Improvement of the gravimetric model of quasigeoid in Slovakia

Marcel Mojzeš, Juraj Janák, Juraj Papčo
Department of Theoretical Geodesy, Slovak University of Technology in Bratislava, Slovak Republic

Abstract. The paper presents two recent approaches of the quasigeoid modelling GMSQ03B and GMSQ03C in the area of Slovakia and their modifications after fitting by GPS/levelling method. The description of data sources follows the two different computational schemes. The statistical testing and fitting using 59 GPS/levelling points are presented. A brief discussion, conclusion and future perspective are summarized in the paper.

Keywords. Gravimetric quasigeoid, data sources, GPS/levelling

1 Introduction

Slovakia is predominantly mountainous country located in central Europe. There is the first part of the Carpathian belt. Slovakia had been systematically measured by detailed gravity measurements from 1956 to 1992 (Kubeš et al., 2001). All area of Slovakia had been covered by gravity observations with approximately homogeneous density varying from 3 to 6 points per square km, that represents more than 200, 000 gravity points. On this basis, using the generalized Molodensky's theory, we have been trying for last nine years to compile the most detailed and accurate model of the quasigeoid. The first attempt was done in 1995 followed by the versions 1996, 1998A and 1998B. Especially the version 1998B after fitting into national vertical datum, known as GMSQ98BF – Gravimetric Model of Slovak Quasigeoid 1998 (Mojzeš and Janák, 1998) and (Mojzeš and Janák, 1999), has frequently been used for various scientific and practical applications.

Since 1998 the gravity database has been revised as far as blunders and systematic errors are concerned (Kubeš et al., 2001). Moreover the wider area of the mean gravity data has meanwhile become available. Some progress in theory and technology of computation has also been made, especially in combining of the terrestrial gravity data with the global geopotential model. All these circumstances have led us to our two recent approaches of the quasigeoid model in Slovakia: GMSQ03B and GMSQ03C and their modifications after fitting GMSQ03BF and GMSQ03CF respectively.

2 Data sources

Three types of input data were used for computation: Terrestrial gravity data (point and mean), elevation data (detailed and global) and global geopotential models. One paragraph is dedicated to describe the sources of a particular data type. As far as the location of all data is concerned, the reference system ETRS89 was used. Normal gravity field applied in computation was GRS80.

We used two sources of gravity data: point refined Bouguer gravity anomalies (232, 280 values) mainly within the Slovakia and mean values of the refined Bouguer gravity anomalies with resolution of $5' \times 7.5'$ in the area $44° < \varphi < 56°$ and $12° < \lambda < 30°$. The first source comes from detailed gravity measurements 1956-1992 mentioned in introduction. Both data sets were transformed to the gravity system GrS-95 (Klobušiak and Pecár, 2004) based on 16 absolute gravity points.

In order to transform the refined Bouguer gravity anomalies into Faye gravity anomalies, the elevation data were needed. We used two digital terrain models (DTM): national DTM within Slovakia and global DTM elsewhere. The first DTM is known as DMR2/ETRS89 and its resolution is 3″ in latitude and 5″ in longitude (Mojzeš, 2002). This resolution approximately corresponds to distance 100 by 100 metres. The vertical datum of mentioned DTM is Kronstadt, Baltic Sea with the abbreviation Bpv. The global DTM we used is well known GTOPO30 (http://edcdaac .usgs.gov/gtopo30.gtopo30.asp). This model was used outside of Slovakia. The unsolved problem we still have with the elevation data is the optimal connection of both national and global DTMs.

The last data type used in our solutions was the global geopotential model. We used two different models. First, the satellite only model GGM01 coming from GRACE dedicated satellite mission in tide-free system (Rummel et al., 2002). The second was well known combined model EGM96 (Lemoine et al., 1998).

3 Computation schemes

Both solutions GMSQ03B and GMSQ03C were in principle computed as so-called gradient solution of the linear Molodensky's problem. Second term of the Molodensky series was approximated by terrain correction as recommended it by Moritz (1980). Correction of the part of the error coming from this approximation was applied in both solutions. The major differences between two solutions are in the manner of how the geopotential model is combined with the terrestrial data and how the integration of the residual gravity anomalies is performed.
The first step, similar in both solutions, was the compilation of the residual Faye anomalies in a regular geographical grid $\Delta\varphi$=20″ and $\Delta\lambda$=30″. First the refined Bouguer gravity anomalies had to be interpolated into mentioned grid. This was done using the Kriging interpolation technique, assuming the anisotropy in geographical coordinates. Then the interpolated refined Bouguer anomalies were transformed into Faye anomalies (free-air plus terrain correction) according to formula

$$\Delta g_F = \Delta g_{RB} + 2\pi G\rho H \qquad (1)$$

where Δg_{RB} are the refined Bouguer gravity anomalies, G is the Newton gravitational constant, ρ is the density of topographical masses (we assumed constant value ρ=2670 kg/m^3) and H stands for normal height in particular grid nodes obtained from DTM. Then the long wavelength part of the free-air gravity anomaly, assuming it approximately identical with the long wavelength part of the Faye anomaly, was subtracted from the Faye anomaly in order to obtain the residual Faye anomaly. The long wavelength parts for particular solutions were computed from global geopotential models GGM01 or EGM96 up to degree and order 20 or 360.

Solution GMSQ03B was computed using a low degree (n=20) spheroid obtained from the geopotential model GGM01. The aim was to avoid the errors coming from the higher-degree geopotential coefficients. Residual Faye anomalies were integrated using modified spheroidal Stokes's function. Modification according to idea of Molodensky (1962) up to degree 20 was used. Integration of the residual Faye anomalies was performed using classical numerical integration up to spherical distance ψ=3°. The truncation bias was estimated by EGM96 geopotential model using coefficients from 21 to 360.

Solution GMSQ03C was computed using a high degree (n=360) spheroid obtained from the geopotential model EGM96. Residual Faye anomalies were integrated using Spherical Stokes's function. Applied integration technique was the Fast Fourier Transformation over the spherical rectangle integration domain identical with the area of input gravity data described above. The GRAVSOFT software package (Tcherning et al., 1992) was used for computation of residual part of the quasigeoid. The truncation bias was neglected.

Let us to describe both solutions mathematically and graphically. The quasigeoid model GMSQ03B consists of four terms

$$\zeta_{GMSQ03B} = \zeta_{GGM01} + \zeta_{res} + \zeta_{tb} + \zeta_{Moritz} \qquad (2)$$

where the first term represents the reference spheroid, the second term is residual part of the quasigeoid obtained from numerical integration of residual Faye anomalies, the third term is estimation of the truncation bias correction and the last term is correction of approximation of the second term of Molodensky's series

$$\zeta_{Moritz} = \pi G\rho\gamma_P^{-1}H_P^2 \qquad (3)$$

computed according to Moritz (1980, Eq.(48-29)). In Eq. (3) the symbols G and ρ are obvious and γ_P is normal gravity and H_P stands for normal height at the point of computation.

The quasigeoid model GMSQ03C consists of three terms

$$\zeta_{GMSQ03C} = \zeta_{EGM96} + \zeta_{res} + \zeta_{Moritz} \qquad (4)$$

where the meaning of particular terms is analogous to Eq.(2). Of course the first two terms of the right hand side were computed from the different data and the second term using even different integration technique. The third term in Eq.(4) computed from Eq.(3) is identical with the last term of Eq.(2).

The final quasigeod model GMSQ03B is shown in Fig.1.

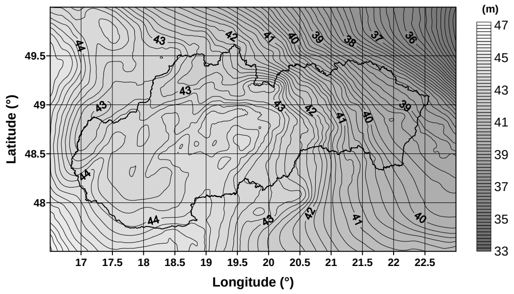

Fig.1 Quasigeoid model GMSQ03B

Another model is not plotted, because the figure would be very similar, rather the difference GMSQ03B-GMSQ03C between both solutions is depicted in Fig.2.

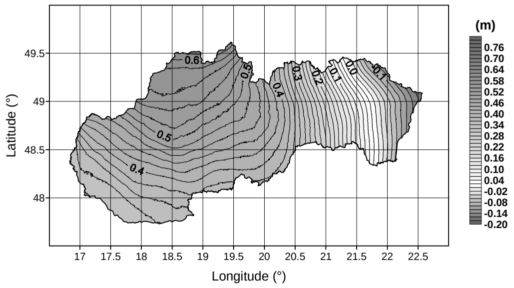

Fig.2 Difference between GMSQ03B and GMSQ03C

Both models are stored as a digital raster in geographical grid with the resolution of 20″×30″.

4 Testing and fitting

As the models of geoid or quasigeoid become more accurate because of the advanced theory and also the quality and quantity of the data, the choice of the verification method and quality of the testing points become very important task. Therefore we chose the independent verification method GPS and levelling. Within the area of Slovakia we chose the 59 testing points shown in Fig.3.

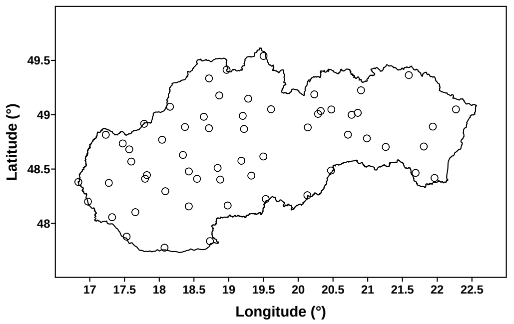

Fig.3 Distribution of the testing points

These points belong to either Central European Geodynamic Reference Network (CEGRN), see (Hefty and Gerhátová, 1997), or Slovak Geodynamic Reference Network (SGRN), see (Hefty, 1996). According to renowned estimates, e.g. (Stangl, 1998) and (Hefty and Mojzeš, 1999), the accuracy of the ellipsoidal height at these points in certain epoch is not worse then 2 centimetres. The accuracy of the normal heights determined by levelling over the Slovakia, in a relative sense, is also about 2 centimetres (ibid.).

The heights of quasigeoid models above the reference ellipsoid GRS80 were interpolated at the location of testing points and compared with the reference values obtained from the simple formula

$$\zeta_{ref} = h - H \tag{5}$$

where h is the ellipsoidal height and H is the normal height according to Molodensky. The basic statistics of the differences is presented in Tab.1.

Table 1 Basic statistics of the set of 59 differences

Quantity	$\zeta_{GMSQ03B} - \zeta_{ref}$	$\zeta_{GMSQ03C} - \zeta_{ref}$
Mean	0.334 m	0.711 m
St. dev.	0.190 m	0.076 m
Range	0.856 m	0.363 m

In order to use the quasigeoid model in geodetic practice, e.g. for estimation of normal heights from GPS measurements, the adaptation of quasigeoid into national vertical datum, so called fitting, is necessary. Incompatibility of the gravimetric models with the national vertical datum is mainly due to influence of global geopotential models in gravimetric solutions, but the differences reflect also other influences and also errors. The fitting is always a "dangerous" process where the original quasigeoid model can be deformed and getting worse, especially between the fitting points. Therefore such a process should be treated with a special care. The best case would be to use the constant shift only to unify the vertical datum. If this is not possible, because the differences show some clear long wavelength features, the fitting surface of lowest possible degree should be used.

For fitting process we used the surface polynomial regression and the fitting points were identical with the testing points (Fig.3). The differences $\zeta_{ref} - \zeta_{GMSQ03B/C}$ were modelled as follows

$$\Delta\zeta = \sum_{p=0}^{P}\sum_{q=0}^{Q} Q_{p,q}\left(\varphi - \varphi_0\right)^p \left(\left(\lambda - \lambda_0\right)\cos\varphi\right)^q \tag{6}$$

In Eq. (6) the φ, λ are the ellipsoidal coordinates of fitting points, φ_0, λ_0 are chosen fixed coordinates and $Q_{p,q}$ are unknown coefficients estimated using the least squares method. The optimal degree of polynomial surface was chosen according to (Anděl, 1998)

$$A_k = \sigma_k^2 \left(1 + \frac{k}{\sqrt[4]{n}} \right) \tag{7}$$

where A_k is the criterion, n is the number of fitting points, k is the degree of polynomial surface and σ_k is the standard deviation of residuals. The optimal degree is the lowest one where the stop criterion A_k decreasing. For both models of quasigeoid in our case, the second degree polynomial surface with 6 coefficients was estimated as the optimal. The coefficient values are shown in Tab.2.

Table 2 Values and standard deviations of the fitting polynomial surfaces

Quasigeoid	GMSQ03B		GMSQ03C	
Coefficient	Value	St. dev.	Value	St. dev.
$Q_{0,0}$ (m)	-0,2903	0,0078	-0,7394	0,0075
$Q_{1,0}$ (m/°)	0,0512	0,0138	-0,0697	0,0122
$Q_{0,1}$ (m/°)	-0,1807	0,0051	-0,0445	0,0048
$Q_{1,1}$ (m/°^{2})	-0,0560	0,0186	0,0015	0,0168
$Q_{2,0}$ (m/°^{2})	-0,0500	0,0261	-0,0576	0,0253
$Q_{0,2}$ (m/°^{2})	-0,0660	0,0054	0,0325	0,0053

Shape of the polynomial surfaces is shown in Fig.4. and Fig.5.

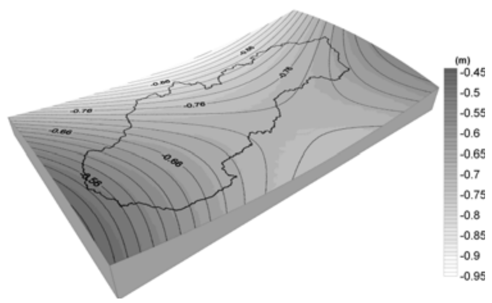

Fig.4 Fitting polynomial surface for GMSQ03B

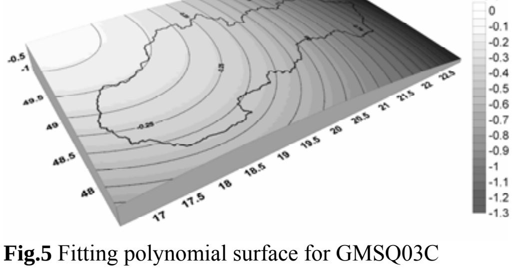

Fig.5 Fitting polynomial surface for GMSQ03C

Names of the quasigeoid models after fitting are GMSQ03BF or GMSQ03CF respectively. The standard deviation of residuals computed at the testing points after fitting is 0.041m for GMSQ03BF and 0.032m for GMSQ03CF. Reader is encouraged to compare these values with the corresponding standard deviations before fitting in Tab.1. It is also interesting to see the residuals obtained at the testing points after fitting process plotted as surface maps, see Fig.6. and Fig.7, or as histograms, see Fig.8. and Fig.9.

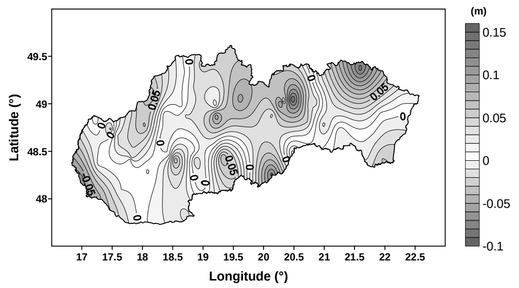

Fig.6 Residuals of GMSQ03BF

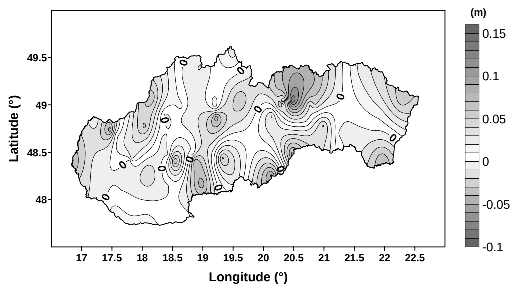

Fig.7 Residuals of GMSQ03CF

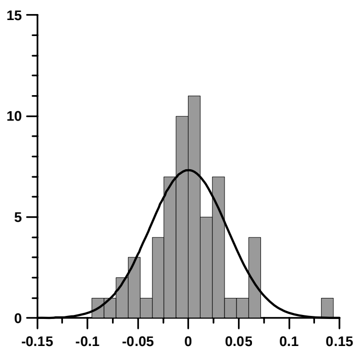

Fig.8 Histogram of GMSQ03BF residuals, x-axis in meters

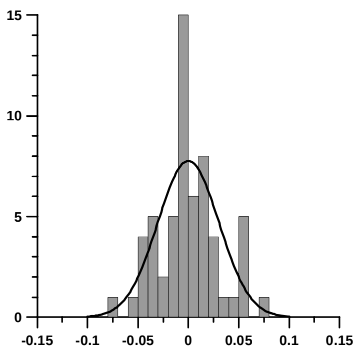

Fig.9 Histogram of GMSQ03CF residuals, x-axis in meters

5 Conclusion and perspective

To rich 1-centimeter accuracy of the quasigeoid model in an absolute sense in a mountainous country, as e.g. Slovakia, is very difficult task. One of the problems is that even the accuracy of reference values at the testing points is sometimes worse than one centimetre. Some improvements can still be done in theory and technology of computation. Presented quasigeoid models can guarantee, as you can see from the results, accuracy better than 5 centimetres in an absolute sense. Of course in many local areas it is much better, also below one centimetre.

Our future investigation, in order to improve the accuracy of the quasigeoid model, will be oriented to rigorous computation of the second term of Molodensky's series, refinement of digital terrain model, improvement of quality of the testing points, optimal numerical integration and careful unification of all reference systems.

Acknowledgement

The project of compilation of new quasigeoid model of Slovakia was partially supported by Research Institute of Geodesy and Cartography in Bratislava and partially supported by Grant Agency of the Slovak Republic (Project VEGA 1/1433/04).

For our computations we use several software products: commercial products SURFER and GRAPHER for interpolation of the input data and creating pictures, and scientific products SHGEO and GRAVSOFT software packages for working with global geopotential models and for numerical integration. We also use some, already mentioned, public data sets: GGM01, EGM96 and GTOPO30.

References

Anděl, J. (1998). Statistical Methods. Matfyz Press, Prague. (in Czech)

Hefty, J. and Ľ. Gerhátová (1997). Implementation of BERNESE software 4.0, processing of connection measurements of selected AGS points with the SLOVGERENET, comparison of processing using the version 3.5 and 4.0 and it's analysis. Technical report, Slovak University of Technology, Bratislava. (in Slovak)

Hefty, J. and M. Mojzeš (1999) Heights in Slovak Geodynamic Reference Network. Proceedings of the workshop "GPS and heights". TU Brno. (in Slovak)

Klobušiak, M. and J. Pecár (2004). Model and algorithm of effective processing of gravity measurements performed with a group of absolute and relative gravimeters. Geodetický a kartografický obzor 50 (92), No. 4-5, Prague, pp. 99-110. (in Slovak)

Kubeš, P., T. Grand, J. Šefara, R. Pašteka, M. Bielik and S. Daniel (2001). Atlas of geophysical maps and profiles. Technical report. National geological institute, Bratislava. (in Slovak)

Lemoine, F.G. et al. (1998). The development of the joint NASA GSFC and the NIMA geopotential model EGM96. NASA Technical Report NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt, Maryland.

Mojzeš, M. and J. Janák (1998). Gravimetric model of Slovak quasigeoid. Proceedings of the Second Continental Workshop on the Geoid in Europe. Reports of the Finnish Geodetic Institute 98:4, pp. 277-280.

Mojzeš, M. and J. Janák (1999). New gravimetric quasigeoid of Slovakia. Bolletino di Geofisica Teorica ed Applicata, Vol. 40, No. 3-4, pp. 211-217.

Mojzeš, M. (2002). Accuracy analysis of the digital terrain model DMR-2 in ETRS-89 reference system. Proceedings of the VIII. International Polish, Czech and Slovak Geodetic Days. Polanica. Zdrój, pp. 45-51. (in Slovak)

Molodensky, M.S., V.F. Eremeev and M.I. Yurkina (1962). Methods for Study of the External Gravitational Field and Figure of the Earth. Israel Program for Scientific Translations, Jerusalem.

Moritz, H. (1980). Advanced Physical Geodesy. Abacus Press, Karlsruhe.

Rummel, R., G. Balmino, J. Johannessen, P. Visser and P. Woodworth (2002). Dedicated Gravity Field Missions – Principles and Aims. Journal of Geodynamics, Vol. 33, 1-2, pp. 3-20.

Stangl, G. (1998). The GPS Campaigns of CERGOP. Reports on Geodesy, No. 9 (39), pp. 39-55.

Tscherning, C.C., R. Forsberg and P. Knudsen (1992). The GRAVSOFT Package for Geoid Determination. Proceedings of the First Continental Workshop on the Geoid in Europe. P. Holota and M. Vermeer (Eds.). Research Institute of Geodesy, Topography and Cartography, Prague, pp. 335-347.