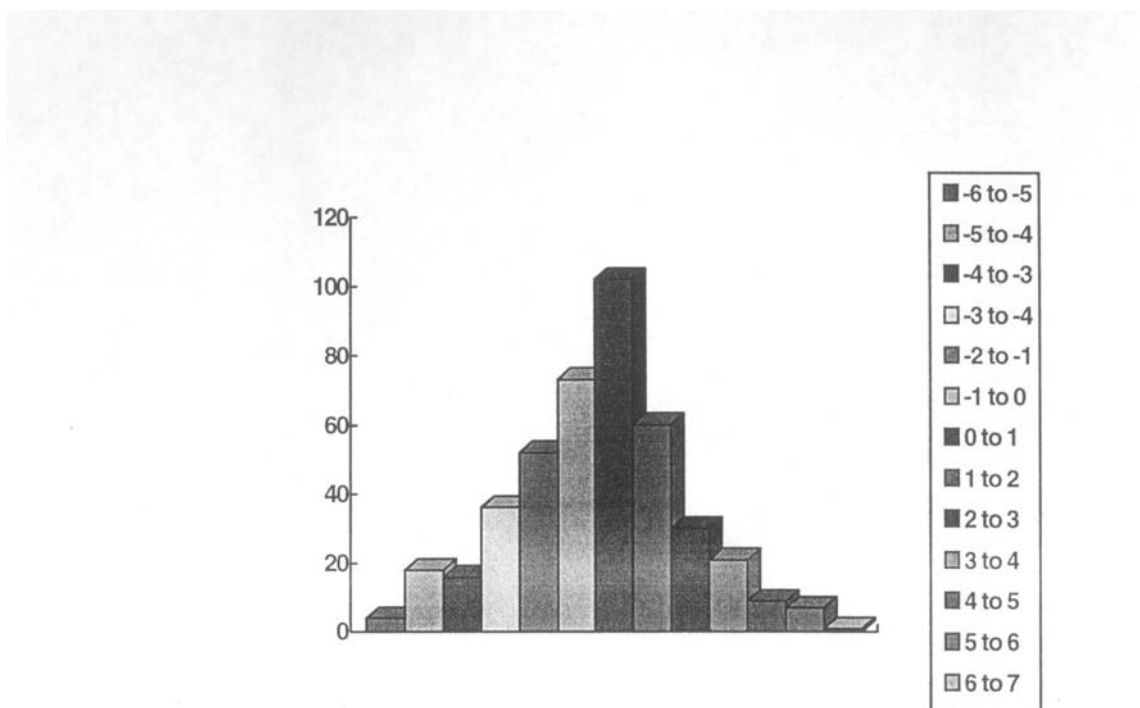


Fig 4 - Histogram of the differences between EGM-X04 and DOP/GPS points (WGS-84).

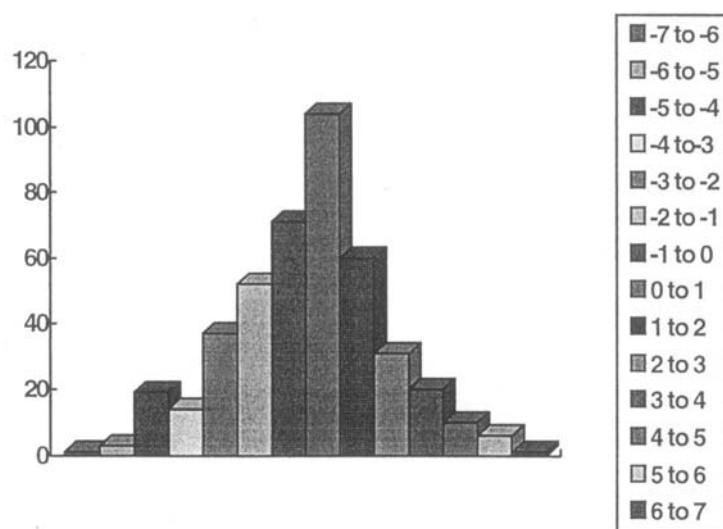


Fig. 5 - Histogram of the differences between EGM-X05 and DOP/GPS points (WGS-84).

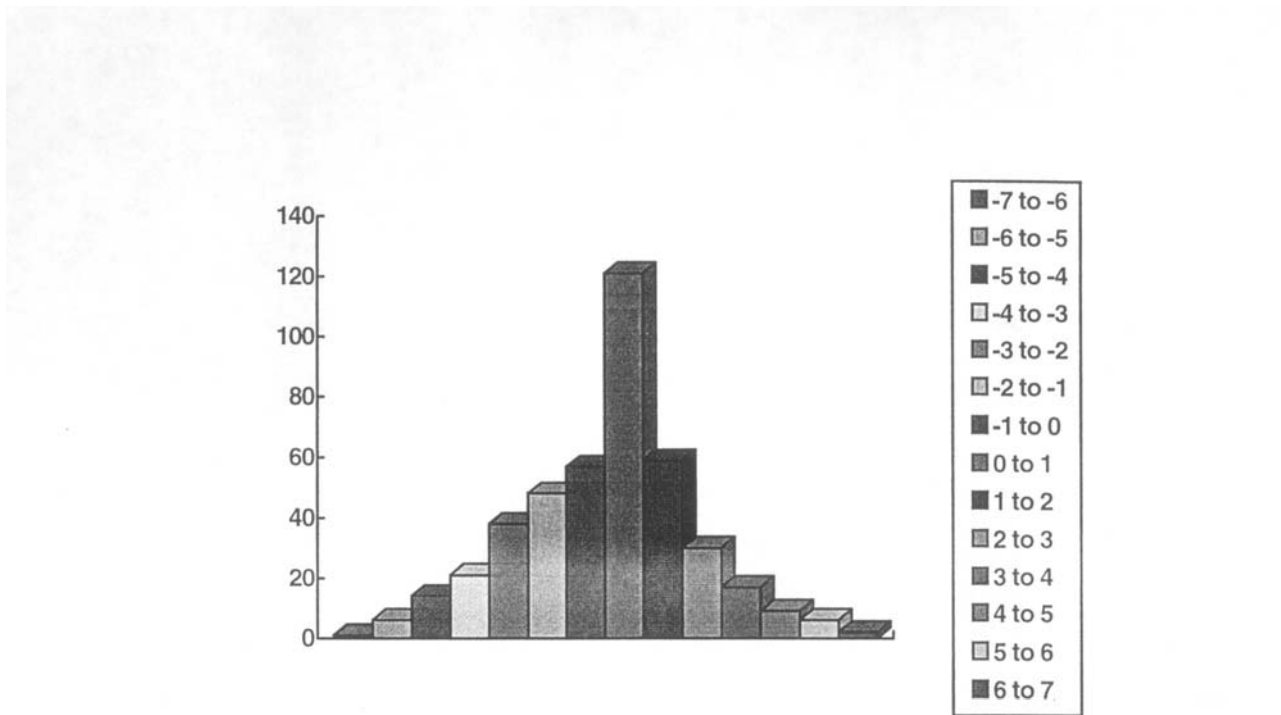


Fig. 6 - Histogram of the differences between EGM-X02 and DOP/GPS points (ITRF90).

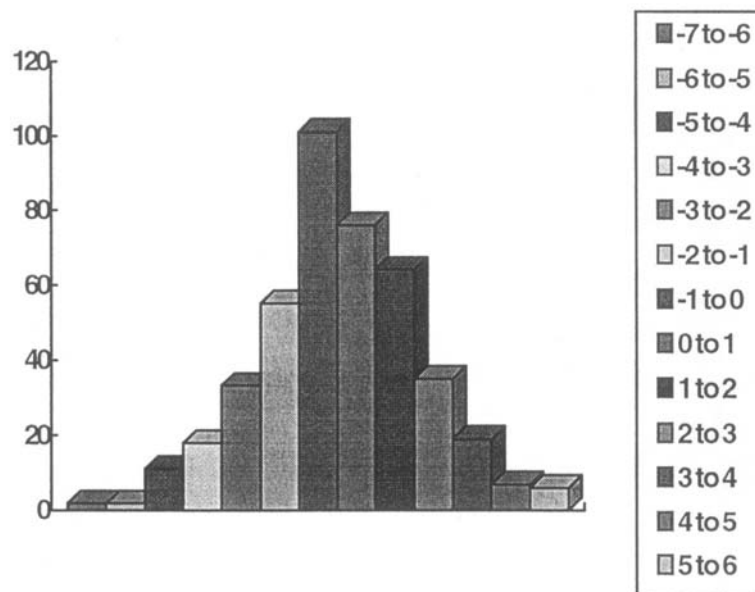


Fig. 7 - Histogram of the differences of JGM3-OSU91A and DOP/GPS points (WGS-84).

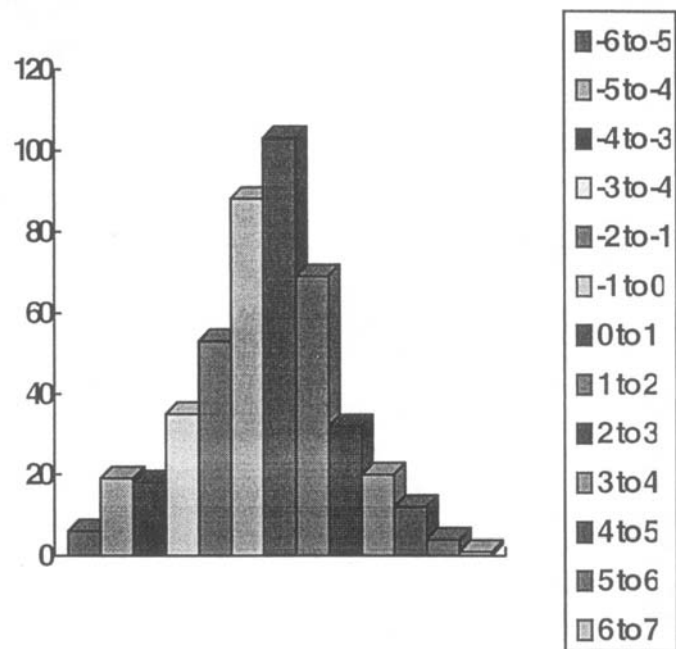


Fig. 8 - Histogram of the differences between EGM96 and DOP/GPS points (WGS-84).

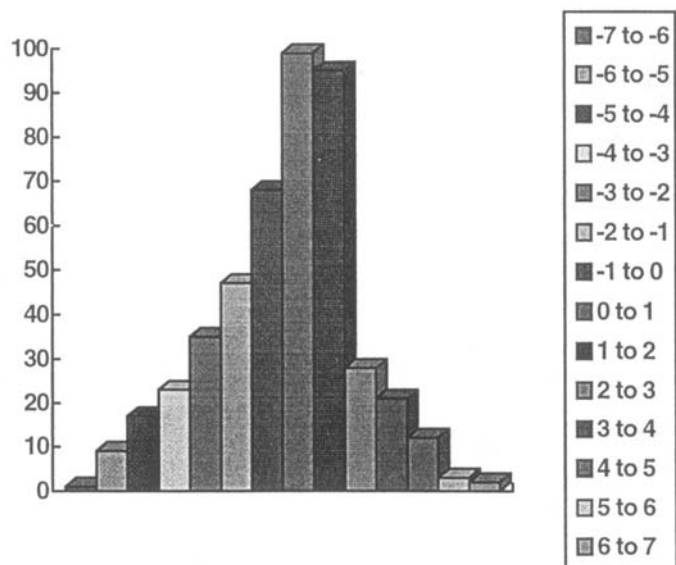


Fig. 9 - Histogram of the differences between EGM96 and DOP/GPS points (ITRF90).

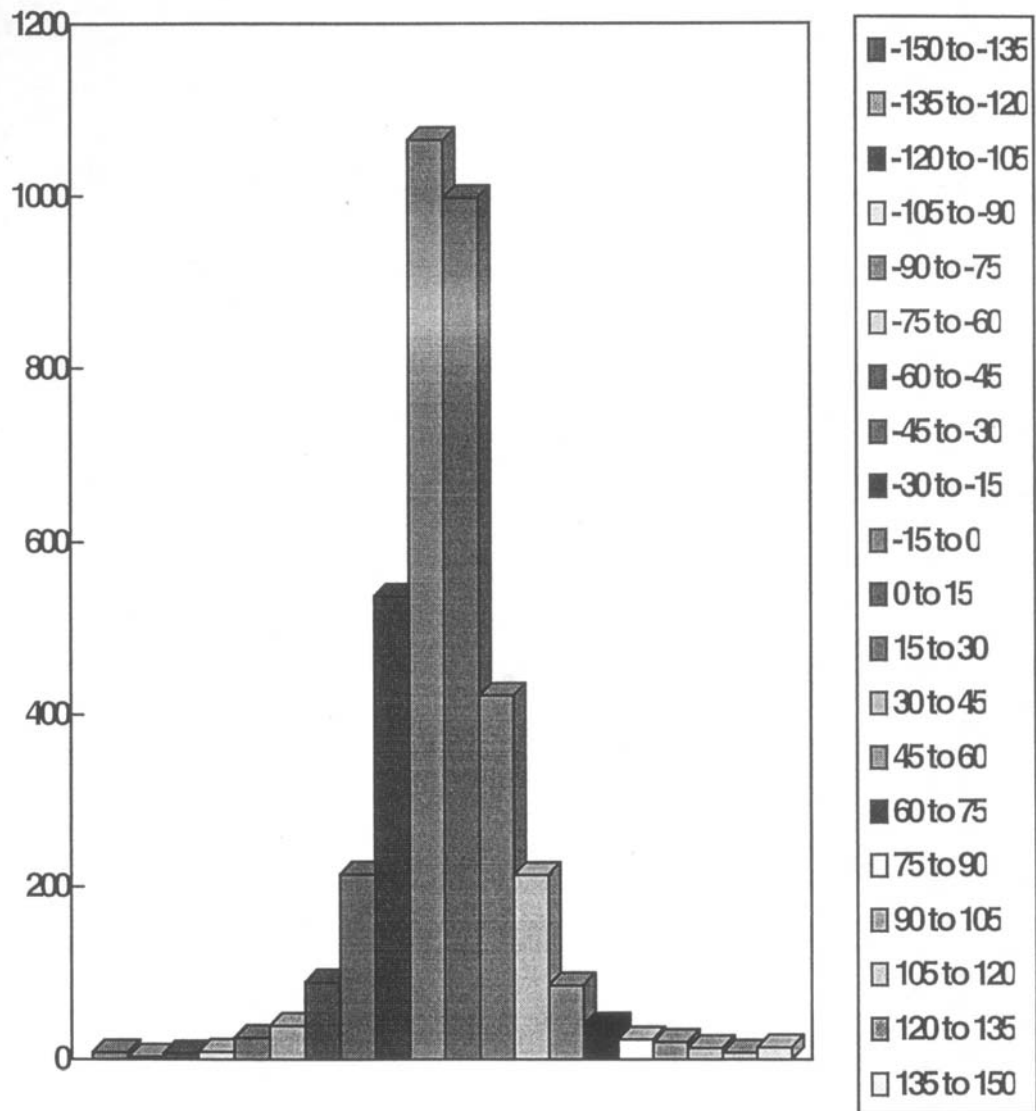


Fig 10 - Histogram of the differences between 30' mean anomalies and JGM-OSU91A.

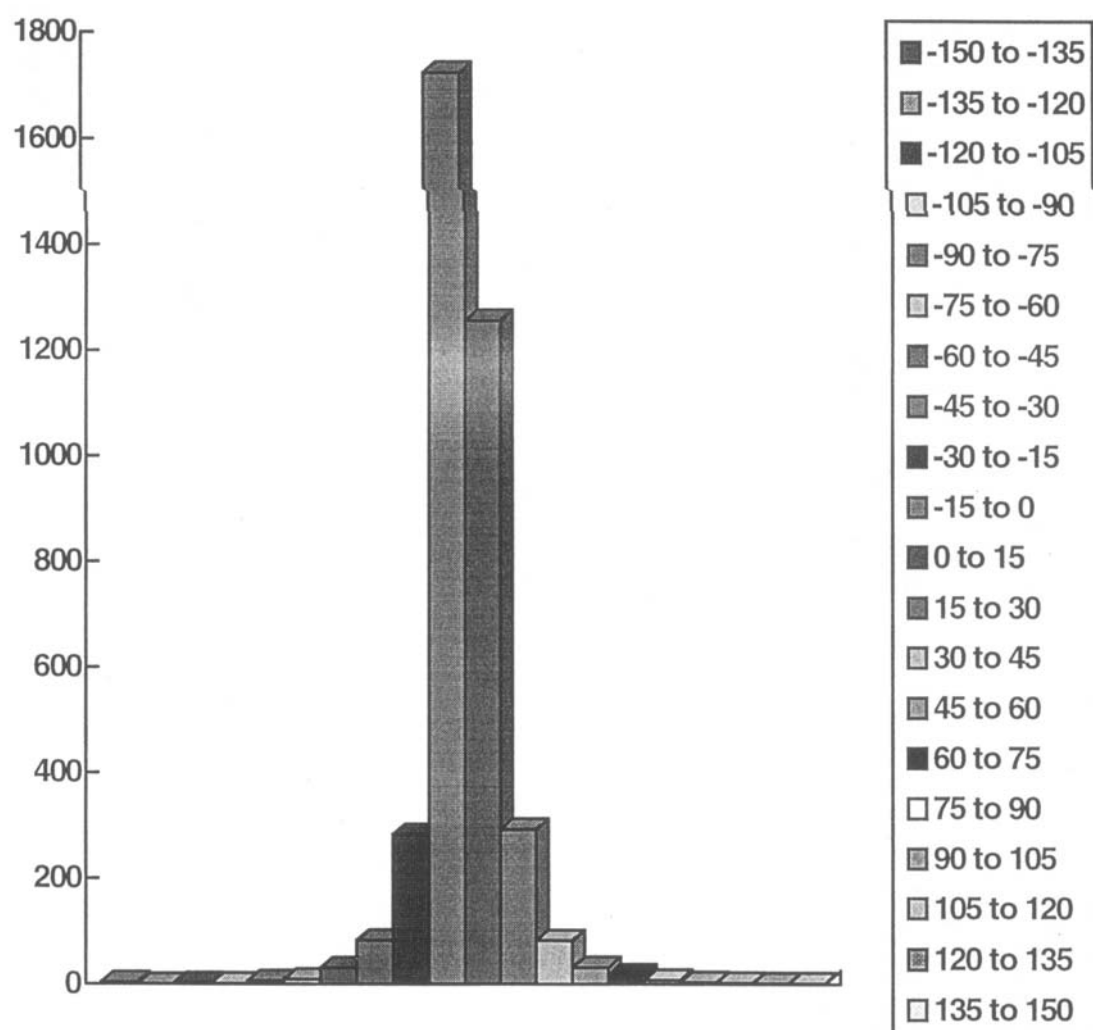


Fig. 11 - Histogram of the differences between 30' mean anomalies and EGM96.

Evaluation of the NASA GSFC and NIMA Joint Geopotential Models for Europe

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Abstract

In 1996 NASA GSFC, NIMA and The Ohio State University (OSU) jointly developed the high resolution geopotential model EGM96. This model is complete to degree and order 360 and combines satellite tracking data with the most up-to-date global $30' \times 30'$ gravity and altimeter data sets. Before the release of EGM96, the precursor models EGM-X01 to EGM-X05 were made available for testing. The EGM models as well as the models OSU91A and JGM3_OSU91A were evaluated using high resolution gravity data, geoid/quasigeoid models, and GPS/leveling data for Europe. The European data sets were collected within the frame of the European Geoid Project, in which the Institut für Erdmessung (IfE) serves as the computing center. This project is supported by IAG.

Comparison of Global Geopotential Models with Gravity and Quasigeoid Data for Europe

In 1990 IfE started to develop high resolution geoid/quasigeoid models for Europe within the frame of the European Geoid Project (see e.g. *Denker et al.* 1994, 1995, 1996). IfE is acting as the computing center in this IAG supported project and has collected numerous gravity and terrain data sets for Europe. At present, about 2.7 million gravity data and about 700 million terrain data are stored in the project data base. The gravity data coverage is displayed in figure 1. From this figure it can be seen that gravity data are available for all European land areas with a resolution of at least 10 km, while still some marine areas exist without any ship gravity data available. For this reason, the terrestrial data were merged with altimetrically derived gravity anomalies from ERS-1 (*Anderson et al.* 1996a and 1996b).

Within the European Geoid Project the primary objective at IfE was the computation of a high resolution quasigeoid (height anomaly) model, as this allows the direct utilization of the data observed at the Earth's surface without any hypothesis about the gravity field in the Earth's interior. Then a subsequent transformation from the quasigeoid to the geoid was performed by introducing a density model (see e.g. *Denker et al.* 1996). The basic modeling strategy at IfE is based on the remove-restore technique, where a global geopotential model is combined with high resolution gravity and terrain data. In this procedure, residual observations reduced for the effect of the global model and the topography are computed first, then the modeling techniques are applied to these residual data, and finally the effects of the global model and the topography are added back. For the computation of the European quasigeoid models at IfE, the field transformation from residual gravity to quasigeoid values was done by the spectral combination technique (see e.g. *Wenzel* 1982) with the numerical evaluation carried out by 1D FFT (according to *Haagmans et al.* 1993). The topography was taken into account using the so-called residual terrain model (RTM) technique according to

Forsberg and Tscherning (1981), where the reference topography was constructed by a $15' \times 15'$ moving average filter.

The residual gravity anomalies were gridded by a fast least squares prediction technique onto a $1.0' \times 1.5'$ grid covering the area from 25°N - 77°N and 35°W - 67.4°E . This yields $3,120 \times 4,096 = 12,779,520$ grid points. For each gridded value also the prediction error was computed. These error estimates were then used for the merging of the gridded terrestrial gravity data with the corresponding altimetrically derived anomalies according to their standard deviations. In the following we will utilize the quasigeoid models EGG.96.01, EGG.96.03 and EGG.97.03, as well as the corresponding gravity data sets. EGG.96.01 is based on the geopotential model OSU91A (*Rapp et al. 1991*), EGG.96.03 uses the JGM3_OSU91A model (JGM-3 complete to degree 70 augmented by OSU91A from degree 71 to 360, *Tapley et al. 1994*), and EGG.97.03 uses the EGM96 model. Furthermore, the quasigeoid models differ in the used gravity data sets. From the analysis of EGG.96.01 an error in one marine gravity data source southwest of Iceland was found, which was then removed from the later computations (the data source was corrected by a bias and was also downweighted). Moreover, the EGG.97.03 quasigeoid model uses an updated version of the altimetrically derived anomalies (*Andersen et al. 1996b*).

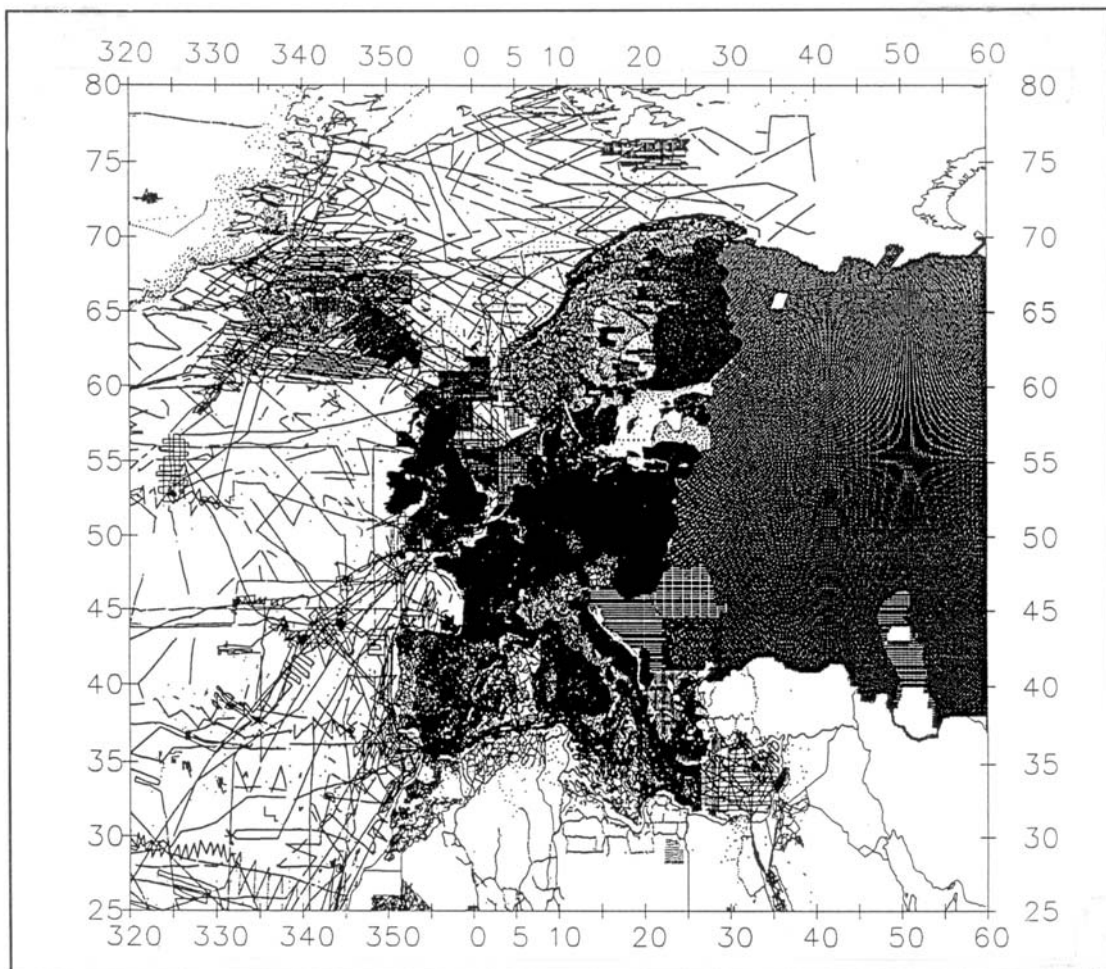


Fig. 1: Locations of Gravity Data Stored in the IfE Data Base. Status January 1997.

In the following, gravity and height anomalies computed from the global models EGM96, EGM-X01 to EGM-X05 (precursor models of EGM96), JGM3_OSU91A as well as OSU91A (all models are complete to degree and order 360) are compared with the respective values from the European Geoid Project data base. In all computations (global model values, IfE gravity and height anomalies) the normal gravity field was defined as GRS80. Moreover, in all comparisons described here, the zero order terms (see e.g. *Torge* 1991, *Rapp* and *Balasubramania* 1992) were neglected. For example, the zero order height and gravity anomaly terms due to differences between the GM values of EGM96 and GRS80 amount to -0.935 m and +0.14 mgal, respectively. The model values were computed at the surface of the Earth using a digital terrain model (DTM). Atmospheric corrections were considered, and all quantities were referred to the same tidal system.

At first we will discuss the evaluation of the above mentioned global models using the EGG.96.01 data sets. Tables 1 - 8 show the results of the comparison of the EGG.96.01 gravity anomaly data set (Δg) with the global geopotential models, while the corresponding results for the height anomalies (ζ) are provided in tables 9 - 16. In addition, the gravity and height anomaly differences are also displayed in graphical form in figures 2 - 5 for the models EGM-X05 and JGM3_OSU91A. The statistics in the tables are given for the complete grid area (columns 2-4), but also limited to those grid points where the standard deviations of the terrestrial gravity anomalies resp. the merged anomalies (terrestrial + altimetric data) are less than 10 mgal (columns 5-7). This was done in order to restrict the comparisons to those regions where reliable data are available.

The residual gravity anomalies were not computed for a small subarea in the southeast of the computation area, because no data were available (see e.g. figures 2 or 4). Therefore, in the anomaly comparisons we have only 12,121,920 grid points, subdivided into 5,844,549 land and 6,277,371 marine grid points. The number of land and marine grid points with a standard deviation of the terrestrial gravity data of less than 10 mgal is 4,373,171 and 3,587,582, respectively. For the marine case the corresponding figure for the merged (terrestrial + altimetric) anomalies increases to 5,573,243 grid points. The comparisons of the EGG.96.01 gravity anomalies with the EGM model values yield standard deviations (std.dev.) in the range of 13.5 - 13.8 mgal for the entire area, more or less independent of the land/sea separation (see tables 1-6). The corresponding mean values range from +0.2 to +0.6 mgal, with the smaller values obtained at sea. However, in this connection one has to consider also the neglected zero order term (0.14 mgal). If only the land points with a data std.dev. of less than 10 mgal are considered, the std.dev. of the differences reduces to 12.7 - 12.9 mgal, with mean values of +0.5 to +0.6 mgal. For the marine areas with a data std.dev. of less than 10 mgal the comparisons yield std.dev. of the differences of 14.4 - 14.6 mgal for the terrestrial data and 13.4 - 13.5 mgal for the merged data, showing a better agreement of the EGM models with the merged data than with the terrestrial data alone. The corresponding mean values are always close to zero (0.0 to +0.2 mgal). In all comparisons discussed in this paragraph, the models EGM-X05 and EGM96 in most cases give slightly smaller std.dev. of the differences than the other test models, especially when considering only the reliable grid points (std.dev. of the data less than 10 mgal). Furthermore, in all anomaly comparisons the EGM models show significantly smaller std.dev. of the differences than the older models OSU91A and JGM3_OSU91A (by about 1 - 6 mgal for the different cases, see tables 7 and 8). However, most of this improvement is due to new gravity data sets used in the EGM models, especially over East Europe, etc., which were not available for the older models (see figures 2 - 5).

The results of the height anomaly comparisons of EGG.96.01 and the global models (see tables 9 - 16) are very much in correspondence with the anomaly comparisons described in the previous paragraph, as in general the height anomaly differences are a low-pass filtered version of the gravity anomaly differences. Again the EGM-X05 and EGM96 models generally yield smaller std.dev. of the differences than the other EGM models, and especially than the OSU91A and JGM3_OSU91A models. However, as can be seen from the graphical display of the gravity and height anomaly differences of EGG.96.01 and EGM-X05 as well as JGM3_OSU91A (figures 2 - 5), most of this improvement is coming from new data sets available for the EGM models, especially over East Europe, etc. Noticeable is also the improvement of the EGM models over the island of Corse (in the Mediterranean), where probably an erroneous data set was used in OSU91A (island data extrapolated from marine data). However, also significant discrepancies exist in several regions between the EGM models and the EGG.96.01 data. Firstly, there is a significant discrepancy in a region southwest of Iceland, which can be seen also in the JGM3_OSU91A comparison. This discrepancy can be attributed to a bias error in one marine gravity source, which was corrected in all gravity and height anomaly models developed after EGG.96.01. Secondly, significant discrepancies between the EGM-X01 to EGM-X05 models and the EGG.96.01 data sets exist in the Gulf of Bothnia (Scandinavia), which are not present in the OSU91A and the JGM3_OSU91A model comparisons. This information was passed to the EGM development team and was corrected in the final EGM96 model (see below). Moreover, larger differences between the EGG.96.01 data and the EGM models can be seen in Bulgaria, Romania and parts of former Yugoslavia, as well as over the Alps and South Norway. The differences in the mountainous regions of the Alps and Norway appear to be correlated with elevation. Such effects are not seen to this extent in the JGM3_OSU91A and OSU91A models. Naturally, also large differences exist between the EGG.96.01 and the EGM models in areas without any accurate and reliable gravity data in the IfE data base, such as Africa, Asia and Greenland. In these regions the EGG.96.01 data sets essentially use the underlying global model values (i.e. OSU91A).

Tables 17 - 20 show the statistics of the gravity and height anomaly comparisons between the EGG.96.03 data sets and the EGM-X05 as well as the EGM96 model. Not all comparisons discussed above could be repeated due to time constraints and because the European data sets developed after EGG.96.01 have only become available recently. The statistics in the tables 17 - 20 show small improvements in terms of std.dev. of the differences as compared to the EGG.96.01 models, which is due to an update of one gravity source (southwest of Iceland) and the change of the underlying global model from OSU91A to JGM3_OSU91A. Due to the downweighting applied to one marine data source southwest of Iceland, also the number of the comparison points changes slightly for some of the marine cases. The gravity and height anomaly differences between EGG.96.03 and EGM-X05 are shown in figures 6 and 7. If one compares these figures with the corresponding EGG.96.01 results shown in figures 2 and 3, the most distinct improvement is southwest of Iceland where one marine gravity data set was corrected. Moreover, long wavelength changes in the height anomaly differences can also be seen, which are due to the change of the underlying global model from EGG.96.01 (OSU91A) to EGG.96.03 (JGM3_OSU91A).

A final gravity and height anomaly comparison was done for the EGG.97.03 data sets and the EGM96 model. The EGG.97.03 gravity data set utilizes an updated set of altimetric gravity anomalies, and the corresponding quasigeoid solution is based on the EGM96 model. The statistics of the differences are given in tables 21 and 22, and a graphical display of the

differences is given in figures 8 and 9. Due to the use of an updated set of altimetric anomalies also the number of marine comparison points with a std.dev. of the merged data of less than 10 mgal changes. We again observe a reduction in the std.dev. of the differences as compared to EGG96.03, which is due to the updated version of the altimetric anomalies and due to the change of the underlying global model in the EGG.97.03 solution. From figures 8 and 9 one can observe that the data problem in the Gulf of Bothnia has been corrected in the final EGM96 model. However, significant gravity and height anomaly discrepancies still exist over Bulgaria, Romania and parts of former Yugoslavia. Furthermore, the larger differences over the Alps and South Norway, which appear to be correlated with elevation, are still existing. These effects should be further investigated, as they are also seen in the GPS/leveling comparisons (see below). Finally, from figures 8 and 9 one can observe, that the EGG.97.03 data and the EGM96 model agree also reasonably well in areas without any gravity data in the IfE data base, which is obviously due to the computation procedure used for the EGG.97.03 model (remove-restore technique, data gridding leads to residual gravity anomalies close to zero in larger data gaps, and thus the final quasigeoid model essentially uses the underlying global model values, i.e. EGM96).

Comparison of Global Geopotential Models with GPS/Leveling Data for Europe

The global geopotential models discussed above were also compared with a number of GPS/leveling data sets available at IfE. In the following we will discuss only the results for the European GPS traverse, running essentially in north south direction from Austria to North Norway. The traverse has a length of about 3000 km (see *Torge et al.* 1989), and thus furnishes a large-scale evaluation of the global models. A statistics of the differences between the GPS/leveling data and different global models is given in table 23. A graphical representation of the differences is provided for selected comparisons in figures 10 and 11. The differences were always computed in the sense GPS/leveling minus model values. The comparisons were done using a bias fit as well as a bias+tilt fit in order to account for inaccuracies in the absolute positioning and for long wavelength errors of all data sets involved (GPS, leveling, quasigeoid). The GPS/leveling data were available in the form of ellipsoidal and normal heights, and the geopotential model values were computed as height anomalies at the surface of the Earth. Furthermore, the GPS/leveling data were also compared with quasigeoid undulations resulting from the European Geoid Project. The zero order terms were again neglected in all computations.

From table 23 it can be seen that for the bias fit the rms values of the differences vary from 0.289 - 0.324 m for the EGM models, whereas the corresponding values for the JGM3_OSU91A and OSU91A models are 0.444 m and 0.318 m, respectively. For the bias+tilt fit the rms differences range from 0.251 - 0.290 m for the EGM models, while for the JGM3_OSU91A and OSU91A models rms values of 0.402 m and 0.260 m were obtained. The adjusted tilt parameters are small in north south direction (max. 0.2 ppm). In west east direction larger values up to 1.3 ppm were found for the EGM models, while values up to 2.5 ppm showed up for the JGM3_OSU91A and OSU91A models. However, due to the small extension of the traverse in west east direction these values are in most cases hardly significant. For the bias+tilt fit the EGM96 model gives the smallest rms difference of all geopotential models of 0.251 m, and the adjusted tilt parameters are -0.20 ppm in north south direction and -0.46 ppm in west east direction (the west east tilt parameter is smaller than for the other EGM models). The poorest rms differences for the bias as well as the bias+tilt

fit were obtained for the JGM3_OSU91A model, indicating that this blended geopotential model has problems in Europe.

From the plot of the differences shown in figures 10 and 11 one can again observe a significant positive discrepancy over the Alps in all geopotential model comparisons. However, for the EGM models this positive discrepancy is more pronounced than for the JGM3_OSU91A and the OSU91A model. As similar effects were already seen in the comparisons of the geopotential models with the gravity and height anomalies from the European Geoid Project, this subject should be further investigated (also in other mountainous regions of the world).

In the comparisons of the GPS/leveling data with the high resolution European quasigeoid models (EGG.96.01 to EGG.97.03) we again observe very long wavelength discrepancies, which have a similar structure than the differences with the global models. However, in this connection one has to consider also that the GPS data are from 1986 and 1987 with a limited GPS satellite constellation and less sophisticated data processing techniques. Nevertheless, when also considering other more up-to-date GPS/leveling data sets not discussed here, it can be stated that the inaccuracies of the global models and long wavelength errors of the terrestrial gravity data still represent the major limiting factor in large-scale geoid/quasigeoid calculations. A significant improvement of this situation can only be expected through a dedicated satellite gravity field mission.

Conclusions

The NASA GSFC and NIMA joint geopotential model EGM96 and the precursor models EGM-X01 to EGM-X05 as well as the models JGM3_OSU91A and OSU91A were evaluated using high resolution European gravity data, quasigeoid models, and GPS/leveling control points. When comparing the global geopotential models with the European gravity and height anomaly data sets, the EGM models yield significantly smaller standard deviations of the differences than the older models JGM3_OSU91A and OSU91A. Most of this improvement is coming from new data sets used in the EGM models, especially over East Europe, etc, which were not available for the older models (OSU91A, JGM3_OSU91A). In the gravity and height anomaly comparisons, in most cases the EGM96 and EGM-X05 models yield slightly smaller standard deviations of the differences than the other EGM models. However, also significant discrepancies exist between the EGM96 model and the European data sets over parts of Southeast Europe and in the mountainous regions of the Alps and South Norway. The differences over the mountainous regions appear to be correlated with elevation (especially over the Alps), and are also seen in the GPS/leveling comparisons. Such effects are not seen to this extent in the OSU91A and JGM3_OSU91A models. Therefore this subject should be further investigated. In the comparisons of the geopotential models with the GPS/leveling data from the European GPS traverse the EGM96 model yields the smallest standard deviation of the differences when doing a bias+tilt fit, while the JGM3_OSU91A model gives the poorest results for the bias as well as the bias+tilt fit. To sum up it can be said that the EGM96 model provides the best gravity field representation over entire Europe available to date.

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Table 1: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM-X01}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.36	.56	.18	.60	.08	.04
Std.dev.	13.60	13.62	13.57	12.83	14.61	13.47
Rms	13.60	13.63	13.57	12.85	14.61	13.47
Min.	-124.99	-105.19	-124.99	-105.19	-124.99	-124.99
Max.	+239.25	+163.03	+239.25	+163.03	+239.25	+239.25

Table 2: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM-X02}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.39	.56	.23	.59	.14	.08
Std.dev.	13.65	13.69	13.61	12.81	14.47	13.41
Rms	13.65	13.70	13.61	12.82	14.47	13.41
Min.	-125.08	-105.77	-125.08	-105.77	-125.08	-125.08
Max.	+233.25	+158.89	+233.25	+158.89	+233.25	+233.25

Table 3: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM-X03}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.37	.60	.16	.61	.04	.05
Std.dev.	13.63	13.69	13.58	12.86	14.59	13.45
Rms	13.64	13.70	13.58	12.87	14.59	13.45
Min.	-124.60	-104.68	-124.60	-104.68	-124.60	-124.60
Max.	+231.74	+162.87	+231.74	+162.87	+231.74	+231.74

Table 4: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM-X04}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.38	.56	.20	.58	.09	.09
Std.dev.	13.62	13.70	13.54	12.78	14.52	13.40
Rms	13.62	13.71	13.54	12.79	14.52	13.40
Min.	-123.99	-105.09	-123.99	-105.09	-123.99	-123.99
Max.	+231.37	+156.10	+231.37	+156.10	+231.37	+231.36

Table 5: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM-X05}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.37	.49	.25	.52	.17	.15
Std.dev.	13.65	13.79	13.53	12.75	14.48	13.37
Rms	13.66	13.79	13.53	12.76	14.48	13.37
Min.	-122.16	-107.30	-122.16	-107.30	-122.16	-122.16
Max.	+227.92	+154.60	+227.92	+154.60	+227.92	+227.92

Table 6: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{EGM96}}$. Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.41	.49	.33	.51	.20	.16
Std.dev.	13.71	13.79	13.64	12.69	14.44	13.40
Rms	13.72	13.80	13.65	12.70	14.44	13.40
Min.	-177.24	-107.23	-177.24	-107.23	-177.24	-177.24
Max.	+226.72	+159.88	+226.72	+159.88	+226.72	+226.72

Table 7: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{JGM3 OSU91A}}$ Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.27	.60	-.04	.84	-.07	-.01
Std.dev.	15.97	17.10	14.84	19.27	16.97	15.31
Rms	15.98	17.11	14.84	19.29	16.97	15.31
Min.	-150.15	-132.14	-150.15	-132.14	-150.15	-150.15
Max.	+233.60	+233.60	+230.00	+233.60	+230.00	+230.00

Table 8: $\Delta g_{\text{EGG.96.01}} - \Delta g_{\text{OSU91A}}$ Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,582	5,573,243
Mean	.40	.91	-.08	1.10	-.09	-.09
Std.dev.	15.89	16.91	14.87	19.17	17.07	15.37
Rms	15.90	16.93	14.87	19.20	17.07	15.37
Min.	-147.87	-129.88	-147.87	-129.88	-147.87	-147.87
Max.	+240.63	+240.63	+231.29	+240.63	+231.29	+231.29

Table 9: $\zeta_{\text{EGG.96.01}} - \zeta_{\text{EGM-X01}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.005	.001	.008	-.013	-.014	-.003
Std.dev.	.759	.903	.576	.603	.558	.482
Rms	.759	.903	.576	.603	.558	.482
Min.	-5.595	-5.495	-5.595	-4.839	-4.862	-4.862
Max.	+8.230	+6.859	+8.230	+4.178	+5.289	+5.289

Table 10: $\zeta_{\text{EGG.96.01}} - \zeta_{\text{EGM-X02}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.007	.001	.014	-.004	-.010	-.004
Std.dev.	.792	.925	.628	.612	.564	.513
Rms	.792	.925	.628	.612	.564	.513
Min.	-5.112	-4.988	-5.112	-4.611	-4.635	-4.635
Max.	+8.844	+6.587	+8.844	+4.281	+5.697	+5.697

Table 11: $\zeta_{\text{EGG.96.01}} - \zeta_{\text{EGM-X03}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.006	.004	.009	-.004	-.017	.000
Std.dev.	.771	.923	.574	.609	.555	.478
Rms	.771	.923	.574	.609	.555	.478
Min.	-5.104	-5.015	-5.104	-4.781	-4.804	-4.804
Max.	+8.250	+6.701	+8.250	+4.345	+5.291	+5.291

Table 12: $\zeta_{\text{EGG},96.01} - \zeta_{\text{EGM-X04}}$. Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.006	.003	.010	-.006	-.015	.002
Std.dev.	.771	.924	.573	.607	.552	.476
Rms	.771	.924	.573	.607	.552	.476
Min.	-5.330	-5.229	-5.330	-4.812	-4.837	-4.837
Max.	+8.257	+6.776	+8.257	+4.190	+5.372	+5.372

Table 13: $\zeta_{\text{EGG},96.01} - \zeta_{\text{EGM-X05}}$. Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.006	-.001	.013	-.009	-.011	.004
Std.dev.	.771	.925	.573	.604	.549	.474
Rms	.771	.925	.573	.604	.549	.474
Min.	-5.465	-5.361	-5.465	-4.782	-4.814	-4.814
Max.	+8.266	+6.813	+8.266	+3.988	+5.440	+5.440

Table 14: $\zeta_{\text{EGG},96.01} - \zeta_{\text{EGM96}}$. Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.009	-.010	.028	-.015	-.005	.007
Std.dev.	.811	.964	.616	.590	.551	.485
Rms	.811	.964	.617	.590	.551	.485
Min.	-5.249	-5.131	-5.249	-4.521	-4.556	-4.556
Max.	+9.272	+6.491	+9.272	+3.810	+6.050	+6.050

Table 15: $\zeta_{\text{EGG},96.01} - \zeta_{\text{JGM3_OSU91A}}$. Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	-.012	-.006	-.018	-.003	-.029	-.007
Std.dev.	.868	1.052	.627	1.129	.721	.617
Rms	.868	1.052	.628	1.129	.722	.617
Min.	-5.264	-5.234	-5.264	-5.234	-5.264	-5.264
Max.	+8.516	+8.516	+4.698	+8.516	+4.698	+4.698

Table 16: $\zeta_{\text{EGG},96.01} - \zeta_{\text{OSU91A}}$. Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,582	5,573,243
Mean	.002	.018	-.014	.044	.006	-.004
Std.dev.	.785	.915	.625	1.077	.772	.647
Rms	.785	.916	.625	1.078	.772	.647
Min.	-5.189	-5.189	-4.414	-5.189	-4.414	-4.414
Max.	+9.539	+9.539	+5.537	+9.539	+5.537	+5.537

Table 17: $\Delta g_{\text{EGG.96.03}} - \Delta g_{\text{EGM-X05}}$ Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,054	5,573,243
Mean	.39	.57	.23	.52	.05	.06
Std.dev.	13.58	13.76	13.42	12.75	14.42	13.33
Rms	13.59	13.77	13.42	12.76	14.42	13.33
Min.	-122.15	-107.80	-122.15	-107.30	-122.15	-122.15
Max.	+228.02	+154.60	+228.02	+154.60	+228.02	+228.02

Table 18: $\Delta g_{\text{EGG.96.03}} - \Delta g_{\text{EGM96}}$ Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,054	5,573,243
Mean	.43	.56	.31	.51	.08	.08
Std.dev.	13.62	13.73	13.51	12.69	14.37	13.35
Rms	13.63	13.74	13.51	12.70	14.37	13.35
Min.	-177.22	-107.23	-177.22	-107.23	-177.22	-177.22
Max.	+226.82	+159.90	+226.82	+159.90	+226.82	+226.82

Table 19: $\zeta_{\text{EGG.96.03}} - \zeta_{\text{EGM-X05}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,054	5,573,243
Mean	.016	-.010	.042	.000	.026	.028
Std.dev.	.691	.826	.516	.548	.525	.454
Rms	.691	.826	.517	.548	.526	.455
Min.	-5.007	-4.947	-5.007	-4.287	-4.319	-4.319
Max.	+6.987	+6.852	+6.987	+4.071	+4.672	+4.672

Table 20: $\zeta_{\text{EGG.96.03}} - \zeta_{\text{EGM96}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,054	5,573,243
Mean	.019	-.019	.058	-.006	.031	.031
Std.dev.	.694	.818	.536	.516	.500	.441
Rms	.694	.818	.540	.516	.501	.442
Min.	-4.787	-4.717	-4.787	-4.026	-4.181	-4.181
Max.	+7.939	+6.502	+7.939	+3.811	+5.281	+5.281

Table 21: $\Delta g_{\text{EGG.97.03}} - \Delta g_{\text{EGM96}}$ Units are mgal.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,121,920	5,844,549	6,277,371	4,373,171	3,587,054	5,728,901
Mean	.27	.45	.11	.52	.08	.09
Std.dev.	11.76	11.34	12.12	12.65	14.10	12.55
Rms	11.76	11.35	12.12	12.67	14.10	12.55
Min.	-172.04	-107.28	-172.04	-107.28	-172.04	-172.04
Max.	+215.56	+159.55	+215.56	+159.55	+215.56	+215.56

Table 22: $\zeta_{\text{EGG.97.03}} - \zeta_{\text{EGM96}}$ Units are m.

Parameter	All	Land (All)	Sea (All)	Land ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ terrestrial}} < 10 \text{ mgal}$)	Sea ($s_{\Delta g \text{ merged}} < 10 \text{ mgal}$)
#	12,779,520	6,445,738	6,333,782	4,373,171	3,587,054	5,728,901
Mean	.001	-.009	.011	.005	.008	.013
Std.dev.	.419	.462	.370	.505	.430	.378
Rms	.419	.462	.370	.505	.430	.379
Min.	-4.226	-4.226	-4.225	-4.226	-4.225	-4.225
Max.	+4.491	+4.491	+2.960	+4.491	+2.960	+2.960

Table 23: Comparison of GPS/Leveling Data From the European GPS Traverse With Different Global Geopotential Models and European Quasigeoid Solutions Developed at IfE.

Geopotential Model	Rms [m]	Min. [m]	Max. [m]	Bias [m]	NS tilt [ppm]	EW tilt [ppm]
EGM-X01	0.324 0.290	-0.643 -0.553	+0.983 +1.125	+0.082 +0.082	- -0.100 $\pm$ 0.074	- -0.824 $\pm$ 0.495
EGM-X02	0.320 0.259	-0.594 -0.449	+0.942 +1.091	+0.119 +0.119	- -0.097 $\pm$ 0.066	- -1.333 $\pm$ 0.442
EGM-X03	0.289 0.258	-0.555 -0.482	+1.154 +1.212	+0.086 +0.086	- -0.019 $\pm$ 0.066	- -1.194 $\pm$ 0.439
EGM-X04	0.293 0.261	-0.523 -0.452	+1.173 +1.227	+0.089 +0.089	- -0.014 $\pm$ 0.067	- -1.240 $\pm$ 0.444
EGM-X05	0.296 0.263	-0.561 -0.493	+1.183 +1.231	+0.088 +0.088	- -0.007 $\pm$ 0.067	- -1.296 $\pm$ 0.449
EGM96	0.304 0.251	-0.805 -0.615	+0.875 +1.114	+0.088 +0.088	- -0.200 $\pm$ 0.064	- -0.458 $\pm$ 0.427
JGM3/ OSU91A	0.444 0.402	-1.111 -1.208	+0.824 +0.696	+0.128 +0.128	- +0.092 $\pm$ 0.103	- -2.214 $\pm$ 0.685
OSU91A	0.318 0.260	-0.732 -0.687	+0.922 +0.792	+0.377 +0.377	- +0.282 $\pm$ 0.066	- -2.532 $\pm$ 0.443
EGG9601	0.283 0.201	-0.350 -0.302	+0.675 +0.508	+0.418 +0.418	- -0.211 $\pm$ 0.051	- -0.704 $\pm$ 0.343
EGG9603	0.300 0.156	-0.435 -0.203	+0.835 +0.432	+0.251 +0.251	- -0.297 $\pm$ 0.040	- -0.690 $\pm$ 0.266
EGG9703	0.287 0.174	-0.409 -0.228	+0.778 +0.465	+0.223 +0.223	- -0.263 $\pm$ 0.044	- -0.635 $\pm$ 0.296

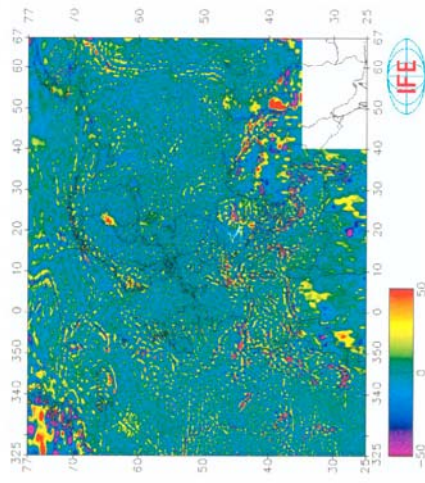


Fig. 2: Gravity Anomaly Differences Minus EGM-X05. Units are mgal.

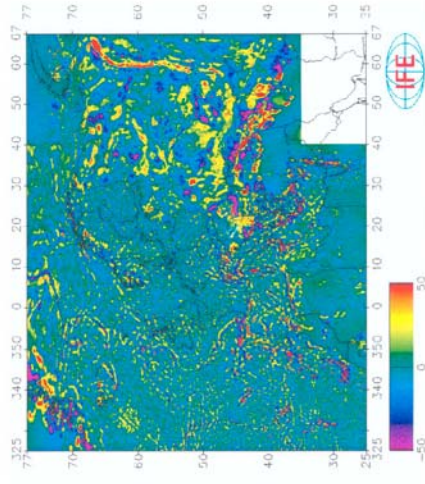


Fig. 4: Gravity Anomaly Differences Minus JGM3-OSU91A. Units are mgal.

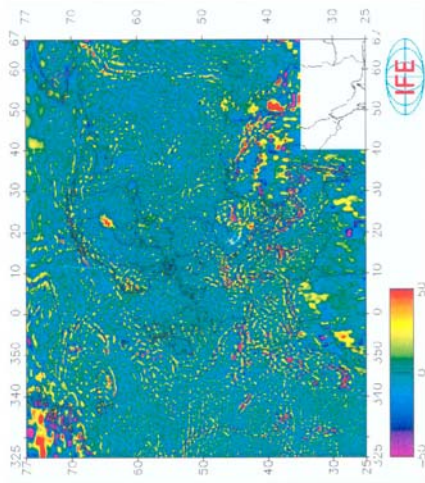


Fig. 6: Gravity Anomaly Differences Minus EGM-X05. Units are mgal.

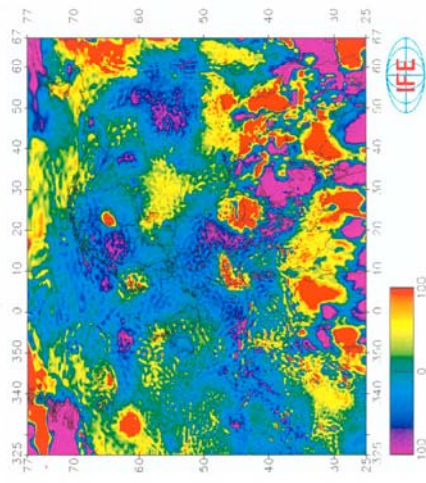


Fig. 3: Height Anomaly Differences Minus EGM-X05. Units are cm.

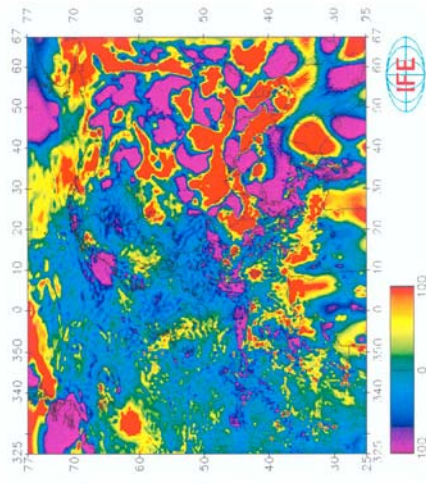


Fig. 5: Height Anomaly Differences Minus JGM3-OSU91A. Units are cm.

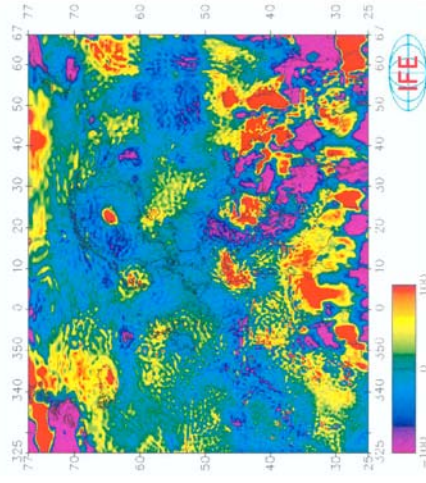


Fig. 7: Height Anomaly Differences Minus EGM-X05. Units are cm.

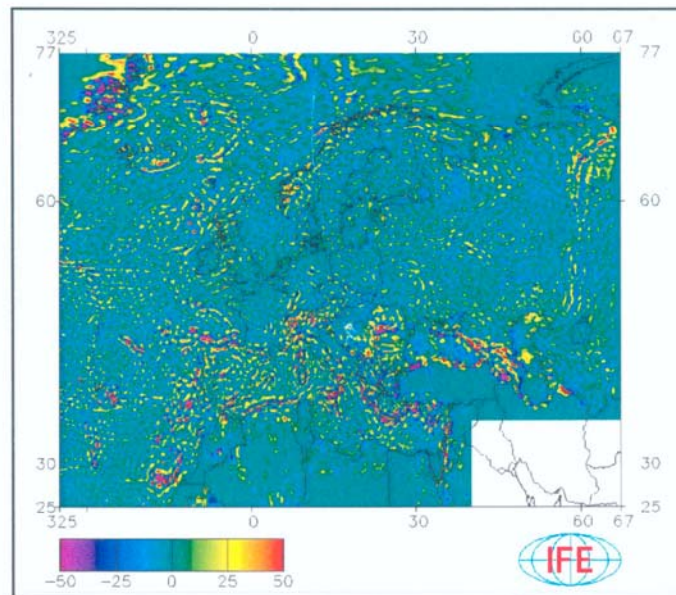


Fig. 8: Gravity Anomaly Differences EGG.97.03 Minus EGM96. Units are mgal.

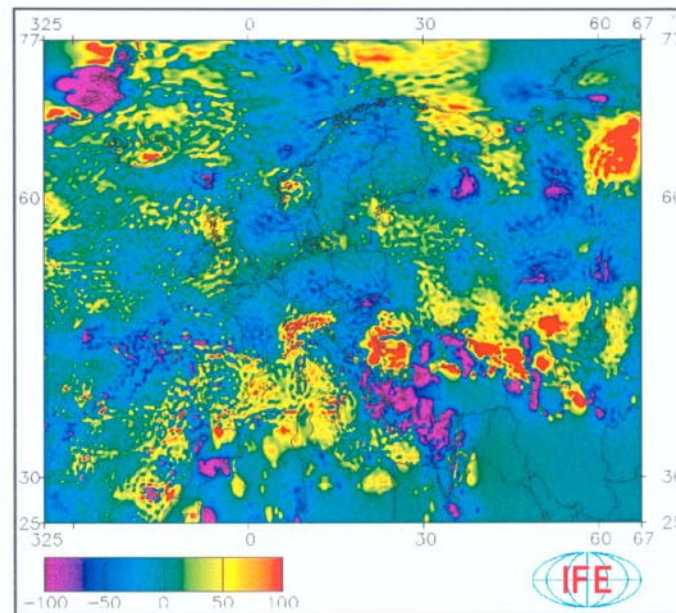


Fig. 9: Height Anomaly Differences EGG.97.03 Minus EGM96. Units are cm.

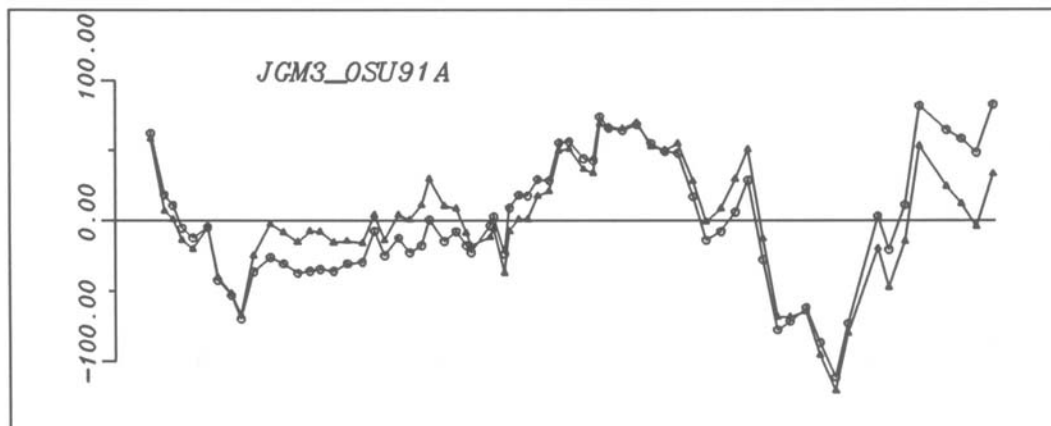
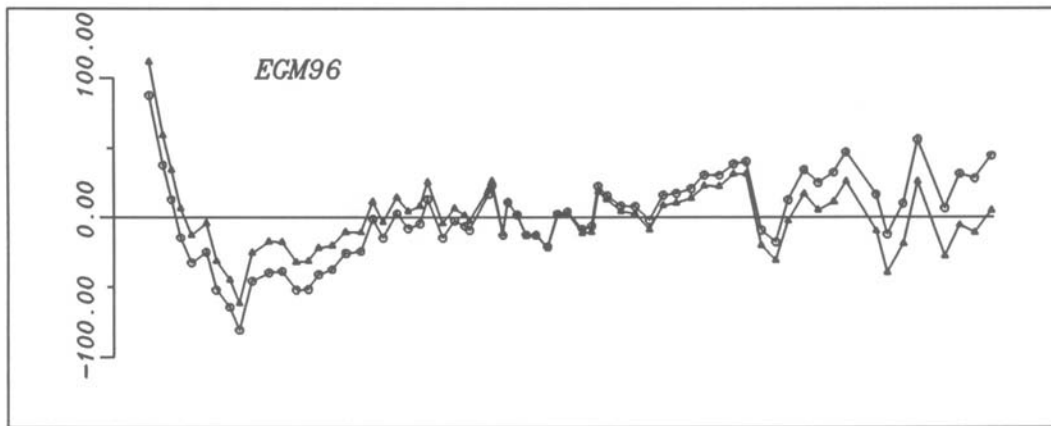
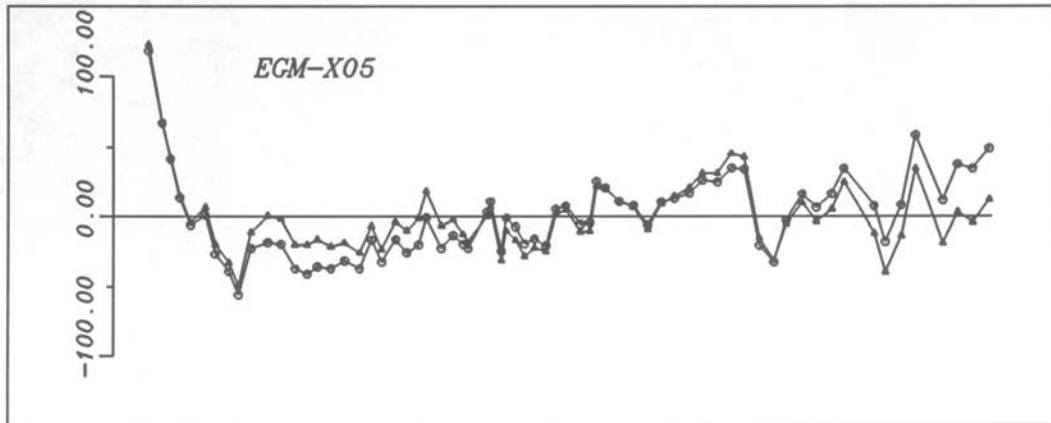


Fig. 10: Comparison of GPS/Leveling Data From the European GPS Traverse With the Geopotential Models EGM-X05, EGM96 and JGM3_OSU91A. Results are shown for the Bias Fit (Circles) as well as for the Bias+Tilt Fit (Triangles). The Differences are Given in the Sense GPS/Leveling Minus Model Values. The Southernmost Station is on the Left Side. The Units are cm.

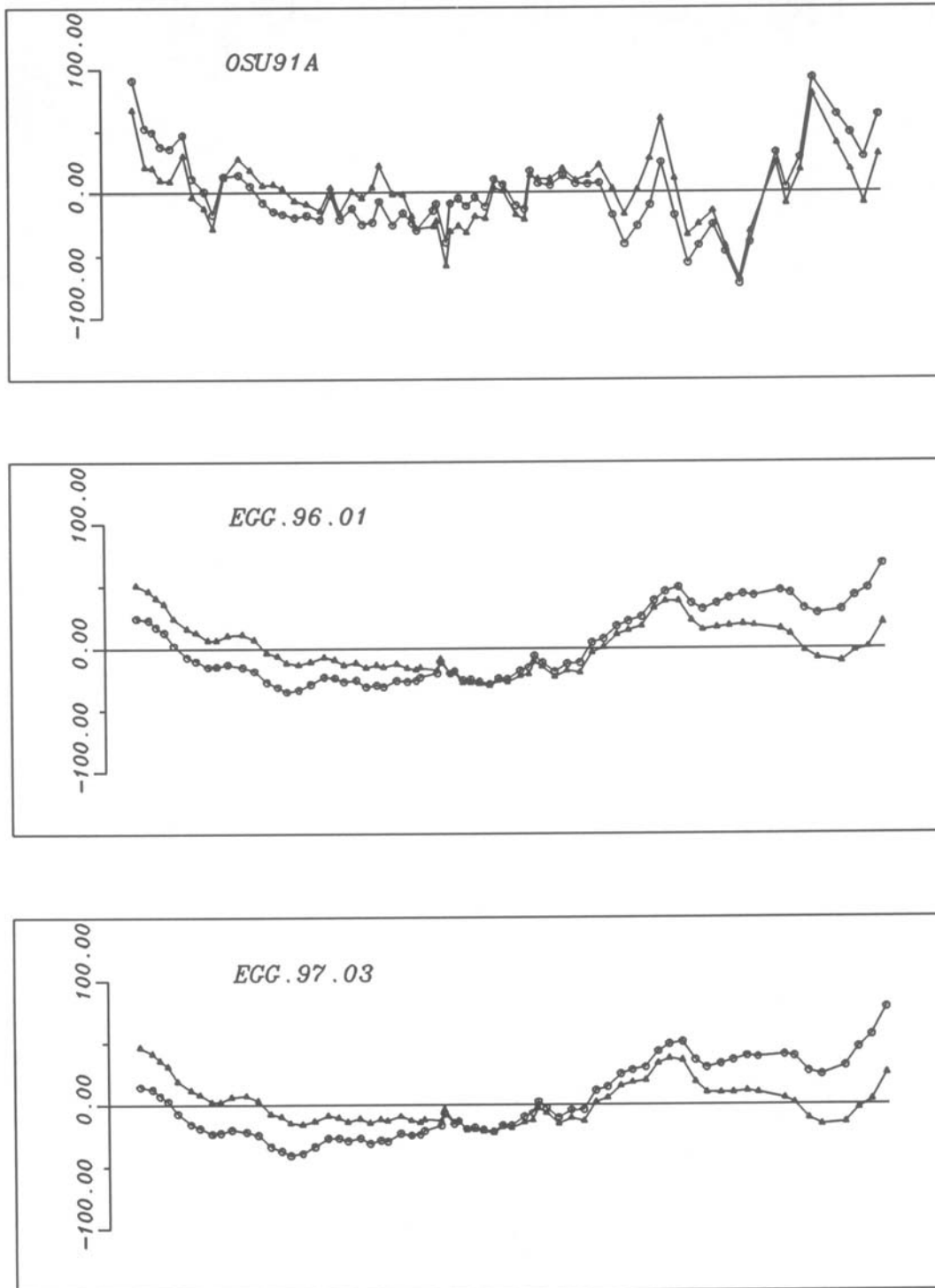


Fig. 11: Comparison of GPS/Leveling Data From the European GPS Traverse With the Geopotential Model OSU91A and the European Quasigeoid Models EGG.96.01 and EGG.97.03. Results are shown for the Bias Fit (Circles) as well as for the Bias+Tilt Fit (Triangles). The Differences are Given in the Sense GPS/Leveling Minus Model Values. The Southernmost Station is on the Left Side. The Units are cm.

Comparison of recent geopotential models with surface, airborne and satellite data in different areas of the earth

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Introduction

The OSU91A geopotential model was for long time the most accurate reference model for many applications in geodesy and other geosciences. The growing accuracy of the satellite born data such as satellite altimeter data calls for more accurate geopotential models. Recently an attempt was set up for a new accurate geopotential model by DMA and NASA/Goddard Space Flight Centre. In the frame of this project five versions of a new geopotential model have been created. A Special Working Group has been established in order to test the quality of these versions and to help the decision for the final, most accurate geopotential model.

The aim of this work was to contribute in the frame of the SWG, in the assessment of the new model by comparing quantities computed from the model with surface, airborne and satellite data in different areas of the Earth. The results of these comparisons showed a clear improvement of the new versions in terms of the mean value and the standard deviation of the reduced observations against OSU91A at least in some test areas. No significant differences between the different versions of EGM were found. It seems that in some areas, EGM-X02 or EGM-X05 are slightly better than the other versions. After communicating the results of the contributions of the SWG members, a final geopotential model was created, named EGM96. This model gives slightly better results than the preliminary EGM versions in the most of the test fields examined in this study.

In the next sections the data used for the comparisons will be described and the results of the comparison will be shown. The results concern the mean value and standard deviation of the original data as well as of the differences between the original data and the same data after the removal of the contribution of OSU91A, the various preliminary versions of EGM and the final EGM96 geopotential model. The empirical covariance functions of free air gravity anomalies referred to OSU91A and to EGM are given for some test areas where the EGM and OSU91A present different behaviour.

Finally, a comparison of the anomaly error degree variances of OSU91A and EGM96 shows that the error estimation of the coefficients of the new model seems to be optimistic especially in the higher degrees.

Data

1. Airborne data

A set of 5,292 point free-air gravity anomalies covering the main part of Greenland bounded by $52.25^\circ \leq \phi \leq 73.68^\circ$, $-58.32^\circ \leq \lambda \leq 20.25^\circ$ is available from (Forsberg and Brozena, 1992).

The measuring height was about 4 km. These values are mean values along the plain tracks with a sampling rate corresponding to about 18 arcmin.

2. Surface data

Arctic zone

Searching the GEODAS CD-ROM (GEODAS-Marine Geological and Geophysical Data from NGDC, published by NOAA.) we have found out that the area bounded by the limits $70^{\circ} \leq \phi \leq 83^{\circ}$, $-10^{\circ} \leq \lambda \leq 20^{\circ}$ is covered by a number of 29,760 point gravity data coming from different Institutions and different surveys. For the main part of these data there is the information that the Gravity System is Potsdam and the reference system is International formula 1930. We assume that also in the cases that this information is missing, the gravity system and the reference gravity field are the same (Potsdam/Int.Form. 1930). However this assumption could introduce compatibility problem when merging all data sources. No information concerning the accuracy of the data is available.

Scandinavia

Two data sets were used for the comparison:

Deflections of the vertical in Scandinavia (300 points) and 13,802 free-air gravity anomalies in the Norwegian Sea, bounded by $61^{\circ} \leq \phi \leq 70^{\circ}$, $0^{\circ} \leq \lambda \leq 17^{\circ}$

The deflections of the vertical have been converted to WGS84 using a 7 parameter datum-shift with errors of 0.1 to 0.2 arcsec. The gravity anomalies are commercial data, with a zero-level uncertainty of 1 - 2 mGal and a internal consistency of about 1 - 2 mGal.

Antarctic zone

From the Gravity Earth System Data CD-ROM (Alpha Release), published by NOAA, March 1992, a file including 57,140 point free air gravity anomalies in the area bounded by $-90^{\circ} \leq \phi \leq -50^{\circ}$, $-180^{\circ} \leq \lambda \leq 180^{\circ}$ was extracted. Error estimates are given for each point, that varying between 0 and 25 mGal (Arabelos and Tscherning, 1996).

Mediterranean

Gridded $5' \times 5'$ free air gravity anomalies have been used in Western and Central Mediterranean. Based on a number of previous studies, the accuracy of this data set is estimated to about 5 mGal (see Arabelos and Tziavos, 1996). In the Eastern Mediterranean, point free air gravity anomalies along ship tracks have been used (see e.g. Arabelos and Tscherning 1988). For the continental Greece, point gravity anomalies have been used, having a mean separation equal to about 5 km. The accuracy estimation of these data based on comparisons between values referred to the same station is equal to about 0.3 mGal (see Arabelos, 1989).

Taiwan

A $3' \times 3'$ grid of free air gravity anomalies covering Taiwan has been used (see Tsuei et al. 1994, 1995).

3. Satellite data

Satellite data from ERS-1/ERM and ERS-1/GM Cycle I have been used for the comparisons in the Mediterranean Sea (see Arabelos and Vermeer, 1996). The time covered by the ERS-1/ERM data set is April 14, 1992 to August 16, 1993 (14 repeat periods, 35 days each, GEM-T2 model for orbit determination).

The ERS-1/GM data contained 69,384 records. The two data sets were selected in the area bounded by the limits $30^\circ \leq \phi \leq 46^\circ$, $-5^\circ \leq \lambda \leq 36^\circ$.

Results

The results in terms of the mean value and standard deviation of the original observations and of the differences between the observations and the corresponding quantities from OSU91A, the EGM-X01,...,EGM-X05 and EGM96 are given in the next tables (1 to 9). All the original data are referred to GRS80. The computations were carried out using the FORTRAN programs GEOCOL and GEOIP (Tscherning et al., 1994).

Table 1. Results of the reduction of gravity anomalies using OSU91A and EGM geopotential models in the Mediterranean Sea. Unit is (mGal).

Western Mediterranean (8,390 5'x 5' grid)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	-1.90	-0.47	-0.17	-0.44	-0.78	-0.50	-0.21	0.34
Std. dev.	24.95	16.22	16.03	15.96	15.97	15.98	16.07	16.07
Central Mediterranean (15,062 5'x 5' grid)								
Differences between original and:								
Original	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	-10.77	-0.16	-0.96	-0.81	-0.91	-0.69	-0.71	-0.90
Std. dev.	50.42	18.15	16.75	16.71	16.81	16.82	16.81	16.62
Eastern Mediterranean (2,085 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	-40.63	-1.75	-1.91	0.36	-1.27	-0.87	-0.73	0.95
Std. dev.	54.05	18.32	19.13	18.73	19.04	19.10	19.01	18.84
Continental Greece (7,504 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	20.35	-3.49	-4.59	-4.90	-4.86	-5.04	-5.27	-5.13
Std. dev.	52.52	33.53	33.33	33.55	33.36	33.31	33.36	33.31

From the statistics of Table 1 it is shown that almost similar results we obtain using OSU91A or EGM models in the Western and Eastern Mediterranean. In the Central Mediterranean EGM models seems to be better than OSU91A: The standard deviation of the observations reduced to

EGM models is about 2 mGal lower than the corresponding quantity of the observations reduced to OSU91A.

In continental Greece the results are very similar using OSU91A or EGM models for the reduction of the free air gravity anomalies.

The results concerning the altimeter data in the Mediterranean Sea are shown in Table 2. From this Table it is clear that in the case of ERS-1/ERM the results in terms of the mean value and standard deviation of the differences (observations - model) are almost identical for both, OSU91A and EGM models, while in the case of ERS-1/GM we have better results using the EGM models.

Table 2. Results of the reduction of altimeter data using OSU91A and EGM geopotential models, in the Mediterranean (entire area). The geopotential component has been computed in a $5' \times 5'$ grid and the individual points have been interpolated using GEOIP program. Unit is (m).

Mediterranean (ERS-1/GM, 65,909)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	32.53	0.41	0.27	0.27	0.29	0.30	0.31	0.37
Std. dev.	13.37	1.12	0.94	0.90	0.95	0.95	0.95	0.90
Mediterranean (ERS-1/ERM, 95,288)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	32.31	0.95	0.92	0.96	0.96	0.97	0.98	1.04
Std. dev.	12.88	3.13	3.12	3.14	3.15	3.15	3.15	3.11

The empirical covariance function of the free air gravity anomalies in Greenland reduced to OSU91A and EGM-X05 are plotted in Fig. 1. From Table 3 as well as from Fig. 1 it is obvious that the airborne data used in this test have been used for the computation of the EGM models. This explains the considerable decrease of the mean value and standard deviation of the reduced to EGM free air anomalies comparing to the corresponding quantities (mean and standard deviation) of the reduced to OSU91A data.

Table 3. Results of the reduction of airborne gravity anomalies using OSU91A and EGM geopotential models, in Greenland. The geopotential component has been computed at the height of each measurement (about 4,000 m). Unit is (mGal).

Greenland (5,292 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	25.11	-2.49	-0.06	-0.12	-0.06	-0.08	-0.10	-0.11
Std. dev.	35.34	27.85	13.53	13.65	13.54	13.50	13.44	13.45

From Table 4 it is shown that in Canada the standard deviation of the free air gravity anomalies reduced to EGM models is slightly higher than the corresponding of the same data reduced to

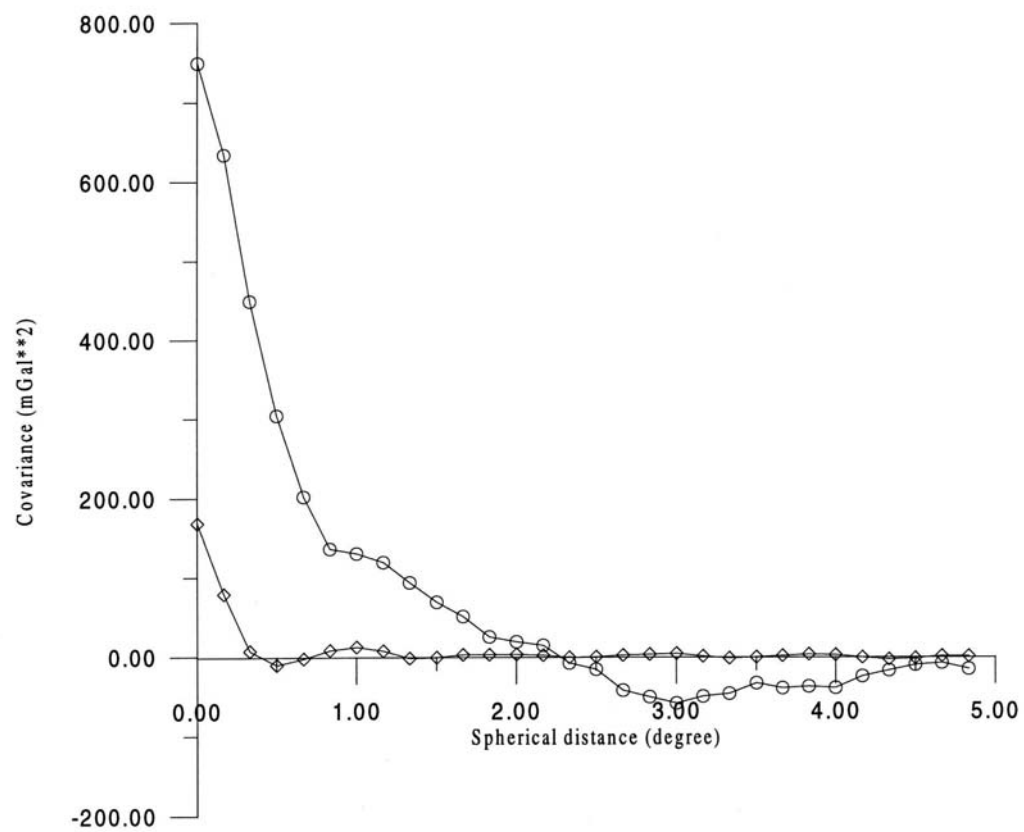


Figure 1. Empirical covariance functions of air borne free air gravity anomalies in Greenland reduced to: OSU91 (circles) and to EGM-X05 (squares) geopotential models

OSU91A. Concerning the mean value, the EGM models fit better the gravity field in this area. The empirical covariance functions of the free air gravity anomalies in Canada reduced to OSU91A and EGM-X05 are plotted in Fig. 2. Considering Fig. 2 no significant differences between the two empirical covariance functions are shown.

Table 4. Results of the reduction of gravity using OSU91A and EGM geopotential models, in Canada. The geopotential component has been computed on a $3' \times 3'$ grid and the individual points have been interpolated using GEOIP program. Unit is (mGal).

Canada (14,177 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	-10.77	0.95	-0.10	-0.23	-0.18	-0.20	-0.23	-0.27
Std. dev.	22.42	12.68	13.41	13.39	13.38	13.34	13.31	13.40

In the case of the Arctic zone (Table 5) we have a decrease of the mean value and of the standard deviation of the reduced to EGM gravity anomalies at the level of 2 mGal with respect to the corresponding mean value and standard deviation of the reduced to OSU91A data.

Table 5. Results of the reduction of gravity using OSU91A and EGM geopotential models, in the Arctic zone. The geopotential component has been computed in a $3' \times 3'$ grid and the individual points have been interpolated using GEOIP program. Unit is (mGal).

Arctic (29,760 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	34.99	-2.80	-0.84	-0.77	-0.75	-0.77	-0.39	-0.38
Std. dev.	24.94	22.22	20.62	20.51	20.57	20.57	20.32	20.43

In the Antarctic zone (Table 6) no significant differences in terms of mean value and standard deviation of the reduced to OSU91A or to EGM models are observed.

Table 6. Results of the reduction of gravity anomalies using OSU91A and EGM geopotential models, in the Antarctic zone. The geopotential component has been computed in 57,140 individual points. Unit is (mGal).

Antarctic (57,140 point values)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	6.14	4.37	4.40	4.36	4.36	4.34	4.17	4.10
Std. dev.	36.72	22.73	21.98	22.02	22.00	21.94	21.99	22.05

Also in Taiwan (Table 7) we have a decrease of the mean value and of the standard deviation of the reduced to EGM gravity anomalies at the level of 2 mGal with respect to the corresponding mean value and standard deviation of the reduced to OSU91A data.

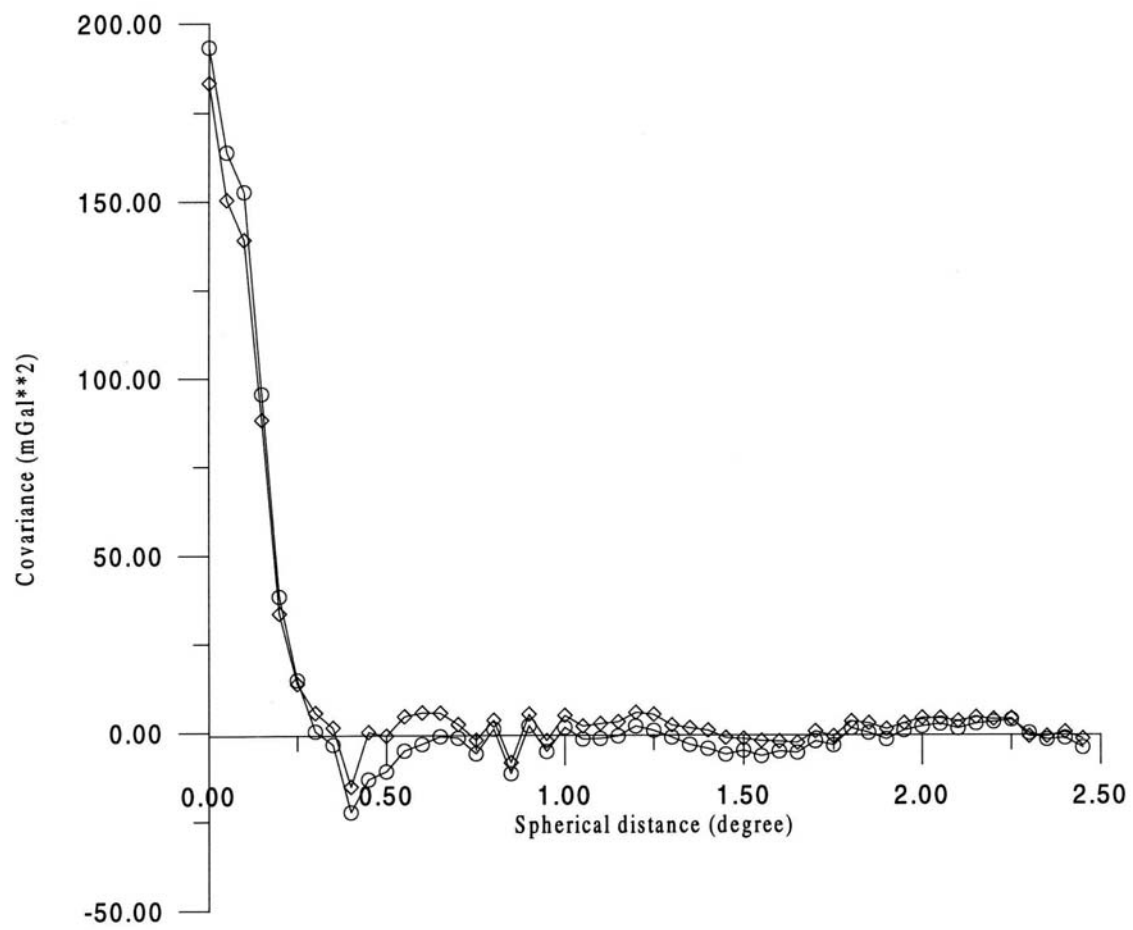


Figure 2. Empirical covariance functions of point free air gravity anomalies in Canada reduced to: OSU91 (cycles) and to EGM-X05 (squares) geopotential models

Table 7. Results of the reduction of gravity anomalies using OSU91A and EGM geopotential models in Taiwan. Unit is (mGal).

Taiwan (4,800 values 3' × 3' grid)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	15.25	1.21	-0.41	-0.79	-0.53	-0.10	-0.04	0.23
Std. dev.	70.13	39.32	37.38	37.19	37.38	37.12	36.82	36.78

The empirical covariance functions of the free air gravity anomalies in Taiwan reduced to OSU91A and EGM-X05 are plotted in Fig. 3. In this figure it is shown that the main features of both covariance functions are almost the same.

Table 8a. Comparison of North-component (ξ) of the deflection of the vertical with values computed from the EGM's in Scandinavia. Units arcsec.

Scandinavia (300 points)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	E°M96
Mean value	-2.56	-0.32	-	-0.29	-0.28	-0.27	-0.25	-0.28
Std. dev.	3.92	2.53	-	2.56	2.55	2.55	2.55	2.52

From Tables 8a and 8b it is shown that in Scandinavian we have no significant improvement in terms of mean value and standard deviation of the reduced to EGM deflections of the vertical in comparison to the mean value and standard deviation of the reduced to OSU91A values.

Table 8b. Comparison of East-component (η) of the deflection of the vertical with values computed from the EGM's in Scandinavia. Unit is arcsec.

Scandinavia (300 points)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	EGM96
Mean value	2.99	-0.19	-	-0.37	-0.35	-0.35	-0.36	-0.33
Std. dev.	4.80	2.74	-	2.61	2.60	2.60	2.59	2.54

Finally, as it is shown in Table 9, we have an improvement at the level of 1 mGal in terms of the standard deviation of the reduced to EGM gravity anomalies in the Norwegian Sea (comparing to the sd of the same anomalies reduced to OSU91A), but the mean value in the case of EGM03, EGM04 and EGM05 was increased by 1.5 mGal.

Table 9. Comparison of free-gravity anomalies with values computed from the EGM's in the Norwegian Sea. Unit is mGal.

Norwegian Sea(13,802 points)								
Differences between original and:								
	Original	OSU91A	EGM01	EGM02	EGM03	EGM04	EGM05	E°M96
Mean value	20.14	1.3	-	0.6	2.8	2.7	2.6	2.6
Std. dev.	22.56	13.0	-	12.0	12.0	11.9	11.9	11.9

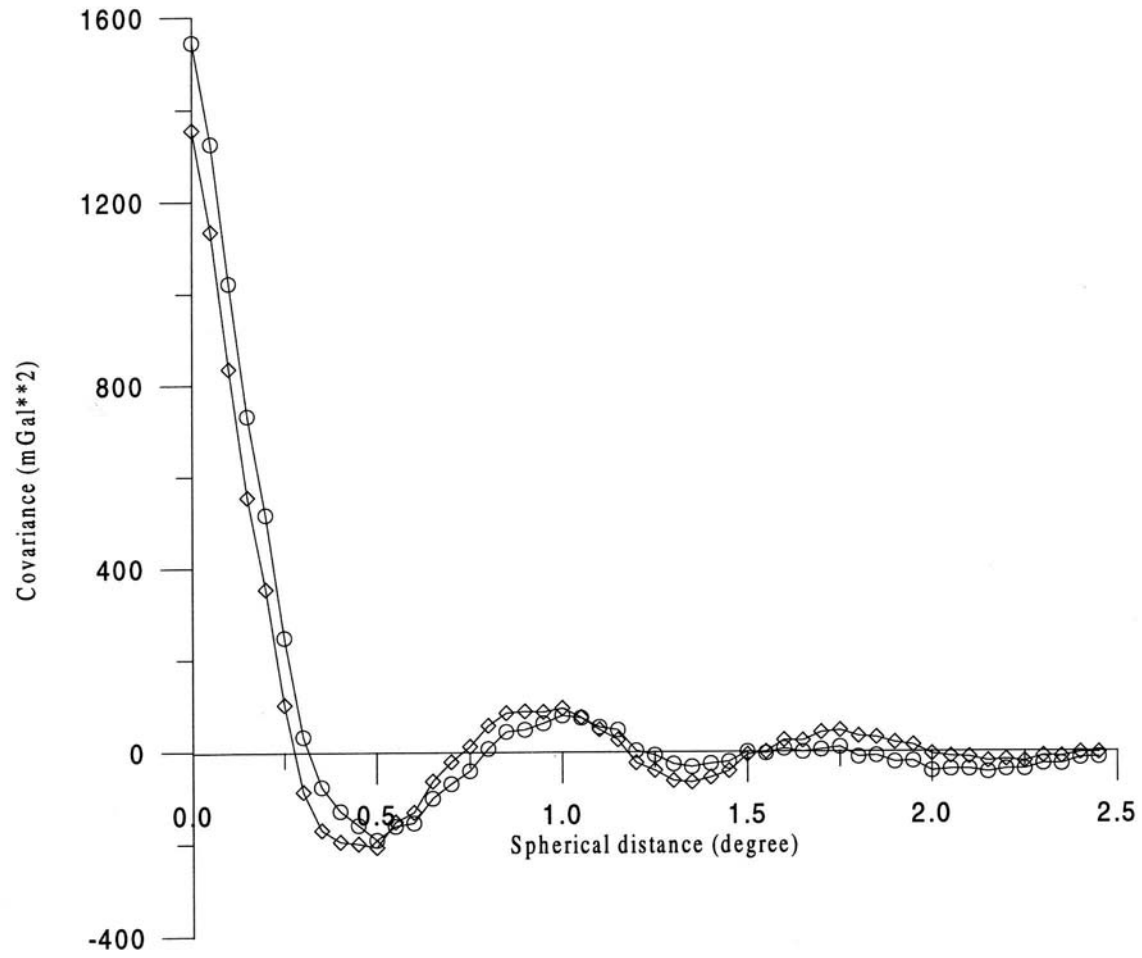


Figure 3. Empirical covariance functions of free air anomalies in Taiwan reduced: to OSU91A (cycles) and to EGM-X05 (squares) geopotential models

Error degree variances

The error estimation of the coefficients of a geopotential model is an essential information e.g. for the computation of analytical expressions for the covariance functions of quantities reduced to this geopotential model. This error estimation can be expressed in terms of the well known error degree variances. Usually, these error degree variances should be scaled by a factor (close to 1) in order to get a good agreement between the empirical and the analytical covariance function. Our experience with OSU91A was that the error estimation of the coefficients of this model was more or less pessimistic since in most test areas we had to multiply the error degree variances by a factor less than 1. In order to compare the error degree variances of the new model (EGM96) with the corresponding ones of OSU91A we have plotted the anomaly error degree variances of both models in Fig. 4.

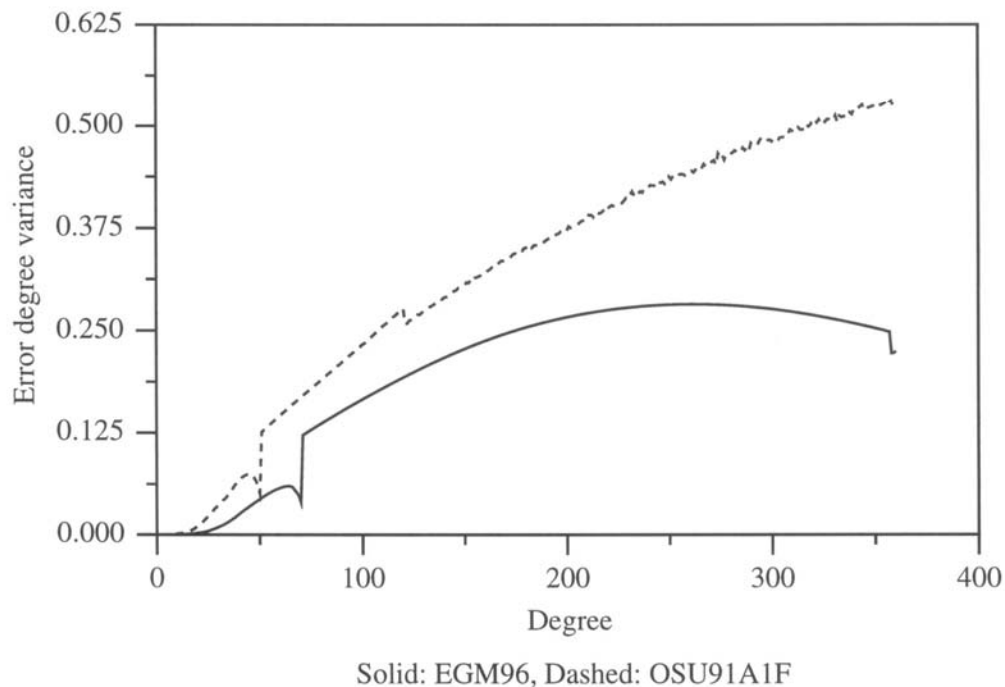


Fig. 4. Anomaly error degree variances of OSU91A1F and EGM96.

From Fig. 4 it is clear that the error estimation of the new model is rather optimistic comparing to the corresponding of OSU91A, especially for the high degrees. This could be due to the new -short wave length data used for the computation of this model.

Conclusion

Concerning the gravity anomalies, from the tests presented in the above Tables it is obvious that a significant improvement of the EGM models over OSU91A has been observed in Greenland. This improvement is obviously due to the fact that the data set used in the test has been also used for the computation of EGM models.

In the other test areas, with the exemption of Canada and Antarctic zone, an improvement in terms of mean value and standard deviation of the reduced to EGM gravity data of the order of 2 mGal has been observed.

In the Antarctic zone the statistics has not been changed using OSU91A or EGM models.

Finally, in Canada we have a small increase of the standard deviation of the reduced gravity anomalies when using the EGM models instead of OSU91A.

With respect to altimeter data in the Mediterranean, a slight improvement has been observed in the case of ERS-1/GM mission (the standard deviation has been decreased from 1.12 to 0.90 using the EGM instead OSU91A). This improvement has not been observed in the case of ERS-1/ERM.

No significant differences between the preliminary EGM versions were found. It seems that at least in some areas, the EGM-X02 or the EGM-X05 are better than the other EGM versions. The final EGM96 model present in the test areas of this study, generally the same behaviour as the preliminary EGM versions.

The comparison of the anomaly error degree variances of EGM96 with corresponding quantities of OSU91A shows that the error estimation of the coefficients of the new model is optimistic especially in the higher degrees.

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COMPARISON OF EGM MODELS IN THE NORDIC REGION AND GREENLAND

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Introduction

This paper summarizes the performance of the EGM spherical harmonic gravity field models for the Nordic region, including the Baltic countries, Iceland and Greenland. These regions are currently included in the geoid and gravity data base activities carried out at KMS (Kort & Matrikelstyrelsen, Denmark), within the framework of the Nordic Geodetic Commission Working Group on Geoid Determination.

For the Scandinavian/Baltic region comparisons are based on a consistent GPS/levelling datum, consistent with the joint European levelling net datum point in Amsterdam (UELN), whereas Iceland and Greenland GPS results are based on local sea-level datums. For the comparisons of GPS levelling data, the statistics of comparisons are quite simple: From observed GPS ellipsoidal heights h_{GPS} and levelled heights H , observed "GPS" geoid heights are obtained by

$$N_{\text{GPS}} = h_{\text{GPS}} - H$$

and compared to the EGM geoid height results by statistics (mean, standard deviation etc.) of the difference $N_{\text{GPS}} - N_{\text{EGM}}$. In addition the local fit to a particular survey is expressed by the residuals relative to a 4-parameter trend surface of form

$$N_{\text{trend}} = a_1 + a_2 \cos\phi \cos\lambda + a_3 \cos\phi \sin\lambda + a_4 \sin\phi$$

which corresponds to the geoid effect of a Helmert transformation.

The actual spherical harmonic geoid computations were carried out in grids of resolution $1/8^\circ$, using the GEOCOL program of the GRAVSOFT package, using the EGM reference ellipsoid parameters in the tide-free system.

The following spherical harmonic models have been tested: The preliminary models EGM-X02 to X05, the final EGM96 model (Lemoine et al., 1996) and the earlier standard reference model OSU91A, all spherical harmonic models complete to degree and order 360. In addition comparison results to detailed gravimetric geoid models, based on local gravity data, are included.

Fig. 1 shows the location of the GPS-levelling comparison points. In addition a number of independent gravity anomaly data were used for supplementary comparisons in Greenland. The results are outlined below on a region-by-region basis.

GPS-levelling comparisons in the Nordic/Baltic region

A number of national and local GPS surveys have been collected in this region as part of the

Table 1. Comparison of EGM models to 329 GPS/levelling in Nordic area. Unit: m.

Model	Mean	Std.dev.	Std.dev. after 4-parameter fit
EGM-X02	-.564	.338	.302
EGM-X03	-.594	.323	.300
EGM-X04	-.593	.320	.297
EGM-X05	-.592	.317	.294
EGM96	-.577	.266	.253
OSU91A	-.585	.379	.354
Local geoid	-.638	.112	.109

Table 2. Examples of geoid model fits to selected GPS surveys. Unit: m.

Area	EGM96			OSU91A			Local geoid		
	mean	stdev	4-fit	mean	stdev	4-fit	mean	stdev	4-fit
IfE N-S line (46 points)	-.60	.19	.16	-.46	.32	.26	-.64	.08	.05
SWET line (34 points)	-.57	.31	.23	-.62	.51	.23	-.63	.11	.05
Denmark (67 points)	-.83	.18	.11	-.69	.13	.11	-.69	.02	.02
Lithuania (36 points)	-.43	.17	.14	-.85	.32	.31	-.59	.09	.08

The results of comparison shows that a significant gravity field model improvement has been obtained in Scandinavia and the Baltic region, with an overall standard deviation of 26 cm. The mean offset of -58 cm is probably primarily due to the zero-level of the levelling (Amsterdam sea-level), which is different from the global zero-level due to sea-surface topography.

It is difficult to see any major difference between the different "EGM-X"-models from the present data, but the final model is significantly better than the "X" models (probably due to a last-minute correction of some errors in the terrestrial gravity data in the Gulf of Bothnia, prior to the final EGM96 release). Compared to OSU91A improvements are significant, seen especially clear in the Baltic area, where gravity data for the East Block were formerly classified, but now have been incorporated in both the EGM models and the local geoid computations.

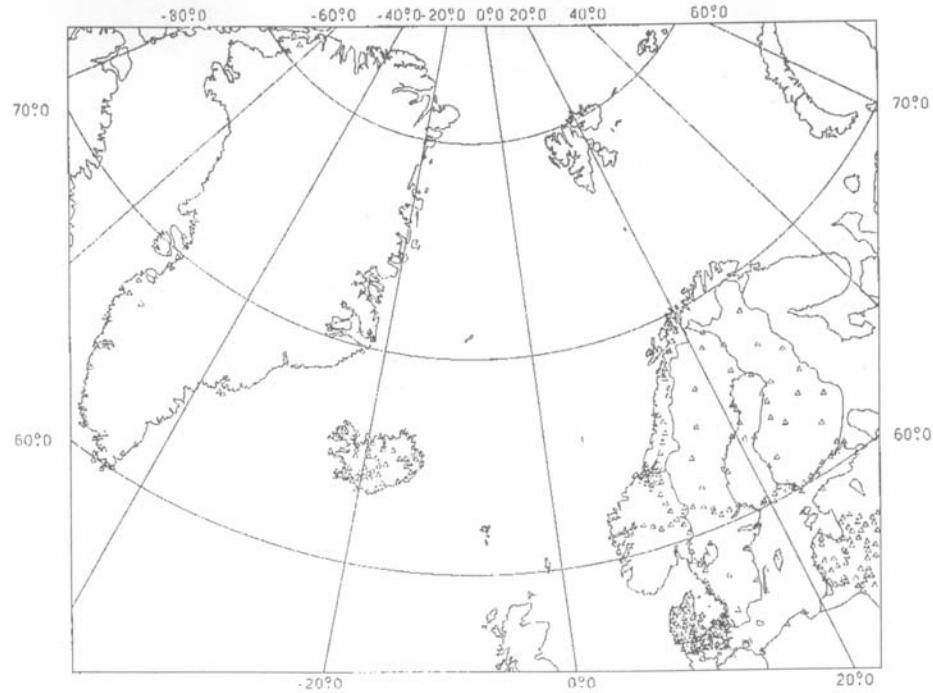


Fig.1. Location of GPS levelling points in Nordic region

NKG Nordic geoid project (Forsberg, 1996). The GPS-levelling data includes additional points in Russia, the Baltic countries and Poland. A majority of these GPS survey points have levelled heights, and in the present compilation it has been attempted to transform all GPS-levelling nets into a consistent datum.

The GPS results has as far as possible been expressed in the ITRF93 system, and national levelling results have been transformed into the UELN (Amsterdam) levelling system, using known constants for the transformations, or - when uncertain - common points between GPS surveys. Eastern european levelling heights, all given in the russian Kronstadt levelling datum, have been assumed to related to the western European UELN (Amsterdam) system by the constant correction (+15 cm) given in Ihde (1993). Normal heights have been used for most stations, and a correction made for land uplift for points in Norway, Sweden and Finland. The "geoid" comparisons are thus in fact quasigeoid comparisons.

The major GPS-levelling surveys which have been used here are:

- NKG "SWET" GPS line (extending roughly W-E along 60N),
- Baltic Sea Level GPS project,
- The Nordic part of the IFE N-S European GPS line,
- National GPS networks of Denmark (REFDK), Sweden (SWEPOS), Lithuania and Latvia, and a local GPS net in Norway (Trondheim).

Overall the material includes 329 stations in the common ITRF93/UELN system, the majority first-order levelled, and a consistency between surveys at the (hopefully) 10 cm level. The SWET and Baltic Sea Level project GPS results (Poutanen, 1995) were used as the basic survey datum reference. Table 1 outlines the fits for all stations, and Table 2 the fits for some selected surveys.

GPS-levelling comparisons - Iceland

Icelandic GPS-levelling data ("isnet") have been provided for 62 points by Landmalingar Islands (Iceland Geodetic Survey). The levelling datum for these points is local average sea level. Results are shown in Table 3, including results of a local NKG FFT geoid solution. Again, neglecting the bias (assumed due to local sea-level datum and long-wavelength errors), the EGM96 perform quite a bit better than OSU91A, and does nearly as good as the local geoid, which has bias problems (probably due to unknown ice thicknesses at time of computation).

Table 3. EGM geoid model comparisons for 62 Iceland GPS points. Unit: m.

Model	Mean	Std.dev.	Std.dev. after 4-parameter fit
EGM-X02	-.674	.240	.202
EGM-X03	-.659	.276	.184
EGM-X04	-.661	.275	.183
EGM-X05	-.677	.280	.193
EGM96	-.869	.247	.187
OSU91A	-.047	.351	.219
Local geoid	-1.27	.153	.130

EGM comparisons of geoid and gravity in Greenland

GPS geoid data in Greenland have been compiled based on GPS results from base nets supporting gravity survey activities in the ice-free parts, carried out 1991-96. Because no levelling nets exist in Greenland, all heights refer to local sea level, and the quality of the heights are of highly varying quality, mostly obtained from local trigonometric networks based on short-duration tidal observations. The most recent GPS networks (1994-96) have been computed in the ITRF-94 system, and the older surveys have here been fitted onto the newer results through common points in the surveys. With the GPS nets mainly based on broadcast orbits, and the uncertainty in the local heights above sea level, the GPS geoid errors are expected to be fairly large.

Comparison results are shown in Table 4, both for all data (62 points), and for the most recent 1996 survey in the rugged fjord region of central East Greenland (17 points). Again it is seen that the EGM models perform much better than OSU91A (a consequence of the inclusion of new airborne and surface gravity data in the new models), and that a local FFT/collocation geoid (GEOID96A; Forsberg and Kenyon, unpublished) do not perform better.

As an alternative test to the "noisy" GPS levelling data, Table 5 shows comparisons of independent newer gravity anomaly data (not included in the data underlying the EGM model development), as well as the comparison of all data in Greenland. Even though a relatively larger part of free-air gravity data are variations at shorter wavelengths, then the results confirm the

pattern of the GPS comparisons: a significant improvement has occurred with the EGM96 model.

Table 4. EGM geoid comparisons to Greenland GPS points. Unit: m.

Model	All data (62 points)			East Greenland (17 points)		
	mean	std.dev.	4-par fit	mean	std.dev.	4-par fit
EGM-X02	-.50	1.09	.93	-.82	.67	.52
EGM-X03	-.45	1.11	.95	-.93	.69	.53
EGM-X04	-.43	1.11	.95	-.93	.69	.54
EGM-X05	-.42	1.12	.96	-.87	.66	.55
EGM96	-.44	1.06	.94	-.74	.61	.53
OSU91A	-1.44	2.25	1.63	-.46	.95	.66
Local geoid	-1.25	1.32	.97	-1.77	.80	.37

Table 6. Comparison of new Greenland gravity anomaly data to spherical harmonic models

Unit: mgal Data statistics - or - difference to model	Ellesmere Island - North Greenland (2530 points)		Oil company data 1994-95 (shelf areas - 4428 points)		NASA Ice cap surface gravity profile (from D. Roman, 219 pts.)		All surface gravity data in Greenland (33885 pts.)	
	mean	sdev	mean	sdev	mean	sdev.	mean	sdev.
Original data	-10.8	64.9	7.3	29.0	26.6	18.4	14.1	39.8
EGM-X02	-4.4	40.7	-1.7	15.5	-1.7	10.5	-3.2	22.1
EGM-X03	-4.5	40.7	-0.4	15.1	-1.3	9.6	-3.1	22.2
EGM-X04	-4.5	40.7	-0.2	15.2	-1.3	9.7	-3.0	22.1
EGM-X05	-4.4	40.4	-0.3	14.9	-1.3	9.7	-3.0	21.9
EGM96	-5.1	39.5	-0.0	16.5	-2.9	10.3	-2.8	21.7
OSU91A	-1.8	47.4	-6.6	28.1	-0.4	19.6	-4.0	25.3

Conclusions

The above comparisons have shown that a significant improvement have been obtained with the new EGM96 model in the Nordic region and Greenland. The new EGM96 model performs very well, and will be a major benefit to geoid determination in this region. From the data it can be summarized that a clear a trend to favor a particular "X" model over any of the others is not existing.

Acknowledgements. I thank the national geodetic agencies of the Nordic and Baltic countries for providing the necessary GPS and levelling data.

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EVALUATION OF THE DMA/GSFC GEOPOTENTIAL MODELS OVER THE BRITISH ISLES

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Abstract

Statistical comparisons of the EGM series, OSU91A and GFZ95A global geopotential models with terrestrial land and marine gravity data, GPS and orthometric height data, and an existing regional gravimetric geoid do not indicate that any model is consistently superior in the vicinity of the British Isles.

Introduction

Whilst the British Isles was not specifically included in the International Geoid Service (IGeS) Special Working Group (SWG) for the evaluation of the joint Goddard Space Flight Center (GSFC) and Defense Mapping Agency (DMA) models (Rapp and Nerem, 1995; Sideris, 1996), the first author had access to them as part of the tests conducted in Australia (described elsewhere in this special issue). Therefore, as the British and Irish data were on-line as part of a project to compute a new gravimetric geoid in this region, it was a relatively straightforward task to conduct tests of the EGM series of geopotential models. The data used as control comprise:

- (i) three sets of terrestrial land gravity observations;
- (ii) three grids of terrestrial marine gravity data;
- (iii) thirty-eight discrete geoid heights derived from the difference between GPS ellipsoidal heights and orthometric heights on the Ordnance Datum Newlyn (Olliver, 1992), and
- (iv) an existing gravimetric geoid (Featherstone, 1992; Featherstone and Olliver, 1993 and 1994).

Computation of geoid and gravity anomalies from a geopotential model

As all the control data are based on GRS80 (Moritz, 1980), the geopotential models must also be evaluated with respect to GRS80, rather than the ellipsoid associated with the model. These differences can be accounted for in gravity space by using the following equations, derived by Vanicek *et al.* (1986) and Kirby and Featherstone (1997). The geoid height (N) with respect to GRS80 is given by

$$N = \frac{GM_{EGM} - GM_{GRS}}{r\gamma_{GRS}} + \frac{GM_{EGM}}{r\gamma_{GRS80}} \sum_{n=2}^{360} \left(\frac{a_{GRS}}{r} \right)^n \sum_{m=0}^n (\delta\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta) \quad (1)$$

and the gravity anomaly (Δg) with respect to GRS80 is given by

$$\Delta g = \frac{GM_{EGM} - GM_{GRS}}{r^2} + \frac{GM_{EGM}}{r^2} \sum_{n=2}^{360} \left(\frac{a_{GRS}}{r} \right)^n (n-1) \sum_{m=0}^n (\delta\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta) \quad (2)$$

The first term on the right hand side of each equation is the zero-degree term, which results from the difference in the value of the geocentric gravitational constant (GM) associated with the EGM and GRS80. The other component of the zero-degree term, which accounts for the difference in the gravity potential between the EGM-derived geoid and that on the surface of GRS80, is sufficiently small so as to be neglected (Kirby and Featherstone, 1997). The zero-degree bias for the EGM series of models and GRS80 is -0.937m in the geoid and -0.144mgal in the gravity anomalies, which indicates the effect a global bias in gravity data has on the geoid.

Normal gravity (γ) is computed on the surface of GRS80 at each point. The term $\bar{P}_{nm}$ is the fully normalised associated Legendre function of degree n and order m . The terms $\delta\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalised spherical harmonic coefficients of the geopotential model, which have been reduced and scaled by the even zonal harmonics of GRS80 according to

$$\delta\bar{C}_{n0} = \bar{C}_{EGM} - \frac{GM_{GRS}}{GM_{EGM}} \left(\frac{a_{GRS}}{a_{EGM}} \right)^n \bar{C}_{GRS} \quad (3)$$

This relationship accounts for the difference in mass, semi-major-axis and flattening (via J_2) of the ellipsoids associated with the EGM and GRS80. Its derivation is given by Vanicek *et al.* (1986) and Kirby and Featherstone (1997).

The triplet (r, θ, λ) comprises the spherical polar co-ordinates of each computation point at the geoid. As only the GRS80 geodetic co-ordinates are supplied for the control data, these must first be transformed to spherical polar co-ordinates. In order to compute the geocentric radius (r) to the geoid, an approximate mean geoid height of the British Isles of 50m is used. This was estimated from the GPS and orthometric height data. Computing the disturbing potential at the geoid only makes a difference of a few millimetres to the geoid and a few microgal to the gravity anomalies because $N \ll r$, but is included for completeness.

The software used to compute the geoid height and gravity anomalies for each geopotential model tested is a modified version of the FORTRAN code by Rapp (1982). It has been rewritten to utilise the GRS80 ellipsoid and compute the zero-degree terms, described above. In the following tests, the geoid height or gravity anomalies have been computed on a $15'$ grid covering the area of interest, then interpolated to each control data point using a bicubic spline. This approach makes use of the fast recursive relations for the associated Legendre functions along parallels, as described by Rizos (1979). The results when using this approach differ by a few millimetres or microgal from a point-by-point solution, but the computation time is reduced by almost two orders of magnitude.

Evaluations using land gravity data

Three separate files of terrestrial land gravity observations have been used as control data during this evaluation, as they represent the complete data-bases maintained in each country. These are:

- 156,284 gravity observations in England, Wales and Scotland (Figure 1a), supplied by Ian Smith of the British Geological Survey (BGS), Nottingham;
- 11,256 gravity observations in Northern Ireland (Figure 1b), supplied by Tony Griffith of the Geological Survey of Northern Ireland; and,
- 20,101 gravity observations in the Republic of Ireland (Figure 1c), supplied by Prof. Tom Murphy of the Dublin Institute for Advanced Studies.

The number of observations correspond to approximately one observation per square kilometre in most areas.

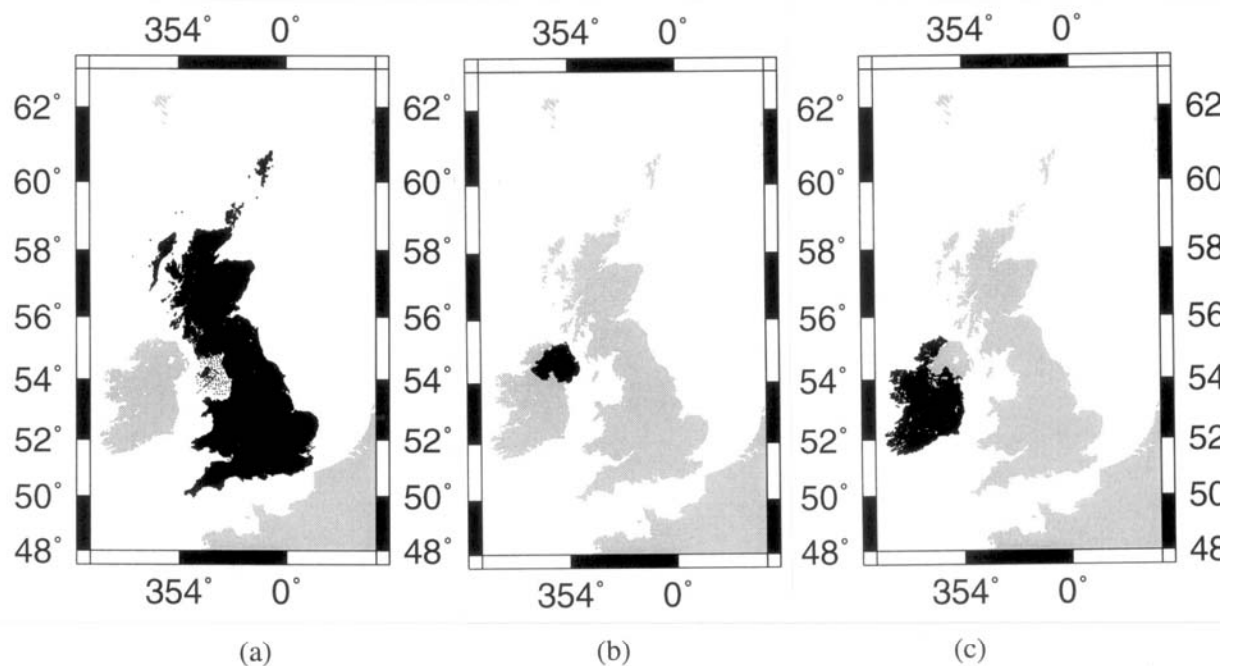


Figure 1. Coverage of the terrestrial gravity observations on land. (Mercator projection).

The origin of the terrestrial land gravity data is described by Featherstone (1992, §4). As the raw observations were available, these data were reprocessed according to the more rigorous geodetic requirements (Heck, 1990): Their horizontal co-ordinates were transformed to a geocentric system using the parameters given by Defense Mapping Agency (1991). The Somigliana-Pizetti normal gravity formula, second-order free-air gravity reduction, and an atmospheric correction were then used to produce free-air gravity anomalies on GRS80 (Moritz, 1980). This is consistent with the land gravity data reductions used for the EGM models (Kenyon and Pavlis, 1995). The estimated standard error of these free-air anomalies is ± 0.3 mgal, generated mostly by the elevation uncertainty.

Table 1. Statistics of the differences between free-air gravity anomalies and geopotential models in England, Wales and Scotland at 156,284 points (units in mgal).

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	120.074	-68.470	1.683	13.465	13.570
<i>EGM-X01</i>	116.429	-69.571	2.204	13.376	13.557
<i>EGM-X02</i>	117.485	-68.963	1.762	13.502	13.617
<i>EGM-X03</i>	117.158	-69.165	1.887	13.406	13.537
<i>EGM-X04</i>	117.759	-69.134	1.809	13.429	13.551
<i>EGM-X05</i>	119.143	-68.820	1.754	13.405	13.519
<i>JGM3_OSU91A</i>	108.384	-73.252	1.391	13.498	13.563
<i>OSU91A</i>	106.885	-74.673	1.181	13.461	13.512
<i>GFZ95A</i>	111.070	-69.796	0.244	13.245	13.248

The statistics in Table 1 refer to the differences between terrestrial land free-air gravity anomalies covering mainland Britain and the gravity anomalies implied by each geopotential model. No gravimetric terrain corrections have been applied to the terrestrial data used in these comparisons. As well as the EGM series of models under investigation (Sideris, 1996),

statistics have also been computed for the OSU91A (Rapp *et al.*, 1991) and the GFZ95A (Gruber *et al.*, 1995) geopotential models. Tables 2 and 3 show the statistics for Northern Ireland and the Republic of Ireland, respectively.

Table 2. Statistics of the differences between free-air gravity anomalies and geopotential models in Northern Ireland at 11,256 points (units in mgal).

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	57.020	-37.372	-1.330	12.856	12.925
<i>EGM-X01</i>	56.549	-38.714	-1.920	12.864	13.006
<i>EGM-X02</i>	58.720	-36.771	0.053	12.920	12.920
<i>EGM-X03</i>	56.801	-38.149	-1.383	12.819	12.893
<i>EGM-X04</i>	56.393	-38.663	-1.618	12.895	12.996
<i>EGM-X05</i>	56.251	-38.021	-1.995	12.812	12.966
<i>JGM3_OSU91A</i>	61.725	-34.428	2.658	13.153	13.395
<i>OSU91A</i>	61.127	-34.711	2.439	13.108	13.333
<i>GFZ95A</i>	62.541	-37.450	3.594	13.845	14.304

Table 3. Statistics of the differences between free-air gravity anomalies and geopotential models in the Republic of Ireland at 20,101 points (units in mgal).

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	54.814	-44.040	-0.582	8.713	8.732
<i>EGM-X01</i>	52.643	-44.358	-1.993	8.525	8.755
<i>EGM-X02</i>	54.194	-43.024	-0.041	8.586	8.586
<i>EGM-X03</i>	52.995	-43.916	-1.293	8.537	8.635
<i>EGM-X04</i>	53.618	-43.868	-1.288	8.511	8.608
<i>EGM-X05</i>	-44.606	54.598	-1.399	8.671	8.783
<i>JGM3_OSU91A</i>	46.736	-43.035	-0.733	8.910	8.951
<i>OSU91A</i>	49.422	-40.336	0.466	8.641	8.653
<i>GFZ95A</i>	44.646	-40.760	0.042	8.826	8.826

From the statistics in Tables 1, 2 and 3, it is unclear which model consistently gives the best agreement with the terrestrial free-air anomalies on land. This is because the variation among the standard deviations of the differences is less than the standard error associated with the control data (± 0.3 mgal). This observation could have been expected as, to the best knowledge of the authors, no significant amounts of new gravity data have been used in this region for the determination of the EGM models compared to OSU91A and GFZ95A. Also, the coverage of the gravity data in this region is homogeneous and is in the public domain.

Evaluations using marine gravity data

The origin of the terrestrial marine gravity data is described by Featherstone (1992, §4). Three separate files of marine gravity data have been used in this evaluation as the data were supplied in this way by Dr Allan Mould of the BGS, Edinburgh. These are:

- 48,904 free-air gravity values covering the western North Sea (Figure 2a);
- 43,124 free-air gravity values covering the North-West Shelf (Figure 2b); and
- 56,803 free-air gravity values covering Southern Britain and surrounding seas (Figure 2c).

These data were compiled and processed by the BGS from ship-track data collected by the BGS and commercial companies, mostly Western Geophysical. The BGS would not supply the raw observations, but a ~4km UTM grid interpolated from the ship-track and land data, described earlier, using a tensioned spline routine written by ARC Geophysics, Milton Keynes. These pre-gridded free-air gravity anomalies were based on GRS67 and thus had to be converted to GRS80 using the formula given by Moritz (1980). The lateral positions of the gravity data points were transformed from the UTM grid to geographical co-ordinates on GRS80. The standard error of these data, quoted by the BGS and based upon a cross-over adjustment, is approximately ± 1 mgal.

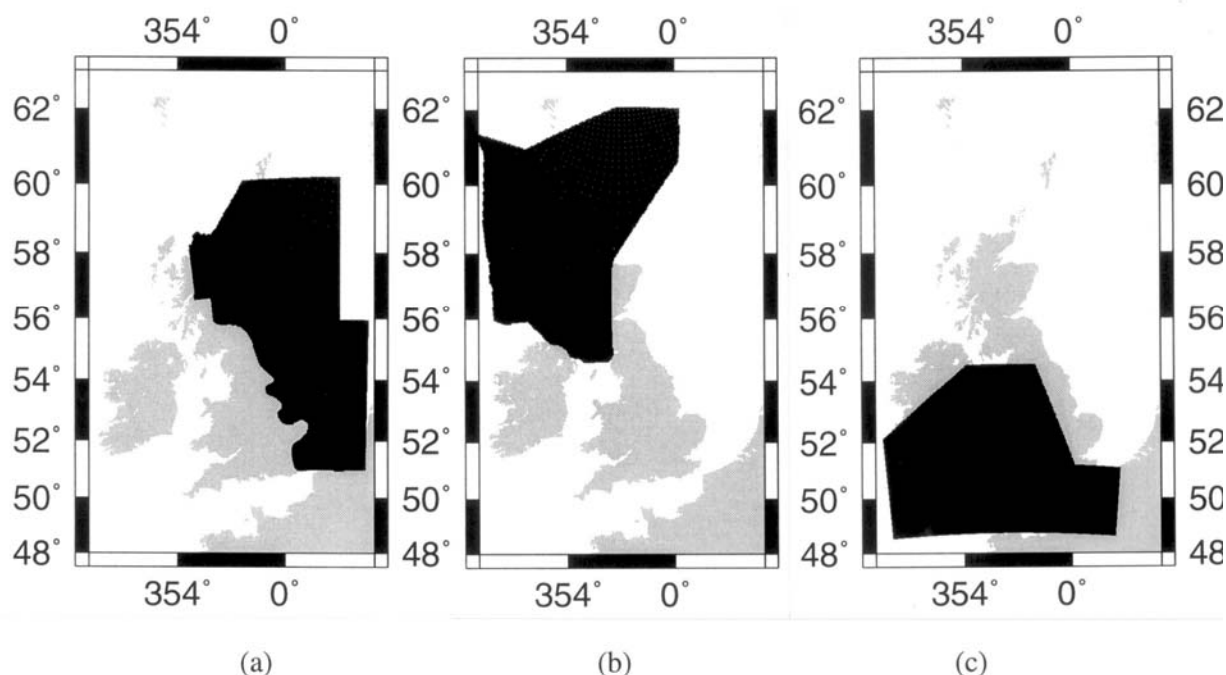


Figure 2. Coverage of the terrestrial gravity data in marine areas. (Mercator projection).

The statistics in Tables 4, 5 and 6 refer to the differences between the terrestrial marine free-air gravity anomalies for each data file and the gravity anomalies implied by each geopotential model. Once again, statistics have also been computed for the OSU91A (Rapp *et al.*, 1991) and the GFZ95A (Gruber *et al.*, 1995) geopotential models.

Table 4. Statistics of the differences between free-air gravity anomalies and geopotential models over the western North Sea at 48,904 points (units in mgal).

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	51.146	-42.204	-0.265	7.697	7.702
<i>EGM-X01</i>	55.999	-39.174	-0.330	7.838	7.845
<i>EGM-X02</i>	55.464	-40.086	-0.805	7.815	7.857
<i>EGM-X03</i>	56.238	-38.951	-0.360	7.796	7.804
<i>EGM-X04</i>	55.732	-39.805	-0.358	7.758	7.766
<i>EGM-X05</i>	53.959	-41.979	-0.334	7.650	7.658
<i>JGM3_OSU91A</i>	57.256	-40.187	-0.865	8.045	8.099
<i>OSU91A</i>	56.407	-40.942	-0.270	8.103	8.107
<i>GFZ95A</i>	56.513	-40.618	0.229	7.711	7.715

Table 5. *Statistics of the differences between free-air gravity anomalies and geopotential models over the North-West Shelf at 43,124 points (units in mgal).*

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	88.017	-69.611	0.717	16.998	17.013
<i>EGM-X01</i>	91.655	-61.525	0.086	16.640	16.641
<i>EGM-X02</i>	91.731	-63.360	0.579	16.722	16.732
<i>EGM-X03</i>	92.901	-61.531	0.478	16.627	16.694
<i>EGM-X04</i>	92.353	-62.863	0.454	16.688	16.694
<i>EGM-X05</i>	88.674	-68.607	0.481	17.019	17.026
<i>JGM3_OSU91A</i>	89.583	-60.567	0.161	16.939	16.943
<i>OSU91A</i>	88.670	-59.321	0.304	16.997	17.000
<i>GFZ95A</i>	107.166	-81.425	-0.991	19.010	19.036

Table 6. *Statistics of the differences between free-air gravity anomalies and geopotential models over Southern Britain and surrounding seas at 56,803 points (units in mgal).*

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	32.415	-37.882	0.868	7.947	7.994
<i>EGM-X01</i>	33.816	-39.022	1.090	7.812	7.937
<i>EGM-X02</i>	34.213	-37.518	1.225	7.937	8.031
<i>EGM-X03</i>	33.407	-39.005	0.817	7.830	7.873
<i>EGM-X04</i>	33.329	-38.735	0.838	7.841	7.885
<i>EGM-X05</i>	32.542	-38.753	0.870	7.930	7.977
<i>JGM3_OSU91A</i>	34.752	-34.614	0.974	8.258	8.301
<i>OSU91A</i>	33.219	-36.375	0.803	8.151	8.190
<i>GFZ95A</i>	31.439	-35.568	0.070	8.091	8.092

From Tables 4, 5 and 6, it is unclear which model consistently gives the best agreement with the terrestrial free-air anomalies at sea. Once again, this is because the variation among the standard deviations of the differences is less than the standard error associated with the control data (± 1 mgal). However, there is a significant increase in the standard deviations (± 17 mgal versus ± 8 mgal), listed in Table 5 as compared to Tables 4 and 6. This is most probably because the majority of the marine data in the North-West Shelf were collected by Western Geophysical and are thus not in the public domain. This illustrates the importance of including observed gravity data in the determination of global models. Another factor that may contribute to the larger standard deviations is the complicated geological structure of this region, whose gravity field is of a relatively short wavelength and thus cannot be accurately modelled by any global model.

Evaluations using GPS and orthometric height data

Two hundred and fifty-six relative GPS baselines between 59 stations in England, Wales and Scotland had been supplied by the Ordnance Survey in 1990. The baseline vectors and their corresponding variance-covariance matrices were network adjusted about the Satellite Laser Ranging (SLR) station at Herstmonceux to yield absolute WGS84 co-ordinates (Olliver, 1992). However, these GPS data were observed using single-frequency instruments at a time when GPS processing algorithms were immature and a full constellation was not available. Nevertheless, these are the only data available to the authors, despite the existence of more

recent survey data which were not supplied by the Ordnance Survey. Orthometric heights of varying tolerances and referred to the Ordnance Datum Newlyn (ODN) were also supplied at these stations. Only the 38 optically levelled stations have been used in this evaluation (Table 7) due to the uncertainties inherent to trigonometrical height determination. The standard error associated with these discrete geometrical geoid control is estimated to be $\pm 10\text{-}20\text{cm}$.

Table 7. *Statistics of the differences between GPS-orthometric height data and geopotential models at 38 points (units in metres).*

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	0.911	-0.594	0.044	0.281	0.284
<i>EGM-X01</i>	1.122	-0.747	0.074	0.266	0.276
<i>EGM-X02</i>	1.094	-0.730	0.003	0.299	0.299
<i>EGM-X03</i>	0.985	-0.622	-0.022	0.255	0.256
<i>EGM-X04</i>	1.005	-0.629	-0.021	0.260	0.261
<i>EGM-X05</i>	1.004	-0.664	-0.021	0.264	0.265
<i>JGM3_OSU91A</i>	1.251	-0.430	0.075	0.328	0.330
<i>OSU91A</i>	1.191	-0.568	0.097	0.309	0.324
<i>GFZ95A</i>	0.740	-0.669	-0.294	0.251	0.387

From the statistics in Table 7, the EGM series of models do appear to provide a small, but not necessarily significant, improvement over the OSU91A, GFZ95A and JGM3_OSU91A models. The control data should not be relied upon too greatly as they are subject to errors in the old, single-frequency GPS survey and the orthometric height data (Featherstone and Olliver, 1993). Nevertheless, the standard deviations of the differences are less than the preliminary results reported for the whole of Europe (Nerem *et al.*, 1995).

Evaluations using the regional OXGEIOD92 gravimetric geoid

The final evaluation is with an existing gravimetric geoid model, called OXGEIOD92, which was computed at the University of Oxford (Featherstone, 1992; Featherstone and Olliver, 1993 and 1994). This model is based on a degree-257 expansion of OSU91A with respect to GRS80, because this is the point at which the global error degree variance exceeds the global degree variance. The terrestrial gravity data used are listed above and were averaged to give a 2' by 4' grid of free-air anomalies. A digital terrain model was also used to compute the gravimetric terrain corrections and primary indirect effect.

The Stokesian integration was conducted in the space domain and used a cap-radius of 1.95° and a Meissl (1971) modification to the integration kernel so as to help reduce the impact of the truncation error. The geoid was computed on a 2' by 4' grid in the area bound by $(-12^\circ \leq \lambda \leq 8^\circ, 48^\circ \leq \phi \leq 63^\circ)$ to give 135,000 point geoid heights above GRS80. The standard error of this gravimetric geoid was coarsely estimated to be $\pm 0.08\text{m}$, by propagating the standard error of the gravity data through Stokes's integral.

The statistics of the difference between OXGEIOD92 and the geopotential models are given in Table 8. From this, OSU91A clearly gives the best comparison to this regional gravimetric geoid. This is most probably because the regional geoid is based on OSU91A and is thus biased towards it. The exact reason for this effect is currently under investigation. However, of the EGM series of models, EGM96 gives the closest agreement, although this is not significant when considering the standard error of OXGEIOD92.

Table 8. Statistics of the differences between OXGEOID92 and geopotential models over the British Isles ($-12^\circ \leq \lambda \leq 8^\circ$, $48^\circ \leq \phi \leq 63^\circ$) at 135,000 points (units in m).

<i>model</i>	<i>max</i>	<i>min</i>	<i>mean</i>	<i>std</i>	<i>rms</i>
<i>EGM96</i>	1.601	-2.199	-0.219	0.320	0.388
<i>EGM-X01</i>	1.619	-1.957	-0.204	0.383	0.435
<i>EGM-X02</i>	1.590	-2.029	-0.204	0.343	0.399
<i>EGM-X03</i>	1.589	-1.912	-0.200	0.352	0.405
<i>EGM-X04</i>	1.598	-1.946	-0.200	0.353	0.406
<i>EGM-X05</i>	1.718	-2.055	-0.201	0.354	0.407
<i>JGM3_OSU91A</i>	1.513	-1.675	-0.106	0.283	0.366
<i>OSU91A</i>	1.609	-1.688	-0.053	0.238	0.244
<i>GFZ95A</i>	1.078	-3.478	-0.215	0.344	0.405

Conclusions

From the statistical tests presented herein, the EGM series of geopotential models generally do make a small improvement upon the existing models OSU91A, JGM3_OSU91A and GFZ95A. However, these do not point conclusively to any single model as being superior. This is most probably because no additional gravity data have been included in the region of the British Isles as compared to the data used to compute OSU91A and GFZ95A. Other factors that have led to this inconclusive study are the dimensions of the test area, which prohibits the study of long wavelength geoid features, and the uncertainties in the control data, which were larger than the statistics of the differences among the models.

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Comparison between the EGM96 model and the French quasi-geoid model.

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Abstract

The GSFC/DMA global geopotential model EGM96, the new French geodetic network and the French quasi-geoid QGF96 were made available approximately at the same time. Despite the small size of the test area, it was interesting to compare them and to evaluate the benefit of EGM96 over OSU91. After a short description of the French quasi-geoid, the paper presents and discusses statistical tests. Provisional results on the tilt of the French levelling datum are also laid out.

The QGF96 quasi-geoid model

The new French quasi-geoid model is described in detail in (Duquenne, 1997). In order to satisfy the needs of both the scientific and surveyors communities, two grids have been computed. The first one, so called "QGF96", is a gravimetric quasi-geoid for scientific purpose. The second grid RAF96 is a fit of QGF96 to the new French geodetic network (RGF93) and suits for levelling by GPS.

QGF96 combines the OSU91A geopotential model, about 270 000 gravity data and a digital terrain model covering west Europe, using the remove-restore technique and the residual terrain model (see for instance Forsberg, 1994). Residual terrain corrections have been computed within the radius of 110 km. The gravity data include gravity anomalies derived from satellite altimeter by Prof. Sandwell in the Atlantic Ocean, Prof. Morelli's anomalies in the Mediterranean Sea, and anomalies from the BGI database. A 220 km radius has been used for Stokes's integration. QGF96 has been checked by a comparison with 979 levelled GPS points of the RGF93 network. Let ζ_{RGF} the height anomaly inferred from levelling and GPS, and ζ_{QGF} the quasi-geoid undulation. The linear part of the discrepancies has been modelled through the observation equation:

$$a + b(\varphi - \varphi_0) + c(\lambda - \lambda_0) \cos \varphi - (\zeta_{RGF} - \zeta_{QGF}) = v \quad (1)$$

where φ and λ are the geographical co-ordinates of the control points, φ_0 and λ_0 the co-ordinates of some reference point, a , b , c three contents and v a residual. A least square adjustment has led to:

$$\begin{aligned} a &= 0.051 \text{ m} & b &= -0.523 \text{ m} / 1000 \text{ km} & c &= 0.852 \text{ m} / 1000 \text{ km} \\ \text{standard deviation of the residuals: } \sigma &= 0.128 \text{ m} \\ \text{largest residuals: } &-0.470 \text{ m} & & +0.724 \text{ m} \end{aligned}$$

b is the south-north component of the tilt of the quasi-geoid against the reference surface of the height datum. It is found in good agreement with the well known systematic error of the French levelling network IGN69 (Kasser, 1989). c is the west-east component. It seems to be surprisingly large and might be interpreted as a long wavelength defect of OSU91A over France, a systematic error of the levelling network in the west-east direction (unknown until now), a shift of a geodetic datum toward the Y direction, a systematic error in gravity data, for instance a shift between land data and Sandwell's data. A combination of these errors is possible. The figure 1 shows the distribution of the residuals. Large errors are confined in three areas. It is now recognised that they are due to various causes:

- improper interpolation of residual anomalies in the Côte d'Azur ($\varphi = 43.5^\circ$, $\lambda = 7.25^\circ$), because of the large variation of the gravity field in this area
- errors in a gravity data set in the Pyrenees ($\varphi = 42.5^\circ$, $\lambda = 2^\circ$)
- lack of data along the coast in the Atlantic Ocean ($\varphi = 44^\circ$, $\lambda = -2^\circ$)

These problems will be corrected as far as possible in a further version of the French geoid model.

The grid RAF96 has been derived from QGF96 by adding the linear part of the bias as described above, and a correction interpolated from the residuals by collocation. RAF96 models the normal height reference surface (IGN69) in the RGF93 geodetic reference frame, which is identical to ETRF89. On the 979 points used to fit RAF96 with the RGF93 network, the discrepancies between RAF96 and RGF93 are at the level of some millimetres. Tests using 160 other control points well distributed over France proved a precision of 2.5 cm and bias at the 1 cm level.

Comparison between EGM96 and QGF96

For EGM96 and OSU91A, height anomalies have been derived from the full set of coefficients and a precise digital terrain model. In this computation, the main parameter (radius of the ellipsoid and the geocentric constant of gravitation) have been fixed to the values recommended in (Rapp, 1991) and (Sideris, 1996). The height anomalies of the models have been converted to GRS80, taking into account the change in the gravitational constant, so that they become compatible as far as possible with each of them and the RGF93 and QGF96 anomalies. Then the differences between the anomalies of the model ζ_{EGM} and the anomalies of QGF96 (ζ_{QGF}), RAF96 (ζ_{RAF}), OSU91A (ζ_{OSU}) and the 979 levelled GPS points of RGF93 (ζ_{RGF}) have been computed. For QGF96 and OSU91A, 160401 points were used in the area $42^\circ \leq \varphi \leq 51.5^\circ$, $-5.5^\circ \leq \lambda \leq 8.5^\circ$. For RAF96, only 90136 points in continental France have been retained. Furthermore a linear regression has been performed in any case so that general trends of errors be extracted. The statistical characteristics of the differences in height anomalies are given in the table 1. In addition, the last row of the table recalls the result of the regression of the 979 RGF93 points to the gravimetric quasi-geoid QGF96.

Table 1: Statistics of differences in height anomalies.

anomalies difference	lin. regr.	a (m)	b (10^{-6})	c (10^{-6})	mean (m)	min. (m)	max. (m)	RMS (m)	std. dev. (m)
$\zeta_{EGM} - \zeta_{QGF}$	none	-	-	-	+0.177	-2.793	+1.205	0.676	0.428
$\zeta_{EGM} - \zeta_{QGF}$		+0.177	+0.006	+0.126	0	-2.208	+1.686	0.426	0.426
$\zeta_{EGM} - \zeta_{RAF}$	none	-	-	-	+0.191	-2.949	+0.150	0.655	0.412
$\zeta_{EGM} - \zeta_{RAF}$		+0.191	+0.506	-0.905	0	-2.089	+0.799	0.333	0.333
$\zeta_{EGM} - \zeta_{OSU}$	none	-	-	-	+0.191	-1.934	+3.070	0.647	0.399
$\zeta_{EGM} - \zeta_{OSU}$		+0.191	-0.162	+0.124	0	-3.428	+1.492	0.394	0.394
$\zeta_{EGM} - \zeta_{RGF}$	none	-	-	-	+0.212	-2.512	+0.104	0.614	0.372
$\zeta_{EGM} - \zeta_{RGF}$		+0.212	+0.462	-0.886	0	-0.685	+1.692	0.303	0.303
$\zeta_{OSU} - \zeta_{RAF}$	none	-	-	-	-0.005	-3.110	+1.160	0.436	0.436
$\zeta_{OSU} - \zeta_{RAF}$		-0.005	+0.687	-0.476	0	-1.464	+2.872	0.387	0.387
$\zeta_{RGF} - \zeta_{QGF}$		+0.051	-0.523	+0.852	0	-0.470	+0.724	0.128	0.128

Discussion

Bias

At a first glance, a bias of about 0.2 m appears between OSU91A and EGM96. As the quasi-geoid QGF96 is derived from OSU91A, the bias is found again between QGF96 and EGM96. The levelled GPS points seem to be in better agreement with OSU91A (bias: -0.051 m) than with EGM96 ($+0.212$ m), but this might be an artefact of the tilt of the French height datum due to systematic errors in the levelling network. The bias (a coefficient) is actually referred to a *mean point* which is not the fundamental benchmark of the levelling network, located in the south of France.

Tilt

In the south-north direction, there is no significant tilt between EGM96, OSU91A and QGF96, while the French height datum shows a tilt of about 0.5 m / 1000 km. This result must be related to the one obtained by a very high precision levelling traverse in 1983 (Kasser, 1989): a 0.29 m / 1000 km bias of the IGN69 datum was found between the Mediterranean and the North Sea.

The slope in the west-east direction between the quasi-geoid issued from EGM96 and OSU91A or QGF96 is small (0.12×10^{-6}). The large tilt of the height datum against QGF96 occurs again against EGM96. Nevertheless it would be adventurous to proclaim the discovery of a tilt of the height reference surface toward east: OSU91A and EGM96 are surely correlated and have more or less the same defect in the area of France, as we shall see in the next section.

Precision

Large discrepancies (about 2 m, see figure 2) appear between both the global models and the gravimetric quasi-geoid or the height anomalies issued from levelling and GPS. The global models are not reliable in high mountainous areas, where elevations exceed 3000 m. The errors are so large that they might spoil the numerical results of the previous sections. Otherwise the precision of EGM96 is estimated at $0.30 \sim 0.43$ m if a linear trend is removed. A processing of $\zeta_{EGM} - \zeta_{RGF}$ excluding the pyrenean and alpine mountains leads to a standard deviation of 0.21 m, after removing a trend.

In continental France, a slight improvement in precision is achieved with EGM96, compared with OSU91A. But large errors of OSU91A in Corsica are removed from EGM96: this is a key point for further determinations of the geoid in this area.

Conclusions

It is not possible to draw general conclusions from a study on a global field model in a so small area as France, and the above results must be seen as a modest contribution to the evaluation of the GSFC/DMA model. The main results of this paper can be summarised as follow:

- the precision of EGM96 in France is $0.30 \sim 0.43$ m
- the new global model shows large errors in rough topography
- the tilt of the French levelling datum referred to the quasi-geoid QGF96 or the OSU91A global model is found again with EGM96. Further investigation are needed on this topic, and it would be judicious to undertake comparisons with others realisations of the (quasi) geoid such as the European model EGG97, or with GPS and tide gauge data.

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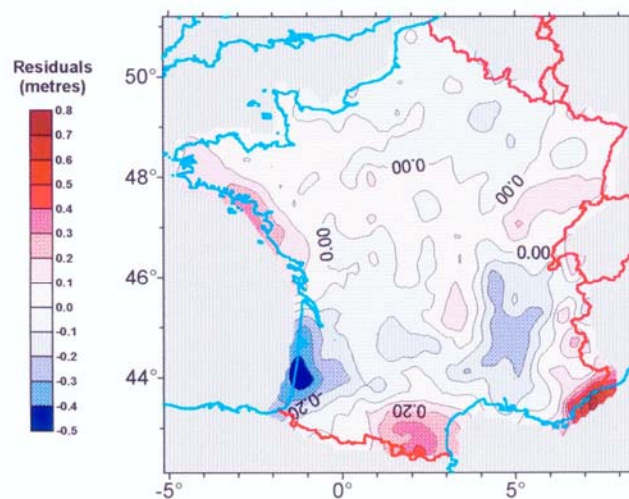


Figure 1:
Residuals of the regression of the quasi-geoid QGF96 to 979 levelled GPS points of the RGF93 network. Contour interval: 0.10 m

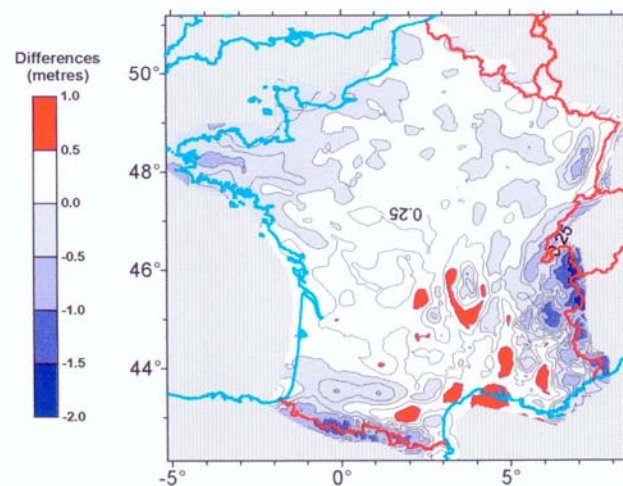


Figure 2:
Comparison of the EGM96 model with the RAF96 height reference surface (after a 3 parameters regression). Contour interval: 0.50 m. Supplementary contours: +/- 0.25 m

GEOPOTENTIAL MODELS VALIDATION AT IGES

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In the framework of the new DMA - NASA global gravity models validation, several tests were performed at IGES in Politecnico di Milano. Different data sets in the Mediterranean area were considered; they are listed in the following:

- a) the Italian quasigeoid (ITALGEO95);
- b) the GEOMED geoid;
- c) the gravity data base used in the computation of the Italian quasigeoid;
- d) the Italian data base of the deflection of the vertical;
- e) GPS/leveling data in Italy;
- f) absolute gravity measurements in Europe.

For each data set and for each model (also the OSU91A model was considered), usual statistics of the observations and of the differences between observed and model values were computed.

The model effects were derived using the f477 program by prof. R.H. Rapp (Rapp 1994).

THE ITALIAN QUASIGEOID

The Italian quasigeoid (Barzaghi et al. 1996) was computed on a regular grid of $3' \times 3'$ over the area

$$36 \leq \varphi \leq 47 \quad 6 \leq \lambda \leq 19 .$$

A slightly modified version of the usual remove-restore technique was applied, based on the OSU91A model, on a $250\text{m} \times 250\text{m}$ DTM and on Fast Collocation. The main problem occurred in the computation was the one related to the reduction of the Corsica Island gravity data base. It was noticed that the OSU91A model did not take into account gravity and geoid signals on that island; this problem seems to be solved with the new models which improved a lot in that area. Furthermore, no data were present in the gravity data base used in the quasigeoid computation on the former Yugoslavia area, which was not taken into account in the comparisons.

The statistics of the original data and of the differences between the ITALGEO95 values and the corresponding values of the various geopotential models are collected in Table 1a and Table 1b. In Fig. 1 the contour plot of the differences between ITALGEO95 and the EGM96 model is presented.

	total values	mean (m)	st. dev. (m)	min (m)	max (m)
ITALGEO95	50231	44.01	5.31	25.37	54.96

Table 1a. - The Italian quasigeoid ($N_{\text{ITALGEO95}}$)

model	total values	mean (m)	st. dev. (m)	min (m)	max. (m)
osu91a	50231	0.17	0.73	-1.85	6.40
x01	50231	-0.11	0.45	-1.61	1.62
x02	50231	-0.12	0.41	-1.75	1.61
x03	50231	-0.13	0.48	-1.53	1.75
x04	50231	-0.13	0.47	-1.52	1.69
x05	50231	-0.14	0.46	-1.59	1.58
EGM96	50231	-0.09	0.43	-1.70	1.86

Table 1b. - The Italian residual quasigeoid ($N_{\text{ITALGEO95}} - N_{\text{MOD}}$)

The maximum value of 6.4 m in OSU91A statistics occurs in the Corsica area, which is missing in the computation of OSU91A, and was introduced as local correction in ITALGEO95 computation.

The residual bias of 16 cm between ITALGEO95 and OSU91A is mostly accounted for by the residual terrain effect, here not included, and reduces to 0.08 m once RTC is taken into account.

Tab.1b shows a bias in residual geoid undulations in comparing OSU91A with the new models ($\Delta N_{\text{OSU}} - \Delta N_{\text{X0}} \sim 25\text{-}30$ cm), partly due to ITALGEO95 estimate based on OSU91A, and a clear improvement in standard deviation (from $\sigma_{\Delta N} \sim 73$ cm to $\sigma_{\Delta N} \sim 40\text{-}47$ cm).

THE GEOMED GEOID

The GEOMED project led to the determination of the geoid over the Mediterranean Sea (Barzaghi et al. 1992; Barzaghi et al. 1993; Arabelos et al. 1994; Barzaghi, Sona 1994). The computation was done in five overlapping areas merged together into a unique grid. The five original grids have regular spacing of $5' \times 5'$ and the following bounds:

$35 \leq \varphi \leq 43$	$-5 \leq \lambda \leq 10$
$30 \leq \varphi \leq 47$	$8 \leq \lambda \leq 20$
$30 \leq \varphi \leq 41.5$	$17 \leq \lambda \leq 36$
$35 \leq \varphi \leq 45$	$5 \leq \lambda \leq 15$
$30 \leq \varphi \leq 40$	$15 \leq \lambda \leq 25$

In each estimate the remove-restore technique was used; the OSU91A model and the TUG87 DTM were considered in the westernmost area while the remaining estimates were computed using the OSU91A and the ETOPO5U DTM. The statistics of the original values and of the differences between GEOMED and model undulations are in Table 2a and Table2b. In Fig.2 are the contour lines of the differences between GEOMED and EGM96.

	total values	mean (m)	st. dev. (m)	min (m)	max. (m)
GEOMED	72144	35.99	11.86	1.29	55.65

Table 2a. - The GEOMED geoid (N_{GEOMED})

model	total values	mean (m)	st. dev. (m)	min (m)	max. (m)
osu91a	72144	-0.11	1.23	-4.73	6.59
x01	72144	-0.32	1.27	-4.62	8.18
x02	72144	-0.31	1.20	-4.60	7.77
x03	72144	-0.29	1.24	-4.47	7.34
x04	72144	-0.29	1.24	-4.46	7.42
x05	72144	-0.29	1.23	-4.59	7.34
EGM96	72144	-0.31	1.17	-4.89	7.26

Table 2b. - The GEOMED residual geoid ($N_{\text{GEOMED}} - N_{\text{MOD}}$)

The final result of the merging procedure has to be considered a relative geoid, as the computation areas have been unified removing the biases between them, and considering the central area as the reference one; therefore the biases between global models and GEOMED are not significant.

THE ITALIAN GRAVITY DATA BASE

The Italian gravity anomaly data base was reduced to an average spacing of 5' among gravity values: gravity is in the IGSN71 system and the normal field is the GRS80 reference field. As pointed out before, the Corsica Island is now properly taken into account by the new models. Two different tests were performed: the former was on the residual values, i.e. observed anomalies minus contributions of the model and of the RTC; the latter was on the observed values minus the model contribution only.

The statistics of the original data and of the differences are in Table 3a, Table 3b, and Table 3c respectively. The map of the point positions and the contour plot of the observed values minus the geopotential model contributions are in Fig.3 - Fig.4.

	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max (mgal)
ITGRAV	16414	7.49	48.03	-162.36	249.35

Table 3a. - The Italian gravity anomaly data base (obs.)

model	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max (mgal)
osu91a	16414	0.48	16.61	-197.34	123.30
x01	16414	0.42	13.96	-181.46	109.53
x02	16414	0.48	14.03	-185.63	107.17
x03	16414	0.35	14.01	-181.22	110.12
x04	16414	0.33	14.26	-183.81	109.69
x05	16414	0.33	14.63	-185.53	111.40
EGM96	16414	0.51	14.47	-188.32	108.59

Table 3b. - Italian residual gravity anomaly (obs.- mod. - RTC)

model	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max (mgal)
osu91a	16414	-6.00	34.03	-242.84	224.43
x01	16414	-6.06	31.34	-236.32	178.48
x02	16414	-6.00	31.67	-242.86	177.32
x03	16414	-6.13	31.24	-235.74	177.82
x04	16414	-6.15	31.37	-240.98	175.91
x05	16414	-6.15	31.52	-247.00	173.89
EGM96	16414	-5.97	31.85	-249.24	176.87

Table 3c. - Italian residual gravity anomaly (obs.- mod.)

A general improvement can be observed in residual gravity anomalies statistics when considering the new geopotential models; the st.dev. improves from 34 mgal to 31.5 mgal (obs-models) and from 16.7 mgal to 14-14.7 mgal once RTC is taken into account.

In order to check the behaviour in regions with high variability in topography and in geoid undulation, local statistics have been computed in the area $44 \leq \varphi \leq 47$, $6 \leq \lambda \leq 13$ comprising Alps and Po Valley; the results are summarized in Table 3d and 3e.

model	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max (mgal)
osu91a	2774	-2.52	22.05	-197.34	114.14
x01	2774	0.79	18.26	-181.46	109.53
x02	2774	-0.10	18.06	-185.63	107.17
x03	2774	1.41	18.27	-181.22	110.12
x04	2774	1.33	18.13	-183.81	109.69
x05	2774	1.47	18.57	-185.53	111.40
EGM96	2774	-0.48	18.54	-188.32	108.59

Table 3d. - Residual gravity anomaly in Northern Italy (obs.- mod. - RTC)

model	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max (mgal)
osu91a	2774	-34.56	55.19	-242.84	138.55
x01	2774	-31.26	51.62	-236.32	139.99
x02	2774	-32.15	52.29	-242.86	136.14
x03	2774	-30.64	51.62	-235.74	140.76
x04	2774	-30.72	52.12	-240.98	137.94
x05	2774	-30.58	52.71	-247.00	135.59
EGM96	2774	-32.53	52.72	-249.24	133.63

Table 3e. - Residual gravity anomaly in Northern Italy (obs.- mod.)

As can be easily observed comparing Tab.3d, 3e with Tab.3b, 3c, the statistics in this restricted area are markedly worse with respect to the whole computation area, particularly before considering the terrain effect.

The set of new models, however, still shows an improvement in comparison with OSU91A.

THE DEFLECTIONS OF THE VERTICAL

The deflections of the vertical over Italy have been measured and collected from IGMI (Istituto Geografico Militare Italiano, the Italian national survey bureau) (Coticchia et al., 1995). They have been measured in the national datum and then transferred to the WGS84 geocentric datum. As for the gravity anomaly data base, two comparisons have been carried out: the former is on the differences between observed minus model and RTC contributions; the latter is on the differences between the observed and the model values. Statistics are in Table 4a, Table 4b and Table 4c: data distribution is in Fig. 5.

model	total values	mean (")	st. dev. (")	min (")	max. (")
ξ	264	-2.63	9.55	-31.60	17.00
η	232	3.71	9.03	-11.50	30.60

Table 4a. - Deflection of the vertical (obs.)

component ξ (")						component η (")				
model	N.val	mean	st.dev	min	max	N.val	mean	st.dev	min	max
osu91a	264	0.05	3.40	-10.07	13.21	232	-0.17	3.01	-10.32	8.37
x01	264	-0.47	2.76	-10.40	10.31	232	0.22	2.59	-6.95	7.78
x02	264	-0.49	2.73	-10.37	10.25	232	0.11	2.61	-7.03	8.00
x03	264	-0.50	2.77	-10.52	10.39	232	0.26	2.61	-7.02	7.95
x04	264	-0.50	2.79	-10.80	10.40	232	0.21	2.63	-6.97	7.65
x05	264	-0.54	2.86	-11.38	10.59	232	0.25	2.72	-7.62	8.38
egm96	264	-0.48	2.78	-11.01	10.10	232	0.14	2.67	-7.50	8.50

Table 4b. - Deflection of the vertical (obs. - model - RTC)

component ξ (")						component η (")				
model	N.val	mean	st.dev	min	max	N.val	mean	st.dev	min	max
osu91a	264	0.24	5.31	-17.74	16.85	232	-0.13	4.66	-16.55	17.68
x01	264	-0.29	5.02	-18.52	15.42	232	0.26	4.38	-13.65	16.69
x02	264	-0.30	4.93	-17.96	15.26	232	0.15	4.42	-14.19	16.74
x03	264	-0.32	5.03	-18.55	15.36	232	0.30	4.39	-13.59	17.00
x04	264	-0.32	4.99	-18.18	15.37	232	0.26	4.40	-13.85	16.81
x05	264	-0.36	4.91	-17.71	15.64	232	0.29	4.44	-13.91	17.01
egm96	264	-0.30	4.87	-17.64	15.57	232	0.18	4.44	-14.09	16.86

Table 4c. - Deflections of the vertical (obs. - model)

The main difference in the statistics of the components ξ , η derived by the new set of models is in the change of sign of the mean values, indicating that an average tilt between OSU91A and the new models is present; this can be explained with a difference in the reference system used for the new global models.

GPS/LEVELING IN ITALY

Several GPS/leveling campaigns have been performed in the Italian area (Surace, 1994; Asili et al. 1995; Achilli et al. 1994). The most relevant among them have been selected for

the comparisons with the geopotential models: the Tyrrhenian geotraverse, the IGMI campaign in the southern part of Italy and measurements covering the Sardinia Island. (see Fig. 6). The statistics of the original values are in Table 5a, Table 6a and Table 7a. The statistic of the observed minus the model values are in Table 5b, Table 6b and Table 7b, and in Table 5c, Table 6c, Table 7c the residual terrain correction (RTC) is also subtracted.

total values	mean (m)	st. dev.(m)	min (m)	max. (m)
28	47.03	2.45	39.60	49.43

Table 5a. - Tyrrhenian Geotraverse (obs.)

model	total values	mean (m)	st. dev.(m)	min (m)	max. (m)
osu91a	28	0.02	0.36	-0.54	0.87
x01	28	0.07	0.22	-0.34	0.66
x02	28	0.10	0.22	-0.29	0.63
x03	28	-0.03	0.22	-0.45	0.54
x04	28	-0.08	0.21	-0.48	0.41
x05	28	-0.09	0.22	-0.56	0.36
EGM96	28	0.18	0.25	-0.29	0.79

Table 5b. - Tyrrhenian Geotraverse (obs. - mod.)

model	total values	mean (m)	st. dev.(m)	min (m)	max. (m)
osu91a	28	-0.49	0.24	-0.92	0.01
x01	28	-0.44	0.20	-0.80	-0.13
x02	28	-0.41	0.20	-0.79	0.02
x03	28	-0.54	0.25	-0.90	-0.13
x04	28	-0.59	0.30	-1.07	-0.11
x05	28	-0.60	0.32	-1.15	-0.10
EGM96	28	-0.33	0.20	-0.67	0.01

Table 5c. - Tyrrhenian Geotraverse (obs. - mod. - RTC)

total values	mean (m)	st. dev.(m)	min (m)	max. (m)
29	42.41	2.74	36.37	48.21

Table 6a. - IGMI campaign (obs.)

model	total values	mean (m)	st. dev. (m)	min (m)	max. (m)
osu91a	29	-0.65	0.50	-1.66	0.79
x01	29	-0.73	0.40	-1.43	0.35
x02	29	-0.67	0.31	-1.40	0.18
x03	29	-0.89	0.48	-1.53	0.43
x04	29	-0.96	0.46	-1.61	0.28
x05	29	-0.99	0.47	-1.67	0.29
EGM96	29	-0.64	0.40	-1.36	0.36

Table 6b. - IGMI campaign (obs. - mod.)

model	total values	mean (m)	st. dev. (m)	min (m)	max. (m)
osu91a	29	-1.29	0.48	-1.89	-0.10
x01	29	-1.38	0.39	-1.96	-0.54
x02	29	-1.31	0.36	-2.01	-0.71
x03	29	-1.54	0.43	-2.12	-0.46
x04	29	-1.60	0.42	-2.29	-0.61
x05	29	-1.64	0.44	-2.40	-0.60
EGM96	29	-1.29	0.32	-1.90	-0.53

Table 6c. - IGM1 campaign (obs. - mod. - RTC)

total values	mean (m)	st. dev.(m)	min (m)	max. (m)
29	46.99	0.84	45.36	48.16

Table 7a. - Sardinia Island (obs.)

model	total values	mean (m)	st. dev.(m)	min (m)	max. (m)
osu91a	29	-0.74	0.30	-1.40	-0.26
x01	29	-1.38	0.29	-1.83	-0.80
x02	29	-1.22	0.29	-1.89	-0.74
x03	29	-1.45	0.30	-1.87	-0.84
x04	29	-1.50	0.30	-1.92	-0.87
x05	29	-1.49	0.29	-1.92	-0.89
EGM96	29	-1.11	0.28	-1.70	-0.59

Table 7b. - Sardinia Island (obs. - mod.)

model	total values	mean (m)	st. dev.(m)	min (m)	max. (m)
osu91a	29	-1.39	0.27	-1.88	-0.65
x01	29	-2.02	0.21	-2.64	-1.63
x02	29	-1.87	0.23	-2.37	-1.39
x03	29	-2.10	0.21	-2.70	-1.68
x04	29	-2.14	0.21	-2.76	-1.74
x05	29	-2.14	0.20	-2.75	-1.73
EGM96	29	-1.76	0.21	-2.27	-1.28

Table 7c. - Sardinia Island (obs. - mod. - RTC)

The differences among the average biases of Tab.5, Tab.6 and Tab.7 are not significant, as the GPS values come from different campaigns which were not jointly adjusted.

However, a general behaviour can be found through the GPS campaigns and also in the comparisons with ITALGEO95 and GEOMED geoid estimates (see Tab.1b and 2b): it is quite evident the grouping of X03, X04, X05 test models, as well as a similarity in the behaviour of the final EGM96 model with the X02 model.

THE ABSOLUTE GRAVITY DATA BASE

Absolute gravity measurements in the European area have also been taken into account in our tests. The normal GRS80 gravity field have been removed from the observed values and

then the model contribution have been subtracted. The locations of the absolute gravity stations are plotted in Fig. 7 and the corresponding statistics are in Table 8a and Table 8b.

total values	mean (m)	st. dev.(m)	min (m)	max. (m)
43	31.13	61.54	-126.60	132.39

Table 8a. - Gravity anomaly derived from absolute gravity measurements (obs.)

model	total values	mean (mgal)	st.dev.(mgal)	min (mgal)	max. (mgal)
osu91a	43	5.59	37.02	-93.49	79.53
x01	43	5.94	38.79	-101.62	85.89
x02	43	5.05	39.02	-105.45	84.59
x03	43	5.91	39.00	-103.23	86.03
x04	43	5.26	38.94	-104.98	86.22
x05	43	5.01	38.72	-105.55	86.31
EGM96	43	4.77	38.58	-106.26	83.89

Table 8b. - Absolute gravity measurements (obs. -mod.)

For this kind of measurements it is not possible to take into account the residual terrain effect, as it is not available a suitable DTM on Europe.

CONCLUSIONS

A general improvement of the new set of geopotential models in comparison with OSU91A model is clearly shown by the statistics reported through the paper, in terms of mean and standard deviation of residual quantities.

The final model EGM96 proves to be the best among the new models in terms of geoid undulation, at least in global comparisons with ITALGEO95 and GEOMED (comparisons with geotraverses give a not so clear result), but the same conclusion cannot be stated if the different functional Δg is considered.

Acknowledgement

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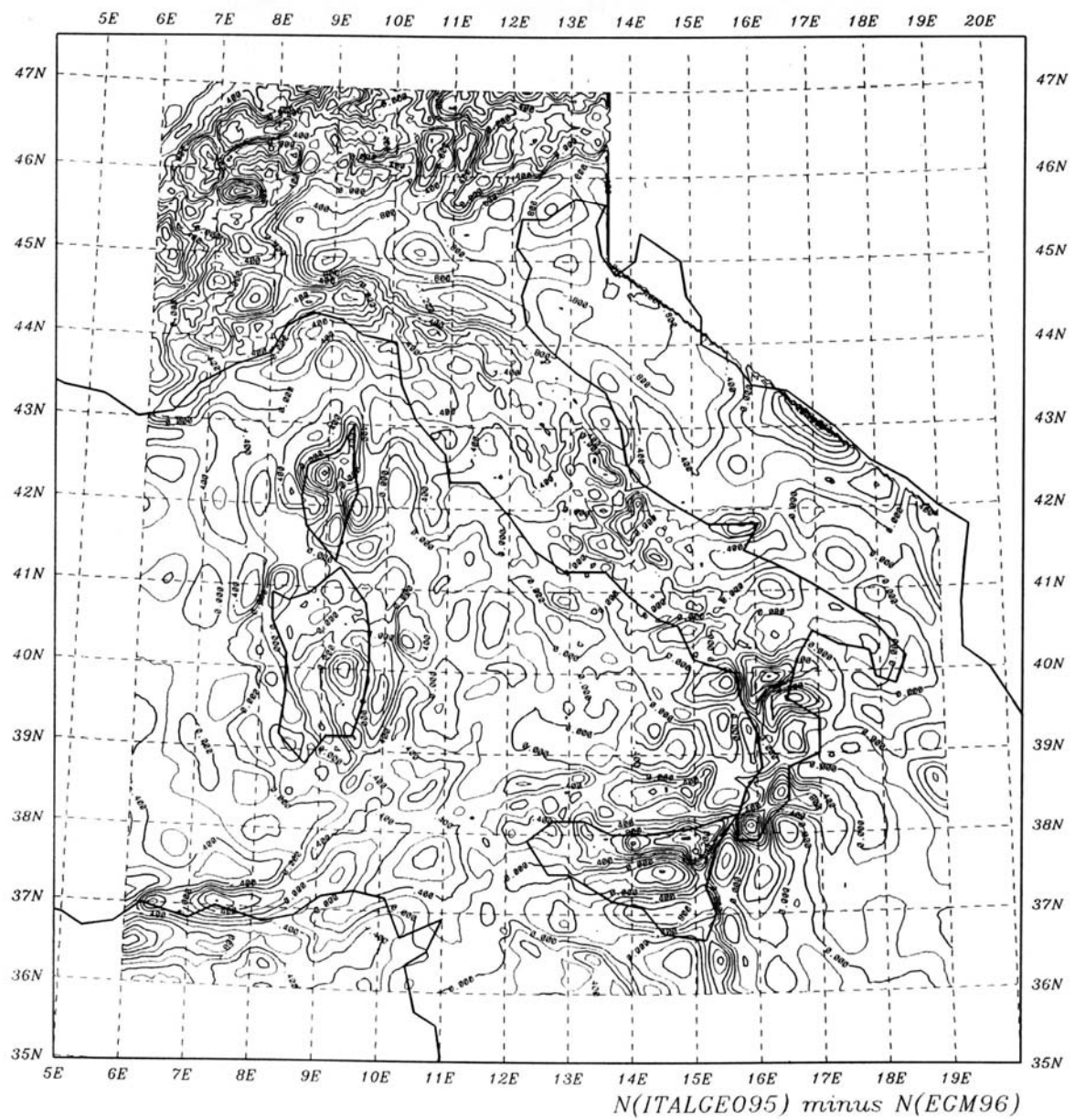


Fig. 1: N ITALGE095 minus EGM96

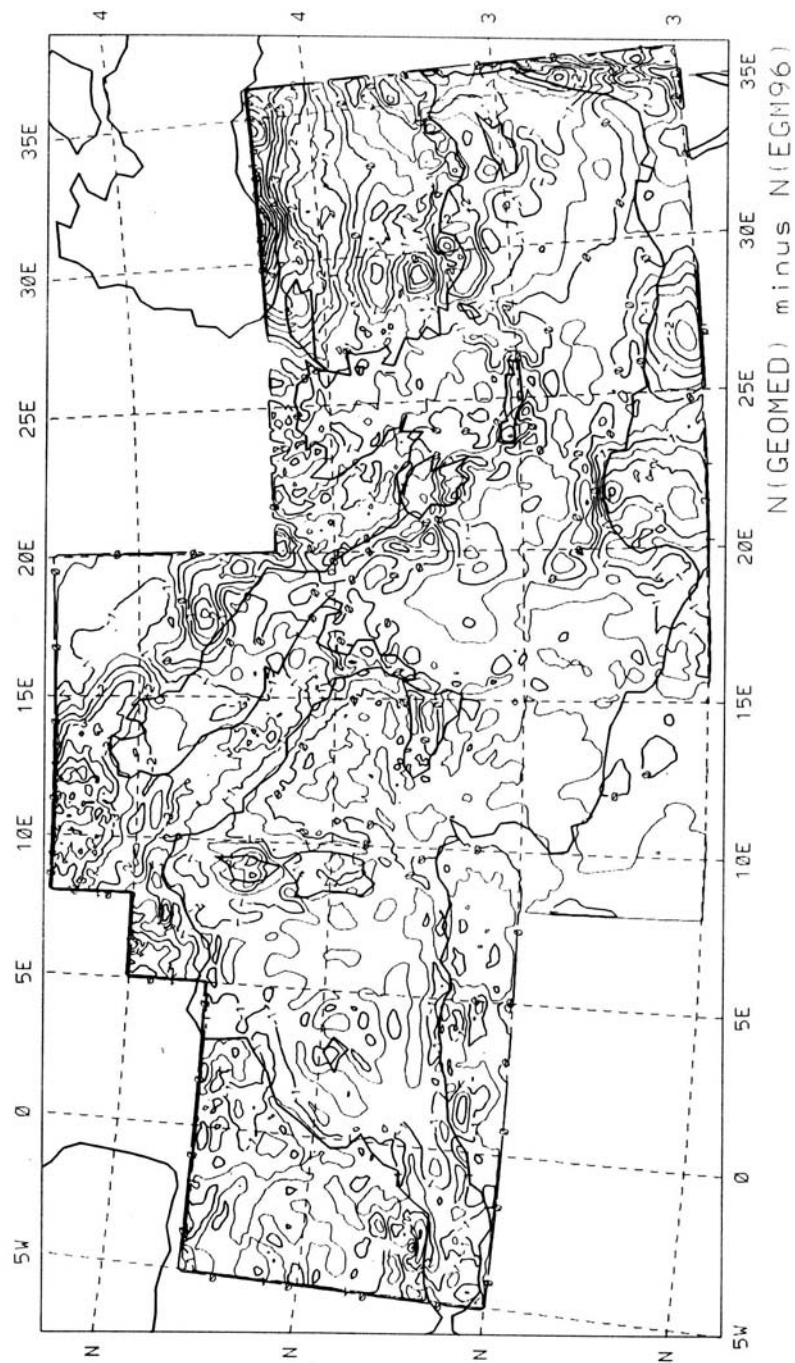


Fig. 2: N GEOMED minus EGM96

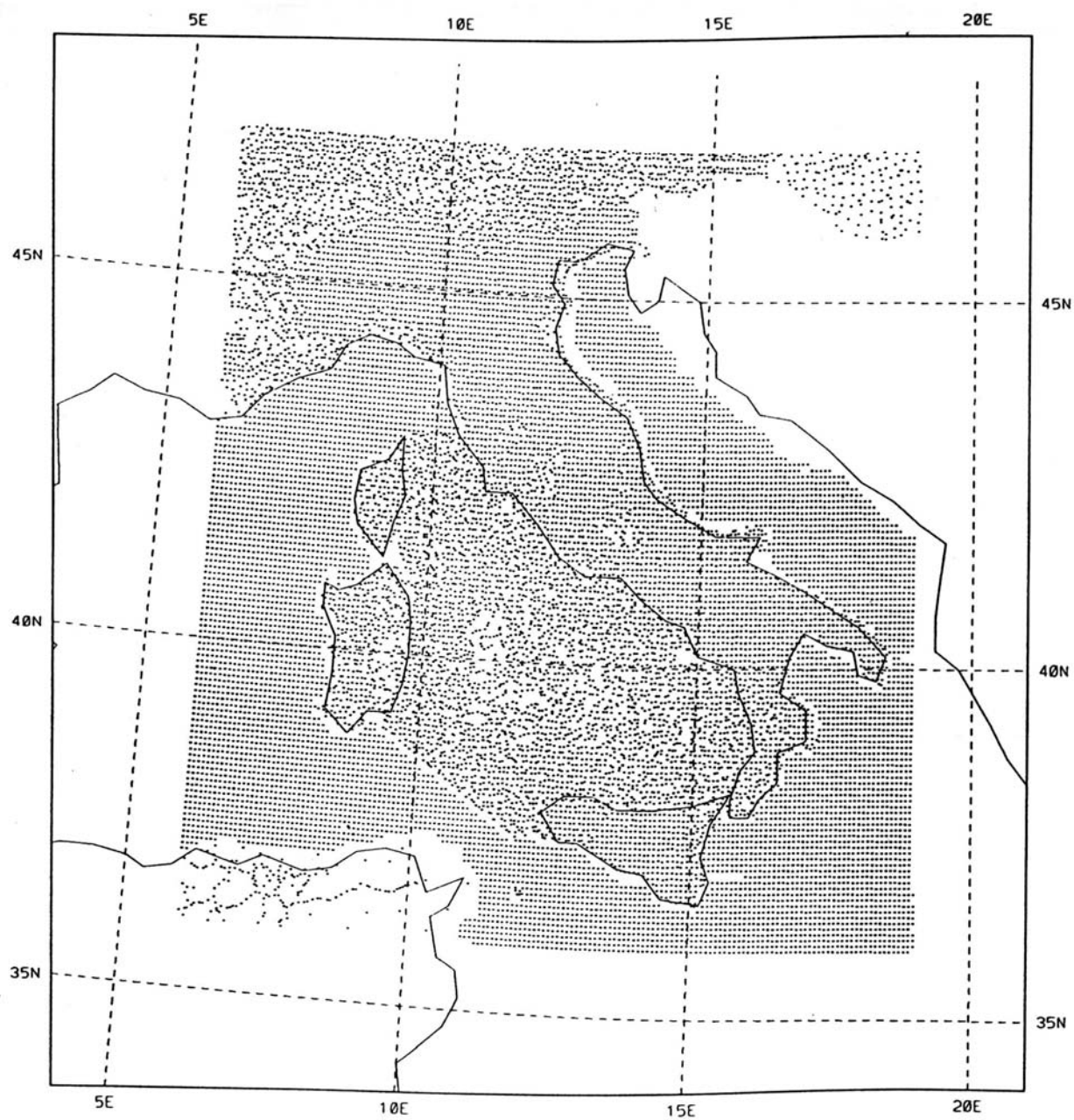


Fig. 3: gravity anomaly data base

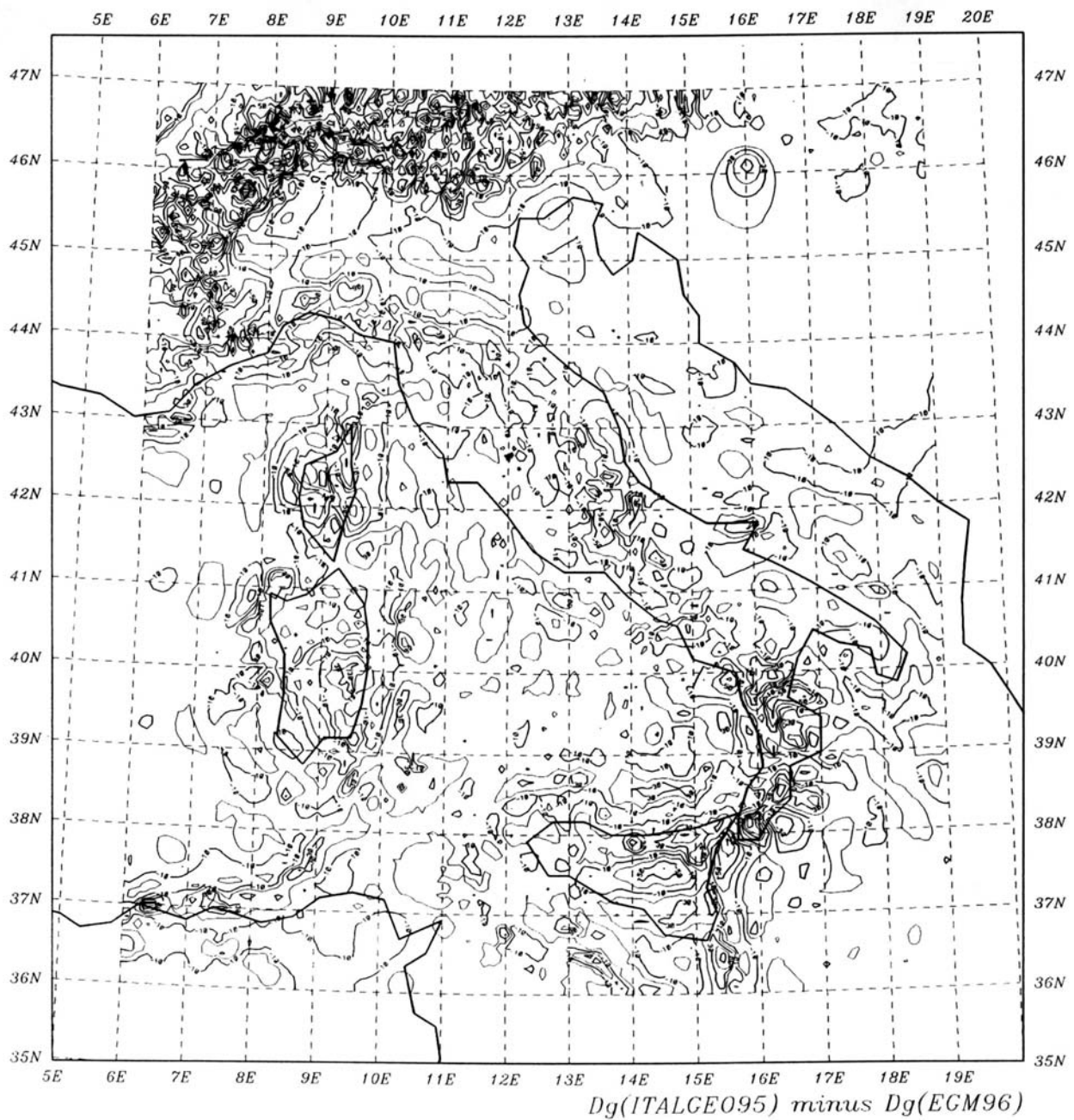


Fig. 4: Δg observed minus EGM96

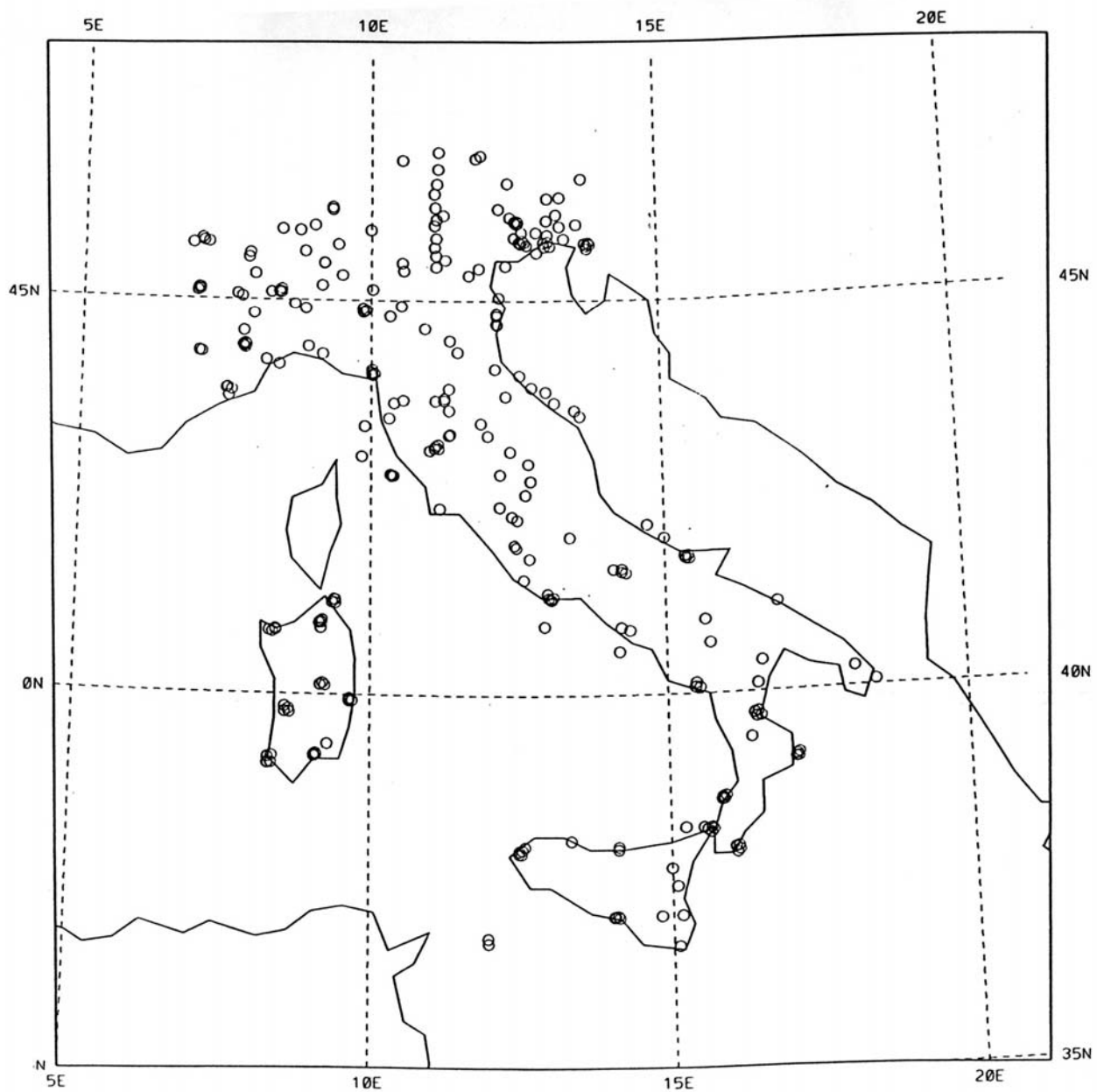


Fig. 5: vertical deflection data base

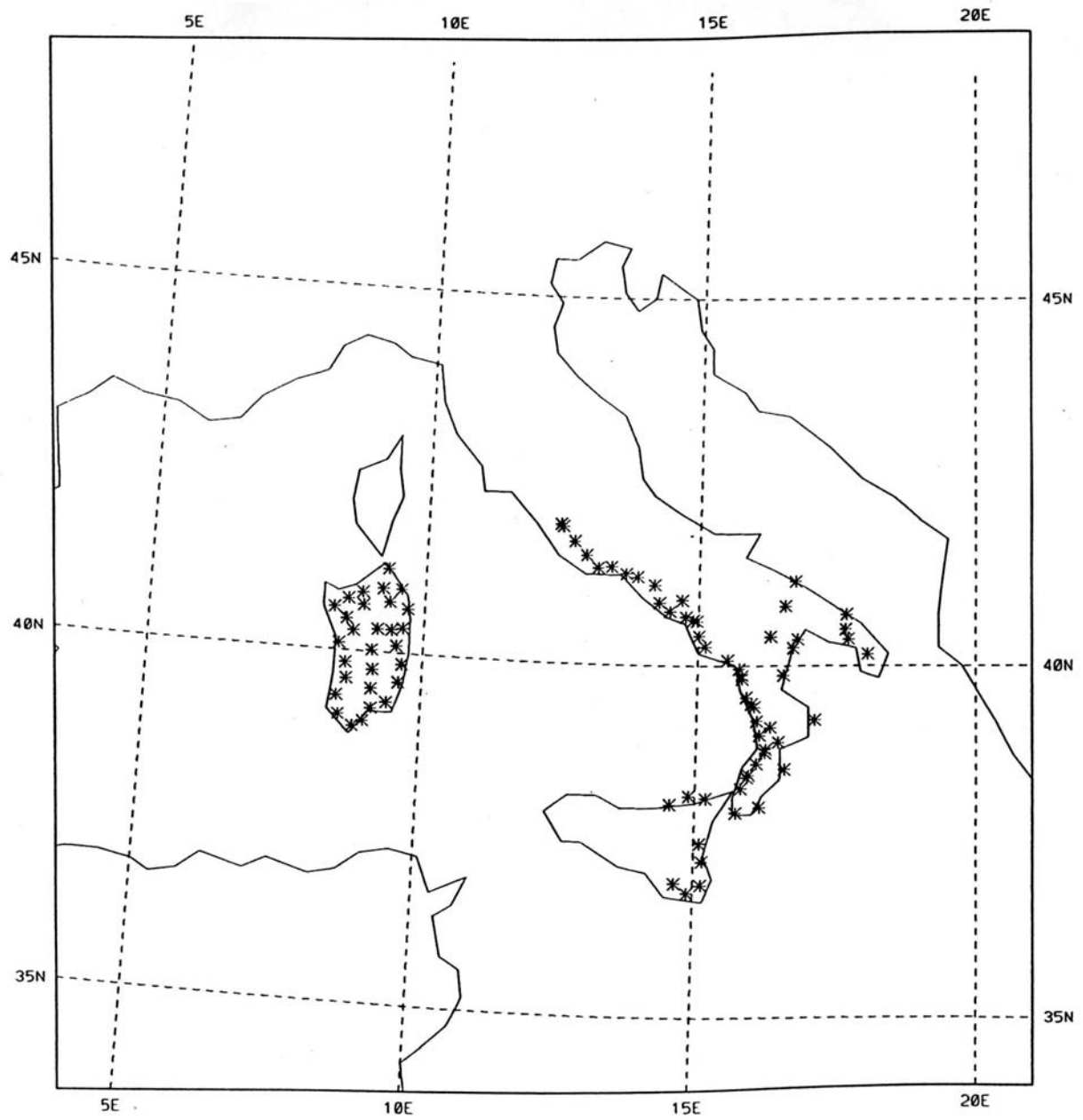


Fig. 6: GPS campaigns

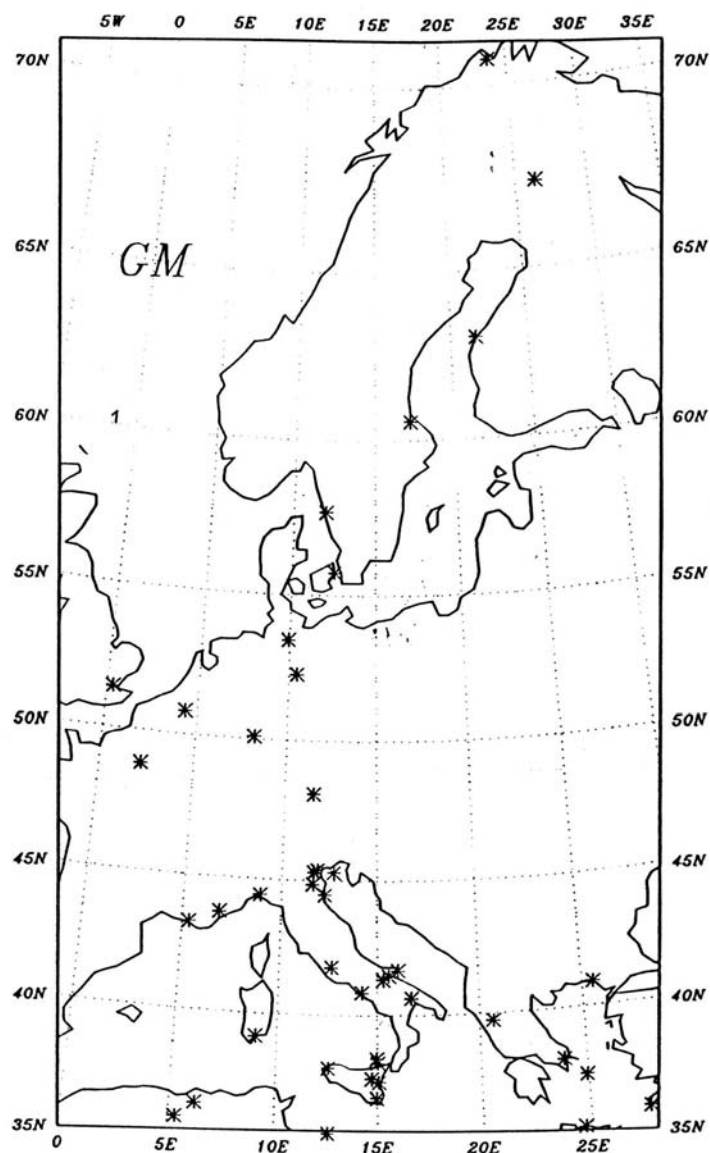


Fig. 7: absolute gravity data base

Evaluation of the new GSFC/DMA geopotential models in the Hellenic area and the Mediterranean sea

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Abstract

The evaluation of the new geopotential models developed by the U.S. Defense Mapping Agency (DMA) and the NASA Goddard Space Flight Centre (GSFC) is described in this study. The focus is the behavior of (a) the five preliminary models (EGM-X01 through EGM-X05), (b) the proposed final optimal geopotential solution EGM96 (c) the OSU91A geopotential model and (d) the combined geopotential solution JGM3_OSU91A, in the Hellenic area and the surrounding sea region and the Mediterranean sea as well. The data sets used in the numerical tests are: (a) point free-air gravity anomalies irregularly distributed in the continental Hellenic area, (b) gridded point free-air gravity anomalies for the Mediterranean sea, (c) gridded satellite altimetric heights in three individual areas of the Mediterranean sea, (d) GPS/leveling measurements available in stations located in the continental Hellenic area and in several islands, (e) deflections of the vertical for the Hellenic area and (f) the most accurate geoid solution for the Hellenic area. The results show that the EGM-X01 through EGM-X05 models give, generally, better results than the solutions OSU91A and JGM3_OSU91A, but it is very difficult to distinguish and nominate which model is the best for our test areas, since there is no a clear out performance of one of the models over the others. Additionally, it should be noted that some unexpected results were found in the Eastern Mediterranean, where the OSU91A and the JGM3_OSU91A solutions are marginally superior when sea gravity data are considered.

1. Introduction

The U.S. Defense Mapping Agency (DMA) and the NASA Goddard Space Flight Centre (GSFC), were decided to jointly develop, with the assistance of the Ohio State University and the U.S. Naval Surface Warfare Centre, an improved potential coefficient model complete to degree and order 360. The intent was to produce a new accurate global geoid model with an absolute accuracy between ± 0.5 m and ± 1.0 m. Besides the need to determine an accurate global geoid undulation model, the new improved geopotential model will help to determine a reference surface for a uniform World Height System (Rapp and Nerem, 1994). In addition the new geoid estimation will help important studies in ocean circulation (sea surface topography), the geodetic positioning through GPS, and will meet the accuracy requirements of charting, mapping, navigation, oceanography and orbit determination (Sideris, 1995).

Due to different data selection, data weighting and estimation techniques, six preliminary models have been first determined and released by GSFC (responsible person Dr. N. Pavlis). In the IUGG Assembly in Boulder, August 1996, has been decided the International Geoid Service (IGeS) and its president Prof. Fernando Sansò to provide leadership to the evaluation effort of these preliminary models. Then, a Special Working Group (SWG) was created, chaired by Prof. M.G. Sideris, in order to carry out the actual model evaluation procedure. The author of the present report, as a member of the above mentioned SWG, was responsible to test these preliminary geopotential models in the Hellenic area and the Mediterranean sea.

In the IAG International Symposium GraGeoMar (Gravity, Geoid and Marine Geodesy) held in Tokyo from September 30 to October 5, 1996, the results of the tests performed by the members of the aforementioned SWG and others individuals on the evaluation of the six geopotential solutions were presented by Sideris (1996). After the extensive testing of these preliminary models and according to the results obtained and the conclusions drawn, a final best global geopotential model was computed by GSFC (Pavlis et al., 1996), further on called EGM96.

In this study the evaluation of all the above mentioned geopotential models is attempted. The statistical analysis performed is primarily aiming to assess which of the new geopotential models is most suitable for the Hellenic area and the Mediterranean sea and if the final solution EGM96 proposed by GSFC as the optimal one really outperforms the other models in our test areas. The entire research was based on the two following fundamental approaches: In the frame of the first approach sea and land free-air gravity anomalies, geoid heights and deflections of the vertical were converted to corresponding residual values by subtracting the contribution of each geopotential model. Statistical analysis was then performed to these differences mainly focusing on the determination of the mean value, minimum/maximum values, standard deviation (SD) and root mean square (rms) error of the aforementioned residuals. In the frame of the second approach, two sets of stations were available, which were coordinated in terms of WGS84 and also had orthometric heights. For each station the orthometric height was subtracted from the ellipsoidal height to determine the "geometric" geoid height. The ellipsoidal height at each station was derived from GPS measurements. Then, the model geoid height was calculated at each station. The differences between the "geometric" geoid heights and the model ones were statistically analyzed.

2. Geopotential models and data sets

The geopotential models tested in the frame of this report are:

- The widely used, during the last decade, in physical geodesy applications "OSU91A" geopotential solution (Rapp et al., 1991).
- The five preliminary geopotential solutions of the joint GSFC/DMA Gravity Improvement Project Team, designated by "EGM-X01", "EGM-X02", "EGM-X03", "EGM-X04", "EGM-X05".
- A composite combination solution, designated by JGM3_OSU91A, that consists of the harmonic coefficients of the JGM3 model (n=2 to 70) augmented by the coefficients of the OSU91A model (n=71 to 360). This composite model represents the state-of-the-art before the joint project activities and is included in the following comparisons. The above models are called in the sequel "EGM models".
- The final optimal EGM96 geopotential solution.

The coefficients and their standard deviations of all the geopotential models are consistent with values of GM and "a" equal to (Pavlis, 1997):

$$GM = 3986004.415E+8 \text{ m}^3/\text{s}^2$$

$$a = 6378136.3 \text{ m.}$$

These values were also used in the numerical tests of this study.

The following data sets have been analyzed individually in the present report:

"emgrav" 8833 point 5'x5' free-air gravity anomalies (73x121 matrix) in the eastern Mediterranean covering the area $31^{\circ} \leq \phi \leq 37^{\circ}$, $26^{\circ} \leq \lambda \leq 36^{\circ}$ and referred to the Geodetic Reference System 1980 (GRS80). These gravity values were derived by combining data from the following data sources: (1) free-air anomaly maps; (2) individual point sea surveys; and (3) different national and international data banks (e.g., GEOMED bank). From the irregular representation of the original gravity data the above mentioned 5'x5' grid was finally resulted.

"cmgrav" 5184 point 5'x5' free-air gravity anomalies (72x72 matrix) in the central Mediterranean covering the area $32^{\circ} \leq \phi \leq 38^{\circ}$, $15^{\circ} \leq \lambda \leq 21^{\circ}$ and referred to the Geodetic Reference System 1980 (GRS80). These gravity values were mainly derived from free-air anomaly maps and national and international data banks. From the irregular representation of the original gravity data the above mentioned 5'x5' grid was finally resulted.

"wmgrav" 6480 point 5'x5' free-air gravity anomalies (60x108 matrix) in the western Mediterranean covering the area $37^{\circ} \leq \phi \leq 42^{\circ}$, $1^{\circ} \leq \lambda \leq 9^{\circ}$ and referred to the Geodetic Reference System 1980 (GRS80). These gravity values were mainly derived from free-air anomaly maps and national and international data banks. From the irregular representation of the original gravity data the above mentioned 5'x5' grid was finally resulted.

"emalt", *"cmalt"*, *"wmalt"* Files containing altimetric heights from the ERS-1 Geodetic Mission (GM) covering the same geographic test areas and grids as the previous three gravity files. This was made to meet the requirements of other applications performed in the same test area, i.e., to use simultaneously gravity and altimetry in order to recover quantities related to the gravity field (e.g., gravity anomalies, geoid heights, etc.). The mean distance between neighboring tracks of the ERS-1/GM is about 5 km in the Mediterranean sea, and from the original along track altimeter data the 5'x5' grids were produced according to an appropriate procedure and using the program "geogrid.f" of the "gravsoft" software (Tscherning et al. 1992).

"pgrav" 1759 point free-air gravity anomalies distributed in the continental part of the Hellenic area and bounded by the limits $38.5^{\circ} \leq \phi \leq 40.5^{\circ}$, $20^{\circ} \leq \lambda \leq 24^{\circ}$. The reference system is the GRS80. These gravity values are used in the specific numerical test in their original irregular distribution (mean distance between individual points 5-10 km).

"ksieta.dat" 78 ξ -components and 72 η -components of deflections of the vertical distributed in the Hellenic area and referred to GRS80. All these observed astrogeodetic data were derived from first-order astrogeodetic observations at stations belonging to the first-, second- and third-order trigonometric networks.

"geoid86" An astrogravimetric geoid solution for the Hellenic area ($34^{\circ} \leq \phi \leq 42^{\circ}$, $18^{\circ} \leq \lambda \leq 28^{\circ}$ to GRS80 with a 0.05 resolution (Tziavos 1984; 1986). The data used to compute this solution were point and mean 6'x10' free-air gravity anomalies for the Hellenic area and the surrounding regions, deflections of the vertical and the OSU81 model as the reference surface. Empirical local covariance functions were used for the data and their errors and the estimated external accuracy of the final solution reached the level of 40 cm in an absolute sense.

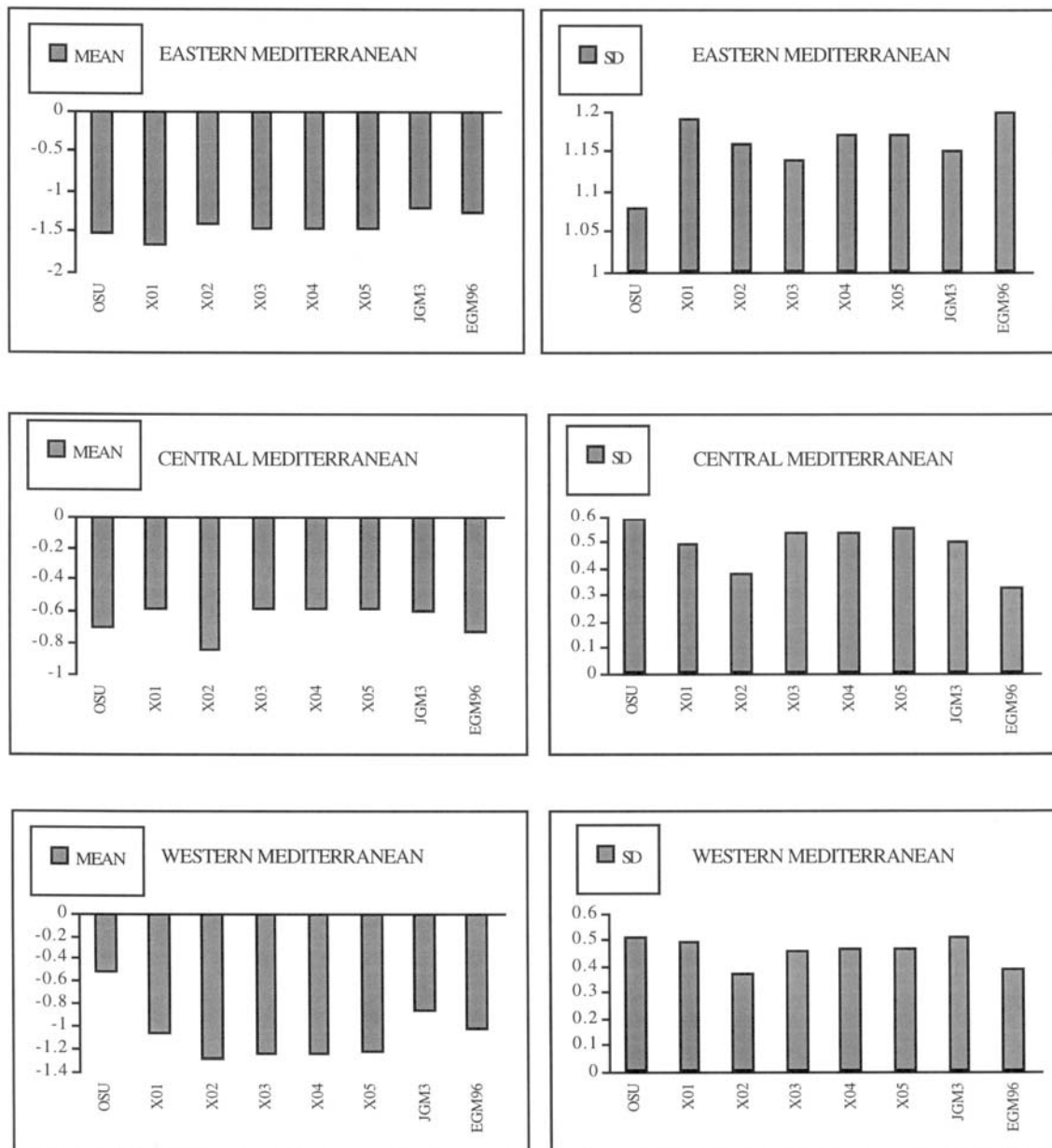
3. Numerical tests and results

The following numerical tests were carried out in order to assess the validity of the previously mentioned geopotential models in our test areas.

1st numerical test: Unadjusted 5'x5' altimetric geoid heights from the ERS-1 GM distributed in the three individual test areas of the Mediterranean sea are compared with corresponding heights derived from the tested geopotential models. The such derived differences are statistically analyzed and the results are summarized in Table 1. In Figure 1 the mean value and the corresponding sd from each comparison are depicted. It is apparent that the EGM models give similar results, which are slightly better than those obtained by OSU91A and JGM3_OSU91A models in the central test area in terms of the mean value and the sd of the differences. The solution EGM96, proposed as the optimal one by GSFC, gives superior results than the other solutions in the central and western test areas of the Mediterranean. It is noticing that the overall fit of the OSU91A is also quite good in the three test subareas and in several cases (eastern) better than that of the EGM models only in terms of sd. Since all the solutions are affected by a significant bias varying from an average value of 65 cm in the central part to a 1.20 m in the western part and 1.40 m to the central part, all the solutions give quite similar results in terms of the rms value.

2nd numerical test: Geoid heights of the combined astrogravimetric geoid solution for the Hellenic area, which is considered as the most accurate current solution for the region, are compared with corresponding geoid heights from the tested geopotential models and the statistical results of their differences are given in Table 2. The representations of such differences are shown also in Figure 2. Quite similar results are obtained by the EGM models, but the results from the OSU91A and JGM3_OSU91A models are superior in terms of sd. This can be mainly attributed to the fact that the gravity data used in the combined geoid solution for the Hellenic area were also used in the construction of the OSU91A model, and, eventually, more recent gravity data sets were used in the construction of the EGM models. It is noticing that the bias (mean value) detected in the EGM models (close to 90 cm) is considerably smaller than that of the OSU91A and JGM3_OSU91A models (about 1.10 m) and consequently the rms values are better for the EGM models than those found for the OSU91A model and the combined solution. From Figure 2 is apparent that the representations of all the geoid height differences are very similar, while the major discrepancies are observed in the north-west and south-west parts of the Hellenic area.

3rd numerical test: The 1st numerical test is repeated here, but now, instead of altimetric geoid heights, sea 5'x5' point-free air gravity anomalies are used. These observed gravity data are compared with corresponding values calculated from the tested geopotential model and the statistics of such differences is given in Table 3. All the EGM models present very similar difference representations; as examples the corresponding representations from X02, X05 and EGM96 models are shown in Figures 3, 4 and 5. In the central test area the EGM models outperform the two other solutions. The EGM96 solution outperforms all other models in the central test area. Between the other EGM models the EGM-X02 model gives slightly better results. In the western and eastern test subareas the results from the OSU91A and the JGM3_OSU91A models are better, but this is also attributed to the fact that these gravity data sets were used to the construction of the OSU91A model. Another numerical test concerns the statistical analysis of point gravity anomalies distributed in the Hellenic area (file "pgrav"). The same statistical analysis was performed and the results are tabulated in Table 4. In this test, where the gravity data are more accurate than the sea gridded data used previously, the improvement of the results with the EGM models over the OSU91A and the combined solution is quite astonishing.



OSU means the OSU91A solution.

* JGM3 means the combined JGM3_OSU91A model.

Fig. 1. Statistical results of ERS-1 GM geoid height differences for the three test subareas in the Mediterranean sea. Unit: [m].

Table 1. Statistics of observed and reduced to GSFC/DMA geopotential models altimeter heights (ERS-1/GM) in the Mediterranean sea. Unit: [m]

Eastern Mediterranean									
N		N (observed) - N (model)							
Statistics	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	8833	8833	8833	8833	8833	8833	8833	8833	8833
mean	15.55	- 1.55	- 1.68	- 1.44	- 1.50	- 1.50	- 1.49	- 1.22	- 1.30
min.	0.04	- 6.91	- 8.17	- 8.13	- 7.90	- 8.02	- 7.93	- 7.41	- 8.18
max.	36.03	4.14	2.28	2.75	2.47	2.38	2.31	4.28	2.77
sd	7.05	1.08	1.19	1.16	1.14	1.17	1.17	1.15	1.20
rms	17.08	1.89	2.06	1.85	1.89	1.90	1.89	1.68	1.77
Central Mediterranean									
N		N (observed) - N (model)							
Statistics	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	5184	5184	5184	5184	5184	5184	5184	5184	5184
mean	27.80	- 0.71	- 0.60	- 0.85	- 0.59	- 0.59	- 0.60	- 0.61	- 0.74
min.	14.42	- 3.14	- 3.19	- 3.37	- 3.20	- 3.28	- 3.31	- 3.39	- 3.17
max.	41.82	1.69	1.50	0.97	1.52	1.47	1.46	1.47	1.28
sd	4.04	0.59	0.49	0.38	0.54	0.54	0.55	0.50	0.33
rms	28.10	0.93	0.77	0.94	0.80	0.80	0.81	0.79	0.81
Western Mediterranean									
N		N (observed) - N (model)							
Statistics	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	6480	6480	6480	6480	6480	6480	6480	6480	6480
mean	45.21	- 0.54	- 1.08	- 1.30	- 1.26	- 1.26	- 1.25	- 0.88	- 1.04
min.	40.58	- 2.41	- 4.22	- 3.93	- 4.27	- 4.42	- 4.48	- 2.64	- 4.13
max.	49.77	1.44	0.63	0.41	0.41	0.46	0.42	0.99	0.38
sd	2.15	0.51	0.49	0.37	0.46	0.47	0.47	0.51	0.39
rms	45.26	0.74	1.19	1.35	1.34	1.35	1.34	1.01	1.11

Table 2. Statistics of differences between geoid heights from the combined geoid solution for the Hellenic area and corresponding heights derived from the GSFC/DMA geopotential models. Unit: [m]

N		N (observed) - N (model)							
Statistics	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	357	357	357	357	357	357	357	357	357
mean	29.93	- 1.16	- 0.88	- 0.92	- 0.92	- 0.92	- 0.91	- 1.03	- 0.96
min.	0.73	- 6.53	- 6.76	- 6.67	- 6.80	- 6.85	- 6.92	- 7.74	- 7.02
max.	45.66	3.51	4.93	4.56	4.60	4.63	4.64	3.77	4.70
sd	10.46	1.44	1.63	1.58	1.56	1.57	1.57	1.82	1.67
rms	31.70	1.85	1.85	1.83	1.81	1.81	1.81	2.09	1.94

Table 3. Statistics of observed and reduced to OSU91A and GSFC/DMA geopotential models 5'x5' point free-air gravity anomalies in the Mediterranean sea. Unit: [mGal]

Eastern Mediterranean									
Statistics	Δg	Δg reduced to							
	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	8833	8833	8833	8833	8833	8833	8833	8833	8833
mean	- 24.02	- 3.87	- 4.32	- 3.04	- 3.72	- 3.70	- 3.68	- 2.58	- 2.71
min.	- 212.34	- 86.87	- 104.97	- 104.97	- 104.67	- 111.85	- 112.93	- 82.77	- 114.56
max.	160.73	79.71	71.93	71.93	72.92	74.27	72.78	83.77	74.77
sd	61.20	16.69	23.36	23.68	23.22	23.98	24.26	17.16	24.23
rms	65.74	17.13	23.76	23.88	23.51	24.26	24.53	17.35	24.38
Central Mediterranean									
Statistics	Δg	Δg reduced to							
	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	5184	5184	5184	5184	5184	5184	5184	5184	5184
mean	- 6.48	0.64	0.76	- 1.04	0.50	0.43	0.37	1.00	- 0.23
min.	- 140.03	- 95.13	- 100.74	- 100.33	- 100.83	- 104.74	- 108.44	- 94.68	- 105.08
max.	142.36	121.35	121.08	118.06	120.67	120.16	118.91	120.50	116.74
sd	35.46	16.18	14.99	14.74	14.98	14.91	14.89	16.11	14.69
rms	36.05	16.20	15.01	14.78	14.99	14.91	14.90	16.14	14.69
Western Mediterranean									
Statistics	Δg	Δg reduced to							
	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	6480	6480	6480	6480	6480	6480	6480	6480	6480
mean	7.15	0.88	0.28	- 0.52	- 0.56	- 0.54	- 0.36	0.38	0.68
min.	- 74.39	- 58.65	- 100.95	- 102.07	- 101.29	- 106.16	- 109.61	- 58.39	- 106.94
max.	114.80	69.79	74.76	72.90	73.18	74.41	74.06	70.25	72.95
sd	25.98	13.66	14.95	14.79	14.89	14.97	15.05	13.56	14.97
rms	26.95	13.69	14.95	14.80	14.90	14.98	15.05	13.56	14.98

Table 4. Statistics of observed and reduced to OSU91A and GSFC/DMA geopotential models point free-air gravity anomalies in the continental Hellenic area. Unit: [mGal]

Statistics	Δg	Δg reduced to							
	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	1759	1759	1759	1759	1759	1759	1759	1759	1759
mean	28.25	14.28	11.37	9.45	10.84	10.66	10.62	14.34	10.91
min.	- 167.20	- 114.54	- 110.75	- 109.06	- 111.15	- 110.60	- 109.82	- 114.23	- 107.50
max.	276.92	290.52	278.35	277.50	278.15	278.41	279.81	290.64	280.68
sd	56.44	36.12	36.18	36.02	36.22	36.21	36.30	36.01	36.12
rms	63.12	38.84	37.92	37.24	37.81	37.75	37.82	38.76	37.71

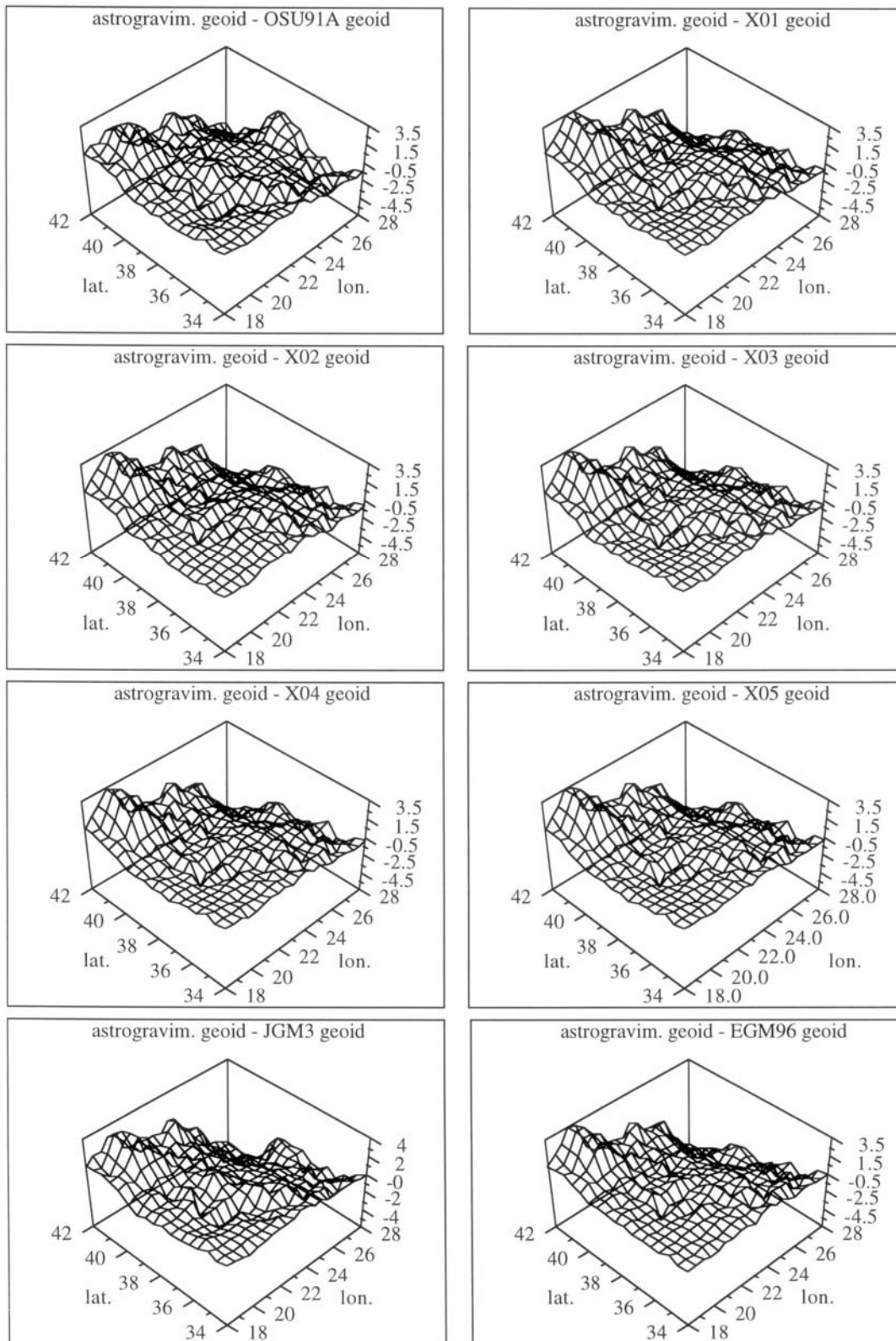


Fig. 2. Representations of geoid height differences between a combined geoid solution for the Hellenic area and corresponding geoid heights calculated from the OSU91A model and the GSFC/DMA models. Unit: [m]

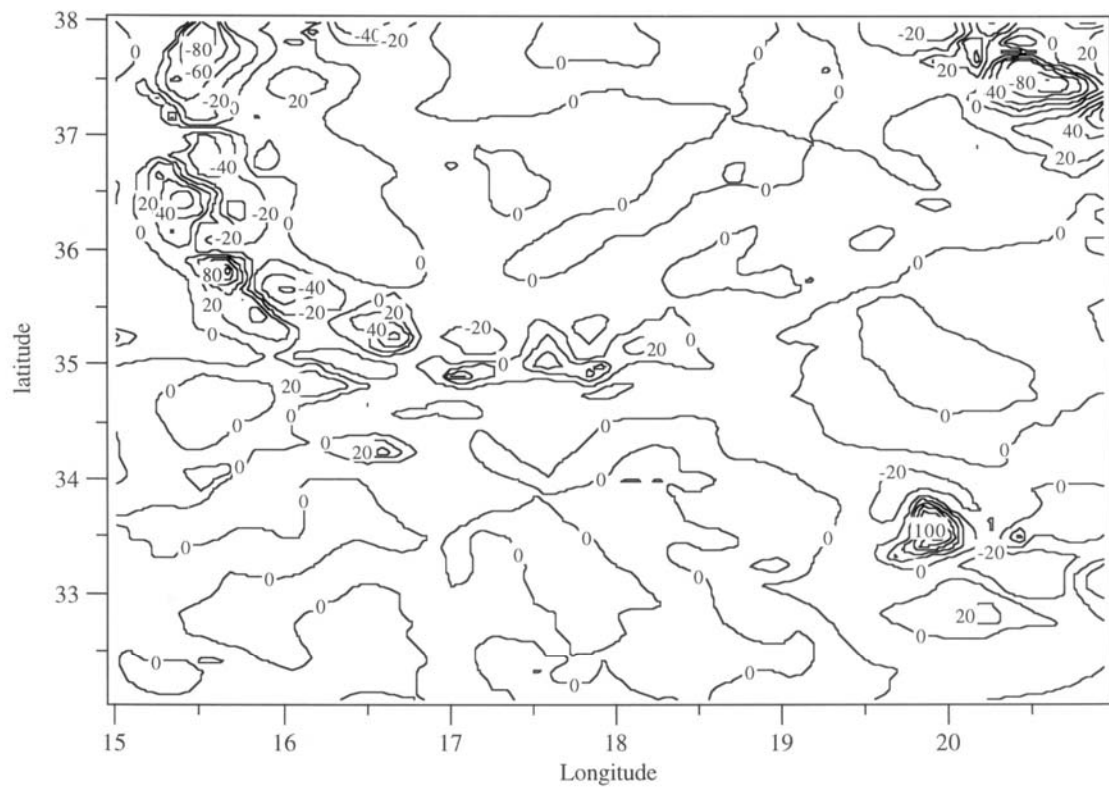


Fig. 3. Gravity anomaly differences between observed gravity and gravity from EGM-X02 in the central Mediterranean. Contour interval: 20 mGal.

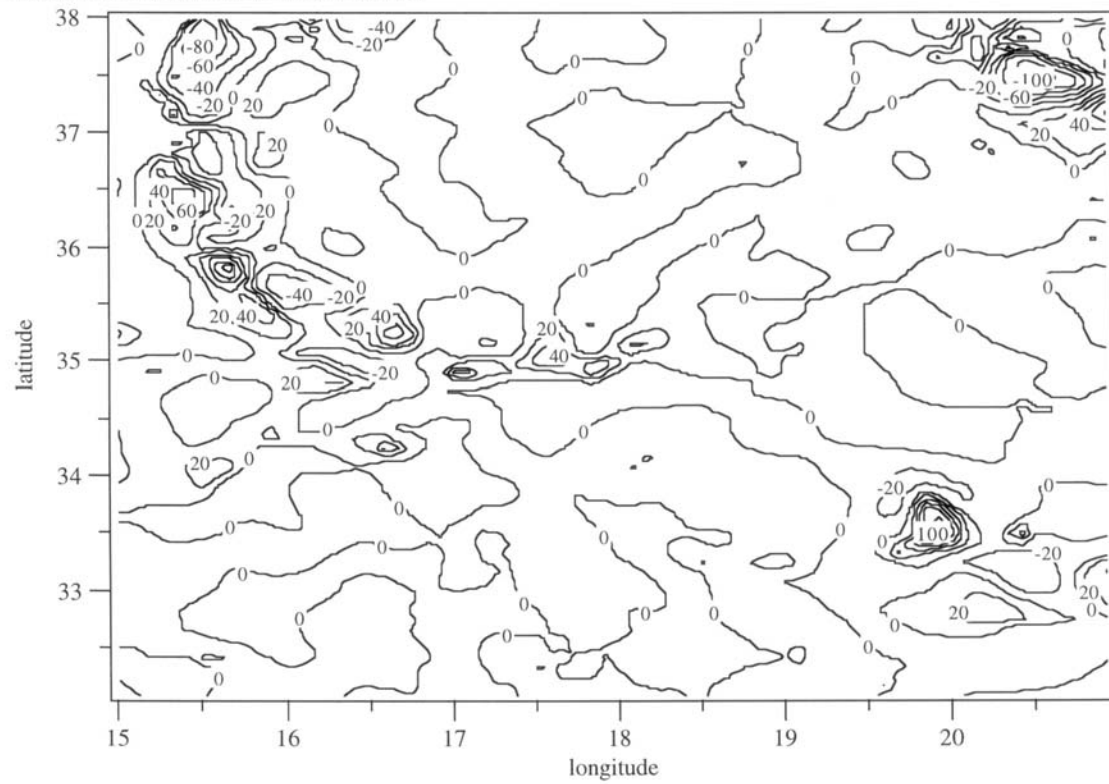


Fig. 4. Gravity anomaly differences between observed gravity and gravity from EGM-X05 in the central Mediterranean. Contour interval: 20 mGal.

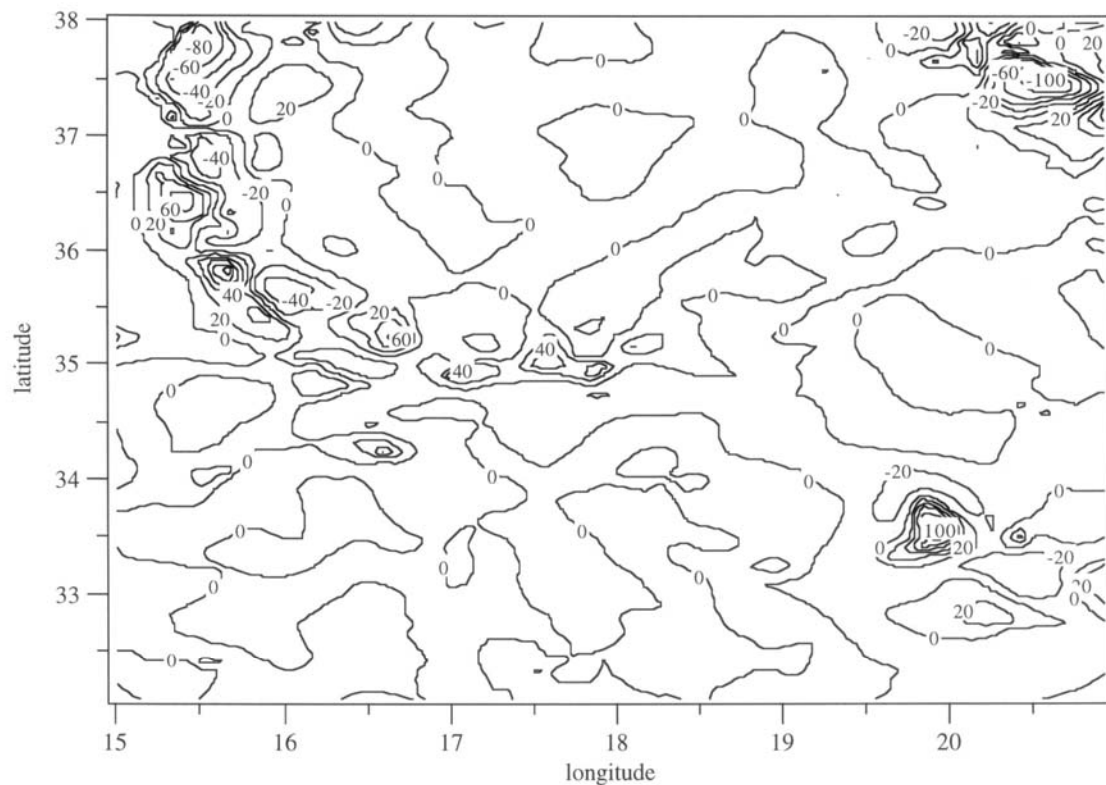


Fig. 5. Gravity anomaly differences between observed gravity and gravity from EGM96 in the central Mediterranean. Contour interval: 20 mGal.

4th numerical test: In this numerical test the available data set of deflections of the vertical for the Hellenic area (file "ksieta.dat") is compared with corresponding components computed from the OSU91A and EGM models. The statistical results are tabulated in Tables 5 and 6. It is obvious a clear out performance of the EGM-X01 through EGM-X05 models over the OSU91A and JGM3_OSU91A models. All the EGM models perform well either considering the mean value or the sd of the differences of both components. More specifically, the best model when comparing the sd is the EGM-X05 for the ξ -component and the EGM-X03 for the η -component. When considering the mean value the preferred model is the EGM-X02 for the ξ -component and the EGM-X04 for the η -component. Generally, the results from the EGM models are practically equivalent, though the fit of the OSU91A model is quite good.

Table 5. Statistics of differences between astrogeodetic deflections of the vertical (ξ -components) and the corresponding values derived from the OSU91A and the GSFC/DMA geopotential models in 78 stations of the Hellenic area. Unit: ["]

Statistics	ξ observed	Differences between ξ -astrogeodetic and ξ -geopotential model							
		OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	78	78	78	78	78	78	78	78	78
mean	- 7.86	- 0.44	- 0.55	- 0.34	- 0.53	- 0.50	- 0.41	- 0.51	- 0.30
min.	- 24.840	-13.89	- 12.24	- 12.46	- 12.30	- 12.09	- 11.65	- 14.84	- 11.77
max.	7.94	14.56	14.70	15.38	14.69	14.82	14.78	14.85	14.85
sd	7.08	4.90	4.78	4.97	4.79	4.80	4.77	5.07	4.82
rms	10.55	4.91	4.81	4.95	4.82	4.82	4.79	5.09	4.84

Table 6. Statistics of differences between astrogeodetic deflections of the vertical (η -components) and the corresponding values derived from the OSU91A and the GSFC/DMA geopotential models in 72 stations of the Hellenic area. Unit: ["]

Statistics	η	Differences between η -astrogeodetic and η -geopotential model							
	observed	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	72	72	72	72	72	72	72	72	72
mean	- 2.72	0.78	0.71	0.83	0.58	0.58	0.61	1.25	0.97
min.	- 23.73	- 8.20	- 8.82	- 8.29	- 8.66	- 8.85	- 8.93	- 7.72	- 8.31
max.	13.66	12.46	14.88	14.73	14.56	14.45	14.39	12.89	14.76
sd	8.04	4.59	4.43	4.46	4.42	4.44	4.44	4.56	4.45
rms	8.44	4.62	4.46	4.51	4.43	4.45	4.45	4.70	4.52

5th numerical test. This last test is composed from two individual numerical tests. In the first one the geoid heights in 23 GPS stations are compared with corresponding geoid heights calculated from the OSU91A model, the combined solution and the EGM models. The coordinates of the stations refer to the WGS84 system and ITRF89 datum. The results from the statistical analysis of the differences are summarized in Table 7. The out performance of the EGM models over the OSU91A solution is very clear. Considering the EGM models, the EGM96 is the best model in terms of sd. When considering the mean value the JGM3_OSU91A is the preferred model, though distinguishing it from EGM-X01 through EGM-X05 at a practical level is difficult. It is worth noting that the EGM models are practically equivalent when comparing the mean value. In the second numerical experiment of the present test the geoid heights from GPS/leveling of a 200 km traverse containing 10 stations are compared with corresponding geoid heights derived from the same geopotential models. The statistical results are given in Table 8. In terms of sd the best models are the EGM96 and EGM-X02, but at the same about level (3 cm) are the sd of the models EGM-X03, EGM-X04 and EGM-X05. When considering the mean value the preferred model is the EGM-X03 (12 cm), but the mean values of the EGM-X04 and EGM-X05 models are practically equivalent.

It is worth noting that in the numerical experiments of this test the GPS/leveling geoid heights and the EGM geoid models were considered "as is" without applying an empirical 4-parameter datum shift model (see Mainville et al., 1992).

Table 7. Statistics of differences between geoid heights from GPS/leveling and corresponding heights derived from the OSU91A and the GSFC/DMA geopotential models in stations located in the Hellenic area. Unit: [m]

Geoid height differences between GPS/lev. and different geopotential models								
Statistics	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_OSU91A	EGM96
No	23	23	23	23	23	23	23	23
mean	- 0.732	- 0.603	- 0.554	- 0.690	- 0.712	- 0.716	- 0.455	- 0.676
min.	- 2.314	- 1.642	- 1.402	- 1.737	- 1.835	- 1.870	- 1.904	- 1.783
max.	0.477	0.689	0.537	0.813	0.792	0.716	0.698	0.436
sd	0.674	0.519	0.499	0.554	0.567	0.564	0.678	0.461
rms	0.985	0.788	0.739	0.877	0.902	0.904	0.804	0.812

Table 8. Statistics of differences between geoid heights from GPS/leveling and corresponding heights derived from the OSU91A and the GSFC/DMA geopotential models in stations belonging to a 200 km traverse in the Hellenic area. Unit: [m]

Geoid height differences between GPS/lev. and different geopotential models								
Statistics	OSU91A	EGM-X01	EGM-X02	EGM-X03	EGM-X04	EGM-X05	JGM3_ OSU91A	EGM96
No	10	10	10	10	10	10	10	10
mean	0.243	0.275	0.540	0.120	0.180	0.208	0.526	0.562
min.	0.174	0.208	0.477	0.053	0.111	0.141	0.458	0.500
max.	0.315	0.324	0.576	0.164	0.224	0.251	0.589	0.598
sd	0.048	0.036	0.028	0.034	0.034	0.033	0.043	0.030
rms	0.247	0.278	0.541	0.124	0.183	0.211	0.527	0.563

4. Conclusions

Based on the data sets used in the present report to evaluate the performance of the preliminary GSFC/DMA geopotential models within Greece and the Mediterranean sea one has to note that in general the new models are an improvement over the OSU91A geopotential model. The fact that several comparisons with sea gravity data show that OSU91A model gives better results can be mainly attributed to the fact that these data sets were used in the construction of the OSU91A solution, while in the construction of the EGM models, eventually, other current data sets replaced the older ones. With respect to the models (EGM-X01 through EGM-X05, JGM3_OSU91A, EGM96) it is worth noting that all models perform well in our test areas and it is difficult to distinguish which is better. A slight out performance of the solution EGM96 over the other models, detected in several tests in the central and western test areas of the Mediterranean, cannot lead to a general conclusion.

More specifically, according to the statistical analysis performed and the data used in our test areas the following conclusions can be drawn:

- The comparisons based on ERS-1 GM geoid heights in three individual test areas in the Mediterranean sea show that all the geoid models are affected by a significant bias. After subtracting the bias, the EGM models give in general slightly better results and the EGM96, EGM-X01 and EGM-X02 are the preferred models. It is worth noting that the OSU91A model fits also well in the areas under consideration.
- The comparisons based on the geoid solution for the Hellenic area show an out performance of the EGM models over the OSU91A in terms of the rms values.
- The statistical analysis based on the sea gridded free-air anomalies shows that OSU91A and the combined solution JGM3_OSU91A models give better results in the eastern and western test areas for the reasons explained before. In the central test area the results from the EGM96 model are superior to those obtained by all other models.
- The results of the EGM models show an improvement over the OSU91A and JGM3_OSU91A results considering the point gravity data comparisons in the continental Hellenic area.

- The comparisons based on the GPS/leveling network and the GPS/leveling traverse show an improvement of the EGM models over the OSU91A model and the combined solution. The comparison results show virtually no significant difference between EGM models.
- The comparisons with deflections of the vertical show better results when using EGM models instead of OSU91A and JGM3_OSU91A models.

Acknowledgments

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QUASIGEOID OF THE CENTRAL EUROPE FROM THE MULTINATIONAL DATA

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Abstract

In agreement with Dr. Gerhardt Soltau from Institut für Angewandte Geodäsie in Leipzig, Germany, the specialists of the Military Topographic Institute of the TS AČR combined the quasigeoids of the Czech Republic and of the territory of the new countries of the Federal Republic of Germany [1] and transformed the common data in the geodetic system ETRF 89.

With regard to the topicality of these solved tasks the proper amplification of this solution and co-operation in definition of the common quasigeoid for the whole Central European Region appear as very prospective for military geodesy and GPS navigation.

Enclosed is the schema of the preliminary solution of the surface of this quasigeoid in ETRF 89.

USED PROCEDURES

The data published in [1] were densified by the additional quotes taken from the enclosed maps and combined with the astro-gravimetric data of the quasigeoid from the Czech Republic territory [2]. United data was in the system S-1942 (PULKOVO).

The following operation transformed this data from the system S-1942 into the ETRF 89. The parameters of the used Helmert transformation were determined independently from the values at the identical points of both systems.

Third step was the interpolation of the isolines of the quasigeoidal surface and their drawing (see Fig.1)

The enclosed schema of the surface of the joint quasigeoid is only of the demonstrative character. It should support the proposal of the TS AČR for establishing of the common working group in the framework of the GGWG which will secure the elaboration of the regional surface of the quasigeoid in the system WGS 84 using the data of proper countries.

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Fig. 1. The surface of the common quasigeoid in ETRF 89 created from the national data.

TESTS FOR ACCURACY OF RECENT GEOPOTENTIAL MODELS

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Abbreviations: GMT – Geopotential Model Testing; GMTN – Geopotential Model
Testing Network; SST – Sea Surface Topography; T/P – TOPEX/POSEIDON

1. Introduction

The methodology for Geopotential Model Testing (GMT) developed by Burke et al. (1995) was applied to the recent EGM models EGM-X01, ..., EGM-X05, EGM96 (Sideris 1995 and 1996, Rapp 1996), and to the previous JGM-3/OSU91A, for comparison. The methodology requires geocentric positions and normal Molodensky heights of the testing sites on the Earth's surface be known as accurate as possible, as well as, the four primary constants defining the level ellipsoid and its gravity field. No hypothetical quantities as the orthometric heights and/or, geoid heights are used in GMT. However, a global coverage by the testing sites is needed.

2. Primary constants used for GMT

An essential progress was made in refining the geoidal potential W_0 (Burša et al., 1996) on the basis of ERS-1/TOPEX/POSEIDON altimeter data (AVISO, 1995) and the sea surface topography model POCM 4B to degree $n = 360$ based on the global circulation model (Rapp and Zhang, 1995):

$$W_0 = (62\,636\,855.8 \pm 0.5) \text{ m}^2 \text{ s}^{-2} . \quad (1)$$

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It is adopted as given inside GMT, besides the other three, i.e. the geocentric gravitational constant

$$GM = (398\,600\,441.8 \pm 0.8) \times 10^6 \text{ m}^3 \text{ s}^{-2} \quad (2)$$

(Ries et al., 1992), the angular velocity of the Earth's rotation

$$\omega = (7\,292\,115.8 \pm 0.8) \times 10^{-11} \text{ rad s}^{-1} \quad (3)$$

and the second zonal Stokes parameter

$$J_2 = (1\,082\,635.9 \pm 0.1) \times 10^{-9} \quad (4)$$

in the zero-frequency tide system (IAG SC3, 1995). Constants (1), (2) and (3) are independent of the tide systems. Because of GMT in the mean tide system, constant (4) should be corrected for the direct permanent tidal effect

$$\delta J_2 = 30.8 \times 10^{-9} \quad (5)$$

(IAG SC3, 1995).

From constants (1) – (5) the three derived can be computed as follows: the geopotential scale factor

$$R_o = \frac{GM}{W_o} = (6\,363\,672.58 \pm 0.05) \text{ m} , \quad (6)$$

the semimajor axis of the level ellipsoid in the mean tide system

$$a = (6\,378\,136.70 \pm 0.05) \text{ m} , \quad (7)$$

and its flattening

$$\alpha = 1 / (298.252\,34 \pm 0.000\,02) . \quad (8)$$

Geopotential scale factor (6) is independent of the tide systems. Parameters (7) and (8) define the mean level ellipsoid, i.e. its figure reflects both the direct as well as indirect zero frequency tidal effects.

The four fundamental constants GM , ω , J_2 , W_o or GM , ω , a , α make it possible to compute the actual potential at any site on the physical Earth's surface normal Molodensky height and/or the geopotential number of which is known.

The testing accuracy is limited by the errors in normal Molodensky heights and in the fundamental constants above. The last amounts $\pm 0.5 \text{ m}^2 \text{ s}^{-2}$ in the geopotential and/or $\pm 0.05 \text{ m}$ in the radius of the local equipotential surface as determined by the geopotential model tested.

Table I. Radial distortions of the equipotential surfaces passing through the oceanic testing sites (20 768 sites).

Geopotential model	Mean [m]	rms [m]
JGM-3/OSU91A	0.002 ± 0.002	± 0.176
EGM-X01	-0.001 ± 0.001	± 0.196
EGM-X02	0.005 ± 0.001	± 0.159
EGM-X03	0.004 ± 0.001	± 0.189
EGM-X04	0.003 ± 0.001	± 0.187
EGM-X05	-0.001 ± 0.001	± 0.141
EGM96	-0.001 ± 0.001	± 0.141

Table II. Radial distortions of the equipotential surfaces passing through the testing sites. Territory: U.S.A. (1835 testing sites)

Geopotential Model	Mean [m]	rms [m]
JGM-3/OSU91A	0.510 ± 0.017	± 0.549
EGM-X01	0.558 ± 0.018	± 0.515
EGM-X02	0.593 ± 0.018	± 0.499
EGM-X03	0.576 ± 0.018	± 0.497
EGM-X04	0.582 ± 0.018	± 0.493
EGM-X05	0.585 ± 0.018	± 0.489
EGM96	0.601 ± 0.012	± 0.506

3. Geopotential model testing network

The GMTN consists of 20 768 testing sites over oceans (Fig. 1), 1 835 testing sites covering the territory of the United States (Fig. 2), and 209 testing sites in the central part of Europe (Fig. 3). The data for the first year of T/P AVISO altimetry (cycles 6 – 18, Oct. 92 – Dec. 93) project (AVISO, 1995) was used for determining the geocentric positions of oceanic testing sites. Sea surface topography model POCM 4B to degree $n = 360$ (Rapp et al., 1996) was used for computing the SST heights to stand for normal Molodensky heights. The testing sites on the U.S.A. territory are identical with the GPS/leveling sites. However, orthometric Helmert heights available had to be transformed into normal Molodensky heights. In the central part of Europe the testing sites are identical with the GPS/leveling sites normal Molodensky heights of which are available.

Table III. Radial distortions of the equipotential surfaces passing through the testing sites. Territory: Central Europe (209 testing sites)

Geopotential Model	Mean [m]	rms [m]
JGM-3/OSU91A	-0.001 ± 0.019	± 0.275
EGM-X01	0.095 ± 0.014	± 0.198
EGM-X02	0.044 ± 0.014	± 0.205
EGM-X03	0.034 ± 0.014	± 0.199
EGM-X04	0.036 ± 0.014	± 0.198
EGM-X05	0.057 ± 0.013	± 0.195
EGM96	0.089 ± 0.013	± 0.186

4. Final results

The GMT methodology is based on the difference δW between the geopotential value at the testing site computed as function

$$W = W(\text{GM}, \omega, J_2^{(o)}, W_o, x_j, H_q) \quad (9)$$

and geopotential value computed from the model:

$$\delta W = W - W(\text{model}) ; \quad (10)$$

x_j stands for GPS coordinates, H_q for normal Molodensky height of the testing site.

Instead of δW radial distortion δR of the equipotential surface passing through the testing site can be used in GMT.

Distortions δW and/or δR are due to the geopotential model tested if the errors in fundamental constants (1) – (4), as well as, the errors in geocentric coordinates x_j and in normal Molodensky heights H_q of the testing site are small enough. The errors in the fundamental constants adopted contribute to δR as follows: GM (± 13 mm), ω (± 1 mm), $J_2^{(o)}$ (± 0.6 mm), W_o (± 50 mm). The errors in x_j and H_q are believed to amount some few centimeters. However, there can be differences in heights due to different Vertical Datums used. That is why, the non-zero biases in δW and/or in δR , i.e. the mean values computed over GMTN parts connected to the same tide gauge station, are interpreted as due to Vertical Datum shifts rather than due to the geopotential model. Numerical δR -results are summarized in Tabs I – III and in Figs 4 – 15.

However, there are distortions an order in magnitude larger compared to *rms*. For example, from 20768 oceanic testing sites 32 exhibit

Table IV. List of GMT sites (crossover centers) over oceans at which $\delta R > 1\text{m}$.

No.	Inner No.	Φ [deg]	Λ [deg]	Altimeter height <i>rms</i> [m]	Ocean depth [m]	δR [m]
1	678	6.46	133.79	± 0.028	2360	-1.17
2	1124	9.72	194.87	± 0.022	2050	-1.14
3	1243	11.30	138.46	± 0.025	3620	-1.89
4	1244	11.30	139.90	± 0.021	3950	-1.20
5	1620	14.46	156.43	± 0.021	5560	1.55
6	1630	14.46	165.77	± 0.017	3030	-1.11
7	3112	29.09	185.89	± 0.019	3340	-2.02
8	4524	44.24	169.72	± 0.021	5170	1.20
9	5217	49.67	168.29	± 0.021	2750	-1.09
10	6485	-0.06	342.18	± 0.014	4670	2.18
11	7494	-8.18	52.96	± 0.027	3040	-1.08
12	8008	-11.38	179.42	± 0.014	3510	-1.33
13	8046	-11.38	208.17	± 0.026	4000	-1.03
14	8050	-11.38	211.04	± 0.019	4980	-1.06
15	8271	-12.97	179.06	± 0.016	2150	-1.08
16	9075	-19.14	58.34	± 0.028	3440	1.10
17	9422	-20.65	325.29	± 0.018	2970	-1.86
18	11710	-33.08	357.27	± 0.022	2560	-1.38
19	14655	-45.24	202.42	± 0.027	4870	-1.06
20	15091	-47.99	8.77	± 0.017	3610	-1.45
21	15095	-47.10	9.85	± 0.020	3610	-1.33
22	15122	-47.99	31.81	± 0.022	3900	1.01
23	15128	-47.10	33.61	± 0.015	4910	1.04
24	16447	-51.30	54.75	± 0.021	4130	-1.14
25	17589	-54.24	82.78	± 0.020	4530	-1.06
26	17809	-54.91	261.34	± 0.020	4450	-1.11
27	17890	-54.91	328.89	± 0.023	6570	1.18
28	17892	-54.91	329.60	± 0.020	6570	1.11
29	17894	-54.91	330.32	± 0.011	6020	1.29
30	17896	-54.91	331.04	± 0.008	6600	1.22
31	19109	-58.60	161.46	± 0.023	3990	-1.32
32	20158	-60.20	309.90	± 0.021	3450	-1.34

δR to be about 1 – 2 m. They are listed in Table IV. The altimeter height *rms* computed on the basis of 12 – 13 ascending and descending passes amount there $\pm(0.011 - 0.028)$ m and, the ocean depths are larger than 2 km. That is why, the altimeter data are believed not to be responsible for such large distortions.

5. Conclusions

(1) The radial distortions of the equipotential surfaces passing through the GMT sites due to geopotential models EGM are smaller in general than those due to JGM-3/OSU91 model.

(2) There are no significant differences among EGM-X01, ..., EGM-X05, and EGM96 as regards their accuracy, in general.

(3) Over oceans, EGM96 and/or EGM-X05 are the best, evidently.

(4) The constant difference about 0.60 m over the territory of the U.S.A. is probably due to the Vertical Datum shift (Father's Point/Rimouski, Quebec, Canada) in relation to surface $W = W_0$. It means, the zero point of the tide gauge station above is situated about 0.60 m below the surface $W = W_0$.

(5) The constant difference of about 5 – 10 cm in the central part of Europe cannot be interpreted in an analogous way as above (space position of the tide gauge station Kronstadt) because of small area covered by GMTN there.

(6) The *rms* radial distortions received from EGM96 over oceans amounts ± 0.14 m only. Evidently, because the TOPEX data were used in creating EGMs.

(7) The *rms* distortion over the territory of the U.S.A. comes out ± 0.5 m, in the central part of Europe about ± 0.2 m.

Acknowledgements

The authors gratefully acknowledge the assistance of Professor Richard H. Rapp who made available POCM 4B model of SST and the discussion on its consistency with TOPEX altimeter system, of Dr. John C. Ries for his interest and the lively discussion on TOPEX altimeter calibration error, of Dr. Erricos C. Pavlis and Professor Michael G. Sideris who made available the geopotential coefficients sets, and last, but not least, of Dr. Patrick Vincent, who made available the TOPEX altimeter data and who kindly instructed us during its treatment.

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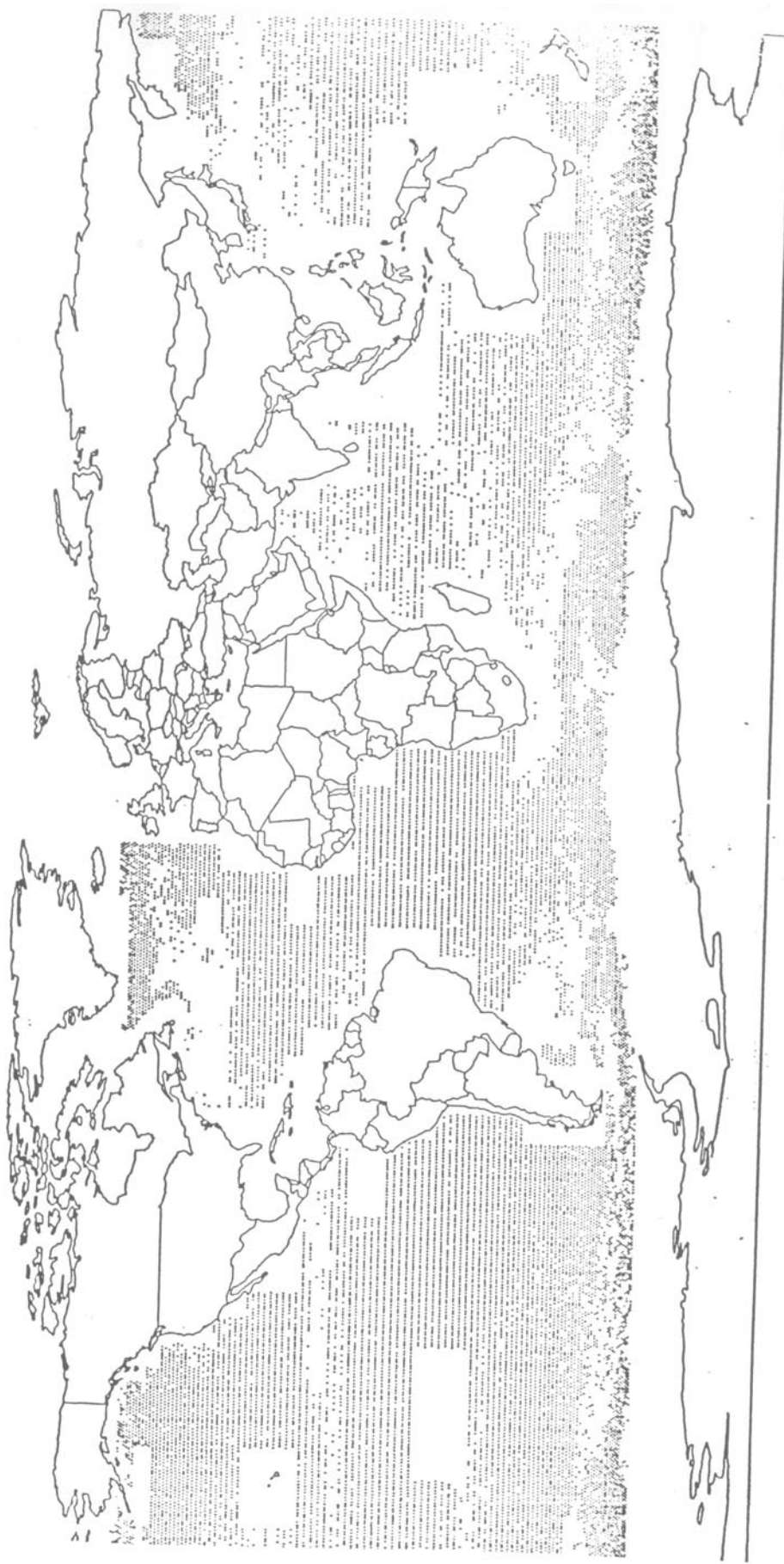


Fig. 1. The location of 20768 GMT sites over oceans

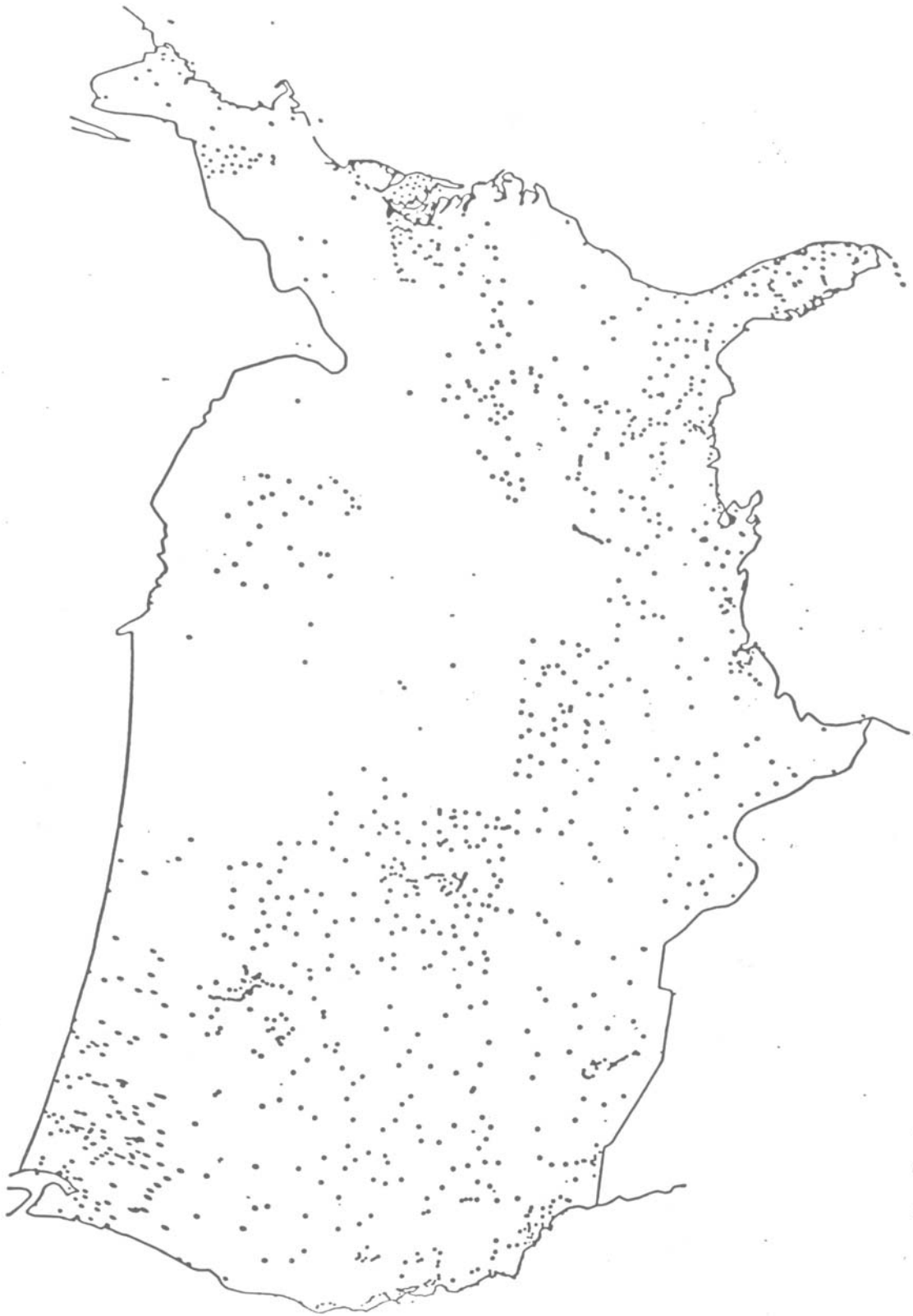


Fig. 2. The location of 1835 GMT sites on the territory of the U.S.A.

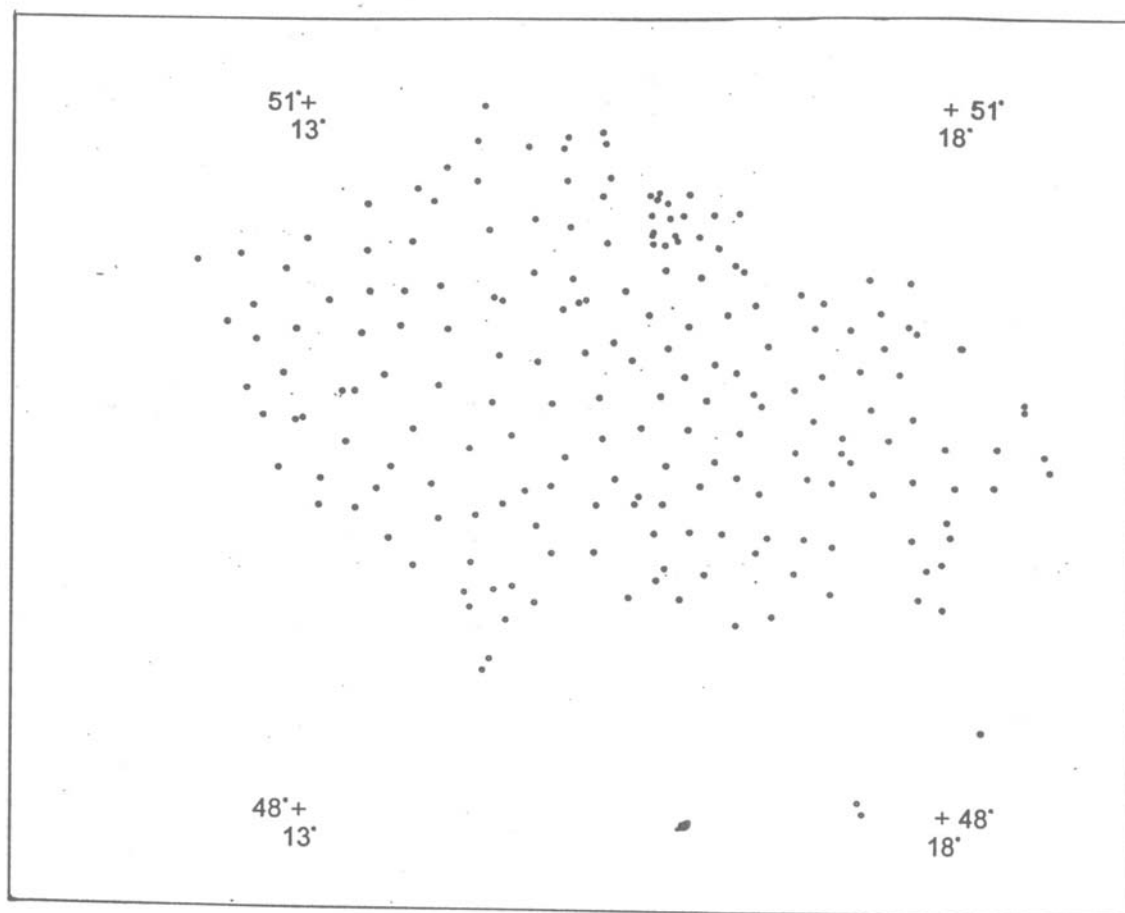


Fig.3. The location of 209 GMT sites in the central part of Europe

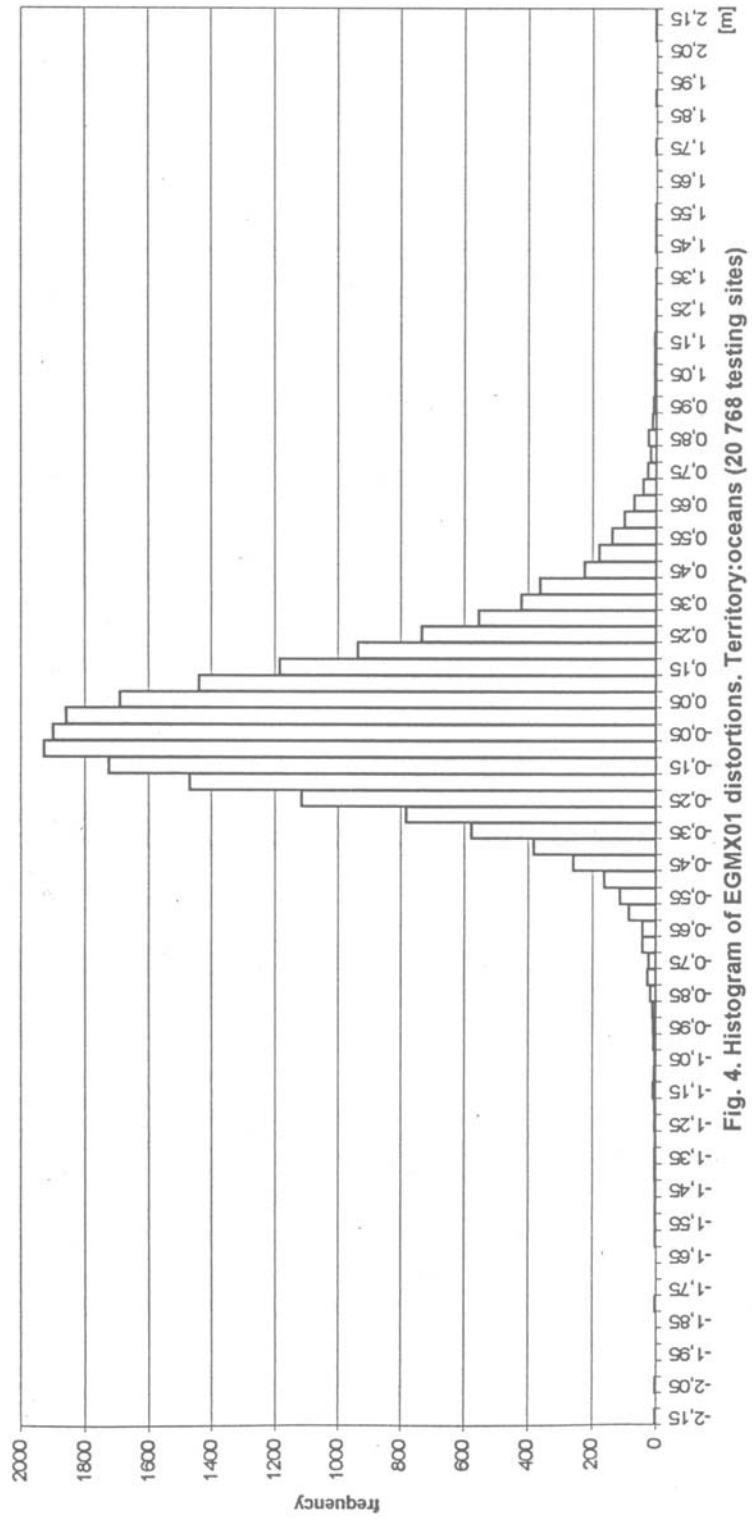


Fig. 4. Histogram of EGMX01 distortions. Territory:oceans (20 768 testing sites)

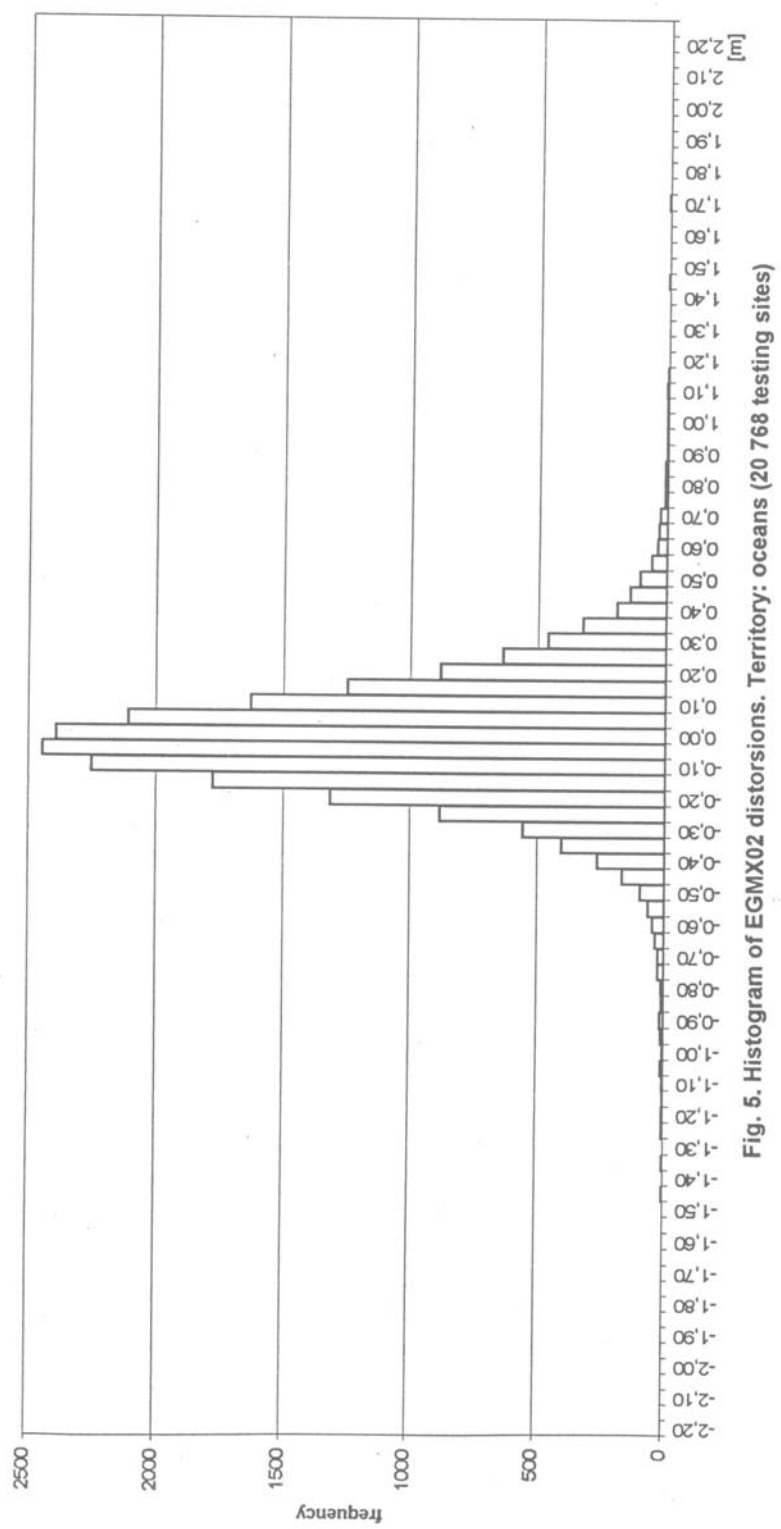


Fig. 5. Histogram of EGMX02 distortions. Territory: oceans (20 768 testing sites)

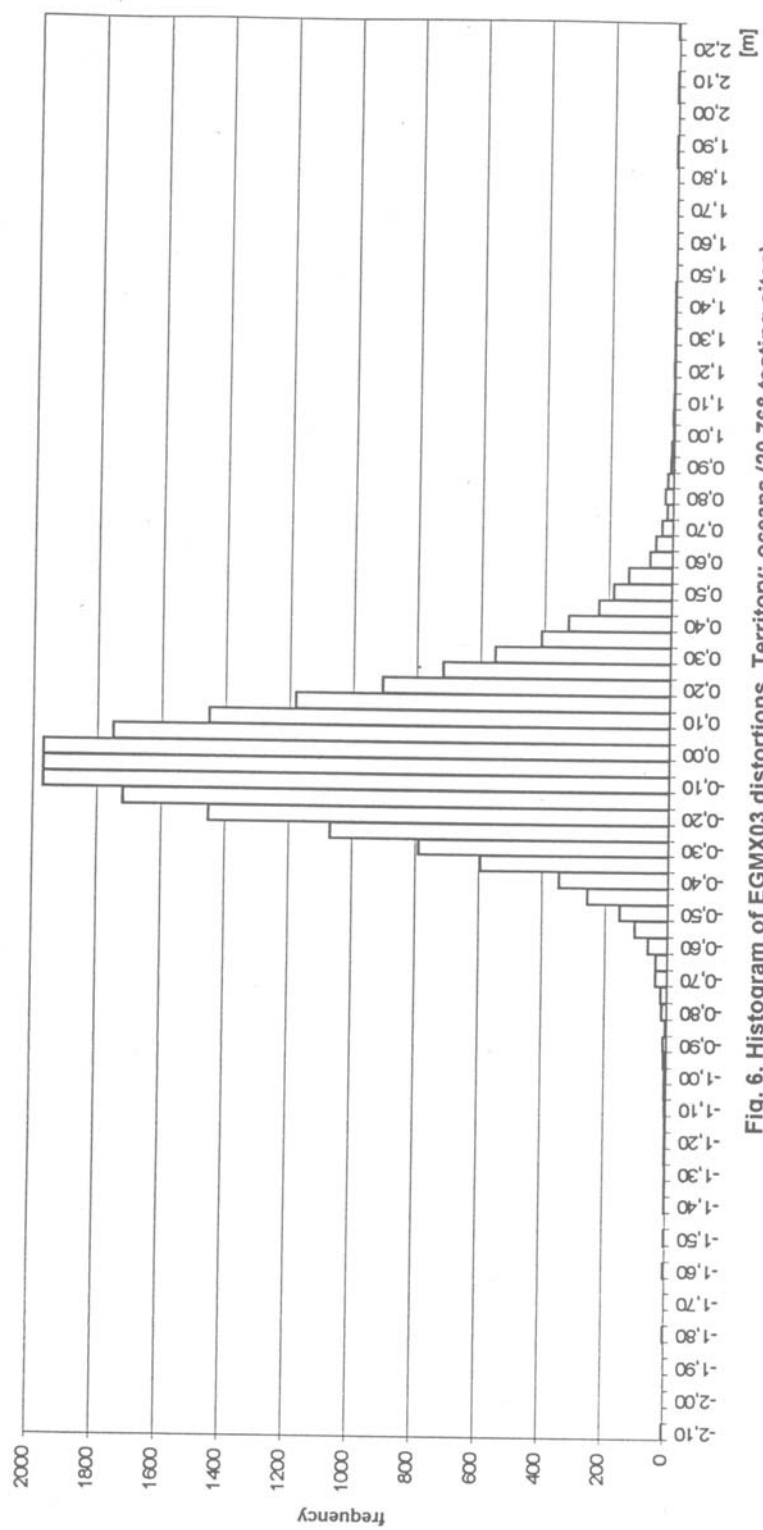


Fig. 6. Histogram of EGMX03 distortions. Territory: oceans (20 768 testing sites)

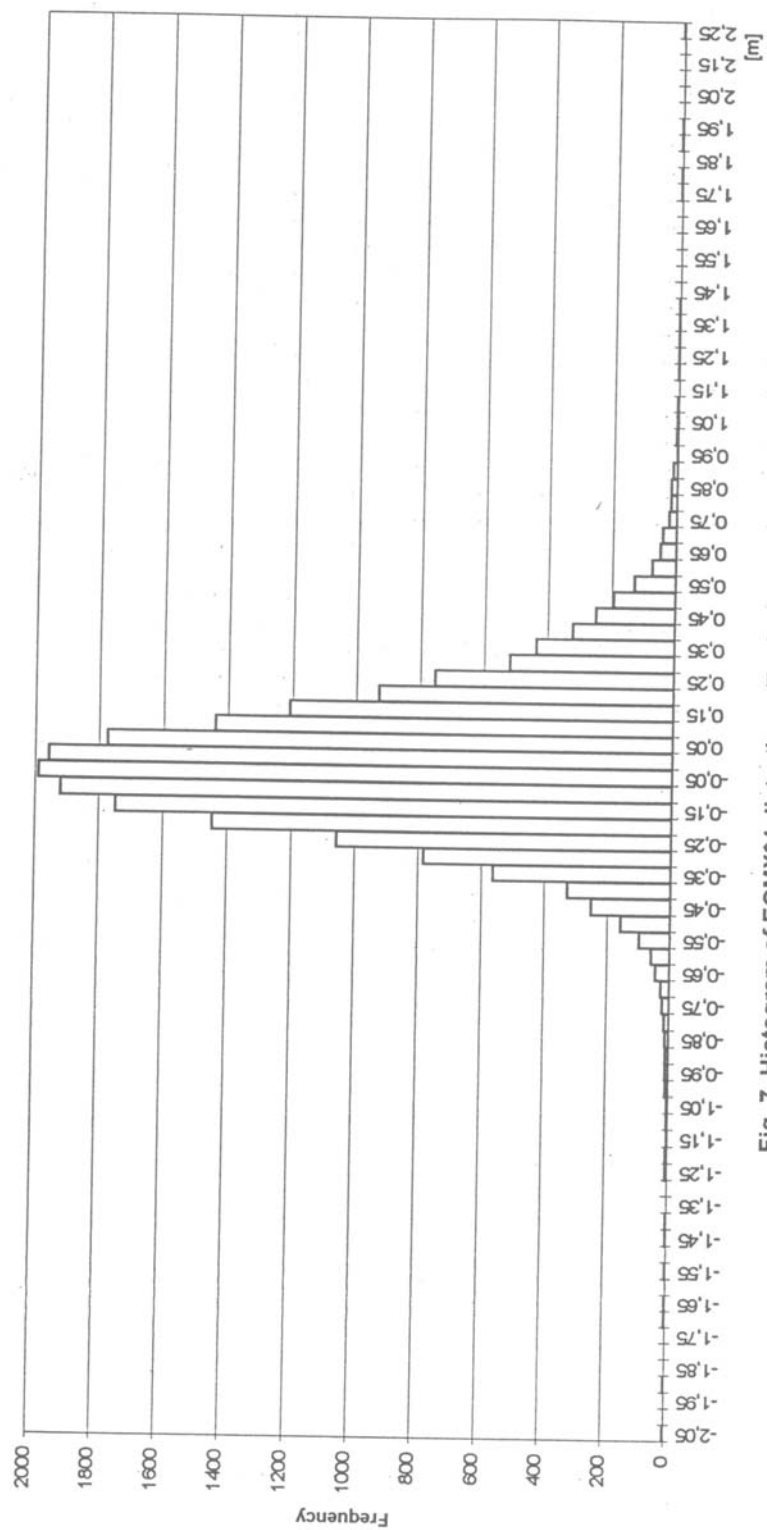


Fig. 7. Histogram of EGMX04 distortions. Territory: oceans (20 768 testing sites)

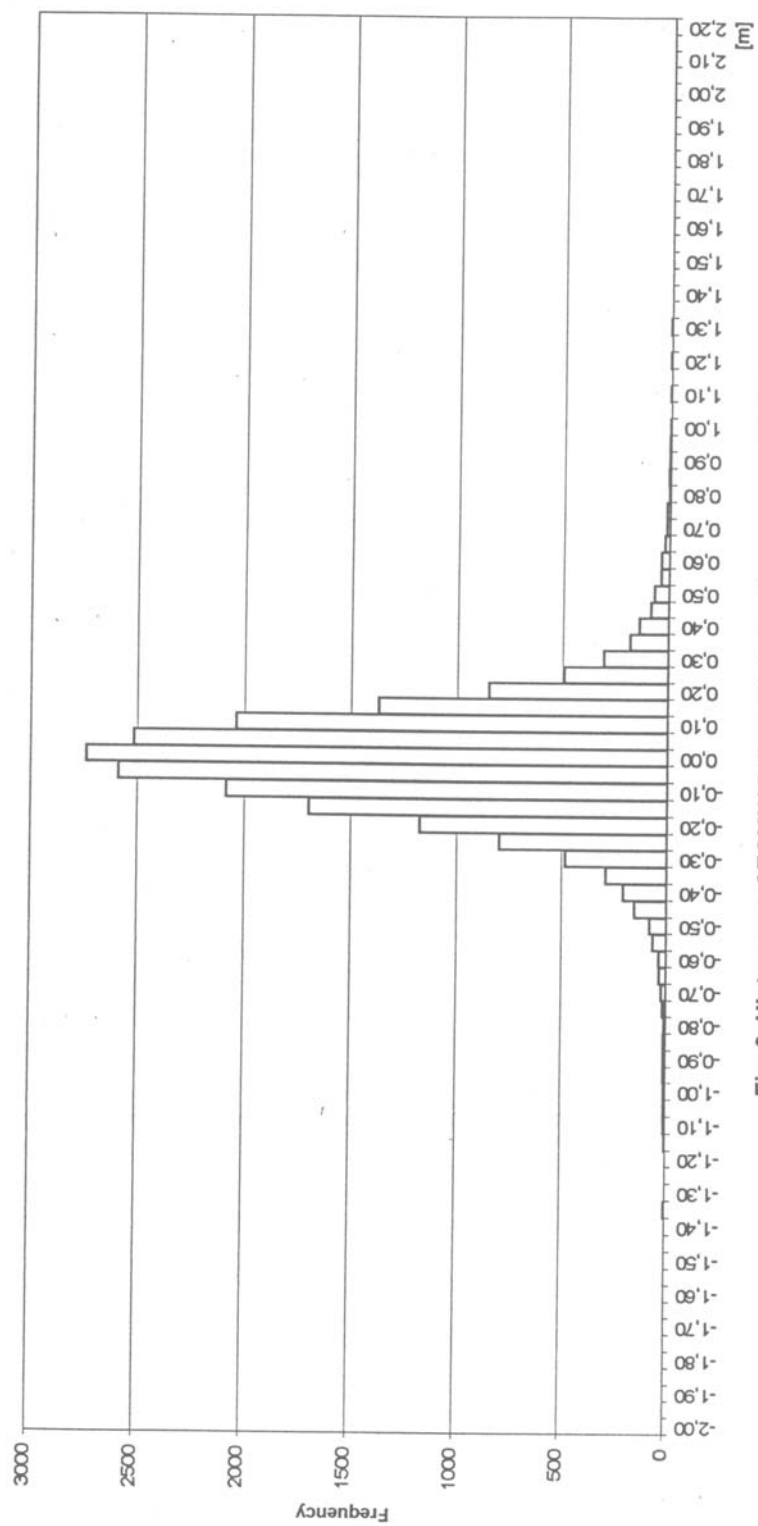


Fig. 8. Histogram of EGMX05 distortions. Territory:oceans (20 768 testing sites)

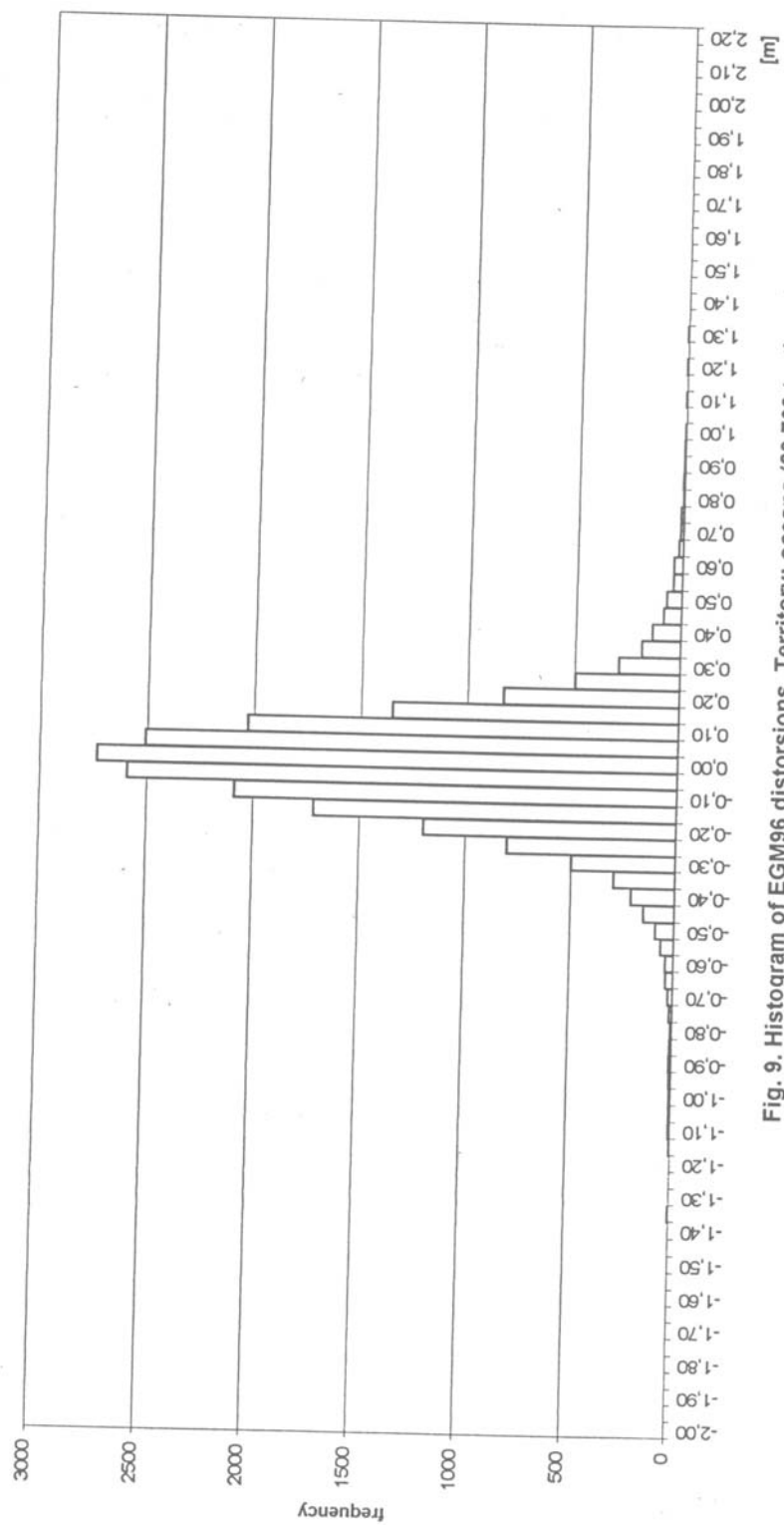


Fig. 9. Histogram of EGM96 distortions. Territory: oceans (20 768 testing sites)

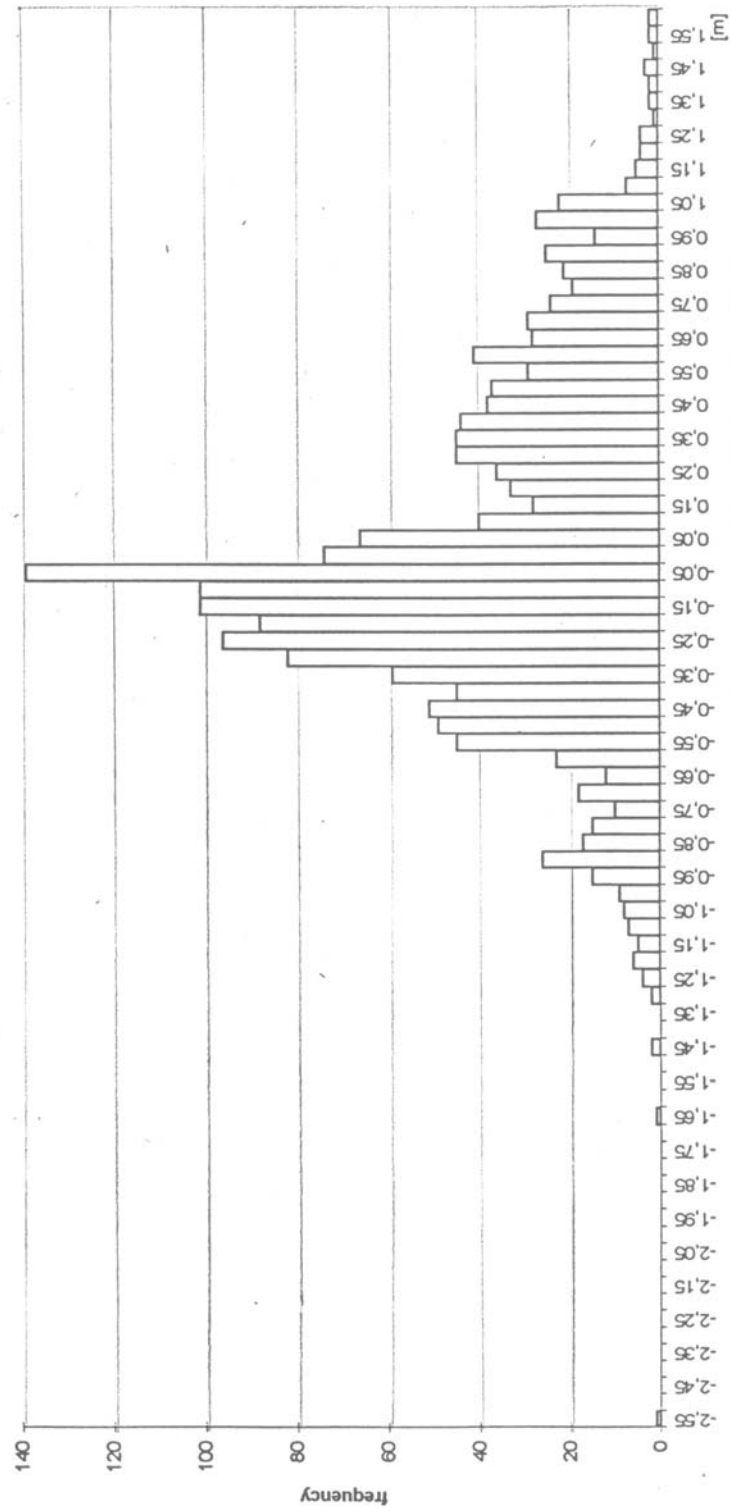


Fig. 10. Histogram of EGMX01 distortions. Territory: USA (1835 testing sites)

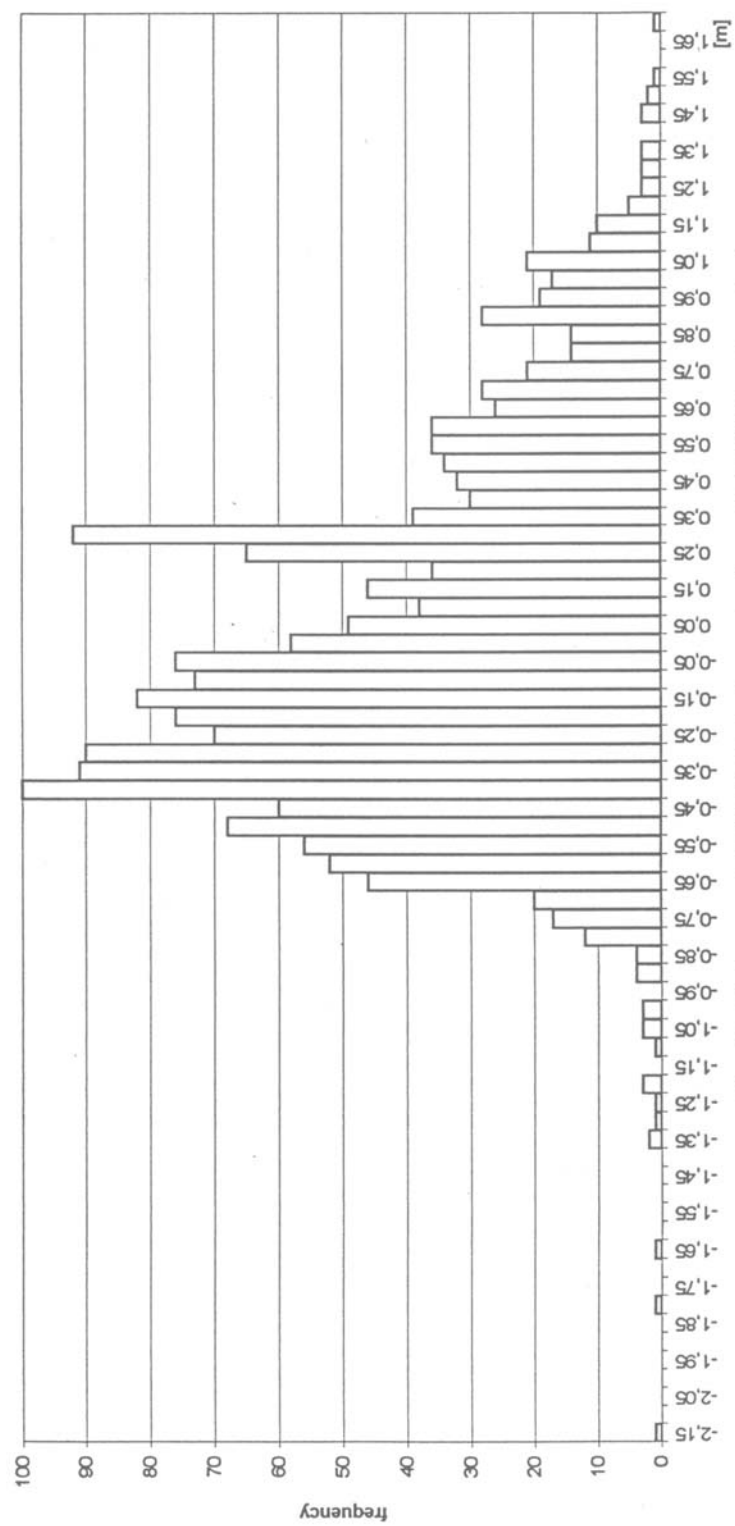


Fig. 11. Histogram of EGMX02 distortions. Territory: USA (1835 testing sites)

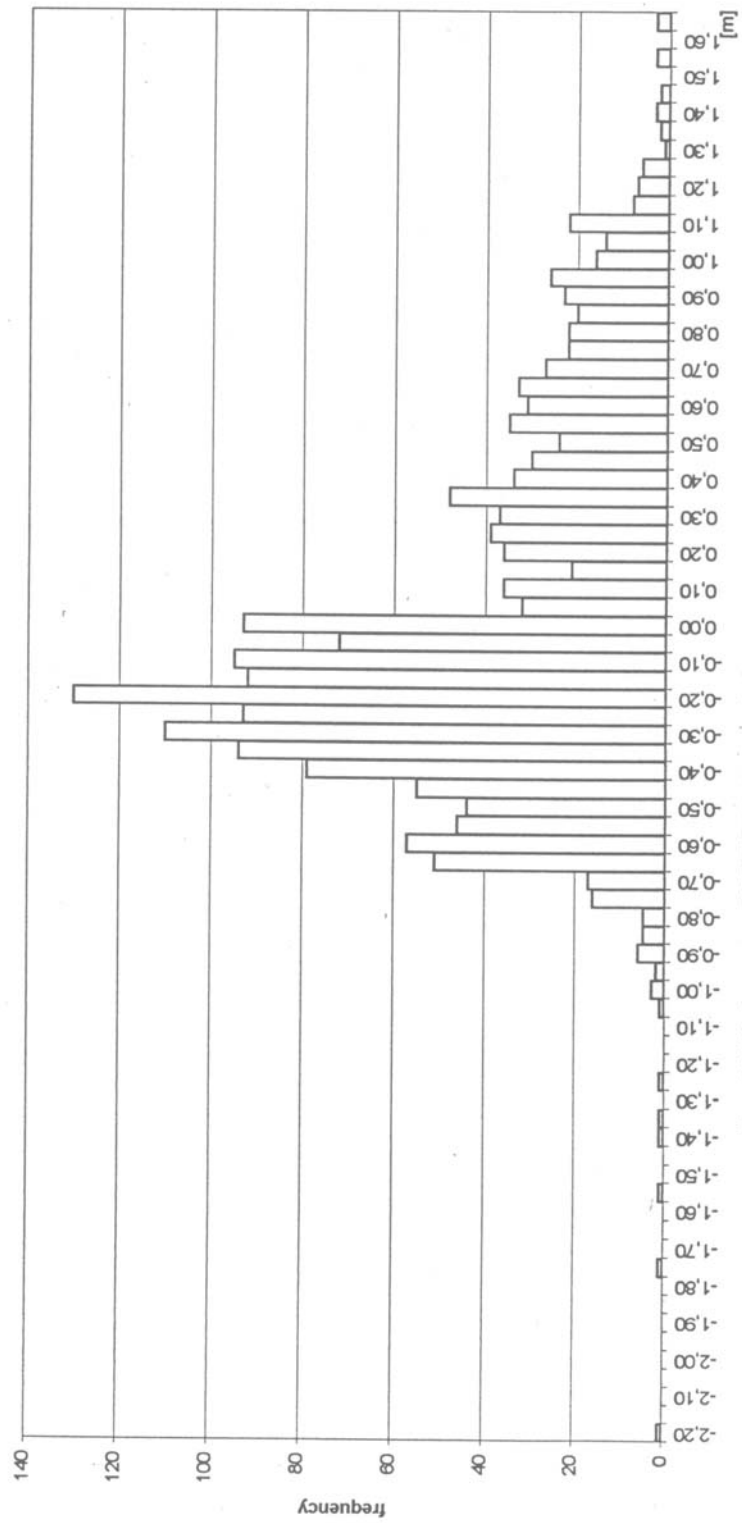


Fig. 12. Histogram of EGMX03 distortions. Territory: USA (1835 testing sites)

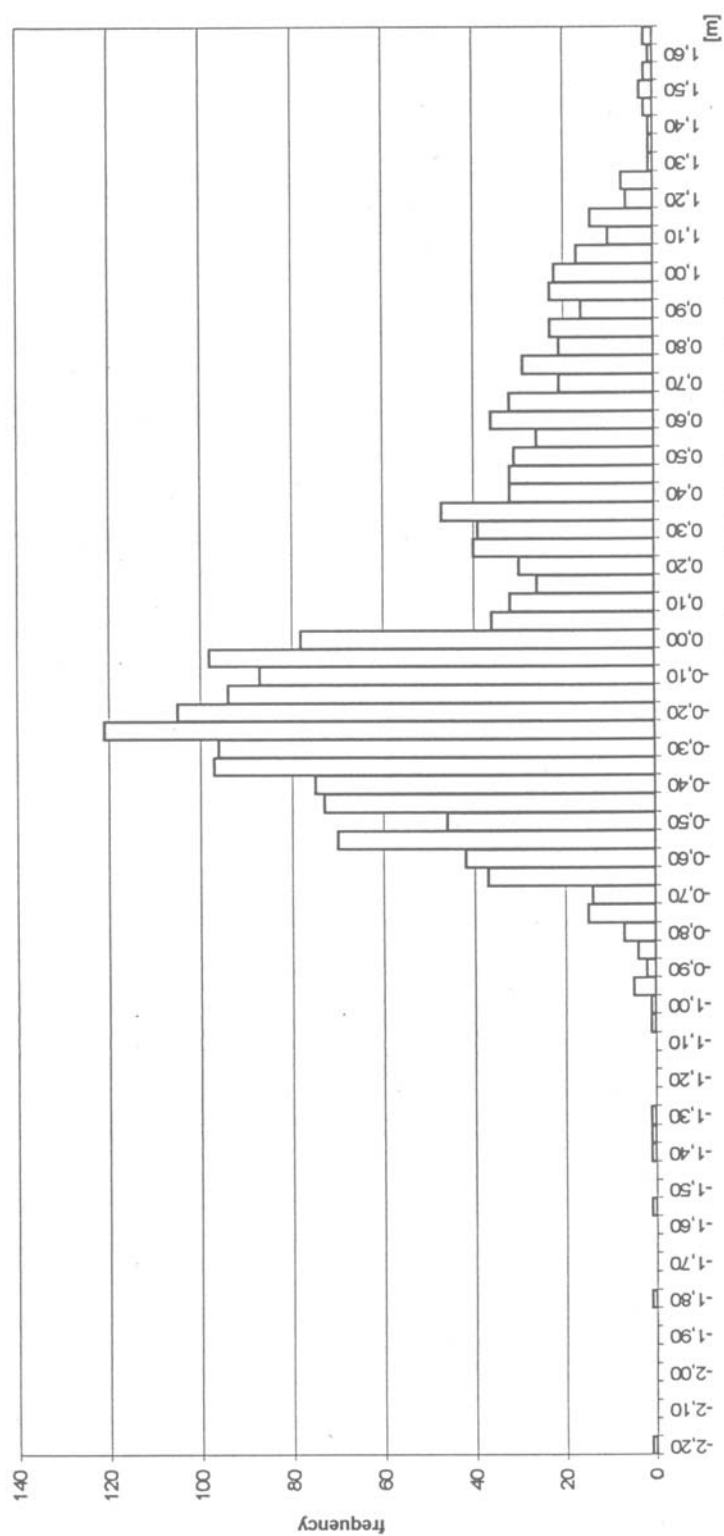


Fig. 13. Histogram of EGMX04 distortions. Territory USA (1835 testing sites)

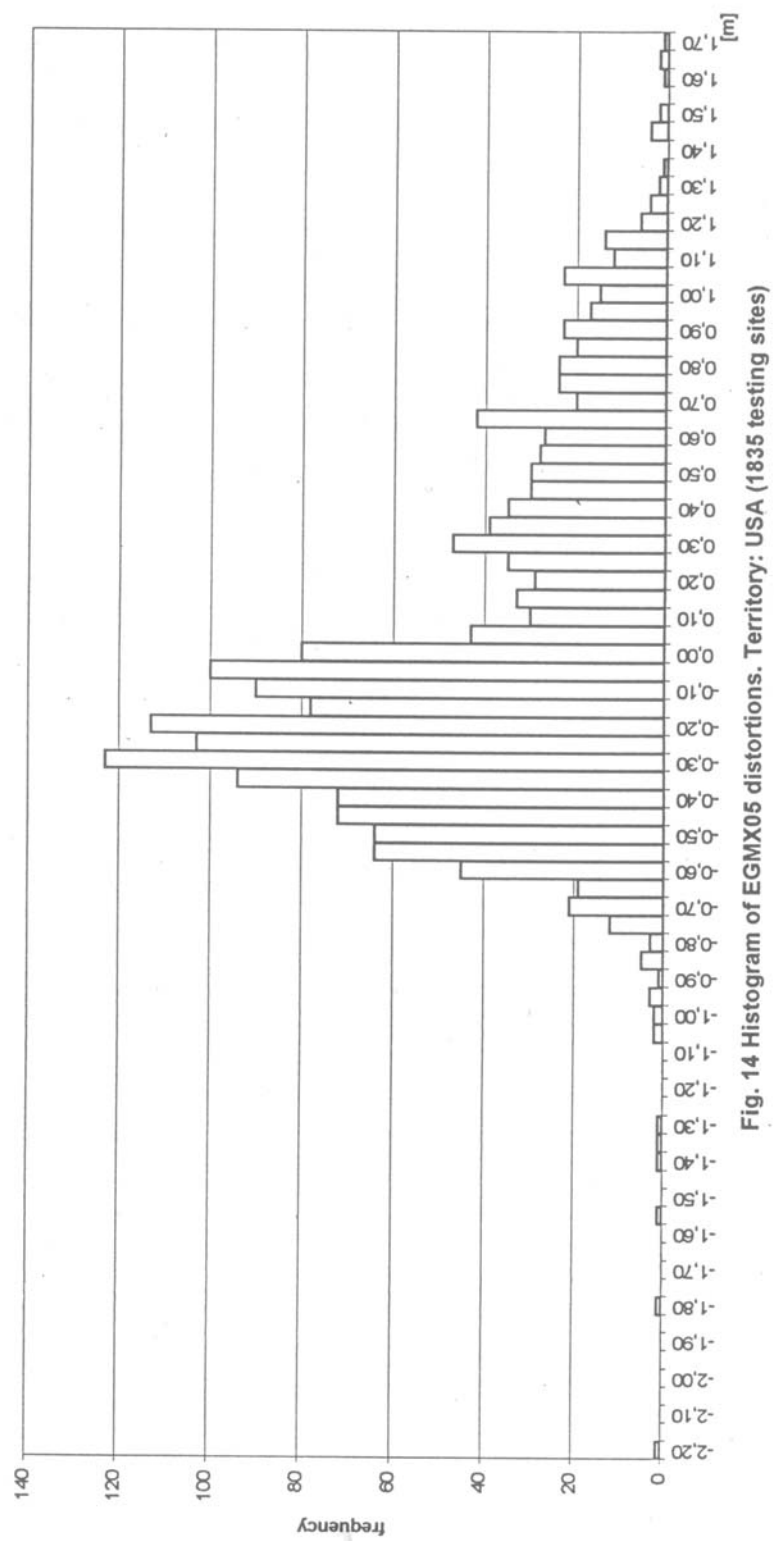


Fig. 14 Histogram of EGMX05 distortions. Territory: USA (1835 testing sites)

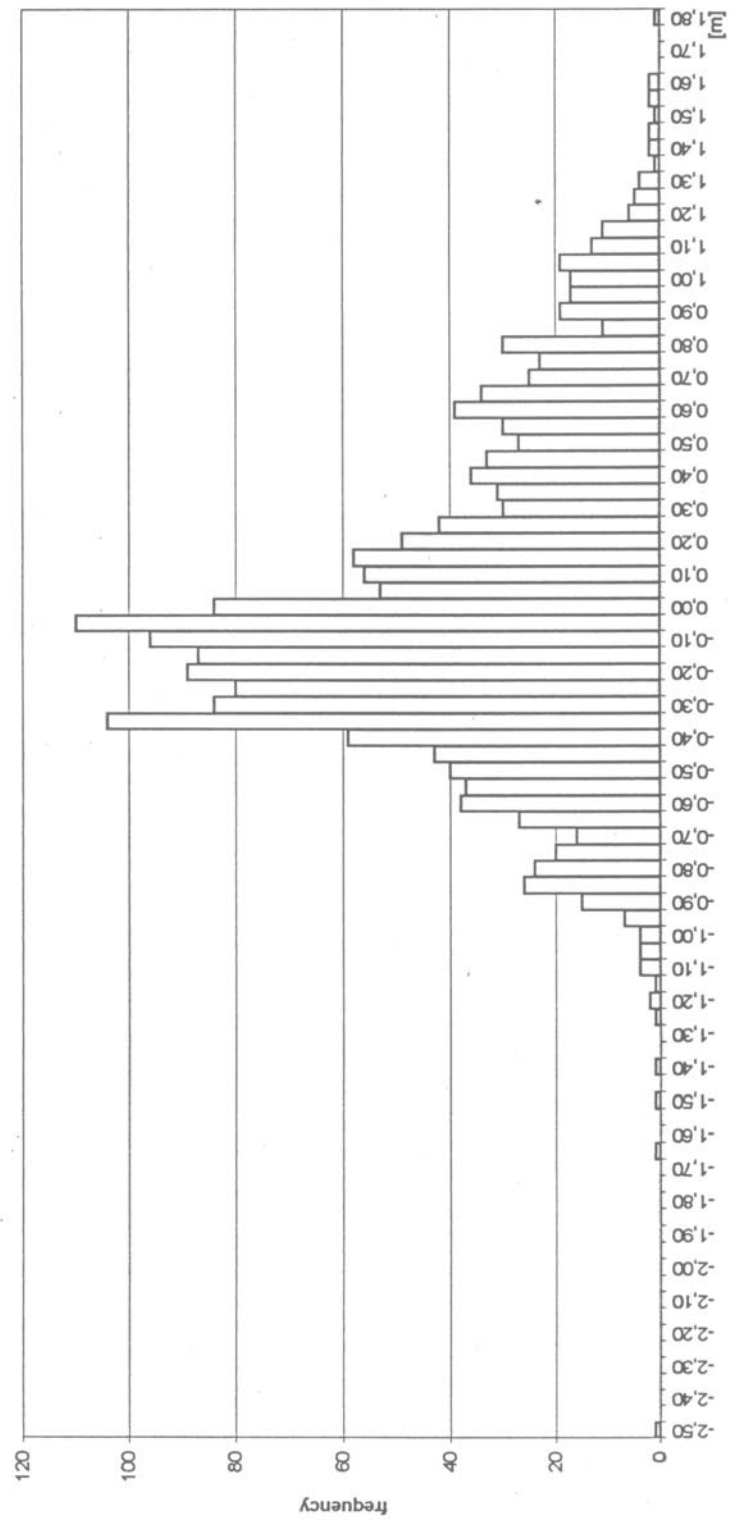


Fig. 15. Histogram of EGM96 distortions. Territory: USA (1835 testing sites)

Testing and Evaluation of the GSFC/DMA EGM in China

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Abstract

This paper presents the results of the comparisons of real data — the measured point and gridded mean free air gravity anomalies and geoid undulations in the Chinese territory with those computed from the GSFC/DMA EGM. It also demonstrates that the fitness of the EGM to the Chinese local geopotential field is much better than that of any other existing geopotential model.

1 Introduction

The testing and evaluation of the GSFC/DMA Earth's geopotential model (EGM) is carried out according to the Appendix VI-A of Annex V to the Protocol for Scientific and Technical Cooperation in Surveying and Mapping between NBSM and USGS/DMA.

This paper will discuss the fitness of the GSFC/DMA EGM-X01~EGM-X05, and the latest EGM96 in free air gravity anomalies and geoid undulations to the real measured gravity data and geoid undulations in the Chinese territory.

2 Data Used in the Testing and Evaluation

2.1 GPS/Leveling Data

27 GPS/Leveling points in Chinese national A order GPS network are used in the evaluation (see Fig. A). The relative accuracy of the side in the network is 0.5×10^{-8} level. The network was observed in 1992. The elevations refer to the China yellow sea 1985 datum, and is with the accuracy of 0.01m level.

Several local GPS networks are located in Beijing, Guiyang, Hainan, Sichuan 1 to Sichuan 4, Xingjiang, Xiamen and Yantai, respectively (see Fig. B). The total number of the local GPS/Leveling points is 238.

2.2 Gravity Anomaly Data

The Chinese $30' \times 30'$ gridded mean free air gravity anomaly set and 15 blocks of $30' \times 30'$ gridded mean gravity anomalies in China are used in the evaluation and testing. The latter are specially chosen and prepared for the evaluation (see Fig. A).

2.3 Earth's gravity model

Not only EGMX01 to EGMX05 and EGM96 but also JGM3OSU and OSU91A models are used in this evaluation and testing. The gravity anomalies and geoid undulations computed from the models have been compared with those in section 2.1 and 2.2.

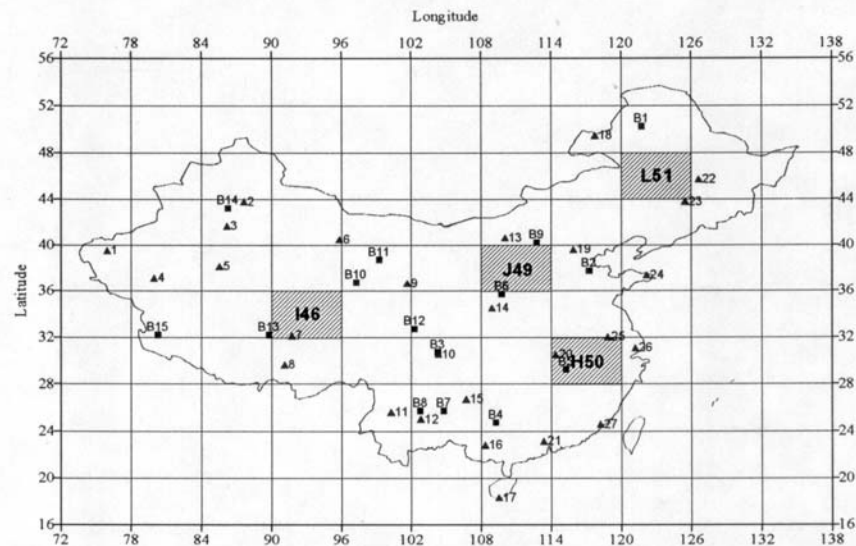


Fig.A The distribution of A order GPS/Leveling points, 4 sheets of 1:1m scale map of point gravity values and 15 blocks of 30' x 30' gridded mean gravity anomaly
 ▨ 1:1m scale map; ▲ A order GPS/Leveling point; ■ 15 blocks of 30' x 30'

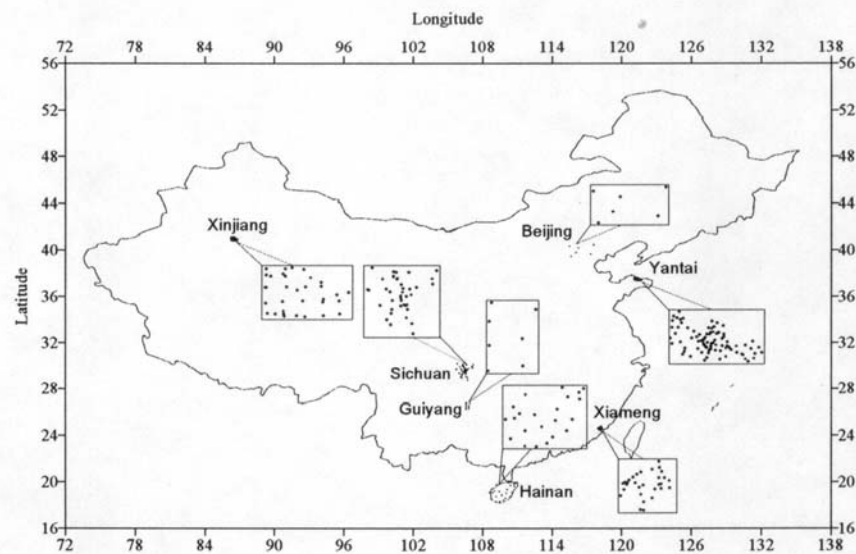


Fig.B The Distribution of Local GPS Networks in China

3 The Formulae Used in the Evaluation

3.1 Formulae for the computation of geoid undulations from EGM

The disturbing potential T can be expressed as follows:

$$T(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=2}^{NMAX} \left(\frac{a}{r} \right)^n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta) \quad (1)$$

where r , θ , λ , are the geocentric radius distance, colatitude and longitude respectively;

GM is the gravitational constant; a is the semi-major axis;

$\bar{C}_{nm}$ and $\bar{S}_{nm}$ are fully normalized geopotential coefficients;

$\bar{P}_{nm}(\cos\theta)$ are fully normalized associated Legendre functions.

If the geopotential coefficients are given, the geoid undulations can then be determined using Bruns' formula, i.e.,

$$N(r, \theta, \lambda) = \frac{T}{\gamma} = \frac{GM}{r\gamma} \sum_{n=2}^{NMAX} \left(\frac{a}{r} \right)^n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta) \quad (2)$$

where γ is normal gravity at computation point.

3.2 Gravity anomalies computed from EGM

According to the linearized Molodensky Boundary Value Problem, the fundamental boundary condition relating the gravity anomaly Δg to disturbing potential take the form, (Moritz, 1980)

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2T}{r} \quad (3)$$

The equation (1) is substituted into (3), and we get

$$\Delta g(r, \theta, \lambda) = \frac{GM}{r^2} \sum_{n=2}^{NMAX} \left(\frac{a}{r} \right)^n (n-1) \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos\theta) \quad (4)$$

3.3 The geoid undulations computed from GPS/Leveling

The relationship between geoid undulation N , ellipsoid height h and orthometric height H for a point at the surface of the Earth is known as

$$N = h - H \quad (5)$$

where h can be determined by GPS techniques, and H is obtained from conventional leveling or short range trigonometric leveling. So N can be taken as a measured geoid undulation and can be compared with those computed from EGM for the evaluation.

GPS techniques refer to the Geodetic Reference System 1980 (GRS80), the equatorial radius of which is 6378137.0m, i.e., the geoid undulation obtained from GPS/Leveling refers to the GRS80 ellipsoid. The geoid undulation computed from the GSFC/DMA EGM refers to a reference ellipsoid whose equatorial radius is 6378136.3m. Taking into account the effect of the different equatorial radius in the computation is necessary.

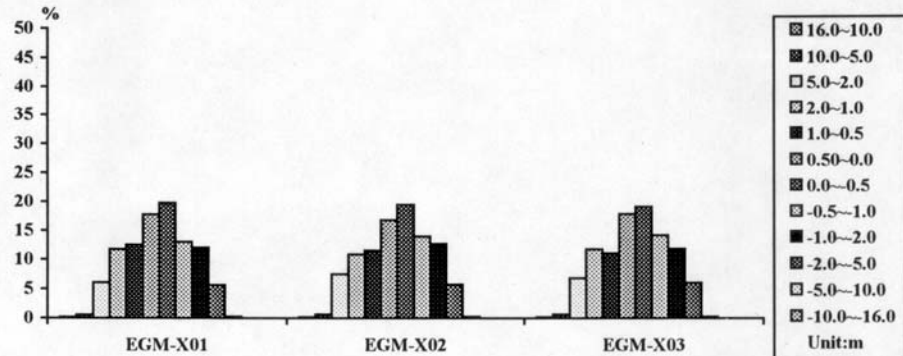


Fig. 1a The ranges of differences between $30' \times 30'$ gridded mean geoid undulations computed from EGM-X01~05, EGM96 JGM3OSU and those from OSU91A on the mainland of China

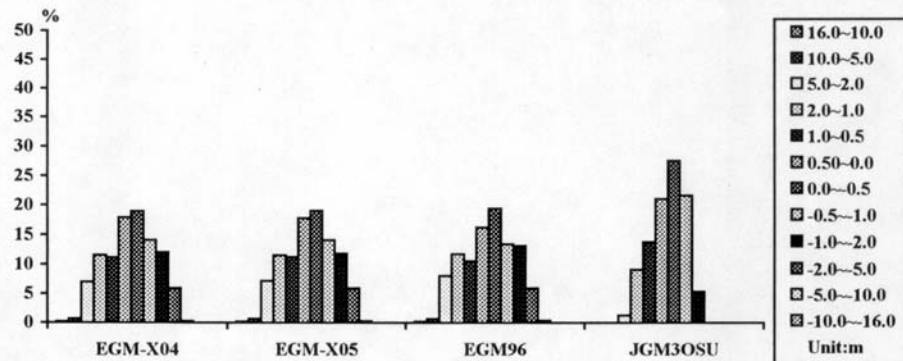


Fig. 1b The ranges of differences between $30' \times 30'$ gridded mean geoid undulations computed from EGM-X01~05, EGM96, JGM3OSU and those from OSU91A on the mainland of China

4 The Testing and Evaluation in Geoid Undulation

4.1 The comparison of $30' \times 30'$ gridded mean geoid undulations

In this test and evaluation, the $30' \times 30'$ gridded mean geoid undulations (4260 blocks on the mainland of China) computed from the OSU91A model have been compared with those from the GSFC/DMA EGMX01~05 and EGM96 as well as the JGM3OSU respectively. The histogram of

the results of the comparison is shown in Fig. 1a and Fig. 2a. The ranges of geoid undulation differences are listed in Tab. 1a and Tab. 1b. The statistics of the comparisons are listed in Tab. 1c. From Tab. 1c, we can see that the standard deviations of geoid undulations differences between the OSU91A model and the EGM-X01 to EGM-X05, EGM96 and JGM3OSU are $\pm 1.531\text{m}$, $\pm 1.617\text{m}$, $\pm 1.585\text{m}$, $\pm 1.606\text{m}$, $\pm 1.606\text{m}$, $\pm 1.681\text{m}$ and $\pm 0.748\text{m}$ respectively.

Tab. 1a: The ranges of differences between $30' \times 30'$ gridded mean geoid undulations computed from EGM-X01~05, EGM96, JGM3OSU and those from OSU91A on the mainland of China

Range (Unit:m)	EGM-X01		EGM-X02		EGM-X03	
	Percent %	Numbers of Grid	Percent %	Numbers of Grid	Percent %	Numbers of Grid
16~10	0.2113	9	0.2347	10	0.2347	10
10~5	0.6103	26	0.6808	29	0.6573	28
5~2	6.1033	260	7.5587	322	6.8310	291
2~1	11.807	503	10.915	465	11.784	502
1~0.5	12.535	534	11.596	494	11.103	473
0.5~0	17.770	757	16.760	714	17.863	761
0~0.5	19.835	845	19.530	832	19.108	814
-0.5~-1	13.004	554	13.990	596	14.201	605
-1~-2	12.089	515	12.699	541	11.901	507
-2~-5	5.7277	244	5.7512	245	6.0329	257
-5~-10	0.2817	12	0.2582	11	0.2582	11
-10~-16	0.0000	-	0.0000	-	0.0000	-

Tab. 1b: The ranges of differences between $30' \times 30'$ gridded mean geoid undulations computed from EGM-X01~05, EGM96, JGM3OSU and those from OSU91A on the mainland of China

Range (Unit:m)	EGM-X04		EGM-X05		EGM96		JGM3OSU	
	Percent %	Numbers of Grid	Percent %	Range (Unit:m)	Percent %	Numbers of Grid	Percent %	Numbers of Grid
16~10	0.2347	10	0.2347	16~10	0.2347	10	0.2347	10
10~5	0.6573	28	0.6573	10~5	0.6573	28	0.6573	28
5~2	7.0188	299	7.2066	5~2	8.0751	344	7.0188	299
2~1	11.572	493	11.549	2~1	11.760	501	11.572	493
1~0.5	11.173	476	11.338	1~0.5	10.657	454	11.173	476
0.5~0	17.863	761	17.723	0.5~0	16.197	690	17.863	761
0~0.5	18.967	808	19.037	0~0.5	19.366	825	18.967	808
-0.5~-1	14.108	601	14.084	-0.5~-1	13.427	572	14.108	601
-1~-2	12.089	515	11.877	-1~-2	13.169	561	12.089	515
-2~-5	6.0094	256	5.9624	-2~-5	6.0094	256	6.0094	256
-5~-10	0.2817	12	0.3052	-5~-10	0.4225	8	0.2817	12
-10~-16	0.0000	-	0.0000	-10~-	0.0000	-	0.0000	-

Tab. 1c: The statistics from the comparison of $30' \times 30'$ gridded mean geoid undulations computed from EGM-X01~05, EGM96, and JGM3OSU with those from OSU91A on the main land of China (Unit:m)

Model	Max	Min	Mean	RMS	Std
EGM-X01	16.084	-6.199	0.046	± 1.532	± 1.531
EGM-X02	16.940	-6.040	0.063	± 1.618	± 1.617
EGM-X03	16.533	-5.973	0.058	± 1.586	± 1.585
EGM-X04	17.000	-6.121	0.059	± 1.607	± 1.606
EGM-X05	16.845	-6.213	0.061	± 1.607	± 1.606
EGM96	16.936	-6.404	0.070	± 1.682	± 1.681
JGM3OSU	2.495	-1.918	0.014	± 0.748	± 0.748

4.2 The comparison of point geoid undulations obtained from China national A order GPS/Leveling network

The comparison of point geoid undulations have been made at 27 points of the China national A order GPS/Leveling network, which are evenly distributed on the mainland of China. The accuracy of the GPS positioning and leveling at these points is with cm level.

The statistic results of the differences between the point geoid undulations from GPS/Leveling and those from EGM are listed in Tab. 2. It demonstrates that there is a significant improvement of the GSFC/DMA EGM, (X01~X05), as compared with the existing geopotential models, e.g. the OSU91A model and JGM3OSU.

To analyze the slop and bias between the two kinds of geoid undulations, i.e. one obtained from GPS/Leveling and the other from EGM, a following linear fitting formula is used based on their differences.

$$\Delta N = \alpha + \beta \Delta B + \gamma \Delta L \quad (6)$$

Where $\Delta N = N_{GPS} - N_M$, N_{GPS} is geoid undulation obtained by GPS/Leveling, and N_M is computed from EGM, $\Delta B = B - B_0$, $\Delta L = L - L_0$, here the point No. 22 in Fig. A is set as B_0 , L_0 . α is a bias in the vertical, β , γ are a slop in latitude and longitude, respectively. The solution of equation (6) is listed in Tab. 3. The model with minimum standard deviation and slop is EGM96. However the fitness and the reliability for the Chinese local geoid undulation to those obtained from any EGM (X01~X05) and EGM96 are better than those from the OSU91A model and JGM3OSU.

Tab. 2: The statistics from the comparison of point geoid undulations computed from EGM-X01~X05, EGM96, OSU91A, JGM3OSU with those from the China national A order GPS/Leveling network (Unit:m)

Model	Max	Min	Mean	RMS	Std
EGM-X01	2.028	-1.321	0.106	± 0.818	± 0.811
EGM-X02	1.844	-1.046	0.004	± 0.689	± 0.689
EGM-X03	2.103	-1.314	0.039	± 0.753	± 0.752
EGM-X04	2.110	-1.427	-0.007	± 0.752	± 0.752
EGM-X05	2.113	-1.367	-0.008	± 0.755	± 0.755
EGM96	2.096	-0.934	-0.060	± 0.668	± 0.665
OSU91A	5.622	-1.700	0.687	± 1.659	± 1.510
JGM3OSU	4.941	-1.367	0.782	± 1.594	± 1.389

Tab. 3: The statistics from the comparison of point geoid undulations computed from EGM-X01~X05, EGM96, OSU91A, JGM3OSU with those from the China national A order GPS/Leveling after a linear fitting

Model	Max (Unit:	Min (Unit:	Std (Unit:	Bias (Unit:	Slop (Unit: ppm)	
					B-Direction	L-Direction
EGM-X01	1.556	-1.724	± 0.757	0.268	-0.33108034	0.02738281
EGM-X02	0.980	-1.671	± 0.640	-0.152	-0.25306352	0.08837264
EGM-X03	1.004	-1.677	± 0.698	-0.261	-0.30915288	0.05125223
EGM-X04	0.996	-1.669	± 0.684	-0.265	-0.32824340	0.08570506
EGM-X05	0.931	-1.662	± 0.681	-0.304	-0.34722224	0.07865781
EGM96	0.769	-1.856	± 0.628	-0.309	-0.24515932	0.03503531
OSU91A	2.988	-3.591	± 1.315	-0.113	0.21645612	-0.55948090
JGM3OSU	2.379	-2.710	± 1.138	-0.264	0.11585502	-0.62018654

4.3 Comparisons of relative geoid undulations between GPS/Leveling points in 10 local GPS networks in China

Comparisons have been made for the relative geoid undulations obtained from two adjacent GPS/Leveling points in 10 local GPS networks in China (see Fig. B).

The statistic results of the comparisons are shown in Tab. 4. Generally, the differences between the relative geoid undulations obtained from the EGM (X01~X05) and EGM96 and those from GPS/Leveling are small as compared with those from the OSU91A model or JGM3OSU. But there is one exception, which is in the Beijing GPS network. The standard deviation of the differences there from the EGM is larger than those from the OSU91A model and JGM3OSU. The causes are not very clear. It could be that the geocentric coordinates in the Beijing GPS network are not precise enough.

5 The Testing and Evaluation in Gravity Anomalies

5.1 The comparisons of the Chinese 30'×30' gridded mean free air gravity anomaly set with those computed from the GSFC/DMA EGM(X01~X05), EGM96, OSU91A and JGM3OSU

The comparisons of the Chinese 30'×30' gridded mean free air gravity anomaly set with those computed from the GSFC/DMA EGM-X01 to EGM-X05, EGN96 as well as the OSU91A model and JGM3OSU have been made respectively. Their histograms, ranges of the gravity anomaly differences and statistics are shown in Fig. 2a, Fig. 2b, Tab. 5a, Tab. 5b, Tab. 5c.

From the above results, the model with minimum standard deviation obtained from the differences of the 30'×30' gridded mean gravity anomaly is EGM-X03. The standard deviations for the gravity anomalies differences between the Chinese set and EGM (X01~X05) and EGM96 are almost equivalent to those for the existing geopotential models, the OSU91A model and JGM3OSU. The differences of 30'×30' gridded mean gravity anomalies between EGM (X01~X05) EGM96 and Chinese 30'×30' set are distributed in 89% on the range from -25 mGal to +25mGal.

Tab. 4: The statistics of the comparisons of relative geoid undulations from EGM-X01~X05, EGM96, OSU91A, and JGM3OSU with those from GPS/Leveling in 10 local GPS networks (238 points) (Unit:m)

Name	Numbers	Model	Bias	Max	Min	Std
Beijing	6	EGM-X01	-9.851	1.434	-0.907	±0.790
		EGM-X02	-10.142	1.478	-0.901	±0.800
		EGM-X03	-10.052	1.408	-0.869	±0.769
		EGM-X04	-10.053	1.422	-0.849	±0.775
		EGM-X05	-10.054	1.387	-0.819	±0.756
		EGM96	-9.908	1.371	-0.796	±0.734
		OSU91A	-9.290	0.668	-0.641	±0.487
		JGM3OSU	-9.720	0.845	-0.829	±0.549
Guiyang	7	EGM-X01	-14.139	0.201	-0.294	±0.167
		EGM-X02	-14.024	0.212	-0.296	±0.174
		EGM-X03	-14.009	0.215	-0.311	±0.174
		EGM-X04	-13.979	0.200	-0.296	±0.170

Continue from Tab. 4

		EGM-X05	-13.956	0.208	-0.302	±0.176
		EGM96	-13.650	0.250	-0.345	±0.193
		OSU91A	-15.275	0.232	-0.374	±0.245
		JGM3OSU	-14.963	0.203	-0.342	±0.209
Hainan	22	EGM-X01	1.072	1.372	-1.086	±0.758
		EGM-X02	1.268	1.585	-1.127	±0.839
		EGM-X03	0.792	1.453	-1.168	±0.800
		EGM-X04	0.789	1.492	-1.190	±0.819
		EGM-X05	0.896	1.474	-1.141	±0.806
		EGM96	0.942	1.511	-1.202	±0.830
		OSU91A	0.903	1.660	-1.324	±0.957
		JGM3OSU	0.939	2.253	-1.808	±1.269
Sichuan 1	12	EGM-X01	-23.350	0.194	-0.204	±0.132
		EGM-X02	-23.447	0.202	-0.205	±0.140
		EGM-X03	-23.221	0.200	-0.218	±0.139
		EGM-X04	-23.278	0.204	-0.238	±0.159
		EGM-X05	-23.285	0.206	-0.244	±0.166
		EGM96	-23.111	0.274	-0.261	±0.181
		OSU91A	-22.347	0.229	-0.525	±0.194
		JGM3OSU	-23.254	0.289	-0.263	±0.174
Sichuan 2	19	EGM-X01	-23.657	0.273	-0.255	±0.180
		EGM-X02	-23.624	0.273	-0.199	±0.159
		EGM-X03	-23.532	0.285	-0.261	±0.191
		EGM-X04	-23.583	0.256	-0.243	±0.172
		EGM-X05	-23.582	0.241	-0.218	±0.159
		EGM96	-23.393	0.290	-0.242	±0.190
		OSU91A	-22.391	0.730	-0.422	±0.351
		JGM3OSU	-23.487	0.884	-0.558	±0.441
Sichuan 3	9	EGM-X01	-23.548	0.387	-0.390	±0.223
		EGM-X02	-23.692	0.335	-0.381	±0.211
		EGM-X03	-23.452	0.397	-0.437	±0.239
		EGM-X04	-23.504	0.395	-0.364	±0.220
		EGM-X05	-23.497	0.394	-0.340	±0.211
		EGM96	-23.403	0.390	-0.457	±0.244
		OSU91A	-22.724	0.520	-1.109	±0.501
		JGM3OSU	-23.746	0.632	-1.195	±0.554
Sichuan 4	11	EGM-X01	-23.671	0.228	-0.321	±0.162
		EGM-X02	-23.725	0.248	-0.401	±0.194
		EGM-X03	-23.594	0.212	-0.350	±0.173
		EGM-X04	-23.613	0.235	-0.307	±0.156
		EGM-X05	-23.602	0.216	-0.289	±0.145
		EGM96	-23.520	0.239	-0.397	±0.191
		OSU91A	-22.897	0.507	-1.002	±0.487
		JGM3OSU	-24.057	0.605	-0.950	±0.479
Xinjiang	33	EGM-X01	-66.910	0.227	-0.233	±0.127
		EGM-X02	-67.054	0.306	-0.321	±0.203
		EGM-X03	-67.246	0.251	-0.282	±0.155
		EGM-X04	-67.186	0.253	-0.289	±0.161
		EGM-X05	-67.143	0.270	-0.307	±0.171
		EGM96	-66.675	0.370	-0.358	±0.233
		OSU91A	-66.594	0.677	-0.552	±0.392
		JGM3OSU	-67.321	0.641	-0.538	±0.381

Continue from Tab. 4

Xiamen	30	EGM-X01	-32.393	0.146	-0.327	± 0.112
		EGM-X02	-32.590	0.133	-0.322	± 0.106
		EGM-X03	-32.319	0.144	-0.328	± 0.113
		EGM-X04	-32.322	0.135	-0.321	± 0.107
		EGM-X05	-32.321	0.131	-0.323	± 0.105
		EGM96	-32.242	0.129	-0.323	± 0.103
		OSU91A	-32.487	0.299	-0.466	± 0.204
		JGM3OSU	-32.868	0.267	-0.423	± 0.179
Yantai	86	EGM-X01	-7.385	0.240	-0.256	± 0.113
		EGM-X02	-7.856	0.232	-0.230	± 0.108
		EGM-X03	-7.529	0.246	-0.252	± 0.115
		EGM-X04	-7.553	0.246	-0.245	± 0.115
		EGM-X05	-7.571	0.245	-0.244	± 0.116
		EGM96	-7.408	0.201	-0.204	0.099
		OSU91A	-7.596	0.467	-0.445	± 0.208
		JGM3OSU	-7.992	0.338	-0.311	± 0.154

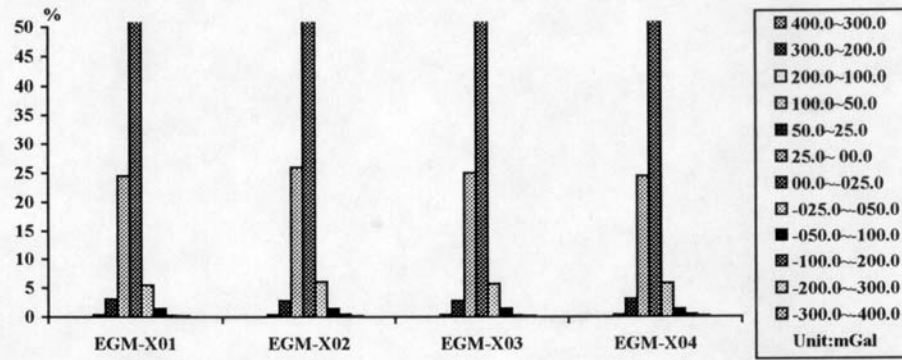


Fig. 2a: The comparison of Chinese 30'x30' gridded mean gravity anomalies set with those obtained from EGM-X01~X05, EGM96, JGM3OSU, and OSU91A

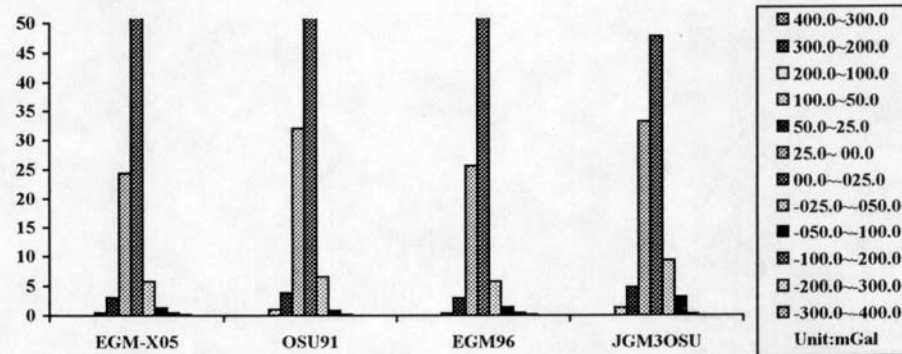


Fig. 2b: The comparison of Chinese 30'x30' gridded mean gravity anomalies set with those obtained from EGM-X01~X05, EGM96, JGM3OSU, and OSU91A

Tab. 5a: The ranges of differences between the Chinese 30'×30' gridded mean gravity anomalies set and those from EGM (X01~X05), EGM96, OSU91A, JGM3OSU

	EGM-X01		EGM-X02		EGM-X03		EGM-X04	
Range (Unit:mGal)	Percent %	Numbers of grid	Percent %	Numbers of Grid	Percent %	Numbers of Grid	Percent %	Numbers of Grid
400~300	0.0000	-	0.0000	-	0.0000	-	0.0000	-
300~200	0.0000	-	0.0000	-	0.0000	-	0.0000	-
200~100	0.0000	-	0.0000	-	0.0000	-	0.0000	-
100~50	0.3756	16	0.3991	17	0.3521	15	0.3991	17
50~25	3.0751	131	2.8404	121	2.8404	121	3.0751	131
25~0	24.5070	1044	25.9390	1105	24.9765	1064	24.4601	1042
0~25	64.6009	2752	62.6761	2670	64.1549	2733	64.2958	2739
-25~-50	5.4930	234	6.0329	257	5.6808	242	5.7512	245
-50~-100	1.4085	60	1.4085	60	1.4554	62	1.3615	58
-100~-200	0.3286	14	0.4695	20	0.3286	14	0.4460	19
-200~-300	0.2113	9	0.2347	10	0.2113	9	0.1878	8
-300~-400	0.0000	-	0.0000	-	0.0000	-	0.0235	1

Tab. 5b: The ranges of differences between the Chinese 30'×30' gridded mean gravity anomalies set and those from EGM (X01~X05), EGM96, OSU91A, JGM3OSU

	EGM-X05		EGM96		OSU91A		JGM3OSU	
Range (Unit:mGal)	Percent %	Numbers of grid	Percent %	Numbers of grid	Percent %	Numbers of Grid	Percent %	Numbers of Grid
400~300	0.0000	-	0.0000	-	0.0000	-	0.0000	-
300~200	0.0000	-	0.0000	-	0.0000	-	0.0000	-
200~100	0.0000	-	0.0000	-	0.0235	1	0.0000	-
100~50	0.4695	20	0.3756	16	1.3146	56	1.1972	51
50~25	3.0986	132	3.0047	128	4.8122	205	5.1174	218
25~0	24.4601	1042	25.6808	1094	33.2160	1415	33.5211	1428
0~25	64.1784	2734	62.9577	2682	47.8169	2037	47.4883	2023
-25~-50	5.7746	246	5.8216	248	9.3427	398	9.3427	398
-50~-100	1.3380	57	1.4789	63	3.2160	137	3.0516	130
-100~-200	0.4695	20	0.4460	19	0.2582	11	0.2817	12
-200~-300	0.2113	9	0.2347	10	0.0000	-	0.0000	-
-300~-400	0.0000	-	0.0000	-	0.0000	-	0.0000	-

Tab. 5c: The statistics of the comparison between the Chinese 30'×30' gridded mean gravity anomalies set with those obtained from EGM (X01~X05), EGM96, OSU91A, JGM3OSU (Unit:mGal)

Model	Max	Min	Mean	RMS	Std
EGM-X01	90.440	-286.614	-4.901	±21.219	±20.645
EGM-X02	86.247	-299.443	-4.996	±21.995	±21.421
EGM-X03	87.180	-288.216	-4.961	±21.249	±20.662
EGM-X04	87.095	-300.110	-4.966	±21.903	±21.333
EGM-X05	86.980	-299.454	-4.970	±22.035	±21.467

Continue from Tab. 5c

EGM96	86.599	-298.425	-4.997	±22.152	±21.581
OSU91A	103.476	-141.939	-4.798	±22.788	±22.277
JGM3OSU	99.387	-143.815	-4.776	±22.817	±22.311

5.2 The comparisons of 15 blocks measured 30'×30' gridded mean free air gravity anomalies with those computed from EGM

The 15 blocks of 30'×30' gridded mean free air gravity anomalies are specially prepared by the Chinese side for the test and evaluation. They are chosen from different parts of China, and with different topography: plains, hilly areas, mountainous area, and plateaus. The comparison has been made between the anomalies of the blocks and those computed from EGM. The statistic results are tabulated in Tab. 6. The fitness of gravity anomalies obtained from GSFC/DMA EGM to those in the blocks is much better than those from the OSU91A and JGM3OSU.

Tab. 6. The comparisons of the 15 blocks of 30'×30' gridded mean free air gravity anomalies with those computed from EGM (X01~X05), EGM96, OSU91A, and JGM3OSU (Unit:mGal)

Model	Max	Min	Mean	RMS	Std.
EGM-X01	35.0	-55.9	6.6	±22.1	±21.1
EGM-X02	39.0	-53.3	7.3	±23.8	±22.6
EGM-X03	36.4	-55.7	7.1	±22.8	±21.6
EGM-X04	40.2	-56.4	6.8	±23.4	±22.4
EGM-X05	40.4	-57.8	6.2	±23.7	±22.8
EGM96	42.6	-58.6	6.2	±24.8	±24.0
OSU91A	80.3	-65.2	10.6	±33.7	±31.9
JGM3OSU	82.6	-66.8	9.7	±33.1	±31.6
30'×30' set	42.8	-58.5	10.7	±25.6	±23.2

5.3 Comparison of point gravity values in 4 sheets of 1:1m map with those computed from EGM

Comparisons with those computed from GSFC/DMA EGM of point gravity values are made, which are distributed in the 4 sheets of 1:1m scale map in China (see Fig. A). The statistic results of the comparison are listed in Tab. 7. The models with minimum standard deviation are EGM-X03 in H50, EGM-X03 in I46, EGM-X05 in J49, and EGM-X05 in L51 respectively. Thus, the models with maximum standard deviation are JGM3OSU in H50, EGM-X05 in I46, OSU91A in J49, and EGM-X02 in L51 respectively.

Tab. 7 The statistic comparison of point gravity anomaly with that from geopotential model

Name	Number s	Model	Max	Min	Mean	RMS	Std
H50	7969	EGM-X01	101.595	-64.925	11.871	±22.640	±19.278
		EGM-X02	101.171	-64.897	10.874	±22.286	±19.453
		EGM-X03	101.935	-64.174	11.767	±22.603	±19.298
		EGM-X04	102.059	-65.247	11.786	±22.972	±19.719
		EGM-X05	114.199	-80.505	12.664	±23.407	±19.685

Continue from Tab.7							
I46	1849	EGM96	103.355	-66.534	9.938	±22.308	±19.972
		OSU91A	101.568	-65.399	11.758	±23.413	±20.246
		JGM3OSU	117.346	-89.307	15.803	±26.293	±21.014
		EGM-X01	160.229	-58.428	5.315	±27.475	±26.956
		EGM-X02	160.954	-63.921	2.230	±28.227	±28.138
		EGM-X03	160.108	-57.206	4.343	±27.331	±26.984
		EGM-X04	158.014	-54.676	3.505	±27.620	±27.397
		EGM-X05	206.665	-110.658	-3.855	±40.180	±39.994
		EGM96	162.880	-62.026	3.577	±28.655	±28.431
J49	8367	OSU91A	157.264	-54.771	4.127	±27.911	±27.604
		JGM3OSU	207.186	-100.093	1.307	±38.410	±38.388
		EGM-X01	180.078	-129.634	8.846	±51.247	±50.478
		EGM-X02	183.047	-130.713	10.980	±52.286	±51.120
		EGM-X03	183.028	-127.656	10.951	±51.909	±50.740
		EGM-X04	185.508	-127.590	10.684	±52.580	±51.483
		EGM-X05	161.287	-111.022	26.600	±42.534	±33.190
		EGM96	182.293	-129.903	11.293	±52.846	±51.626
		OSU91A	185.278	-129.623	10.553	±52.891	±51.828
L51	8273	JGM3OSU	158.751	-120.424	24.842	±41.944	±33.797
		EGM-X01	95.177	-57.441	26.737	±42.356	±32.851
		EGM-X02	102.214	-60.508	29.113	±45.139	±34.496
		EGM-X03	95.916	-57.155	27.459	±42.870	±32.922
		EGM-X04	97.788	-58.099	27.579	±43.295	±33.375
		EGM-X05	97.079	-44.651	32.934	±41.451	±25.170
		EGM96	101.467	-63.006	27.586	±45.081	±35.663
		OSU91A	100.254	-59.093	27.924	±44.164	±34.216
		JGM3OSU	95.360	-42.448	31.593	±41.194	±26.435

6 Conclusion

This paper has described the comparisons of the measured geoid undulations and gravity anomalies in China with those computed from EGM. According to the above results, the model with the best fitting to the long distance geoid undulations in the Chinese territory is EGM96, the model with the best fitting to the local geoid undulations is EGM-X02, and the model with the best fitting to the local gravity field in the Chinese territory is EGM-X03.

Generally speaking, the geoid undulations and gravity anomalies computed from EGM96 have been significantly improved in the representation of the Chinese local gravity field as compared to other existing model.

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Analysis of EGM Models in New Zealand

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1 INTRODUCTION

In October 1996 a new global geopotential model was released through a joint project between the NASA Goddard Space Flight Centre (GSFC) and the U.S. Defence Mapping Agency (DMA), being referred to as the Earth Geopotential Model of 1996 (EGM96). This paper summarises the testing of the pre-release models (EGM-X02 through X05) and EGM96 for New Zealand. This investigation formed part of the work undertaken by the IAG Special Working Group (SWG) established to test the EGM models.

2 EVALUATION APPROACH

Two approaches were undertaken to determine which of the new models was most suitable for New Zealand. In brief they were :

Approach ONE: A nationwide set of stations that were coordinated in WGS84 and also had orthometric heights were chosen (Figure 1). For each station the orthometric height (H) was subtracted from the ellipsoidal height (h) to obtain the geometric geoid height ($N_{GEO} = h - H$). Then at each station the model geoid height (N_{EGM}) was calculated using software based on Rapp (1982). The difference between the geoid heights at each station was then calculated ($\Delta N = N_{GEO} - N_{EGM}$) and analysed statistically.

Approach TWO: The land free-air gravity anomalies for New Zealand were converted to residual gravity anomalies by subtracting the model's gravity anomalies. Statistical analyses was then performed on these residuals to determine which model performed best.

3 RESULTS

3.1 Approach ONE

WGS84 Coordinates

The stations used in this analysis form part of the New Zealand Datum Investigation work which is being undertaken by LINZ. The processing of the GPS data and the determination of ITRF93 coordinates for the stations is described in Pearse and Morgan (1995). The cartesian coordinates were converted to geodetic coordinates on the WGS84 reference frame (Appendix A).

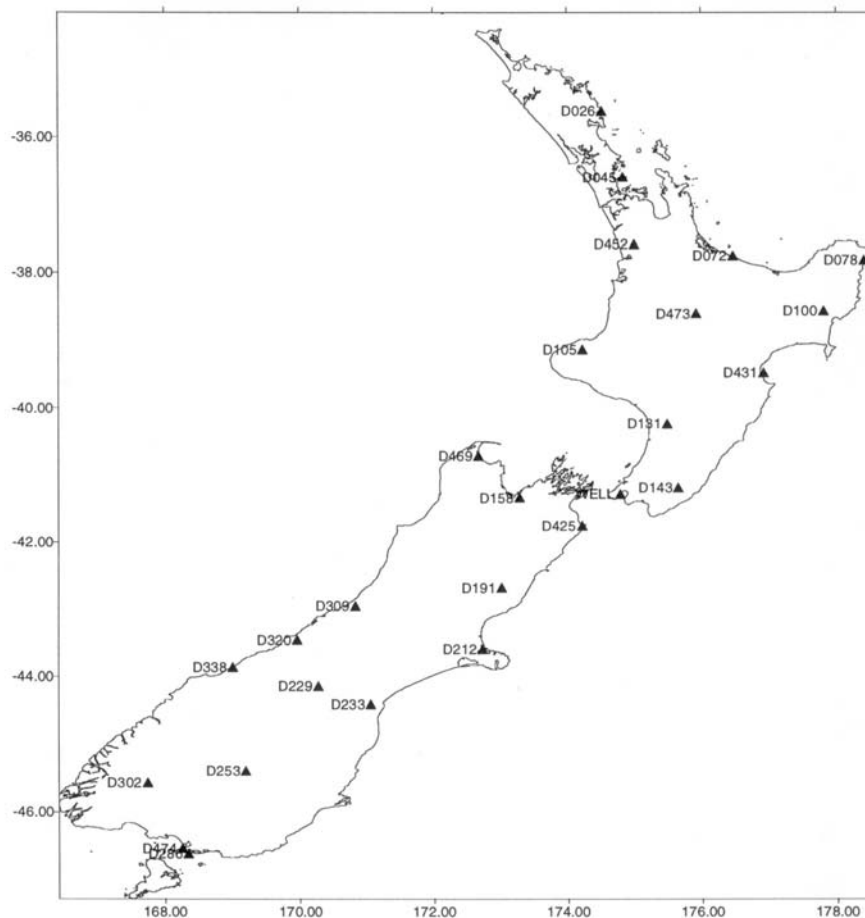


Figure 1 : Location of test stations for comparing models using Approach One.

Orthometric Heights

New Zealand does not have a single nationwide height datum for levelling networks. There are a number of tide gauges distributed around the coast of both the North and South Island's which are used to define a series of Mean Sea Level (MSL) datums. Consequently some benchmarks in the precise levelling networks can have heights in terms of more than one tide-gauge, though the maximum height difference is less than 0.25 metres (Gilliland, 1987). It should be noted that there are no tide gauges along the west coast of the South Island.

Unfortunately only one of the stations used in this analysis had orthometric height in terms of the 1st order precise levelling network. Most stations were either 3rd or 4th order. The specifications for order in New Zealand height levelling networks are normally given in terms of the maximum allowable difference between a forward and backward levelling run, ie. the close error (Appendix B).

The ΔN values at each North Island station are contained in Table 2. From Table 2 it is evident that all models tested (OSU91A, JGM3_OSU91A and the five EGM's) produce ΔN values that can vary by up to $\pm 1\text{m}$. It should be noted that these differences could be generated by errors in any one, or all, of the three heights (h , H or N_{EGM}).

North Island Results

Station ID	OSU91A	JGMOSU	EGM-X02	EGM-X03	EGM-X04	EGM-X05	EGM96
D026	38.224	38.468	38.356	38.404	38.444	38.488	38.316
D045	35.802	36.140	35.085	35.256	35.254	35.259	35.104
D072	29.319	29.640	29.110	29.181	29.185	29.145	29.218
D078	21.265	21.434	21.337	21.209	21.214	21.229	21.267
D100	21.313	21.513	21.747	21.696	21.710	21.654	21.738
D105	22.383	22.336	22.649	22.861	22.879	22.922	22.955
D131	12.897	12.737	13.022	13.358	13.201	13.003	13.139
D143	16.396	16.153	16.422	16.305	16.552	16.723	16.809
D431	18.463	18.526	18.009	18.293	18.149	18.066	18.151
D452	31.864	32.192	31.518	31.804	31.768	31.758	31.684
D473	26.585	26.807	27.098	27.231	27.309	27.314	27.404
WELL	13.649	13.409	12.816	12.665	12.774	12.867	13.086

Table 1 : Geoid heights (N_{EGM}) generated from respective models for each North Island station (units = metres). JGMOSU is the combined JGM3 and OSU91A model.

If one assumes that there is no datum bias between the three height networks then the model that best represents the test stations would have an average ΔN closest to zero and also the smallest standard deviation.

In terms of the average then EGM-X03 is the best model, though distinguishing it from OSU91A, EGM-X04 or EGM-X05 at a practical level is difficult. When considering the standard deviation then EGM-X02 is the preferred model. It is worth noting that JGM3_OSU91A, EGM-X03, EGM-X04, EGM-X05 and EGM96 are practically equivalent when comparing the standard deviation.

Station ID	OSU91A	JGMOSU	EGM-X02	EGM-X03	EGM-X04	EGM-X05	EGM96
D026	0.205	-0.039	0.073	0.025	-0.015	-0.059	0.113
D045	-0.595	-0.933	0.122	-0.049	-0.047	-0.052	0.103
D072	0.011	-0.310	0.220	0.149	0.145	0.185	0.112
D078	0.629	0.460	0.557	0.685	0.680	0.665	0.627
D100	0.872	0.672	0.438	0.489	0.475	0.531	0.447
D105	0.255	0.302	-0.011	-0.223	-0.241	-0.284	-0.317
D131	-0.550	-0.390	-0.675	-1.011	-0.854	-0.656	-0.792
D143	-0.643	-0.400	-0.669	-0.552	-0.799	-0.970	-1.056
D431	-0.147	-0.210	0.307	0.023	0.167	0.250	0.165
D452	0.249	-0.079	0.595	0.309	0.345	0.355	0.429
D473	0.623	0.401	0.110	-0.023	-0.101	-0.106	-0.196
WELL	-0.809	-0.569	0.024	0.175	0.066	-0.027	-0.246
<i>minimum</i>	-0.809	-0.933	-0.675	-1.011	-0.854	-0.970	-1.056
<i>maximum</i>	0.872	0.672	0.595	0.685	0.680	0.665	0.627
<i>average</i>	0.008	-0.091	0.091	0.000	-0.015	-0.014	-0.051
<i>std. dev.</i>	0.561	0.474	0.408	0.451	0.457	0.468	0.500

Table 2 : Geoid height differences ($\Delta N = N_{\text{GEO}} - N_{\text{EGM}}$) for respective models at each North Island station (units = metres). Note : std. dev. is the standard deviation at the one sigma level.

South Island Results

Station ID	OSU91A	JGMOSU	EGM-X02	EGM-X03	EGM-X04	EGM-X05	EGM96
D158	15.495	15.368	15.086	14.644	14.665	14.604	15.002
D191	13.035	13.213	13.389	12.734	12.733	12.720	13.261
D212	11.820	12.163	12.076	11.439	11.443	11.427	11.915
D229	8.367	9.144	9.019	8.128	8.143	8.132	8.854
D233	8.381	9.017	8.705	7.920	7.903	7.862	8.491
D253	6.481	7.124	6.779	6.225	6.200	6.252	6.746
D286	3.184	3.490	3.542	3.331	3.303	3.388	3.611
D302	3.840	4.443	4.503	4.070	4.142	4.121	4.479
D309	11.696	12.346	11.595	10.715	10.692	10.651	11.345
D320	8.501	9.309	9.135	8.305	8.307	8.289	8.966
D338	5.716	6.553	5.997	5.371	5.362	5.327	5.893
D425	12.011	11.848	11.140	10.916	10.896	10.933	11.25
D469	19.859	19.679	18.744	18.351	18.400	18.383	18.634
D474	3.386	3.721	3.740	3.525	3.484	3.573	3.802

Table 3 : Geoid heights (N_{EGM}) generated from respective models for each South Island station (units = metres)

Station ID	OSU91A	JGMOSU	EGM-X02	EGM-X03	EGM-X04	EGM-X05	EGM96
D158	0.702	0.829	1.111	1.553	1.532	1.593	1.195
D191	0.522	0.344	0.168	0.823	0.824	0.837	0.296
D212	0.146	-0.197	-0.110	0.527	0.523	0.539	0.051
D229	1.921	1.144	1.269	2.160	2.145	2.156	1.434
D233	0.571	-0.065	0.247	1.032	1.049	1.090	0.461
D253	1.561	0.918	1.263	1.817	1.842	1.790	1.296
D286	0.728	0.422	0.370	0.581	0.609	0.524	0.301
D302	2.105	1.502	1.442	1.875	1.803	1.824	1.466
D309	2.495	1.845	2.596	3.476	3.499	3.540	2.846
D320	2.470	1.662	1.836	2.666	2.664	2.682	2.005
D338	1.763	0.926	1.482	2.108	2.117	2.152	1.586
D425	-0.318	-0.155	0.553	0.777	0.797	0.760	0.443
D469	-0.982	-0.802	0.133	0.526	0.477	0.494	0.243
D474	0.595	0.260	0.241	0.456	0.497	0.408	0.179
<i>minimum</i>	-0.982	-0.802	-0.110	0.456	0.477	0.408	0.051
<i>maximum</i>	2.495	1.845	2.596	3.476	3.499	3.540	2.846
<i>average</i>	1.020	0.617	0.900	1.455	1.456	1.456	0.986
<i>std. dev.</i>	1.056	0.778	0.790	0.933	0.932	0.954	0.833

Table 4 : Geoid height differences ($\Delta N = N_{\text{GEO}} - N_{\text{EGM}}$) for respective models at each South Island station (units = metres). Note : std. dev. is the standard deviation at the one sigma level.

For the South Island stations the range between the minimum and maximum geoid height differences (Table 4) is over twice the size of the North Island values, ie. -1 to 3.5 m.

When a comparison of the average for the South Island geoid height differences is made there appears to be a bias of approximately 1 m between the three height datums. The two stations (D309, D320) which contain the largest geoid height differences for all models are both on the west coast. A few possible reasons for the discrepancy are that firstly there are no tide gauges along the west coast to control the orthometric height of these stations. Secondly the

Southern Alps, which is the main range in the South Island, runs within approximately 15 km of these stations and rises from sea level to between 2300 to 3700 m.

As approximately half of the stations in the South Island had geoid height differences of greater than 1 m the deletion of one or more suspect stations from the analysis was not feasible.

In comparing the standard deviations the best model could be either JGM3_OSU91A or EGM-X02, with EGM96 having the third smallest standard deviation. What is also interesting to note is that EGM-X03 through X05 perform no better than OSU91A. But the tests are inconclusive because of the possible presence of large errors in the data sets.

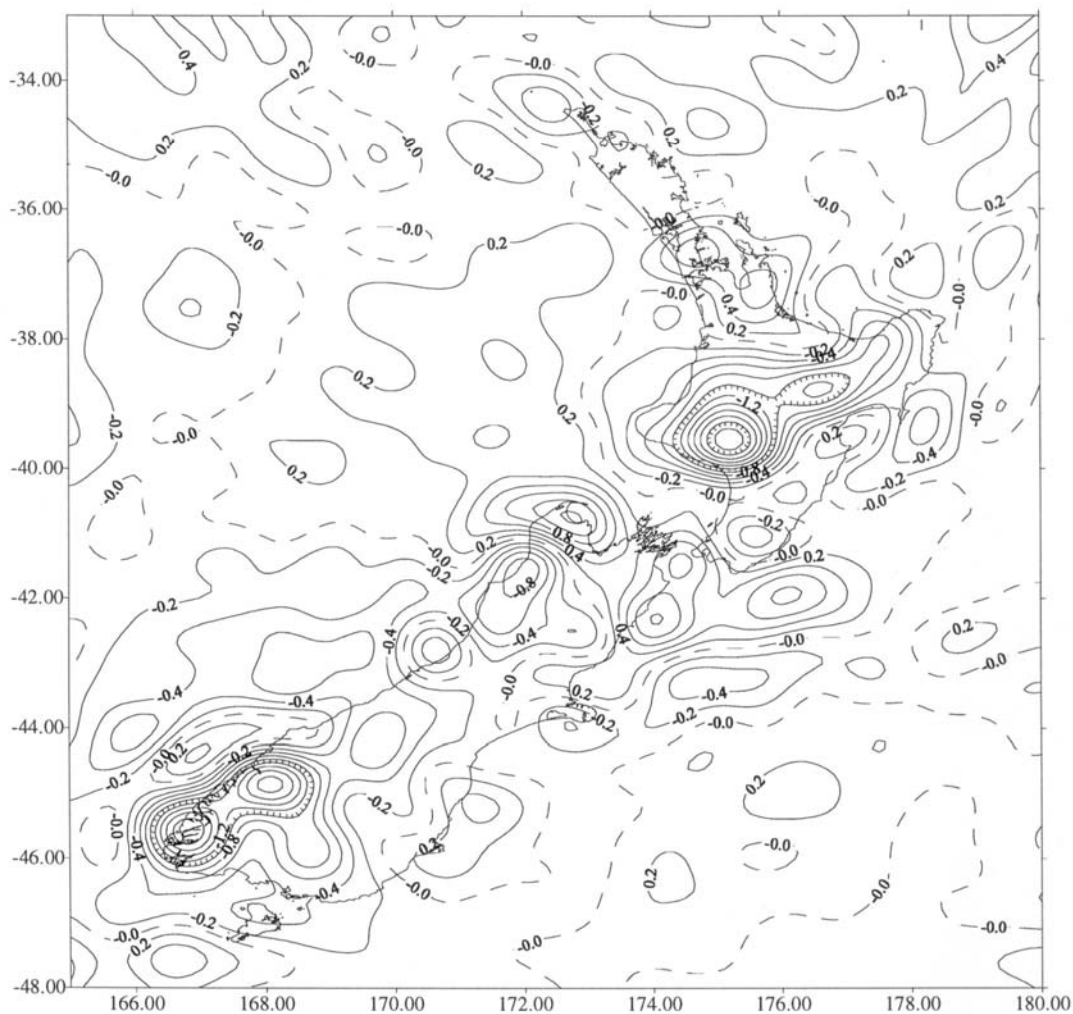


Figure 2 : Geoid height differences across New Zealand generated by subtracting EGM96 from OSU91A for a $0.1^\circ \times 0.1^\circ$ grid. (units = metres)

From Table 1 and 3 it was surprising to see that the differences in N_{EGM} values between OSU91A and EGM96 could vary from basically 0.0m (D078) up to 1.2m (D469). To

investigate the size and location of the maximum difference between the two N_{EGM} values across New Zealand, a $0.1^\circ \times 0.1^\circ$ grid of N values were generated from each model. The difference between these two surfaces (Figure 2, sense OSU91A minus EGM96) revealed that variations ranged from -2.357 m ($S39.5^\circ$, $E175.0^\circ$) to 1.233m ($S40.5^\circ$, $E173.0^\circ$), with a mean of -0.024 m and a standard deviation of 0.341m.

The probable reason for the high frequency variations, since they are located primarily on land, is the application of different power law constraints to the satellite only solutions (R. Forsberg, pers. comm., 1996). OSU91A used Kaula's power law, divided by the square root of two, which underestimates the power in the Earth gravity field, as determined from surface gravity observations (Lemoine *et al.*, 1996). Lemoine *et al.* (*ibid.*) state that EGM96 used a power law constraint derived from the signal of the surface gravity data to spectrally constrain the coefficients of the EGM96 satellite-only solution. As the satellite-only solution is the foundation of the high degree quadrature solution, it important that the spectrum of the errors in this satellite-only model be consistent with the signal observed by surface gravity data. Another reason for the variations is that data used to derive the EGM96 coefficients contains terrain corrections when OSU91A does not.

From the use of a more appropriate power law constraint, EGM96 appears to be able to represent smaller scale variations in the geoid than OSU91A.

3.2 Approach TWO

Gravity data

The gravity data used in this analysis is the onshore data only from Gilliland (1988). The data is free-air gravity anomalies referenced to the International Gravity Standardisation Net 1971 (IGSN71) and the Geodetic Reference System 1980 (GRS80), corrected for atmospheric effects and terrain correction.

Residual gravity anomalies

Using a program based on Rizos (1979), a grid (0.1° interval) of gravity anomalies was generated for each model. From the grid of gravity anomalies a value at each gravity station was interpolated and then subtracted from the observed free-air gravity anomaly to give the residual free-air gravity anomalies at each gravity station.

Results

From comparing the average residual free-air anomaly for each island (Table 5) it is clear that there is a bias between the North Island and the South Island of between -5 to -10 mGal.

Using the same assumptions as were used in Approach One, that is that the average should be zero and the standard deviation should be a minimum, then the South Island gravity data appears to be poorly recovered by the models tested. However, the bias in the South Island could be due to errors in either the orthometric heights, or the assumed mean crustal density, used to reduce the observed gravity to free-air gravity anomalies.

Data errors in the South Island aside, the New Zealand wide results show that the JGM3_OSU91A model performs the worst out of the tested models in terms of both the average and the standard deviation. Comparing the standard deviations of the EGM-X02, EGM-X03, EGM-X04, and EGM-X05 models against OSU91A, in general OSU91A performs the worst. However, when comparing the averages OSU91A is superior to EGM-X02. There

is also no real practical difference between EGM-X03, EGM-X04 and EGM-X05 for either the averages or standard deviations. When comparing standard deviations EGM96 is marginally superior to OSU91A.

	OSU91A	JGMOSU	EGM-X02	EGM-X03	EGM-X04	EGM-X05	EGM96
NZ WIDE							
Number of stations	34763	34763	34763	34763	34763	34763	34763
Minimum	-96.994	-98.844	-109.490	-109.081	-110.813	-111.752	-112.903
Maximum	193.732	195.038	192.361	195.442	195.460	196.703	193.639
Average	-4.415	-6.145	-5.188	-3.365	-3.501	-3.358	-5.466
Standard Deviation	28.752	28.960	28.563	28.213	28.141	28.192	28.459
NORTH ISLAND							
Number of stations	17467	17467	17467	17467	17467	17467	17467
Minimum	-96.994	-98.844	-109.490	-109.081	-110.813	-111.752	-112.903
Maximum	146.787	148.970	146.520	149.693	145.877	143.746	143.194
Average	-0.171	-1.639	-0.241	-0.873	-1.242	-1.407	-1.877
Standard Deviation	21.138	21.112	20.130	20.296	20.264	20.378	20.527
SOUTH ISLAND							
Number of stations	17296	17296	17296	17296	17296	17296	17296
Minimum	-83.208	-86.385	-77.678	-73.331	-73.579	-75.099	-78.279
Maximum	193.732	195.038	192.361	195.442	195.460	196.703	193.639
Average	-8.700	-10.696	-10.183	-5.882	-5.783	-5.329	-9.091
Standard Deviation	34.261	34.560	34.365	34.224	34.157	34.212	34.295

Table 5 : Statistical analysis of the residual free-air gravity anomalies for each respective model by island and the country as a whole (units = mGal). Note : the standard deviation is at the one sigma level.

4 COMPARISONS

Based on this limited data set (especially for Approach One) to evaluate the performance of the new models within New Zealand one has to note that the EGM-X02 through X05 models are an improvement over OSU91A. Unfortunately EGM96 is only marginally superior to OSU91A. The superior performance of the models in the North Island is possibly attributable to the North Island being topographically less undulating than the South Island and/or the control in the North Island being more precise. Though the possibility of there being a bias in the reduction of the raw gravity observations to IGN71 due to errors in the reference gravity stations or in the height network in the South Island needs to be investigated.

In nominating which model is best for New Zealand it is difficult to distinguish between EGM-X03, X04 or X05. When the results of both approaches are considered together EGM-X04 is only marginally superior. When OSU91A and EGM96 are compared, using this small data set in New Zealand, EGM96 is marginally superior.

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APPENDIX A

The orthometric heights contained in the table were extracted from the LINZ Geodetic Database by J. Hall during January 1995. The latitude, longitude and ellipsoidal height are in terms of GRS80.

Station ID	Latitude	Longitude	Ellipsoidal Height (h)	Orthometric Height (H)	Order of H
	dec. deg.	dec. deg.	metres	metres	
North Island					
D026	-35.61725	174.51436	174.519	136.090	3
D045	-36.60022	174.83205	141.907	106.700	2
D072	-37.75947	176.46641	95.930	66.600	4
D078	-37.82454	178.40710	360.584	338.690	3
D100	-38.57515	177.80485	323.545	301.360	3
D105	-39.13400	174.22821	263.238	240.600	2
D131	-40.24020	175.48831	143.727	131.380	1
D143	-41.18014	175.65217	590.954	575.200	3
D431	-39.47899	176.91724	119.396	101.080	2
D452	-37.58939	175.00002	319.113	287.000	4
D473	-38.61605	175.91503	760.488	733.280	3
WELL	-41.27489	174.78295	37.820	24.980	2
South Island					
D158	-41.33162	173.28090	791.897	775.700	4
D191	-42.68742	173.01028	405.657	392.100	4
D212	-43.58897	172.72733	510.526	498.560	3
D229	-44.13885	170.27760	1005.888	995.600	3
D233	-44.40057	171.05735	397.352	388.400	4
D253	-45.38765	169.19770	1681.042	1673.000	4
D286	-46.61496	168.33880	269.062	265.150	2
D302	-45.56212	167.73892	411.445	405.500	4
D309	-42.95325	170.82966	919.511	905.320	3
D320	-43.45580	169.96033	114.831	103.860	3
D338	-43.86082	169.00359	14.639	7.160	3
D425	-41.74905	174.21381	254.693	243.000	4
D469	-40.71300	172.67221	169.677	150.800	4
D474	-46.53693	168.25344	176.531	172.550	3

APPENDIX B

Specifications for orthometric height levelling networks in New Zealand. Supplied by G. Rowe (pers. comm.)

Order	Specification
1st	spirit levelling with a maximum close error of $\pm 2 \text{ mm} \sqrt{d}$, where d is the distance in km. Average close error of 3 mm
2nd	spirit levelling with a maximum close error of $\pm 7 \text{ mm} \sqrt{d}$, where d is the distance in km. Average close error of 5 mm
3rd	spirit levelling which does not meet 1st or 2nd order specifications. trigonometric levelling that is well controlled with short or low elevation observations
4th	trigonometric levelling that is poorly controlled with long or high elevation observations