

FOREWORD

Dear reader,

this number of the IGeS Bulletin is coming out after the special number dedicated to the Working Group for the evaluation of the EGM '96 performances.

A few paper came in Milan too late to be published in that number, therefore you find here a last section devoted to them.

Subsequently a section for new geoids, new theories follows.

Please note that we are publishing a paper on Informatics and Geoid.

The idea is that papers like that should be more and more present the Bulletin, for which reason I invite all of you to think whether you have, also in your practical experience, material on this item which we will be happy to publish.

Finally a section comes on international internal business and organization of the Service.

Please note that **A NEW GEOID SCHOOL IS ADVERTISED** for **OCTOBER 19-24, 1998** in **BANDUNG (INDONESIA)**.

Tests of the DMA/GSFC geopotential models over Australia

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Abstract

The EGM series of geopotential models have been compared with terrestrial free-air gravity anomalies, GPS-levelling data, and an existing gravimetric geoid model in the Australian region. While no significant amounts of new gravity data were included in these geopotential models, a small overall improvement upon OSU91A, JGM3-OSU91A and GFZ95A was observed.

Introduction

The International Geoid Service's Special Working Group on the Goddard Space Flight Center (GSFC) and Defense Mapping Agency (DMA) Model Evaluation commissioned tests at selected areas around the Earth, where the models would be compared statistically with surface gravity and geoid height data. This report describes the procedures used and results obtained from such tests using data on the Australian continent and in the surrounding seas.

The models tested were:

- EGM-X01 through X05;
- EGM96;
- JGM3-OSU91A;
- OSU91A [Rapp et al. (1991)];
- GFZ95A [Gruber et al. (1996)].

Each model was expanded to spherical harmonic degree and order 360, and scaled to the GRS80 reference ellipsoid [Moritz (1980)] as Australian data are given relative to this particular spheroid.

The JGM3-OSU91A model is a composite of JGM3 [Tapley et al. (1996)] from degrees 2 to 70, and OSU91A from degrees 71 to 360, and was provided with the EGM-X models. OSU91A

was included in comparisons as it is extensively used throughout Australia [*e.g.* Steed and Holtznagel (1994)], and GFZ95A was included as it is a recent global model computed by an independent group.

Control data

The models were compared with the following terrestrial gravity and geoid data:

- Free-air gravity anomalies on land derived from the Australian Geological Survey Organisation's (AGSO) 1992 data-base (526,091 observations);
- Marine gravity anomalies derived from the AGSO data-base (111,396 observations);
- Marine gravity anomalies on a two-minute grid from ERS-1, Geosat/ERM and TOPEX/Poseidon satellite altimeter observations compiled by Sandwell et al. (1995) (3,178,940 points);
- geometrical geoid heights from the difference between GPS ellipsoidal heights comprising the Australian Fiducial Network (AFN) and Australian National Network (ANN) and optically levelled Australian Height Datum (AHD) heights (59 points);
- AUSGEOID93 gravimetric geoid heights on a ten-minute grid over continental Australia (34,240 points) [Steed and Holtznagel (1994)].

The AGSO gravity data comprised free-air anomalies from the database re-validated by Featherstone et al. (1997), using second-order free-air and atmospheric corrections. They have an estimated accuracy of 1.2–1.8mgal on land [*ibid.*] and an estimated accuracy of 3–5mgal at sea.

Seventy-three AFN and ANN stations are distributed evenly over the continent and were observed using dual-frequency GPS receivers between 1992 and 1994 [Manning and Harvey (1994)]. Fifty-nine of these were co-located with Australian Height Datum (AHD) heights obtained from third-order optical levelling. Only these optically levelled stations were used in the analysis.

The statistics of the control data are shown in Table 1.

Data	min	max	mean	std. dev.	RMS
AGSO land (mgal)	-121.030	172.785	0.410	25.296	25.299
AGSO marine (mgal)	-241.436	292.128	-3.428	34.523	34.693
Sandwell satellite (mgal)	-297.200	295.400	-3.892	30.787	31.032
AFN/ANN/AHD (m)	-32.241	71.301	14.738	26.873	30.649
AUSGEOID93 (m)	-39.500	75.760	16.793	27.616	32.321

Table 1: Statistics of the gravity and geoid height data used as control.

Computation of the potential fields

The geoid heights and gravity anomalies were calculated relative to the GRS80 ellipsoid from each model's coefficients and includes the zero-degree term, accounts for the mass difference between the EGM and GRS80, and a scaling term, which accounts for the difference in flattening and semi-major axis length [Kirby and Featherstone (1997)]. The zero-degree bias for the EGM series of models is -0.937m in the geoid and -0.144mgal in the gravity anomaly. Tests were also performed excluding the zero-degree term in order to illustrate its effect.

The software used to compute the geoid height and gravity anomalies from the geopotential coefficients is a modified version of the FORTRAN code developed by Rapp (1982). In the tests, the geoid height and gravity anomalies were calculated to degree and order 360 at the nodes of a 15-minute grid covering Australia and its surrounding seas. The value of the desired potential field was then obtained at the locations of the surface control data by bicubic interpolation: 15-minutes was deemed suitable for the accurate representation of wavelengths for a 360-degree model [Tziavos (1996)].

This approach also makes use of the fast recursive relations for the associated Legendre functions along parallels [*cf* Rizos (1979)]. The results produced using this technique differ by a few millimetres or microgals from a point-by-point solution, however, the computation time is reduced by almost two orders of magnitude.

Results

In order to determine the fit of the geopotential model to the control data, the model value was subtracted from the control data value at the observation location, and the statistics of the difference calculated for all data points. These statistics are shown in Tables 2 to 11.

Comparing the differences calculated with and without the zero-degree term, it can be seen that its inclusion serves to decrease the mean difference between data and model for the AGSO land and marine gravity data (*cf* Tables 2 and 3, and Tables 4 and 5). The standard deviations remain virtually unchanged, as expected, since the zero-degree term is an approximate bias.

The mean difference for the AFN/ANN/AHD data, however, increases when the zero-degree term is included (*cf* Tables 8 and 9). It is uncertain as to the exact origin of this difference, but it is most probably due to a bias between the geoid as defined by $W_0 = \text{constant}$ and the AHD. This offset agrees with the bias between these datums as estimated by Rapp (1994) and Featherstone (1995).

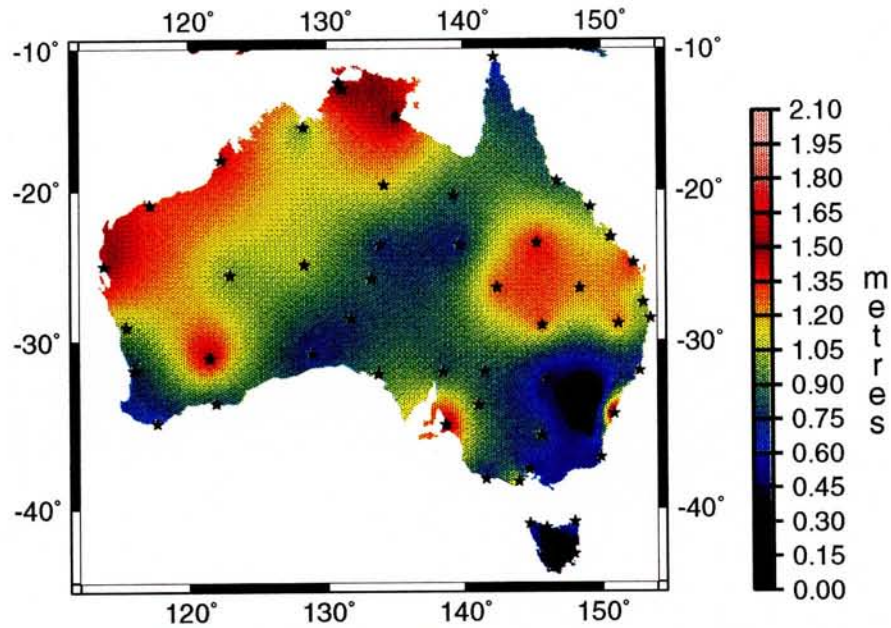


Figure 1: A colour image of the interpolated differences between the geometrical geoid height at the AFN/ANN/AHD stations (black dots) and the EGM96 geoid including the zero-degree term (units in metres; Mercator projection).

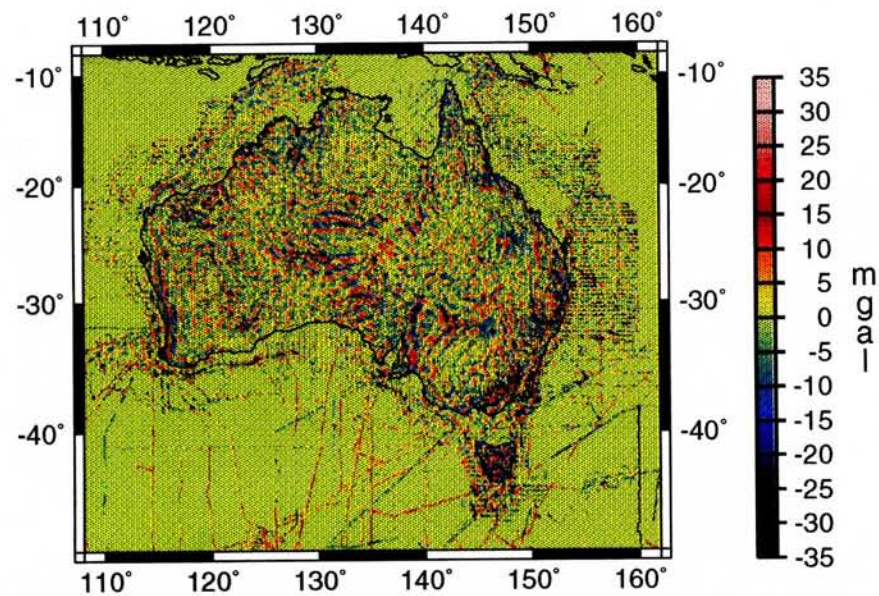


Figure 2: A colour image of the residuals between the AGSO land and marine gravity data and EGM96 (units in mgal; Mercator projection).

Figure 1 shows a colour image of the residuals interpolated between the geometrical geoid height at the 59 AFN/ANN/AHD stations and the EGM96 geoid (including the zero-degree term). The large differences (from the mean value of 0.921m) in West Australia and Queensland, for example, are most likely due to errors in levelling traverses, since the GPS measurements are estimated to have an accuracy of approximately 0.1m. Furthermore, the larger difference observed in Tasmania can be partly explained by the 0.4m offset between the AHD on the Australian mainland and the AHD in Tasmania [Rapp (1994)].

Figure 2 shows a colour image of the residuals between the terrestrial gravity anomalies and EGM96. No long wavelength gravity residuals are correlated with the geoid residuals in Figure 1, which suggests that the reason for the latter differences is due to the AFN/ANN/AHD control data. Several ship tracks in the south of Figure 2 exhibit biases and tilts over their length, which is due to errors in these data. This is to be expected because these tracks are widely spaced, thus preventing a rigorous cross-over adjustment.

AGSO land data

Considering the individual models, the results (through the standard deviations) show that the fit of the EGM models is worse than OSU91A, GFZ95A and the hybrid model, however the margin is only a fraction of a milligal. GFZ95A has the best fit of all models, with $\sigma = 10.658\text{mgal}$. The best fitting EGM solution is EGM-X05 with $\sigma = 11.050\text{mgal}$, the worst is EGM-X03 with $\sigma = 11.162\text{mgal}$, while the new model, EGM96, has a standard deviation of 11.097mgal . Given that the AGSO land data may be in error by over 1mgal , the differences are indistinguishable.

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-97.019	113.489	-1.044	11.144	11.192
EGM-X02	-96.140	114.050	-0.957	11.114	11.155
EGM-X03	-97.192	113.143	-1.193	11.162	11.225
EGM-X04	-96.047	112.196	-1.047	11.071	11.121
EGM-X05	-96.212	110.894	-0.948	11.050	11.091
EGM96	-95.661	111.747	-0.929	11.097	11.136
JGM3-OSU91A	-101.184	120.141	-0.965	10.928	10.970
OSU91A	-99.663	122.731	-0.974	10.865	10.908
GFZ95A	-97.591	122.461	-0.424	10.658	10.667

Table 2: Statistics of the difference between the AGSO land gravity data and the stated geopotential model including the zero-degree term (mgal).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-97.253	113.174	-1.188	11.145	11.208
EGM-X02	-96.373	113.727	-1.102	11.115	11.169
EGM-X03	-97.425	112.829	-1.338	11.163	11.243
EGM-X04	-96.280	111.874	-1.192	11.073	11.137
EGM-X05	-96.439	110.562	-1.093	11.051	11.105
EGM96	-95.901	111.414	-1.074	11.099	11.151
JGM3-OSU91A	-101.311	119.809	-1.109	10.928	10.985
OSU91A	-99.804	122.385	-1.132	10.866	10.924
GFZ95A	-98.169	122.043	-0.573	10.658	10.673

Table 3: Statistics of the difference between the AGSO land gravity data and the stated geopotential model without the zero-degree term (mgal).

AGSO marine data

The EGM models exhibit a better fit to the marine data than the others, by over a milligal in the standard deviation. Of these, EGM-X05 shows the best fit with $\sigma = 15.244\text{mgal}$, while EGM-X03 has the worst with $\sigma = 15.437\text{mgal}$. EGM96 has a good fit with $\sigma = 15.252\text{mgal}$. GFZ95A has the poorest fit of all the models with a σ of 17.429mgal . Again, when considering the estimated accuracy of the marine gravity data, these differences are virtually indistinguishable.

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-237.522	201.452	-0.489	15.436	15.443
EGM-X02	-236.825	198.207	-0.633	15.361	15.374
EGM-X03	-236.575	202.103	-0.380	15.437	15.442
EGM-X04	-237.396	197.286	-0.405	15.309	15.314
EGM-X05	-237.803	195.559	-0.407	15.244	15.249
EGM96	-237.997	194.267	-0.507	15.252	15.260
JGM3-OSU91A	-236.230	213.884	-1.236	16.368	16.414
OSU91A	-236.502	213.471	-1.262	16.288	16.337
GFZ95A	-234.649	287.360	-0.932	17.429	17.454

Table 4: Statistics of the difference between the AGSO marine gravity data and the stated geopotential model including the zero-degree term (mgal).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-237.717	200.978	-0.633	15.435	15.448
EGM-X02	-237.021	198.444	-0.776	15.361	15.380
EGM-X03	-236.771	201.629	-0.524	15.437	15.446
EGM-X04	-237.592	197.374	-0.549	15.309	15.319
EGM-X05	-237.982	195.822	-0.551	15.244	15.254
EGM96	-238.177	194.530	-0.651	15.252	15.265
JGM3-OSU91A	-236.446	213.524	-1.379	16.367	16.425
OSU91A	-236.732	213.099	-1.419	16.288	16.349
GFZ95A	-234.951	287.148	-1.081	17.428	17.462

Table 5: Statistics of the difference between the AGSO marine gravity data and the stated geopotential model without the zero-degree term (mgal).

Sandwell's satellite data

Again, the EGM models show a better fit than their predecessors, by over half a milligal in the standard deviation. EGM-X05 has the best fit ($\sigma = 14.819\text{mgal}$), and EGM-X01 the worst ($\sigma = 14.909\text{mgal}$). EGM96 has a very good fit ($\sigma = 14.820\text{mgal}$), while GFZ95A has a relatively poor fit ($\sigma = 21.891\text{mgal}$).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-260.456	302.363	0.258	14.909	14.911
EGM-X02	-262.917	307.199	0.237	14.901	14.903
EGM-X03	-258.885	303.159	0.273	14.899	14.902
EGM-X04	-266.507	303.529	0.312	14.840	14.843
EGM-X05	-266.027	303.158	0.320	14.819	14.822
EGM96	-265.382	304.098	0.325	14.820	14.824
JGM3-OSU91A	-237.406	272.286	-0.188	15.500	15.501
OSU91A	-241.672	273.338	-0.195	15.501	15.502
GFZ95A	-244.069	321.265	-0.661	21.891	21.991

Table 6: Statistics of the difference between the Sandwell altimeter gravity field and the stated geopotential model including the zero-degree term (mgal).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-260.600	302.219	0.114	14.909	14.909
EGM-X02	-263.061	307.055	0.093	14.901	14.901
EGM-X03	-259.029	303.015	0.129	14.899	14.900
EGM-X04	-266.651	303.385	0.168	14.840	14.841
EGM-X05	-266.171	303.014	0.176	14.819	14.820
EGM96	-265.526	303.954	0.181	14.820	14.821
JGM3-OSU91A	-237.550	272.412	-0.332	15.500	15.504
OSU91A	-241.830	273.180	-0.352	15.501	15.505
GFZ95A	-244.218	321.116	-0.809	21.891	22.107

Table 7: Statistics of the difference between the Sandwell altimeter gravity field and the stated geopotential model without the zero-degree term (mgal).

AFN/ANN/AHD stations

The model that best fits the GPS-estimated geoid heights is EGM-X01 ($\sigma = 0.425\text{m}$), while EGM-X03 has the poorest fit ($\sigma = 0.483\text{m}$). Interestingly, EGM96 shows an excellent fit by comparison, with a σ of 0.409 m . Neither OSU91A nor JGM3.OSU91A are favourable, whereas GFZ95A shows the best fit of all models ($\sigma = 0.342\text{m}$). However, these control data will be contaminated by any difference between the true geoid and the AHD, and thus should not be used as an unequivocal indicator.

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-0.179	1.901	0.922	0.425	1.015
EGM-X02	-0.375	2.166	0.947	0.483	1.063
EGM-X03	-0.125	1.743	0.947	0.428	1.040
EGM-X04	-0.157	1.763	0.943	0.432	1.037
EGM-X05	-0.174	1.766	0.948	0.435	1.043
EGM96	0.024	1.788	0.921	0.409	1.008
JGM3.OSU91A	-0.316	2.359	0.982	0.480	1.093
OSU91A	0.036	2.214	1.099	0.477	1.198
GFZ95A	0.398	1.916	1.072	0.342	1.125

Table 8: Statistics of the difference between the AFN/ANN/AHD geometrical geoid height and the stated geopotential model including the zero-degree term (metres).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-1.115	0.964	-0.015	0.425	0.426
EGM-X02	-1.313	1.229	0.010	0.483	0.483
EGM-X03	-1.062	0.805	0.010	0.428	0.428
EGM-X04	-1.094	0.826	0.006	0.432	0.432
EGM-X05	-1.112	0.828	0.011	0.435	0.435
EGM96	-0.913	0.851	-0.017	0.409	0.410
JGM3.OSU91A	-1.254	1.421	0.044	0.480	0.481
OSU91A	-0.989	1.189	0.074	0.476	0.482
GFZ95A	-0.563	0.955	0.111	0.342	0.360

Table 9: Statistics of the difference between the AFN/ANN/AHD geometrical geoid height and the stated geopotential model without the zero-degree term (metres).

AUSGEOID93

The OSU91A and JGM3.OSU91A models show a markedly better fit to AUSGEOID93 ($\sigma = 0.236\text{m}$ and $\sigma = 0.382\text{m}$). A similar result is observed for the British tests, described elsewhere in this issue. This can be expected to some extent because AUSGEOID93 was calculated using a remove-restore technique referenced to this model [Steed and Holtznagel (1994)]. Of the GSFC/DMA models, EGM-X01 exhibits the best fit, with a σ of 0.387m , while EGM-X02 has the poorest fit of 0.419m . The GFZ95A model does not fit AUSGEOID93 well.

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-1.237	2.818	0.914	0.387	0.993
EGM-X02	-1.039	2.482	0.910	0.419	1.001
EGM-X03	-1.275	2.791	0.931	0.415	1.019
EGM-X04	-1.257	2.769	0.931	0.411	1.018
EGM-X05	-1.248	2.873	0.932	0.412	1.019
EGM96	-0.968	2.718	0.887	0.401	0.974
JGM3.OSU91A	-1.558	2.652	0.932	0.382	1.008
OSU91A	-0.706	2.529	0.988	0.236	1.015
GFZ95A	-2.111	3.094	0.879	0.434	0.980

Table 10: Statistics of the difference between AUSGEOID93 and the stated geopotential model including the zero-degree term (metres).

GGM	min	max	mean	std. dev.	RMS
EGM-X01	-2.174	1.881	-0.023	0.387	0.388
EGM-X02	-1.976	1.545	-0.027	0.419	0.420
EGM-X03	-2.212	1.854	-0.006	0.415	0.415
EGM-X04	-2.194	1.832	-0.006	0.411	0.411
EGM-X05	-2.185	1.936	-0.005	0.412	0.412
EGM96	-1.905	1.781	-0.050	0.401	0.404
JGM3.OSU91A	-2.495	1.715	-0.005	0.382	0.382
OSU91A	-1.732	1.503	-0.038	0.236	0.239
GFZ95A	-3.073	2.132	-0.082	0.434	0.442

Table 11: Statistics of the difference between AUSGEOID93 and the stated geopotential model without the zero-degree term (metres).

Discussion

The size of the Australian continent has permitted a study of the long wavelength characteristics of the GSFC/DMA models. The lateral extent of these self-consistent data-sets allows an evaluation of the models down to a spherical harmonic degree of approximately 10.

Considering all five control data-sets, no single one of the GSFC/DMA models is consistently superior. This was to be expected because the majority of the Australian gravity data have been in the public domain for a number of years.

If we were to pick a winner, it would be EGM-X05 as this has performed most promisingly in three of the five tests. Generally, however, all the EGM models show a small improvement over OSU91A.

Acknowledgements

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FINAL REPORT ON TESTING ACCURACY OF GEOPOTENTIAL MODELS EGMX01-X05, EGM 96

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1. INTRODUCTION

At the end of 1996 the main GMT (Geopotential Model Testing) activity of WG GGT concerning the most recent geopotential models EGM-X01, EGM-X02, EGM-X03, EGM-X04, EGM-X05 and EGM96 (Sideris 1995 and 1996, Rapp 1996) resulted in the report by (Bursa et al.1996). However, some refining additional activities followed and, that is why, the Final Report should be presented. It repeats the most important basic information, however, all the numerical results have been slightly changed. Some previous inconsistencies in rms, Tabs I-III, have been corrected. The GMT network was enlarged for the territory of Canada (Mainville, 1997) and the Baltic region in Europe (Chen and Kakkuri, 1995).

2. PRIMARY CONSTANTS USED FOR GMT

An essential progress was made in refining the geoidal potential W_0 (Burša et al.,1997) on the basis of ERS-1/TOPEX/POSEIDON altimeter data (AVISO, 1995) and the sea surface topography model POCM 4B to degree $n = 360$, which is based on the global circulation model (Rapp et al., 1996):

$$W_0 = (62\,636\,855.8 \pm 0.5) \text{m}^2 \text{s}^{-2}. \quad (1)$$

It is adopted as given in GMT along with the following three constants, i.e. geocentric gravitational constant

$$GM = (398\,600\,441.8 \pm 0.8) \times 10^6 \text{m}^3 \text{s}^{-2} \quad (2)$$

(Ries et al.,1992), the angular velocity of the Earth's rotation

$$\omega = 7\,292\,115 \times 10^{-11} \text{rad s}^{-1} \quad (3)$$

and the second zonal Stokes parameter

$$J_2 = (1\,082\,635.9 \pm 0.1) \times 10^{-9} \quad (4)$$

in the zero-frequency tide system (IAG SC3, 1995). Constants (1), (2) and (3) are independent of the tide systems. Since GMT in the mean tide system is desired, the constant (4) should be corrected for the direct permanent tidal effect

$$\delta J_2 = 30.8 \times 10^{-9} \quad (5)$$

(IAG SC3, 1995). From the constants (1) - (5) additional three are computed as follows: the geopotential scale factor

$$R_0 = \frac{GM}{W_0} = (6\,363\,672.58 \pm 0.05) \text{ m} , \quad (6)$$

the semimajor axis of the level ellipsoid in the mean tide system

$$a = (6\,378\,136.696 \pm 0.050) \text{ m} , \quad (7)$$

and its flattening

$$\alpha = 1/(298.252336 \pm 0.000\,020). \quad (8)$$

Geopotential scale factor (6) is independent of the tide systems. Parameters (7) and (8) define the mean level ellipsoid, i.e. its figure reflects both the direct as well as indirect zero frequency tidal effects. The four fundamental constants GM, ω, J_2, W_0 , or GM, ω, a, α make it possible to compute the actual potential at any location on the physical Earth surface for which the Molodensky normal height and/or the geopotential number is known. The testing accuracy is limited by errors in the Molodensky normal heights and in the fundamental constants above. The latter amounts to about $\pm 0.5 \text{m}^2 \text{s}^{-2}$ in the geopotential and/or $\pm 0.05 \text{m}$ in the radius of the local equipotential surface as determined by the geopotential model tested.

3. GEOPOTENTIAL MODEL TESTING NETWORK (GMTN)

The GMTN consists of 20 768 testing sites over oceans, 1 830 testing sites covering the territory of the United States, 963 testing sites on the territory of Canada, 220 testing sites in the central part of Europe and 25 testing sites in the Baltic region. The data for the first year of T/P AVISO altimetry (cycles 6-18, Oct.92-Dec.93) project (AVISO, 1995) was used for determining the geocentric positions of oceanic testing sites. Sea surface topography model POCM 4B to degree $n = 360$ (Rapp et al., 1996) was used for computing the SST heights to stand for the Molodensky normal heights. The testing in U.S.A. and Canada coincide with the GPS/leveling stations. However, the Helmert orthometric heights available had to be transformed into the Molodensky normal heights. In Canada only 963 of 1482 Helmert orthometric heights could safely be transformed into the Molodensky normal heights. In the central part of Europe the testing sites are identical with the GPS/leveling sites the Molodensky normal heights of which are available.

4. FINAL RESULTS

The GMT methodology (Burke et al., 1996) is based on the difference δW between the geopotential value at the testing site computed as function

$$W = W(GM, \omega, J_2, W_0, x_j, H_q) \quad (9)$$

and geopotential value computed from the model:

$$\delta W = W - W(model); \quad (10)$$

x_j stands for GPS coordinates, H_q for the Molodensky normal height of the testing site. Instead of δW , the radial distortion δR of the equipotential surface passing through the testing site can be used in GMT.

Differences δW and/or δR are due to distortions of the geopotential model tested if the errors in fundamental constants (1) - (4), as well as, the errors in geocentric coordinates x_j and in the Molodensky normal heights H_q of the testing site are small enough. The errors in the adopted fundamental constants contribute to δR as follows: GM (± 13 mm), ω (± 1 mm), W_0 (± 50 mm). The errors in x_j and H_q are believed to amount some few centimeters. However, there can be differences in heights due to different Vertical Datums used. That is why, the non-zero biases in δW and/or in δR , i.e. the mean values computed over GMTN parts connected to the same tide gauge station, are interpreted as due to Vertical Datum shifts rather than due to the geopotential model. Numerical δR -results are summarized in Tabs I-V.

However, there are some differences an order of magnitude larger than *rms*. For example, from 20 768 oceanic testing sites 32 exhibit δR to be about 1 - 2 m. The altimeter height *rms*

computed on the basis of 12-13 ascending and descending passes amount to $\pm(0.011 - 0.028)$ m and, the ocean depths here are larger than 2 km. That is why, the altimeter data are believed not to be responsible for such large differences.

5. CONCLUSIONS

(1) The radial distortions of the equipotential surfaces passing through the GMT sites due to geopotential models EGM are smaller in general than those due to JGM-3/OSU91 model.

(2) There are no significant differences among EGM-X01,..., EGM-X05, and EGM96 in regards to their accuracy, in general.

(3) Over oceans, EGM96 and/or EGM-X05 are the best, evidently.

(4) The constant difference about 0.53 m over the territory of the U.S.A. and 0.65 m over the territory of Canada are probably due to the Vertical Datum shift (Father's Point/Rimouski, Quebec, Canada) in relation to surface $W = W_0$. It means that the implied zero point of the tide gauge station above is situated about 0.60 m below the surface $W = W_0$.

(5) The constant difference of about 18 cm in the central part of Europe cannot be interpreted in an analogous way as above (space position of the tide gauge station Kronstadt) because of small area covered by GMTN there.

(6) The *rms* radial distortions received from EGM96 over oceans amounts ± 0.14 m only. Evidently, because the TOPEX data were used in creating EGMs.

(7) The EGM96 *rms* distortion over the territory of the U.S.A. comes out ± 46 cm, over the territory of Canada ± 35 cm, in the central part of Europe about ± 19 cm, in the Baltic region ± 33 cm.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance of Professor Richard H. Rapp who made available POCM4B model of SST and discussion on its consistency with TOPEX altimeter system, of Dr. John C. Ries for his interest and the lively discussion on TOPEX altimeter calibration error, of Dr. Erricos C. Pavlis and Professor Michael G. Sideris who made available the geopotential coefficients sets, and last, but not least, of Dr. Patrick Vincent, who made available the TOPEX altimeter data and who kindly instructed us during its treatment.

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Table I. Radial distortions of the equipotential surface passing through the oceanic testing sites (20 768 sites)

geopotential model	mean [m]	rms [m]
JGM-3/OSU91A	0.002 ± 0.001	± 0.176
EGM-X01	-0.001 ± 0.001	± 0.196
EGM-X02	0.005 ± 0.001	± 0.159
EGM-X03	0.004 ± 0.001	± 0.189
EGM-X04	0.002 ± 0.001	± 0.187
EGM-X05	-0.001 ± 0.001	± 0.141
EGM96	-0.001 ± 0.001	± 0.141

Table II. Radial distortions of the equipotential surface passing through the testing sites.
Territory: U.S.A. (1 830 sites)

geopotential model	mean [m]	rms [m]
JGM-3/OSU91A	0.441 ± 0.012	± 0.496
EGM-X01	0.486 ± 0.011	± 0.476
EGM-X02	0.522 ± 0.011	± 0.448
EGM-X03	0.506 ± 0.011	± 0.454
EGM-X04	0.512 ± 0.011	± 0.449
EGM-X05	0.516 ± 0.010	± 0.446
EGM96	0.530 ± 0.011	± 0.465

Table III. Radial distortions of the equipotential surface passing through the testing sites.
Territory: Central Europe (220 sites)

geopotential model	mean [m]	rms [m]
JGM-3/OSU91A	0.103 ± 0.018	± 0.272
EGM-X01	0.195 ± 0.013	± 0.197
EGM-X02	0.144 ± 0.014	± 0.205
EGM-X03	0.136 ± 0.014	± 0.201
EGM-X04	0.137 ± 0.013	± 0.199
EGM-X05	0.157 ± 0.013	± 0.194
EGM96	0.185 ± 0.013	± 0.188

Table IV. Radial distortions of the equipotential surface passing through the testing sites.
Territory: Baltic region (25 sites)

geopotential model	mean [m]	rms [m]
JGM-3/OSU91A	0.218 ± 0.060	± 0.300
EGM-X01	0.005 ± 0.073	± 0.366
EGM-X02	0.046 ± 0.071	± 0.353
EGM-X03	0.003 ± 0.075	± 0.375
EGM-X04	0.004 ± 0.072	± 0.368
EGM-X05	-0.003 ± 0.072	± 0.359
EGM96	-0.001 ± 0.066	± 0.330

Table V. Radial distortions of the equipotential surface passing through the testing sites.
Territory: Canada (963 sites)

geopotential model	mean [m]	rms [m]
JGM-3/OSU91A	0.624 ± 0.019	± 0.587
EGM-X01	0.659 ± 0.012	± 0.375
EGM-X02	0.675 ± 0.012	± 0.369
EGM-X03	0.671 ± 0.012	± 0.362
EGM-X04	0.664 ± 0.012	± 0.361
EGM-X05	0.659 ± 0.012	± 0.360
EGM96	0.646 ± 0.011	± 0.352

Evaluation of the EGM96, the EGG97 and the GEONZ97 (gravimetric) Geoids in the North Sea Area.

by

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Abstract: ERS-1, Topex altimeter heights as well as OSU95 Mean Sea Surface Heights have been compared to the EGM96 geoid, the EGG97 geoid and the GEONZ97 gravimetric geoid in the North Sea area. The altimeter and MSS data were converted from the Topex reference system to WGS84. The comparison showed mean absolute differences of the order of 0.2 m and standard deviations of 0.35 m for the differences with respect to EGM96 and 0.23 - 0.26 m for differences with respect to EGG97 and GEONZ97. This shows that the EGG97 geoid and the GEONZ97 gravimetric geoid both are significant improvements as compared to EGM96 in this area.

1. Introduction.

The EGG97 geoid has been calculated using local data to improve the EGM96 geoid. The geoid has been evaluated for the European land areas, see Denker et al. (1997, Table 1). Furthermore a North Sea Geoid Model, GEONZ97, (Bruijne et al., 1997) has been computed having as an intermediate result a gravimetric geoid computed from local gravity data and the EGM96 spherical harmonic model. The GEONZ97 model covers "only" the North Sea and the Netherlands, while the EGG97 model covers all of Europe. Obviously the EGM96 model is global.

In this brief report an evaluation of the three geoids in two rectangles using altimeter data and mean sea surface heights will be described. The comparison is for all three geoids made in the lat.-long.-rectangle corresponding to the area of the GEONZ97, and for the EGM96 and the EGG97 geoids as well in a larger rectangle covering the North Sea.

2. Altimeter Data for the evaluation.

NASA has published a CD-ROM with so called pathfinder data. ERS-1 (repeat 1 - 18) and Topex (repeat 1 - 154) altimeter heights are interpolated into normal points, and the residuals with respect to the OSU 95 Mean Sea Surface (Yi, 1996) are stored for each track together with the height of this surface so that the original height with respect to the Topex reference ellipsoid ($a = 6378136.3$ m, $1/f = 298.257$) can be restored.

ERS-1, Topex and OSU95 MSS data in the area $50^\circ < \text{latitude} < 60^\circ$ and $-5^\circ < \text{longitude} < 15^\circ$ was selected. This gave 2143 ERS-1 values, 229227 Topex values and 168+1692 OSU95 MSS "normal point heights".

The EGG97 geoid is given with respect to the GRS80 reference ellipsoide, so the altimeter heights were transformed to this ellipsoid by first converting all positions to (X,Y,Z), and then converting the positions to latitude, longitude and ellipsoidal height with respect to GRS80. The GEONZ97 gravimetric geoid is in a system similar to the Topex system, since its evaluation is based on the use of EGM96.

3. Geoid heights.

Geoid heights were obtained by evaluating the EGM96 spherical harmonic model complete to degree 360 (Lemoine et al., 1996) using the GRAVSOF program GEOCOL12 (Tscherning et al.,1993). The EGG97 grid was evaluated using the program EGGPRO provided with the EGG97 CD-ROM. The GEONZ97 gravimetric geoid was also given as a grid, and was evaluated using the GRAVSOF program geoiip.

The EGG97 geoid is shown in Figure 1, and the differences with respect to the EGM96 geoid are shown in Figure 2.

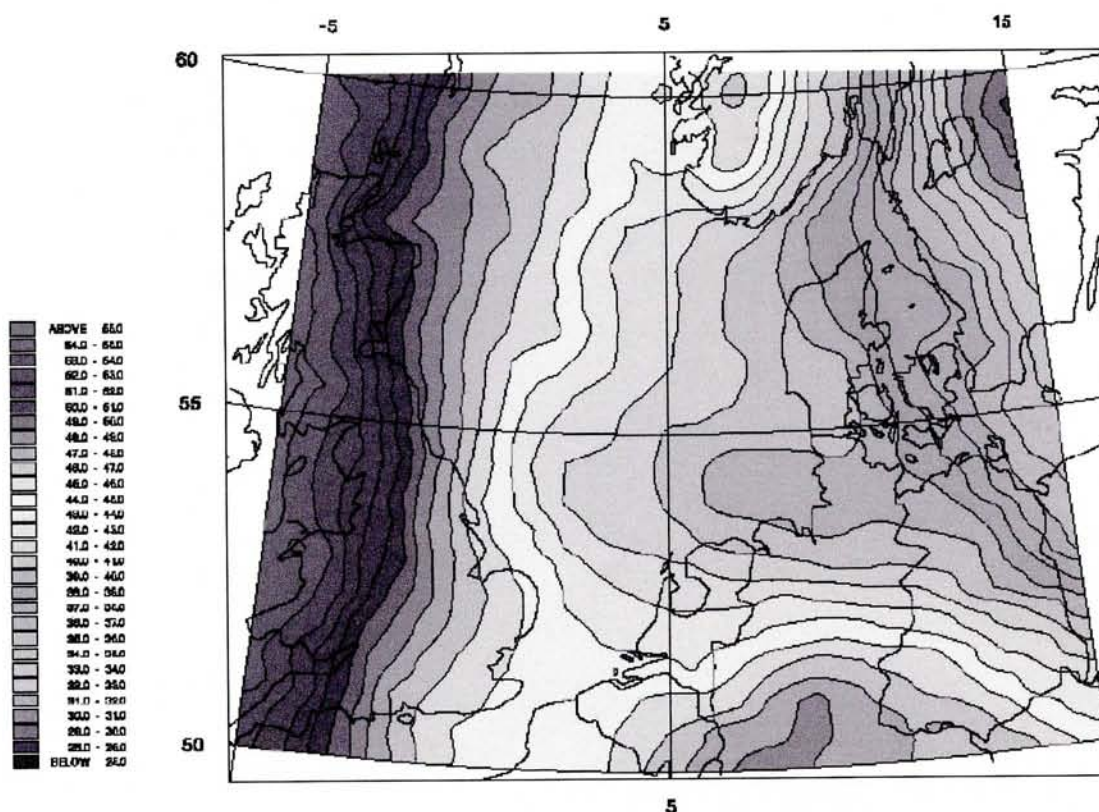


Fig. 1. EGG97 Geoid, Units m.

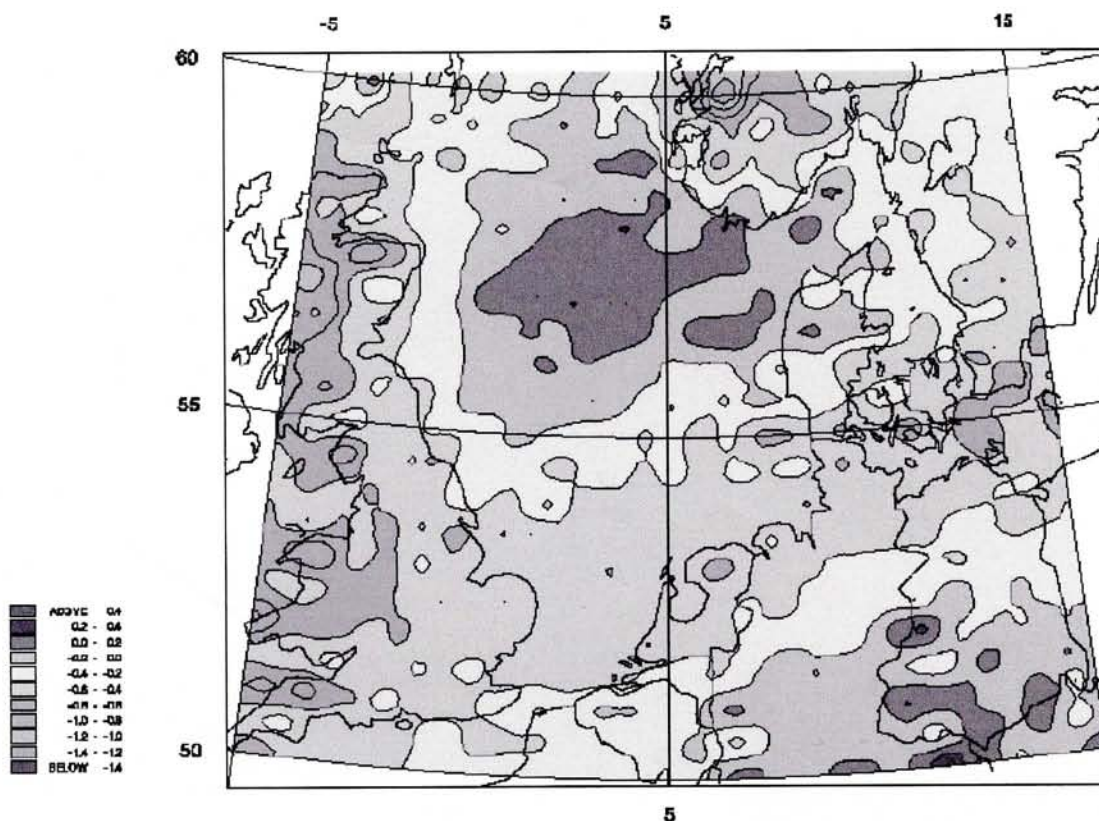


Fig. 2. Differences EGM96 and EGG97 Geoids, Units m.

The mean value of the differences is -0.35 m, the standard deviation of the differences is 0.25 m, the minimum difference is -1.33 m and the maximal difference is 0.45 m. This is a regular grid of 861 points spaced 0.5° in latitude and longitude.

The differences between EGG97 and GEONZ97 which have a 0.10 m mean value (after conversion to GRS80) and a standard deviation of 0.14 m are shown in Fig. 4.

4. Result of Comparison.

The differences between the altimeter heights and the MSS heights and their standard deviations are given in Table 1 for the larger rectangle.

Table 1	EGM96		EGG97	
Units: m	Mean	Stan.dev.	Mean	Stan.dev
Topex	-0.29	0.42	-0.31	0.34
ERS-1	0.06	0.33	0.10	0.25
OSU95/ERS	-0.20	0.30	-0.19	0.15
OSU95/Topex	-0.16	0.31	-0.19	0.18

Table 2 shows the result of the comparison for all 3 geoids in the smaller rectangle.

Table 2	EGM96		EGG97		GEONZ97	
Units: m	Mean	Stan.dev.	Mean	Stan-dev.	Mean	Stan.dev.
Topex	-0.29	0.35	-0.29	0.26	-0.20	0.25
ERS-1	0.09	0.32	0.13	0.24	0.22	0.23
OSU95/E RS	-0.18	0.28	-0.16	0.12	-0.07	0.11
OSU95/T opex	-0.16	0.26	-0.17	0.14	-0.07	0.13

The histogram of the differences showed that the Topex data contained relatively many outliers. In fact land data (on lakes) had not been removed completely. Also data in areas with a large tidal signal showed large residuals. Figure 3 shows the location of the differences for

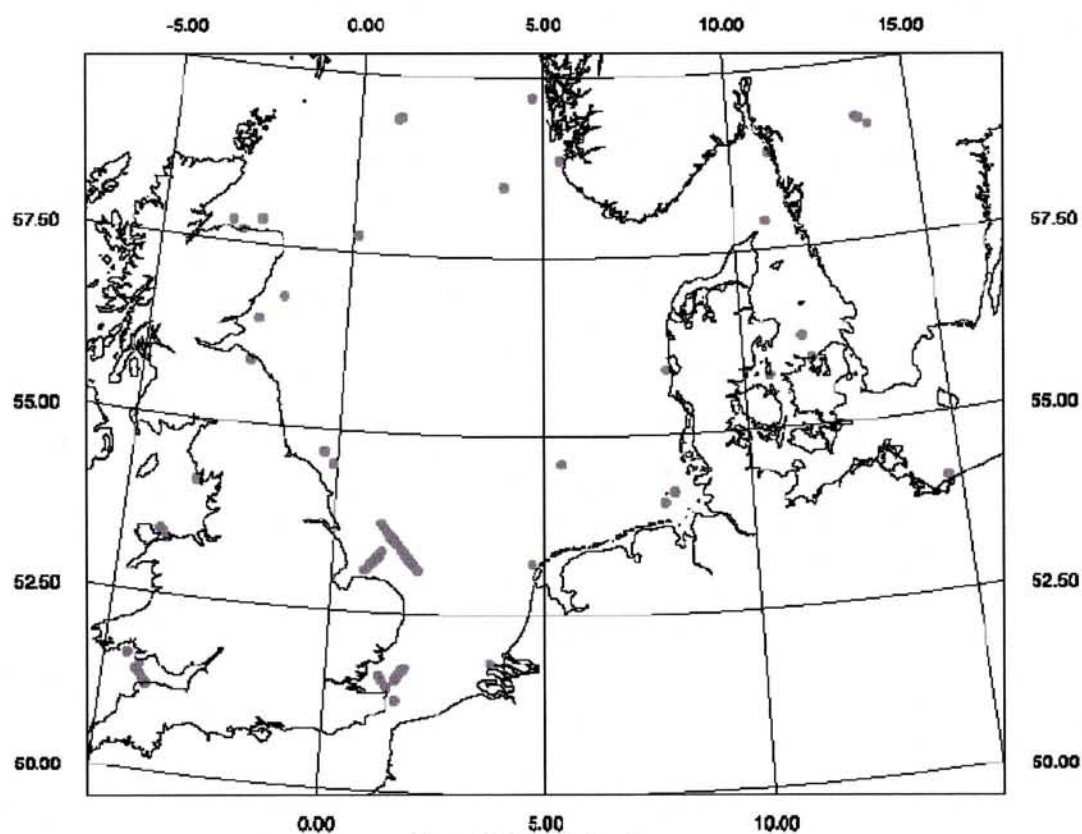


Figure 3 Topex data outliers.

which the numerical value is larger than 1.5 m.

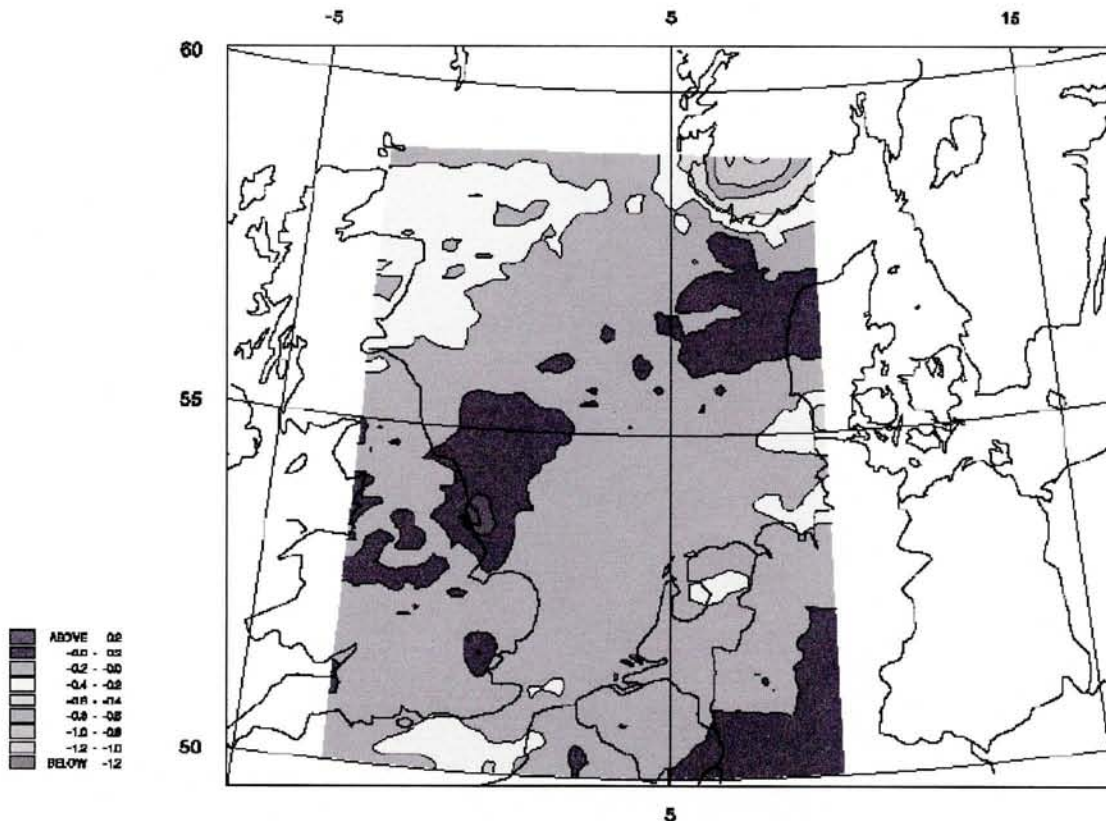


Fig. 4. Differences GEONZ97 Gravimetric and EGG97 Geoids, Units m.

5. Conclusion.

First of all is remarkable how small the biases are. This is probably due to the good progress made both in orbit improvement and tidal analysis. There is however a difference between ERS-1 and Topex which probably is due to that no joint cross-over analysis was made like when creating the OSU95 MSS (see Yi, 1995, section 2.11)..

The agreement with the OSU95 MSS is also very good, with standard deviation of differences of 0.15 - 0.18 m.

Most important is the fact that EGG97 and GEONZ97 (gravimetric geoid) represent real improvements as compared to EGM96. Whether the biases are due to stationary sea-surface topography, system inconsistencies or long wavelength errors in the gravity field models need to be explained. A future space borne gravity field mission should be able to help in solving this problem.

Finally it should be mentioned that GEONZ97 gravimetric geoid has been modified as to agree with Topex data in order to create a precise reference surface for oceanographic studies. For details see (Bruijne et al., 1997).

Acknowledgement: Thanks to Institut für Erdmessung, Hannover and IAG for providing the

CD-ROM with the EGG97 model, and thanks to NASA for providing the Pathfinder data. The access to the ERS-1 data is within the framework of the ESA AO ERS-1 Project AFRICAR, and ESA is thanked for making the data available. The digital grid representing the GEONZ97 geoid was provided by the colleagues at DUT and at Rijkswaterstaat.

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QUASIGEOID COMPUTATION IN JAEN

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Abstract

The Fast Collocation method has been used for the computation of a quasigeoid solution in Jaen (Spain), combining a geopotential model complete to degree and order 360, gravity anomalies and topographic information. Topographic effects have been calculated from 100m x 100m heights using a RTM reduction with a reference terrain model of 10'x10' mean heights. The prediction has been done in a 3'x3' grid. The gravimetrically determined quasigeoid has been compared with geoid undulations provided by GPS/leveling. The discrepancies between them amount to about ± 4 cm.

Keywords: quasigeoid, gravity anomaly, GPS, DTM, Fast Collocation

Introduction

Most of the Iberian Peninsula is occupied by the Meseta, a large plateau that is almost completely surrounded by mountain ranges. The region of Andalusia is located in the South. Jaen is an area in the North of Andalusia. An estimate of the quasigeoid in Jaen has been calculated, [3].

This quasigeoid has been computed from a validated gravity data set covering the area $37.3 \leq \varphi \leq 38.5$, $-4.3 \leq \lambda \leq -2.3$ and a Digital Terrain Model (D.T.M.) of the land area $37.1 \leq \varphi \leq 38.7$, $-4.5 \leq \lambda \leq -2.1$, having a grid mesh of 100m. The method used to compute the quasigeoid is the Remove-Restore and Fast Collocation technique. The terrain effect has been computed with the available DTM. The OSU91A model has been used as a reference field in order to remove and restore the long wavelength components

of gravity and geoid respectively. The final result is a gravimetric quasigeoid of Jaen on a $3' \times 3'$ grid in the area with limits $37.3 \leq \varphi \leq 38.5$, $-4.3 \leq \lambda \leq -2.3$.

A comparison between the gravimetric quasigeoid and GPS/leveling data has been performed in order to test the quality and accuracy of the quasigeoid calculated.

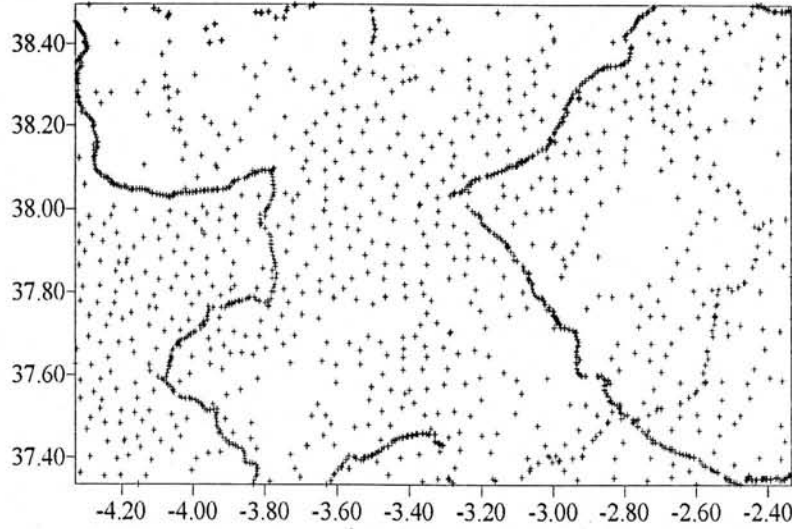


Figure 1: Distribution of gravity anomalies.

1 Data Used in This Work

1.1 Gravity Data

Instituto Geográfico Nacional [4] supplied Spanish gravity data covering the Iberian Peninsula and Baleares Islands. A number of 27691 point gravity values are available with an accuracy of 0.1 mgal. Initially, gravity anomalies in the Geodetic Reference System(1980) have been computed because they were calculated with the International Gravity Formula(1967). The value of Δg_{1967} is changed by [6]:

$$\Delta g_{1980} = \Delta g_{1967} - (0.8316 + 0.0782 \sin^2 \varphi - 0.0007 \sin^4 \varphi) \quad (1)$$

From this data base a set of 1874 free-air anomalies is collected. Figure 1 shows the distribution of gravity anomalies in Jaen.

1.2 Spherical harmonic coefficients

For this work, the high-degree global geopotential model OSU91A [7] has been used. This model is considered complete up to degree and order 360. Gravity anomalies can be computed in spherical approximation from a geopotential coefficient set by:

$$\Delta g = G \sum_{n=2}^{n_{max}} (n-1) \sum_{m=0}^n [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} m\lambda] \bar{P}_{nm}(\cos\theta) \quad (2)$$

where θ , λ are the geocentric colatitude and longitude of the point at which Δg is to be determined; $\bar{C}_{nm}$, $\bar{S}_{nm}$ are the fully normalized spherical geopotential coefficients of the anomalous potential; $\bar{P}_{nm}$ are the fully normalized associated Legendre functions; n_{max} is the maximum degree of the geopotential model and G is the mean gravity.

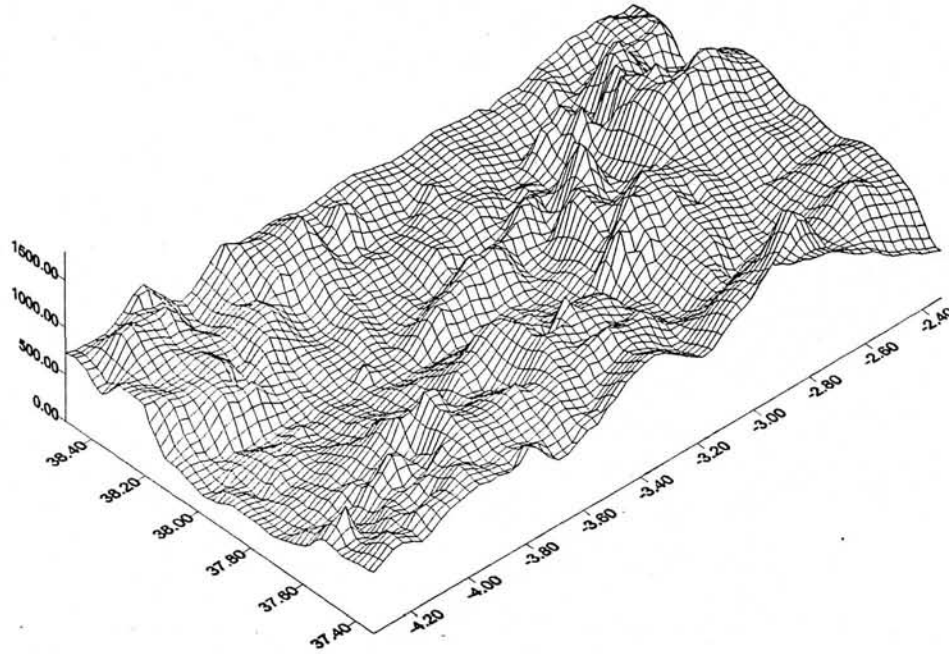


Figure 2. Topography of Jaen

1.3 Topographic Data

The DTM was provided by Servicio Geográfico del Ejército. This data base contains 3001 x 2001 heights on a regular UTM grid of 100m x 100m spacing. Figure 2 shows the topography in the area of Jaen.

1.4 GPS/leveling data

The GPS survey was performed in October 1996. Measurement of the network baselines was carried out by collecting on each point for two hours with two double frequency P code receivers (fast-static survey). The baselines lengths from 3 to 30 Km long. This small network contains 9 benchmarks of the first order leveling network and 3 points of the first order geodetic network.

2 QUASIGEOID COMPUTATION

Fast Collocation and the remove-restore procedure were used to compute the quasigeoid estimate [1]. In order to obtain a smoothed gravity field as input for the collocation procedure gravity data must be reduced for the geopotential model and terrain effect. The spherical harmonic coefficient set OSU91A has been used to remove the long wavelength component of the gravity data. The residual terrain effect was computed using the available DTM and with respect to a 10'x10' reference grid. Only residual topography distance of 38km was taken into account. To select this value, we computed for some points contributions of the residual terrain model to different distances from calculation point and we chose the value beyond which the results were quite similar, showing that the effect was negligible beyond this distance [2]. To compute the effect of topography TC program has been used [9]. Finally a set of 1874 residual gravity anomalies is available. In order to detect outliers in this data set a comparison of the difference between the observed and the predicted values has been done [8].

A gravity anomaly in a point P, Δg_p , is predicted from a set of values $\Delta g_i, i = 1, 2, \dots, n$ in the neighbourhood space as regularly as possible in all direction by,

$$\overline{\Delta g_p} = \sum_{i=1}^{\infty} a_i \Delta g_i \quad (3)$$

where

$$a_i = C_{ip}^T (C_{ij} + D_{ij})^{-1} \quad (4)$$

with,

C_{ip} : the covariance between the observation i and the predicted value p

C_{ij} : the covariance of the observations

D_{ij} : the covariance of the observation errors (associated to the points i and j).

The error estimate is then computed as,

$$\sigma_p^2(\Delta g_p - \overline{\Delta g_p}) = C_0 - C_{ip}^T (C_{ij} + D_{ij})^{-1} C_{jp} \quad (5)$$

A measurement is rejected or considered suspected if,

$$|\Delta g_p - \overline{\Delta g_p}| > k(\sigma_p^2(\Delta g_p - \overline{\Delta g_p}) + \sigma_p^2(\Delta g_p))^{1/2} \quad (6)$$

where we have taken $k = 3$ and $\sigma_p^2(\Delta g_p)$ is the error variance of the observation Δg_p .

By using this technique 1.1% outliers has been detected.

The gridding of Δg_r to produce a 3'x 3' grid of residual anomalies have been done with GEOGRID program of GRAVSOFT package. To estimate ζ_r via Fast Collocation the empirical and model covariance functions of the gravity anomalies are needed. The covariance function model used in this work was [5]:

$$C(P, Q) = a \sum_{n=2}^N c_n \left(\frac{R^2}{rr'} \right)^{n+2} P_n(\cos(\psi)) + \sum_{n=N+1}^{\infty} \frac{A(n-1)}{(n-2)(n+24)} \left(\frac{R_B^2}{rr'} \right)^{n+2} P_n(\cos(\psi)) \quad (7)$$

where, ψ is the spherical distance between the points P and Q of the radial distances r and r' , P_n are the Legendre polynomials, R is the mean radius of the Earth, R_B the radius of the Bjerhammar sphere, c_n are the error anomaly degree variances associated with the model coefficients.

The free parameters are: a , the factor to scale the error degree variances, A , the scale factor of the degree variances and the summation limit N , its value reflects the degree to which the spherical harmonic coefficient information is considered reliable for the area.

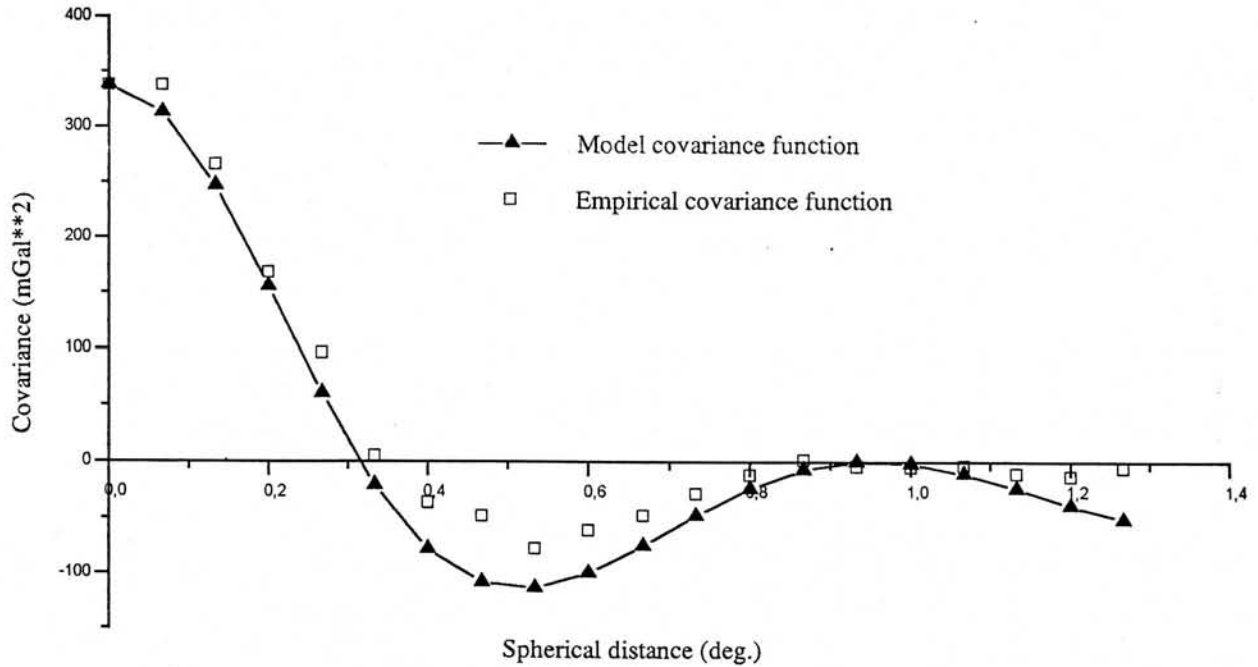


Figure 3: The empirical and the covariance functions of residual gravity.

The covariance functions used in the prediction of ζ_r via Fast Collocation are plotted in Figure 3. For the analytical approximation of the empirical covariance functions and for computation of height anomalies COVFIT [5] and FASTCOLB programs have been used.

The last step to get the final product is the restore of the model and of the residual terrain effect on the 3'x 3' residual quasigeoid grid (Figure 4):

$$\zeta = \zeta_r + \zeta_{rtc} + \zeta_m \quad (8)$$

The numerical results of the estimation procedure are summarized in Table 1.

3 Comparison with GPS/leveling data

A prediction of quasigeoid via collocation has been done over the 9 benchmarks. A comparison between the quasigeoid solution and GPS/leveling derived undulations values showed standard deviations discrepancies of $\pm 7\text{cm}$.

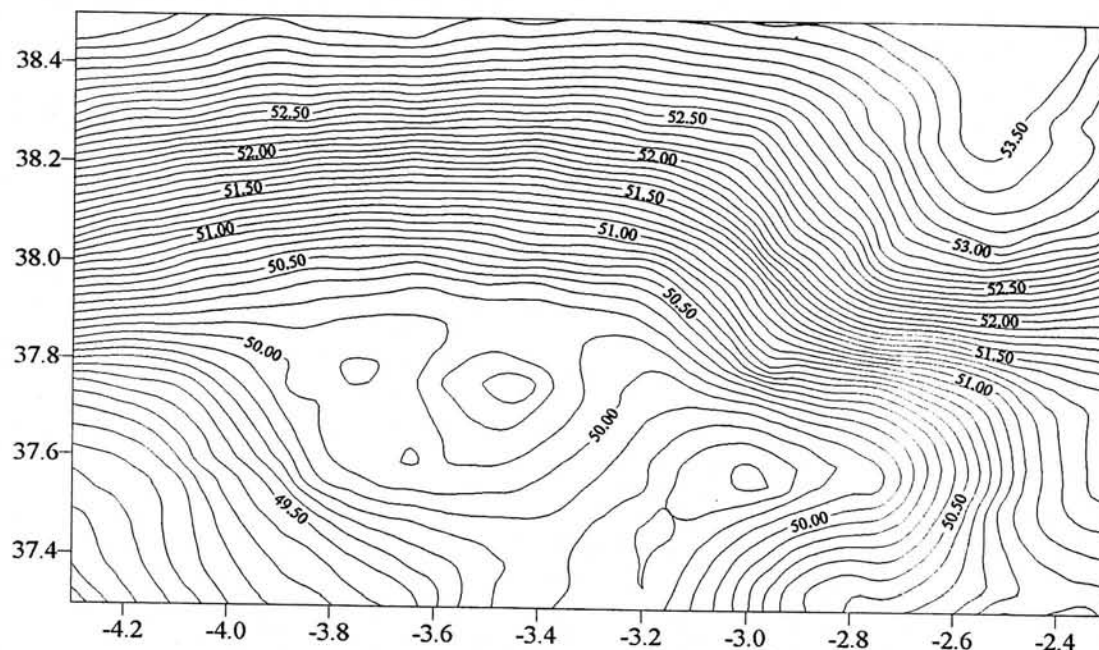


Figure 4: Quasigeoid solution in Jaén. (in meters).

To minimize the long wavelength error the systematic datum difference between the gravimetric quasigeoid and the GPS/leveling values was removed by a four parameter transformation. The use of datum shift fit eliminates the possible tilt of the gravimetric quasigeoid as well, yielding geoid fit only $\pm 4\text{cm}$ which shows a good fit between the quasigeoid of Jaen and GPS/leveling data has been reached.

4 Conclusions

The accuracy of the gravimetric quasigeoid calculated over the Spanish territory of Jaen has been checked comparing the gravimetric quasigeoid with the GPS/leveling undulations in 9 benchmarks. The gravimetric quasigeoid has been computed via Fast Collocation by using Remove-Restore procedure. The mean value of the standard deviations of the discrepancies after a four parameter datum shift fit amounts to $\pm 4\text{cm}$.

This work has been a first test in the calculation of a gravimetric quasigeoid of Jaen

via Fast Collocation. The result obtained has been satisfactory, so in the near future the gravimetric quasigeoid of Andalusia will be computed using several techniques.

In order to do a second test of the quasigeoid solution of Jaen a new GPS survey in the benchmarks of the Spanish first order vertical net will be carried out in January 1998.

Table 1. Statistical summary of remove-restore procedure

	Δg_{fa} (mGal)	$\Delta g_{fa} - \Delta g_m$ (mGal)	Δg_r (mGal)	ζ_r (m)	ζ (m)
n° of points	1874	1874	1853	1025	1025
mean	-2.16	-11.29	-1.38	0.28	51.18
st. dev.	30.19	23.07	18.48	0.37	1.41
max.	135.32	145.54	64.45	1.27	53.78
min.	-77.78	-81.56	-66.83	-0.43	48.52

5 Acknowledgements

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Crossover Adjustment using Array Algebra ¹

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Abstract

The existence of regular structures permits to use very efficient numerical solution techniques. The regular structure of the crossover adjustment suggests itself for an application of these techniques. In this paper the crossover problem is analyzed and in connection with the Kronecker-separability a very efficient computational strategy is developed. The only requirements to the data set are completeness and homogeneity within each used track; this means that each track must be completely defined with the same accuracy over the explored area and all the crossover-points must exist. But no restriction is imposed on the distance between satellite tracks, especially no equi-distant tracks are required. The use of array algebra and Kronecker-products allows a very efficient approach and reduces the computational effort dramatically.

Key Words: Crossover Adjustment, Regular Structures, Kronecker-Separability, Generalized Inverses.

Introduction

In the frame of the GEOMED project various problems related to crossover adjustments have been investigated. The crossover technique relies on the principle, that two measurements at the same point are used to calibrate the measurement equipment or to determine corrections to the measurement arrangements. Especially, many satellite measurement arrangements with intersection points between the ascending and descending tracks allow the application of this technique. The number of crossover points (point of intersection between ascending and descending track), which can be used for altimeter calibration depends on the satellite orbit (repeat time) and also on the size of the explored area. Generally, the number of crossover points increases

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quickly because each individual ascending (descending) track causes a number of intersections with the descending (ascending) tracks. With each new track only a few model parameters are added. Because the number of intersections is larger than the number of parameters, this model results in an overdetermined equation system and a least squares adjustment procedure can be applied. Due to the fact that only relative measurements (differences) are used within the adjustment, only relative parameters can be determined uniquely. If parameters with absolute information such as bias and tilt parameters have to be predicted, the resulting normal equations are well-known to be singular and some additional constraints have to be fixed to solve this adjustment problem. In practice there exist three ways to overcome the rank deficiency:

- a number of arbitrary unknowns (some tracks) are fixed (*Master Arcs Approach*).
- with the help of a generalized inverse an unconstrained solution can be determined and after this step constraints are used to fit the system to a well-defined reference frame. The pseudoinverse solution combines these two steps within one procedure (*Free Adjustment*).
- the third possibility to overcome the singular problem is to introduce the reference frame as pseudo-observations and to stabilize with this information the whole system. A special target function with a proper weighting between real observations and pseudo-observations allows to solve this hybrid system (*Combined Solution*).

It should be mentioned that the first two approaches don't affect the observations and the observational residuals. Only the fitting of a so called *free surface* is affected by the choice of the additional parameters. The pseudo-observations within the combined solution, in turn, are constraining the configuration of the real observations. Therefore, one has to be very careful when it comes to choose the weight factor.

Within this study we observe a special distribution of crossover points, namely the complete system. This means, that all points of all used tracks can be determined. Because of its shape such an area is called *Crossover Diamond* (cf. Fig 1).

The curvature of the sphere slightly perturbs the regular structure of a lozenge. This latitude and size dependent distortion results in slightly different distances between the crossover points (1-2% cf. *SCHRAMA (1989)[7]*, pp. 25-27). Numerically, this small distortion brings a lot of problems, because the reason for this perturbations is caused by the transition from the plane to the sphere. Corresponding to this transition the rank deficiency of the planar, local solution changes to the new situation in the spherical case. Some of the vanishing eigenvalues in the planar approximation become very small but significant numbers depending on the size and the latitude of the crossover diamonds and cause numerical troubles (cf. *FÜRST et al. (1992)[4]*). For small areas and equatorial regions, therefore, the planar approximation is usually applied to overcome this numerical problems. The result of this approximation step (cf. section 2) is a regular planar grid, with straight parallel lines as projected images of the satellite orbits. The intersections of this not necessarily equi-distant lines - also repeat or near repeat arcs are allowed - define the crossover points. If all the intersections exist and are

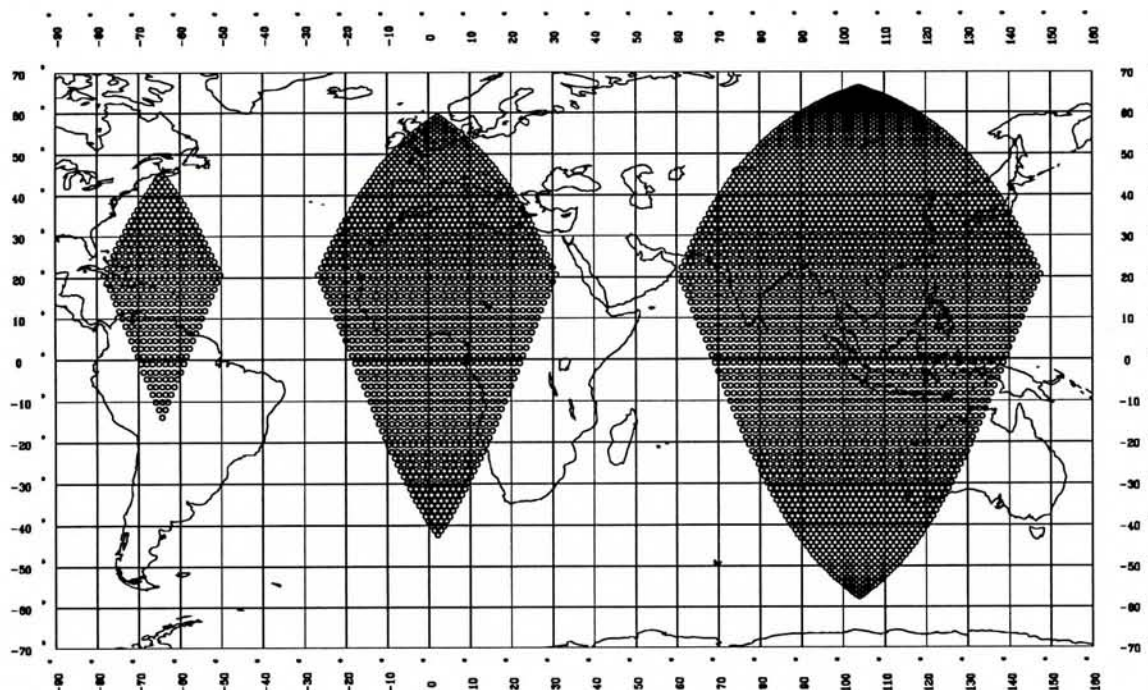


Figure 1: Crossover diamonds (simulated GEOSAT ERM tracks).

defined with the same accuracy, the resulting structure is denoted as *gridded* crossover points.

In numerical mathematics there exist several techniques which utilize the regular structure of a problem. Extremely efficient techniques, like DFFT (Discrete Fast Fourier Transforms), Toeplitz and circulant matrix techniques are able to decrease the storage requirements and to speed up the solution time both with sequential techniques and with parallel approaches. Especially multi-dimensional problems often allow a splitting into lower dimensional subtasks. The powerful tool of *Array Algebra* helps to treat this kind of problems very efficiently. Using this mathematical framework, the two dimensional crossover problem can be separated into two one-dimensional subtasks. The principles of array algebra, especially the *Kronecker-product*, are summarized in the next section. As an example of the application of array algebra to the crossover adjustment an altimeter approach will be used. Therefore, a short introduction to the problem formulation is given in section 2. The very efficient application of the array algebra to solve this problem is illustrated in section 3. It will be demonstrated how the effort of crossover adjustment - also for a lot of tracks - can be reduced by this technique to the capacity of a pocket computer !

1 Mathematical Principles

If a matrix C can be split up into two matrices A and B

$$C = A \otimes B \quad (1)$$

where $\otimes$ defines the *Kronecker-product*

$$A \otimes B = \begin{pmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{pmatrix} \quad (2)$$

the matrix C is called a *Kronecker-separable-matrix*. The Kronecker-product allows a very sufficient and clear arrangement of multi-dimensional problems, but it is restricted to *gridded* data. The special restrictions for *gridded* data sets are defined at the end of this section. The properties of the Kronecker-product (cf. KOCH (1988)[9], p. 23-24, or BLAHA (1977)[1]) are as follows:

- (a) $0 \otimes A = A \otimes 0 = 0$
- (b) $\alpha A \otimes \beta B = \alpha\beta A \otimes B$
- (c) $(A_1 + A_2) \otimes B = A_1 \otimes B + A_2 \otimes B$
- (d) $A \otimes (B_1 + B_2) = A \otimes B_1 + A \otimes B_2$
- (e) $(A \otimes B) \otimes C = A \otimes (B \otimes C)$
- (f) $(A_1 \otimes B_1)(A_2 \otimes B_2) = A_1 A_2 \otimes B_1 B_2$
- (g) $(A \otimes B)^T = A^T \otimes B^T$
- (h) $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$
- (k) $(A \otimes B)^- = A^- \otimes B^-$

They open up the possibility of an efficient treatment of adjustment problems. If the standard Gauß-Markov model

$$\begin{array}{lll} Ax = \ell + v & \dots & \text{deterministic model} \\ \Sigma(\ell) = \sigma^2 P^{-1} & \dots & \text{stochastic model} \end{array} \quad (4)$$

can be separated into

$$\begin{array}{lll} (A_y \otimes A_x)x = \ell + v & \dots & \text{deterministic model} \\ \Sigma(\ell) = \sigma^2 (P_y^{-1} \otimes P_x^{-1}) & \dots & \text{stochastic model} \end{array}, \quad (5)$$

then the adjusted unknowns $\tilde{x}$

$$\tilde{x} = (A^T P A)^{-1} A^T P \ell \quad (6)$$

can be computed by

$$\tilde{x} = \left((A_y^T P_y A_y)^{-1} A_y^T P_y \otimes (A_x^T P_x A_x)^{-1} A_x^T P_x \right) \ell \quad (7)$$

(cf. e.g. SNAY (1978)[3]). At first sight it can be recognized, that the solution (inversion) of (6) is split up into two solutions (inversions) of much smaller systems. If the size of the matrices A_y (A_x) is fixed by the size $m_y \times n_y$ ($m_x \times n_x$), the squared system $(A^T P A)$ of (6) consists of $n_y * n_x$ unknowns and the squared systems $(A_y^T P_y A_y)$ and $(A_x^T P_x A_x)$ of (7) have only dimensions n_y and n_x .

The next step is to try to bypass the computation of the Kronecker-product in formula (7), because the explicit determination needs a lot of data storage and also computational time. The way around is accomplished by a mapping procedure. If instead of the $(m_y * m_x)$ -dimensional vector ℓ a $(m_x \times m_y)$ -matrix L is defined by the sequential column-wise mapping operator

$$\ell = \text{vec } L := \begin{pmatrix} \ell_{11} \\ \ell_{21} \\ \ell_{31} \\ \vdots \\ \ell_{m_x 1} \\ \vdots \\ \ell_{1m_y} \\ \ell_{2m_y} \\ \ell_{3m_y} \\ \vdots \\ \ell_{m_x m_y} \end{pmatrix} \iff \begin{pmatrix} \ell_{11} & \ell_{12} & \dots & \ell_{1m_y} \\ \ell_{21} & \ell_{22} & \dots & \ell_{2m_y} \\ \ell_{31} & \ell_{32} & \dots & \ell_{3m_y} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{m_x 1} & \ell_{m_x 2} & \dots & \ell_{m_x m_y} \end{pmatrix} \quad (8)$$

and the same procedure is used for the mapping of the $(n_x * n_y)$ -vector of unknowns x to the $(n_x \times n_y)$ -matrix X , then the mapping relation (cf. KOCH (1988)[9], pp. 48)

$$(B_1 \otimes B_2) x = B_2 X B_1^T \quad (9)$$

allows both the reformulation of the deterministic model (5) in terms of

$$A_y X A_x = L + V \quad (10)$$

and the solution (7)

$$\tilde{X} = (A_x^T P_x A_x)^{-1} A_x^T P_x L P_y A_y (A_y^T P_y A_y)^{-1} \quad (11)$$

Note that the computation of the Kronecker-product in (7) is avoid and at the same time a lot of storage is saved, because no explicit computation of large matrices is required. Instead of this, a sum of small matrix operations do the same job.

For a small example ($m_y = 3$, $n_y = 2$, $m_x = 4$, $n_x = 3$) with six $(n_y * n_x)$ unknowns Fig. 2 illustrates graphically the efforts necessary for the computation of both solution methods.

This separation technique brings about also benefit for the error propagation and all post-processing computations. Especially the use of (3h) and (3k), resp., in connection with (9) offers a very efficient approach for multi-dimensional adjustment problems. For these problems the mapping processes have to be extended and special operators (e.g. *Rauhala-Matrix-Multiplication* cf. RAUHALA (1980)[2]) must be defined. This framework of transformations and operations is denoted as *Array Algebra*.

$$\tilde{x} = (A^T A)^{-1} A^T \ell$$

$$\tilde{X} = (A_x^T A_x)^{-1} A_x^T L A_y (A_y^T A_y)^{-1}$$

Figure 2: Graphical illustration of computations: default solution vs. Kronecker-solution.

The typical restriction of array algebra is the need for a gridded observational structure without inhomogeneities and gaps. But this grid has not to be uniform (e.g. equidistant) and it is not restricted to a particular coordinate system (rectangular, polar) or to the number of dimensions. If the problem formulation allows the use of array algebra techniques, the number of operations and also the amount of computer storage decreases significantly.

Especially recurrent measurements on board of a satellite produce a lot of data which are tied to the position of the satellite. For small areas the decreasing and increasing satellite tracks form a two-dimensional grid. Therefore, this structure encourages us to watch out an array algebra approach.

2 Crossover Adjustment of Altimeter Data

The precise analysis of the altimeter measurements is one of the typical problems using the crossover technique and therefore, these task is used as an example to demonstrate the new solution strategy.

In the case of altimeter measurements the crossover-points are used to improve the radial component of the satellite orbit. The altimeter observation ℓ_i

$$\ell_i = H_i^0 + h_i = H_i^0 + d_i + \epsilon_i \quad (12)$$

can be split up into a track-independent part H_i^0 , only depending on the measurement location (ϕ_i, λ_i) , and a residual part h_i which is track dependent. The residual part h_i consists of a part due to the radial orbit error d_i and the stochastic measurement inaccuracy ϵ_i . Introducing the crossover observation $\Delta\ell_{ij}$ as the difference of observations at the same point $(\phi_i = \phi_j, \lambda_i = \lambda_j)$ using track i and track j , the track-independent part cancels and only the residual part h_i resp. h_j has to be taken into account is left,

$$\Delta\ell_{ij} = h_i - h_j = d_i - d_j + \epsilon_i - \epsilon_j. \quad (13)$$

It is well-known that the radial orbit error can be sufficiently modelled by either a time- or a distance-dependent polynomial,

$$d_i = \sum_{\ell=0}^k \alpha_{j\ell} (t_i - t_i^0)^\ell = \sum_{\ell=0}^k \beta_{j\ell} (\psi_i - \psi_i^0)^\ell, \quad (14)$$

with

k ... degree of chosen polynomial
 α, β ... polynomial coefficients
 t_i, t_i^0 ... time parameter
 ψ_i, ψ_i^0 ... spherical distance parameter .

Since only differences of measurements are used as observations and since these differences are invariant with respect to certain transformations, the corresponding adjustment problem is singular. The singularity is characterized by $2k$ vanishing eigenvalues in the planar case, while in the spherical case the rank deficiency is just k (k ... degree of chosen polynomials). For small areas, it is sufficient to approximate the spherical situation by the corresponding planar one, which is usually done by choosing the latitude λ_i as distance parameter for the polynomials:

$$d_i = \sum_{\ell=0}^k \gamma_{j\ell} (\lambda_i - \lambda_i^0)^\ell \quad (15)$$

(c.f. RUMMEL (1993)[5], SCHRAMA (1989)[7]). In many applications only low degree polynomials such as bias and tilt models are used. Therefore, each satellite track introduces only two unknowns to the adjustment problem, but it brings a lot of observations,

$$\Delta\ell_{ij} = (a_i + b_i\Delta\lambda_i) - (c_j + d_j\Delta\lambda_j) + (\epsilon_i - \epsilon_j) \quad (16)$$

with

$\Delta\ell_{ij}$... height difference at the crossover point of track i and j
 a_i, b_i ... bias and tilt parameter of the descending track i
 c_j, d_j ... bias and tilt parameter of the ascending track j
 $\Delta\lambda_i, \Delta\lambda_j$... relative geographical longitudes of crossover point within track i and track j
 ϵ_i, ϵ_j ... observation residuals ,

because each intersection between this track and crossing tracks can be used. This results a strong overdetermined least squares adjustment problem

$$Ax = \ell + v \quad (17)$$

with a rank deficiency of four (4). These sparse observation equations yield a block diagonal structured normal equation system with sparse off-diagonal blocks (cf. SCHARLER (1992)[6], pp. 60). For an efficient solution strategy the unknowns x are split up into two groups $x_1(a_i, b_i)$ and $x_2(c_j, d_j)$ (ascending and descending track parameters)

$$(A_1 \ A_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \ell + v. \quad (18)$$

The particular solution $\hat{x}_1$ and $\hat{x}_2$ of this singular problem can be computed by

$$\begin{aligned}\hat{x}_1 &= \left(A_1^T \left(I - A_2 (A_2^T A_2)^{-1} A_2^T \right) A_1 \right)^{-} A_1^T \left(I - A_2 (A_2^T A_2)^{-1} A_2^T \right) \ell \\ \hat{x}_2 &= (A_2^T A_2)^{-1} A_2^T (\ell - A_1 \hat{x}_1)\end{aligned}\tag{19}$$

with an arbitrary generalized inverse $()^-$. It should be mentioned, that it is sufficient to determine the generalized inverse only within the first group of unknowns due to the fact that two fixed tracks solve the fitting problem. The adjusted observational residuals are determined by

$$v = A_1 \tilde{x}_1 + A_2 \tilde{x}_2 - \ell .\tag{20}$$

The particular solution $\hat{x}$ and a complete base in the nullspace of A ($S^\perp(A)$) - summarized as columns of G - can be used to determine the solution with the best fit, in the sense that the solution vector x_{min} has minimal norm. This problem can be solved by an adjustment, resulting in the overdetermined equation system

$$Gy + \hat{x} = x\tag{21}$$

and the target function

$$\|x\| \dots min.\tag{22}$$

The solution of this standard least squares adjustment problem is given by

$$x_{min} = - \left(I - G (G^T G)^{-1} G^T \right) \hat{x}.\tag{23}$$

It should be mentioned that the use of the pseudoinverse in the two step approach of formula (19) minimizes only the norm of $\hat{x}_1$ and is therefore not equivalent to the minimum norm solution x_{min} (23), where the minimization is done with respect to both groups of unknowns.

3 Application of Array Algebra

Without any restriction of generality, the application of array algebra to the crossover adjustment is demonstrated by an example:

A small area with five descending and four ascending tracks is chosen (cf. Fig. 3). The orbit corrections of this 9 tracks, using a bias and tilt model, result in 18 unknowns.

The structure of the sparse observation equation matrix A is given by

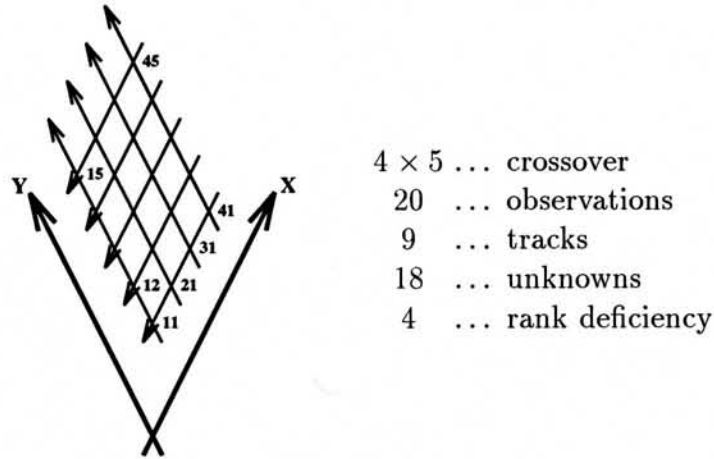


Figure 3: Planar test area.

A	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5	c_1	d_1	c_2	d_2	c_3	d_3	c_4	d_4
Δh_{11}	1	x_1										-1	$-y_1$					
Δh_{21}	1	x_2											-1	$-y_1$				
Δh_{31}	1	x_3													-1	$-y_1$		
Δh_{41}	1	x_4															-1	$-y_1$
Δh_{12}			1	x_1								-1	$-y_2$					
Δh_{22}			1	x_2									-1	$-y_2$				
Δh_{32}			1	x_3											-1	$-y_2$		
Δh_{42}			1	x_4													-1	$-y_2$
Δh_{13}					1	x_1						-1	$-y_3$					
Δh_{23}					1	x_2							-1	$-y_3$				
Δh_{33}					1	x_3									-1	$-y_3$		
Δh_{43}					1	x_4											-1	$-y_3$
Δh_{14}							1	x_1				-1	$-y_4$					
Δh_{24}							1	x_2					-1	$-y_4$				
Δh_{34}							1	x_3							-1	$-y_4$		
Δh_{44}							1	x_4									-1	$-y_4$
Δh_{15}									1	x_1	-1	$-y_5$						
Δh_{25}									1	x_2			-1	$-y_5$				
Δh_{35}									1	x_3					-1	$-y_5$		
Δh_{45}									1	x_4							-1	$-y_5$

if the default numbering scheme of the unknown parameters is applied. Let us now reorder the parameter as follows: First alternate bias a_i and tilt b_i and collect all the unknowns of descending tracks, with a *track by track* sorting scheme. The second part of unknowns consists of all parameters belonging to the ascending tracks. Here the ordering scheme starts with all bias parameters of all ascending tracks and after this all the tilt parameters are collected. The sorting scheme can be denoted as *type by type*. The resulting structure of the reordered observation equation matrix A is given

by

A	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5	c_1	c_2	c_3	c_4	d_1	d_2	d_3	d_4
Δh_{11}	1	x_1									-1				- y_1			
Δh_{21}	1	x_2										-1				- y_1		
Δh_{31}	1	x_3											-1				- y_1	
Δh_{41}	1	x_4												-1				- y_1
Δh_{12}			1	x_1							-1				- y_2			
Δh_{22}			1	x_2								-1				- y_2		
Δh_{32}			1	x_3									-1				- y_2	
Δh_{42}			1	x_4										-1				- y_2
Δh_{13}					1	x_1					-1				- y_3			
Δh_{23}					1	x_2						-1				- y_3		
Δh_{33}					1	x_3							-1				- y_3	
Δh_{43}					1	x_4								-1				- y_3
Δh_{14}							1	x_1			-1				- y_4			
Δh_{24}							1	x_2				-1				- y_4		
Δh_{34}							1	x_3					-1				- y_4	
Δh_{44}							1	x_4						-1				- y_4
Δh_{15}									1	x_1	-1				- y_5			
Δh_{25}									1	x_2		-1				- y_5		
Δh_{35}									1	x_3			-1				- y_5	
Δh_{45}									1	x_4				-1				- y_5

(25)

It is mentioned in section 2 that the observation equations have not full column rank. The nullspace of the reordered bias and tilt model is given by

$$G^T = \begin{pmatrix} 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 \\ & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & x_1 & & x_2 & & x_3 & & x_4 \\ y_1 & & y_2 & & y_3 & & y_4 & & y_5 & & & & & & & & 1 & 1 & 1 & 1 \\ & y_1 & & y_2 & & y_3 & & y_4 & & y_5 & & & & & & & x_1 & x_2 & x_3 & x_4 \end{pmatrix} \quad (26)$$

(cf. *SCHRAMA (1989)[7]*, pp. 27).

Using this numbering scheme within the design matrix, the regular structure in the first part, but also the regular structure in the second part can be easily seen and can be resolved with the help of

$$\begin{aligned}
 A_1 = (I_x \otimes A_x) \quad \text{with} \quad A_x = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ 1 & x_4 \end{pmatrix} \quad I_x = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \\
 A_2 = (A_y \otimes I_y) \quad \text{with} \quad A_y = \begin{pmatrix} -1 & -y_1 \\ -1 & -y_2 \\ -1 & -y_3 \\ -1 & -y_4 \\ -1 & -y_5 \end{pmatrix} \quad I_y = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}
 \end{aligned} \tag{27}$$

leading to the Kronecker separated observation equation

$$(I_x \otimes A_x \quad A_y \otimes I_y) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \ell + v. \tag{28}$$

Introducing again the column-wise mapping from l to L (cf. section 1)

$$L = \begin{pmatrix} \Delta h_{11} & \Delta h_{12} & \Delta h_{13} & \Delta h_{14} & \Delta h_{15} \\ \Delta h_{21} & \Delta h_{22} & \Delta h_{23} & \Delta h_{24} & \Delta h_{25} \\ \Delta h_{31} & \Delta h_{32} & \Delta h_{33} & \Delta h_{34} & \Delta h_{35} \\ \Delta h_{41} & \Delta h_{42} & \Delta h_{43} & \Delta h_{44} & \Delta h_{45} \end{pmatrix} \tag{29}$$

and the same for x_1 and x_2

$$\begin{aligned}
 X_1 &= \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \end{pmatrix} \\
 X_2 &= \begin{pmatrix} c_1 & d_1 \\ c_2 & d_2 \\ c_3 & d_3 \\ c_4 & d_4 \end{pmatrix},
 \end{aligned} \tag{30}$$

the mapping relation (9) can be used to transform the observation equations into the simple form

$$A_x X_1 + X_2 A_y^T = L + V. \tag{31}$$

The corresponding nullspace G is given by

$$G = \begin{pmatrix} -A_y \otimes I_2 \\ I_2 \otimes A_x \end{pmatrix} \quad \text{with} \quad I_2 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}. \tag{32}$$

The main task of the whole adjustment procedure (cf. first part of Eq. (19)) is the computation of the generalized inverse for the solution of the descending track parameters

$$\left(A_1^T \left(I - A_2 (A_2^T A_2)^{-1} A_2^T \right) A_1 \right)^-. \tag{33}$$

Introducing the Kronecker separated matrices A_1 and A_2 from Eq. (27) and using the properties of the Kronecker-product (3), it can be easily shown that the matrix (33), which has to be inverted, is equivalent to

$$\left(I_x - A_y (A_y^T A_y)^{-1} A_y^T \otimes A_x^T A_x \right)^{-} . \quad (34)$$

Due to property (3k) for Kronecker-products the inversion can be split into two tasks:

$$\left(I_x - A_y (A_y^T A_y)^{-1} A_y^T \right)^{-} \otimes (A_x^T A_x)^{-1} \quad (35)$$

The second part - $(A_x^T A_x)^{-1}$ - has no rank deficiency and can be solved uniquely, but for the first part a rank deficiency is expected and a generalized inverse must be computed. Therefore, the definitions for the generalized inverse (cf. eg. *BEN-ISRAEL et al. (1974)[8]*, pp. 7)

$$\left. \begin{array}{l} \text{(a)} \quad AA^-A = A \quad \Rightarrow \quad A^- \\ \text{(b)} \quad A^-AA^- = A^- \\ \text{(c)} \quad (AA^-)^T = AA^- \\ \text{(d)} \quad (A^-A)^T = A^-A \end{array} \right\} \Rightarrow A^+ \quad (36)$$

have to be considered, where the first property (a) reflects that each idempotent matrix A is a generalized inverse of itself. A look to the first part of Eq. (35) shows that the matrix $(I_x - A_y (A_y^T A_y)^{-1} A_y^T)$ possesses this property, moreover this matrix is the projection matrix $\Pi_{A_y}^\perp$,

$$\Pi_{A_y}^\perp = I_x - A_y (A_y^T A_y)^{-1} A_y^T , \quad (37)$$

which projects a vector into the orthogonal column space of A_y . Therefore, the projector $\Pi_{A_y}^\perp$ is equivalent to its generalized inverse

$$\Pi_{A_y}^\perp = (\Pi_{A_y}^\perp)^- , \quad (38)$$

and fulfills also properties (35b-d). Consequently, $\Pi_{A_y}^\perp$ is also its own pseudoinverse $(\Pi_{A_y}^\perp)^+$

$$\Pi_{A_y}^\perp = (\Pi_{A_y}^\perp)^+ . \quad (39)$$

Unfortunately, the pseudoinverse of that part of the problem is not equivalent to the pseudoinverse solution x_{min} of the whole problem (cf. section 2).

Using the aforementioned properties, the solution of (35) is given by

$$\left(I_x - A_y (A_y^T A_y)^{-1} A_y^T \right) \otimes (A_x^T A_x)^{-1} , \quad (40)$$

or with the help of the orthogonal projector $\Pi_{A_y}^\perp$ by

$$\Pi_{A_y}^\perp \otimes (A_x^T A_x)^{-1} . \quad (41)$$

If this solution of the generalized inverse is introduced into Eq. (19), the first part of the unknowns $\hat{x}_1$ can be computed by

$$\hat{x}_1 = \left(\Pi_{A_y}^\perp \otimes (A_x^T A_x)^{-1} A^T \right) \ell . \quad (42)$$

Employing the mapping relation (9) and the symmetry of $\Pi_{A_y}^\perp$ is equivalent to

$$\hat{X}_1 = (A_x^T A_x)^{-1} A_x^T L \Pi_{A_y}^\perp. \quad (43)$$

The remaining unknowns x_2 can be computed in a consistent manner with the help of the second part of Eq. (19) by

$$\hat{X}_2 = (L - A_x \hat{X}_1) A_y (A_y^T A_y)^{-1}. \quad (44)$$

The observational residuals follow:

$$V = A_x \hat{X}_1 + \hat{X}_2 A_y^T - L \quad (45)$$

In addition to this adjustment the minimum norm solution (23) must be computed. For the minimum norm solution x_{min} the Kronecker-separability of G (32) can be used, but, because of the size of the problem (four unknowns), there is only little to gain.

Summary and Conclusions

With the help of the Kronecker-separability the effort of the crossover adjustment with gridded data can be decreased drastically. The solution of the crossover problem is usually divided into two tasks. First the parameters of the ascending tracks and then the parameters of the descending tracks are computed or vice versa. The default equations

$$\begin{aligned} \tilde{x}_1 &= \left(A_1^T \left(I - A_2 (A_2^T A_2)^{-1} A_2^T \right) A_1 \right)^{-1} A_1^T \left(I - A_2 (A_2^T A_2)^{-1} A_2^T \right) \ell \\ \tilde{x}_2 &= (A_2^T A_2)^{-1} A_2^T (\ell - A_1 \tilde{x}_1) \\ v &= A_1 \tilde{x}_1 + A_2 \tilde{x}_2 - \ell \end{aligned}$$

need a lot of space and operations, also if sparsity is utilized. The observation equations have a size of $(t_x * t_y) \times p * (t_x + t_y)$, with $t_x(t_y)$ number of tracks in $x(y)$ direction and p the order of the polynomial. The resulting normal equations form a sparse, symmetric system with $(p * (t_x + t_y))$ unknowns. For example, using 100 ascending and descending tracks yields 10,000 observation equations with 400 unknown parameters, if only a bias and tilt model is applied. Compared to that the Kronecker-separability and the use of array algebra reduces the size of the matrices to 100 rows and 2 columns.

$$\begin{aligned} \tilde{X}_1 &= (A_x^T A_x)^{-1} A_x^T L \Pi_{A_y}^\perp \\ \tilde{X}_2 &= (L - A_x \tilde{X}_1) A_y (A_y^T A_y)^{-1} \\ V &= A_x \tilde{X}_1 + \tilde{X}_2 A_y^T - L \end{aligned}$$

All the necessary matrices, which have to be inverted have the size of two. If instead of a bias and tilt model a higher order polynomial model is used, the order of this polynomial defines the size of the systems. In conclusion, the effort of the whole crossover computation can be neglected also for large and densely covered areas.

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The definition of a regional geopotential field for precise gravity field modelling

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1. Introduction

The comparisons between the Italian geoid ITALGEO95 (R. Barzaghi et al., 1996), derived via remove-restore technique and Fast Collocation, and the available GPS/levelling data, lead to standard deviations of 20 cm. It is necessary improve the ITALGEO95 accuracy, defining a new geoid computation method, also in order to carry out a joined estimation based on gravimetric and GPS/levelling data.

A second iteration on gravity residuals to compute geoid refinements failed. In fact, the gravity residuals, used in computing ITALGEO95, obtained subtracting the global geopotential model OSU91A up to degree 360 and the Residual Terrain Correction (RTC) from the observations, have a covariance function with a short correlation length with respect to the Italian area ($36^\circ \leq \varphi \leq 47^\circ$, $6^\circ \leq \lambda \leq 19^\circ$). In practice, the function is equal to zero just after 0.6° . Such a covariance function loses its physical meaning and has numerical instabilities when it is used for predicting values over the whole area. Furthermore, these gravimetric residuals have not uniform statistical feature in the Italian area.

Then a revisited remove-restore procedure has been proposed; the gravimetric signal has been spectrally analysed in an empirical way and it has been modelled introducing an intermediate step. The observed values have been RTC reduced (high frequency component) and the OSU91A model has been subtracted up to degree 150; in this way, data with medium wavelength component have been obtained. They have been further smoothed through a moving average operator (with 20' size and 5' sampling rate) and then using collocation $\Delta\hat{g}_R$ and $\hat{N}_R$ (we define them as "regional signal") have been computed. This procedure allows the definition of a regional geopotential field, properly

tuned on the medium wavelength of the area under investigation, to be used in the remove-restore process.

The statistics of the gravity residuals show that the regional model is more effective in representing the local field; furthermore the improvements in the statistic of the gravimetric residuals reflects also in the improvements on geoid undulations.

This procedure has been applied to the whole Italian area and in some local zones.

2. Refining the ITALGEO95 estimates.

As a first attempt to improve the accuracy of the ITALGEO95 geoid, obtained with remove-restore technique and the Fast Collocation, we have tried to estimate a second order geoid component from the gravity residuals.

Since ITALGEO95 has been computed using gridded data Δg_r^G , possible smoothing have been introduced through the gridding.

To improve and refine the geoid estimate we can then apply the following procedure:

- 1) Fast Collocation estimation of the residual point values Δg_r^i from the gridded values Δg_r^G (over the whole Italian area) and evaluation of the second order residuals $\delta \Delta g_r^i$

$$\Delta g_r^G \xrightarrow{\text{Fast Collocation}} \Delta g_r^i \Rightarrow \delta \Delta g_r^i = \Delta g_r^i - \Delta g_r^G;$$
- 2) Collocation to predict $\delta \hat{N}_i$ from the second order residual values $\delta \Delta g_r^i$ on local areas ($\sim 3^\circ \times 3^\circ$);
- 3) Estimation of the refined ITALGEO95 quasigeoid as

$$\hat{N} = N(\text{ITALGEO95}) + \delta \hat{N}_i.$$

This method has produced unreliable estimates of Δg_r^i , because of the covariance function structure of Δg_r^G . The covariance function of Δg_r^G has a very short correlation length with respect to the width of the computation window ($36^\circ \leq \varphi \leq 47^\circ$, $6^\circ \leq \lambda \leq 19^\circ$): the function is practically zero just after 0.6° . When using such covariance function to predict Δg_r^i , we tested different cut off at different values of spherical

distance ψ_c . The prediction Δg_r^i doesn't stabilise, when increasing ψ_c , due to the particular feature of the covariance of Δg_r^G . The sharp variations that we got in the estimates imply that no reliable values can be obtained in such away.

Thus it turned out that no improvement in geoid estimate can be reached in this way, for these reasons we had to give up this procedure and introduce some change in the geoid estimation methodology.

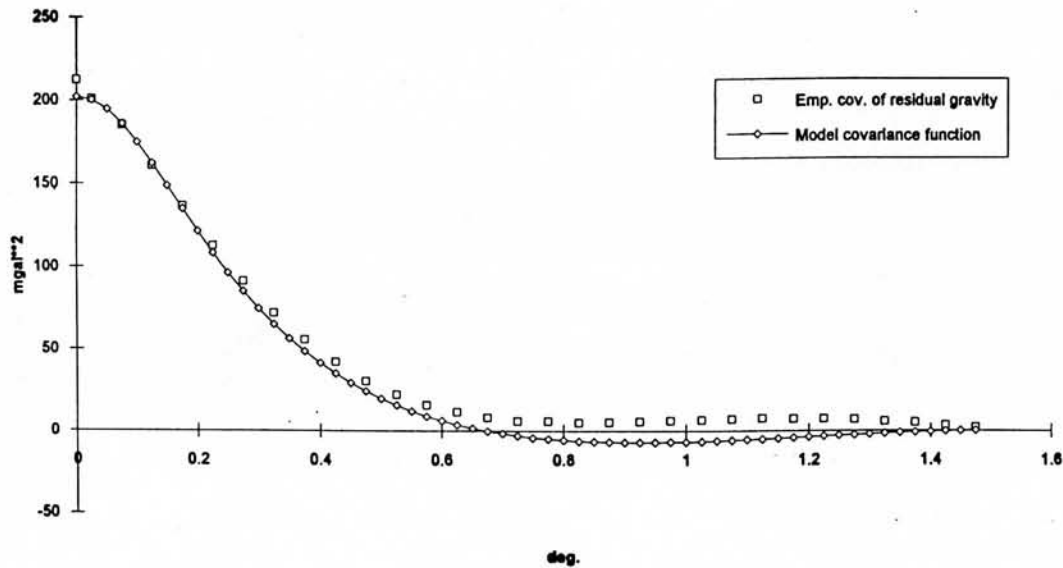


Fig.2.1 Covariance function of ITALGEO95 residuals

3. A new procedure for geoid estimate

It has been carried out a review of the remove-restore procedure used for the ITALGEO95 geoid computation, based on the definition of a regional geopotential field, matching local gravimetric data and information coming from the low degree harmonics of global models.

The gravimetric signal has been analysed in spectral terms and it has been modelled introducing an intermediate remove-restore step. The gravity high frequency components have been taken by the RTC, while the geopotential model OSU91A up to degree 150 has been used as low frequency reference field. The medium wavelength

components, that is the regional ones, has been obtained from the observations by the Collocation technique.

The basic scheme of this method has been applied following these steps:

a) gravity residual computation:

$$\Delta g_R = \Delta g_{\text{obs}} - \Delta g_M(\text{OSU91A-150}) - g_{\text{rtc}}$$

b) moving average on the Δg_R (window size of 20' and resolution of 5') to obtain a regular mean regional field Δg_R^G

c) from this mean regional field using Fast Collocation, prediction of the $\Delta \hat{g}_R$ and $\hat{N}_R$ values were obtained. In this case, it is reasonable to use the Collocation method because the covariance function of the residuals Δg_R^G is a smooth function with a suitable correlation length (with respect to the computation area)

d) the new reference model is then:

$$\Delta g_M(\text{NEW}) = \Delta g_M(\text{OSU91A-150}) + \Delta \hat{g}_R$$

$$N_M(\text{NEW}) = N_M(\text{OSU91A-150}) + \hat{N}_R$$

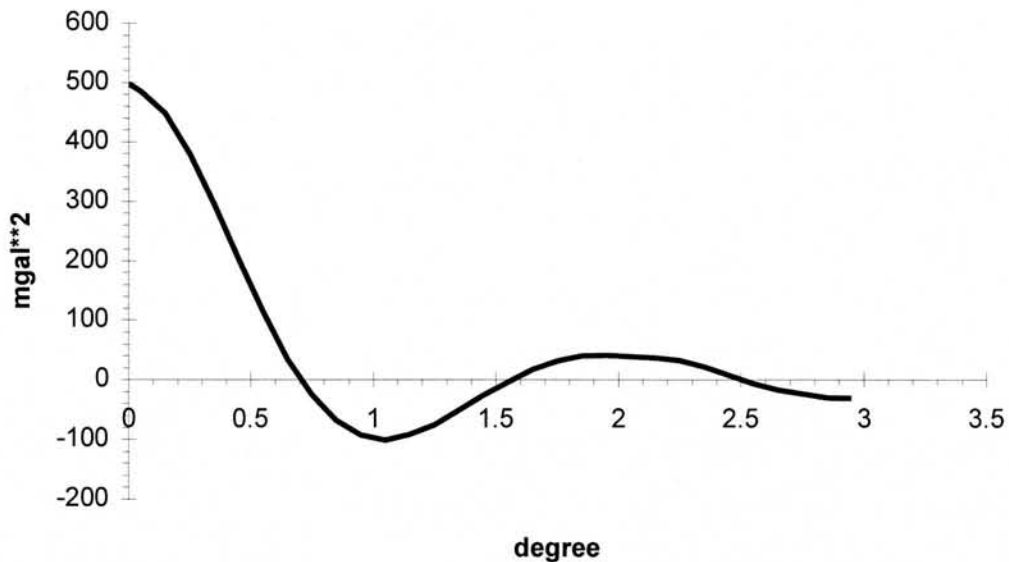
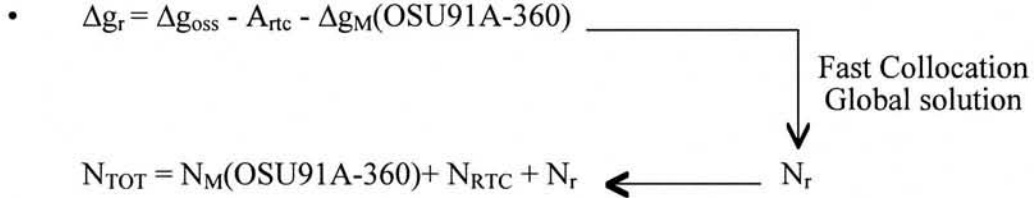


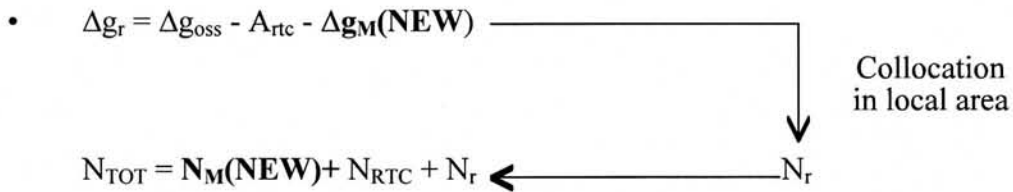
Fig. 3.1. Empirical covariance function of the averaged data Δg_R^G

In the end, a possible scheme of the remove-restore technique is the same that in ITALGEO95 geoid estimation, but from the model component which is now tuned on the gravity data contained in the computation area:

* Procedure used in the ITALGEO95 computation:



* New procedure:



4. Applications and comparisons

The remove step of the new procedure has been applied to compute the gravity residual values Δg_r for a subset of the database used for the ITALGEO95 computation (fig. 4.1).

We have subtracted from the observations the NEW model $\Delta g_M(NEW)$ and the RTC component; the same computation has been done using the global geopotential models OSU91A (R.H. Rapp, 1994) and EGM96 (1997) (up to degree 360), thus obtaining three sets of gravity residuals, $\Delta g_r(NEW)$, $\Delta g_r(OSU91A)$ and $\Delta g_r(EGM96)$; statistics of these residuals are collected tab. 1.

As it is expected the statistical proprieties of the $\Delta g_r(NEW)$ are better than the ones related to OSU91A and EGM96 models and the plots of the residuals show clearly this better statistical behaviour (fig.4.2).

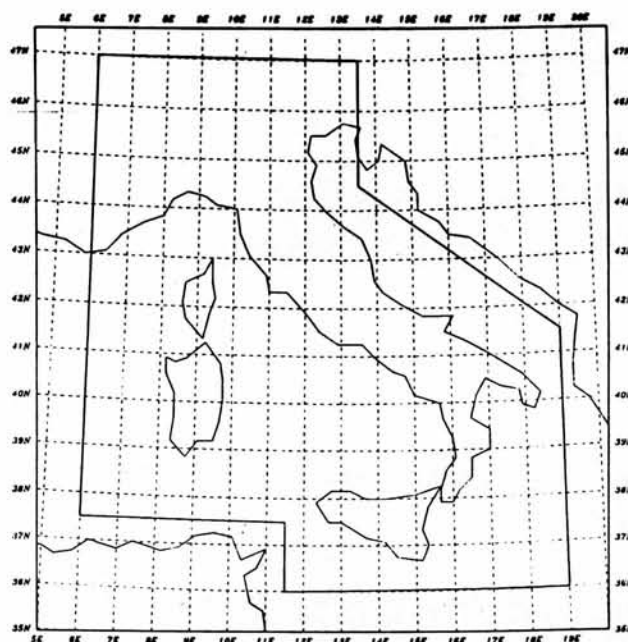


Fig. 4.1 ITALGEO95 computation area.

Tab. 1. Statistics of gravity residuals

	$\Delta g_r(\text{OSU91A})$ [mGal]	$\Delta g_r(\text{EGM96})$ [mGal]	$\Delta g_r(\text{NEW})$ [mGal]
n	22858	22858	22858
E	-0.14	0.02	0.09
σ	17.69	14.89	11.37
min	-197.34	-188.32	-166.74
max	129.55	105.46	91.19

In table 2 and 3 the statistics respectively of the $1^\circ \times 1^\circ$ means and standard deviations of the gravity residuals have been reported, and in fig. 4.3 and 4.4. these values have been plotted. We can see again that the $\Delta g_r(\text{NEW})$ residuals are more homogeneous, while the $\Delta g_r(\text{OSU91A})$ and $\Delta g_r(\text{EGM96})$ residuals show a strong correlation with the Italian topography: we can easily recognise the Alps, the Padania plain, the Apennine, the Sicily, Sardinia and Corsica Islands.

(In the contour plot of $\Delta g_r(\text{OSU91A})$ there are high values in correspondence to the Corsica Island, because of mismodelling in the global geopotential model OSU91A; this reflects loosely into the $\Delta g_r(\text{NEW})$ residual since $\Delta g_M(\text{NEW})$ is based partially on OSU91A).

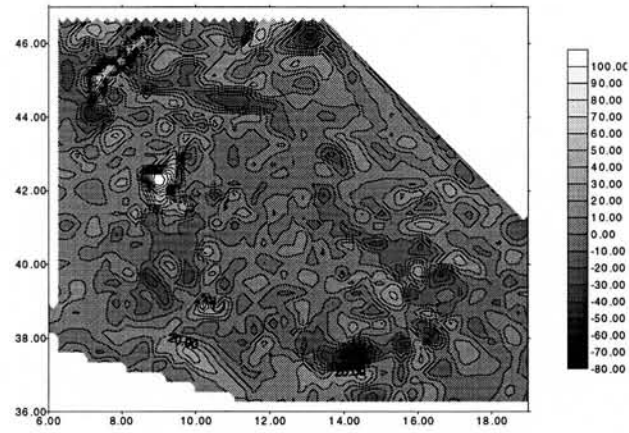
Tab. 2. Statistics of the $1^\circ \times 1^\circ$ gravity residuals means

	$\Delta g_r(\text{OSU91A})$ [mGal]	$\Delta g_r(\text{EGM96})$ [mGal]	$\Delta g_r(\text{NEW})$ [mGal]
n	122	122	122
E	0.64	0.61	0.43
σ	9.12	5.86	5.19
min	-21.92	-13.56	-17.85
max	33.91	16.14	29.47

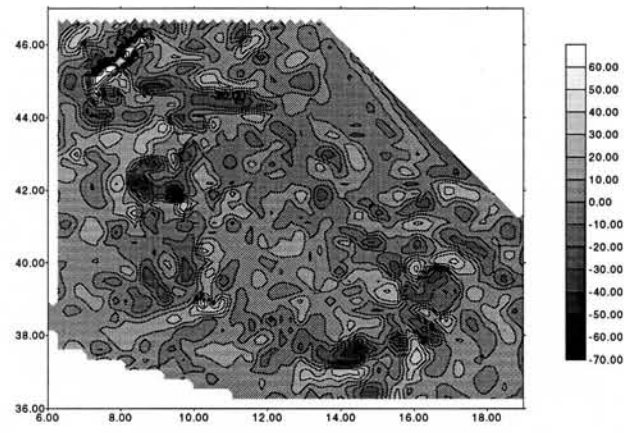
Tab. 3. Statistics of the $1^\circ \times 1^\circ$ gravity residuals standard deviations

	$\Delta g_r(\text{OSU91A})$ [mGal]	$\Delta g_r(\text{EGM96})$ [mGal]	$\Delta g_r(\text{NEW})$ [mGal]
n	122	122	122
E	12.61	11.77	9.16
σ	7.08	5.59	4.49
min	0.96	2.73	1.58
max	41.46	39.86	26.40

a) OSU91A



b) EGM96



c) NEW

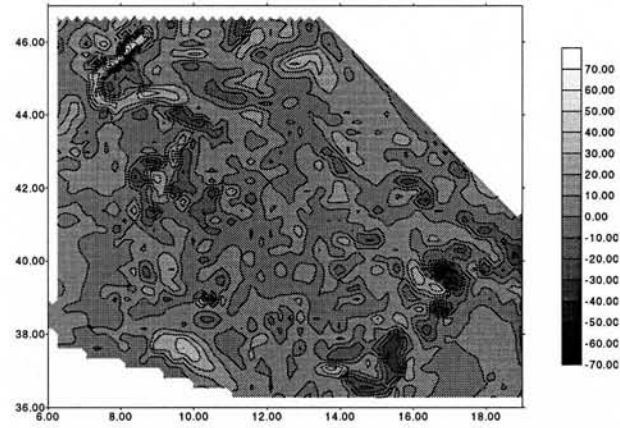
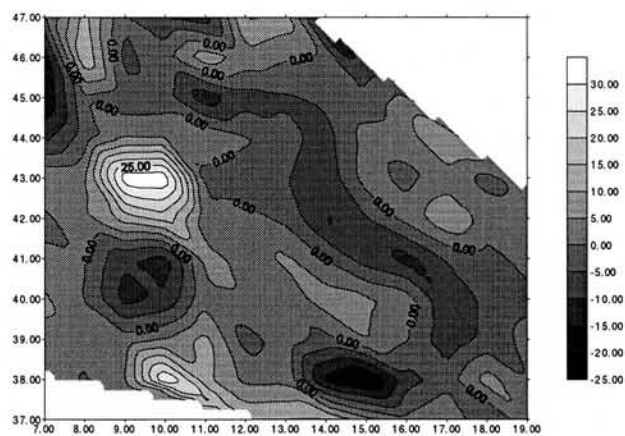
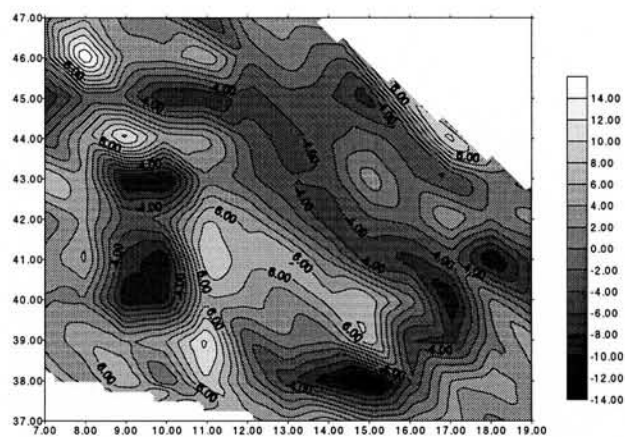


Fig. 4.2. Plots of the residuals $\Delta g_{\text{OBS}} - A_{\text{RTC}} - \Delta g_{\text{M}}$.

a) OSU91A



b) EGM96



c) NEW

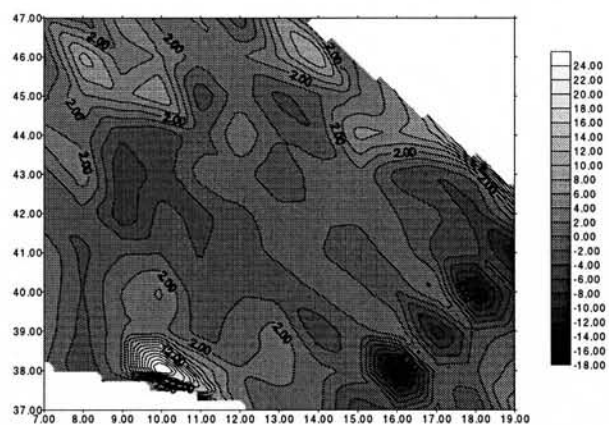
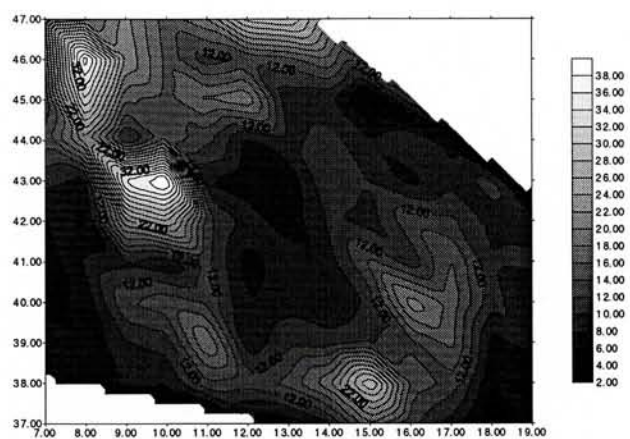
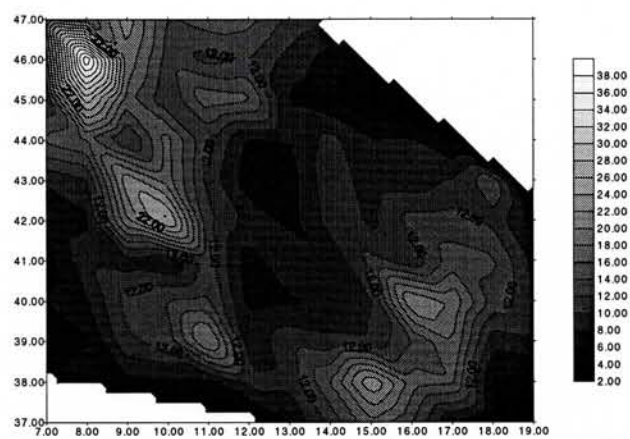


Fig. 4.3. Plots of 1° x 1° means of the gravity residuals.

a) OSU91A



b) EGM96



c) NEW

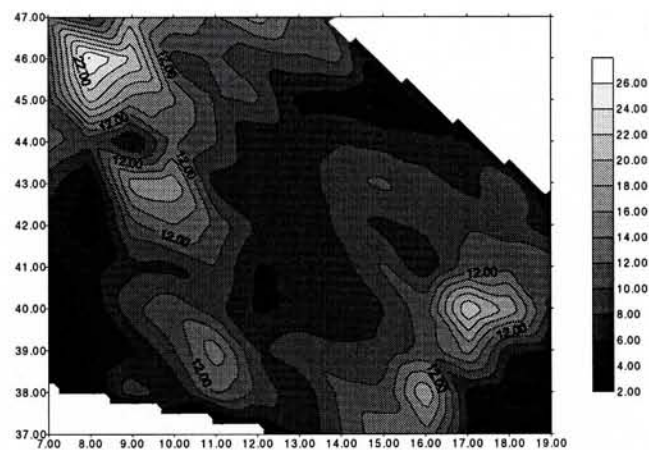


Fig. 4.4. Plots of $1^\circ \times 1^\circ$ standard deviations of the gravity residuals.

Furthermore, the empirical covariance functions of the gravity residuals have been computed and compared: we can see in fig. 4.5 that the empirical function of $\Delta g_r(\text{NEW})$ displays a more regular feature.

We have then repeated these comparisons in three local areas ($\sim 3^\circ \times 3^\circ$) using the whole set of available data (fig. 4.6).

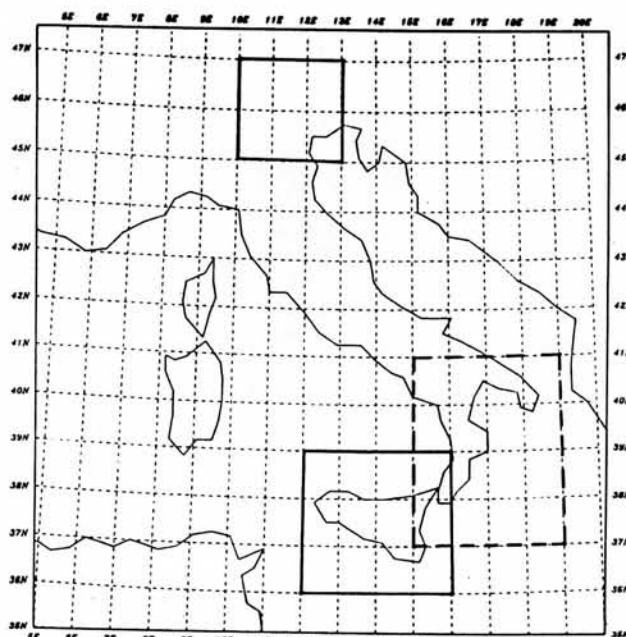


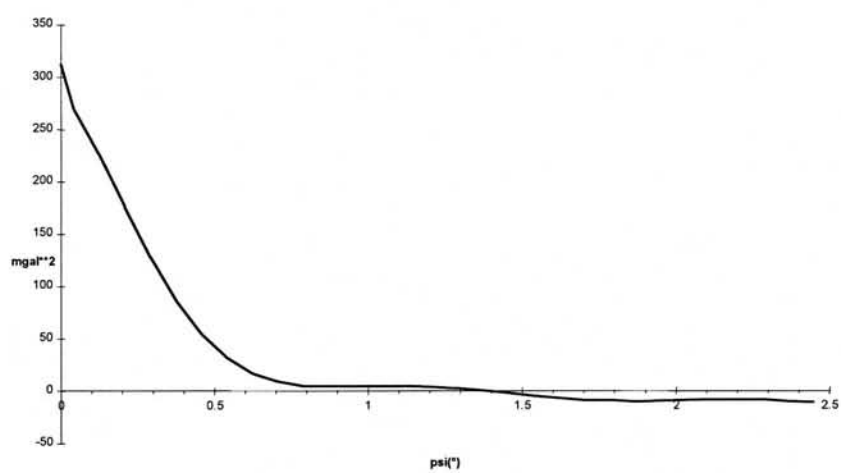
Fig.4.6 Local areas: the Alpj, Calabria and Sicily

Even in local areas, the new procedure improves the gravity residuals statistics: we obtained lower mean and standard deviation values (tab. 4). An important behaviour which must be stressed is that the covariance functions of the $\Delta g_r(\text{NEW})$ (fig. 4.7) are similar with each other and with the Italian covariance function, while this is not true using the OSU91A and EGM96 residuals.

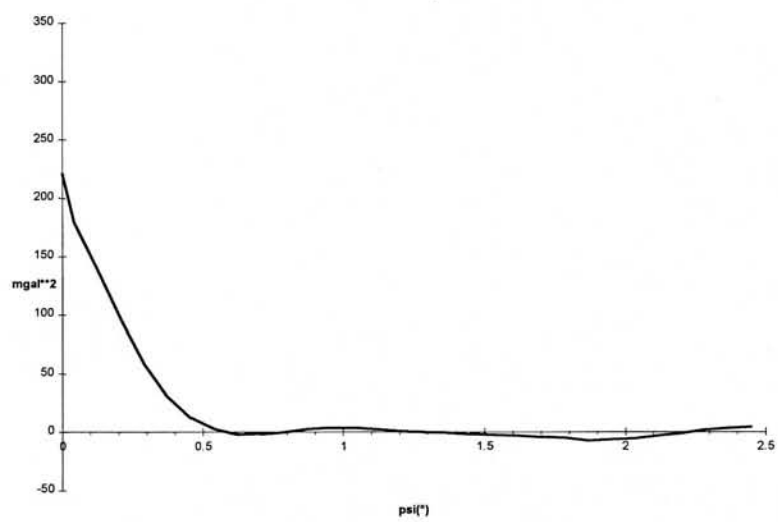
Comparisons have been also carried out in one selected area on N. Along the Adige valley, we have several GPS/leveling points; there we can derive $N_{\text{GPS}} = h - H$ and compare these values with the ones from the different models.

The statistics of the differences $\Delta = N_{\text{GPS}} - N_{\text{M}} - N_{\text{RTC}}$, as obtained from the three different models, NEW, OSU91A, EGM96, are described in tab. 5.

a) OSU91A



b) EGM96



b) NEW

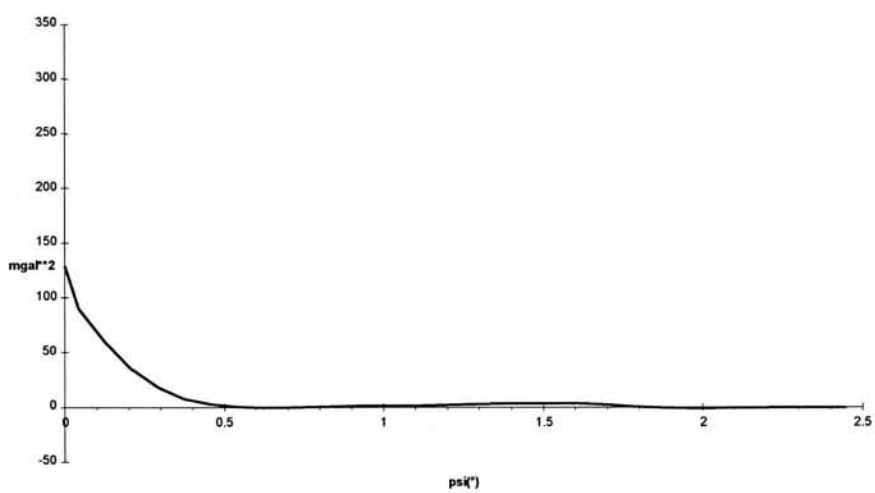
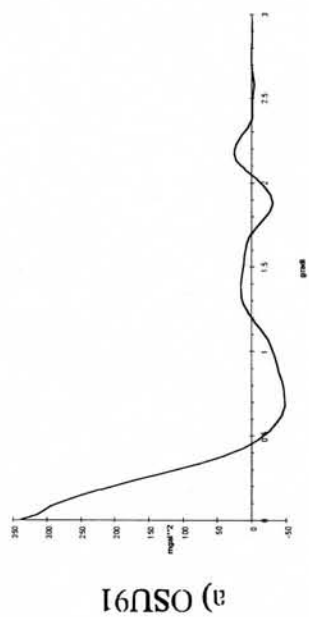
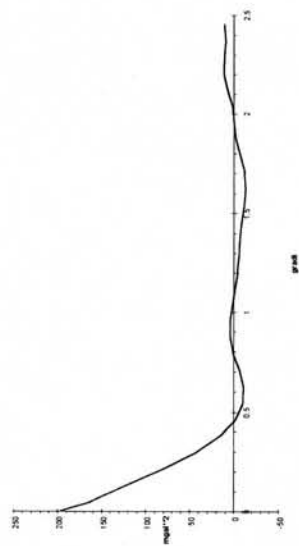


Fig. 4.5 Residuals covariance function on the Italian area

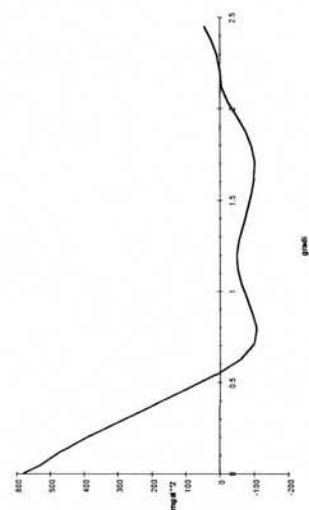
ALPI



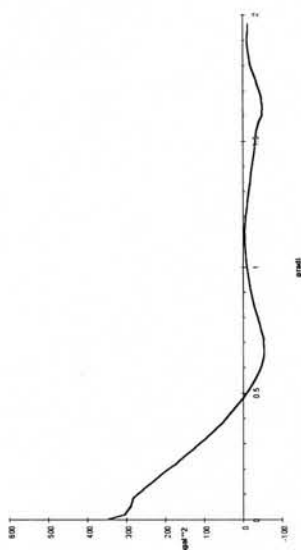
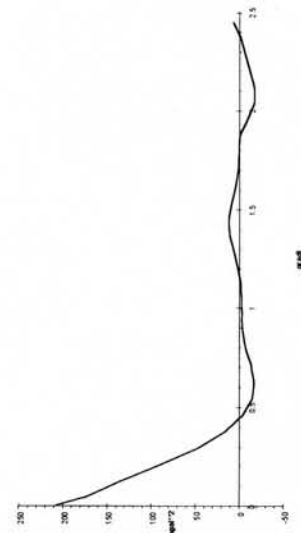
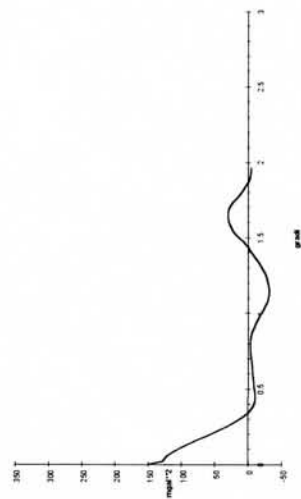
CALABRIA



SICILIA



b) EGM96



c) NEW

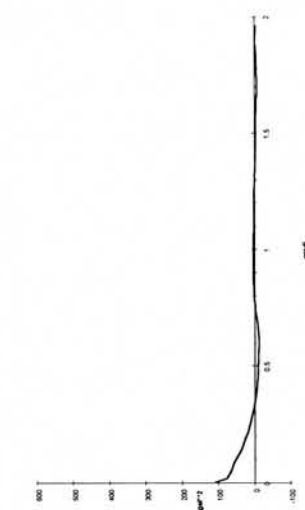
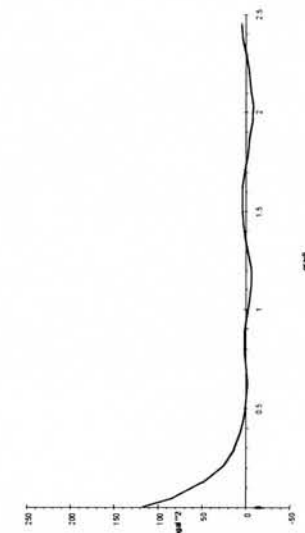
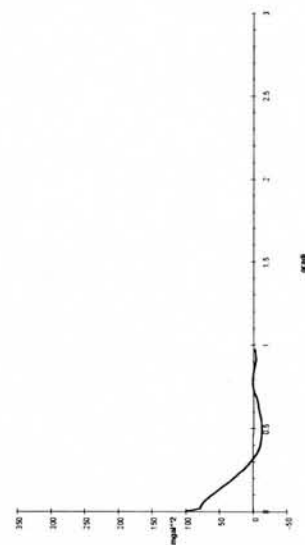


Fig 4.7 Covariance function of the gravity residuals in the local areas.

A sharp improvement is reached, at least with respect to the OSU91A model: EGM96 performs a little but better than the regional model. This can be related either to the use of OSU91A in the definition of the low frequency component of the regional model or to mismodelling in $\hat{N}_R$. Such a problem must be further investigated to get a uniform improvement using the new model both in Δg_T and N_T .

tab. 4. Statistics of local gravity residuals

AREA		OSU91A -360	EGM96 - 360	NEW
SICILIA	n	11671	11671	11671
	E	-9.95	-7.17	-1.33
	σ	24.04	18.61	10.44
	min	-91.76	-72.64	-58.40
	max	48.84	45.72	49.49
ALPI	n	8738	8738	8738
	E	2.20	1.45	1.08
	σ	18.37	12.17	9.94
	min	-82.11	-46.26	-46.65
	max	61.74	50.80	45.43
CALABRIA	n	15684	15684	15684
	E	-4.98	-5.98	-0.94
	σ	14.09	14.40	10.83
	min	-67.39	-56.94	-72.49
	max	45.57	47.16	41.68

Tab. 5. Statistics of differences $N_{GPS} - N_M - N_{RTC}$

	OSU91A [m]	NEW [m]	EGM96 [m]
n	102	102	102
E	-0.20	0.19	-0.39
σ	0.71	0.31	0.24
min	-1.19	-0.40	-0.81
max	1.30	0.89	0.05

5. Conclusions

The new procedure, that is the definition of a regional geopotential field from the local data, improves the gravity residual statistics. The residual field is more homogeneous and this is important when considering local estimates. So that it seems reasonable to assume the new model as reference model to reduce the gravity data.

However, these improvements in terms of gravity don't imply an improvement in term of geoid undulation, at least in a cross check with EGM96 model (i.e. the $\Delta g_r(\text{NEW})$ have better statistical proprieties with respect to $\Delta g_r(\text{EGM96})$ but the converse is true when considering $N_r(\text{NEW})$ and $N_r(\text{EGM96})$).

This could be connected to the estimating procedure used in the definition of the regional model and must be carefully checked before applying the remove-restore procedure based on the regional model itself.

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New IGeS formulas for the Terrain Corrections

F. Sansó, C. Sciarretta, F. Vespe

1 Introduction

Following its own internal programs, IGeS is undertaking a review of the whole process leading to the computation of a local geoid. Among other items, also the problem of the Terrain Correction (T.C.) has been tackled in the framework of a cooperation within IGeS of different Italian institutions (Politecnico di Milano, Telespazio, Agenzia Spaziale Italiana). The material presented here is a new theory for the computation of the terrain correction and some numerical results coming from the first experiments.

2 Why new T.C. formulas?

Let us first of all remark that the residual terrain correction problem, which is actually mostly used in geoid computations, can be traced back to a double T.C., so that for the moment we will concentrate on this last problem. Moreover we will work here in plane approximation, although formulas in spherical approximation are as easily derived.

First of all let us examine what is the state of the art and what are the actual open problems in the authors' opinion.

Basically there are nowadays two methods for computing T.C. (in planar approximation):

- a) the prisms method, employing a prism-like approximation to the field of terrain heights $H(\xi)$, and exploiting closed formulas for the gravity (and potential) contributions of each prism ([2])
- b) the Fourier Transform approach, which after a series expansion of the Newton's integral kernel, exploits the capability of F.T. of handling in a fast and simple way convolutions; ([3]).

Both methods present some drawbacks:

- a) in the prism approach
- there are limitations in the computation due to the relative length of computation time; these prevent us from computing T.C. for very dense D.T.M. (e.g. 10 m spatial resolution) in relatively large areas (e.g. $5^\circ \times 5^\circ$); already

the computation of a T.C. point in the middle of a $1^\circ \times 1^\circ$ zone with 10 m resolution requires handling some 10^8 prisms;

- the rather rough pattern of prisms (with edges and vertices) combined with a non optimal spatial resolution of D.T.M., can produce high frequency error patterns specially if the computation point is close to an edge;
- the computation point often has to be moved from the real surface to the top of the relevant prism, if we don't want to compute corrections sometimes even for buried points, or for points floating in the air,
- b) in the F.T. approach one has to use typically the approximation (cfr. Fig. 1 for the symbols)

$$T.C.(\xi) = \mu \int \frac{(H_\xi - H_\eta)^2}{[\ell^2 + (H_\xi - H_\eta)^2]^{3/2}} d_2\eta \simeq \quad (1)$$

$$\simeq \mu \int \frac{H_\eta^2}{\ell^3} d_2\eta - 2\mu H_\xi \int \frac{H_\eta}{\ell^3} + \mu H_\xi^2 \int \frac{1}{\ell^3} d_2\eta$$

$$(\mu = G\rho, \rho = \text{mass density})$$

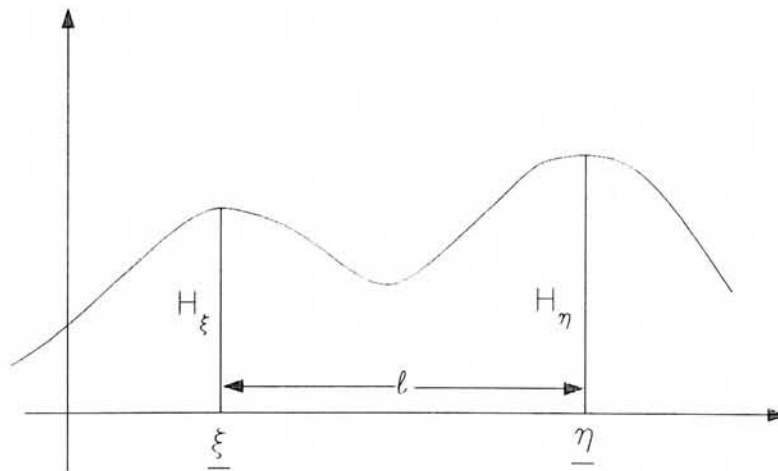


Fig. 2.1 - the planar approximation set up.

The approximation (2.1) is justified only if

$$|H_\xi - H_\eta| \ll \ell$$

and furthermore the splitting of the formula in 3 terms (in order to exploit the convolution theorem) is giving rise to 3 integrals individually diverging, what has always been a concern in the F.T. approach.

Remarks One could observe that also in the F.T. approach the final computation of the integrals has to be performed through a discretization, which goes back to something similar to the use of a prism approximation of the D.T.M.; however since the F.T. algorithm is so fast, it can handle huge amounts of data so that the spatial resolution of the computation can be pushed to the extreme, with the available data, strongly reducing the possible edge effects.

3 The new formulas for T.C.

New formulas for T.C. have been elaborated by the authors, keeping in mind the following points:

- smoothing the "true" surface $z = H_0(\xi)$, i.e. approximating it by a differentiable surface $H(\xi)$, doesn't affect significantly the gravity field since the difference $H_0(\xi) - H(\xi)$ will appear basically as a small, irregular, high frequency noise with very localized effects ([5]); this at least as far as our formulas don't put any constraint on the shape, particularly on the inclination, of $H(\xi)$;
- the approximation error should be controlled at any phase of the approximation procedure and in particular the use of diverging formulas should be forbidden.

Furthermore we will assume that the body, of which we are computing the newtonian attraction, has a finite extent so that $H(\xi) = 0$ outside a disk of diameter D in the plane. Our proposal is then based on an integral identity and its subsequent approximations.

The identity we will work with, is (cfr. Fig. 3.1 for the symbols)

$$\begin{aligned}
 g(\underline{x}) = & 2\pi\mu H_\xi - \pi\mu H_\xi^2 (D^2 + H_\xi^2)^{-\frac{1}{2}} + \\
 & -\mu\frac{1}{2} \int_0^{2\pi} d\theta \int_0^D d\ell \frac{\ell(H_\xi - H_\eta)^2}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{3}{2}}} + \\
 & +\mu\frac{1}{2} \int_0^{2\pi} d\theta \int_0^D d\ell \frac{(H_\xi - H_\eta)^3 H'_\eta}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{3}{2}}} = \\
 & = B_1 - B_2 - T_2 + T_3 \quad ,
 \end{aligned} \tag{2}$$

where (cfr. Fig. 3.1)

$$H'_\theta = \frac{\partial}{\partial \ell} H(\underline{\xi} + \ell \underline{e}) = \underline{e} \cdot \nabla H(\underline{\eta})$$

$$(\underline{\xi} + \ell \underline{e} = \underline{\eta}; \quad |\underline{e}| = 1)$$

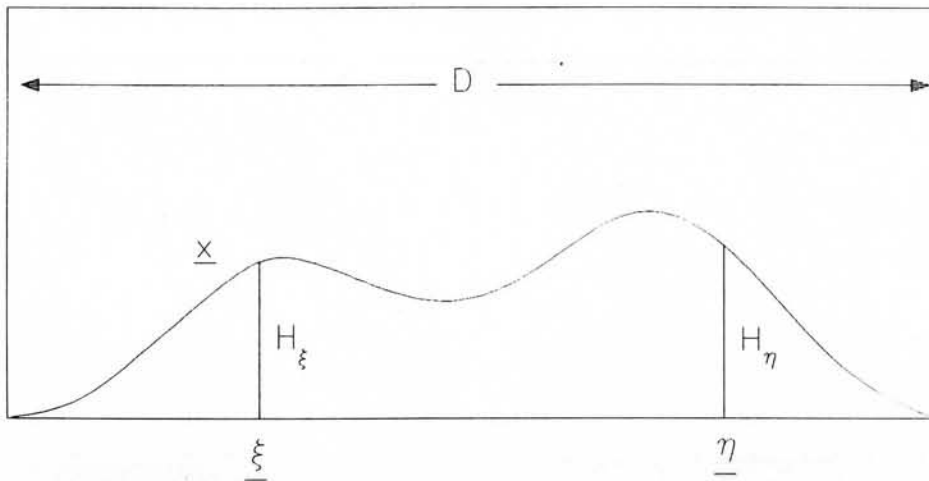
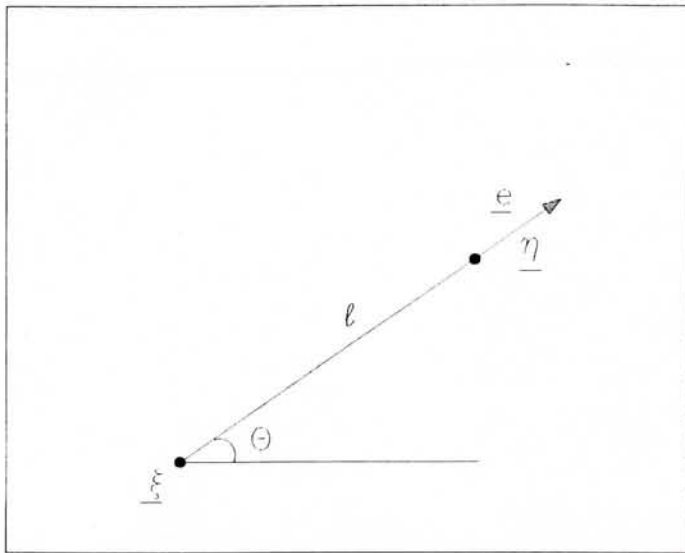


Fig. 3.1: meaning of symbols in (2), (3).

The proof of (2) runs in two steps as follows:

$$\begin{aligned}
g(\underline{x}) &= \mu \int d_2 \eta \int_0^{H_\eta} \left\{ -\frac{\partial}{\partial H_\xi} \frac{1}{[\ell^2 + (H_\xi - H)^2]^{\frac{1}{2}}} \right\} dH = \\
&= \mu \int_0^{2\pi} d\theta \int_0^D \left\{ \frac{1}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{1}{2}}} - \frac{1}{[\ell^2 + H_\xi^2]^{\frac{1}{2}}} \right\} \ell d\ell \\
&= \mu \int_0^{2\pi} d\theta \int_0^D \left\{ \frac{\ell - (H_\xi - H_\eta) H'_\eta}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{1}{2}}} - \frac{\ell}{[\ell^2 + H_\xi^2]^{\frac{1}{2}}} \right\} d\ell + \\
&\quad + \mu \int_0^{2\pi} d\theta \int_0^D \frac{(H_\xi - H_\eta) H'_\eta}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{1}{2}}} d\ell = \\
&= 2\pi \mu H_\xi + \mu \int_0^{2\pi} d\theta \int_0^D \frac{(H_\xi - H_\eta) H'_\eta}{[\ell^2 + (H_\xi - H_\eta)^2]^{\frac{1}{2}}} d\ell ,
\end{aligned} \tag{3}$$

the last step being justified considering that if $H_\xi > 0$ (what we assume) then $H_\eta = 0$ when $|\underline{\xi} - \underline{\eta}| = D$.

It is interesting to note that (3) is in itself an identity with no approximation.

Now, noting that

$$(H_\xi - H_\eta) H'_\eta = -\frac{1}{2} \frac{d}{d\ell} (H_\xi - H_\eta)^2 ,$$

(3) can be integrated in $d\ell$ by parts yielding the identity (2).

Before continuing let us remark that in (2) B_1 represents the usual Bounguer correction, B_2 is a new term which basically accounts for the extension of T_2 from $\ell = D$ to $\ell = +\infty$, while T_2 and T_3 are respectively 2^{nd} and 3^d order terms in H (also considering that H'_η is left free to attain any value).

We now reason on possible approximations of T_2 and T_3 . To this aim we will further split the integrals in $d\ell$ into an inner and an outer part, i.e.

$$\int_0^D ... d\ell = \int_0^{\epsilon_0} ... d\ell + \int_{\epsilon_0}^D ... d\ell .$$

Naturally this splitting could be better done by using a smooth $\epsilon(\ell)$ like in Fig. 3.2

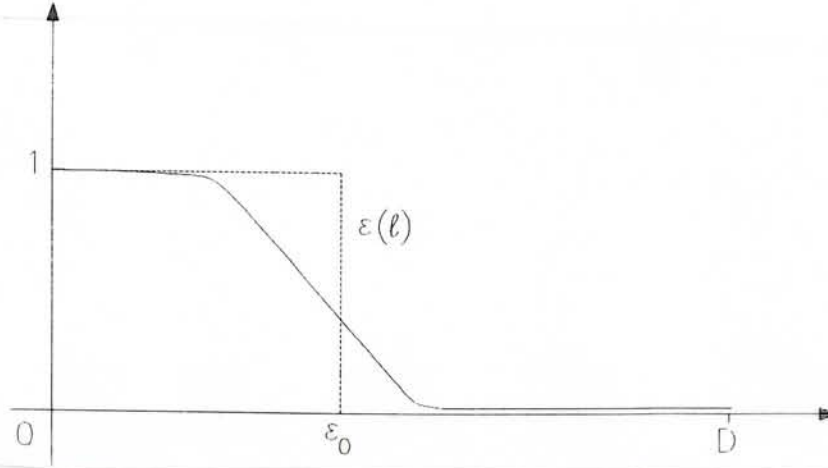


Fig. 3.2: An example of smooth split function.

In this way we derive the following approximations:

$$T_{2i} = \mu \frac{1}{2} \int_0^{2\pi} d\theta \cdot \int_0^{\epsilon_0} d\ell \frac{\left(\frac{H_\xi - H_\eta}{\ell}\right)^2}{\left[1 + \left(\frac{H_\xi - H_\eta}{\ell}\right)^2\right]^{\frac{3}{2}}} \simeq \quad (4)$$

$$\simeq \mu \frac{1}{2} \epsilon_0 \int_0^{2\pi} d\theta \frac{|\nabla H_\xi|^2 \cos^2 \theta}{[1 + |\nabla H_\xi|^2 \cos^2 \theta]^{\frac{3}{2}}}$$

$$T_{2o} = \mu \frac{1}{2} \int_0^{2\pi} d\theta \cdot \int_{\epsilon_0}^D d\ell \frac{\left(\frac{H_\xi - H_\eta}{\ell}\right)^2}{\left[1 + \left(\frac{H_\xi - H_\eta}{\ell}\right)^2\right]^{\frac{3}{2}}} \simeq \quad (5)$$

$$\simeq \mu \frac{1}{2} \int_0^{2\pi} d\theta \int_{\epsilon_0}^D d\ell \frac{(H_\xi - H_\eta)^2}{\ell^3} =$$

$$= \mu \frac{1}{2} \int d_2 \eta K_{\epsilon_0, D}^2(\underline{\epsilon} - \underline{\eta}) (H_\xi - H_\eta)^2$$

where the Kernel $K_{\epsilon_0, D}^2$ is given by

$$K_{\epsilon_0, D}^2(\underline{\xi}) = \begin{cases} \frac{1}{|\underline{\xi}|^3} & \epsilon_0 \leq |\underline{\xi}| \leq D \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Let us observe that a rational choice of ϵ_0 could be done by looking at the variogram of H_ξ and seeing where, for instance,

$$E \{ (H_\xi - H_\eta)^2 \} \sim \frac{1}{10} \ell^2 \Rightarrow \ell = \epsilon_0 \quad .$$

Moreover let us note that in the term T_{2i} we are not requiring that the inclination of the terrain at ξ be small or at least smaller than 1.

We remark also that the term T_{2o} is what is often used as the whole T.C. (although with a different meaning of ϵ_0).

As for the term T_3 we get

$$\begin{aligned} T_{3i} &= \mu \frac{1}{2} \int_0^{2\pi} d\theta \int_0^{\epsilon_0} d\ell \frac{\left(\frac{H_\xi - H_\eta}{\ell} \right)^3 H'_\eta}{\left[1 + \left(\frac{H_\xi - H_\eta}{\ell} \right)^2 \right]^{\frac{3}{2}}} \simeq \\ &\simeq \mu \frac{1}{2} \epsilon_0 \int_0^{2\pi} d\theta \frac{|\nabla H_\xi|^4 \cos^4 \theta}{[1 + |\nabla H_\xi|^2 \cos^2 \theta]^{\frac{3}{2}}} \\ T_{3o} &= \mu \frac{1}{2} \int_0^{2\pi} d\theta \int_{\epsilon_0}^D d\ell \frac{\left(\frac{H_\xi - H_\eta}{\ell} \right)^3 H'_\eta}{\left[1 + \left(\frac{H_\xi - H_\eta}{\ell} \right)^2 \right]^{\frac{3}{2}}} \simeq \\ &\simeq \mu \frac{1}{2} \int_0^{2\pi} d\theta \int_{\epsilon_0}^D d\ell \cdot \ell \frac{(H_\xi - H_\eta)^3}{\ell^4} H'_\eta = \\ &= \mu \frac{1}{2} \int d_2 \eta (H_\xi - H_\eta)^3 \underline{K}_{\epsilon_0, D}^3 \cdot \nabla H_\eta \end{aligned}$$

where the vector Kernel $\underline{K}_{\epsilon_0, D}^3$ is given by

$$\underline{K}_{\epsilon_0, D}^3(\underline{\xi}) = \begin{cases} -\frac{\underline{\xi}}{|\underline{\xi}|^4} & \epsilon_0 \leq |\underline{\xi}| \leq D \\ 0 & \text{otherwise} \end{cases}$$

Concluding this paragraph two remarks are in order.

First of all one could think of increasing the accuracy of the approximations for T_{2o} and T_{3o} , although this seems not to be necessary if ϵ_0 is judiciously chosen. Second, one could object that our terms T_{2i}, T_{3i}, T_{3o} depend in fact from ∇H which might not be a well defined quantity; however one should remember that we are working with a smoothed D.T.M. for which the gradient is certainly defined. The numerical experiments confirm that this seems to be not a critical point.

4 Numerical experiments

Basically we have run two numerical experiments to perform a first check of our formulas; the gravity field has been generated from synthetic terrain models with the Gravsoft software ([4]) and the T.C. reconstructed from our formulas has been computed and compared with the original field. To compute our correction formulas the $|\nabla H|$ has been estimated most simply through the discrete recipe

$$|\nabla H|_{ik}^2 = \frac{1}{4\Delta^2} \left[(H_{i+1,k} - H_{i-1,k})^2 + (H_{i,k+1} - H_{i,k-1})^2 \right] , \quad (7)$$

where Δ is the grid pass.

The situation of experiment 1 is illustrated in Fig. 4.1, Fig. 4.2, Fig. 4.3, while the experiment 2 is illustrated in Fig. 4.4, Fig. 4.5, Fig. 4.6.

In each figure we find the definition of the field represented, the maximum modulus of the values and other specific informations.

In particular in Fig. 4.1 and Fig. 4.4 "dim" is the dimension along the two axes, $(\Delta x, \Delta y)$ is the pixel dimension, radius is ϵ_0 ; in Fig. 4.2(b) and in Fig. 4.5(b) the full terrain correction according to our formulas is represented (H^2 here means what we called B_2 in (2)); in Fig. 4.3 and Fig. 4.6 the differences are represented and on the line "mean diff" the mean differences is given together with their r.m.s. Willing to summarize the results we can look at the following table (4.1) where the power of each signal is evaluated as

$$Pow = E \{X\}^2 + \sigma^2(X) ;$$

(values in mGals)	<i>Exp1</i>	<i>Exp2</i>
Power simulation	0.37	1.73
Max value simulation	3.78	7.96
Power differences	0.14	0.33
Max value differences	1.46	2.61

Table 4.1

As we can see the results for the moment cannot be defined neither complete nor conclusive; however the figures of Table 4.1 seem to be encouraging, although for the moment, particularly the choice of ϵ_0 has not been optimized in any way.

The quadratic term B_2 seems as a matter of fact to be mostly disturbing, although in principle it should not.

5 Perspectives

Concluding this first report on the new IGeS approach to T.C. computations we can mostly underline the points object of future work:

- the choice of ϵ_0 has to be optimized;
- the F.F.T. algorithm has to be applied to the computations of T_{20} , T_{30} ;
- the gradient estimation has to be more closely analyzed;
- wider numerical tests have to be conducted in a variety of cases;
- formulas for potential corrections have to be studied.

Despite these perspectives of a large work to be performed we think that the actual idea seems to be effective and could, in future, become the basis of a new IGeS software.

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Figure Captions

Fig. 4.1 - Furst simulated test DTM.

Fig. 4.2 - a) The T.C. from Gravsoft corresponding to Fig. 4.1.

- **b)** The T.C. computed with new formulas corresponding the Fig. 4.1.

Fig. 4.3 - a) Difference between a) and b) in Fig. 4.2.

- **b)** Comparison of Gravsoft with 2^{nd} order term only.

- **c)** The same as a) with relation error.

- **d)** The same as b) with relative error.

Fig. 4.4 - Real test DTM

Fig. 4.5 - a) The T.C. from Gravsoft corresponding to Fig. 4.4.

- **b)** The T.C. computed with new formulas corresponding the Fig. 4.4.

Fig. 4.6 - a) Difference between a) and b) i Fig. 4.5.

- **b)** Comparison of Gravsoft with 2^{nd} order term only.

- **c)** The same as a) with relation error.

- **d)** The same as b) with relative error.

TEST DTM

max height: 500m

dim: 4800m x 4800m

delta x: 27m, delta y: 27m

“radius”: 250 m

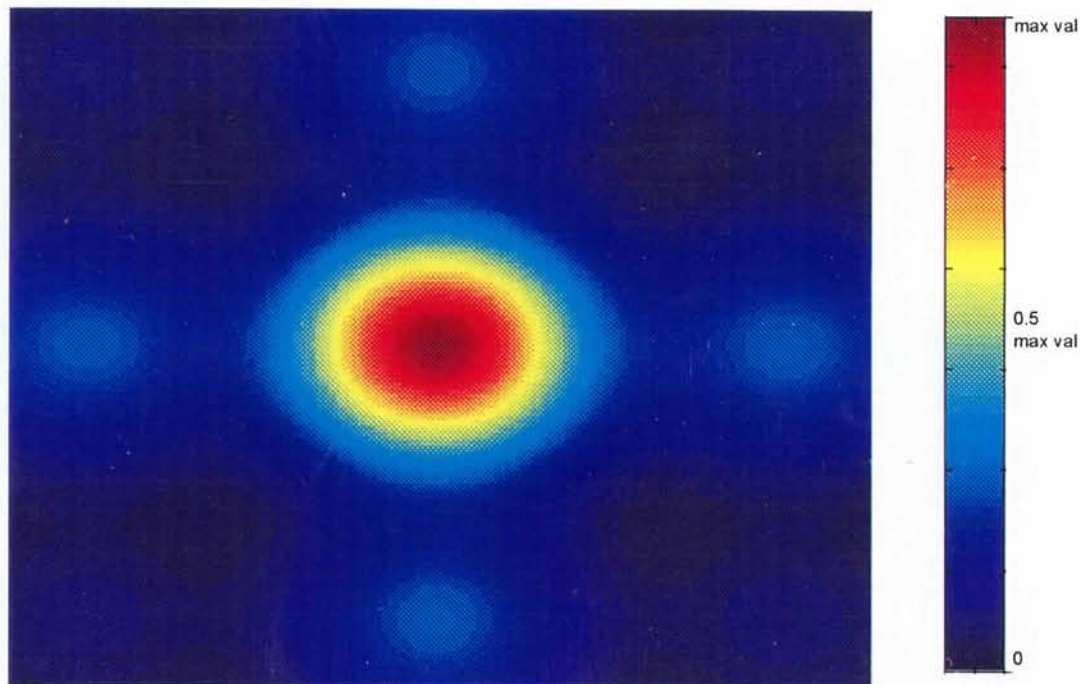
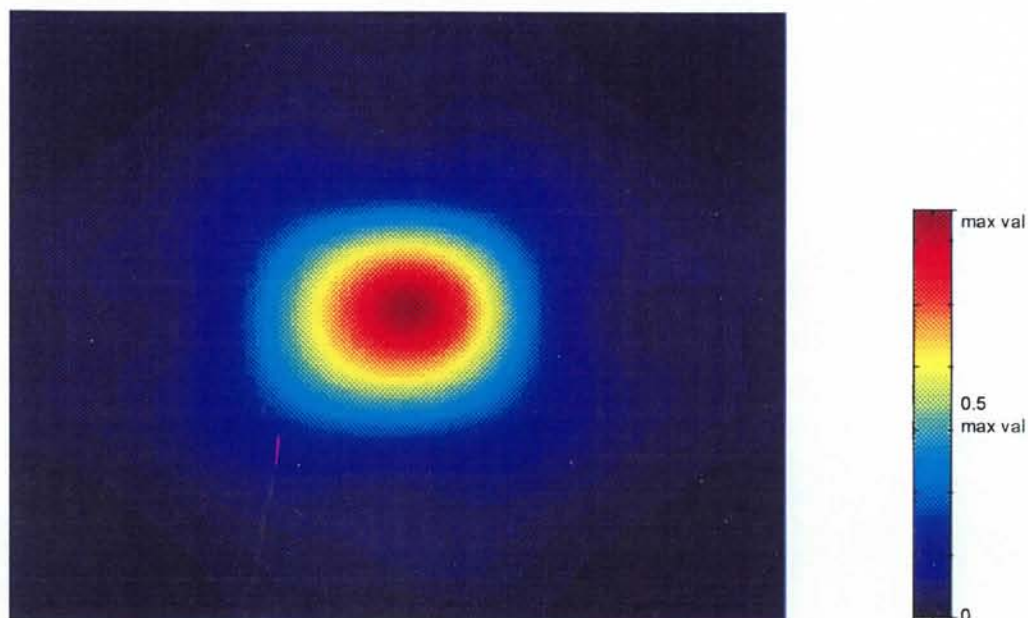


Fig. 4.1

TC from Gravsoft:
max val 7.96 mgal, mean val 0.98+/- 1.43 mgal

a)



TC from formulas:
TC = "2nd ord" int + "2nd ord" ext - "3rd ord" int + "3rd ord" ext + "H²"
max val 10.30 mgal, mean val 1.10+/- 1.70 mgal

b)

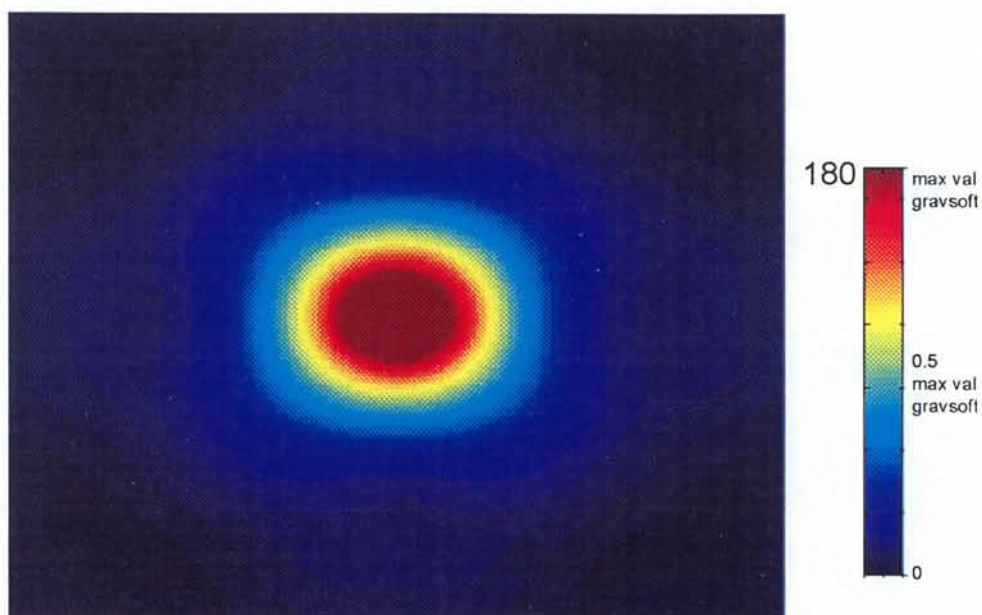
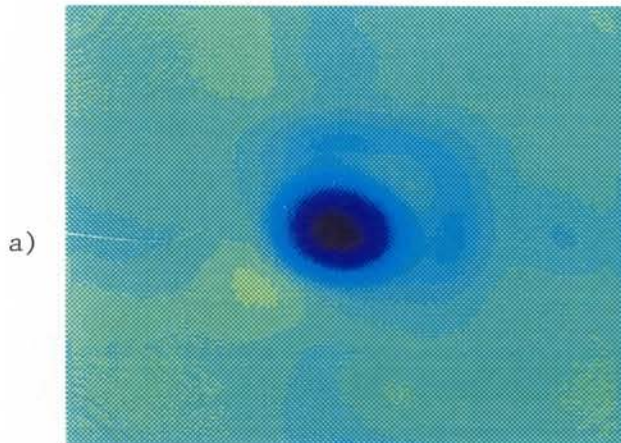
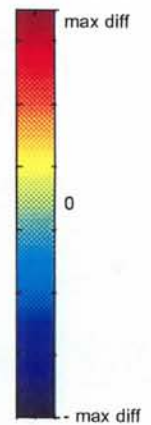
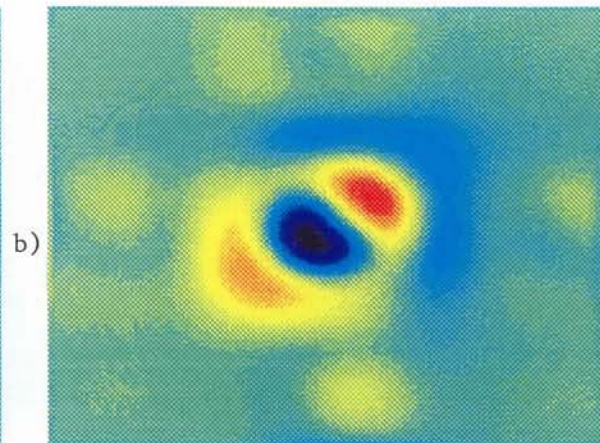


Fig. 4.2

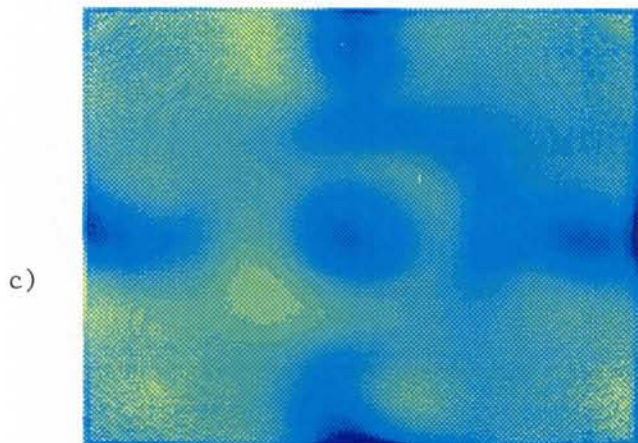
$\text{diff} = \text{TC}_{\text{gravsoft}} - \text{TC}_{\text{formulas}}$
 $\text{max abs (diff)} = 2.61 \text{ mgal}$
 $\text{mean (diff)} = -0.12 \pm 0.31 \text{ mgal}$
 $\text{plot: diff} / 2.61$



$\text{diff} = \text{TC}_{\text{gravsoft}} - \text{TC}_{\text{"2nd ord"}}$
 $\text{max abs (diff)} = 0.64 \text{ mgal}$
 $\text{mean (diff)} = 0.01 \pm 0.09 \text{ mgal}$
 $\text{plot: diff} / 0.64$



$\text{diff} = (\text{TC}_{\text{gravsoft}} - \text{TC}_{\text{formulas}}) / \text{TC}_{\text{gravsoft}}$
 $\text{mean (diff)} = -9 \pm 11 \%$



$\text{diff} = (\text{TC}_{\text{gravsoft}} - \text{TC}_{\text{"2nd ord"}}) / \text{TC}_{\text{gravsoft}}$
 $\text{mean (diff)} = 1 \pm 9 \%$

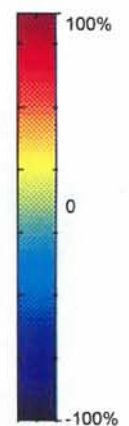
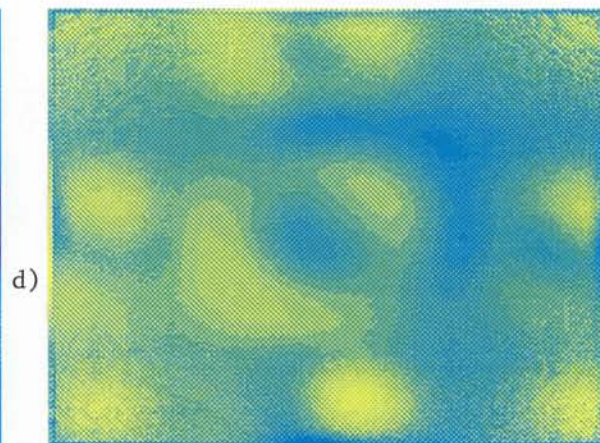


Fig. 4.3

TEST DTM

max height: 218m

dim: 4800m x 4800m

delta x: 80m, delta y: 80m

“radius”: 60 m

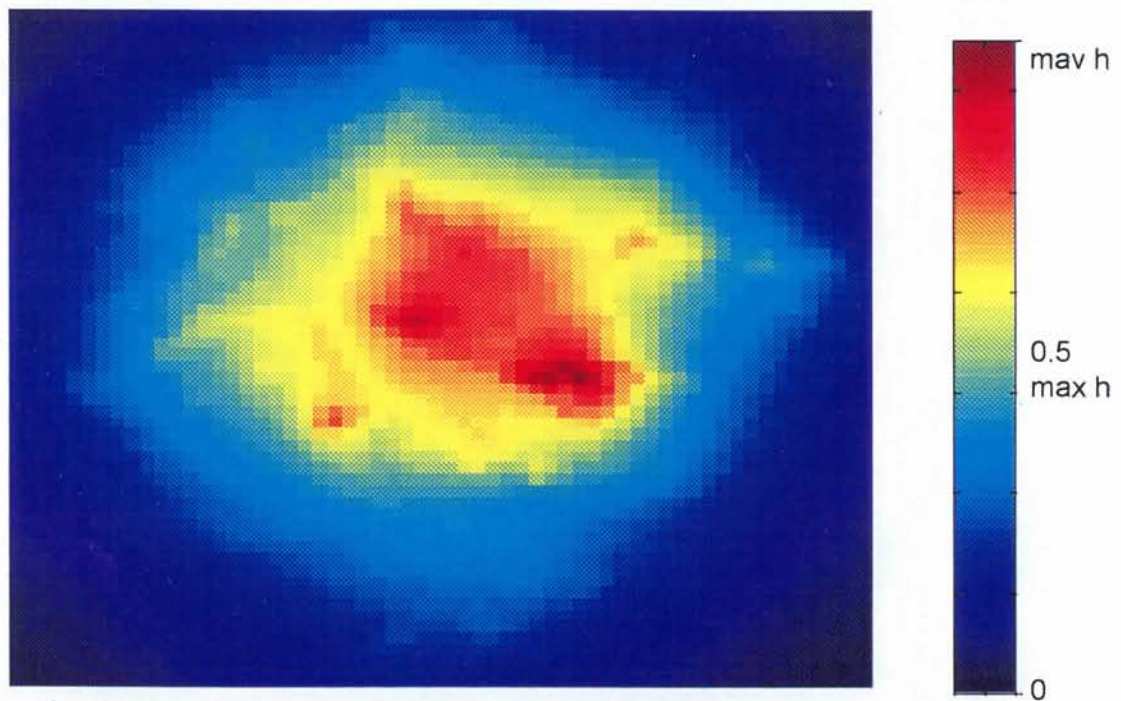
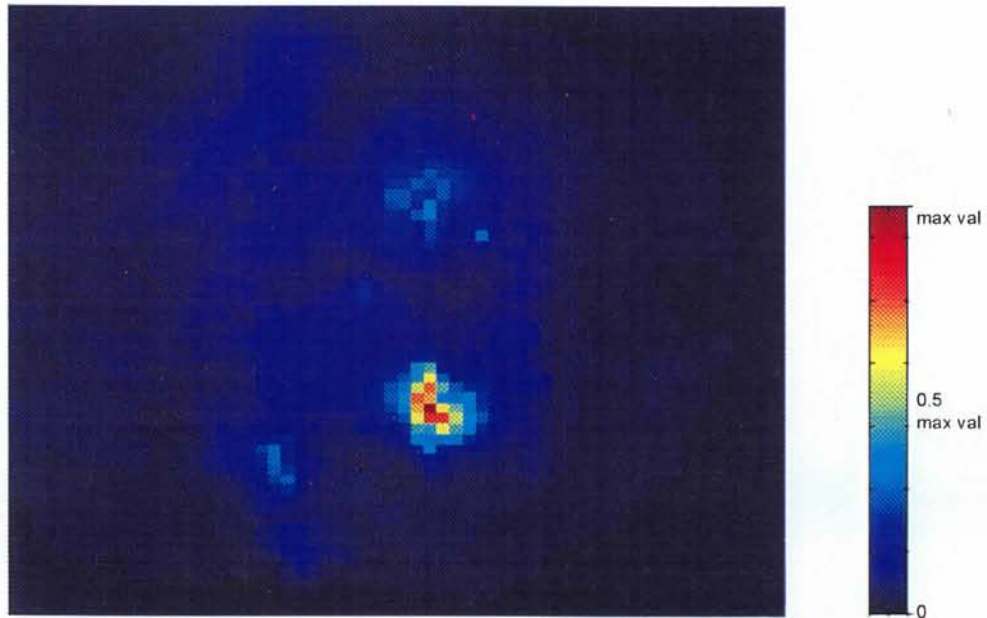


Fig. 4.4

TC from Gravsoft

max val: 3.78 mgal, mean val: 0.27+/-0.26 mgal

a)

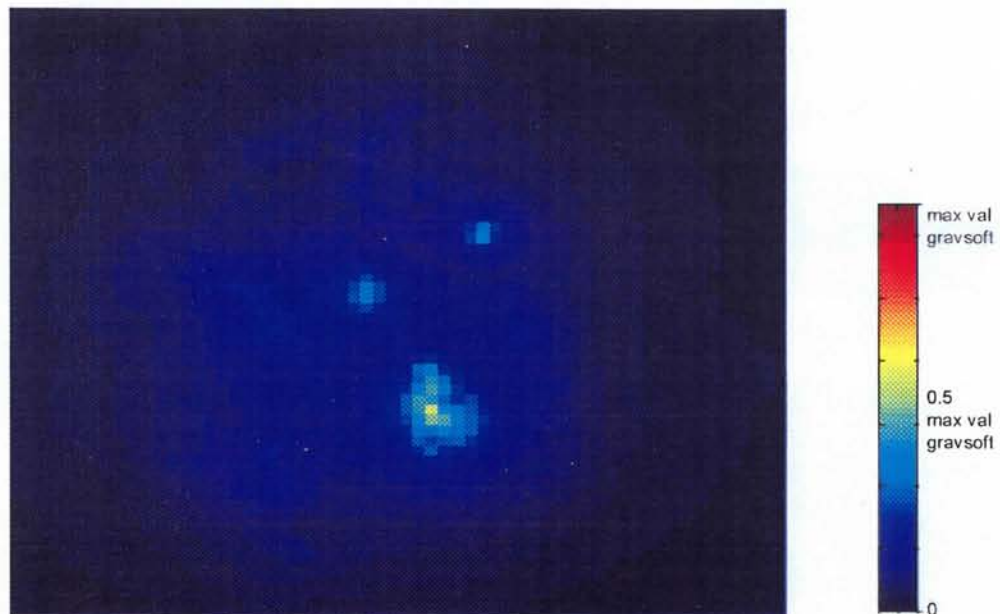


TC from formulas

TC = "2nd ord" int + "2nd ord" ext - "3rd ord" int + "3rd ord" ext + "H²"

max val: 2.32 mgal, mean val: 0.27+/-0.21 mgal

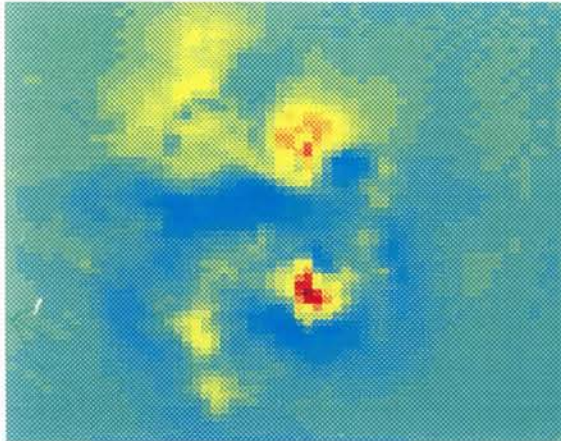
b)



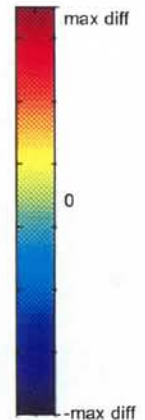
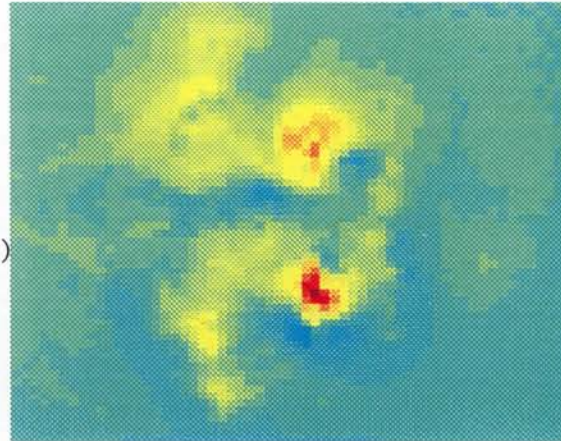
$\text{diff} = \text{TC}_{\text{gravsoft}} - \text{TC}_{\text{formulas}}$
 $\text{max abs (diff)} = 1.46 \text{ mgal}$
 $\text{mean (diff)} = 0. \pm 0.14 \text{ mgal}$
 $\text{plot: diff} / 1.46$

$\text{diff} = \text{TC}_{\text{gravsoft}} - \text{TC}_{\text{"2nd ord"}}$
 $\text{max abs (diff)} = 1.89 \text{ mgal}$
 $\text{mean (diff)} = 0.06 \pm 0.16 \text{ mgal}$
 $\text{plot: diff} / 1.89$

a)



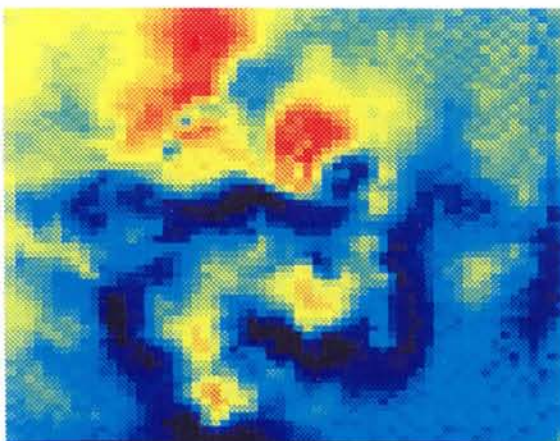
b)



$\text{diff} = (\text{TC}_{\text{gravsoft}} - \text{TC}_{\text{formulas}}) / \text{TC}_{\text{gravsoft}}$
 $\text{mean (diff)} = -14 \pm 39 \%$

$\text{diff} = (\text{TC}_{\text{gravsoft}} - \text{TC}_{\text{"2nd ord"}}) / \text{TC}_{\text{gravsoft}}$
 $\text{mean (diff)} = 6 \pm 36 \%$

c)



d)

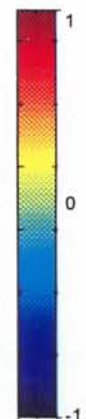
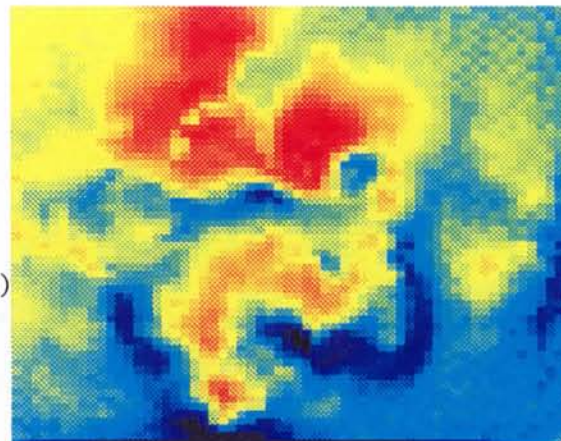


Fig. 4.6

A Database Approach for Geodetic Data Management

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1. INTRODUCTION

A database approach to manage geodetic data means that using a database management system (DBMS in short) used to structure and interact with geodetic data (bathymetry, gravimetry, GPS, altimetry, radar, geoid estimation) can improve the set of operation that you can do on these data and the performance of these operations.

What geodetic community do with these data set? A geodetic user, for example, selects records belonging to a file or to multiple files of the same kind (maybe with different format) with a spatial search predicate. This means that the user selects the records identifying geodetic observations located in squared, circular or polygonal zones. Also, the user can retrieve all records on a particular grid with a size defined.

Data belonging to geodetic archives are characterised by a spatial component expressed as latitude, longitude, elevation. In general, geodetic data has a fixed format because every geodetic file is made up by records with fixed format. The format means that the spatial component is part of the information that composed the whole data. For example, the Bureau Gravimetrique International offers to users two distinct data format: the Compressed Gravimetric Data File (CGDF) and the Archive Files.

Also, the search predicate can be not geographical but can regard qualitative attribute of the descriptive geodetic record. For example a user can select all records belonging to a BGI CGDF file with elevation type equal to land and free air anomaly > 52.8 microrgal.

2. THE FILE APPROACH: PROBLEMS

Now geodetic archives are available via internet, but very often archive means a well organised hard disk with a fixed tree-directory structure; these archives are organised under the temporal point of view, because the minimum downloading unit is a file connected with measurement made in a particular day or week; also you can find files connected with the location of the measuring instrument (Fig. 1).

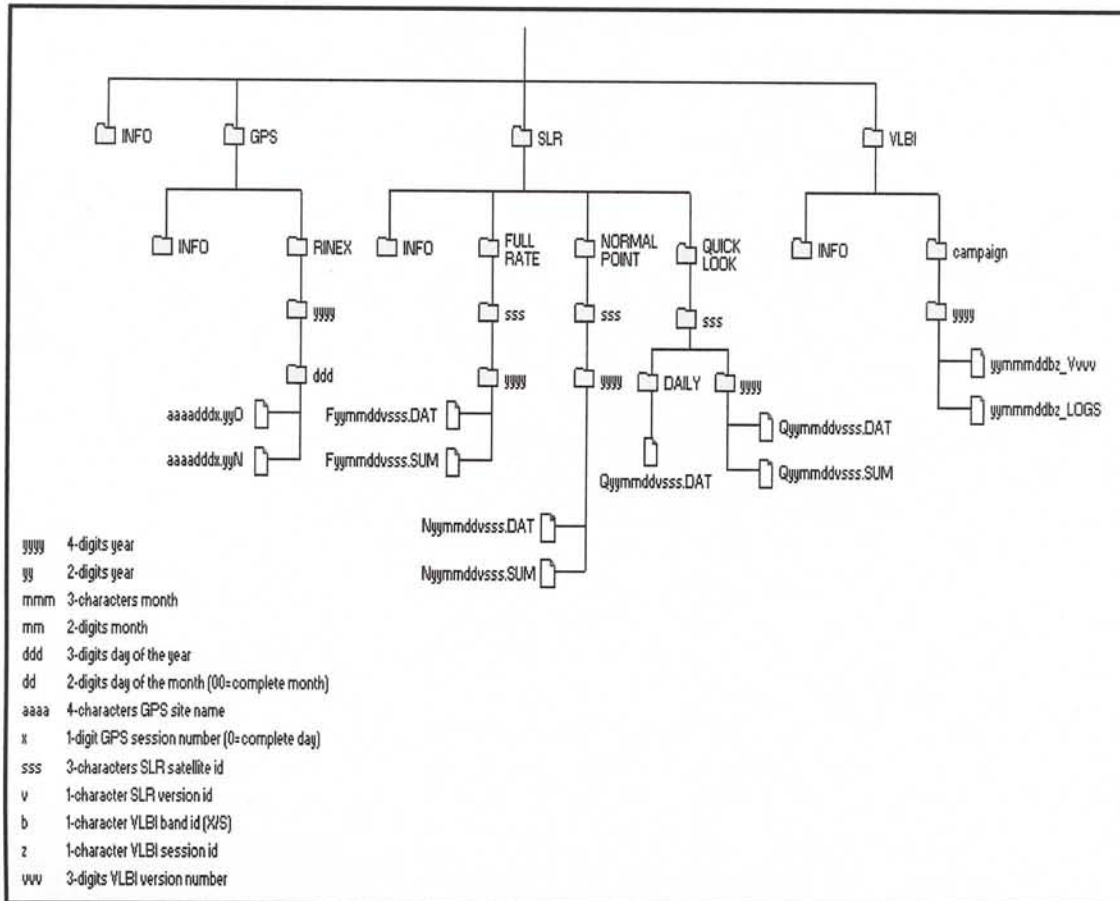


Fig. 1 - Example of a well organised tree-directory structure coming from the GEODAF www server.

When the user downloads a single file from an archive, he has to implement a procedure (sometimes using the Fortran language) to extract from this file, the necessary records.

The searching predicates are expressed in the procedure itself; if the user want more searching predicates he has to built more procedures (Fig. 2).

Of course, the procedures are strictly connected with format of the original file. If you want to work with more files coming from more archives, you note that formats aren't the same; so you must have a procedure that in some way merges the n different formats. But this procedure is strictly connected with the n involved formats, for another set of n formats you have to implement another procedure for merging data.

C ----- C --- Read the ASCII Data C --- C --- GHER : R.Schoenauen (4/1/94) C ----- REAL*4 A(500000),VALEX,XMIN,XMAX INTEGER IMAX,JMAX,KMAX,I,J,K CHARACTER*50 NAME WRITE (6,'(A)') 'Read the ASCII Data' WRITE (6,'(A)') '-----' WRITE (6,'(A)') WRITE (6,'(A)') 'File to read ?' READ (5,'(A)') NAME OPEN (UNIT=10,FILE=NAME, FORM='FORMATTED') READ (10,102) IMAX,JMAX,KMAX READ (10,100) VALEX,XMIN,XMAX CALL READIT (A,IMAX,JMAX,KMAX) CLOSE (10) 100 FORMAT (3E12.5) 102 FORMAT (3I5) END C ----- SUBROUTINE READIT (A,IMAX,JMAX,KMAX) C ----- REAL*4 A(IMAX,JMAX,KMAX) INTEGER IMAX,JMAX,KMAX DO 10 K = KMAX,1,-1 DO 20 J = JMAX,1,-1 READ (10,101) (A(I,J,K),I=1,IMAX) 20 CONTINUE 10 CONTINUE 101 FORMAT (5E12.5) END	C ----- C --- Read the ASCII Data C --- C --- This program is OK for data arrays C --- with fixed dimensions (167,63,34) C --- C --- GHER : R.Schoenauen (4/1/94) C ----- REAL*4 A(167,63,34) REAL*4 VALEX,XMIN,XMAX INTEGER IMAX,JMAX,KMAX,I,J,K CHARACTER*50 NAME WRITE (6,'(A)') 'Read the ASCII Data' WRITE (6,'(A)') '-----' WRITE (6,'(A)') WRITE (6,'(A)') 'File to read ?' READ (5,'(A)') NAME OPEN (UNIT=10,FILE=NAME, FORM='FORMATTED') READ (10,102) IMAX,JMAX,KMAX READ (10,100) VALEX,XMIN,XMAX DO 10 K = KMAX,1,-1 DO 20 J = JMAX,1,-1 READ (10,101) (A(I,J,K),I=1,IMAX) 20 CONTINUE 10 CONTINUE CLOSE (10) 100 FORMAT (3E12.5) 101 FORMAT (5E12.5) 102 FORMAT (3I5) END
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Fig. 2 - Fortran language sample codes to read ASCII data from the Mediterranean Oceanic Data Base archives.

3. THE DATABASE APPROACH: BENEFITS

A database approach can organise a geodetic data management system in a different way. We discuss the basic differences between the file and the database solution.

3.1 Data redundancy

In the file approach, each user defines, implements and maintains the files needed for its specific applications.

The redundancy in storing the same data multiple time leads to several problems:

- there is the need to perform a single logical update multiple times;
- Storage space is wasted on storing the same data repeatedly.

The data redundancy in defining and storing data means difficult to maintain data consistency.

In the database approach, only one repository of data is maintained that is defined once and accessed by multiple users. So, if you have to permit multiple access to the same data stored once time, the database solution grants the data consistency.

3.2 Data definition.

The data structure is a part of the application programs themselves; these programs are constrained to work with only one specific data structure declared in the application programs and the declared structure represents the format of the file coming from a particular archive connected with the program.

The database system contains not only the database itself but also a complete definition of the database; data structure is defined in a system catalogue which the system refers to obtain information about the structure of the particular database.

A database system works equally with any number of database applications.

3.3 Data/Application relationship.

The structure of data is embedded in the application programs, so if you change the data structure you must change all programs that use these data.

A database is program-independent because programs that access data are written independently of any specific data structure; this structure is stored, in the database environment, separately from the access programs.

3.4 Data model.

Programs are needed to access data; in these programs the structure of the data itself must defined, translating the record format and declaring some variables.

Database systems provide a data model (there are many data models available, the relational is one of the most diffuse) used to obtain a

conceptual representation of the data involved; this representation doesn't include many of the details of how data is stored.

In a relational environment when you have to deal with different files of the same subject coming from different archives with different formats you can define one time the structure of relation connected with a format, and this is possible by data model.

3.5 Multiple views.

Using the file approach, each program provides a different view of the managed data. If multiple views of the same data are needed, multiple access programs have to be implemented. Different view for different users means that every user could extract different information from the same available data. A view can be defined as a set of data stored on the db, or data obtained with processing on these data. In a file approach multiple view means multiple procedures that produce particular output or processing. Each database user can require a different perspective (view) of the database. This must provide facilities for defining multiple views.

4. DATA TREATMENT WITH A DBMS.

A typical project for geodetic data treatment using a DBMS follows three main levels:

1) IMPORTING: The user has to define a structure with a data model supported by the used DBMS for every file data format used.

This is a definition that you have only one time. The importing routines are available for data format and very often you can implement there routine with a GUI (Graphical User Interface).

2) MERGING: The merging process of n different files doesn't require a particular procedure because the user imports one file in the data base structure defined for this file format.

3) EXTRACTING: To extract information from these files the user has to implement and execute a query, using a high level query language available in the used DBMS. A query definition only request *what* retrieves *from where* and *with* a searching predicate, but the user has not to specify *how* to implement the query itself.

5. CONCLUSIONS.

A general improvement of the data base systems for geodetic data management is clearly shown by the previous notes reported through the paper, in terms of data redundancy for multiple data views on multiple files coming from different data storage systems (software speaking).

Furthermore, if a data base system offers web facility, a user can query the data base via internet, downloading only the useful data. In a such way the user can avoid to retrieve the entire large file for a further local data extraction.