Outlier detection in CHAMP kinematic orbit data to be used in gravity field determination

C. Gruber

Institute of Navigation and Satellite Geodesy, Graz University of Technology,
gruber@geomatics.tu-graz.ac.at

March 1, 2005

Abstract

CHAMP orbit data have become available from dynamic, reduced dynamic and kinematic (purely geometric) approaches. All of them have been used in gravity field determination. However, the kinematic orbits enjoy higher acceptance since they do not employ any a-priori gravity field model. Unfortunately, kinematic orbits suffer from higher noise levels and outliers. Hence, improving kinematic orbits will directly influence the quality of the gravity field model. In this paper, a pre-processing strategy for kinematic orbit data will be presented. Based on orbit smoothing, outliers will be detected and excluded from further processing.

1 Introduction

The energy balance method is an established approach to determine the lower spherical harmonic coefficients (in the case of CHAMP up to degree/order ≈ 70) of the Earth gravity field (cf. *Badura et al.* 2004). Due to this approach the quality of the solution depends to a high degree on an accurate knowledge of the velocity of the spacecraft. If kinematic orbits shall be used, a strategy has to be developed to derive those velocities with sufficient accuracy. In the case of CHAMP, where a geoid solution at *dm* precision is envisaged, the velocity RMS must not be larger than $10^{-1} mm/s$ in order to meet this requirement. For the case of simulated orbits this can usually be achieved by standard numerical differentiation. Unfortunately, kinematic orbits suffer, compared to simulated or reduced dynamic orbits from a much higher noise level due to low confidence in the GPS configuration, i.e. poor satellite geometry or a low number of observations for a certain epoch (cf. *Švehla et al.* 2003). Althought the overall orbital data is of unpreceedented quality, still outliers have to be recognized in order to ensure the introduction of normally distributed kinetic energy as input for the balance equations.

The idea of how to detect outliers and to exclude them from further gravity field processing is the correlation of the data by a simple model. Filtering the orbital data with the spectral properties of the model could then be used to define a criterion for outliers.

2 The error model

We assume to estimate kinetic energy from orbit positions and their numerically derived velocities as,

$$T = \frac{1}{2}\left(\frac{G^2}{R^2} + \dot{r}^2\right) \tag{1}$$

with G being the angular momentum, R the radius from the satellite to the Earth center and $\dot{r}$ the radial velocity. The expression in brakets in eq. (1) can be assigned to $\dot{q}^2$, where $\dot{q}$ represents generalized momentum.

Understanding the force model as

$$U = -\int_{r_0}^{r} \boldsymbol{\mathcal{K}}(\boldsymbol{q})d\boldsymbol{q} \tag{2}$$

with q describing the shortest path from $\boldsymbol{r}_0$ to $\boldsymbol{r}$ in the Hamiltonian sense.

The total force $\mathcal{K}$ can be approximated as

$$\boldsymbol{\mathcal{K}} = \nabla V_{\oplus} + \nabla V_{\odot ☾} + \boldsymbol{F}_s \tag{3}$$

with the conservative gradients of the Earth potential field, as well as of third bodies and the surface force $\boldsymbol{F}_s$, representing energy dissipation in the upper atmosphere. The surface force can be further analyzed by

$$\boldsymbol{F}_s = \boldsymbol{ac} + \boldsymbol{b} \tag{4}$$

leading to a linear correction of the (in the case of CHAMP) mismeasured dissipative accelerations $\boldsymbol{ac}$. The unknown bias $\boldsymbol{b}$ can be approximated if the derivatives of the along track, cross track and radial velocities are set equal to eq. (3) cf. *Gruber et al.* (2005), or after integration along the orbit by applying low degree polynomials (*Badura et al.* 2004).

After determination of the dissipative forces a representive energy constant over a certain time span can be found by

$$E = \frac{1}{n}\sum_{i=1}^{n}(T_i + U_i) \tag{5}$$

or by following $\|\mathcal{L}\|_1$ and using instead of the mean value the median.

From interchanging eq. (5), a general reference momementum $\dot{q}_r$ can be calculated

$$\dot{q}_r = \sqrt{2(E - U(q))} \tag{6}$$

Differences in position from the reference momentum and the change in position per epoch Δr of the satellite are obtained,

$$\delta r = \dot{q}_r \Delta t - \|\Delta \boldsymbol{r}\| \tag{7}$$

and can be treated by treshold values to define outliers.

3 Data preparation

The kinematic position data in the terrestial reference frame (ITRF) has been transformed into a non rotating frame and numerically differentiated. From analysis of the inclination of the orbital plane, gross outliers can be easily detected with standard methods, e.g. regression based technics (cf. M-estimation, *Huber* 1973) and the covariance information for these positions has been adjusted.

The gradient forces can be obtained from any a priori gravity field of the Earth as well as the solid Earth tides. The accelerometer data has been rotated into the orbital plane by the available attitude information as well as the orientation of the local orbital tripod in each point.

Fig. 1 shows the kinematic residuals from eq. (7) during a 12 hour time span (30 *sec* sampling), revealing a very good accordance between the kinematic orbit and the used model.

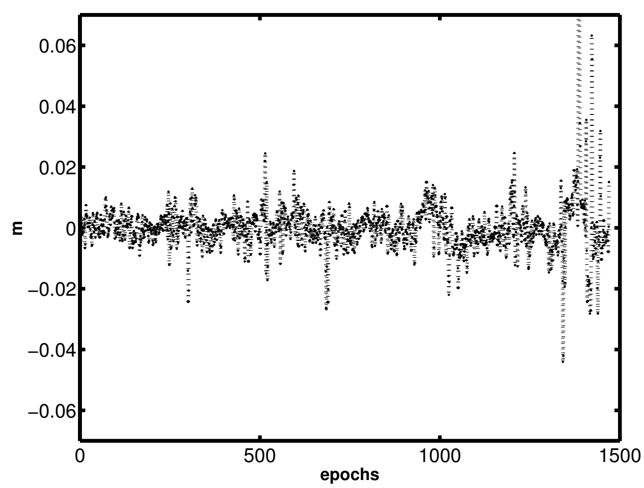

Figure 1: Kinematic position residuals between the used geopotential model and the measured data.

4 The Outlier detection

The kinematic residuals from eq. (7) will be split in the following by a suitable filter into a smooth dynamic innovation, $f(\delta r)$, with respect to the used geopotential coefficients of the reference field in eq. (2) and higher frequency components $g(\delta r)$, consisting of omitted geopotential spectrum, noise and outliers. Instead of filtering in spectral domain by impulse response filters, where hardly use of the available covariance information can be made, $f(\delta r)$ shall be approximated by a functional model with smooth characteristics in order to approximate the continuous properties of the osculating ellipses. The term continuous in this context underlines the fact that small changes within the geopotential field of the Earth will introduce only small changes to the orbital geometry of the spacecraft. Since the lower frequency part of the used reference field, e.g. EGM96 (*Lemoine et al.* 1998) is in good accordance to this by its parameterization with spherical harmonics (SH), the occurence of spikes in the data should be fully ascribed to outliers from the determination of the kinematic velocities.

A similiar approach can be found in SLR data processing, known as normal point generator (*Sinclair* 1997). The range observations are reduced by predicted satellite state vectors from force models and the differences are then approximated by smooth trend functions. The RMS of the residuals to the trendfunction will then be used as a rejection level for the observations.

4.1 Definition of a trend model

In our model the trend function is establised by harmonical base functions up to a finite spectral resolution and the corresponding amplitudes shall be determined by an optimized fit to the data. In order to make the functional model insensitive to outlier peaks, the available covariance information will be used as an a priori observation dispersion as well as a robust estimation approach (cf. *Koch/ Levenhagen* 2002) that shall fit the trend function. Thus a spectrally limited harmonic analysis of the kinematic residuals shall best approximate low frequencies as the innovative signal, and gain outliers as their signal counterparts beyond a to be defined treshold.

4.2 Estimation of the model parameters

The functional model can then be described by a Fourier series, but the coefficients shall be determined by an estimation procedure in order to incorporate covariance information.

$$
\begin{aligned}
f(\delta r) \Leftrightarrow \mathcal{F}(\omega) &= a \cdot \cos(2\pi n \tau) + ib \cdot \sin(2\pi n \tau) \\
(a + i \cdot b) &= (A'Q^{-1}A)^{-1}A'Q^{-1}f(\delta r) \qquad (8)
\end{aligned}
$$

The linear parameter estimation, minimizing the norm of the residuals shows deficiencies when outliers are in the data. We therefore applied an iterative approach where assumed outliers are iteratively downweighted during the estimation process.

4.3 Definition of the spectral resolution of the trend model

In oder to supplement the low geopotential frequencies used in eq. (2) as well as to account for unmodelled effects such as ocean tides, it has to be defined what spectral resolution of the trend function should be envisaged.

From analysis of Fig. 1 it can be seen that in order to remove a suitable trend function a development up to a few principal frequencies would be sufficient. The corresponding frequency of the flattening term of the Earth (J_2) transforms within a 12 *hours* time span, i.e. rougly 8 revolutions, to 16 harmonic cycles or an upper frequency of roughly $(46 \cdot 60)^{-1}$ Hz if the revolution duration is $\approx 92min$. If the trend function is limited to that degree, only a possible mismodeling of this particular geopotential frequency is being taken care of by the trend function.

If we regard the used gravity field model as a rough approximation only, then the trend function should be able to absorb deficiencies in all employed geopotential frequencies in order to maintain independence from the model. On the other hand it is clear that during a given timespan only a limited number of geopotential frequencies (e.g. mainly zonal terms) should truly affect the satellites position. We therefore assume the isotropic commission error from error degree variances (RMS) of the SH - spectrum as representative measure to analyse the effect on the orbit positions. According to Kaula's rule of thumb (*Kaula* 1966),

$$\sigma_l = \sqrt{\sum_{m=0}^{l} \bar{C}_{lm}^2 + \bar{S}_{lm}^2} = \sqrt{2l+1} \quad \frac{10^{-5}}{l^2} \tag{9}$$

approximates the spectral density (PSD) of the fully normalized geopotential coefficients (C, S) and together with the transfer function

$$\lambda_l = \frac{GM}{R} \left(\frac{a_\oplus}{R}\right)^l \tag{10}$$

we obtain error degree variances for the gravity potential at satellite height,

$$\sigma_{U_l} = \sqrt{\lambda_l^2 \sigma_l^2}, \tag{11}$$

with GM the gravity constant of the Earth, $a_\oplus$ mean Earth radius, l geopotential degree, R geocentric radius of the satellite.

The equivalent error model according to *Rapp* (1978) reads,

$$\sigma_l(\Delta g) = \sqrt{\alpha_1 \frac{l-1}{l+1} s_1^{l+2} + \alpha_2 \frac{l-1}{(l-2)(l+2)} s_2^{l+2}} \tag{12}$$

where $\alpha_1 = 3.404, \alpha_2 = 140.03, s_1 = 0.998006, s_2 = 0.914232$

and the corresponding transfer function

$$\lambda_l = \left(\frac{ae}{R}\right)^l \frac{R}{l-1}. \tag{13}$$

Error propagation of σ_{U_l} into eq. (7) yields the reference position error per degree

$$\sigma_{l,\delta r} = \frac{\Delta t}{\sqrt{2(E - U(q))}} \cdot \sigma_{U_l} \tag{14}$$

due to the used geopotential model. Fig. 2 shows the position error per degree, comparing the two different error degree variance models, namely Kaula and Rapp,

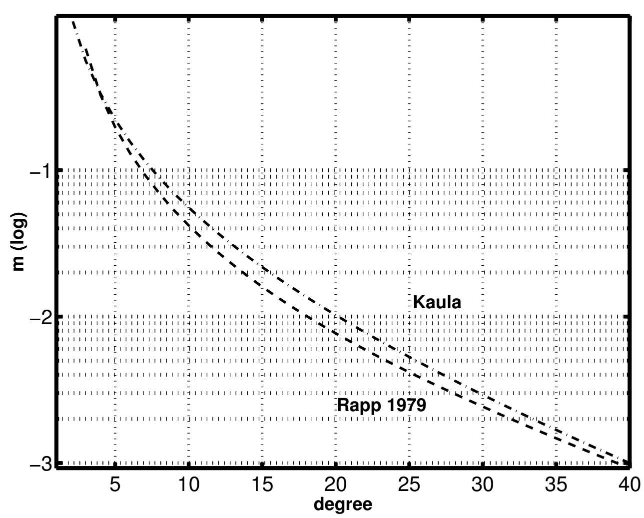

Figure 2: position error per degree according to error degree variances of the SH expansion from different empirical models.

It is understood, that in order to approach a *cm*- level for the filtering of the kinematic position residuals the spectral range of the trend function should be in the case of Kaula's rule equivalent to geopotential frequencies up to DEG/ORD 20. On the other hand, the higher the resolution of the trend function becomes, the more difficult it gets to robustly estimate the corresponding parameters. A proposed rigorous treatment would be the statistical assessment of the applied model, i.e. testing for significance of the model parameters for each estimation. The outliers could then be statistically detected by data snooping. For details cf. (*Kern*, 2005). In our approach we must admit a lot better quality to the reference field than the error PSD of the coefficients indicate and therefore limit the resolution of the trend function to an equivalent geopotential degree of $l_{\max} = 6$. This can be understood in the following manner: the innovative difference between the used reference model and the solution belonging to the kinematic orbit data shall be found far below the degree variance models of Kaula and Rapp. This is certainly a critical aspect and we might therefore loose independend gravity field information within the spectral bandwith (BW) of the used reference geopotential model beyond ($l > 6$) that would possibly be detected as outliers. Nevertheless, it turns out after geopotential recovery of the filtered kinematic orbit (cf. section 5), the named spectral BW comes out different from the used reference model, inspite of being the weakest part in terms of infiltration of a- priori information. The system proofes to be still capable in recovering an independend solution. Fig. 3 shows the trend function, approximating the kinematic position residuals.

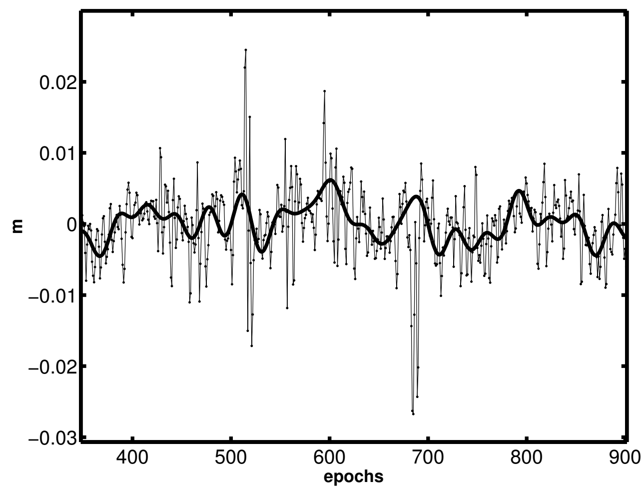

Figure 3: Fit of the trend function with a spatial resolution up to 15 epochs (DEG/ORD 6).

4.4 Definition of a treshold for outliers

Once the trend model has been fitted, the innovative part can be subtracted from δr, yielding the noise and outlier function $g(\delta r)$. To distinguish outliers from remaining gravity field signal a treshold value has to be defined, taking into account the remaining signal power that can be expected from the omission error of the geopotential development of the reference field,

$$\Delta\delta r = \frac{\Delta t}{\sqrt{2(E - U(q))}} \cdot \Delta U \tag{15}$$

where ΔU has been calculated by

$$\Delta U = \int_{r_0}^{r} \nabla V_{l,\infty} d\boldsymbol{q} \tag{16}$$

from EGM96 but could be simultaneously obtained again from e.g. Kaula's rule of thumb. Fig. 4 shows the omission error according to eq. (15) from a signal beyond DEG/ORD $l > 40$ in terms of orbit deviations.

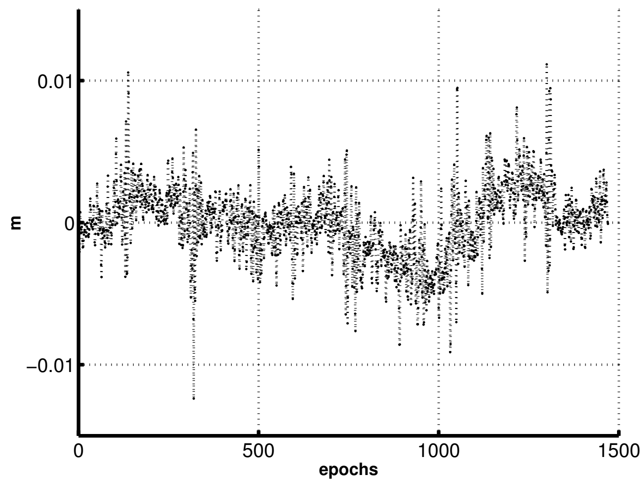

Figure 4: Range of the omission error in terms of orbit deviations

It is evident that the omission of geopotential signal beyond DEG/ORD 40 represents a clear limitation for an outlier treshold. Reduction of the omission error by a newly trend reveals remaining deviations of less than 1 *cm*. The treshold value has therefore been set to 1.25 *cm*.

Fig. 5 shows the total situation after removal of the trend function. No assumeable geopotential information should be found beyond the treshold values.

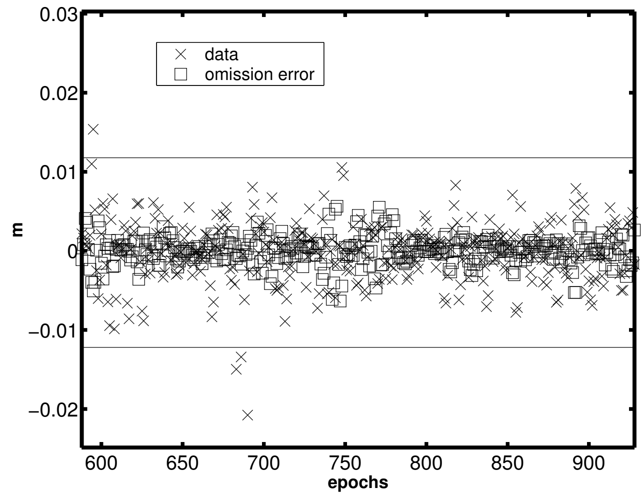

Figure 5: Kinematic residuals after trend removal and omission error within a treshold.

5 Model test

After the filtering with EGM96, modified error variances for the kinetic energy have been introduced into a LS estimation of the full geopotential coefficients from DEG/ORD 2 to 50, applying the energy balance equation (cf. *Jekeli* 1999). Fig. 6 shows the difference PSD's with respect to other gravity field models.

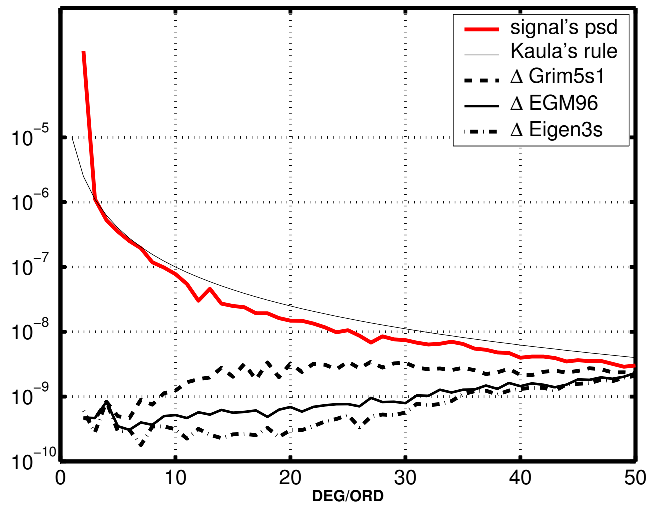

Figure 6: Comparison of a filtered solution by EGM96 to existing geopotential models: GRIM5S1, EGM96, EIGEN3.

It can be seen that the solution fits best to EIGEN3S, although not being used for the outlier detection. A similiar result is obtained if EIGEN3S shall be used for the filtering. If GRIM5S1 has been used, a slight deterioration of the result can be observed (not plotted) which is in accordance to the general deficiency of the approach in the BW above $l > 6$, as being stated earlier. In order to obtain acceptable results a good reference solution is thus necessary.

6 Conclusions and Outlook

The method presented in this paper is an alternative approach, compared to the use of dynamic orbits, in order to detect outliers, by their kinematic correlation to external gravity field information.

In an ideal case, the a-priori covariance information would represent the configuration of the observation space. Projection onto the parameter space would then give an immediate result for the geopotential model. Unfortunately this is not the case and the available covariances give therefore a too optimistic picture concerning the outliers.

Once a global gravity field solution has been processed, the a-posteriori covariances for the velocities (and thus the positions) can be computed and their differentiation being repeated. A question under investigation is, whether an iterative approach without use of any a-priori information will converge. An evaluation strategy could moreover be defined, assessing the quality of the approach and the error probability of successfully detection (or failure) of outliers.

7 Acknowledgement

The Author would like to thank the Geoforschungszentrum Potsdam (GFZ) for providing the accelerometry data as well as Drs. Dražen Švehla and Markus Rothacher who made available the kinematic orbit data. Valuable discussions and input from Dr. Michael Kern has been gratefully appreciated.

The Authors Adress:
Christian Gruber
Graz University of Technology
Institute for Navigation and Satellite Geodesy
Steyrergasse 30/III, A-8010 Graz, Austria
phone +43(0)316 873-6357
fax +43(0)316 873-6845
email chrs.gruber@gmx.net

References

Badura T., Gruber C., Klostius R., Sakulin C. CHAMP *gravity field processing applying the Energy Integral Approach* Joint CHAMP/GRACE Science Meeting,GFZ, July 6-8, 2004, IN: Arne K. Richter (ed.) Advances in Geosciences 2004.

Gerlach C. et al., *A* CHAMP*-only gravity field model from kinematic orbits using the energy integral,* Geophysical Research Letters, VOL. 30, NO. 20, 2037, doi:10.1029/2003GL018025, 2003.

Gruber C., Tsoulis D., Sneeuw N., CHAMP *Accelerometer calibration by means of the equation of motion and an a-priori gravity field.* Zeitschrift für Vermessungswesen, ZfV, in print, 2005.

Huber P.J. *Robust regression: Asymptotics, conjectures and Monte Carlo*, Ann. Stat., **1**, 799-821, 1973.

Jekeli C., *The determination of gravitational potential differences from satellite-to-satellite tracking,* Cel. Mech. Dyn. Astron., 75, 85-101, 1999.

Kaula W.M. *Thoery of Satellite Geodesy, Applications of Satellites to Geodesy.* Dover Publications, Inc. Mineola, New York (edition 2000).

Kern M., *External Calibration and Validation Methods for the Satellite Mission* GOCE, Final technical report in the frame of External ESA Fellowship at Graz University of Technology 2005.

Rapp RH.: *Potential Coefficient and Anomaly Degree Variance Modelling Revisited.* Tech. Rep. No. 293, The Ohio State University, Department of Geodetic Science, Columbus, Ohio 1979.

Reigber Ch., Lühr H., Schwintzer P.: *First Champ Mission Results,* Springer Verlag, Berlin Heidelberg 2003

Sinclair A.T.: *Data Screening and Normal Point Formation.* Royal Greenwich Observatory, Cambridge, UK 1997.

Švehla D, Rothacher M, *Kinematic and Reduced-Dynamic Precise Orbit Determination of* CHAMP *satellite over one year using zero-differences.* EGS-AGU-EUG Joint Assembly 06-11 April 2003, Nice, France. Geophysical Research Abstracts, European Geophysical Society Vol. 5. ISSN:1029-7006