On the Potential of Wavelets for Filtering and Thresholding Airborne Gravity Data

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Abstract. Wavelets can be used in the decomposition and analysis of airborne gravity data. In this paper, multiresolution analysis is applied to de-noise gravity disturbance and different de-noising techniques are studied. The first objective is testing the usefulness of wavelets for analyzing and filtering airborne gravity data. The second one is a comparison between the usage of the wavelet transform and other well known low-pass filters. The gravity disturbances are filtered using wavelet thresholding to remove the noise introduced by the dynamics of the aircraft. Two procedures have been tested. The first one is de-noising using minimax and universal techniques. The second one is a combination of wavelet thresholding and filtering at certain levels. Comparison to independent reference data is performed in the area of interest to determine the external accuracy of this approach. Results from both cases and from the low-pass filter approach are compared. The results of testing different de-noising techniques show that the combination of thresholding and filtering can reach RMS values equal to 25 mGal in comparison to the results from the 90s low-pass filter.

Keywords. Wavelet multiresolution analysis; airborne gravimetry; filtering; universal, hard and soft thresholding.

1 Introduction

Wavelet analysis is a comparatively young branch in signal processing. It is developed to overcome some of the problems of Fourier analysis. Wavelet expansions allow better local description and decomposition of signal characteristics. Filtering airborne gravimetry data has been introduced in a number of publications. Glennie and Schwarz (1997), Schwarz and Glennie (1998), Glennie and Schwarz (1999), Wei and Schwarz (1998), and Glennie (1999) low-pass filtered gravity disturbances using an FIR filter to specified cut-off filter lengths, which range from 30s to 120s.

Bruton et al. (1999) used wavelets for the analysis and de-noising of kinematic geodetic measurements. Wavelets have been used as a de-noising tool for removing stochastic noise from different geodetic signals. They used SURE (Stein Unbiased Estimate of Risk) soft thresholding and tested the Daubechies family in the decomposition of the signal. Abdel-Hamid et al. (2004) improved the performance of MEMS-based sensors using multiresolution analysis. They concentrated on enhancing the performance of Kalman-filtering INS/GPS integration techniques.

Our objective in this paper is to check the effectiveness of using wavelets in de-noising gravity disturbance. Different thresholding techniques will be tested. Minimax thresholding, fixed thresholding (universal), and thresholding combined with filtering at certain levels will be used. Hard and soft thresholding have been used in parallel with the different thresholding techniques.

In the next section, wavelets are introduced as a filtering tool. Then the system, flight and data characteristics are described. This is followed by the analysis of the data using wavelets and FFT. Following that the results of de-noising the gravity disturbance data using different wavelet thresholding techniques are compared with reference data and a comparison is made between the different thresholding and filtering techniques. The paper ends with conclusions, recommendations and future work with respect to the suitability, accuracy and efficiency of the methods used.

2 Wavelets as a Filtering Tool

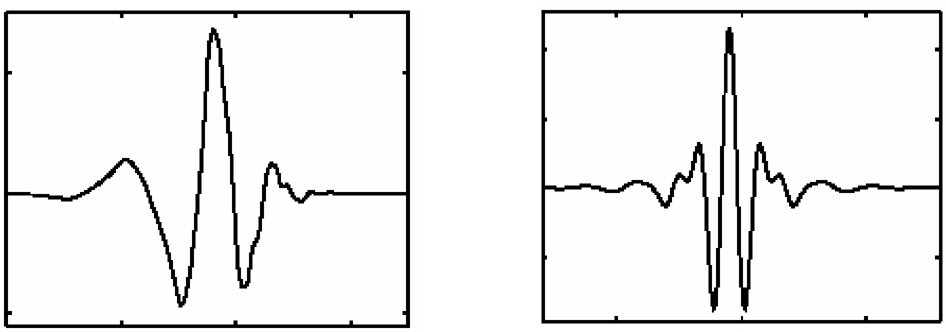
A wavelet base is a set of building blocks to construct or represent a signal or function; Burrus et al. (1998). The discrete wavelet transform (DWT) coefficients $\omega_{j,k}$ of a signal or a function $f(t)$ are calculated by the inner product

$$\omega_{j,k} = \langle f(t), \psi_{j,k}(t) \rangle \qquad (1)$$

where $\psi_{j,k}$ is the wavelet expansion function, and both j is the scale and k is the translation and both are integer indices. The inverse wavelet transform is used for the reconstruction of the signal from the wavelet coefficients $\omega_{j,k}$ by

$$f(t) = \sum_j \sum_k \omega_{j,k} \psi_{j,k}(t) \qquad (2)$$

Equations (1) and (2) are named analysis and synthesis, respectively. Wavelets used in this paper have energy concentrated in time, continuous null moments, and decrease quickly towards zero when the input tends to infinity. Meyer and Daubechies wavelets have been used in this research (Figure 1). Both the wavelet and scaling function for Meyer are defined in the frequency domain and have a closed form. Meyer is not compactly supported, nevertheless, it introduces a good approximation leading to FIR filters, and then allowing DWT, (Misiti et al., 2002). Daubechies (db) wavelets have no explicit expression except of db1 (Haar wavelet).



a b

Fig. 1 Daubechies 4 wavelet (a) and Meyer wavelet (b) (Misiti et al., 2002)

2.1 Wavelet Thresholding

Wavelet thresholding is a technique used to remove random noise and outliers from the signal before reconstruction. Wavelet absolute coefficients larger than a certain specified threshold δ are the ones that should be included in reconstruction. The reconstructed function can be show as, (Ogden, 1997):

$$\hat{f}(t) = \sum_j \sum_k I_{\{|\omega_{j,k}|>\delta\}} \omega_{j,k} \psi_{j,k}(t) \qquad (3)$$

where $I_{\{|\omega_{j,k}|>\delta\}}$ is the indicator function of this set. This represents a keep or kill wavelet reconstruction. This thresholding is a kind of nonlinear operator on the wavelet coefficients vector. This leads to a resultant vector of estimated coefficients $\hat{\omega}_{j,k}$ to be used in the reconstruction process. The problem is always about finding the proper thresholding value. In order to make this decision, two main parameters have to be taken into account. These parameters are the sample size ***n*** and the noise level σ^2. The estimated coefficients can be determined from either hard thresholding or soft thresholding. In case of hard thresholding the estimated coefficients will be:

$$\hat{\omega}_{j,k} = \begin{cases} \omega_{j,k} & if \left|\omega_{j,k}\right| \geq \delta \\ 0 & otherwise \end{cases} \qquad (4)$$

In case of soft thresholding, the estimated coefficients will be:

$$\hat{\omega}_{j,k} = \begin{cases} \omega_{j,k} - \delta & if \left|\omega_{j,k}\right| \geq \delta \\ 0 & if \left|\omega_{j,k}\right| \leq \delta \\ \omega_{j,k} + \delta & if \ \omega_{j,k} < -\delta \end{cases} \qquad (5)$$

This can also be illustrated in the following figure.

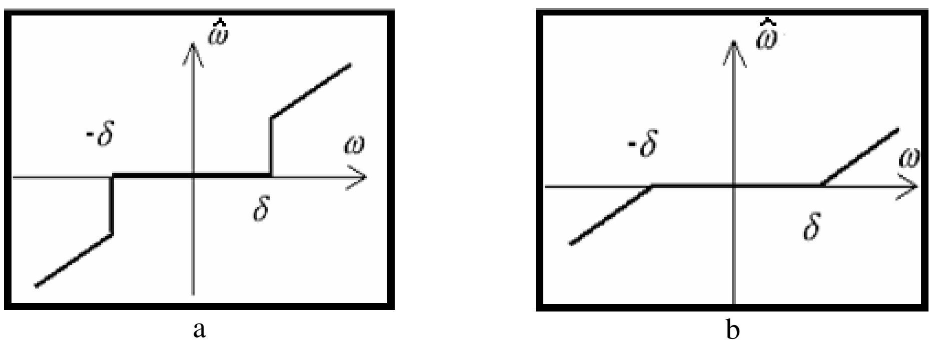


Fig. 2 (a) Hard and (b) soft thresholding

Choosing the value of the threshold is a very fundamental problem in order to avoid oversmoothing or undersmoothing.

Minimax thresholding has been applied in this research. This technique depends on the sample size ***n*** and it minimizes a bound for the risk involved in estimating a function. Minimax threshold has no closed form, but it can be approximated numerically. Another thresholding technique has been used in this research; it is called universal thresholding (fixed thresholding). This is an alternative to the minimax technique. The thresholding value is chosen equal to $\sqrt{2 \log n}$. This thresholding value is larger than the corresponding value in the minimax estimate, for the same sample size; for more details, see Donoho and Johnstone (1994).

2.2 Wavelet Level-dependent Thresholding and Filtering

Filtering is adjusting all the coefficients of certain level or levels to zero:

$$\hat{\omega}_{j,k} = \begin{cases} \omega_{j,k} & if \ j \leq j_{max} \\ 0 & if \ j \geq j_{max} \end{cases} \qquad (6)$$

In this research, filtering has been combined with thresholding for de-noising gravity disturbances. First, either minimax or universal thresholding is applied to the signal. This is followed by applying filtering to a number of the high wavelet decomposition levels. The choice of the wavelet decomposition level to be filtered depends on the minimum wavelength of signal that can be reconstructed from each level. Sometimes both filtering and level-dependent thresholding are used, which means that a possibly different threshold value is chosen for each wavelet level *j*, (Barthelmes et al., 1994).

3 System, Flight and Data Description

The data used originated from a project collected by the University of Calgary on 9, 10, and 11 of September, 1996. Only the data of the second day has been used in this research. The hardware used consists of three main components: a strapdown INS, GPS master and remote stations, and data acquisition. The INS system is a Honeywell LASEREFIII (LRFIII) navigation grade inertial system. Various types of GPS receivers have been used. The major requirements were low noise and high reliability. The data was collected over the Rocky Mountains; the area covered was 100 km * 100 km. This area was covered by 14 lines in day 2, as shown in Figure 3.

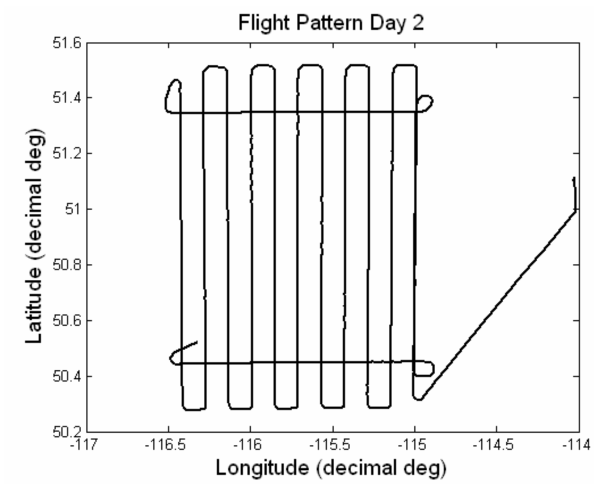


Fig. 3 Flight pattern for day 2 (September 10th) of the Kananaskis Campaign

This area was chosen because of the high variability of the gravity field and the availability of dense ground gravity values to be used as reference. The reference data were upward continued gravity disturbances with RMS equal to 1.5 mGal. The heights of the terrain in the area vary from 800 m to 3600 m. Average ellipsoidal flight height was 4357 m. The flight speed was 360 km/hr. The data used in the de-noising procedure has been resampled to 1 Hz (01 h bandlimit).

4 Analysis and De-noising of Results

4.1 Wavelet versus FFT analysis

The data to be analyzed is a gravity disturbance with 1h bandlimit, introduced as sub output from the KINematic Geodetic Software for Position and Attitude Determination (KINGSPAD) software, figure 4.

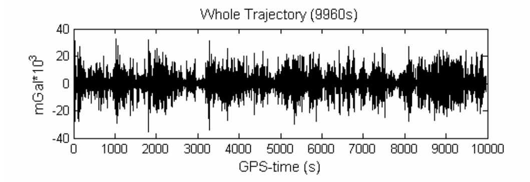
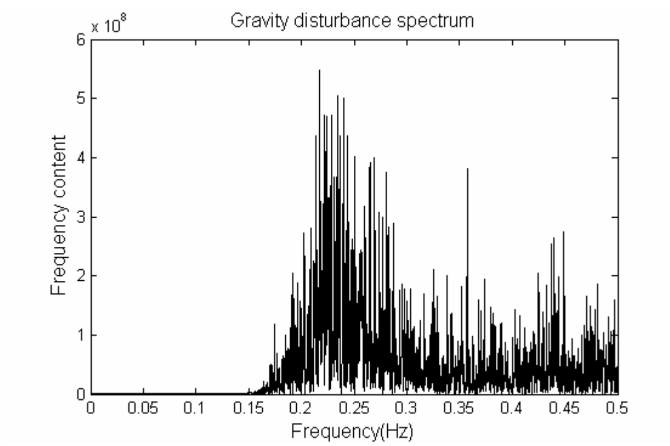
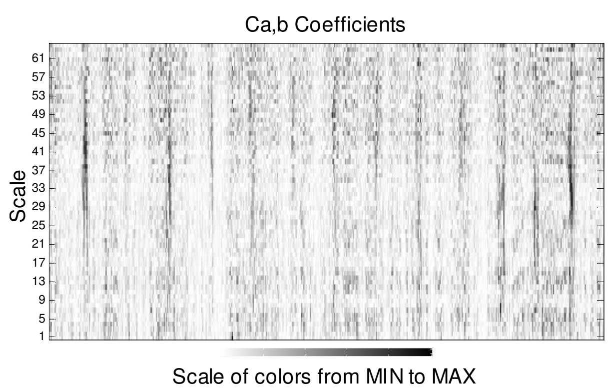


Fig. 4 Gravity disturbance with 1 Hz sampling rate

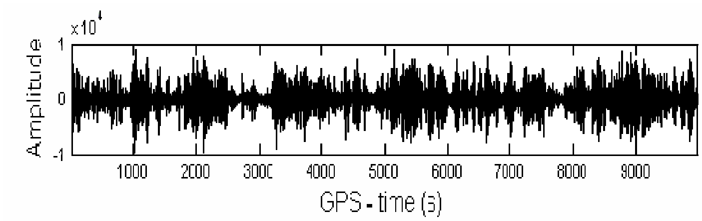
The gravity disturbance analysis (Figure 5a) starts with the demonstration of (1h bandlimit) signal using Fourier analysis. The FFT was used to visualize the different frequency contents of the signal. This spectrum shows that there are different signals at different frequencies, but the problem is that it is difficult to localize the errors, and separate them from the signal (figure 5a). The deficiency of time localization by FFT analysis leads to the use of wavelet transform analysis.



(a)



(b)



(c)

Fig. 5 (a) FFT spectrum shows a number of undesired frequencies. **(b)** Time-frequency analysis and localization using continuous wavelet transform. **(c)** Coefficients Line - $\omega_{j,k}$ for scale j = 32 (frequency = 0.031).

The usage of the continuous wavelet transform allows time-frequency localization; this can be seen in Figure 5b. Continuous here means that it is not decimated to dyadic format (decreasing the number of coefficients into half at each approximation or detailed level) as in the traditional discrete cases. However, all the wavelet decomposition coefficients are taken into account. Darker shades show high frequencies, different from the expected gravity disturbances frequencies (depending on a priori information). These high frequencies are interpreted as errors at certain time and scale. In wavelet analysis, different types of errors can be tracked through the whole trajectory and interpreted corresponding to different aircraft dynamics. Also, by the decomposition of the signal into several levels, stochastic errors and outliers can be easily detected and removed using different thresholding techniques. For example, the takeoff and different maneuver periods between lines can be easily identified. Figure 5c is a sample of the coefficients at scale 32 showing clearly the maneuvers between lines.

4.2 Wavelet versus low-pass filter

Wavelet transforms have been also used in de-noising and smoothing of gravity disturbances. The wavelet techniques are compared to the output of a low-pass filter with four cut-off lengths, which are 30s, 60s, 90s, and 120s. The difference between output computed raw data and the reference data shows the presence of noise and outliers, as shown in figure 6. The four low-pass filters are an output from the KINGSPAD and AGFILT software, developed by the University of Calgary.

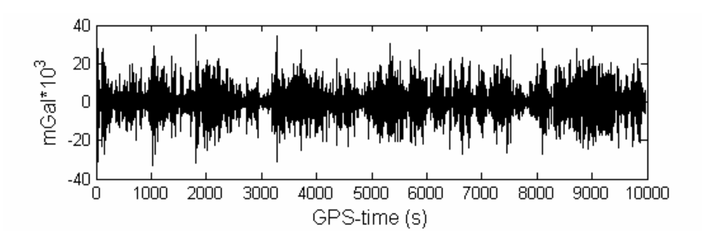
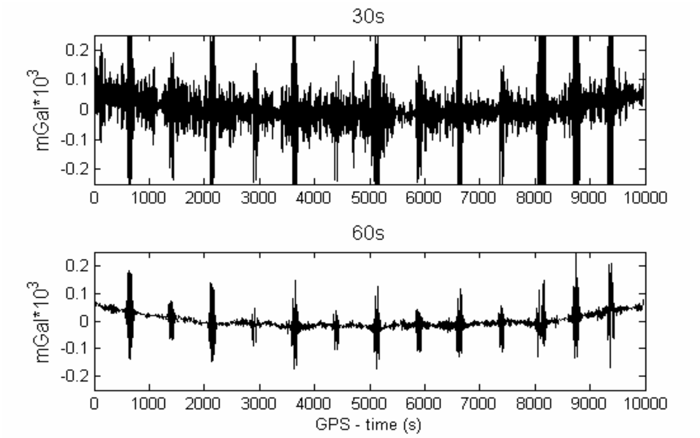
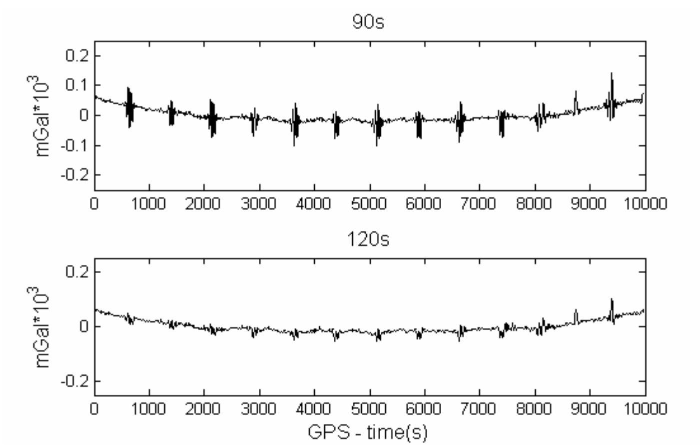


Fig. 6 Deference between reference and bandlimited 1h data

The difference between the output from the four low-pass filters and the reference data is shown in Figure 7. The RMS of the difference between the low-pass filtered data and the reference has been computed.



(a)

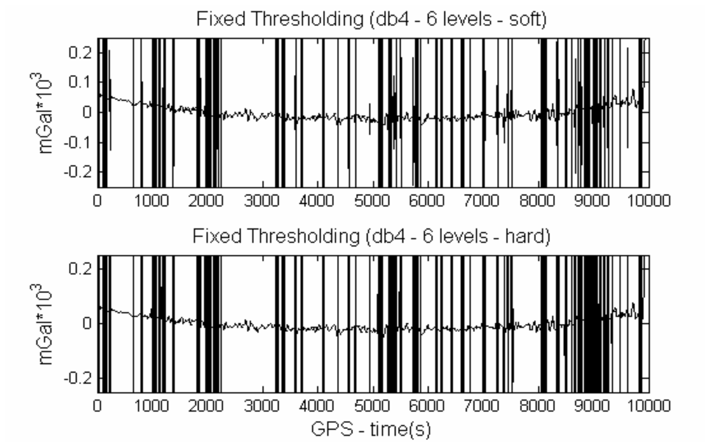


(b)

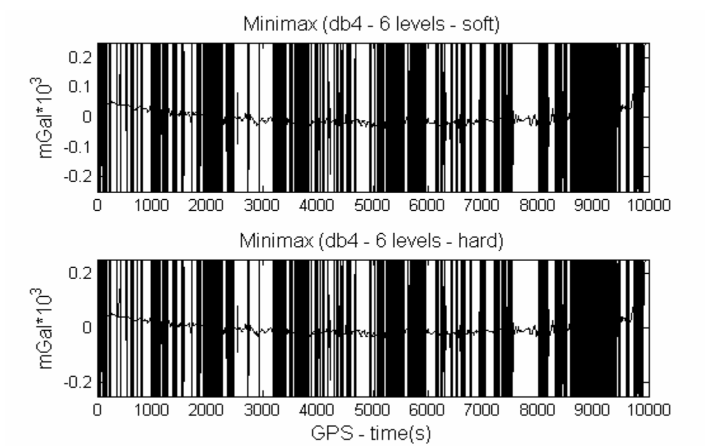
Fig. 7 (a) Difference between reference data and 30s, and 60s low pass filter output. **(b)** Difference between reference data and 90s, and 120s low pass filter output.

The same data was de-noised using different thresholding and filtering techniques, using the two wavelet families shown in Figure 1. Finally, they are compared with the results obtained from the four low-pass filters.

The universal and minimax thresholding has been applied to the gravity disturbance data. Both of them have been applied with soft and hard thresholding. The results are compared to the reference data. The RMS of the residuals ranges between 712 and 1700 mGal, which is not acceptable (Figure 8).



(a)

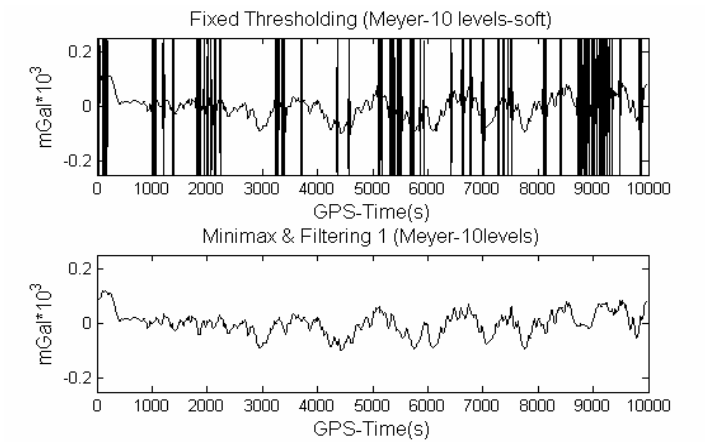


(b)

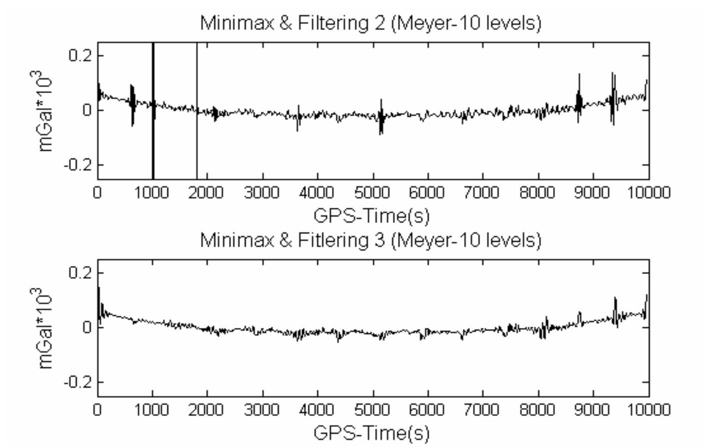
Fig. 8 **(a)** Difference between the gravity disturbances de-noised by universal (fixed) thresholding and the reference data **(b)** Difference between the gravity disturbances de-noised by minimax thresholding and the reference data

Two modifications have been made to reduce this difference. Another wavelet family has been tested and filtering has been applied to the data in sequence with the two thresholding techniques mentioned before. After different trials and tests, it has been found that the usage of minimax soft thresholding, by Meyer wavelets followed by filtering at certain high levels of the wavelet decomposition, is the optimum in our case study.

In the following figures it can be seen that universal thresholding has been tested with Meyer wavelets. The RMS was 729 mGal, which is still not acceptable. This is followed by three trials of applying the minimax soft thresholding, and applying filtering by removing a whole decomposition level at a certain scale. The value of the RMS decreased from 1846 mGal, because of inappropriate choice of the filtering level, at the first trial to 25 mGal at the third trial. This value is close to the best result introduced by low-pass filtering (120s). This can be illustrated in figure 9.



(a)



(b)

Fig. 9 (a) Meyer wavelets with fixed soft thresholding (up), and minimax and filtering (1st trial) down. **(b)** Meyer wavelets with minimax and filtering 2^{nd} (up), and 3^{rd} (down) trial

Table 1 RMS of the difference between de-noised gravity disturbances and reference data

Method	RMS (mGal)
Low pass filter (30s)	99.52
Low pass filter (60s)	37.45
Low pass filter (90s)	26.68
Low pass filter (120s)	23.71
Minimax & filtering 1	1846.80
Minimax & filtering 2	82.86
Minimax & filtering 3	25.52

The choice of the thresholding value is depending on the sampling rate and the signal that can be reconstructed at each level. It is automated through one of the two techniques mentioned above, either universal or minimax thresholding. Applying filtering to certain levels is very dangerous, needs a prior information and is depending on trial and error. The same procedure was applied to one of the 14 lines, which is line 6 between 4725 s and 5280 s. The first and last 10 points, which contain a lot of outliers because of the dynamics introduced from the turning maneuvers of the aircraft, have been removed. Similar results were obtained The only difference is that, in case of line 6, less usage of filtering was required because of the stability of the aircraft during this period. It is worth mentioning here that a trend has been removed from the difference of the de-noised and the referenced data, in both cases (low-pass filter, and different wavelet procedures). The RMS errors from different trials and approaches for the difference between the de-noised gravity disturbance and reference data are summarized in table 1.

5 Conclusions

Wavelets can be used in the decomposition and analysis of airborne gravimetry data. Minimax and fixed thresholding techniques as an automated procedure are not enough for de-noising airborne gravity data, because of the need of filtering high frequency levels. The combination of these automated thresholding techniques with filtering is more effective in de-noising and bandlimiting the gravity disturbance. Soft thresholding proved to be more effective in this study than hard thresholding. Different wavelets have different effect on the analysis and de-noising of the signal. This can be easily noticed from the improvement in the accuracy when using Meyer wavelets instead of Daubechies. Decomposition of the signal to higher levels and applying thresholding and filtering is effective especially in the case of outliers. Wavelet de-noising reached the same accuracy as the approximation introduced by the low-pass filters with 120s cut-off, and better than the 30s, 60s, and 90s low pass filters. Trial and error is required for determining the levels to be removed (filtering) in case of combining both filtering and thresholding. Further research is under investigation to automate the filtering and thresholding procedures by combining them with priory information about the range of the frequency required.

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