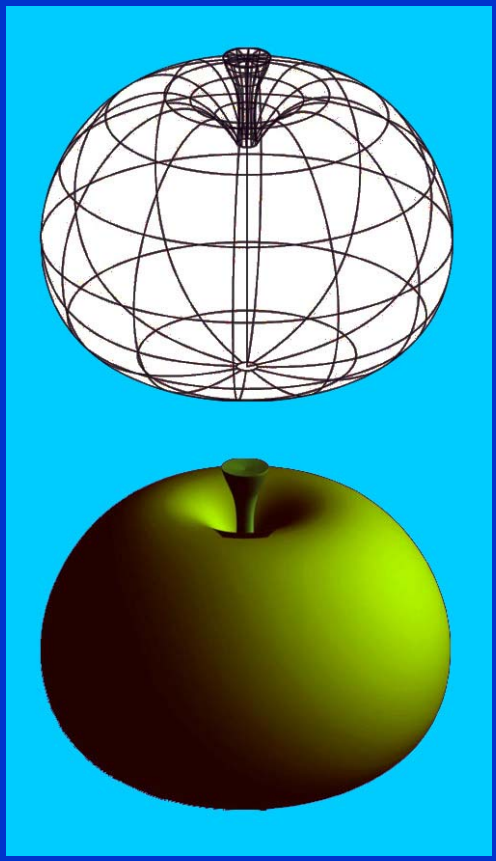


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External Quality Evaluation Reports of EGM08

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Foreword

The Joint Working Group (JWG) between the International Gravity Field Service (IGFS) and the Commission 2 of the International Association of Geodesy (IAG), entitled "**Evaluation of Global Earth Gravity Models**", was officially established in 2005. The main objective of this JWG is to study standard validation/calibration techniques for global geopotential models, and to perform quality assessment procedures of GRACE, CHAMP and GOCE based satellite-only and combined solutions for the static part of Earth's gravity field. The external data sets that are commonly used for such purposes include GPS and leveling height data, airborne and surface gravity data, mean oceanographic sea-surface-topography (SST) models and altimetric data, orbit data from other geodetic and altimetric satellites, and astro-geodetic vertical deflections. The initial membership of the JWG included 24 scientists from 15 countries, which has finally increased to 30 scientists from 20 countries due to the strong international interest in evaluating the PGM2007A model, a preliminary version of the official Earth Gravitational Model 2008 (EGM2008).

The IGFS/IAG JWG has successfully coordinated the evaluation of both PGM2007 and EGM2008, in close collaboration with the EGM development team from the U.S. National Geospatial-Intelligence Agency (NGA). This joint evaluation project was carried out through three phases: the implementation and testing of the NGA software for spherical harmonic synthesis using ultra-high degree geopotential models (2006-2007), the evaluation of the PGM2007 model (2007-2008), and finally the evaluation of the official EGM2008 model (2008-2009). Most of the results of the above tasks are publicly available at the official webpage of the working group: http://users.auth.gr/~kotsaki/IAG_JWG/IAG_JWG.html.

The first splinter meeting of the JWG was held on July 31, 2006 in Istanbul during the first IGFS international symposium, and it marked the end of Phase 1. The PGM2007A model was released to the members of the JWG in July 2007, initiating the beginning of Phase 2. A total of thirty evaluation reports for PGM2007A were completed and published at the JWG's website by December 2007. Phase 3 started right after the official release of EGM2008 at the EGU General Assembly in April 2008. The first results of the EGM2008 evaluation tests were presented by the working group members in a dedicated session during the IAG international symposium 'Geoid, Gravity and Earth Observation' that was held in Chania, Greece, June 23-27, 2008.

This special issue of *Newton's Bulletin* consists of 25 peer-reviewed evaluation papers of EGM2008 (and partially of PGM2007A), which are grouped into four different sections according to the geographical region of the evaluation tests: Global, the Americas, Europe and Africa, and Asia, Australia and Antarctica. Their results provide a thorough external assessment of EGM2008, using a variety of geodetic data and testing methodologies.

We are grateful to all people who made the publication of this special issue possible. First of all, we would like to express our deep appreciation to all contributing authors of the evaluation papers for their interest and dedication to the project. The success of this project is primarily attributed to their continuous participation and close cooperation. Secondly, we would like to thank the development team of EGM2008 for their support and continuous collaboration towards the successful completion of this international project. Last but not least, the IGFS and the Commission 2 of the IAG are acknowledged for their effective international leadership, guidance and coordination.

Special thanks are due to the International Geoid Service (IGeS) and the Bureau Gravimétrique International (BGI) for the publication of this special issue of *Newton's Bulletin*.

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Evaluation of the EGM2008 Gravity Field by Means of GPS-Levelling and Sea Surface Topography Solutions

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Abstract

The new EGM2008 global gravity field model is evaluated by comparisons of geoid heights computed from the model with those available at GPS levelling stations in various regions and by computing sea surface topography solutions from the difference between the mean sea surface and the geoid from this model. In order to identify how good the model performs the same tests also are performed for other recent global gravity field models. The evaluation method, in particular, has to take into account the omission error when truncating global gravity field models at chosen degrees and orders and when comparing them with observed quantities. The procedures applied for the computation of the omission error as well as the general methods applied for the evaluation are described in the paper. For testing the models there are available GPS levelling heights in six different regions (Europe, Germany, USA, Japan, Canada, Australia). Some of these data sets seem not to be adequate for evaluation purposes, because they exhibit long wavelength structures in the geoid height differences. Nevertheless, from the results of the geoid height comparisons one can conclude that the EGM2008 model performs best in most of the regions. Similar results are derived from the sea surface topography solutions based on different global gravity field models. The dynamic ocean topography determined with the EGM2008 geoid seem to show more realistic and more detailed features than solutions based on other global gravity field models. In summary, from the tests performed one can state that EGM2008 represents a significant improvement of the global gravity field in terms of quality and resolution.

1 Introduction

In spring 2008 the new ultra high resolution gravity field model EGM2008 has been made available by the National Geospatial-Intelligence Agency (NGA) (see Pavils et al, 2008). The Inter-Commission Working Group between Commission 2 of the International Association of Geodesy (IAG) and the International Gravity Field Service (IGFS) has been asked to perform an independent evaluation of this new model. For a more detailed description of the purpose and tasks of this working group it is referred to:
http://users.auth.gr/~kotsaki/IAG_JWG/IAG_JWG.html.

In the context of this working group an extensive analysis of the EGM2008 gravity field by comparing model derived geoid heights against independently observed geoid heights at GPS levelling stations in various areas of the world and by the determination of ocean topography solutions in different ocean basins has been performed. The following chapters provide a description of the evaluation technique as well as selected results. In particular, chapter 2 describes the technique and data sets applied for gravity field evaluation specifically taking into account the problem of omission. Evaluation results are shown in chapters 3 and 4

applying the techniques described in chapter 2. Finally, in chapter 5 a summary is given and final conclusions about the quality of the EGM2008 are drawn.

Comparing quantities of global gravity field models with other gravity field observations per definition is a chicken-and-egg problem. First of all, gravity field modellers try to use the best available data sets in their solutions, which implies, that these data cannot be used anymore for comparison purposes in order to warrant independence. Second, any gravity field quantity as observed on the Earth's surface contains the full spectral signal power, while any global model is limited by its spectral resolution, i.e. the maximum degree and order of the spherical harmonic series of the model. When comparing such data we have to take into account this problem, which commonly is related to the "omission error". In order to determine the omission error one would need perfect knowledge of the global gravity field. If we would know it perfectly, there would be no further need to perform gravity field modelling by spherical harmonics.

A discussion of potential techniques to be applied for the evaluation of global gravity fields and their limitations is provided in Gruber (2004) and Gruber et al (2006). In the quality analysis of EGM2008 (it is referred to the following chapters) it is tried to reduce the effect of these limitations by using independent data sets, which have not been used in the model determination, and by filtering the terrestrial and altimetric data sets in order to make them spectrally consistent. We will see later that this can only be done to some extent, because we do not have a reduction model up to infinity. In other words one can state that at least a part of the omission error remains a problem when comparing model derived quantities with observed data.

2 Evaluation Technique and Data Sets

As discussed in the introductory section, for evaluating the EGM2008 model, we compare it with independent geoid heights on GPS levelling stations and we determine sea surface topography solutions by subtracting the model geoid from a mean sea surface. In the sequel both approaches are explained in detail. For comparison, the same procedures are applied to other global gravity field models in order to identify, which of them performs better or worse.

2.1 Basic Relations

The evaluation approach with GPS-levelling data requires geoid heights computed from the model under test as well as independently observed geoid heights, while for the evaluation with the sea surface topography (dynamic ocean topography) apart from the model geoid heights a mean sea surface from altimetry is needed. The relation between the quantities involved is illustrated in Figure 1 and explained in the following.

From GPS positioning and satellite altimetry we get geometric heights above the reference ellipsoid. Over land from conventional levelling we get orthometric (normal) heights (physical heights), and by the difference between the geometric and orthometric (normal) heights we get an observed geoid (quasi-geoid) height (solid black line at land in Figure 1). From the global gravity field model we compute geoid heights referring to the same reference ellipsoid by spherical harmonic synthesis (dotted black line at land in Figure 1). Now both geoid heights can be compared and used for evaluation of the global model. Over the ocean the situation is very similar. From the differences between the mean altimetric sea surface heights and the geoid heights, computed from the global model, we get an estimate of the dynamic ocean topography (dotted black line at ocean in Figure 1). This dynamic ocean

topography can be compared to an alternatively estimated surface, e.g. from an ocean circulation model (solid black line at ocean in Figure 1).

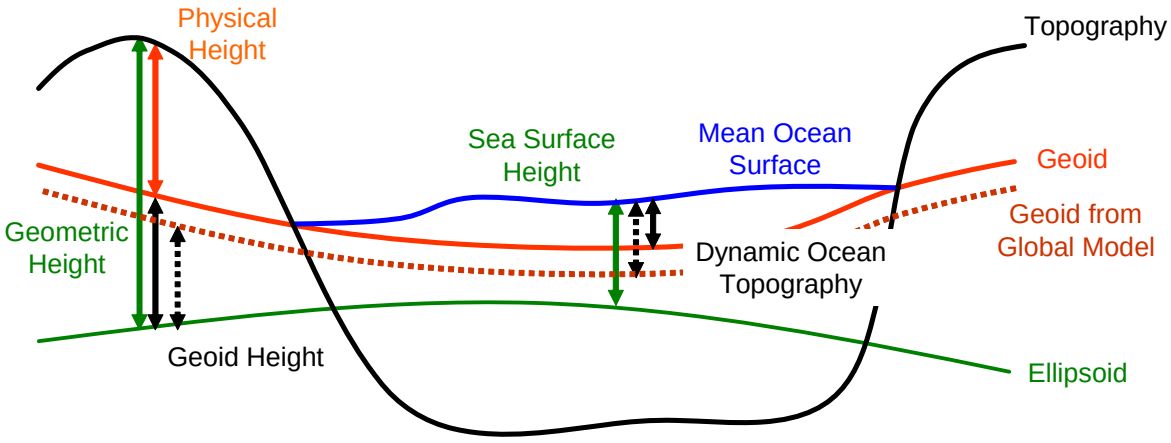


Figure 1: Quantities and height systems involved in global gravity field validation by means of geoid and sea surface topography.

2.2 The Problem of Omission

As described in the introduction, the omission error plays a significant role for evaluating global models with independently observed data. For this reason a more detailed description of this problem is provided. Figure 2 shows an overview of the situation we have.

The horizontal axis in Figure 2 shows the spectral domain (i.e. degree of spherical harmonic series) from zero to high frequencies, while the vertical axis specifies the spatial domain from pointwise observations to block-mean values. The diagonal line from top left to bottom right shows the maximum spatial resolution, which can be represented by a spherical harmonic series up to a specific degree. The blue stars represent some examples for this. E.g. with a series up to degree and order 60 we can represent a spatial resolution or grid size of 3 degrees. Other examples are degree 360 corresponding to a 30’ grid and degree 2160 corresponding to a 5’ grid. A point observation contains the full spectral range without any limit (this is indicated by the red line in Figure 2). The green horizontal bar represents 5’x5’ block mean values as they have been used for computing the EGM2008 model. During estimation of the model one has to take care of frequencies above the maximum degree in order to avoid aliasing of the high frequencies to the estimated spectrum. This aliasing problem also can be regarded as a kind of omission problem for spherical harmonic analysis. For the model evaluation we solve the spherical harmonic series to a chosen degree and based on various spatial grids or on dedicated points by spherical harmonic synthesis. This is shown by the yellow bars and the black vertical line in Figure 2. The yellow bars show examples for solving the series up to degree and order 360 for global spatial grids with 10’, 30’ or 2 degrees resolution, respectively. In case the bar is above the diagonal line (in the white space) we can regard this as a kind of under-sampling, while the signal is over-sampled in case the bar does not reach the diagonal (in the light blue space). For a synthesis on a chosen point (e.g. coordinates of a GPS levelling point) the spectral content is limited to the maximum degree and order of the spherical harmonic series (black line at the bottom of Figure 2). When we now compare both results (from the model and the observation) we suffer from the omission error, which is represented by the blue line in Figure 2.

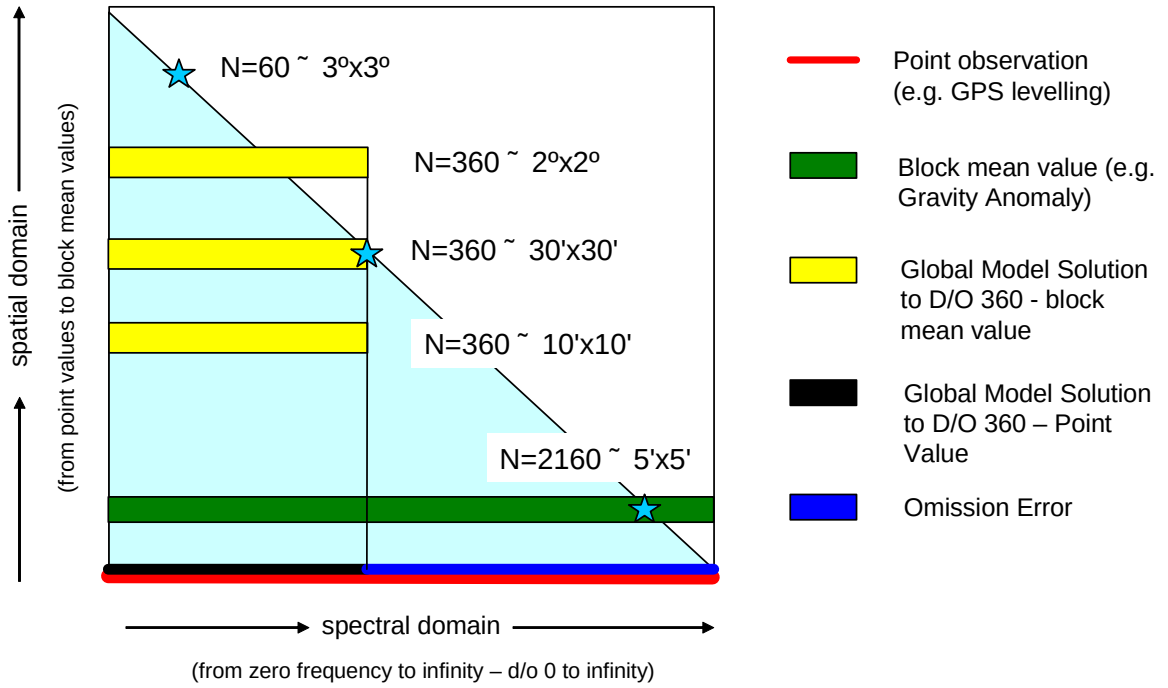


Figure 2: Description of omission error

From the situation described above we can conclude that in any conversion between space and frequency domain the omission error has to be taken into account by appropriate means. The problem is now to find the appropriate way to take the omission error into account. This in fact is not trivial, because one would need to know the global gravity field to infinity at any point in the world. In other words, as explained before, one can regard the omission error problem as a chicken-and-egg problem. For the actual evaluation of the EGM2008 model the following chapter describes the way of how the omission error is approximated in this study.

2.3 Model Evaluation Method

As described in the introduction two methods are applied in this study to estimate the quality of the EGM2008 global gravity field model. The first one compares model geoid heights with GPS levelling derived geoid heights and the second one estimates dynamic ocean topography solutions and checks them for consistency. The following paragraphs describe in detail how the computations are performed.

GPS Levelling Geoid Heights

Several GPS levelling geoid height data sets are available (see chapter 2.4). All data sets provide geoid heights for points on the Earth surface. We assume that all geoid heights refer to the tide free system, i.e. we use the tide free version of the EGM2008 coefficient data set. Then the following processing steps are performed:

1. The EGM2008 tide free model is solved for geoid heights at the location of the GPS levelling point (latitude, longitude and orthometric height) up to a selection of maximum degrees and orders n_{max}. The applied normal field corresponds to the one used for the GPS levelling geoid heights.
2. The omission error is approximated per point from a solution of the EGM2008 tide free spherical harmonic series for degree and order n_{max}+1 to 2190 (note: EGM2008

contains the full coefficient set up to degree 2160 and a selected set of coefficients for degrees 2161 to 2190).

3. The omission error is subtracted from the GPS levelling geoid height in order to make them spectrally consistent (at least approximately).
4. Geoid height differences are computed between the model geoid heights and the reduced GPS levelling geoid heights. The difference is regarded as a quality estimate for the global model.
5. For each regional GPS levelling data set a mean value of the differences is computed and subtracted in order to take into account inconsistencies in the height system definitions (or zero potential definition).
6. Finally the RMS of the "un-biased" geoid height differences is computed for each region and shown in chapter 3 for various models.
7. In addition geoid height slope differences between all points in a region are computed and a RMS per distance class is computed (for a more detailed description see Gruber et al, 2006). Results of this test are shown in chapter 3, too.

Dynamic Ocean Topography (Sea Surface Topography) Solutions

In order to compute sea surface topography solutions the geoid is subtracted from an altimetric mean sea surface (geodetic approach of sea surface topography determination). Again the tide free version of the EGM2008 model is used for this procedure.

1. The mean sea surface is converted to the tide free system. Note: Sea surface heights from altimetry are always given in a mean tide system, because sea level includes the contribution of permanent tides.
2. The mean sea surface model is converted to spherical harmonics up to very high degree and order applying numerical quadrature (degree and order 1800 for this study). Land areas are filled with geoid data from a global model (EGM96) in order to reduce strong leakage at coastal areas.
3. Mean sea surface heights and geoid heights are solved for a geographical grid at sea level by spherical harmonic synthesis up to degree n_{max}, which again is varying for different test cases. The same reference ellipsoid is used for both. By this approach both data sets are filtered in the same way, which means in other words, that the omission error can be reduced significantly when building the difference.
4. The difference between the filtered mean sea surface and geoid heights are computed for various ocean basins. Coastal areas are blended and not taken into account because of possible leakage from land.
5. Sea surface topography solutions are inspected visually in order to check the solutions for plausibility. Note: For the evaluation of EGM2008 we did not compare these results with oceanographic solutions, because from preliminary analyses it was found that the oceanographic model results deviate significantly. This needs further investigation. The results from these computations are shown in chapter 4.

2.4 Data Sets Applied

GPS levelling and dynamic ocean topography comparisons are performed for a set of gravity field models. The following models have been used:

- EGM96: Combined model complete to degree and order 360 (Lemoine et al, 1998). This is the well known fore runner of EGM2008.

- GGM02C: Combined model complete to degree and order 200 (Tapley et al, 2005). This represents one of the earlier GRACE based combined solutions and mostly applies the surface data sets as they have been provided by the EGM96 project.
- EIGEN-GL04C: Combined model complete to degree and order 360 (Förste et al., 2007). This model is based on a recent GRACE satellite solution, some new surface and altimetry data sets (e.g. Arctic gravity, GFZ in house mean sea surface), but also data from the EGM96 project.
- EIGEN-5C: Most recent combined model from GFZ/CNES complete to degree and order 360 (Förste et al, 2008).
- ITG-GRACE03S: Satellite-only GRACE model, which was applied as basis for the EGM2008 model (Mayer-Gürr, 2007). Note, this model is the only satellite-only model applied in the present tests. Therefore, we will experience some different behaviour as compared to all others.
- PGM2007A: This is the preliminary EGM2008 model complete to degree and order 2160 with some additional coefficients up to degree 2190, which was released in summer 2007 for evaluation purposes. It is based on a different satellite-only model and some preliminary surface data compared to EGM2008 (Pavlis et al, 2007). The model is included here in order to find out, if the final EGM2008 model shows differences to this preliminary solution. Note, this model is not published.
- EGM2008: This is the final combined model complete to degree and order 2160 with some additional coefficients up to degree and order 2190 (see Pavlis et al, 2008). This model represents the state-of-the-art of global high resolution modelling and is evaluated against all other models mentioned above in order to identify its performance.

Six different regional GPS levelling data sets are applied for comparison purposes. The point distributions for the different regions are shown in Figure 3. The following data sets have been used:

- Australia (Johnston & Manning, 1998): 197 points.
- Germany (Ihde & Sacher, 2002): 675 points in Germany.
- Europe (Kenyeres et al, 2006): 1233 data points from the EUVN-DA project (European Vertical Network – Densification A).
- Canada (Veronneau, 2007): 430 points from new Canadian network
- Japan (Nakagawa, 2003): 837 points along first order levelling lines.
- USA (NGS, 1999): Geoid on benchmark data set edited for some points exhibiting large deviations (finally 5168 points are taken).

For the dynamic ocean topography tests the Goddard Space Flight Center mean sea surface model was used (Wang et al, 2001). The model has been augmented with EGM96 geoid heights over land and transformed into a spherical harmonics up to degree and order 1800.

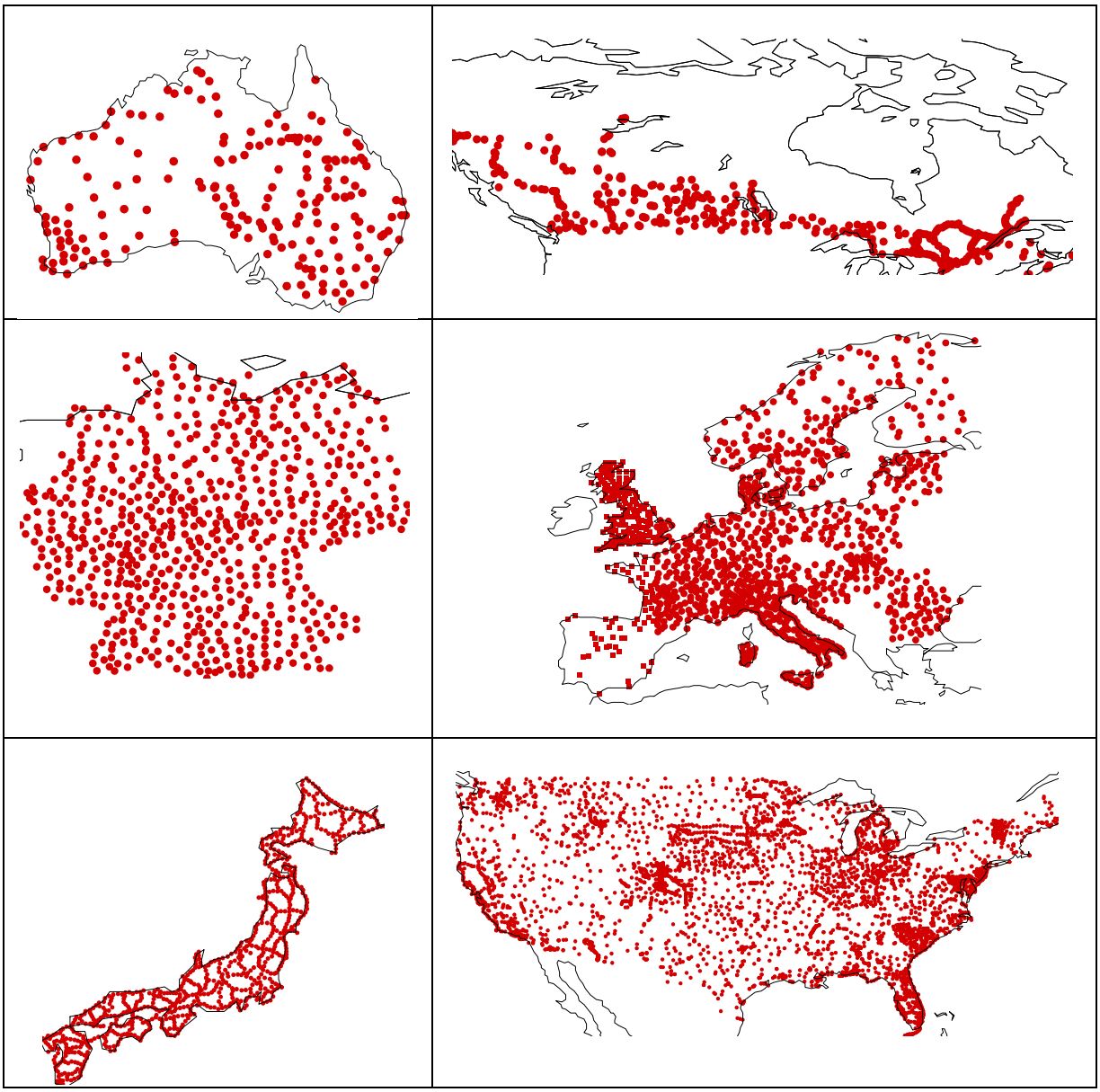


Figure 3: Distribution of geoid data points of the six regional GPS levelling Data sets. Australia (top left), Canada (top right, Germany (mid left), Europe mid right), Japan (bottom left), USA (bottom right).

3 EGM2008 Evaluation by Means of GPS Levelling Data

The results of the comparison of model derived geoid heights with GPS levelling geoid heights applying various degrees and orders of model truncation are shown in the following figures.

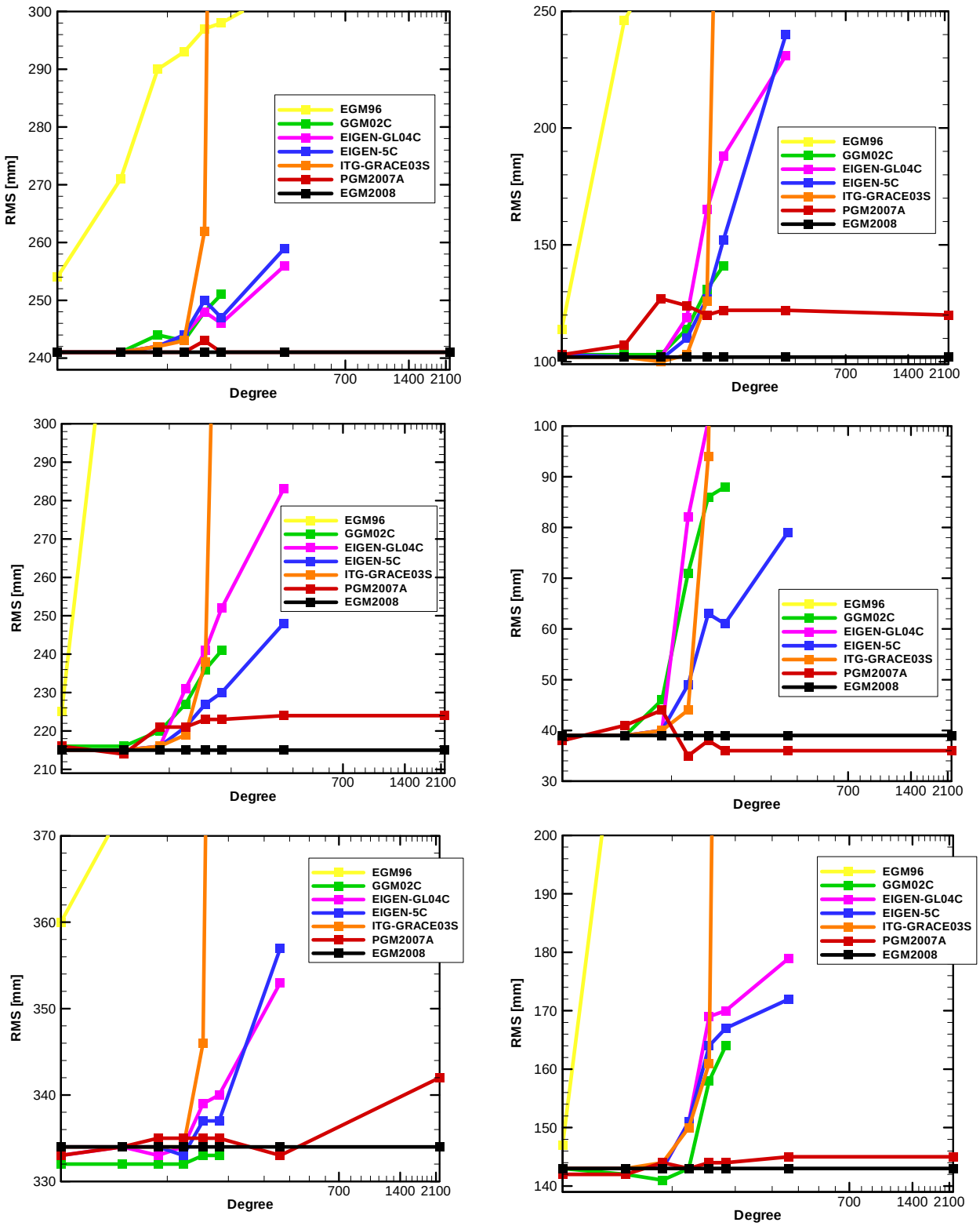


Figure 4: RMS of geoid height point differences (after subtracting the mean value) for various truncation degrees and various regions. All in [mm]. Top left: Australia, top right: Japan, mid left: Europe, mid right: Germany, bottom left: USA, bottom right: Canada.

Figure 4 shows the RMS of geoid height differences (after subtracting the mean difference) for the models under test truncated at degree and order 30, 60, 90, 120, 150, 180, 360 and 2160 respectively. Each square in the plots represents the RMS geoid height difference for the color coded model (see legend in each plot) for a region under test after eliminating a bias. Note: As explained above the omission error was estimated from the EGM2008 model and

subtracted from the observed GPS levelling geoid heights. For this reason the comparison is slightly biased towards the EGM2008 model, which is visible by the identical RMS values for each degree of truncation for this model.

Generally, one can conclude that all models exhibit a significant improvement with respect to the EGM96 model (coded in yellow). EGM96 shows much larger deviations to the GPS levelling geoid heights than any other model under test (for some figures RMS values are even outside the axis range). Another common feature in all figures is that the satellite-only GRACE model (ITG-GRACE03S coded in orange), which forms the basis for EGM2008, up to degree and order 90 is very close to the EGM2008 solution, while for higher degrees RMS values increase drastically. This indicates, that the EGM2008 model is dominated by the satellite-only information up to this degree, while surface and altimeter data start to contribute more and more for higher degrees. Nevertheless the satellite-only model performs quite well up to degree 90 and represents one of the best models available from GRACE-only information. The other combined models applied for comparisons show some different behavior (green, pink and blue curves). All three models are based on a GRACE satellite-only model and contain surface and altimeter data from various sources (many of them different to those used for EGM2008). For these models RMS of geoid height differences start to increase at degree 60 or 90 and are finally significantly above the EGM2008 curve. From this we can conclude that EGM2008 is based on significantly better surface data and that the transition from the satellite information to the surface data contribution is smooth. From the improvements from the pink to the blue curve one can see that the transition between the two data sources plays an important role, because both models are based mostly on the same input data, but for the more recent model (blue curve) the modeling has been improved. Finally when comparing the preliminary EGM2008 model (red curve) with the final one (black curve), one can identify, that for most regions the final model is slightly better than the preliminary one. Only for the German data set the preliminary model is slightly better, but on a very low level (2 mm RMS). This indicates that the final model is slightly superior to the preliminary model.

Regarding the level of RMS differences one can see that some GPS levelling data sets probably are not good enough for evaluation purposes. RMS differences for EGM2008 vary between 3.8 cm (for Germany) and 33.4 cm (for USA). In this sense, the global model can be applied for identifying problems in the GPS levelling geoid heights, specifically when regarding to long wavelength patterns in geoid differences. For this reason, for the both regions mentioned above the height differences are shown geographically. Figure 5 shows the results for the US GPS levelling data set for two models truncated at degree and order 360. From the differences one can see that both models show similar behavior in terms of the general structures. This is an indicator, that the GPS levelling data set is contaminated by some long term feature, which does not agree well with the model results. Excellent results exhibit the German data set. For this, we have RMS differences on the level of a few centimeters. Looking at the results in Figure 6 we can clearly see, that from left to right we get significant improvement. While for EGM96, there is a strong long wavelength feature in the differences, the result for EIGEN-5C is significantly better, but still shows some pattern in the differences. Finally, applying the EGM2008 model only some small systematic feature from North-West to South-East is left, which could be either a problem in the GPS levelling data or the EGM2008 model. From these very small differences, we can also conclude that the data set is well suited for evaluating global models (in contrast to the results from the US data set). For the other regions results vary (figures are not shown). Some data sets seem to be applicable for global model evaluation, some of them show problems like the US data set. In principle one can identify the level of suitability of the data sets from the level of the RMS

differences shown on the vertical axes in Figure 4. The sequence from best to worst comparison results per region is: Germany – Japan – Canada – Europe – Australia – USA.

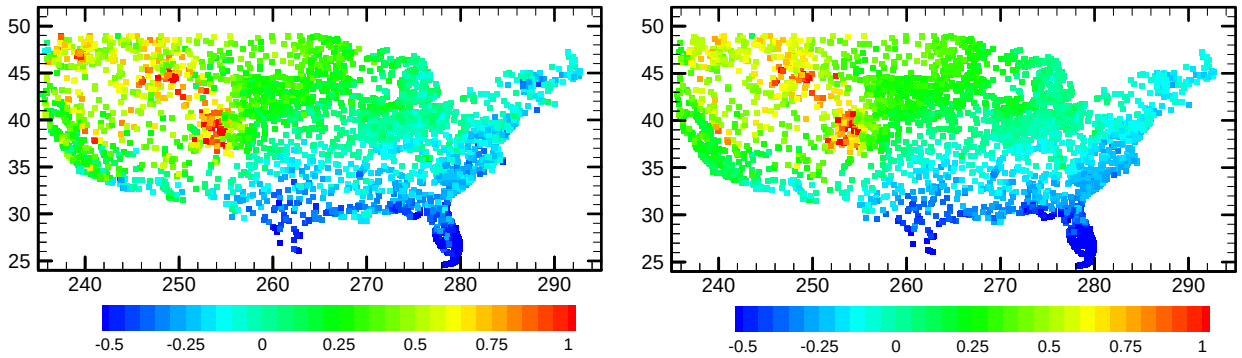


Figure 5: Geoid height differences for US GPS levelling data set: left: EIGEN-5C model, right: EGM2008. Both models are truncated at degree and order 360. All differences are given in [m].

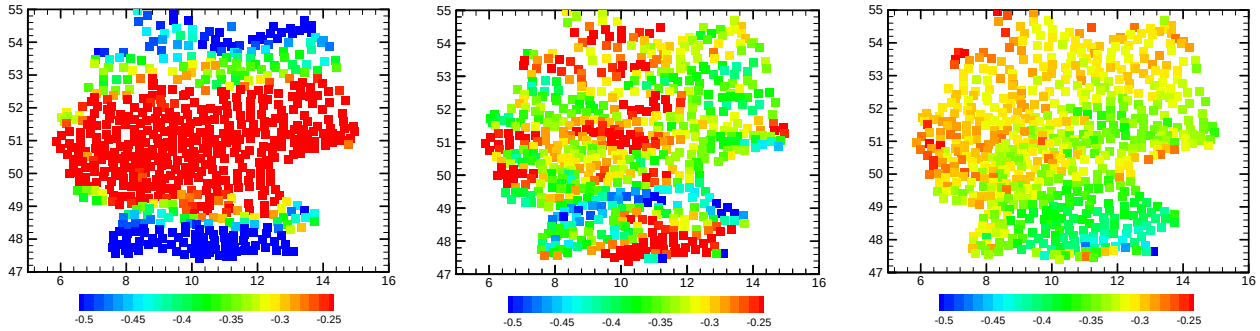


Figure 6: Geoid height differences for the German GPS levelling data set: left: EGM96, middle: EIGEN-5C model, right: EGM2008. All models are truncated at degree and order 360. All differences are given in [m].

In order to identify how the models perform over different distances, geoid slopes from the models and the GPS levelling data have been computed. These slope differences have been put into distance classes and RMS values of slope differences per distance class have been computed. Some results of this test are shown in Figure 7, for two regions and different degrees of truncation of the global models. Figure 7 supports the conclusions from the height differences tests described above. Up to degree 90 nearly no differences are visible for the GRACE based models. Only EGM96 exhibits much larger geoid slope differences (Figure 7 top left). By increasing the degree and order of truncation, differences between the solutions become more and more visible. The earlier GRACE model EIGEN-GL04C (pink curve) performs worse than all other GRACE based models for Canada (up to degree 150) and for Germany (up to degree 120 and 360) (see Figure 7 top right, bottom left and bottom right). Major improvements could be reached with the EIGEN-5C model (blue curve), which is computed by an improved combination technique, but which is based on more or less the same data as EIGEN-GL04C. This again shows that the proper combination technique of satellite and surface information is crucial for the final model performance. Also the GGM02C model performs partly better than EIGEN-GL04C, which probably has the same reason. Remarkable well performs the satellite-only GRACE model also up to degree and order 150 (orange curve) for the geoid slope differences. Finally, from Figure 7 we can conclude that the EGM2008 model (black curve) performs slightly better than the preliminary model (red curve), specifically for the longer distances, which are somehow related to the lower degrees and orders. This could be a hint to the effect of the replaced satellite-only model on the final solution.

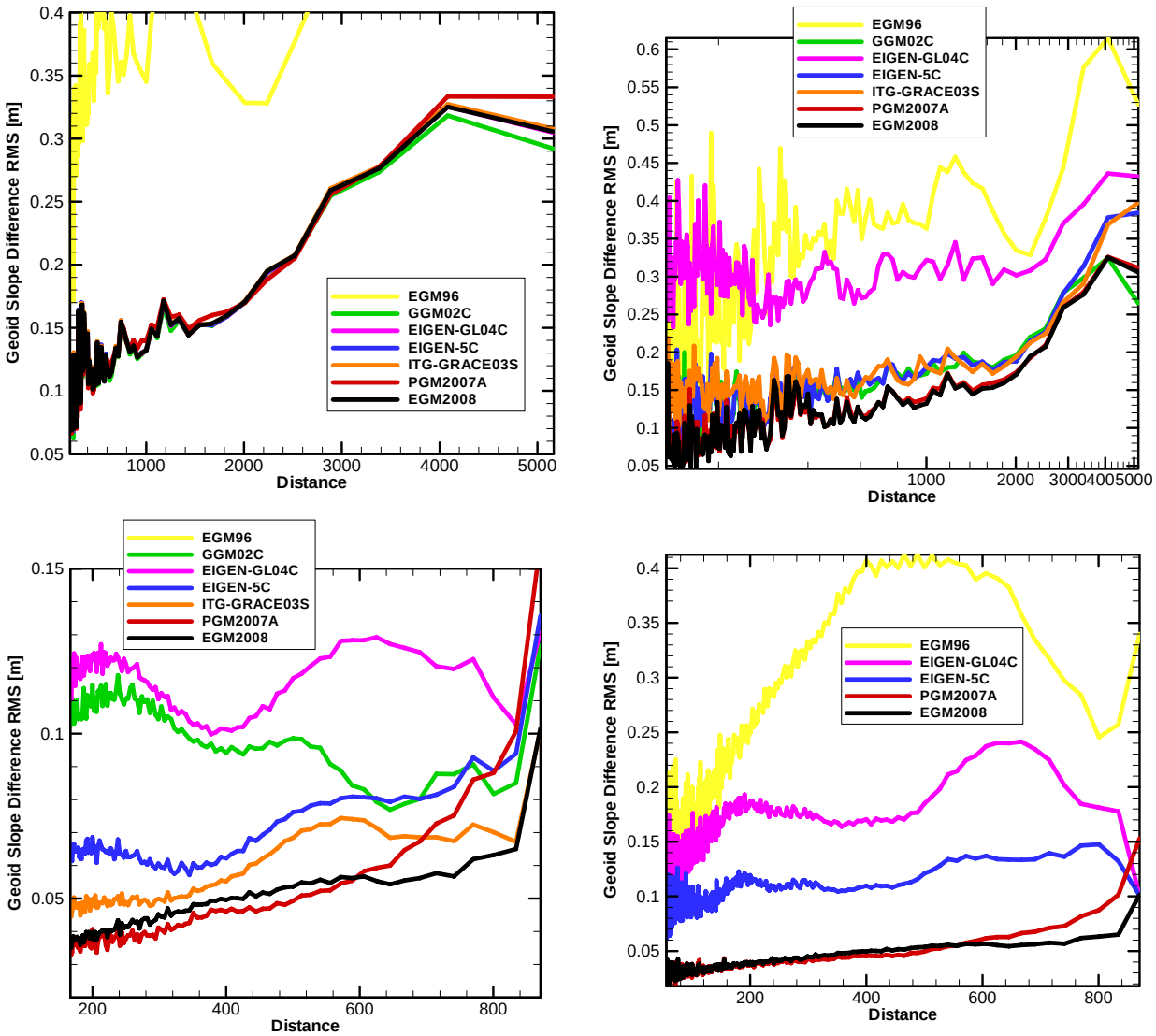


Figure 7: RMS of geoid slope differences per distance class: top left: for Canada, global models truncated at degree 90; top right: for Canada, global models truncated at degree 150; bottom left: for Germany, global models truncated at degree 120; bottom right: for Germany, global models truncated at degree 360.

4 EGM2008 Evaluation by Means of Ocean Topography Solutions.

Figure 8, Figure 9 and Figure 10 show solutions for the dynamic ocean topography computed by the procedure as described in chapter 2.3. For comparison purposes the North Atlantic was selected, because there the structure of the sea surface topography is better known than in other regions. Figure 8 shows the results for the global gravity field models available up to degree and order 360, while Figure 9 shows the dynamic ocean topography computed from the full resolution of PGM2007A and EGM2008.

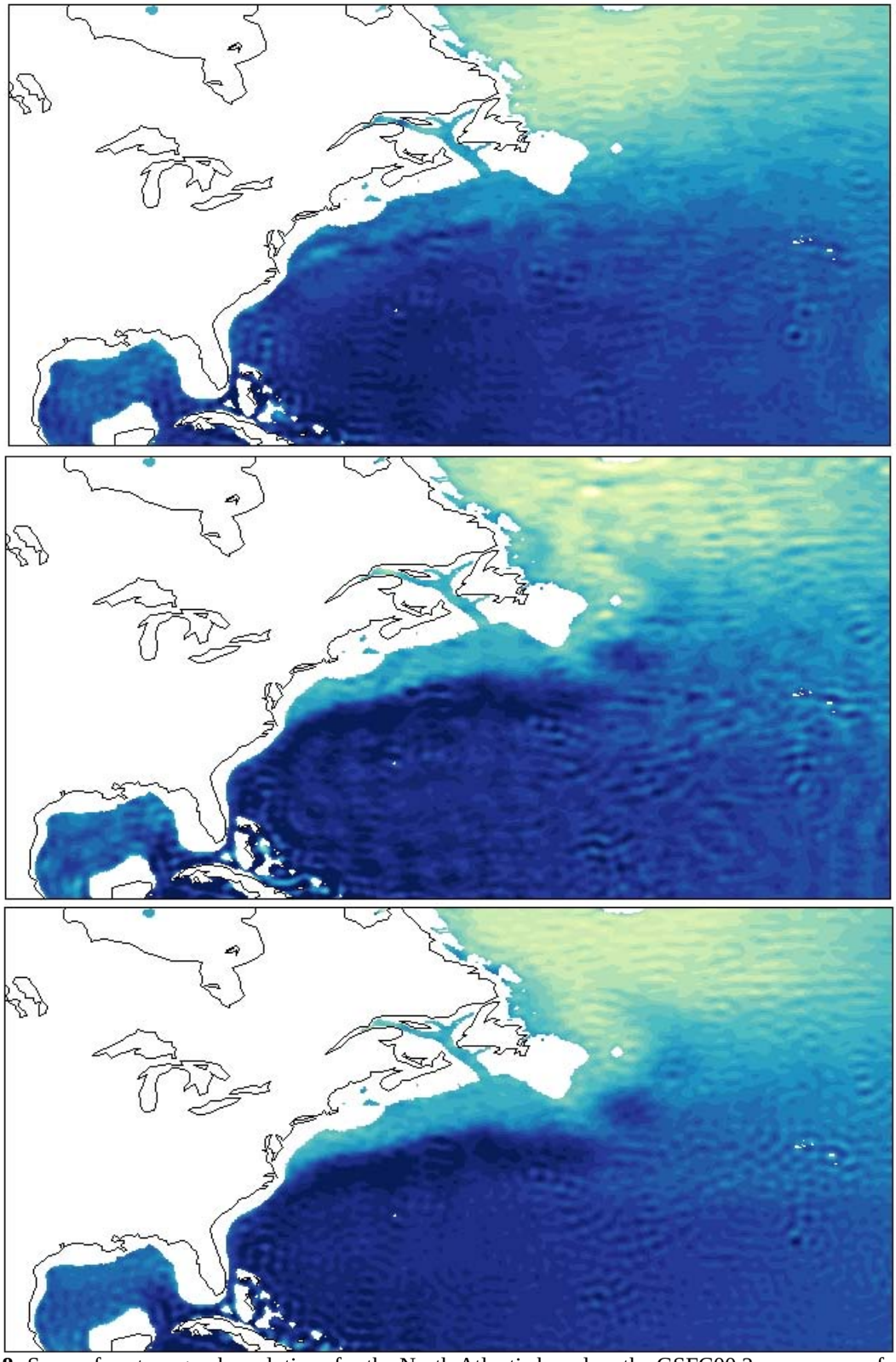


Figure 8: Sea surface topography solutions for the North Atlantic based on the GSFC00.2 mean sea surface and different global gravity field models up to degree and order 360 (full resolution of gravity models).: top: EGM96 solution; middle: EIGEN-GL04C solution; bottom: EIGEN-5C solution.

From Figure 8 we can identify that from top to bottom the structure of the dynamic ocean topography models becomes more and more realistic. This supports the results from the GPS levelling comparisons. Nevertheless, also for the bottom figure there is still some ringing pattern visible, which could be caused by truncating the spherical harmonic series of the mean

sea surface at degree and order 360. Even more realistic solutions are computed from the EGM2008 preliminary and final models (see Figure 9). There the ring structures disappear and well known ocean features become visible. The final EGM2008 solution (bottom figure) shows a slightly rougher structure than the preliminary solution. The reason for this is unknown, but has probably to do with some different altimeter data sets applied for the final model. From this we can not conclude, which of the two solutions is superior to the other. Finally for completeness, Figure 10 shows the global sea surface topography solution for the final EGM2008 model. In general, from the sea surface topography solutions one can draw similar conclusions than for the geoid height tests. There is visible a clear improvement with respect to the previous global gravity field models, which can be addressed to new data sets, to a better satellite-only solution, and partly also to the higher resolution of the global model.

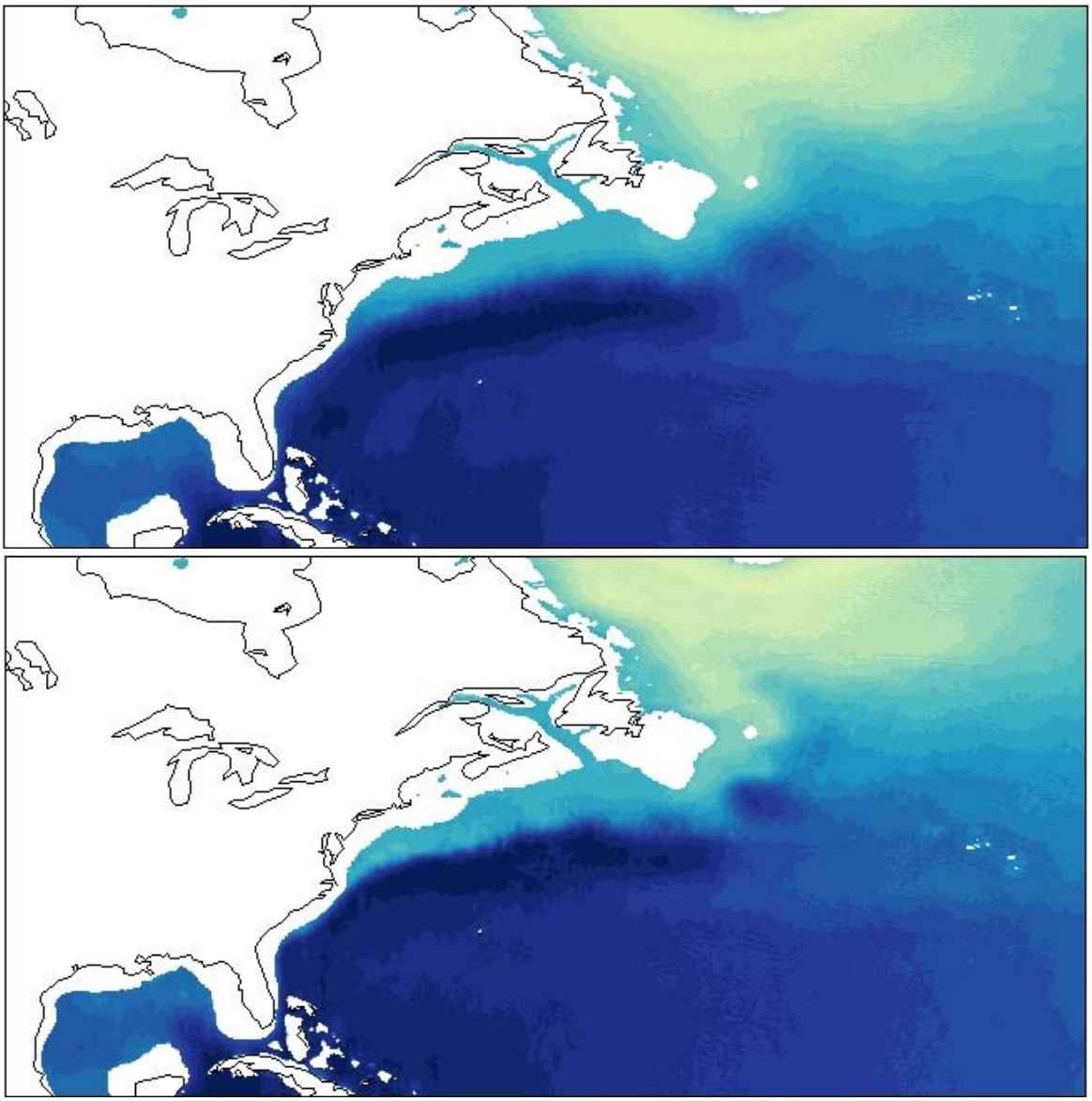


Figure 9: Sea surface topography solutions for the North Atlantic based on the GSFC00.2 mean sea surface and different global gravity field models up to degree and order 2190 (full resolution of gravity models).: top: PGM2007A solution; bottom: EGM2008 solution.

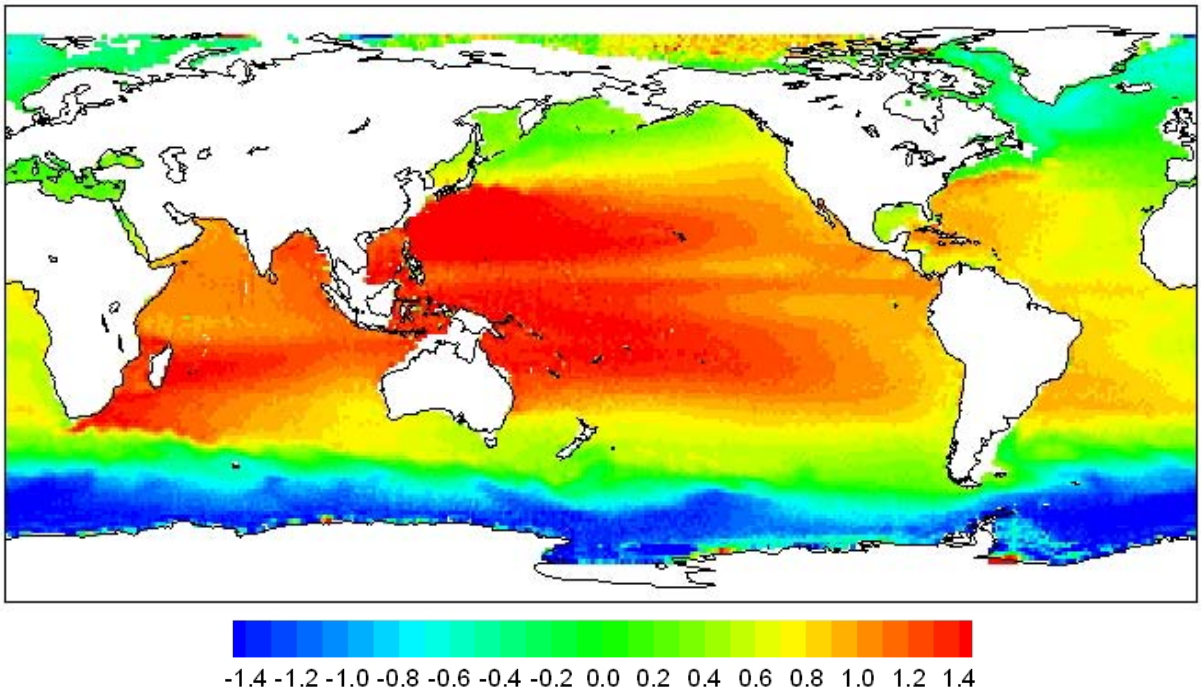


Figure 10: Global sea surface topography solution for the EGM2008 model up to full resolution [m].

5 Summary and Conclusions

The purpose of this paper is the evaluation of the EGM2008 global gravity field model by means of comparisons with GPS levelling geoid heights and sea surface topography solutions. In order to make such comparisons one has to take into account the problem of the omission error when comparing point observations with band limited information from global models (by truncation of the spherical harmonic series). In the approach applied, the very high degree harmonics of the EGM2008 solution are taken in order to compute the omission error. This may cause all comparisons to be slightly biased towards the EGM2008 model. For all other models in principle coefficient sets from two different models are mixed (model under test up to the degree of truncation, EGM2008 from truncation degree up to degree and order 2190). Nevertheless, it is assumed that the computations performed provide a rather fair information about the quality of the EGM2008 model.

The comparisons with the GPS levelling data clearly show that the new EGM2008 solution (also the preliminary model) performs significantly better than any other model, which has been used in the tests. This is true for all truncation degrees above 90. Up to degree and order 90 the solution is dominated by the satellite-only GRACE model, which also seems to perform very well in this spectral range. Not all data sets seem to be adequate to perform such an evaluation, because the GPS levelling data themselves, seem sometimes to have systematic errors. Therefore, for the final conclusions it is important to select adequate data sets. In our case we assume that the German, Canadian and Japanese data sets are suitable for such tests. The sea surface topography solutions computed by subtracting the geoid from a mean sea surface show more realistic features for the EGM2008 solution than for the others. This could be partly caused by effects of truncating the mean sea surface topography at degree 360 for the global gravity field models available with this resolution. Nevertheless, the solutions based on EGM2008, at least in the region investigated (North Atlantic), shows new, realistic and more detailed features than any other model. For this reason, also from this evaluation one can conclude that EGM2008 provides the best high resolution geoid.

Looking into the future, GOCE will provide more detailed satellite derived information for the global gravity field. Therefore we can expect with future updates of EGM2008, which will incorporate GOCE data as well, that the model can be further improved specifically in the range of wavelengths between degree 90 and 250.

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Evaluation of the EGM2008 Gravity Model

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The EGM2008 gravity model was evaluated using three techniques that assess the model performance on the modeling of satellite dynamics and the geoid accuracy over the ocean and land at different wavelengths. The tests include orbit tests with SLR and GRACE data, GPS leveling tests, and ocean circulation and marine geoid tests.

1. Orbit tests

Satellite orbit fit is one traditional measure of the gravity model accuracy in primarily the long-wavelength. This is a particularly demanding test for the GRACE-based gravity models because Earth gravity models had previously depended on the tracking to various geodetic satellites to determine the low degree part of the field, which led to these fields being noticeably tuned to their particular orbit inclinations. Satellite Laser ranging (SLR) data from a global network of well-determined tracking stations can provide an unambiguous and precise measurement of the satellite orbit accuracy, especially for compact spherical (cannonball) satellites such as Starlette, Stella, Ajisai, LAGEOS-1 and -2. These satellites, along with the BE-C satellite, are an important resource for measuring the long-term variations of the Earth's gravity field [Cheng and Tapley, 2004].

Table 1. Orbit characterization of satellites used in test

Satellite	*a* (m)	*e*	*i* (deg)	Arcs	Obs	Stations
Lageos 1	12266414	0.00396	109.86	119	557	13
Lageos 2	12165376	0.00141	52.65	116	550	13
Ajisai	7868998	0.00138	50.01	119	1233	15
Starlette	7332571	0.02007	49.84	119	709	14
Stella	7181361	0.00147	98.28	119	355	13
BEC	7492969	0.02577	41.16	119	964	9

A series of 3-day orbit fits to the SLR tracking of six satellites during the year 2003 was used to evaluate the performance of several gravity fields, including the EGM-96 [Lemoine et al., 1998], PGM2007A, EGM2008, GGM02C [Tapley et al., 2005], GGM03S [Tapley et al., 2007], EIGEN-GL04C (GFZ04C), EIGEN-GL05C (GFZ05C) [Förste et al., 2007], and ITG03S [Mayer-Guerr, 2007]. Table 1 lists the orbit characterization [semi-major axis (*a*), eccentricity (*e*) and inclination (*i*)] at 1 January 2003, the number of arcs, the 3-day average number of observations and the average number of tracking station for the satellites used in this analysis. The measurement and

force models were consistent with that used for RL 04 GRACE gravity solution [Bettadpur, 2007] based on the IERS2003 standard except for the gravity model, and the ITRF2005 station coordinate with corresponding EOP series were used. In addition to the same FES2004 ocean tide and ocean pole tide models, the same Atmosphere-Ocean De-aliasing (AOD) time series used in the RL04 GRACE processing were used in the SLR orbit fits.

The sensitivity of a satellite to the gravitational perturbation is altitude dependent. The maximum degree and order of the gravity field used are 20x20 for LAGEOS-1 and -2, and 70x70 for BEC, Starlette, Stella and Ajisai. The orbit fits were performed both with and without the adjustment, every 3-days, of once-per-revolution (1-cpr) empirical accelerations for the radial and cross-track components. When the empirical accelerations are not adjusted, more of the long-wavelength gravity model error signals are preserved in the SLR residuals. The drag coefficient, *Cd* for Starlette, Stella, Ajisai and BEC, and the empirical along-track acceleration, *Ct*, for LAGEOS-1 and -2, were adjusted every 12 hours.

The RMS of the SLR residuals should reflect the relative performance of the various gravity field models at the longer wavelengths. Figures 1 and 2 compare the SLR residual RMS from 3-day orbit fits from using EGM96 (red circles), PGM2007A (blue circles) and GGM02C (open circles) for LAGEOS-2 and Starlette. The pattern is similar for LAGEOS-1, Ajisai, Stella and BEC. Table 2 compares the results for the one year average RMS for 3-day orbit fits without the adjustment of once-per-revolution empirical accelerations using different gravity fields, including EGM96, PGM2007A, EGM2008, GGM02C, GGM03S, GFZ04C, GFZ05C and ITG03S.

Table 2. Average laser ranging residual RMS (cm) from 3-day orbit fits without adjusting the once-per-revolution (1-cpr) empirical accelerations

Model	Lageos-1	Lageos-2	Ajisai	Starlette	Stella	BEC
EGM96	1.49	1.34	5.60	4.95	9.14	11.07
PGM2007A	1.49	1.29	4.95	3.85	3.02	9.04
EGM2008	1.51	1.38	5.31	4.63	2.91	9.28
GGM02C	1.47	1.28	4.83	3.47	3.22	9.03
GGM03S	1.41	1.30	5.28	3.25	1.76	9.22
GFZ04C	1.44	1.26	4.62	3.06	2.56	8.93
GFZ05C	1.43	1.25	4.67	3.14	2.60	8.95
ITG03S	1.51	1.39	5.20	4.58	2.90	9.27

The results shown in Table 2 and Figures 1 and 2 suggest that in comparison with EGM96, the PGM2007A has slightly improved the lower degree (< 20) portion based on the tests for LAGEOS-2, but significantly improved the higher degree portion based on

the tests from Starlette, Stella, Ajisai and BEC. The performance of the EGM2008 model is very similar to the ITG03S model, which was used as satellite-only basis for EGM2008. One also can see a slight degradation compared to other GRACE models.

Table 3 compares the results with the adjustment of the once-per-revolution empirical accelerations in the orbit fit. The adjustment of the 1-cpr acceleration parameters can remove the effect of errors in the zonal and resonance coefficients, as well as accommodate part of the errors in the nongravitational force models. Comparison of the relative residual RMS for the cases (where the once-per-revolution acceleration parameters were estimated) attempts to isolate the improvements in the gravity coefficients other than the zonal and resonance coefficients.

Based on the orbit fits of these six geodetic satellites using SLR data, the performance of PGM2007A and EGM2008, in general, is at the same level as the other GRACE-based gravity models GGM02C, EIGEN-GL04C (GFZ04C), EIGEN-GL05C (GFZ05C), and ITG-GRACE03S(ITG03S) for degree/order less than 70. However, in comparison with PGM2007A, EGM2008 slightly degraded the orbit fit for all of satellites in the case of without adjusting the 1-cpr acceleration parameters (Table 2). The results were more mixed in the case of adjusting the 1-cpr acceleration parameters (Table 3).

Table 3. Average laser ranging residual RMS (cm) from 3-day orbit fits with adjusting the once-per-revolution (1-cpr) empirical accelerations

Model	Lageso-1	Lageos-2	Ajisai	Starlette	Stella	BEC
EGM96	1.04	0.97	5.24	3.58	6.54	9.04
PGM2007A	0.95	0.86	4.40	1.63	1.91	7.54
EGM2008	0.95	0.86	4.44	1.69	1.59	7.56
GGM02C	0.95	0.86	4.42	1.65	2.14	7.57
GGM03S	0.95	0.88	4.43	1.48	1.52	7.53
GFZ04C	0.95	0.86	4.41	1.64	2.56	7.49
GFZ05C	0.95	0.86	4.42	1.69	1.59	7.48
ITG03S	0.95	0.86	4.32	1.54	1.55	7.52

The last orbit test was to compare the residuals from the GRACE K-Band intersatellite range-rate residuals using EGM2008 with the model currently used for the Release-04 (RL04) processing at CSR. For the month of February 2008, the results were essentially identical, with a fit of 0.407 μ/sec for EGM2008 and 0.409 μ/sec for the RL04 processing.

2. GPS Leveling test

GPS leveling test is a comparison of the geoid undulation derived from GPS leveling data and the geoid undulation (N) calculated from a gravity model and/or terrestrial gravity data. This test is sensitive to the geoid components with wavelengths ranging from the shortest baseline to the longest baseline in the test network, reflecting the quality and treatment of satellite and/or surface gravity data used in the geoid determination. The RMS abort the mean of the ΔN at an area can be used to access the accuracy of the geoid over the land predicted from a gravity model (up to degree and order 360 in this test). The method, namely a 'degree-banded' approach (Huang and Véronneau, 2005) is used to perform high-pass filtering to the surface gravity data to allow the satellite model define the long-wavelength geoid components, and the surface gravity data determine the short wavelength geoid components, thus provides the most sensitive way to evaluate a satellite model without being seriously affected by the omission error of the satellite model, and isolation of the power of gravity signals to a certain degree range. Figure 3 compares the GGM02C, EIGEN-GL04C (GFZ04C), PGM2007A and EGM2008 geoids to 1149 GPS leveling data over Canada and the 6168 data for the US, respectively. The mean of the ΔN is calculated globally over Canada, but state-by-state for the US since there is a state-dependent systematic bias contained in the GPS leveling data over the US. The performance of the models below approximately degree 90 cannot be assessed since the cumulative GRACE model error is smaller than 2 cm that is within the noise range of GPS-leveling and surface gravity data. In the degree range from 90 to approximately 110, the results can be expected to mainly reflect the quality of the GRACE solution used. Above degree 110, the results can be expected to reflect the quality of the surface information incorporated. It is clear that EGM2008 model has significantly improved the geoid in these areas and approached to the noise level of GPS leveling data for both Canada and the US .

3. Marine geoid tests

More accurate mean dynamic ocean topography (DOT) maps can be used to determine the sub-surface geostrophic currents with greater detail. These circulation maps are very useful for evaluating the improvement of the geoid computed from a gravity field, since small changes in the geoid can lead to significant changes in the circulation, especially in the tropics. The DOT in this test is determined from the mean sea surface (CSRMSS98) minus the marine geoid from a test gravity field. The zonal and meridional circulation from a topography map are computed using forward-backward difference between adjacent grids and accounting for the changes in the area of an equi-angle grid away from the equator. See Tapley et al. (2003) for further details about this procedure.

The large-scale zonal and meridional geostrophic currents from various DOT maps are compared to the World Ocean Atlas 2001(WOA01) data relative to 4000 m (courtesy of V. Zlotnicki). The comparison is to degree/order 120, and 400 km smoothing has been

applied. Higher resolution is not supported since the WOA01 data has already had a similar level of smoothing applied. With the accuracy of the GRACE-based gravity solutions, this test may now be limited by errors in the long-term topography. We included the results for EGM96, GG02C, EIGN-GL04C, EIGN-GL05C, GGM03S, ITG-GRACE03S (GRACE-based component of EGM2008), PGM2007A and EGM2008 to evaluate the impact of the combination on the performance. Table 4 summaries the results. There is some degradation in the results from ITG-GRACE03S to EGM2008, but it seems to be relatively minor.

Table 4. Ocean circulation comparisons

Model	Standard Deviation (cm/s)		Correlation	
	Zonal	Meridional	Zonal	Meridional
EGM96	8.18	7.00	0.352	0.288
GGM02C	3.04	3.23	0.914	0.482
EIGEN-GL04C	3.01	3.01	0.916	0.543
EIGEN-GL05C	3.24	3.10	0.903	0.513
GGM03S	2.91	2.97	0.921	0.550
ITG-GRACE03S	2.91	2.94	0.921	0.558
PGM2007A	3.25	3.14	0.920	0.517
EGM2008	2.97	2.99	0.918	0.551

We demonstrate the improvement of EGM2008 over the ocean for smaller wavelengths by comparing it to mean sea surface profiles determined from TOPEX/Poseidon (T/P) data from September 20, 2002 to December 31, 2003, when T/P had been shifted to a new groundtrack between its old groundtrack. These data are used because the groundtrack is different from any of those from previous altimeter missions used to create the gravity models, and so represent new observations of the marine geoid. We create residuals along each satellite pass calculated from MSS - WOA01 DOT – geoid at different wavelength filtering (shorter and longer than 300 km [half-wavelength]). The WOA01 DOT is computed to a reference level of 4000 m, and the geoid is evaluated using coefficients to spherical harmonic degree/order 360. The mean was removed along each altimeter pass before computing the RMS. The results indicate that EGM2008 and PGM2007A perform well at both the longer and the shorter wavelengths. Since the GGM02C model was extended to 360x360 using EGM96, it is not surprising that the results are the same for the short wavelengths. PGM2007A and EGM2008 demonstrate smoother geoid residuals (see Fig. 4), which is likely the reason for the better statistics shown in Table 5.

Figure 4 illustrates the smooth geoid residuals for EGM2008. The residuals are the difference between a ‘high-frequency DOT’ defined as (GSFCMSS00-geoid) and the same DOT smoothed to 900 km. This removes most of the long-wavelength dynamic topography so that smaller scale artifacts can be seen. In previous models, significant ‘striations’ were apparent in such maps.

Table 5. Short-wavelength geoid comparisons (to degree and order 360)

Model	> 300 km	< 300 km
EGM96	9.3	12.7
GGM02C(+EGM96)	8.2	12.7
EIGEN-GL04C	8.7	13.1
EIGEN-GL04C	7.8	12.6
PGM2007A	7.5	11.7
EGM2008	7.6	11.8

Summary

The orbit fit tests show all recent GRACE-based models performing similarly. The GPS leveling test also indicates excellent performance with EGM2008, and the test now appears to be limited by the data errors rather than geoid errors. In the ocean circulation test, PGM2007A and EGM2008 can be used to recover accurate circulation features, as demonstrated by the high correlation with the World Ocean Atlas 2001. EGM2008 also performs best in the short-wavelength marine geoid test, providing smooth short-wavelength marine geoid residuals.

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Fig. 1 and 2 SRL Residual RMS from 3-day Orbit Fit

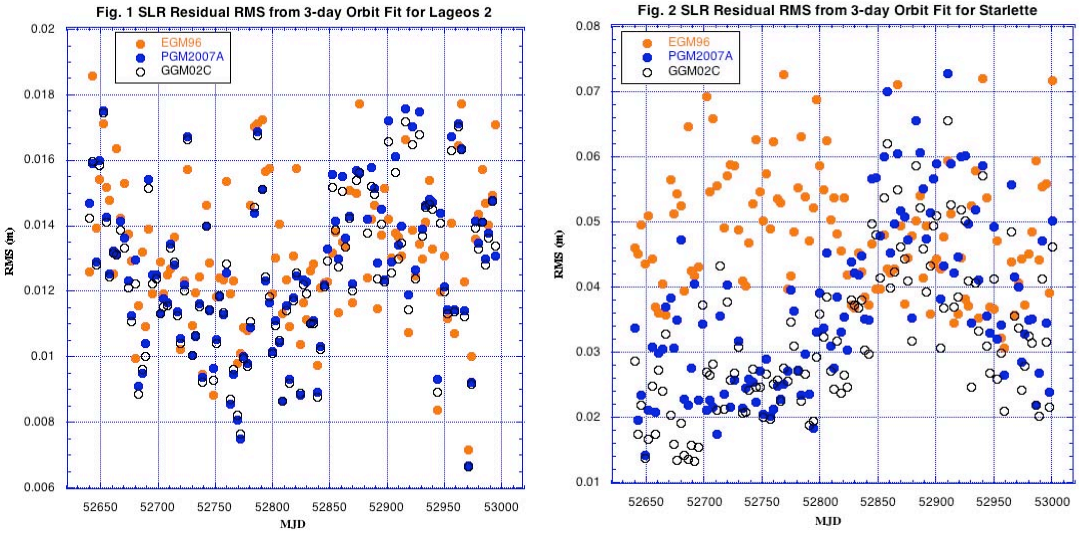


Fig. 3 Degree-banded GPS leveling test over Canada and US

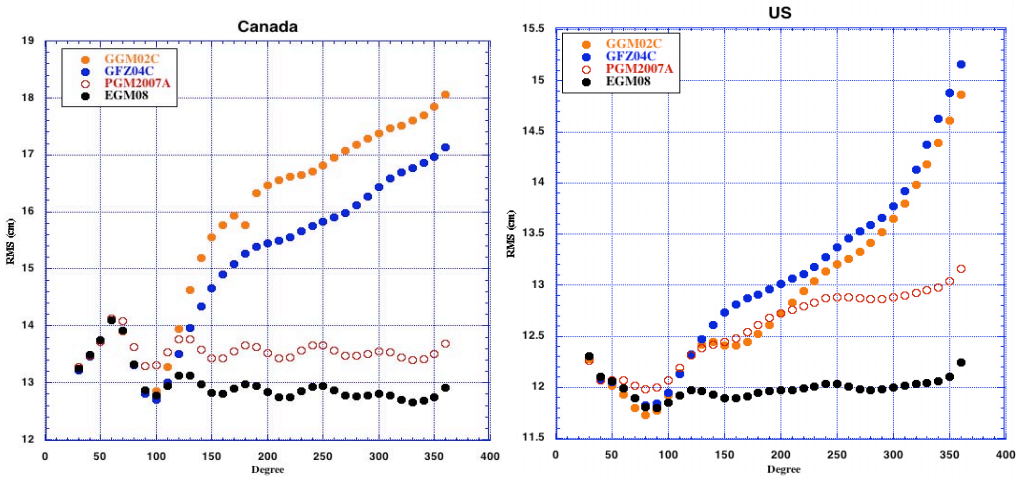
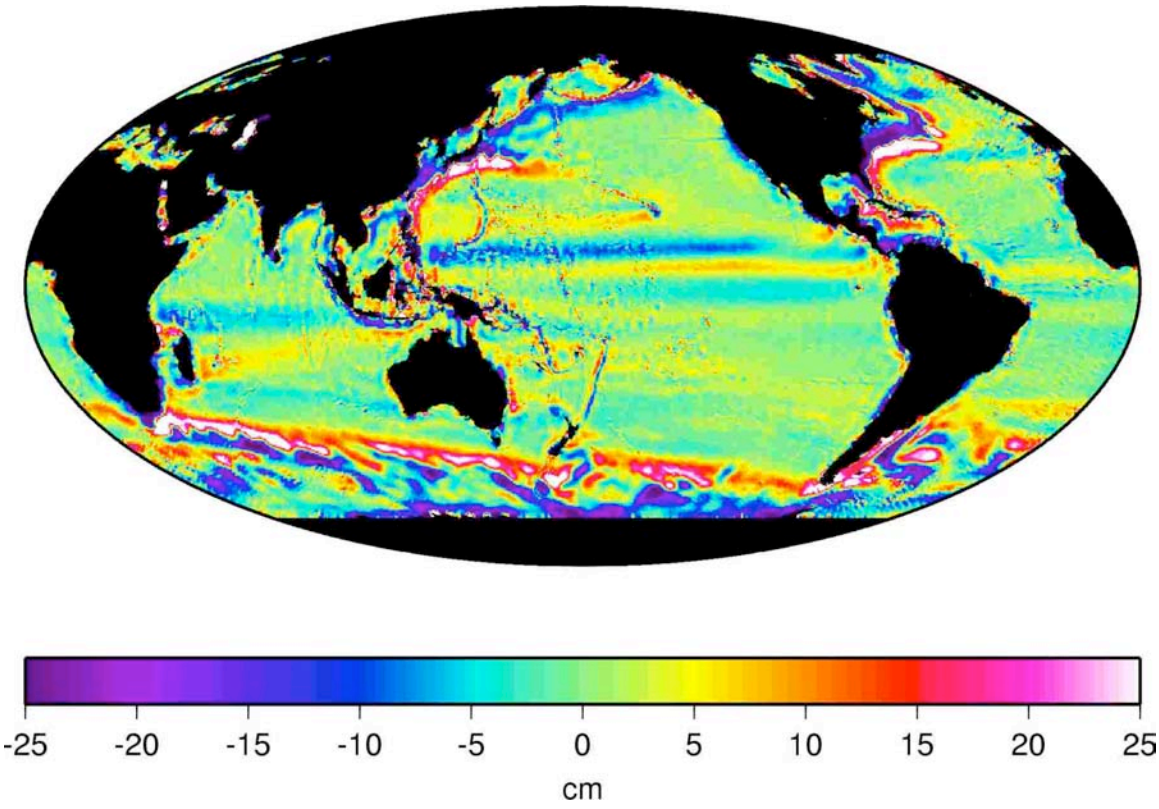


Fig. 4 Short wavelength geoid residuals from EGM2008 gravity model



Evaluation of EGM2008 by comparison with other recent global gravity field models

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Abstract

The new gravity field model EGM2008 has been evaluated by comparisons with other, satellite-only as well as combined global gravity field models. Our evaluation comprises orbit adjustment tests, comparisons of the spectral behaviour, GPS/leveling tests and ocean geoid comparisons.
For the GPS/leveling tests and the ocean geoid comparisons the new EGM2008 model outperforms all other tested models. Orbit adjustment tests for CHAMP, GRACE and other satellites show a very good inner consistency for EGM2008 and its corresponding satellite-only model ITG-GRACE03S. In these tests EGM2008 shows no major performance differences to the other tested models.

Keywords

Earth gravity field model, Global gravity field recovery, GRACE, CHAMP, LAGEOS, EGM2008, ITG-GRACE03S, EIGEN-5, GPS/leveling, Ocean geoid

Introduction

The recently released new global gravity field model EGM2008 (Pavlis et al. 2008) with a resolution of maximum degree and order of 2159 (including additional coefficients extending to degree 2190 and order 2159) represents a milestone in the development of global gravity field models. It is of general interest to evaluate this new high resolution model by comparison with other recent global gravity field models. In this paper we present results of the comparison of the new EGM2008 with other satellite-only as well as combined models. Our evaluation of EGM2008 comprised orbit computations, comparisons of the spectral behaviour, GPS/levelling tests and comparisons of the ocean geoid.

The global gravity field models included in our evaluation of EGM2008 were the following satellite-only and combined models:

- **EGM96** (Lemoine et al. 1998)
  This is a combined gravity field model of maximum degree/order 360. This pre-CHAMP model has been composed from various SLR and other satellite data and terrestrial gravity data from gravimetry and altimetry. Since the launch of CHAMP and GRACE, EGM96 is no longer state of the art. But since this model represented a standard during the last decade, we included it in some of our evaluation tests.

- **GGM02C** (Tapley et al. 2005),
  This model is a combination of the coefficients of the GRACE-only model GGM02S (Tapley et al. 2005) with EGM96 and has a maximum degree/order of 200. It was computed by the Center for Space Research (CSR) at the University of Texas at Austin.

- **GGM03S** (Tapley et al. 2007)
  This recently released GRACE satellite-only model has a maximum degree/order 180. It can be considered as an upgrade of GGM02S including the latest GRACE CSR processing standards release 4 (Bettadpur 2007)

- **ITG-GRACE03** (Mayer-Gürr et al. 2007)
  This is a GRACE satellite-only model of a maximum degree/order of 180, published by the Institute for Theoretical Geodesy (ITG) of the University of Bonn. The coefficients of this model including their variance/covariance matrix have been used as satellite-part in the combination of terrestrial and satellite data for EGM2008.

- **JEM01-RL03B** (by courtesy of Mike Watkins and Dah-Ning Yuan, 2008)
  This GRACE satellite-only model is of a maximum degree/order of 120. The originator of this model is NASA/JPL in Pasadena. JEM01-RL03B has been computed in accordance to the JPL Level-2 Processing Standards (Watkins and Yuan 2007). The coefficients of this model including their variance/covariance matrix have been used as satellite-part of PGM2007A, a preliminary version of EGM2008.

- **EIGEN-GL04C** (Förste et al. 2008a)
  This combined gravity field model has been released in 2006. It is an outcome of the joint gravity field processing between GRGS Toulouse and GFZ Potsdam. The satellite-part of EIGEN-GL04C is based on GRACE and LAGEOS data and the maximum degree/order of this model is 360.

- **EIGEN-5C** and **-5S** (Förste et al. 2008b)
  The combined gravity field model EIGEN-5C is an upgrade of EIGEN-GL04C and has a maximum degree/order of 360. The model is again a combination of GRACE and LAGEOS mission data of a maximum degree/order 150 (= **EIGEN-5S**) plus 0.5 x 0.5 degrees gravimetry and altimetry surface data. The combination of the satellite and surface data has been done by the combination of normal equations, which are obtained from observation

equations for the spherical harmonic coefficients. The included satellite data have been processed by GFZ Potsdam (GRACE for February 2003 - January 2007) and GRGS Toulouse (GRACE for August 2002 - January 2007 and LAGEOS for January 2002 – December 2006). The satellite data processing has been done in accordance to the GRACE GFZ Level-2 Processing Standards for Release 4 (Flechtner 2007). This comprises for instance arc lengths of 1 day for GRACE and 10 days for LAGEOS including the usage of EIGEN-GL04C as a-priori gravity field.

The used surface data are identical to those included in EIGEN-GL04C except of new gravity anomaly data sets for Europe (H. Denker, IfE Hannover, 2007, personal communication), the Arctic Gravity Project gravity anomaly data (Forsberg and Kenyon 2004) as updated in 2006 and newer Australian gravity anomalies (W. Featherstone, Curtin University of Technology, personal communication, 2008)

Spectral behaviour

Figure 1 shows a comparison of the spectral behaviour of EGM2008 versus some other recent global gravity field models. This plot displays the degree variance differences of the long-to-medium wavelengths range up to degree and order 240 for EGM2008 in comparison to EIGEN-5C, EIGEN-5S, GGM03S and ITG-GRACE03S.

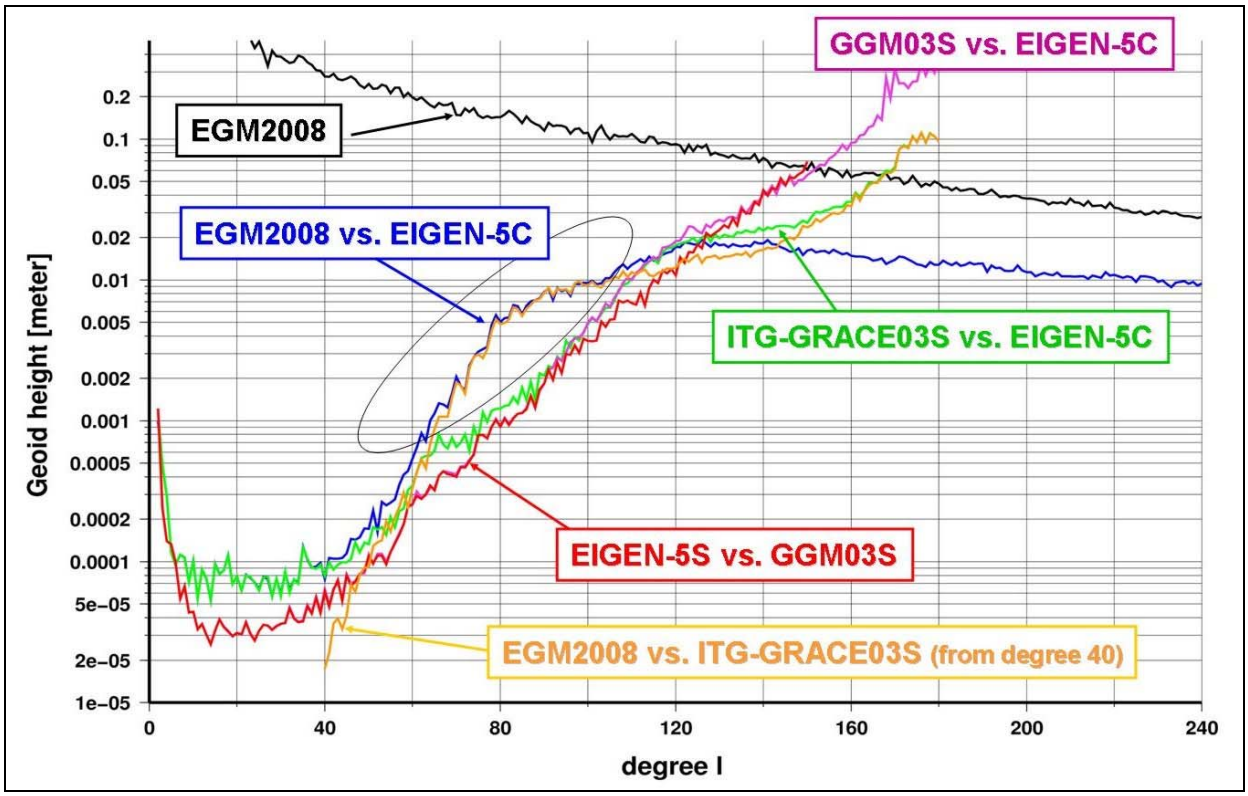


Figure 1: Degree variances in terms of geoid heights for EGM2008 in comparison to other global gravity field models

When looking into the degree variance behaviour of EGM2008 (figure 1, blue and yellow lines), we see noticeable differences to the other displayed gravity field models (green, red and purple lines): Whereas the EIGEN-models, GGM03S and ITG-GRACE03S have a very similar spectral behaviour up to degree/order 120, the EGM2008 degree variances show a remarkable different behaviour between degree 70 and 100 (this is marked by the blue circle in figure 1). This “bump” should be caused by the combination of the terrestrial data with GRACE data, since ITG-GRACE03S doesn’t show this behaviour (see the yellow line). It is remarkable that even the ITG-GRACE03S model, on which the EGM2008 is based, shows a behaviour which is very similar to the other models. Compared with the combined model EIGEN-5C, we suspect that the “bump” in the degree variance spectrum of

EGM2008 between degrees 70 and 100 is caused by a stronger weighing of the terrestrial data (which exhibits more signal power in this frequency band) versus the satellite data.

Orbit adjustment tests

One measure of the long-to-medium wavelength accuracy of a gravity field model is the fit of observations to the adjusted satellite orbit. In this study we calculated satellite laser ranging (SLR) residuals for various satellites in two different manners:

- For CHAMP and GRACE after an orbit determination by using GPS-SST and accelerometer data (CHAMP and GRACE) and K-Band Range-Rate data (GRACE), where the SLR measurements were not included into the orbit adjustment.
- For a number of other satellites after on orbit determination by using SLR data (including PRARE and DORIS observations for ERS-2 and ENVISAT, respectively).

The used satellites, the number of the included SLR observations and the measurement time periods are given in table 1. Table 2 gives the parameterization of the adjusted orbits except of CHAMP and GRACE. For CHAMP and GRACE the orbit fits were performed with the adjustment of the following empirical parameters in addition to the orbital elements, GPS ambiguities and clocks:

- One scaling factor and one bias parameter per arc for each of the three acceleration components,
- Additionally for CHAMP: one thruster parameter per arc and thruster pair, one scaling factor per component and arc for the Lorentz force on the accelerometer proof mass,
- Additionally for GRACE K-band: Range bias 1/rev per arc, range acceleration and range-rate bias per revolution

Satellite	**Number of included SLR observations / Data period / Tested arcs: Number and lengths**
GFZ-1	2029 / October 1995 / 5 x 3 days
STELLA	1528 / October 1997 / 5 x 3 days
STARLETTE	1815 / October 1997 / 5 x 3 days
AJISAI	6760 / October 1997 / 5 x 3 days
LAGEOS-1	3140 / October 1997 / 3 x 6 days
LAGEOS-2	2591 / October 1997 / 3 x 6 days
ERS-2*	7944 / September – October 1997 / 6 x 6 days
ENVISAT**	10176 / July 2002 / 7 x 4...8 days
WESTPAC	1587 / July – August 1998 / 5 x 6 days
JASON	20003 / Nov ... Dec 2004 / 6 x 10 days
CHAMP	358 / October 2001 / 4 x 1.5 days
GRACE	592 / September 2002 / 4 x 1.5 days

Additionally included other observations for ERS-2 resp. ENVISAT:
* 24837 PRARE-Range and 24435 PRARE-Doppler observations
** 130404 DORIS observations

Table 1: SLR data and test arcs of the orbit computation tests

The orbit fit results for CHAMP and GRACE are given in table 3. We carried out our orbit computations for these two satellites with different maximum degrees of the spherical harmonic coefficients to investigate possible degree-related differences between the tested gravity field models. First of all, the obtained best orbit fit residuals of all tested models are of the same order of magnitude of about 5 cm. This finding indicates that EGM2008 has no major performance differences with the other tested models for the investigated degree range up to 150. But when looking in detail we see noticeable differences between the tested models:

- The residuals for EGM2008 and ITG-GRACE03S decrease continuously for CHAMP as well as for GRACE when increasing the maximum used degree from 70 up to 150. This finding indicates a good inner consistency of both gravity field models with respect to CHAMP and GRACE and is in contrast to all the other tested models, which show a different behaviour for GRACE. Here with the other models the best orbit adjustment results were already reached around degree 90, but the residuals rise again by a few millimetres higher maximum degrees are used.
- The best orbit fit results are obtained with GGM02C in the case of CHAMP and with the EIGEN models and GGM03S in the case of GRACE.

Satellite	**Cd* and Cr****	**Empirical accelerations**
GFZ-1	Cd: 1/6h	1/rev per arc for along- and cross-track
STELLA	Cd: 1/day	1/rev per arc for along- and cross-track
STARLETTE	Cd: 1/day	1/rev per arc for along- and cross-track
AJISAI	Cd: 1/day	1/rev per arc for along- and cross-track
LAGEOS-1	—	1/rev per arc for along- and cross-track + 1/day for along-track
LAGEOS-2	—	1/rev per arc for along- and cross-track + 1/day for along-track
ERS-2	Cd: 1/day	1/rev per arc for along- and cross-track
ENVISAT	Cd: 1/6h Cr: 1/arc	1/rev per arc for along- and cross-track
WESTPAC	Cd: 1/day	1/rev per arc for along- and cross-track
JASON	Cd: 1/6h Cr: 1/arc	1/rev per arc for along- and cross-track

*Cd = Scaling factor for the atmospheric drag
**Cr = Scaling factor for the solar radiation pressure

Table 2: Parameterization (in addition to the solved-for orbital elements) of the adjusted orbits, except of CHAMP and GRACE

It should be of interest to compare the results between combined models and their corresponding satellite-only models:

- In the case of CHAMP, EGM2008 gives noticeable larger residuals for all tested maximum degrees than ITG-GRACE03S. The differences between the residuals of both models are larger than one millimetre. For GRACE, EGM2008 shows a better performance than ITG-GRACE03S between degree 70 and 110, but ITG-GRACE03S gives again smaller residuals for degrees 120 and 150. From our point of view, these results for CHAMP and those of the degrees 120 and 150 for GRACE indicate a slight degradation of EGM2008 versus ITG-GRACE03S for the tested degree range which could be due to a not-optimum combination of the terrestrial data with the satellite components.
- In contrast to EGM2008/ITG-GRACE03S, the differences between the corresponding orbit fit residuals for the pair EIGEN-5S/C are significantly smaller (only a few tenths of millimetres except of degree 70 for CHAMP)

The orbit adjustment results for the other tested satellites are given in table 4. Again we used different maximum degrees for the gravitational spherical harmonic coefficients to investigate possible degree-related differences between the tested gravity field models. For most of the satellites we tested maximum degrees of 70 and 120 while for GFZ-1 the orbit adjustment tests has been carried out for a wider range comprising maximum degrees between 50 and up to 150.

Satellite	max. degree used	GGM02C	GGM03S	JEM1-RL03B	EIGEN-GL04C	EIGEN-5S	EIGEN-5C	ITG-GRACE03S	EGM2008
CHAMP	70	17.62	16.71	19.66	17.94	17.68	17.86	15.97	16.74
	80	9.39	9.51	9.48	9.50	9.66	9.69	8.69	9.34
	90	6.12	6.10	6.15	6.12	6.14	6.14	5.96	6.02
	100	5.89	5.96	5.95	5.97	6.01	6.03	5.58	5.97
	110	5.48	5.53	5.51	5.61	5.60	5.59	5.39	5.57
	120	5.32	5.45	5.35	5.44	5.55	5.51	5.38	5.51
	150	5.19	5.44	--	5.41	5.56	5.49	5.30	5.46
GRACE	70	14.11	15.00	14.03	14.93	14.92	14.93	16.06	15.49
	80	7.39	6.54	7.35	6.51	6.43	6.44	8.24	7.27
	90	5.76	5.05	5.75	5.03	4.92	4.93	6.30	5.57
	100	5.40	5.20	5.41	5.20	5.09	5.09	6.12	5.54
	110	5.61	5.31	5.64	5.31	5.20	5.19	5.84	5.45
	120	5.50	5.28	5.55	5.25	5.17	5.15	5.39	5.46
	150	5.54	5.27	--	5.24	5.19	5.14	5.38	5.43

Table 3: SLR residuals (cm) after an orbit determination based on GPS-SST and accelerometer data (CHAMP, GRACE) and K-Band Range-Rate data (GRACE). The SLR data were not included in the orbit adjustment.

For all satellites except of GFZ-1 there is practically no change of the orbit adjustment residuals for EGM2008 as well as for the other tested models when the maximum used degree is decreased from 70 to 120. The differences between the results for degrees 70 and 120 are less than about half a millimetre for all gravity field models, which does not indicate any significant degree dependence. This is not a big surprise, since all these satellites orbit at higher altitudes than CHAMP, GRACE and GFZ-1, resulting in a lesser sensitivity to the gravity field (i.e., the effect of model differences is attenuated).

For GFZ-1 the orbit adjustment results are different. This satellite has a low altitude of about 400 km and is sensitive for spherical harmonic coefficients beyond degree 70 (König et al 1999). Therefore we started with a maximum degree of already 50 and increased it up to 150. First of all, the orbit adjustment residuals for GFZ-1 rise for all tested models including EGM2008 when increasing the maximum used degree up to 150. But this is not a surprise, since all tested models are based on GRACE data which means that these models are "tailored" for GRACE-like, near polar satellite orbits. It is therefore not unusual to find a worse inner consistency for other orbit types like GFZ-1 with its inclination of 51.6°. On the other hand, it's remarkable that the orbit adjustment residuals are nearly identical up to degree 90 for all tested gravity field models. Up to degree 90 the differences between all models are on the scale of a few tenths of millimetres only, which is insignificantly small. But when increasing the maximum degree up to 120 and 150 the differences between the tested models rise up to about 1 cm and EGM2008 gives the worst results. Like our orbit test results for CHAMP and partially for GRACE (see table 3) ITG-GRACE03S performs again better than EGM2008. The best orbit fit results for GFZ-1 at the maximum used degree of 150 were obtained for EIGEN-models and GGM03S.

In the previous section we reported a remarkably different spectral behaviour in the degree range from 70 to 100 for EGM2008 compared to all other tested models, i.e. the "bump" for EGM2008 in figure 1. If such a "bump" has an impact on the quality of EGM2008, this should be visible in the degree dependence of the orbit tests, in particular in comparison with ITG-GRACE03S, whose spectrum looks very similar to the other tested models. But the results for CHAMP and GRACE indicate no rising degradation of EGM2008 with respect to ITG-GRACE03S when increasing the maximum used

Satellite	Max. degree used	GGM02C	GGM03S	JEM1-RL03B	EIGEN-GL04C	EIGEN-5S	EIGEN-5C	ITG-GRACE03S	EGM2008
GFZ-1	150	14.49	14.14	—	13.81	13.62	14.23	14.34	15.00
	120	14.34	13.86	14.05	13.78	13.78	14.10	14.14	14.70
	110	13.16	12.68	12.76	12.62	12.71	12.86	12.99	13.55
	100	14.75	14.10	14.18	13.93	14.14	14.43	14.55	14.93
	90	8.09	8.04	8.05	8.08	8.05	8.05	8.04	8.11
	80	10.81	10.72	10.73	10.76	10.72	10.72	10.70	10.65
	70	12.07	12.03	12.05	12.01	12.01	12.01	12.01	12.02
	60	20.28	20.28	20.24	20.23	20.22	20.22	20.21	20.22
	50	31.79	31.77	31.76	31.76	31.74	31.74	31.73	31.75
STELLA	120	3.22	2.91	3.10	2.93	2.92	2.92	2.96	2.92
	70	3.25	2.91	3.13	2.92	2.91	2.91	2.94	2.89
STARLETTE	120	2.44	2.81	2.49	2.54	2.53	2.53	2.55	2.54
	70	2.43	2.78	2.49	2.54	2.54	2.54	2.56	2.53
AJISAI	120	3.17	3.37	3.18	3.16	3.15	3.15	3.17	3.18
	70	3.17	3.37	3.18	3.16	3.15	3.15	3.17	3.18
LAGEOS-1	120	1.02	1.03	1.02	1.01	1.01	1.01	1.01	1.02
	70	1.02	1.03	1.02	1.01	1.01	1.01	1.01	1.02
LAGEOS-2	120	1.01	1.02	1.01	1.02	1.02	1.02	1.02	1.01
	70	1.01	1.02	1.01	1.02	1.02	1.02	1.02	1.01
ERS-2	120	5.83	5.34	5.59	5.31	5.29	5.29	5.30	5.31
	70	5.85	5.36	5.61	5.32	5.30	5.30	5.32	5.32
ENVISAT	120	4.37	4.27	4.35	4.47	4.48	4.49	4.28	4.27
	70	4.38	4.28	4.37	4.50	4.52	4.52	4.29	4.29
WESTPAC	120	4.33	4.09	4.21	4.12	4.12	4.12	4.12	4.10
	70	4.37	4.10	4.20	4.11	4.11	4.11	4.12	4.09
JASON-1	120	1.84	1.83	1.84	1.83	1.82	1.82	1.81	1.89
	70	1.84	1.83	1.84	1.83	1.82	1.82	1.81	1.84

Table 4: SLR residuals (cm) after orbit determination for various satellites for different maximum used degrees of the spherical harmonic coefficients**.**

degree from 70 up to 100. In the case of GFZ-1 all tested models differ significantly only beyond degree 100 which is almost outside the questionable degree range. All this confirms that the degree dependencies of the orbit adjustment residuals for EGM2008 show no detectable correlation with the "bump" in the degree variance spectrum for all tested satellites.

GPS/leveling comparison

An independent comparison with external data can be made using geoid heights determined point-wise by GPS positioning and leveling ("GPS/leveling"). Table 5 shows the results for several gravity field models in comparison with EGM2008 using GPS/leveling points of the USA (Milbert, 1998), Canada (M. Véronneau, Natural Resources Canada, personal communication 2003), Germany (Ihde et al.

2002), Europe (Ihde, personal communication, 2008) and Australia (G. Johnston, Geoscience Australia and W. Featherstone, Curtin University of Technology, personal communication 2007). For this comparison, height anomalies were calculated from the spherical harmonic coefficient data sets and reduced to geoid heights (c.f. Rapp 1997). The topographic correction was done by using the DTM2006.0 model, which is available in spherical harmonic coefficients (Pavlis et al. 2007). For the comparison with the other gravity field models, the EGM2008 coefficients were used only till degree/order 360 (blue column). It is obvious that EGM2008 fits best in comparison with all other tested models. Only for Australia the EIGEN-5C model reaches the same level of the fit. To show the excellent performance of EGM2008 when the full resolution is applied, we additionally computed the GPS/leveling residuals for the maximum degree 2190 of this model (last column).

Dataset	EGM96	GGM02C/ EGM96*	EIGEN-CG01C	EIGEN-GL04C	EIGEN-5C	EGM2008 Max. degree 360	EGM2008 Max. degree 2190
Europe (1234)	48	32	37	34	30	27	21
Germany (675)	29	17	22	18	15	14	4
Canada (1930)	36	26	27	25	25	23	13
USA (6169)	38	33	35	34	34	32	25
Australia (201)	30	25	26	24	24	24	22

Table 5: Root mean square (cm) about mean of GPS-Levelling minus model-derived geoid heights (number of points in brackets)
*GGM02C has been filled up to degree/order 360 with EGM96 coefficients

As a further example of the GPS/leveling results we show in figure 3 the plots of the fit of the individual GPS/leveling points for the European data set (Ihde, personal communication, 2008) to the EGM2008 geoid (computed till degree and order 360) in comparison to EIGEN-5C. At first view, the colour patterns of the two plots look similar. But a closer inspection reveals, that noticeable differences can be seen, for instance over Denmark, the Alps, South France or Sardinia. We assume that these differences are mainly caused by the different ground data sets used in EGM2008 and EIGEN-5C. These differences will be a subject for further investigations.

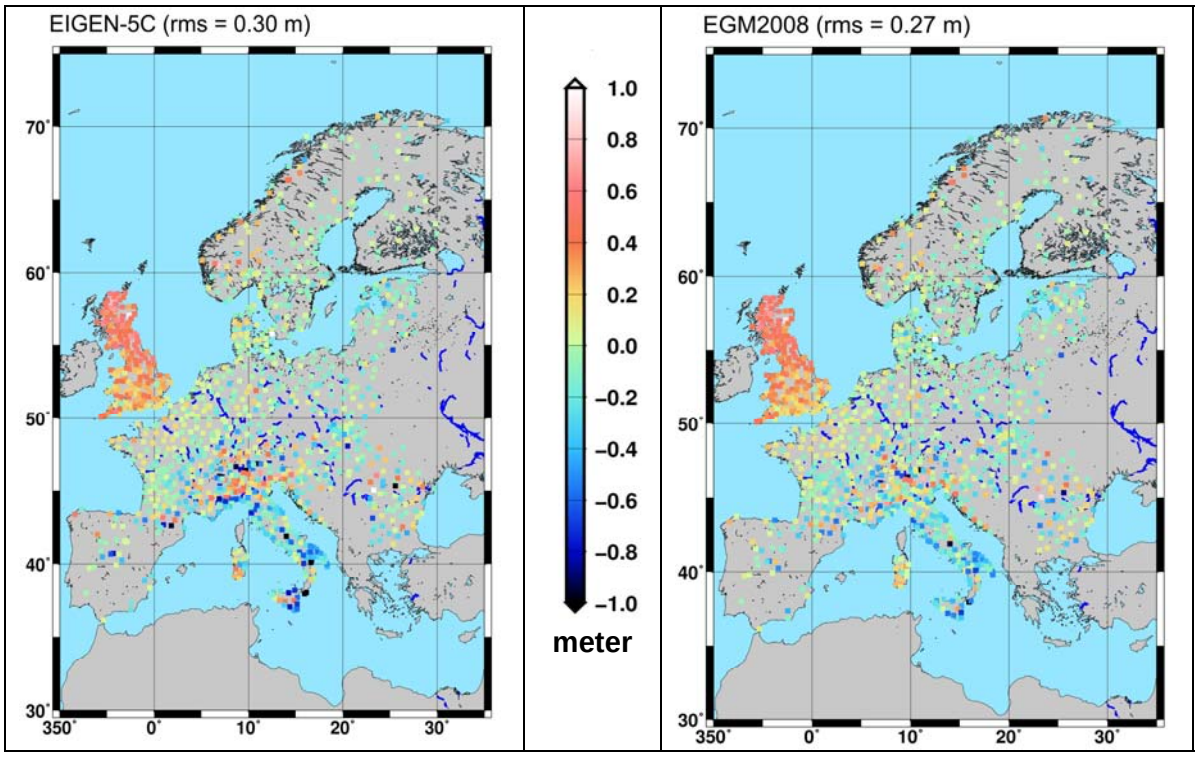


Figure 3: Geoid height differences (m) in Europe between GPS/levelling data and EIGEN-5C (left) resp. EGM2008 (right) up to degree/order 360

Geoid residuals over the oceans

We also computed geoid residuals over the oceans. These residuals are the differences between the geoid of the tested gravity field model and geoid undulations over the oceans derived from GFZ mean sea surface heights (MSSH, T. Schöne and S. Esselborn, 2005, GFZ, personal communication) minus ECCO sea surface topography (Stammer et al. 2002). This ocean geoid undulation data set is the same as used in the computation of EIGEN-GL04C and EIGEN-5C.

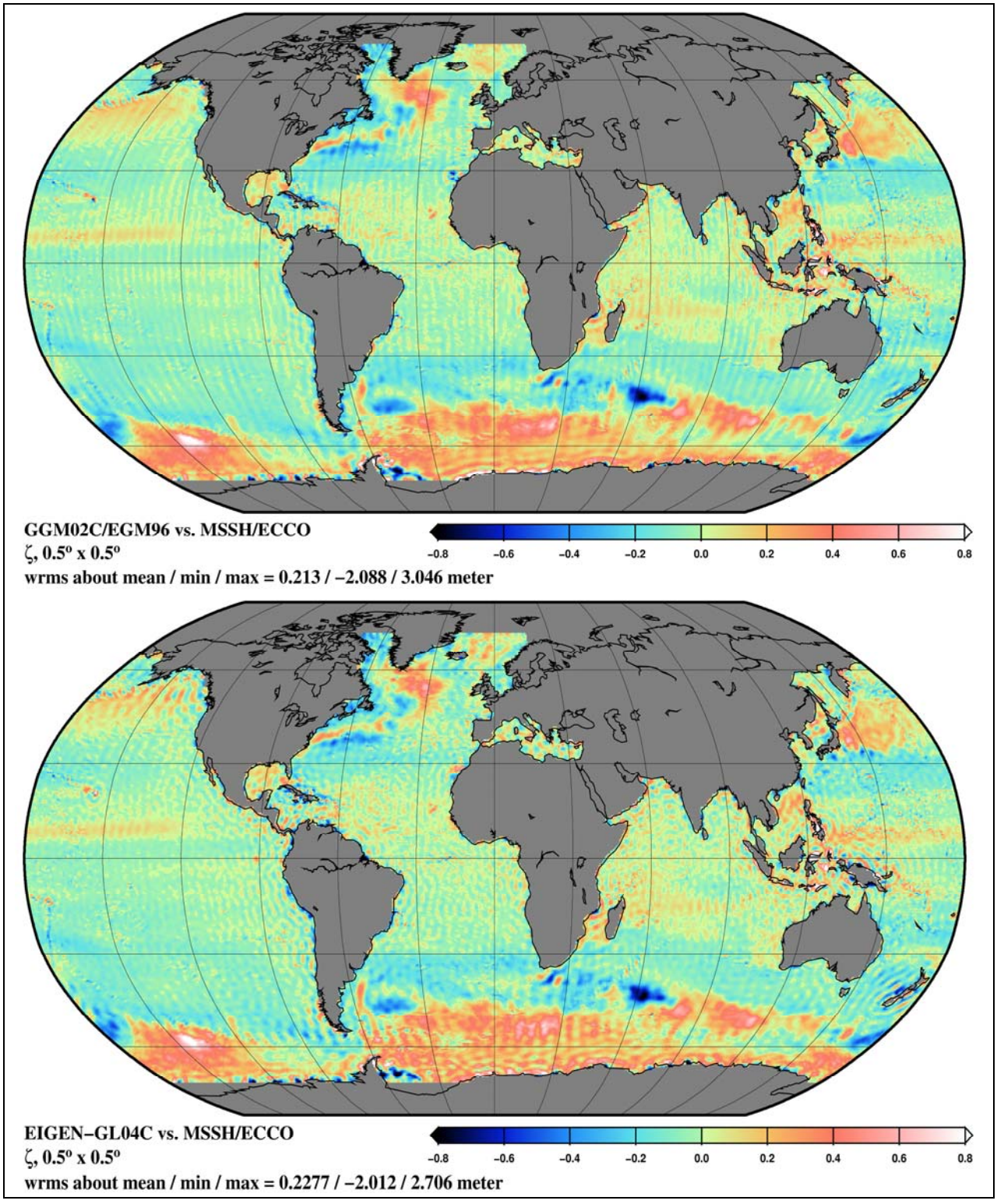


Figure 4: Ocean geoid residuals for GGM02C/EGM96 (top) in comparison to EIGEN-GL04C (bottom)

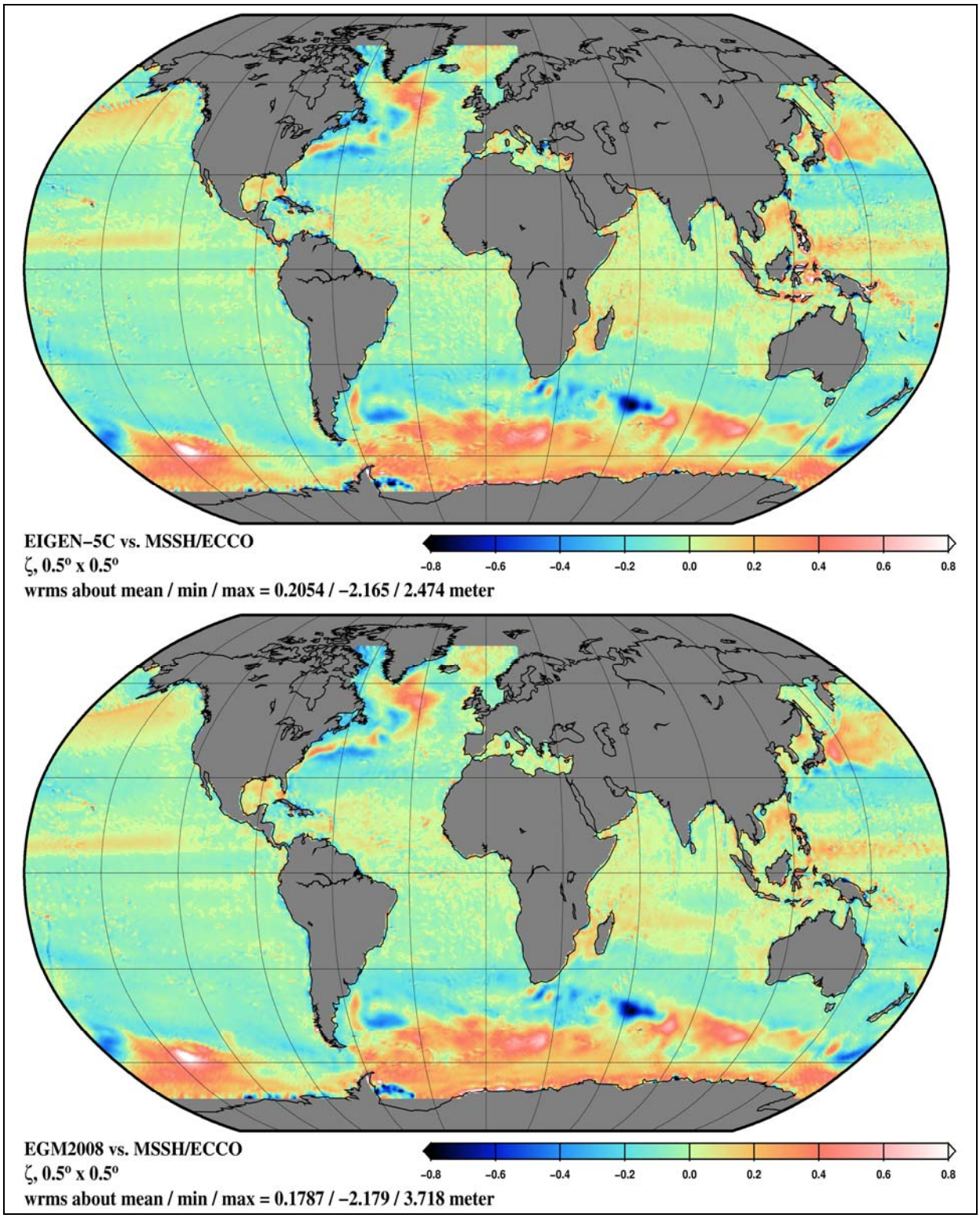


Figure 5: Ocean geoid residuals for EGM2008 (bottom) in comparison to EIGEN-5C (top)

Although the MSSH model is not error-free, our experience is that the calculation of such kind of residuals is an appropriate method to probe a gravity field model for artefacts like stripes and rings over the oceans. Figure 4 and 5 shows gridded maps of corresponding residuals with a resolution of 0.5° x 0.5° for GGM02C, EIGEN-GL04C, EIGEN-5C and EGM2008. Whereas GGM02C and EIGEN-GL04C show the well known unrealistic stripe and ring patterns (see figure 4), EIGEN-5C gives a significant improved picture with only some remaining weak striping patterns, for instance south of Alaska and east of the Philippines (see figure 5). Lastly EGM2008 is obviously free of stripes

and other artefacts such as ringing. This improvement of EGM2008 compared to EIGEN-5C corresponds to a smaller corresponding wrms value of the geoid residuals (see the numbers just below the individual plots in figure 5: **0.1787** m vs. **0.2054** m).

Summary

In our investigation EGM2008 shows the best performance concerning the ocean geoid and comparisons with external GPS/levelling geoid information compared to other tested combined gravity field models. We also carried out orbit adjustment computations for EGM2008 in comparison to other combined and satellite-only gravity field models. In these tests EGM2008 shows no major performance differences to the other tested models. Furthermore, EGM2008 and its corresponding satellite-only model ITG-GRACE03S show a better inner consistency with respect to CHAMP and GRACE than the other gravity field models. We also did not find a correlation with the remarkable "bump" in the degree variance spectrum between degree 70 and 100.

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Evaluation of EGM08 – Globally and Locally in South Korea

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Abstract
The newest global geopotential model, EGM08, is evaluated locally using GPS/leveling data in South Korea and globally in terms of its degree variances (power spectral density). It is found that the agreement between the EGM08 geoid and the geoid undulation derived from GPS/leveling data over 500 irregularly distributed points has a standard deviation of 18.5 cm. This agreement is better than with EGM96 (27.4 cm). From the mean difference between EGM08 and GPS/leveling it is found that the local geoid (national vertical datum of South Korea) is offset from the global geoid by 43.4 cm, which agrees roughly with the 34 cm offset implied by the EGM96 analysis. The degree variances of the EGM08 geoid show that the EGM96 is significantly underpowered at its high degrees. They also imply that the Earth's high-frequency gravitational field does not have constant fractal dimension, which may provoke additional investigation.

Introduction
We start with the spherical harmonic expansion of the disturbing potential, *T*, in terms of spherical polar coordinates, (r,θ,λ):

$$T(r,\theta,\lambda)=\frac{GM}{a}\sum_{n=2}^{\infty}\sum_{m=-n}^{n}\left(\frac{a}{r}\right)^{n+1}C_{nm}\bar{Y}_{nm}(\theta,\lambda), \tag{1}$$

where *GM* is the product of Newton's gravitational constant and Earth's total mass (including the atmosphere), *a* is the radius of the bounding (Brillouin) sphere, the C_{nm} are the (unit-less) Stokes coefficients, and the $\bar{Y}_{nm}(\theta,\lambda)$ are surface spherical harmonics defined by

$$\bar{Y}_{nm}(\theta,\lambda)=\bar{P}_{n,|m|}(\cos\theta)\begin{cases}\cos m\lambda, & m\geq 0\\ \sin|m|\lambda, & m<0\end{cases} \tag{2}$$

The functions, $\bar{P}_{n,|m|}$, are associated Legendre functions, fully normalized so that

$$\frac{1}{4\pi}\iint_{\sigma}\left(\bar{Y}_{nm}(\theta,\lambda)\right)^2 d\sigma = 1 \quad \text{for all } n,m\,, \tag{3}$$

where σ represents the unit sphere. Representation (1) assumes that *M* is also the mass of the normal ellipsoid, whose center, furthermore, coincides with Earth's center of mass, as well as the coordinate origin.

Let *W* be the gravity potential, *U* the normal gravity potential, and $\boldsymbol{x}$ a point in space. Then,

$$W(\boldsymbol{x}) = T(\boldsymbol{x}) + U(\boldsymbol{x})\,. \tag{4}$$

In linear approximation, we have

$$W(\boldsymbol{x}) = T(\boldsymbol{x}) + \left(U(\boldsymbol{x}_0) + \frac{\partial}{\partial h} U(\boldsymbol{x})\Big|_{\boldsymbol{x}=\boldsymbol{x}_0} h \right), \tag{5}$$

where both $\boldsymbol{x}$ and $\boldsymbol{x}_0$ are on a normal to the ellipsoid and *h* is the vertical distance between them. If $\boldsymbol{x} = \bar{\boldsymbol{x}}$ is a point on the geoid and $\boldsymbol{x}_0 = \bar{\boldsymbol{x}}_0$ is the corresponding point on the ellipsoid (Figure 1), then in this case, $h = N$, the geoid undulation, and we have

$$N(\bar{\boldsymbol{x}}) = \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} T(\bar{\boldsymbol{x}}) - \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} (W_0 - U_0)\,, \tag{6}$$

where $\gamma(\bar{\boldsymbol{x}}_0) = -\partial U(\bar{\boldsymbol{x}})/\partial h\big|_{\bar{\boldsymbol{x}}=\bar{\boldsymbol{x}}_0}$ is normal gravity on the ellipsoid, $W(\bar{\boldsymbol{x}}) = W_0$, and $U(\bar{\boldsymbol{x}}_0) = U_0$. The surface, $W(\bar{\boldsymbol{x}}) = W_0$, represents the global geoid (e.g., the level surface that best fits the mean sea surface on a global scale, although one may adopt other definitions), and *N*, given by equation (6), is the geoid undulation with respect to the given normal ellipsoid.

Local Vertical Datum

If the adopted normal ellipsoid is the one that also best fits the global mean sea surface, then $U_0 = W_0$ and we obtain the geoid undulation of the *global* geoid with respect to the best-fitting ellipsoid:

$$N(\bar{\boldsymbol{x}}) = \frac{1}{\gamma(\bar{\boldsymbol{x}}_0)} T(\bar{\boldsymbol{x}})\,. \tag{7}$$

If the ellipsoid is not optimum (such as the WGS84 ellipsoid, by today's accuracy standards), then the geoid undulation of the global geoid with respect to this ellipsoid is

$$N^{(\mathrm{WGS84})}\left(\bar{\boldsymbol{x}}\right)=N\left(\bar{\boldsymbol{x}}\right)-\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0\right)}\left(W_0-U_0^{(\mathrm{WGS84})}\right). \tag{8}$$

We may also consider a local geoid, which defines a national vertical datum. The origin point for the vertical datum often does not lie on the global geoid, since it is tied to a local mean sea level point. In that case, the local geoid does not coincide with the global geoid; and, the geoid undulation of the local geoid with respect to the non-optimum ellipsoid is given by (see also Figure 1)

$$\begin{aligned} N_{\mathrm{local}}^{(\mathrm{WGS84})}\left(\bar{\boldsymbol{x}}\right)&=\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0\right)}T\left(\bar{\boldsymbol{x}}\right)-\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0\right)}\left(W_0^{\mathrm{local}}-U_0^{(\mathrm{WGS84})}\right)\\ &=N\left(\bar{\boldsymbol{x}}\right)-\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0\right)}\left(W_0-U_0^{(\mathrm{WGS84})}\right)+\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0\right)}\left(W_0-W_0^{\mathrm{local}}\right) \end{aligned} \tag{9}$$

The offset given by the second term on the right side, $-\left(W_0-U_0^{(\mathrm{WGS84})}\right)\big/\gamma$, is about $-53\,\mathrm{cm}$, based on the current best value, $W_0=62636856.33\,\mathrm{m}^2/\mathrm{s}^2$, and the parameter values of the WGS84 normal ellipsoid. The offset given by the third term, $\left(W_0-W_0^{\mathrm{local}}\right)\big/\gamma$, depends on a local determination of W_0^{local} (see below).

In principle, the infinite spherical harmonic series (1) for the disturbing potential converges to the true value only on or above the Brillouin sphere (neglecting the atmospheric and other extra-terrestrial masses). Therefore, we should not express the geoid undulation in terms of the series by substituting (1) into (6) because this would require the evaluation of the series at a point on the geoid, $\bar{\boldsymbol{x}}$, which is likely inside the bounding sphere. However, in practice, the infinite series must be truncated to some finite degree, $n_{\max}$, since one can determine only a finite number of coefficients, C_{nm}, from a given finite set of data on a bounding sphere. As such, the concern about formal series convergence disappears, although there is still the question of whether the truncated series accurately represents the disturbing potential. On the other hand, we can now use the truncated series anywhere in free space, even below the bounding sphere and on or above the Earth's surface, since the truncated series is harmonic anywhere in free space. Still, in order to respect theory, we cannot use the series *inside* the Earth, specifically on the geoid, which would be the case in equation (6), because the disturbing potential is not harmonic there.

If we apply equation (5) to two points again on a normal to the ellipsoid, but now $\boldsymbol{x}=\bar{\boldsymbol{x}}^e$ is a point on the Earth's surface and $\boldsymbol{x}_0=\bar{\boldsymbol{x}}_0^e$ is a point on the *telluroid*, which is a surface such that $U\left(\bar{\boldsymbol{x}}_0^e\right)=W\left(\bar{\boldsymbol{x}}^e\right)$, then the distance between these is called the height anomaly ($h=\zeta$) and is given (in linear approximation) by

$$\zeta\left(\bar{\boldsymbol{x}}^e\right)=\frac{1}{\gamma\left(\bar{\boldsymbol{x}}_0^e\right)}T\left(\bar{\boldsymbol{x}}^e\right). \tag{10}$$

Now, we can substitute legitimately the series for T (truncated at $n = n_{\max}$), because $\bar{x}^e$ is not below the Earth's surface, and get

$$\zeta\left(\bar{x}^e\right) = \frac{GM}{a\,\gamma\left(\bar{x}_0^e\right)} \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left(\frac{a}{r^e}\right)^{n+1} C_{nm} \bar{Y}_{nm}(\theta, \lambda), \tag{11}$$

where r^e is the radius to the point, $\bar{x}^e$. Without significant error at this stage of the analysis, we may also approximate, $\gamma\left(\bar{x}_0^e\right) \approx GM/a^2$, which simplifies the antecedent factor of the series to a, and which can further be approximated by $a \approx R$, where R is a mean Earth radius.

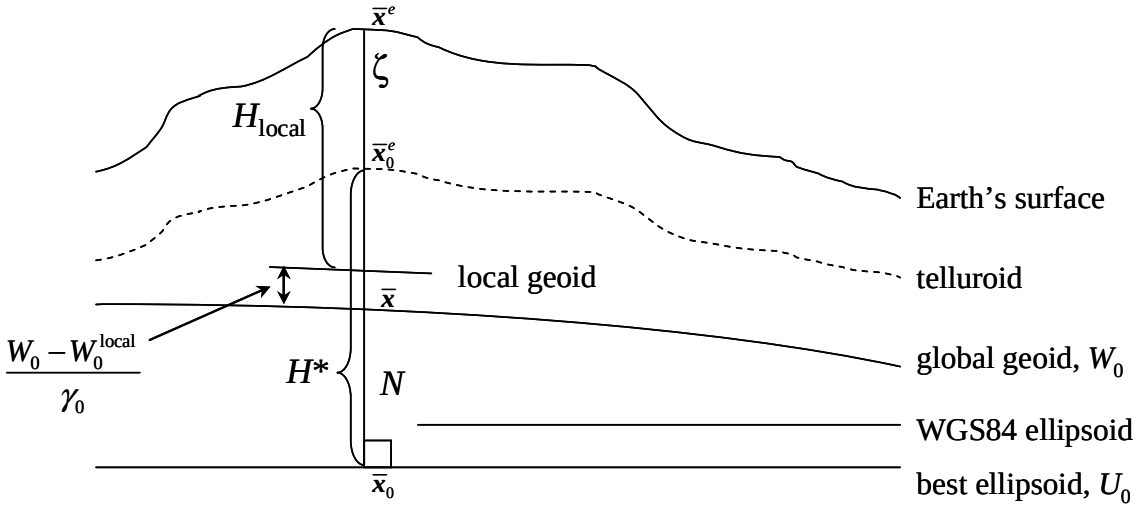


Figure 1: Geometry of geoid, ellipsoid, telluroid, and Earth's surface.

We note that the difference, d, between the height anomaly and the geoid undulation is

$$d = \zeta - N = H - H^* \approx H \frac{\Delta g_B}{\gamma_0}, \tag{12}$$

where H is the orthometric height (with respect to the global geoid), Δg_B is the Bouguer gravity anomaly and γ_0 is an average value of normal gravity on the ellipsoid. World-wide, this difference, for the EGM96 model, has a mean value of about -5 cm, with a standard deviation of about 22.3 cm and an extreme absolute value of 311 cm (Lemoine et al., 1998, p.5-14).

Assessment Using GPS/Leveling Data

For a local point-wise assessment of a global model for the geoid undulation, such as EGM96 or EGM08 (Pavlis et al., 2008), one may consider using a set of GPS/leveling data, which yield the geoid undulation of the local geoid with respect to the WGS84 ellipsoid:

$$N_{\text{local}}^{(\text{WGS84})} = h^{(\text{WGS84})} - H_{\text{local}} \,. \tag{13}$$

The points where such data are available for the Republic of South Korea (J. Kwon, personal communication, 2008) are shown in Figure 2. The data set contains 500 points where both the GPS height above the WGS84 ellipsoid, $h^{(\text{WGS84})}$, and the orthometric height with respect to the national vertical datum, H_{local}, have been measured.

In view of equation (9), we must consider in our evaluations that the local geoid differs from the global geoid ($W_0^{\text{local}} \neq W_0$). Fell and Tanenbaum (2001) report a bias of about $\left(W_0 - W_0^{\text{local}}\right)\big/\gamma_0 = 90$ cm (i.e., the orthometric heights with respect to the national vertical datum are about 90 cm smaller than with respect to the EGM96 geoid, assumed to be the global geoid). This bias was determined using 27 GPS/leveling stations in South Korea. Although not explicitly stated in their paper, we assume that the EGM96 geoid undulation was computed with respect to the WGS84 ellipsoid, rather than the best-fitting ellipsoid. Hence, the 90 cm bias should be modified by the -53 cm difference between these two ellipsoids, so that the bias between vertical datums (global and national) is only 37 cm. In addition, we must assume a mean tide system because the GPS/leveling data presumably were not corrected for tidal effects.

Evaluating the geoid undulation according to

$$N\left(\bar{\boldsymbol{x}}_0\right) = R\sum_{n=2}^{n_{\max}}\sum_{m=-n}^{n}\left(\frac{a}{r^e}\right)^{n+1} C_{nm}\bar{Y}_{nm}\left(\theta,\lambda\right), \tag{14}$$

where r^e is the radial distance to the ellipsoid ($a = 6378137$ m) and according to

$$N\left(\bar{\boldsymbol{x}}_0\right) = R\sum_{n=2}^{n_{\max}}\sum_{m=-n}^{n}\left(\frac{a}{r^e}\right)^{n+1} C_{nm}\bar{Y}_{nm}\left(\theta,\lambda\right) - d\left(\bar{\boldsymbol{x}}_0\right), \tag{15}$$

using software supplied for the evaluation of EGM96 (that includes the correction, $-d$, equation (12)), we found in South Korea only differences with a standard deviation of about 1 cm. Thus, we can safely neglect for present purposes the topographic bias (the mismodeling associated with evaluating a harmonic function inside the masses on the geoid). Table 1 compares the geoid undulation computed using equation (14) for the models EGM96 and EGM08 against the geoid undulation computed from GPS/leveling data, according to equation (13). The standard deviation of the difference includes both commission and omission errors of the spherical harmonic model, and also errors in the GPS/leveling data. The mean difference accounts for different ellipsoids and geoids.

Considering EGM96, the mean difference of 19 cm (Table 1) implies the following difference between the local vertical datum (LD) and the global geoid:

$$
\begin{aligned}
19 \text{ cm} &= N_{\text{EGM96}} - \left(h_{\text{WGS84}} - H_{\text{LD}}\right) = N_{\text{EGM96}} - \left(\left(h_{\text{best ellip}} - 53 \text{ cm}\right) - H_{\text{LD}}\right) \\
&= \left(N_{\text{EGM96}} - h_{\text{best ellip}}\right) + 53 \text{ cm} + H_{\text{LD}} \\
&= -H_{\text{global geoid}} + 53 \text{ cm} + H_{\text{LD}} \\
H_{\text{global geoid}} - H_{\text{LD}} &= 34 \text{ cm}
\end{aligned}
\tag{16}
$$

which agrees approximately with the modified result by Fell and Tanenbaum (2001). For the EGM08 geoid comparison to the GPS/leveling data, the mean difference of 9.6 cm similarly implies

$$H_{\text{global geoid}} - H_{\text{LD}} = 43.4 \text{ cm}. \tag{17}$$

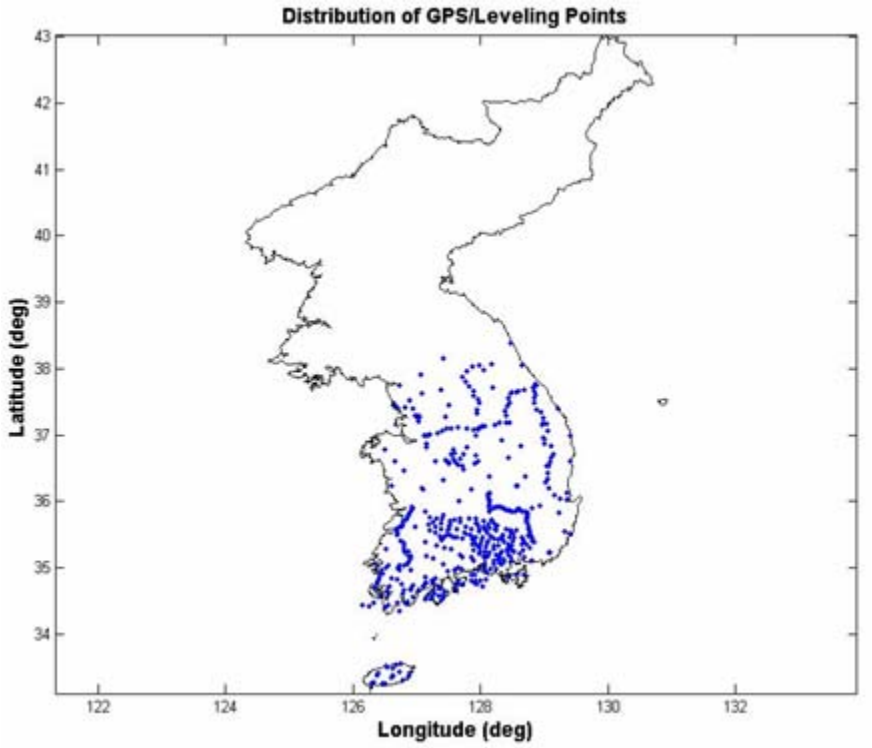


Figure 2: Points where GPS/leveling data are available.

Table 1: Statistics for the differences between geoid models and GPS/leveling data at 500 points shown in Figure 2. Units: [m].

model ($n_{\max}$)	minimum	maximum	mean	st.dev.
EGM96 (360)	-0.52	1.43	0.189	0.274
EGM08 (2160)	-0.54	1.17	0.096	0.185

Degree Variances at High Degrees

As might be expected, the agreement between EGM08 and the GPS/leveling data is better than for EGM96. However, it is also interesting to know the reason for the remaining difference between these two types of geoid undulations. Certainly, errors in the GPS heights and in the leveling data contribute, but the extent of this is not known; and common wisdom would dictate that aside from the occasional bad data neither GPS height errors nor orthometric height errors would contribute at the level of one or two decimeters. Therefore, we assume that the differences are due primarily to errors in the spherical harmonic model.

We consider these model errors in terms of degree variances, which may be separated into commission and omission errors. By putting a bound on the omission error, we will be able to provide some assessment of the commission error, since by assumption Table 1 essentially provides the total error. Quantifying the omission error relies on a statistical model for the harmonic coefficients. That is, the high-degree coefficients are assumed to behave in a stochastic sense with a particular standard deviation (or, variance) for each degree (and zero average). Scaled to the geoid undulation, the degree variance is given by

$$\left(\sigma_N^2\right)_n = R^2 \left(\frac{a}{r^e}\right)^{2n+2} \sum_{m=-n}^{n} C_{nm}^2 \,, \tag{18}$$

where r^e is taken as constant (e.g., $r^e = a$). W.M. Kaula promoted and developed this idea; and an often-used rule-of-thumb (Kaula, 1966, p.98), bearing his name, provides a rough, global estimate of the standard deviation of the omission error:

$$\sigma_{\text{omission}}^{(n_{\max})} = \sqrt{\sum_{n=n_{\max}+1}^{\infty} \left(\sigma_N^2\right)_n} \approx \frac{64}{n_{\max}} \text{ [m]}. \tag{19}$$

Hence, if $n_{\max} = 360$, then $\sigma_{\text{omission}}^{(180)} = 18$ cm. Many other researchers, notably Rapp (1979), have devised refinements of this model. Figure 3 shows the degree variances of the geoid according to the EGM96 and EGM08 models, as well as Kaula's rule.

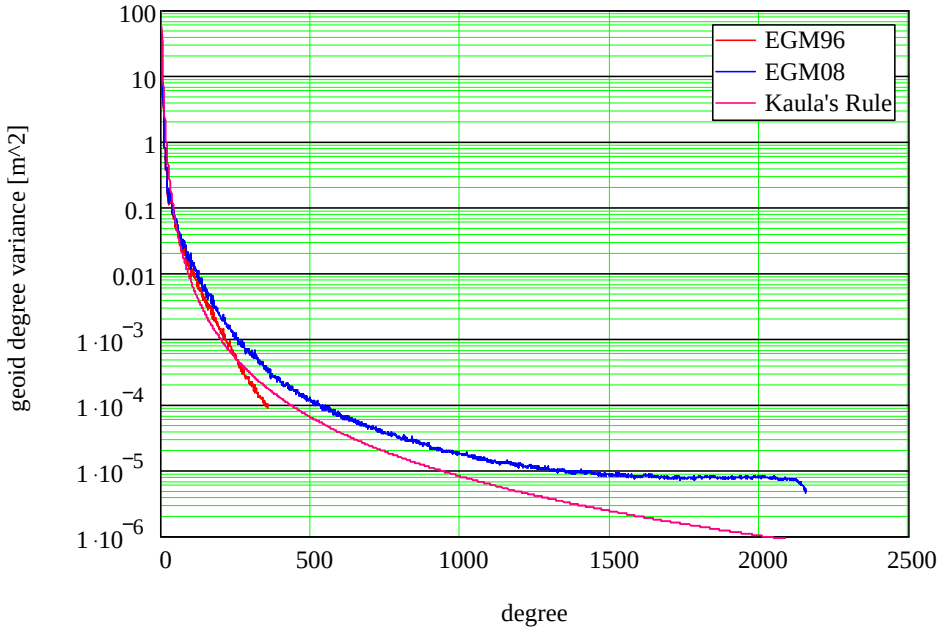


Figure 3: Degree variances of the geoid according to various models.

A significant feature of the new EGM08 model is the demonstration that the EGM96 model is under-powered at the high degrees, as already predicted earlier by some investigators (e.g., Jekeli, 1999). We can transform from degree variance to power spectral density (psd), Φ, using the relationship

$$\Phi_N(f_n) = \frac{2\pi R^2}{n}\left(\sigma_N^2\right)_n, \tag{20}$$

where $f_n = n/(2\pi R)$ is cyclical frequency. Plotting the psd's, now with frequency also on a logarithmic scale, we find a second striking feature (Figure 4). While Kaula's rule significantly overestimates the psd of the geoid undulation at the lower frequencies, it underestimates the psd at the high frequencies, although the power attenuation is essentially correct, at least up to wavelengths of about 30 km (frequency, 3×10^{-5} cy/m). It has been suggested that the Earth's gravitational potential, like its topography and that of other planets, behaves like a fractal (e.g., Turcotte, 1987). That is, its psd obeys a power law, essentially of the form:

$$\Phi(f) = bf^{-\alpha}, \quad f \geq f_0, \tag{21}$$

with constants, α, b, where α is related to the fractal dimension of the fractal. Clearly, the EGM08 model indicates that for frequencies, 2×10^{-6} cy/m $\leq f \leq 3\times10^{-5}$ cy/m, the psd is a power law. If one postulates that the gravitational field is a fractal (with constant fractal dimension) at high frequencies (as often modeled, e.g., by Jekeli (2003), on the basis of local gravity anomaly and topographic data sets), one might question the significant upturn in the

EGM08 psd at frequencies, $f \geq 3\times10^{-5}$ cy/m, corresponding to harmonic degrees greater than about 1200. On the other hand, if accurate at these higher frequencies, EGM08 shows that the Earth's gravitational potential does not possess the same fractal dimension at all (higher) frequencies.

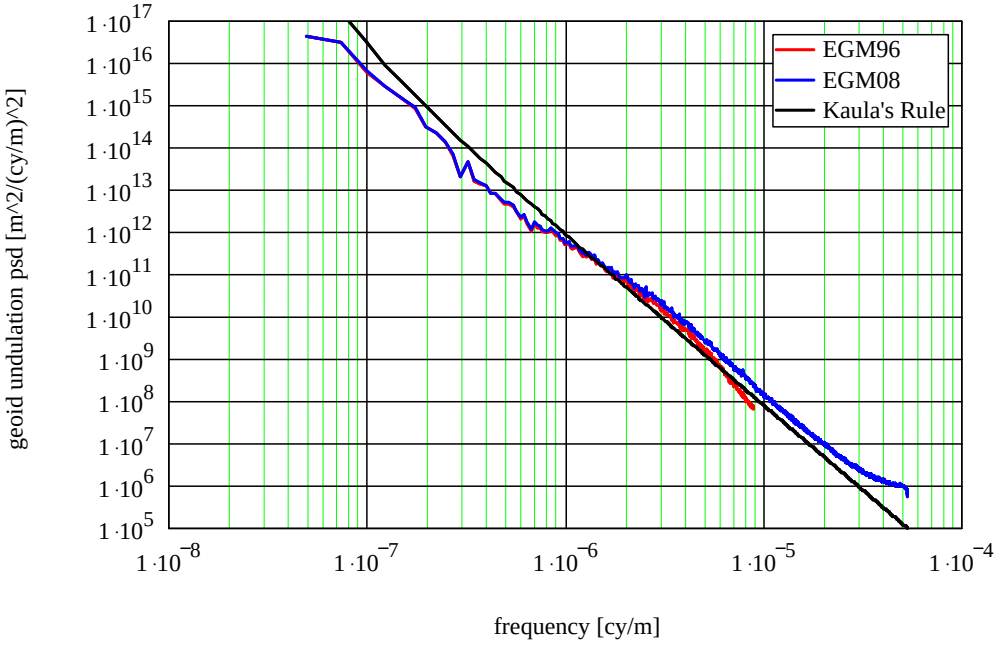


Figure 4: PSDs of the geoid undulation according to various models.

A power law model for the psd that more accurately fits the frequency band, 3×10^{-6} cy/m $\leq f \leq 3\times10^{-5}$ cy/m (harmonic degrees, 120 – 1200), is given by

$$\Phi_N(f) = 4.044\times10^{-12}\, f^{-3.898}\ \text{m}^2/(\text{cy/m})^2. \tag{22}$$

Assuming that this new model (shown in Figure 5) yields a reasonable approximation to the standard deviation of the omission error, we find

$$\sigma_{\text{omission}}^{(n_{\max})} = \sqrt{\frac{1}{2\pi}\int_{f_{\max}}^{\infty} \Phi_N(f)\, f\, df} = 0.041\ \text{m}, \quad \text{for } n_{\max} = 2160. \tag{23}$$

Since the omission and commission errors are independent, we have

$$\left(\sigma_{\text{commission}}^{(n_{\max})}\right)^2 = \sigma_{\text{total}}^2 - \left(\sigma_{\text{omission}}^{(n_{\max})}\right)^2. \tag{24}$$

Using the standard deviation in Table 1 as the total (i.e., disregarding errors in the GPS/leveling data) and assuming the omission error model , equation (23) is representative of EGM08 in South Korea, the commission error in EGM08 for South Korea is about 18.0 cm. If the new omission error model, equation (23), underestimates the high-degree degree variances (high-frequency psd), as indicated in Figure 5, then it is more difficult to estimate the omission error (and hence, the commission error). Furthermore, it should be emphasized that this model, or Kaula's rule, or any other model extrapolated from the degree variances, is a global model for the standard deviation of the omission error. For a local computation of the geoid undulation from a particular model (like EGM08), the standard deviation of the omission error may be significantly lower or higher.

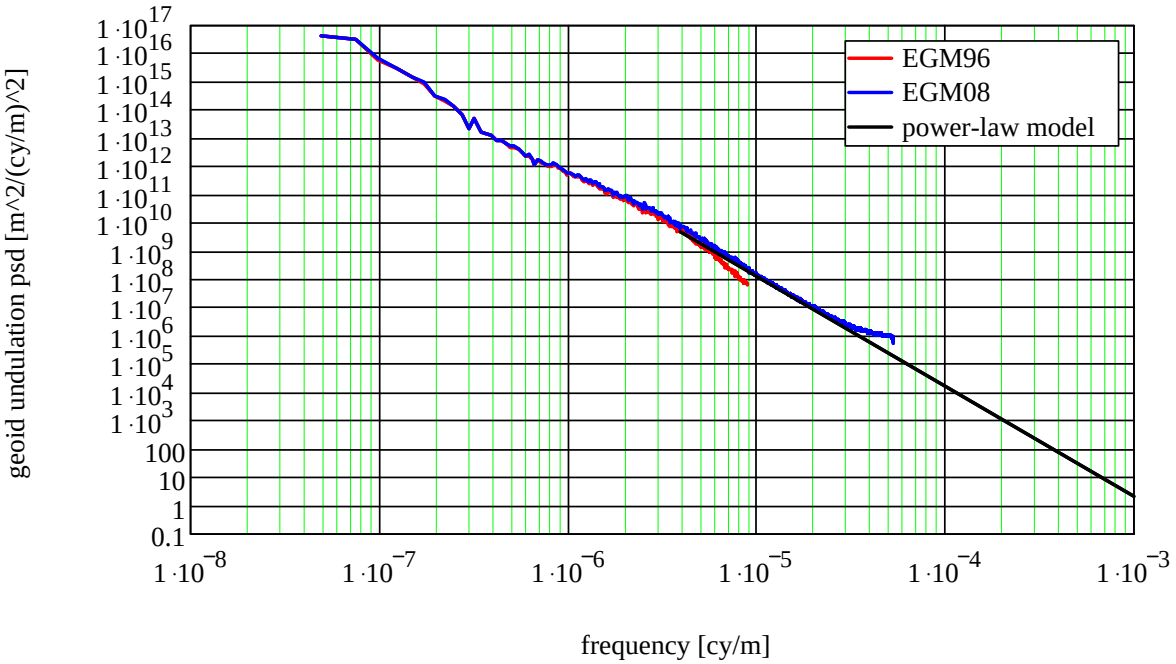


Figure 5: Global power spectral density of the geoid according to the EGM96 and EGM08 models, and their power-law extension.

Summary Discussion

A brief local assessment of EGM08 has been performed using GPS/leveling data in South Korea. These data are available at 500 points somewhat unevenly distributed over the South Korean peninsula. However, they cover many types of terrain. The accuracy of these data is not available (at present), but in terms of standard deviation the EGM08 geoid agrees better (18.5 cm) with these geometrically derived undulations than does the EGM96 geoid (27.4 cm). The mean differences (gravimetric model minus geometric measurement) also provide an indication of the offset of the local vertical datum from the global geoid. EGM08 implies a 43.3 cm offset, compared to the EGM96 model, which implies a 34 cm offset.

We note that EGM96 was published as a *non-tidal geoid* model; whereas EGM08 is offered both as a zero-tide and a non-tide geoid model. We performed our computations of EGM08

using its zero-tide version. The difference between the two is (in terms of geoid undulation; see Jekeli (2000, ch.6); Lemoine et al. (1998, ch.11))

$$N_{\text{zero-tide}} - N_{\text{non-tide}} = -0.099 k_2 \left(3\sin^2\psi - 1\right) \text{ [m]}, \tag{25}$$

where $k_2 = 0.29$ is a Love's number and ψ is the geocentric latitude. In South Korea, this latitude is $\psi \approx 36°$, and, hence, $N_{\text{zero-tide}} - N_{\text{non-tide}} = -0.001$ m. We see that the mean difference, $\text{mean}\left(N_{\text{EGM96}} - N_{\text{EGM08}}\right) = 9.3$ cm, in South Korea is not due to the difference in tidal correction. The reason for this mean difference is not known to the authors at this time.

The degree variances of the EGM08 geoid show that the EGM96 spectrum is significantly underpowered at its high degrees. Furthermore, it is shown that Kaula's rule is remarkably close in describing the fractal dimension of the field in the frequency band, 2×10^{-6} cy/m $\le f \le 3\times10^{-5}$ cy/m (corresponding to harmonic degrees, 80 – 1200). At higher frequencies, EGM08 departs from this fractal dimension. However, if the power-law attenuation were an accurate representation of the high-degree variances, then the omission error for EGM08 would be approximately 4.1 cm (global standard deviation), which if it further holds for South Korea would imply a commission error in EGM08 for South Korea of about 18.0 cm (st.dev.), assuming relatively insignificant errors in the GPS/leveling data.

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RESULTS OF EGM08 GEOPOTENTIAL MODEL TESTING AND ITS COMPARISON WITH EGM96

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1. Introduction

The knowledge of geopotential models accuracy and its problematic areas is necessary for all users. In case of World Height System (WHS) development, the geopotential model accuracy limits

- WHS accuracy;
- determination of the geoidal geopotential W_0;
- connection of local vertical datum to WHS;
- computation of the geopotential values W;
- height computations.

What do we need for geopotential model testing?

- Theory of testing and its applications
- value of the geoidal geopotential W_0
- four primary constants defining the level ellipsoid and its gravity field
- geopotential model evaluation and monitoring network (GMEMN), which covers (if possible) the whole Earth´s surface.

The theory for Geopotential Model Testing (GMT), developed by Burke et al. (1995), has applied to the recent EGM08 model, and for comparison purposes, also to the previous EGM96 one. The methodology requires the four primary constants defining the level ellipsoid and its gravity field and accurate geocentric positions as well as normal Molodensky heights of the testing sites on the Earth's surface. Over the oceans, geocentric positions, altimetric heights observed by TOPEX/POSEIDON (TP) or Jason 1 as well as h_{SST}, obtained from a sea surface topography model (POCM4B), are also needed. No hypothetical quantities such as, for example, the orthometric heights and/or geoid heights are used in this GMT. Although a global coverage of the testing sites is preferable, regional testing networks are also useful for GMT

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2. Primary constants used for GMT

Essential progress was made in refining the geoidal potential W_0 (Burša et al., 1999a, 2001/2002) on the basis of TP altimeter data and the sea surface topography (SST) model POCM 4B (360 by 360) of Rapp et al. (1996), based on the global circulation model:

$$W_0 = (62\ 636\ 856.0 \pm 0.5)\ \mathrm{m}^2 \cdot \mathrm{s}^{-2}. \qquad (1)$$

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The above value has been adopted for our GMT, the other three adopted constants were: the geocentric gravitational constant

$$GM = (398\,600\,441.8 \pm 0.8) \times 10^6\ \mathrm{m^3 \cdot s^{-2}}, \quad (2)$$

(Ries et al., 1992), the angular velocity of the Earth's rotation

$$\omega = (7\,292\,115.8 \pm 0.8) \times 10^{-11}\ \mathrm{rad \cdot s^{-1}} \quad (3)$$

(IAG SC3 Rep., 1995) and the second zonal Stokes parameter

$$J_2 = (1\,082\,635.9 \pm 0.1) \times 10^{-9}, \quad (4)$$

(IAG SC3 Rep., 1995) which is in the zero-frequency tide reference system. Note that the constants (1), (2) and (3) are independent of the tide reference system. Since some GMT data are in the mean tide system (oceans) or in the tide-free system (continents), the constant (4) should be corrected to account for the tidal reference system.

From constants (1) - (4), three derived constants can be computed as follows: the geopotential scale factor

$$R_0 = \frac{GM}{W_0} = (6\,363\,672.56 \pm 0.05)\ \mathrm{m} \quad (5)$$

the semimajor axis of the level ellipsoid in the mean and tide-free reference systems, respectively

$$a_{mean} = (6\,378\,136.68 \pm 0.05)\ \mathrm{m}, \quad a_{tide\text{-}free} = (6\,378\,136.55 \pm 0.05)\ \mathrm{m}, \quad (6)$$

and its flattening

$$\alpha_{\mathrm{mean}} = 1 / (298.252\,34 \pm 0.000\,02), \ \alpha_{\mathrm{tide\text{-}free}} = 1 / (298.257\,69 \pm 0.000\,02). \quad (7)$$

The geopotential scale factor (5) is also independent of the tide systems. Parameters (6) and (7) define the mean and tide-free level ellipsoids.

The four fundamental constants GM, ω, J_2, W_0 or GM, ω, α, a make it possible to compute the actual potential at any site on the physical surface of the Earth with known geocentric position, thus the normal Molodensky height and/or the geopotential number can also be considered known. The GMT is limited by the errors of the normal Molodensky heights and uncertainties of the fundamental constants. The fundamental constant uncertainties contribute about $\pm 0.5\ \mathrm{m^2 \cdot s^{-2}}$ error to the geopotential and/or ±0.05 m error to the radius of the local equipotential surface, determined by the tested geopotential model.

3. Geopotential model evaluation and monitoring network (GMEMN)

The GMEMN consists of 31,557 testing sites, covering about 82% of the Earth´s surface (Tab. 1, Fig. 1). The altimetry data of TP AVISO Altimetry project was used for determinations of geocentric positions and altimeter heights of repeat cross-over points, forming 20,768 oceanic testing heights. The POCM 4B (360 by 360) SST model (Rapp et al., 1996) has been used for computations of SST heights in order to obtain the normal Molodensky heights over the oceans. No bias or systematic low frequency errors (possibly due to SST) were removed here. This may be attempted in the future work. The testing sites on the continents are identical with the GPS/leveling sites. However, where only orthometric Helmert heights were available, they had to be first transformed into the normal Molodensky heights. In case of Australia, the height system was uncertain, so we have also assumed orthometric Helmert heights as a first approximation.

Table 1 Regions and number of testing sites assembled in the past and used for of GMEMN used for geopotential model testing

Territory/blocs	Number of testing site	Territory/blocs	Number of testing site
U.S.A. and Canada	6 479	France	973
Mexico	686	Spain	305
Australia	866	Belgium	42
Argentina	32	Greece	5
Brazil	182	Baltic Region	25
Chile	45	Portugal	122
Uruguay	10	Czech Republic, Hungary, Slovak Republic, Poland	654
Venezuela	21	Federal Republic of Germany, Netherlands, Scandinavia, Latvia, Lithuania	324
Indonesia	18	Oceans (repeat cross-over points)	20 768

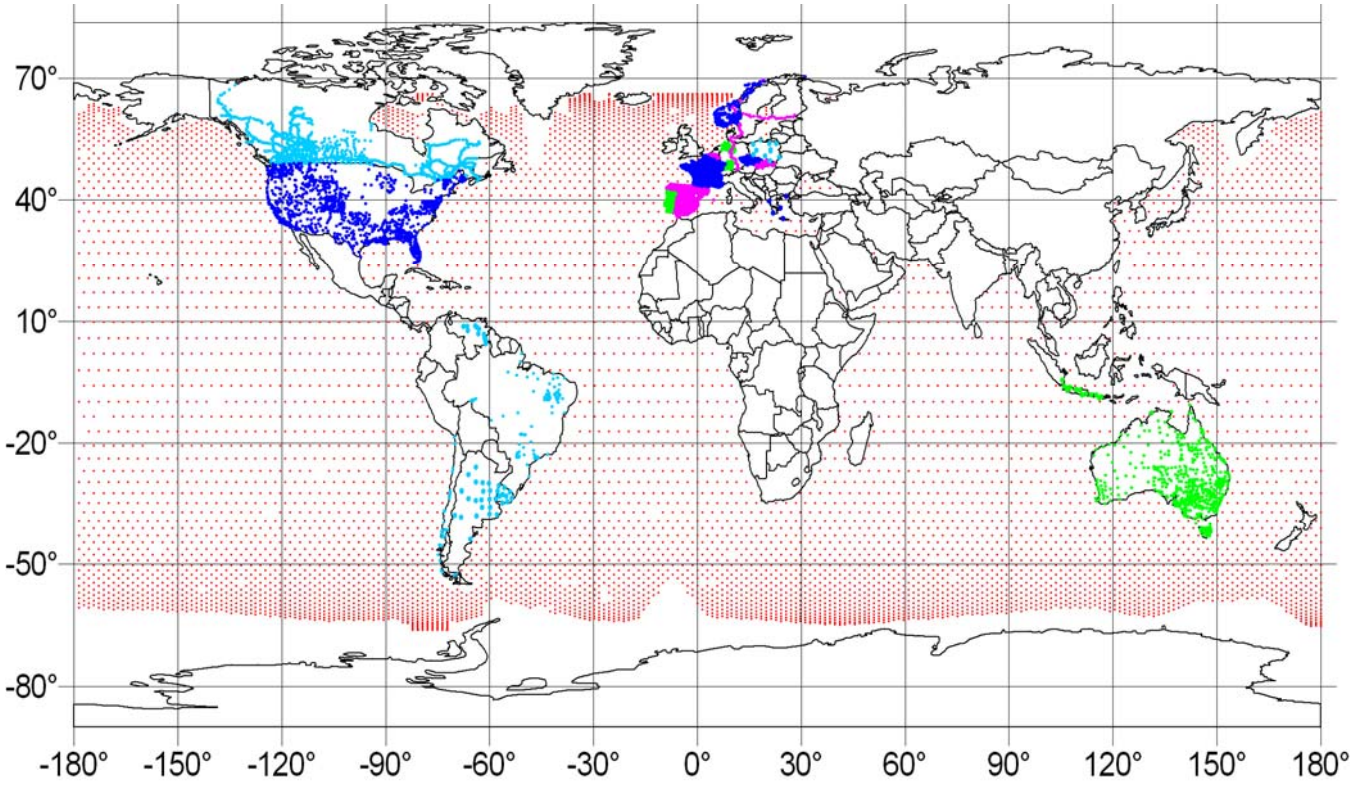


Fig. 1 Geopotential Model evaluation and monitoring network

4. The first results

The GMT methodology is based on the difference δW between the geopotential value at the testing site, computed as a function of positions x_j, normal Molodensky heights H_q and the four fundamental constants

$$W = W(GM,\ \omega,\ J_2^{(0)},\ W_0,\ x_j,\ H_q) \tag{8}$$

(for an explicit functional expression, see Burša et al., 1999b), and the geopotential value, computed from the tested geopotential model:

$$\delta W = W - W(\text{model}). \tag{9}$$

Instead of δW, the corresponding radial distortions δR ($\delta R = -GM\ \delta W / W^2$) of the equipotential surface, passing through the testing site, can be used in GMT. The distortions δW and/or δR are due to the tested geopotential model, if the errors in fundamental constants (1) - (4), as well as, the errors in geocentric coordinates x_j and in normal Molodensky heights H_q of the testing sites are small enough. The uncertainty in the adopted fundamental constants contributes to δR as follows: GM (±13 mm), ω (±l mm), $J_2^{(0)}$ (±0.6 mm), W_0 (±50 mm). The errors in x_j and H_q are believed to amount to a few centimetres only in most cases. However, there are regions, such as those outside Europe and North America, where leveling height errors are likely much larger. Furthermore, there are differences in heights due to different Local Vertical Datums used. That is why, the non-zero biases in δW and/or in δR, i.e. the mean values, computed over GMEMN regions connected to the same tide gauge station, are interpreted as being due to Local Vertical Datum shifts. Therefore it is necessary to exclude mean values $\delta\overline{W}$ and/or $\delta\overline{R}$, i.e., to use standard deviations (Std) rather than RMS. Then, the $\delta\overline{R}$ and the corresponding Std of tested geopotential model obtained over an area covered by testing sites, can be evaluated as follows:

$$\delta\overline{R} = \frac{\sum_1^n \delta R}{n} \quad \text{and} \quad Std = \sqrt{\frac{\sum_1^n \left(\delta\overline{R} - \delta R\right)^2}{n-1}}\ , \tag{10}$$

where n is the number of testing sites within a region. Note that the mean values of (10) can also be used for connecting Local Vertical Datums to a World Height System.

4.1. Numeral results of testing

The first results of the application of the above theory are summarised in **Table 2**. Here, the geopotential models EGM96 and EGM08 were evaluated in selected regions only.

Conclusions for geopotential model evaluation:

a) geopotential model EGM96
This model gave Std ranging from **±0.128 m (oceans) up to ±1.601 m (Venezuela)**. The mean Std value was **±0.500 m**.

b) geopotential model EGM08
This model gave the best Std's, ranging from **±0.071 m (oceans) up to ±0.935 m (Venezuela)**. The mean Std value was **±0.334 m**.

Table 2 First results of geopotential models EGM96 and EGM08 testing.

Territory	Standard deviation [m]		Territory	Standard deviation [m]	
	EGM96	EGM08		EGM96	EGM08
Oceans	**± 0.128**	**±0.071**	**Portugal**	**± 0.347**	**±0.235**
U.S.A.	**± 0.398**	**± 0.283**	**Slovakia**	**±0.451**	**±0.292**
Canada	**± 0.363**	**±0.225**	**Spain**	**± 0.312**	**±0.164**
Australia	**± 0.445**	**±0.323**	**Indonesia**	**±0.650**	**±0.442**
France	**± 0.368**	**±0.116**	**Mexico**	**± 0.613**	**±0.400**
Greece	**± 0.324**	**±0.154**	**Argentina**	**± 0.783**	**± 0.656**
Czech republic	**±0.185**	**±0.113**	**Brazil**	**± 0.884**	**± 0.762**
Hungary	**±0.137**	**±0.064**	**Chile**	**± 0.946**	**±0.696**
Baltic region	**± 0.232**	**±0.123**	**Uruguay**	**± 0.614**	**±0.564**
Poland	**±0.226**	**±0.075**	**Venezuela**	**± 1.601**	**±0.935**

Table 3 Relative **comparison of geopotential models EGM96 and EGM08.**

EGM08 Std decrease (%) with respect to EGM08			
Territory	**EGM96 → EGM08**	**Territory**	**EGM96 → EGM08**
Oceans	**-44.5 %**	**Portugal**	**-32.3 %**
U.S.A.	**-28.9 %**	**Slovakia**	**-35.3 %**
Canada	**-38.0 %**	**Spain**	**-47.4 %**
Australia	**-27.4 %**	**Indonesia**	**-32.0 %**
France	**-68.5 %**	**Mexico**	**-34.7 %**
Greece	**-52.5 %**	**Argentina**	**-16.2 %**
Czech republic	**-38.9 %**	**Brazil**	**-13.8 %**
Hungary	**-53.2 %**	**Chile**	**-26.4 %**
Baltic region	**-47.0 %**	**Uruguay**	**-8.1 %**
Poland	**-66.8 %**	**Venezuela**	**-41.6 %**

4.2. Numeral results of comparisons of geopotential models EGM96 and EGM08

The first results of Std comparisons of the geopotential models EGM96 and EGM08 are shown in **Table 3**.

Conclusions of geopotential models EGM96 and EGM08 evaluation (Std comparison):

The EGM08 Std's are 33.9 % smaller than EGM96 ones. The highest Std decrease was seen in France (**-68.5%),** Poland **(-66.8 %),** Hungary **(-53.2 %)** and Greece **(-52.5 %).**

5. Conclusions

- Our GMT technology is ready for testing of a geopotential model with harmonic expansions up to degree/order *n*=2190
- It is necessary to enlarge the GMEMN (e.g. in Asia, Africa,…)
- **amsignificant global decrease of EGM08 Std's is evident : the mean standard deviation value is ±0.339 m, (a decrease of about -39% with respect to EGM96!)**
- The highest precision (in terms of Std): oceans (±0.071 m)
  Hungary (±0.064 m)
  Poland (±0.075 m)
- The lowest precision (in terms of Std): Venezuela (±0.935 m)
- The highest Std decrease of EGM08 (wrt EGM96): France (-68.5 %)
  Poland (-66.8 %)
- The lowest Std decrease of EGM08 precision (wrt EGM96): Uruguay (-8.1 %)
  Brazil (-12.2%)
- The observed GMT technology distortions can be used for improvements of the EGM08 geopotential model.

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Evaluation of PGM2007A by comparison with globally and locally estimated gravity solutions from CHAMP

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Abstract. New gravity field models incorporate GRACE data as the best available data source for the low to medium wavelength but validation is difficult since comparisons with existing GRACE models will always be biased. Maybe the best independent data set on a global scale is therefore the CHAMP data. It is known that the accuracy of the CHAMP solutions is approximately one order of magnitude worse than the one of GRACE-only solutions. On the other hand and considering e.g. the degree difference RMS between PGM2007A and GGM02S, also discrepencies between these models occur which cannot solely be explained by numerical inaccuracies. Consequently, it has been investigated if the CHAMP solutions can serve as an indicator. This research shows results of the evaluation of the preliminary gravity field model PGM2007A with a CHAMP solution derived from two years of kinematic orbits. The quality of the global CHAMP solutions is further improved by a local refinement with Slepian functions which can make better use of the information in areas of high data density, e.g. in high-latitude areas. However, it is concluded that despite the data density the poorer quality of the CHAMP data is preventing a definite assessment of the quality of the PGM2007A.

Keywords. CHAMP, GRACE, energy balance approach, spherical harmonics, Slepian functions

1 Introduction

The evaluation of the preliminary gravity field model PGM2007A with CHAMP data will focus on the low to medium degree part of the spectrum due to the restricted spatial and spectral resolution of the CHAMP mission. It is a comparison between a single satellite but independent solution and PGM-2007A which contains besides GRACE also altimetric and terrestrial data. Three different data sets will be considered in the comparison. Besides PGM-2007A, the GRACE-only solution GGM02S provided by UTCSR (*Tapley et al.*, 2005) and a two year CHAMP-only solution (*Weigelt*, 2007) is used.

Before we start with the description of the data processing and the validation approach, all statistical quantities used throughout the paper are stated in section 2 for the sake of completeness. Section 3 introduces a short review of the CHAMP data processing strategy. The primary measurements are positions, velocities and accelerations which need to be related to a gravity field quantity. For this, the so-called energy balance approach is used in order to derive pseudo-potential observations along the orbit (*Jacobi*, 1836; *Jekeli*, 1999; *Gerlach et al.*, 2003). Subsequently, a global spherical harmonic analysis is performed in order to derive the global satellite-only solution. Since the data is not equally distributed, a local refinement in areas with high data density can make better use of the available information (*Garcia*, 2002; *Weigelt*, 2007). Here, the Slepian functions are employed and a proof of concept is presented in section 3.3.2. Subsequently, section 4 presents the global and local validation results in an attempt to indicate whether the CHAMP data agrees better with PGM2007A or GGM02S.

2 Tools of analysis

The quantification of the differences between CHAMP, GGM02S and PGM2007A is done in term of statistical quantities. For completeness, all the necessary formulas are given here with short explanations. They will be used extensively in section 4.

2.1 Spatial domain

Besides the maxima and minimum values and their location, values of interest are the mean, the standard deviation and the root mean square (RMS), arithmetic as well as area-weighted. The mean is given as:

$$\text{arithmetic:} \qquad \mu = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1}$$

$$\text{area weighted:} \qquad \mu_w = \frac{\sum_{i=1}^{N} w_i \cdot x_i}{\sum_{i=1}^{N} w_i}, \tag{2}$$

where x_i are the observations, N the number of observations and w_i the weights which are here determined

by calculating the area of a Voronoi cell around each data point. A Voronoi cell is characterized by an area in which any point is closer to the data point than to any neighboring data point. It is also referred to as Thiessen-polygons, Voronoi diagram or Dirichlet decomposition (*Barber et al.*, 1996).

The standard deviation is defined as the square root of the second moment of the mean, i.e. the square root of the variance:

$$\text{arithmetic:} \qquad \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2} \tag{3}$$

$$\text{area weighted:} \qquad \sigma_w = \sqrt{\frac{\sum_{i=1}^{N} w_i \cdot (x_i-\mu_w)^2}{\sum_{i=1}^{N} w_i}}. \tag{4}$$

The RMS is defined very similar:

$$\text{arithmetic:} \qquad \text{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}x_i^2} \tag{5}$$

$$\text{area weighted:} \qquad \text{RMS}_w = \sqrt{\frac{\sum_{i=1}^{N} w_i \cdot x_i^2}{\sum_{i=1}^{N} w_i}}. \tag{6}$$

In case of a zero mean, standard deviation and RMS will coincide.

Two, more rarely, used quantities are the skewness and kurtosis (*Webster and Oliver*, 2001). The skewness coefficient is defined formally as the third momentum of the mean divided by the third power of the standard deviation:

$$g = \frac{1}{\sigma^3}\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^3. \tag{7}$$

It is a measure of the asymmetry of the observations. Symmetric distributions have $g = 0$. Comparisons between means of different data sets are unreliable if the data is skewed. The kurtosis gives an estimate of the peakedness of a distribution:

$$k = \frac{1}{\sigma^4}\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^4. \tag{8}$$

For normal distributions $k = 0$; flatter distributions have $k < 0$ and more peaked ones $k > 0$.

2.2 Spectral domain

The spherical harmonic coefficients $\bar{K}_{lm}$ are two-dimensional quantities which are derived, e.g. in a least-squares adjustment or by quadrature. As output, the covariance matrix of the unknowns $Q_{\hat{x}\hat{x}}$ might be available. Taking the diagonal elements, the variance of the coefficients can be represented but correlations are neglected.

$$\text{diag}\,(Q_{\hat{x}\hat{x}}) = \text{VAR}\,\{\bar{K}_{lm},\bar{K}_{lm}\} = \sigma_{lm}^2 \tag{9}$$

The variance primarily represents the internal accuracy of the estimation, i.e. the fit of the model to the data. For a comparison with external data the difference between two signal spectra is more adequate:

$$\Delta_{lm} = \bar{K}_{lm}^2 - \bar{K}_{lm}^1. \tag{10}$$

The advantage of the latter is that it can also be used when $Q_{\hat{x}\hat{x}}$ is not available. Both of them are two-dimensional representations of signal and noise.

The most common way to determine a one-dimensional error spectrum from spherical harmonic coefficients is to derive degree-specific quantities. The first one to be mentioned is the error degree variance:

$$\sigma_l^2 = \sum_{m=-l}^{l}\sigma_{lm}^2 \qquad \forall \quad l \in [2\ldots L]. \tag{11}$$

The summation in this case is from $-l$ to l, where the negative degrees denote the sine and the positive the cosine coefficients. The error degree variance represents the total error power in the coefficients per degree and is a quadratic quantity. Dividing it by the number of coefficients $(2l+1)$ and taking the square root, an average standard deviation for the coefficients of a specific degree l can be derived. The result is the root mean square of the error spectrum per degree:

$$\text{RMS}_l = \sqrt{\frac{\sigma_l^2}{2l+1}} = \sqrt{\frac{1}{2l+1}\sum_{m=-l}^{l}\sigma_{lm}^2}. \tag{12}$$

The RMS_l is a representative standard deviation if and only if the error spectrum is isotropic, i.e. it is independent of the order m. Order specific components can also be derived but do not have any physical meaning. Equations (11) and (12) can be applied analogously to the signal difference spectrum (equation 10) yielding difference degree variances and difference degree RMS. These quantities will be used in the discussion of the global solutions in section 4.1.

3 Data Processing

The gravity field from CHAMP-data is recovered using the energy balance approach which yields pseudo-potential observations along the orbit (cf. section 3.1), followed by a brute-force spherical harmonic analysis on the sphere, see section 3.2. More details about the data processing can be found in *Weigelt* (2007). In section 3.3, we show that a local refinement could provide additional information in selected areas and introduce the framework of a Slepian analysis.

3.1 Energy balance approach

The basic idea is to separate orbit determination and gravity field recovery into three steps. The first step is the derivation of the position data which is done kinematically and provided by the Institute for Astronomical and Physical Geodesy (IAPG), TU Munich (*Švehla and Rothacher*, 2005). The data is considered independent from a priori information because no dynamical model is used in the calculation. Since the kinematic derivation yields positions only, velocities have to be derived numerically by a 4^{th} order central difference Taylor differentiator (*Khan and Ohba*, 1999). Subsequently, pseudo-potential observations are derived from the position data. The utilized energy balance approach is based on the law of energy conservation. The basic formula is given as:

$$T = E^{\text{kin}} - U - Z - \int \left(\mathbf{f} + \sum_i \mathbf{g}_i \right) d\mathbf{x} + c, \quad (13)$$

where T is the disturbing, U the normal and Z the centrifugal potentials. The latter two can be calculated pointwise from the position data. The normal potential is derived from the *WGS*84-constants which are given in table 1. The kinetic energy E^{kin} is derived from the velocity of the satellite. The integral contains all known time-variable gravitational accelerations $\mathbf{g}_i$ which are derived from models and are summarized in table 2. Non-gravitational accelerations $\mathbf{f}$ are measured using the accelerometer onboard CHAMP and calibration parameters are determined together with the integration constant c using a comparison of the pseudo-observables with the disturbing potential along the orbit derived from a known *a priori* model (here EGM96). Necessary transformations between the Earth-fixed and inertial frame are done in accordance with the IERS Conventions 2003 (*McCarthy and Petit*, 2003). Overall, two years of data from April 2002 till February 2004 are used for the calculation.

Tbl. 1. Constants in the calculation

	name	value
GM	grav. parameter	$3.986004415 \cdot 10^{14}\,\mathrm{m^3/s^2}$
R	radius	$6378136.46\,\mathrm{m}$
J_2	SH-coefficient	$1.08262982131 \cdot 10^{-03}$
J_4	SH-coefficient	$-2.37091120053 \cdot 10^{-06}$
J_6	SH-coefficient	$6.08346498882 \cdot 10^{-09}$
J_8	SH-coefficient	$-1.42681087920 \cdot 10^{-11}$
ω	rotation rate	$7.292115085 \cdot 10^{-05}\,\mathrm{rad/s}$

Tbl. 2. Utilized models for known corrections

source	model
astronomic tide	Sun/Moon (point masses) coordinates from DE405
solid Earth tide	IERS Conv. 2003, §6.1
solid Earth pole tide	IERS Conv. 2003, §6.2
ocean tide	FES2004
ocean pole tide	IERS Conv. 2003, §6.3
relativistic corrections	IERS Conv. 2003, §10.2

3.2 Global spherical harmonic analysis

The spherical harmonic analysis with its inherent downward continuation is done using a least-squares approach. The mathematical model connecting the pseudo-observable T with the spherical harmonic coefficients is given as:

$$T(r,\theta,\lambda) = \frac{GM}{R} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left(\frac{R}{r} \right)^{l+1} \bar{K}_{lm} \bar{Y}_{lm}, \quad (14)$$

where G is the gravitational constant, M the mass of the Earth, R the radius, r, θ and λ the spherical coordinates of the calculation point, $\bar{K}_{lm}$ the fully normalized spherical harmonic coefficients and $\bar{Y}_{lm}$ the spherical surface harmonics. The indices of the double summation are the degree l and the order m. For the least squares adjustment the equation can be reorganized into matrix-vector form:

$$\mathbf{l} + \varepsilon = A\mathbf{x}, \quad (15)$$

where $\mathbf{l}$ is the observation vector and is filled with the observations T. It is a stochastic quantity, which is expressed by the model inconsistencies ε. The unknown vector $\mathbf{x}$ is formed by the spherical harmonic coefficients $\bar{K}_{lm}$. All other elements of equation (14) are part of the A-matrix, i.e. for one particular mea-

surement k an element j of A reads:

$$a_{kj} = \frac{GM}{R}\left(\frac{R}{r}\right)^{n_j+1} P_{n_j,m_j}(\cos\theta_k)e^{im_j\lambda_k}. \quad (16)$$

The factor j denotes the column of the design matrix A and stands for a coefficient with one specific combination of degree l and order m. Here, the ordering is in accordance with *Colombo* (1983) which collectes $\bar{C}_{lm}$- and $\bar{S}_{lm}$-coefficients for all degrees in blocks of ascending orders. As an example and considering a maximum degree of $L = 70$, the column $j = 3$ corresponds to $\bar{C}_{20}$, $j = 73$ to $\bar{C}_{21}$ and $j = 143$ to $\bar{S}_{21}$. The least-squares solution is then achieved as:

$$\hat{x} = \underbrace{\left(A^T PA\right)^{-1}}_{N^{-1}} \underbrace{A^T Pl}_{y} = N^{-1}y, \quad (17)$$

where P is the inverse of the cofactor matrix and contains the error information of the observations. The kinematic positions are provided with error information for each data point including correlations between the coordinates but different data points are assumed uncorrelated.

3.3 Local refinement with Slepian functions

The motivation for a local refinement comes from the investigations by *Sneeuw et al.* (2003) who showed among others that the data distribution and the groundtrack influence the accuracy of the monthly CHAMP solutions. A similar effect for the GRACE-mission was discussed by *Yamamoto et al.* (2005) and *Wagner et al.* (2006). It is also known that the orbits are converging towards the poles yielding a much higher data density in these areas. Figure 1 shows the number of data points in 100 km^2 patches vs. the latitude. The increase of the data points per area is clearly visible. Consequently and by utilizing locally supported base functions, one can make better use of the information in the high-latitude areas.

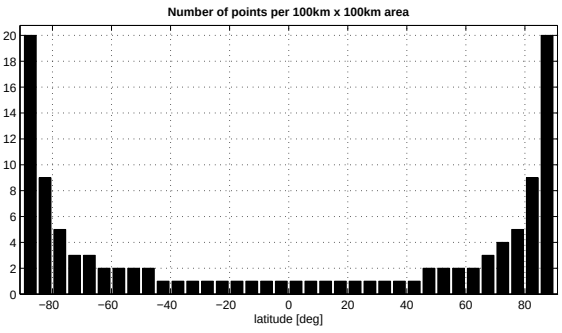


Fig. 1. Number of points per 100 km^2 area vs. latitude

3.3.1 An empirical localizing base function

Consider a function f which is strictly contained within an arbitrarily shaped region ω on the sphere Ω. Since it will not have values outside this area, it is spacelimited. Nevertheless and as any function on a sphere, it can be described by an infinite spherical harmonic expansion:

$$f = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \bar{f}_{lm}\bar{Y}_{lm}, \quad (18)$$

where $\bar{Y}_{lm}$ are again the spherical surface harmonics and $\bar{f}_{lm}$ the spherical harmonic coefficients, both normalized. Practically, a series till infinity cannot be implemented and the series needs to be truncated at a maximum degree L. The function becomes bandlimited.

Strictly speaking, no spherical function can be spacelimited and bandlimited at the same time. However, a set of bandlimited functions can be found, which are optimally concentrated within the area ω, and vice versa a spacelimited function, which is optimally concentrated within an interval $0 \le l \le L$. *Gilbert and Slepian* (1977) showed that this leads to the same description as an algebraic eigenvalue problem. The basic idea is to maximize the ratio between the spacelimited and the unlimited norm and thus the spatial concentration of the bandlimited function:

$$\xi = \frac{\|f\|_\omega^2}{\|f\|_\Omega^2} = \frac{\int\limits_\omega f^2\, d\Omega}{\int\limits_\Omega f^2\, d\Omega}, \quad (19)$$

where ξ is a measure of the spatial concentration. Using the bandlimited spherical harmonic synthesis formula and making use of the orthonormality relation of the normalized spherical surface harmonics, the relation reads

$$\xi = \frac{\sum\limits_{l=0}^{L}\sum\limits_{m=-l}^{l} \bar{f}_{lm}\sum\limits_{n=0}^{L}\sum\limits_{k=-n}^{n} D_{lmnk}\bar{f}_{nk}}{\sum\limits_{l=0}^{L}\sum\limits_{m=-l}^{l} \bar{f}_{lm}^2}, \quad (20)$$

with

$$D_{lmnk} = \int\limits_\omega \bar{Y}_{lm}\bar{Y}_{nk}\, d\Omega. \quad (21)$$

The elements D_{lmnk} can be arranged in a matrix D which is real, symmetric and positive definite. The solution of the spatial localization problem is found as the solution of an algebraic eigenvalue problem

forming an orthogonal set of base functions (*Simons et al.*, 2005):

$$D = G \Xi G^T. \tag{22}$$

Each column $\bar{g}^j$ of G represents an eigenvector and forms a base function $\bar{S}_j$ which can be reconstructed by

$$\bar{S}_j(\lambda,\theta) = \sum_{l=0}^{L} \sum_{m=-l}^{l} \bar{g}^j_{lm} \bar{Y}_{lm}(\lambda,\theta). \tag{23}$$

The corresponding eigenvalue ξ^j indicates the spatial concentration in the area R. Using the fully normalized Legendre functions for the calculation of D_{lmnk}, the eigenvalues will be normalized. $\xi = 1$ represents an optimal and $\xi = 0$ no concentration in the area of interest.

The application of the Slepian functions in physical geodesy is also not new. *Albertella et al.* (1999) considered the Slepian functions as a possible solution to the polar gap problem. Similar to equation (14), the potential along the orbit can be developed in Slepian base functions:

$$T(r,\theta,\lambda) = \frac{GM}{R} \sum_{j=1}^{(L+1)^2} \bar{\beta}_j \bar{S}_j(\lambda,\theta,r) \tag{24}$$

The unknown coefficients $\bar{\beta}_j$ are to be determined in a least-squares adjustment. Practically, not all $(L+1)^2$ Slepian coefficients can be estimated when utilizing local data. Only those with a concentration of 99% or higher will be considered since only those are well supported by the data. Less concentrated base functions will either cause leakage errors or lead to a rank deficient design matrix.

3.3.2 Proof of concept

The concept is proven in a test scenario with real CHAMP data of January 2003. GGM02S will serve as a reference for the comparison of the global spherical harmonic solution and Slepian solution. All solutions as well as the development of the kernel D_{lmnk} and the Slepian base functions are restricted to degree 70. The area of interest is a 23.5° spherical cap over Canada. Since only local data is to be used for the refinement, the long wavelength part cannot be estimated and needs to be reduced beforehand. For this, a global spherical harmonic solution till degree and order 40 has been derived from the data of January 2003 and reduced. After the estimation of the parameter $\bar{\beta}_j$, equation (24) can be used to synthesis the data in the area of interest and compare the results in the spatial domain with GGM02S.

The statistics of the comparison are shown in table 3 and indicate that by the usage of the Slepian functions an improvement has been reached. The maximum and minimum values are reduced by approximately 40% from 4.03 m to 2.39 m and from −3.76 m to −2.21 m, respectively. Since the mean value is close to zero, standard deviation and RMS are almost identical but both are reduced by 42.3% in case of the Slepian solution. The skewness indicates that the differences of the Slepian solution are slightly more asymmetric. Nevertheless, the values for both solutions are close to zero, i.e. the data can be considered unskewed and thus the comparison of the mean values is valid. The kurtosis shows a slightly flatter solution in case of the Slepian func-

Tbl. 3. Statistics of the differences of the global monthly spherical harmonic solution and the Slepian solution to GGM02S in terms of geoid height

quantity	global SH	Slepian
maximum	4.03 m	2.39 m
at $\lambda =$	110.62° W	60.62° W
$\phi =$	52.71° N	45.63° N
minimum	−3.76 m	−2.21 m
at $\lambda =$	106.87° W	106.04° W
$\phi =$	49.38° N	79.38° N
μ	-0.015 m	-0.011 m
μ_w	-0.020 m	-0.015 m
σ	1.384 m	0.855 m
σ_w	1.404 m	0.873 m
RMS	1.383 m	0.856 m
RMS_w	1.404 m	0.873 m
g	-0.026	0.110
k	-0.170	-0.271

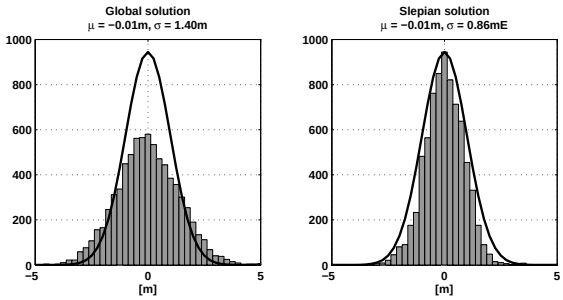


Fig. 2. Histogram of the monthly spherical harmonic solution (left) and the Slepian solution (right) vs. GGM02S in terms of geoid height

tions, i.e. peaks in this solution are flatter than in the global spherical harmonic solution. Practically, there is also no difference between the arithmetic and the area-weighted quantities which suggests that the differences are normally distributed. However, the histograms of both comparisons in figure 2 show that the data in the Slepian solution is closer to be normally distributed than the data of the global spherical harmonic solution.

Figure 3 visualizes the comparison in the spatial domain in terms of geoid height. The differences of the spherical harmonic solution (top) grows with increasing latitude and are more pronounced than the one of the Slepian solution (bottom). The pattern appears to be similar but in the later case the absolute values are approximately half the size of the former.

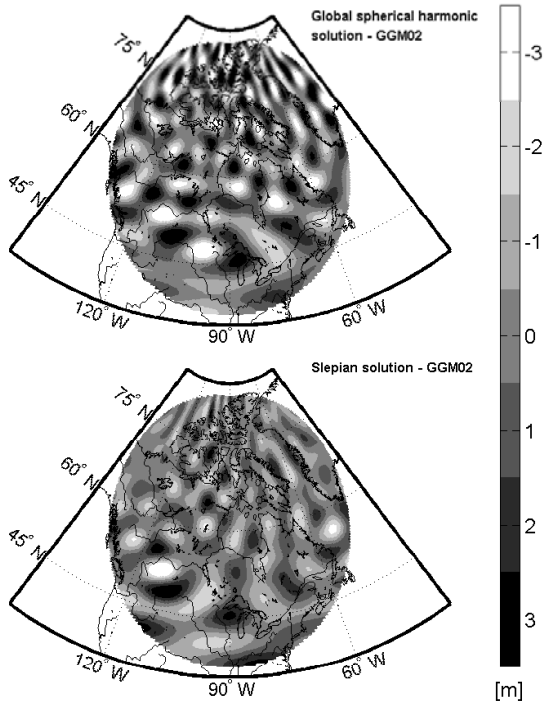


Fig. 3. Difference of the monthly spherical harmonic solution (top) and the Slepian solution (bottom) w.r.t. GGM02S in terms of geoid height

The results show that the improvement is primarily due to a better modelling of the short wavelength part of the spectrum and it can be concluded that better use of the data can be made by a local refinement. In the shown case, the global spherical harmonic analysis is obviously not able to take full advantage of the data density in the high-latitude area.

4 Validation results

After having outlined the data processing and the validation tools, this section will deal with the actual validation of the gravity field model PGM2007A. Naturally, both PGM2007A and GGM02S will outperform the CHAMP solution due to the higher data quality, i.e. a comparison in terms of absolute values does not make sense. Instead, the idea is to use the CHAMP solution as an indicatior by comparing the differences to PGM2007A and GGM02S in the spectral and spatial domain. Consequently, it can only be concluded to which model the CHAMP data fits better.

4.1 Global comparisons

The first comparison is done in the spectral domain. Figure 4 shows error and difference degree RMS of the three different solutions. The bottom

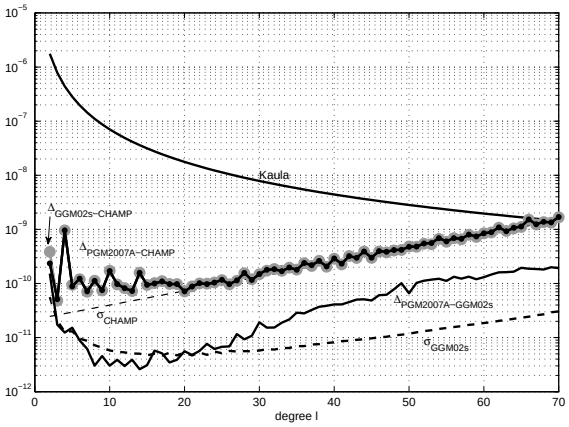


Fig. 4. Error and difference spectra of GGM02S, PGM-2007A and the CHAMP-solution

black solid line shows the difference degree RMS between PGM2007A and GGM02S whereas the dashed black line indicates the error degree RMS of GGM02S. Note the significant discrepancy. For the very low degrees, the difference degree spectrum is following the GGM02S error spectrum but from degree 20 onwards the difference deviates up to one order of magnitude.

The question is if CHAMP can serve now as an indicator. The gray and black line with dot marks are the difference degree RMS between GGM02S and PGM2007A both w.r.t. CHAMP. They are almost identical with the exception of degree 2. Here, the PGM2007A and the GGM02S solution disagree slightly which might be related to the handling of the $\bar{C}_{21}$- and $\bar{S}_{21}$-coefficients. Obviously and for all

degrees, the difference between PGM2007A and GGM02S is smaller than the difference to CHAMP and thus drawing conclusions will be very difficult. It is a first indication that both fields will perform equally in the comparison to the CHAMP data.

The comparison to the CHAMP data in the spatial domain and on a global scale is shown in figure 5 but it is inconclusive. Both solutions show no significantly different pattern. The statistical data and the histograms support this. The most significant difference is in the extreme values. The maximum differs with 1.342 m for GGM02S and 1.326 m for PGM-2007A by 1.6 cm. However, their location is varying and, considering the random nature of the differences in figure 5, this cannot be seen as significant. On the other hand, the minima with -1.338 m for GGM02S and -1.364 m for PGM2007A are at the same location and differ by 2.6 cm and thus show a very slight tendency of the CHAMP data towards the GGM02S solution. Mean value, standard deviation and RMS have the same slight tendency as the minimum but on average the difference for the latter two is 4 mm which is approximately 1% of the signal and thus cannot be considered significant. The skewness shows the difference between GGM02S and CHAMP is slightly more symmetric and the kurtosis indicates that it is also slightly less peaked than the difference between PGM2007A and CHAMP.

Overall, it has to be concluded that the differences are not significant and the GGM02S as well as the PGM2007A solution show the same behavior in the comparison to the CHAMP data. At best, one can say that there is a very slight tendency of the CHAMP data towards the GGM02S-solution.

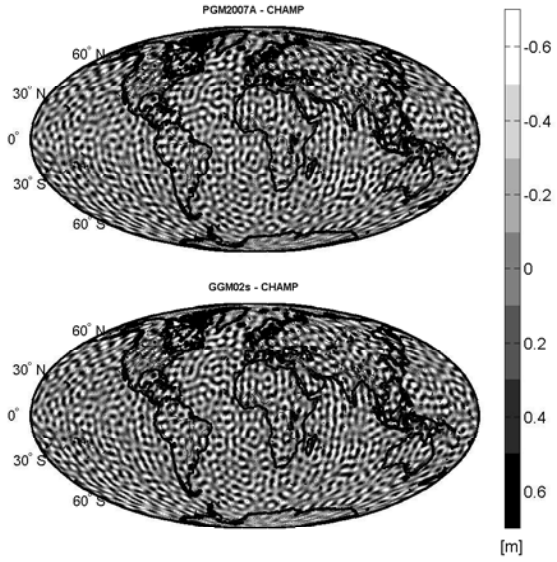


Fig. 5. Spatial comparison between the GGM02S and PGM2007A solution w.r.t. the global CHAMP solution

Tbl. 4. Statistics of the differences of GGM02S and PGM2007A to the CHAMP solution in terms of geoid height

quantity	PGM2007A	GGM02S
maximum	1.326 m	1.342 m
at λ	77.00° E	72.00° E
ϕ	29.00° N	22.00° S
minimum	-1.364 m	-1.338 m
at λ	19.00° E	19.00° E
ϕ	52.00° S	52.00° S
μ	-0.018 m	-0.004 m
μ_w	-0.024 m	-0.009 m
σ	0.320 m	0.317 m
σ_w	0.329 m	0.326 m
RMS	0.321 m	0.317 m
RMS_w	0.330 m	0.326 m
g	0.038	0.029
k	0.239	0.199

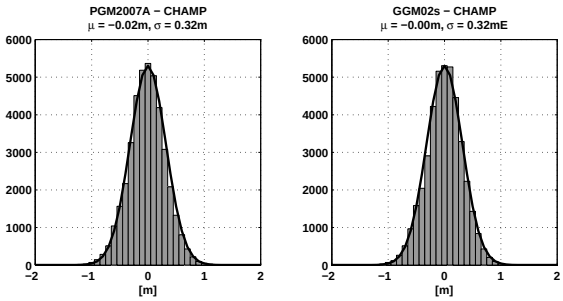


Fig. 6. Histogram of PGM2007A (left) and GGM02S (right) w.r.t. the CHAMP soluition in terms of geoid height

4.2 Local comparison

For the local refinement of the CHAMP solution a latitude band from 60° N to 85° N is chosen and 141 base functions are used in the Slepian adjustment to improve the solution. In the remove step the full global spherical harmonic solution from CHAMP till degree 70 is used. The recovered residual signal has a strength of $\delta N \approx 10$ cm which is added in the spatial domain to the spherical harmonic solution

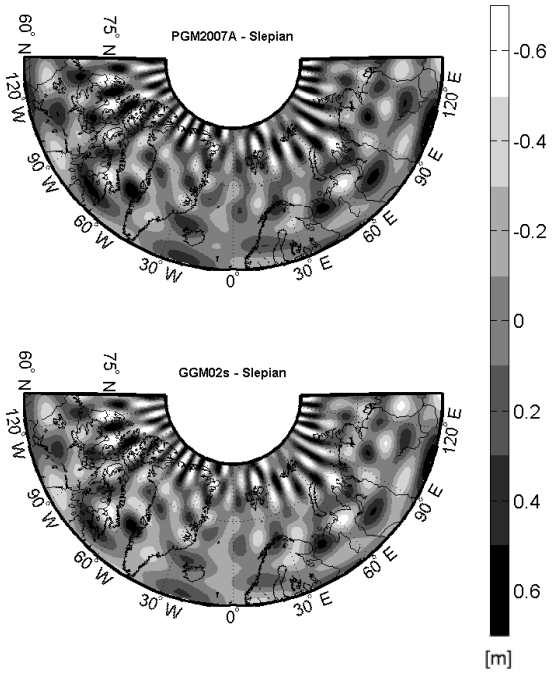


Fig. 7. Spatial comparison between the PGM2007A (top) and GGM02S (bottom) solution w.r.t. the CHAMP solution for a latitude band from 60° N to 85° N

and hereafter called the Slepian solution.

The comparison in the spatial domain of PGM-2007A and GGM02S to the Slepian solution is shown in figure 7 but again shows no significant differences. The top picture, i.e. the comparison between PGM2007A and the Slepian solution, appears slightly darker but both pictures are dominated by the deficiencies in the CHAMP data.

The histograms show nearly identical values for both cases. The difference between PGM2007A and the Slepian solution has a by 2 cm higher mean value

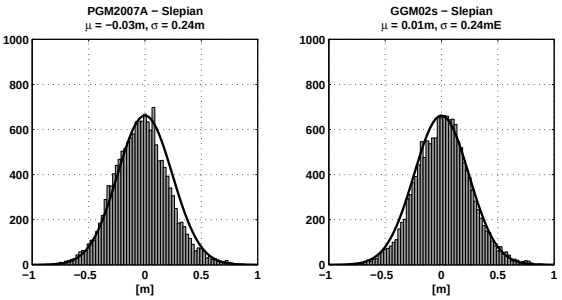


Fig. 8. Histogram of GGM02S and PGM2007A solution w.r.t. the Slepian solution for a latitude band from 60° N to 85° N

which explains the darker impression of the top panel in figure 7. The extremas of both solutions are at the same location and thus comparable. The maximum values are 1.014 m for GGM02S and 1.032 m for PGM2007A and differ by 1.8 cm. The minimum values are −1.207 m for GGM02S and −1.217 m for PGM2007A and thus 1 cm smaller in the difference of the GGM02S to the Slepian solution, i.e. there is again a slight tendency of the CHAMP data towards the GRACE solution. The same can be seen in the mean value. All other statistics are very similar and the differences are not significant.

Tbl. 5. Statistics of the differences of PGM2007A and GGM02S to the CHAMP solution in terms of geoid height for a latitude band from 60° N to 85° N

quantity	PGM2007A	GGM02S
maximum	1.032 m	1.014 m
at λ	89.38° W	89.38° W
ϕ	66.88° N	66.88° N
minimum	−1.217 m	−1.207 m
at λ	176.88° W	176.88° W
ϕ	61.88° N	61.88° N
μ	-0.030 m	-0.006 m
μ_w	-0.031 m	-0.015 m
σ	0.247 m	0.245 m
σ_w	0.318 m	0.313 m
RMS	0.247 m	0.245 m
RMS_w	0.318 m	0.314 m
g	-0.0347	-0.0378
k	0.5799	0.6098

5 Conclusions

In conclusion, one can say that the poorer quality of the CHAMP data is preventing a real statement about the quality of the PGM2007A. At best, one can say that the global as well as the local CHAMP solution agree slightly better with the GRACE-only solution GGM02S than with the PGM2007A. PGM2007A and GGM02S show significant difference from degree 20 to 70 which, however, cannot be verified nor quantified in the comparison to the CHAMP data.

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Evaluation of the GRACE-Based Global Gravity Models in Canada

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Abstract. Four GRACE-based gravity models, especially EGM08, have been evaluated using the surface gravity observations, GPS-levelling, deflections of the vertical, and the recent Canadian gravimetric geoid CGG05**.** The RMS of the differences between the EGM08-predicted gravity anomalies and the observed anomalies is smaller than 5 mGal on sea and lake surfaces, in contrast to about 14 mGal on land in Canada. The RMS increases with increasing elevation on land, exhibiting an evident height-dependent trend, while the RMS decreases with increasing depth on sea and lake surfaces, without a significant trend. The GPS-levelling comparisons suggest that EGM08 models the geoid with an accuracy of 10 cm or better in Canada. It is comparable with the Canadian Gravimetric Geoid 2005 (CGG05). Recent releases of GRACE models show noteworthy improvement over earlier ones. The comparisons between the EGM08-predicted and astronomical deflections of the vertical show the RMS of 1.8 arc-seconds in the north-south direction, and 2.1 arc-seconds in the east-west direction, which are significantly larger than the RMS of differences between the CGG05-predicted and astronomical deflections .

Keywords. GRACE, geoid, gravity, GPS-levelling, deflections of the vertical, EGM08

1 Introduction

A series of satellite-only and combined global gravity models have been developed by a number of scientific teams worldwide since the twin Gravity Recovery And Climate Experiment (GRACE) satellites were launched on March 17, 2002. Two representative GRACE-only models are GGM02S (Tapley et al., 2005) and GL04S1 (Forste et al., 2008) developed by the Center for Space Research, University of Texas, USA and jointly by GFZ, Germany and GRGS, France, respectively. The most recent and revolutionary global model is the Earth Gravitational Model 2008 (EGM08) developed by the National Geospatial-Intelligence Agency (NGA) of USA, superseding its processor EGM96 (Lemoine et. al, 1998). EGM08 combines satellite (GRACE), marine (satellite-altimetry-derived), and land gravity data to model the global gravity field with a geo-spatial resolution of 5 by 5 arcmin (Pavlis et al., 2008). It is complete to degree and order 2159 and contains additional spherical harmonic coefficients up to degree 2190, which account for the transform corrections from ellipsoidal to spherical coefficients. Its accuracy is largely dependent on the accuracy of GRACE, marine and land gravity data and their availability. The fact that the geo-spatial resolution of EGM08 approaches to that of Canadian regional geoid models will certainly have significant impact on the development of a geoid model with one-centimetre accuracy in Canada.

The objective of this study is to evaluate GRACE global models and EGM08, i.e., to examine improvement of the low-degree components and the accuracy of EGM08 using surface gravity, GPS-levelling, deflection of the vertical data and CGG05.

2 Gravity Comparisons

2.1 Surface Gravity Data

The Canadian Gravity Database (CGDB) is a collection of gravity observations on land, sea and lake surfaces, sea and lake bottoms, ice caps and over the air. About 98 percent of the observations were collected during the past 50 years covering entirely Canada and neighboring seas. The accuracy of these data varies from place to place depending on types of instruments, platforms, height reductions, etc. They have a mean error standard deviation of 1.88 mGal. They can be broken down into six different types in reference to observation platform as described in Tables 2.1 and 2.2, and

displayed in Figure 2.1. The free-air gravity anomalies are derived from the observations with reference to GRS80. The following formula was used to add the atmospheric corrections to these free-air anomalies in the gravity database.

$$Dg_A = 0.8658 - 9.727 \times 10^{-5} H + 3.482 \times 10^{-9} H^2 \quad (1)$$

where H is the orthometric height in meter. The atmospheric correction is in mGal.

Figure 2.1 Distribution of gravity observations. Blue dots are on land; yellow dots on water surface; red dots on water bottom; black dots on ice surface; green dots for airborne data.

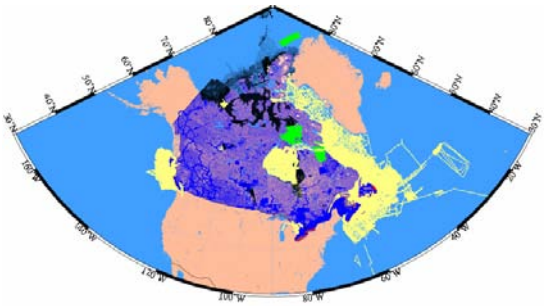


Table 2.1 Types of the gravity data in Canada.

Code	Number	%	Description
Land	215,862	28.80	land
SLS	455,464	60.78	water surface
SLB	6,538	0.87	water bottom
SLI	36,487	4.87	ice surface
IC	802	0.11	ice cap
AIR	34,249	4.57	airborne
All	749,402	100.00	

Table 2.2 Statistical description of the free-air gravity anomalies. Unit: mGal.

Code	Min	Max	Mean	Std	RMS
Land	-184	393	-10	29	31
SLS	-161	241	-6	36	36
SLB	-103	102	-18	26	32
SLI	-168	107	-23	30	38
IC	-98	218	55	43	70
AIR	-100	54	-27	30	40
All	-184	393	-9	34	35

2.2 Computation of the Gravity Anomaly from EGM

The gravity anomaly is computed from the following formula (Heiskanen and Moritz, 1967):

$$\Delta g = -\frac{\partial T}{\partial h} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial h} T \quad (2)$$

where T is the disturbing potential, h is the ellipsoidal height, γ is the normal gravity.

A routine ('Harmonic_synth.f', version 05/01/2006, isw=50, 3 and 7) provided by National Geospatial-Intelligence Agency (NGA) was used in computing point free-air gravity anomalies at the gravity stations on the Earth's surface. Input parameters were set to produce point values in the tide-free system with reference to GRS80 (Moritz, 1992). The gravity anomalies were synthesized from spherical harmonic degree 2 to 2190 of EGM08 as well as its preliminary model PGM07A at each gravity station. The zero-degree term due to the difference between the geopotential constants GM of EGM08 and GRS80 is 0.144 mGal in absolute value and was added to the synthesized anomalies.

2.3 Comparisons

The gravity anomalies synthesized from PGM07A and EGM08 were compared to the observed ones. The results are shown in Table 2.3.

Table 2.3 Differences between the observed gravity anomalies and the gravity anomalies predicted from (PGM)07A and (EGM)08. Unit: mGal.

Data	Model	Min	Max	Mean	Std
Land	07A	-176	174	-1.64	13.75
	08	-177	175	-1.69	13.71
SLS	07A	-202	48	-1.19	4.78
	08	-204	53	-1.14	4.61
SLB	07A	-43	29	-0.78	4.00
	08	-44	28	-0.68	4.17
SLI	07A	-81	50	-0.49	4.47
	08	-89	50	-0.56	4.14
IC	07A	-65	82	-1.30	19.61
	08	-61	82	-2.29	19.61
AIR	07A	-23	39	1.40	4.84
	08	-23	37	1.57	4.61
All	07A	-202	174	-1.16	8.45
	08	-204	175	-1.14	8.36

The RMS of the differences on land is about 14 mGal, which is caused by the land data errors, the

commission and omission errors of PGM07A and EGM08. Huang and Véronneau (2008) suggested that the land data contain a systematic error of about 1.4 in RMS. Pavlis et al. (2008) suggest that the commission error of EGM08 is about 1 - 2 mGal. These estimates indicate that the model omission errors are very likely dominant. This conclusion is consistent with the RMS estimate from the Tscherning-Rapp degree variance model (1974), that is 11.2 mGal for the spherical harmonic components higher than degree 2160. The differences in Figure 2.2 show a linear increase with respect to the elevations of the gravity stations at the rate of about 8.66 mGal/km, and a stronger trend associated with the elevations higher than 1 km.

Figure 2.2 The differences between the gravity anomalies predicted from EGM08 and the gravity anomalies from observations vs. elevations on land.

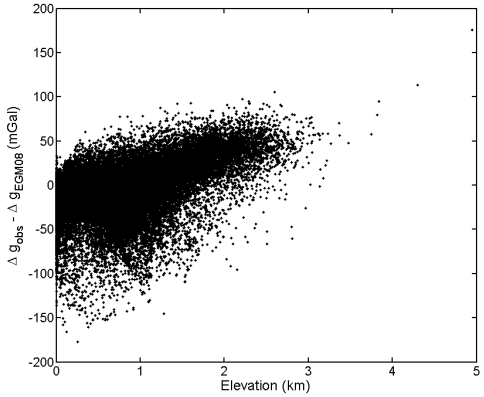
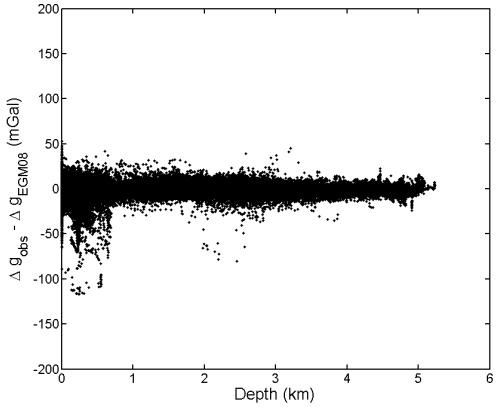


Figure 2.3 The differences between the gravity anomalies predicted from EGM08 and the gravity anomalies from observations vs. depth on sea and lake surfaces.



The comparison over sea and lake surfaces performs significantly better than the one on land. The RMS of the differences on sea and lake surfaces is smaller than 5 mGal which is about one third of the land one. It largely reflects the model omission level over seas where most marine data are located. Figure 2.3 indicates that the differences decrease with increasing depth without showing an evident dependence on depth. This reflects the fact that the separation between water surface and bottom serves as a natural low-pass filter to smooth the gravity field on the water surface: the greater the depth, the smoother the surface field. It can also be noticed that the differences for depths of less than 1 km show a similar magnitude to those on land. They mostly correspond to coastal areas with shallow water where only the altimetry-derived gravity data are used for EGM08.

A comparison between PGM07A and EGM08 suggests that the latter agrees notably better with the observed values than the former.

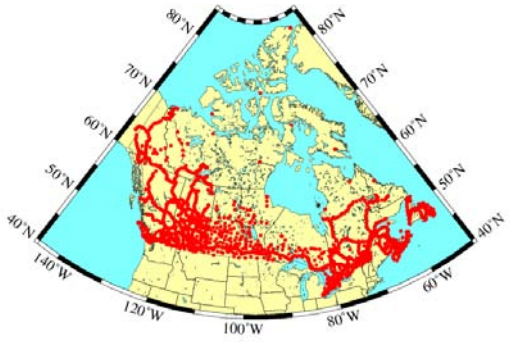
3 GPS-Levelling Comparisons

3.1 GPS-Levelling Data

The geoid heights or height anomalies derived from GPS-levelling data are the highest quality and most reliable data in validating the gravimetric geoid. Two GPS-Levelling data sets are used in this evaluation. The first set is derived from the Canada-wide GPS and levelling adjustments of 2004 and includes 430 stations. The ellipsoidal heights at these stations were estimated from GPS observations collected after 1994, processed with precise IGS orbit products defined in ITRF97 and referred to the GRS80 reference ellipsoid. Their standard deviations vary from 0.2 cm and 7.6 cm with an average of 1.3 cm. The co-located orthometric heights were estimated from levelling measurements made after 1981 to minimize crustal motion effect. A single fixed station in Rimouski, Québec was chosen to define mean sea level for the adjustment, similar to the constraint for the North American Vertical Datum of 1988 (NAVD88). Their standard deviations increase with increasing distance from the fixed station in Rimouski ranging from 0 to 9 cm with a mean standard deviation of 5.4 cm.

The second set consists of all 2579 co-located GPS-levelling stations currently available in Canada. The early GPS observations date back to the late 1980s and the levelling data can trace back to the beginning of the 1900s. The standard deviations of the ellipsoidal heights vary from less than 1 cm to a few decimeters. Recent observations are generally more precise than earlier ones. On the other hand, the orthometric heights can be wrong by a few decimeters largely due to natural changes and human activity. Even though it is difficult to interpret validation results from this data set due to incomplete knowledge on its uncertainty, inclusion of more stations is considered to provide additional information on the quality of validated geoid models.

Figure 3.1 Geographical distribution of 2579 GPS-levelling stations.



3.2 Computation of the Geoid Height from an EGM

The following equation is used to compute the geoid height with respect to GRS80:

$$N_L = \frac{G\delta M}{r_e\gamma_0} - \frac{\delta W}{\gamma_0} + \frac{GM}{r_e\gamma_0}\sum_{n=2}^{L}\left(\frac{a}{r_e}\right)^n \sum_{m=0}^{n} Y_{nm}(\phi,\lambda) + C_{TB} \quad (3)$$

The first two terms on the right side of Equation (3) are the zero-degree corrections. The second term represents the spherical harmonic expansion of the geoid. The last term is the so-called topographic bias that corrects for the analytical downward continuation error within the topography (see e. g. Rapp, 1997). The DTM is synthesized from a spherical harmonic topographic model provided by NGA complete to spherical harmonic degree and order 2190 over the land, while it is assigned as zero over the oceans. The geoid heights were evaluated with respect to the reference ellipsoid defined by GRS80. The geoid potential is defined by W_0 = 62636856.88 m^2s^{-2} which is the same as that for CGG05 (Véronneau and Huang, 2007).

3.3 Comparisons

One objective of the GPS-levelling comparisons is to check the improvement of the GRACE gravity models. GGM02S (Tapley et al., 2005) and EIGEN-GL04S1 (Förste et al., 2008) were chosen to represent early and recent releases of GRACE-only models, respectively. PGM07A uses a JPL GRACE model while EGM08 uses ITG-GRACE03 (Mayer-Gürr, 2007). Therefore, there are four different GRACE models included in the comparisons directly and indirectly. The comparisons were limited with a degree band spanning from spherical harmonic degrees 2 to 90 because the error of GRACE models increases rapidly beyond degree 90. In order to eliminate the omission error effect on the comparisons, GGM02S and GL04S1 were extended to degree and order 2159 with additional coefficients up to degree 2190 from EGM08. The validation results are shown in Tables 3.1 and 3.2.

Table 3.1 Comparisons of EGM models against the GPS-levelling data at 430 stations. Unit: m.

Model	Min	Max	Mean	Std
GGM02S*	-0.672	-0.147	-0.351	0.102
GL04S1*	-0.661	-0.157	-0.354	0.100
PGM07A	-0.741	-0.069	-0.356	0.105
EGM08	-0.669	-0.149	-0.356	0.100
CGG05	-0.668	-0.120	-0.396	0.102

Table 3.2 Comparisons of EGM models against the GPS-levelling data at 2579 stations. Unit: m.

Model	Min	Max	Mean	Std
GGM02S*	-0.923	0.088	-0.375	0.135
GL04S1*	-0.929	0.080	-0.375	0.132
PGM07A	-0.913	0.123	-0.367	0.136
EGM08	-0.922	0.090	-0.380	0.133
CGG05	-0.932	0.067	-0.420	0.134

It is evident that there exist biases of about 36 cm between the GPS-levelling derived geoid heights and the gravimetric geoid heights. These biases mainly represent the separation between the mean sea level at the fixed station in Rimouski and the adopted global sea level. Once determined, these constant biases can be corrected. Ignoring these

biases, the standard deviation gives a proper measure of the level of agreement between the geoid and GPS-levelling.

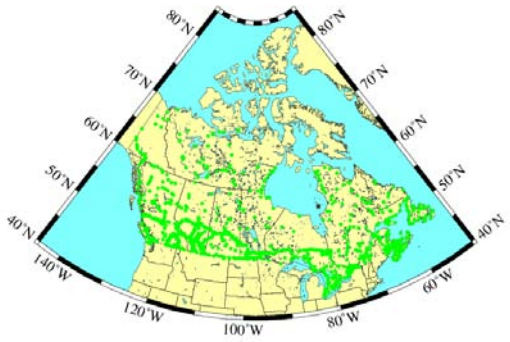
It can be seen that recent GL04S1 shows slight improvement over early GGM02S. EGM08 performs equally well with the extended GL04S1 in terms of standard deviation. This result suggests that the differences between the low degree parts (2 to 90) of EGM08 and GL04S1 are insignificant in term of the GPS-levelling accuracy. In the meantime, EGM08 shows slight improvement over its preliminary model PGM07A. It can partly be attributed to the improvement of ITG-GRACE03 over the early JPL GRACE model used for PGM07A. It is worth pointing out that a weighted scheme was used for EGM08 to combine ITG-GRACE03 and terrestrial gravity mainly within a degree band from degrees 70 to 120 using covariance information associated with both data sources. The direct truncation and extension used here for GGM02S and GL04S1 is not theoretically optimal, and may introduce additional errors.

The second objective is to estimate the accuracy of EGM08. The standard deviation of h-H-N results from errors in ellipsoidal, orthometric and geoid heights. The EGM08 results suggest it can predict the geoid in Canada with an accuracy of better than 10 cm when the errors in the ellipsoidal and orthometric heights are taken into consideration. That accuracy reflects the aggregate level of the EGM08 commission and omission error, and is comparable with that of CGG05 shown in Tables 3.1 and 3.2. The fact that EGM08 compares well with the regional geoid model in terms of precision represents an exceptional achievement.

4 Comparisons with Deflections of the Vertical

4.1 Deflections of the Vertical in Canada

Figure 4.1 Geographical distribution of 939 deflection stations.



Similar to the gravity anomalies, deflections of the vertical are gradients of the anomalous disturbing potential but with respect to horizontal directions instead of the vertical. They are usually defined by their south-north and east-west components that represent geoid slopes in each direction. They are determined by astronomical and geodetic observations:

$$\begin{aligned} \xi &= \Phi - \phi \\ \eta &= (\Lambda - \lambda)\cos\phi \end{aligned} \tag{4}$$

where the pair (Φ, Λ) are astronomical latitude and longitude, and the pair (ϕ, λ) are geodetic latitude and longitude. In Canada, the astronomical observations were collected from 1910 to 1975, and estimated to be accurate to 0.3 to 0.5 arcsec. The geodetic observations are known in ITRF93 but with unknown accuracy. However, they should be more accurate than the astronomical ones in terms of observation technology. Figure 4.1 shows the distribution of 939 Canadian stations with deflections of the vertical. Table 1 gives a statistical description of these data.

Table 4.1 Statistical description of the deflections of the vertical in Canada in arc-second.

	Min	Max	Mean	Std
ξ	-23.410	17.880	0.205	4.501
η	-22.390	24.350	-0.232	6.137

4.2 Computation

Deflections of the vertical on the Earth's surface can be computed by the following formulae (Heinskanen and Moritz, 1967):

$$\begin{aligned} \xi &= -\frac{1}{R}\frac{\partial \varsigma}{\partial \phi} \\ \eta &= -\frac{1}{R\cos\phi}\frac{\partial \varsigma}{\partial \lambda} \end{aligned} \tag{5}$$

where ς represents the height anomaly and can be computed from the geoid height by

$$\varsigma = N - \frac{\Delta g_B}{\bar{\gamma}} H \tag{6}$$

H is the orthometric height, and Δg_B is the Bouguer gravity anomaly. The deflections at a point

were numerically computed from values at nine nodes closest to that point in an evenly-spaced grid of 2 by 2 arcmin.

4.3 Comparisons

Deflections of the vertical predicted from PGM07A and EGM08 were compared to the observations made at stations shown in Figure 4.1. The comparison results are described in Tables 4.2 and 4.3. It can be seen that the differences are significantly larger than the estimated 0.5 arcsec error level of the astronomical latitude and longitude. They are also significantly larger than the differences between the CGG05-derived and astronomical deflections reported in Tables 4.2 and 4.3. CGG05 is determined in the spacing of 2 by 2 arcmin while EGM08 has a spatial resolution of 5 by 5 arcmin. Thus, the omission error of CGG05 is smaller than that of EGM08. The omission error of PGM2007A and EGM08 are most likely a major source for those larger differences. This conclusion is consistent with the RMS estimate from the Tscherning-Rapp degree variance model (1974), that is 1.66 arcsec for the spherical harmonic components higher than degree 2160. One may argue that the performance of deflection tests is not as critical as the GPS-levelling one because of the predominant role of the geoid as height datum.

Like the gravity and GPS-levelling comparisons, the deflection comparisons in Tables 4.2 and 4.3 suggest a slightly better agreement of EGM08 than PGM2007A with the observations.

Table 4.2 Differences between the predicted deflections of the vertical and the observations along the north-south (ξ) direction in arc-second.

Models	Min	Max	Mean	Std
PGM07A	-12.364	9.946	-0.013	1.785
EGM08	-12.429	9.948	-0.014	1.760
CGG05	-5.331	9.835	0.051	1.215

Table 4.3 Differences between the predicted deflections of the vertical and the observations along the east-west (η) direction in arc-second.

Models	Min	Max	Mean	Std
PGM07A	-12.839	14.944	0.203	2.111
EGM08	-12.814	15.266	0.197	2.101
CGG05	-12.703	9.159	0.232	1.643

5. Summary

The two GRACE-only models GGM02S and GL04S1, and the two combined models PGM07A and EGM08 have been evaluated using the surface gravity observations, GPS-levelling, deflections of the vertical, and the recent Canadian geoid model CGG05. The main conclusions can be summarized as:

1. The accuracy of the EGM08-predicted geoid is better than 10 cm in Canada. It is comparable with that of the recent Canadian geoid CGG05.
2. The omission error of EGM08 is about two times larger on land (~14 mGal) than that on sea and lake surfaces (~5 mGal).
3. The recent GRACE-only models show noteworthy improvement over the earlier ones.
4. CGG05 significantly outperforms EGM08 in terms of their validation against deflections of the vertical on land mainly due to its higher spatial resolution.

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EGM08 Comparisons with GPS/Leveling and Limited Aerogravity over the United States of America and its Territories

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Abstract. GPS-derived ellipsoidal heights on leveled bench marks (GPSBM's) have long been used as external data for testing gravimetric geoid models at regional and global scales. Ellipsoidal heights above the NAD 83 datum are available for all regions of the United States and its territories. Vertical datums are also available for all of the conterminous United States (NAVD 88), Alaska (NAVD 88), Puerto Rico (PRVD02), Guam (GUVD04), the Commonwealth of the Northern Marianas Islands (NMVD03), and American Samoa (ASVD04). Hence, it possible to provide estimates of the geoid undulation at a select few points scattered around the world. The more recent vertical datums provide coverage in remote regions and are more internally consistent and accurate. These data were compared to the Earth Gravity Model of 2008 (EGM08) in an effort to assess its quality and utility. Results indicated significant improvements for both the Conterminous United States (CONUS) and Outside CONUS (OCONUS) regions. In CONUS, a significant, meter-level trend was better defined than in EGM96. The remaining signal was also much smaller (under 7 cm SD). No significant trends were determined for OCONUS regions and most had dm-level agreement. Additionally, limited airborne gravity data over the northern Gulf of Mexico region were available. These data had very low cross-over errors (0.87 mgals RMSE) and compared favorably to EGM08 (1 mgal SD). Over all, the EGM08 model is deemed a definite improvement over the antecedent, EGM96, and it will be employed as a reference field in developing forthcoming national models for the United States.

Keywords. EGM, GPSBM, aerogravity, calibration/validation

1 Introduction

The National Geospatial-Intelligence Agency's (NGA) standard reference gravity field model for the past decade has been the venerable Earth Gravity Model of 1996, EGM96 (Lemoine et al. 1998). This model has served as a common reference field in the development of regional models, thereby ensuring high levels of international agreement between regions. NGA has now released an update to this in the form of the Earth Gravity Model of 2008, EGM08 (Pavlis et al. 2008). NGA formed an international working group under the auspices of the International Association of Geodesy to assess the quality and utility of this new model. This report provides that assessment from the national perspective of the United States.

The National Geodetic Survey (NGS) is the civilian counterpart to NGA and is responsible for maintaining the National Spatial Reference System (NSRS). The NSRS is composed of many elements including maintenance of bench marks and reference frames for GPS-derived ellipsoidal heights, orthometric heights, and gravity observations. These data were used here to evaluate EGM08.

The difference between the GPS-derived ellipsoidal heights and orthometric heights on leveled bench marks (GPSBM's) provides a point estimate of the separation between the ellipsoidal and vertical datums. It should be noted that there are several different vertical datums, primarily for geographically separated regions: conterminous United States (NAVD 88), Alaska (NAVD 88), Puerto Rico (PRVD02), Guam (GUVD04), the Commonwealth of the Northern Marianas Islands (NMVD03), and American Samoa (ASVD04). Hence, each of these regions was independently assessed to evaluate EGM08 worldwide.

2 CONUS GPSBM'S

One truth about the GPSBM's determined for conterminous United States (CONUS) is that their realization changes over time, even if the position on the ground doesn't move. Constant new observations on the existing bench marks as well as ongoing adjustments modify the ellipsoidal height for a

given location. The net effect is that the apparent datum separation changes. Hence, comparisons with GPSBM data must be cautiously examined to ensure that an EGM with dm-level errors isn't compared to data with meter-level errors.

In an effort to reduce some of these known errors, NGS recently completed a national readjustment. The results in the shift of the ellipsoidal heights are shown in Figure 1. Note that many states experienced shifts in a mean value as well as increased variability. More significantly, some states show systematic effects (e.g., Minnesota).

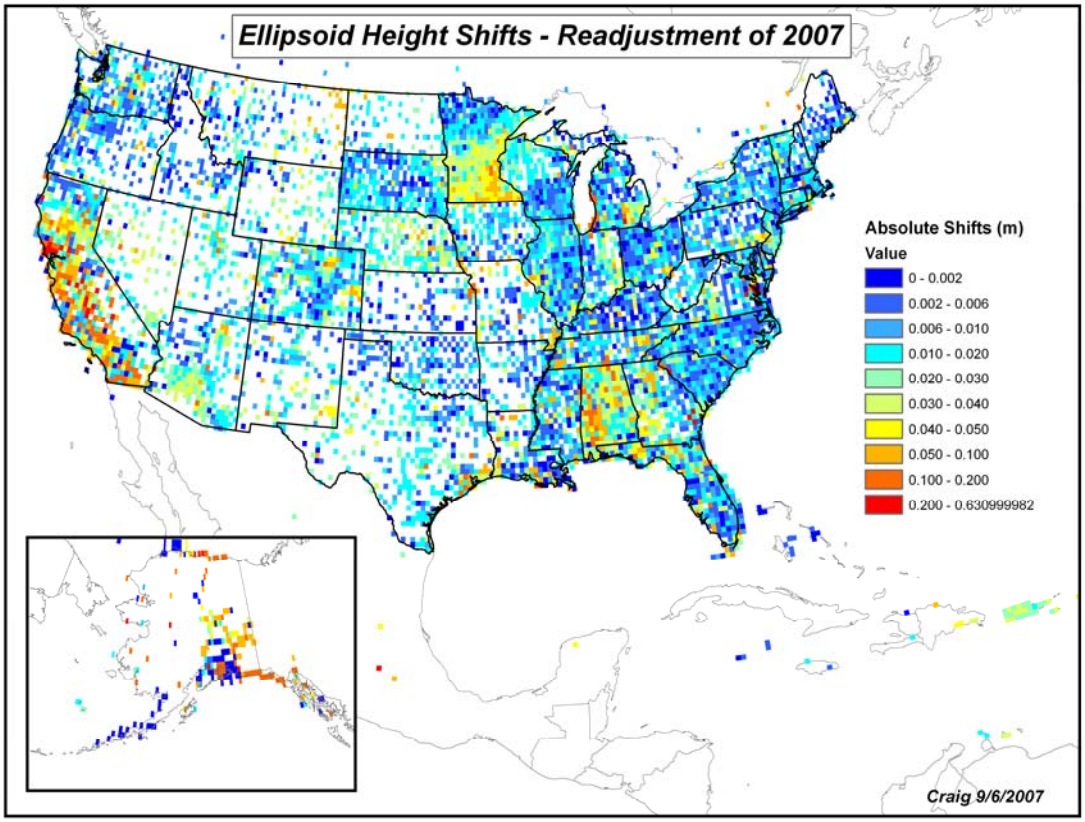


Figure 1. Changes to ellipsoid height coordinates as a result of the National Readjustment of 2007 for CONUS and Alaska. Note dm-level biases, tilts, and standard deviations across numerous states.

GPSBM's were then developed using these revised ellipsoidal heights for comparison to EGM08. Biases were removed state by state and then a standard deviation was determined. These results for CONUS are given in Figure 2. The biases are given on the top and the SD on the bottom. Note the trend in the state biases increases steadily westward. This is a significant feature thought to represent the propagated errors in the development of the North American Vertical Datum of 1988 (NAVD 88). The SD also shows increases westward into the mountainous western states, where a higher variability is not unexpected.

EGM96 and regional gravimetric geoid models developed from it were also compared. These models showed similar magnitude trends in the biases but were about 50% worse in the state SD comparisons. Nationally, EGM08 had about a 7 cm SD, while EGM96 was closer to 11 cm.

These improvements show that adopting EGM08 as a reference field significantly reduced the residual signal. This is highly desirable given amount of variability seen in the residuals across the CONUS region. Fortunately, other U.S. regions outside CONUS do not see such great variability in the systematic errors associated with the respective GPSBM's.

EGM08 was also compared at sites in Alaska, American Samoa, Guam, the Commonwealth of the Northern Marianas Islands, and Puerto Rico where similar improvements were seen (Table 1).

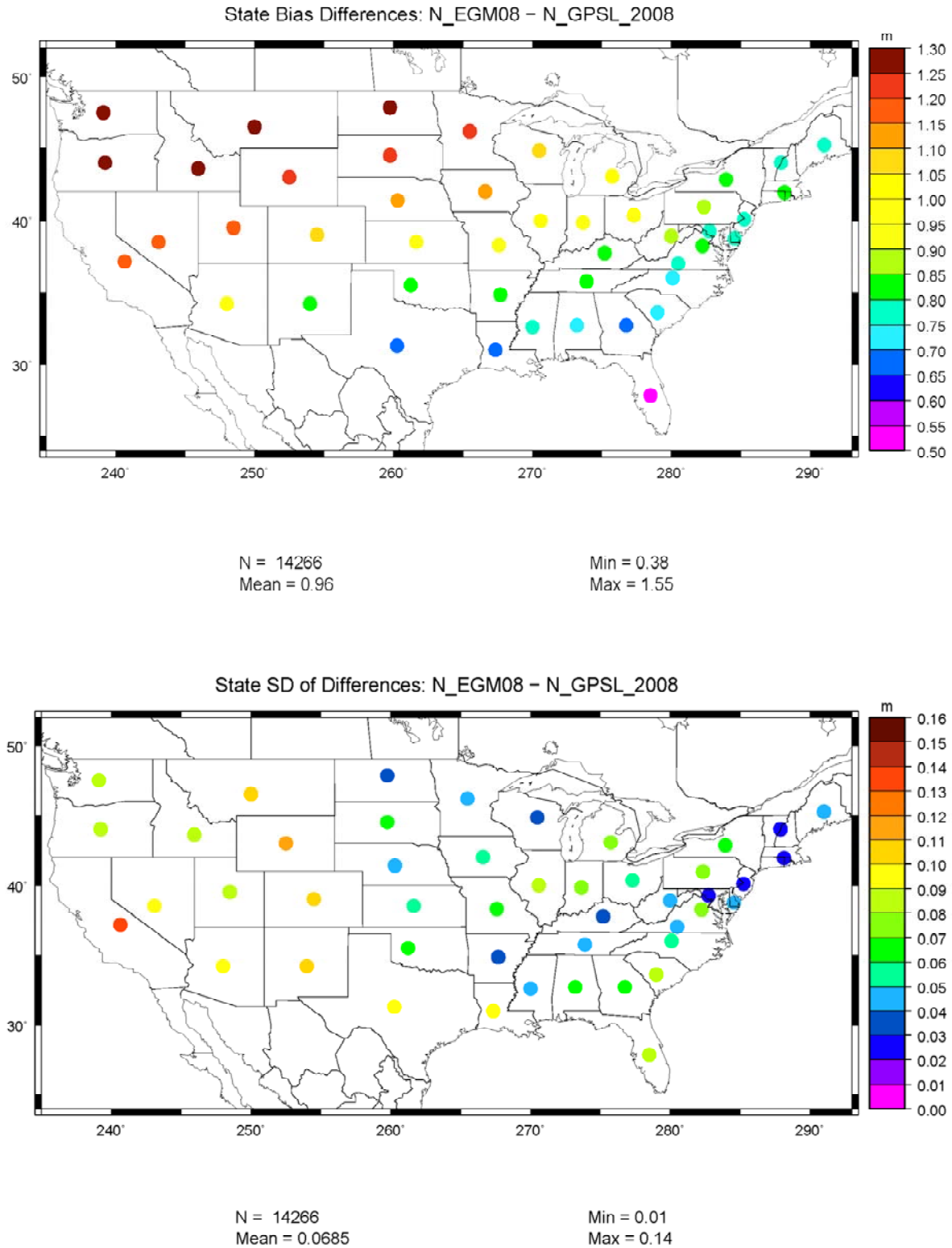


Figure 2. Biases developed from comparisons on a statewide basis between EGM08 and GPSBM's are shown in the top figure, while the SD's are shown in the bottom figure.

Table 1. Comparisons between EGM08 and GPSBM's for U.S. states and territories. Biases largely reflect the tide gage selected as the datum. SD is the significant statsistic and is very good throughout the Pacific.

Region	# Pts	Min (m)	Max (m)	Ave (m)	SD (m)
CON -US	14266	0.229	1.793	0.960	0.069
Alaska	239	0.761	2.551	1.744	0.254
Puerto Rico	29	0.030	0.128	0.078	0.029
Guam	16	-0.773	-0.563	-0.645	0.066
CNMI	54	-0.744	-0.489	-0.570	0.072
Am. Samoa	22	-0.853	-0.514	-0.696	0.110

For most regions, significant biases occurred. This largely reflects the selection of the tide gage used as the datum point. The SD is most relevant and demonstrates that EGM08 generally provides dm-level or better agreement.

Alaska represents an exception for both the bias and the SD for a number of reasons. The tide gage is the same as for CONUS, Father Point/Rimouski in the St. Lawrence Seaway - thousands of kilometers away. Also, errors in NAVD 88 were propagated to the end of the network, which is Alaska. These factors contribute to the significant bias. Frost heave and other factors also contribute high variability in the GPSBM's and create a significant SD. EGM96 and other models based on it have proportional biases and SD's.

3 Aerogravity

Aerogravity profiles have been collected by NGS to validate existing terrestrial data holdings and served a similar purpose for EGM08. Figure 3, 4, and 5 show the differences between EGM08 and aerogravity profiles for over Florida in 2005 and the northern Gulf of Mexico in 2006 and 2008. These data were collected at 35,000 ft (approximately 10 km) at speeds of about 500 kmh. Track spacing was at 10 km and intended to capture a 20 km full wavelength signal – commensurate with along track signal after filtering. Comparisons are made at flight altitude. Note that none of these three data sets were incorporated into EGM08, so they provide independent assessments of the quality of EGM08.

In Figure 3, the profiles over Florida are rougher and track spacing is closer to 20-40 km. A significant bias exists and seems largely due to the features off of Cuba and the Bay of Florida. This may derive from significant issues with ocean topography in the altimetric anomalies used in EGM08. These data were not gridded due to the sparseness and irregular nature of the profiles.

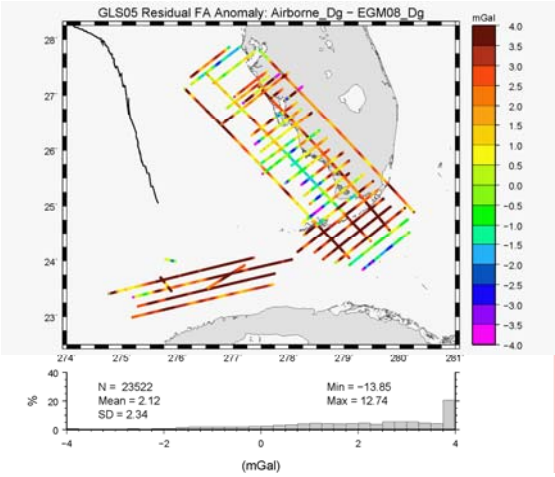


Figure 3. Differences between EGM08 and aerogravity data collected over Florida in 2005 at 35,000 ft.

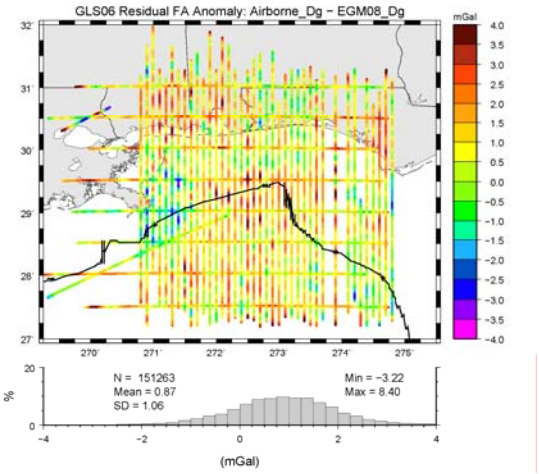


Figure 4. Differences between EGM08 and aerogravity data collected over coastal regions of the northern Gulf of Mexico in 2006 at 35,000 ft.

A more comprehensive and consistent collection occurred in 2006 for the northern Gulf of Mexico. This survey had a very low crossover error (0.87 mgal) and provides an excellent comparison. Figure 4 shows that EGM08 agrees closely with the aerogravity data. (1 mgal SD).

The central portion of this area was re-flown in early 2008 as a means of calibrating a new aerogravimeter purchased by NGS. The earlier surveys were flown by NOAA using the Naval Research Laboratory's instruments. The 2008 survey used the

NGS instrument and yielded a more typical cross-over comparison (2 mgal SD). Track spacing was at 5 km to evaluate omission errors in the collection scheme. The comparison with EGM08 was, likewise, 2 mgal SD and highlights many of the same features seen in the 2006 survey.

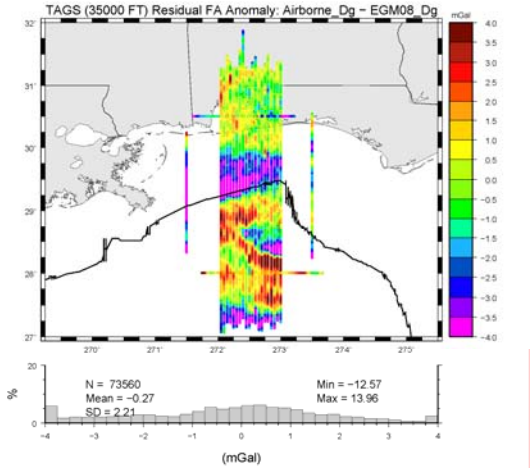


Figure 5. Differences between EGM08 and aerogravity data collected over coastal Alabama in 2008 at 35,000 ft.

In Figures 3, 4, and 5, prominent features exist that seem to point to inconsistencies with features at scales at hundreds of kilometers. These features have relatively low signal (3-4 mgals) but they are seen to span multiple profiles. This supports a supposition that real differences exist and that the features are not just random track noise or derived from filtering. Their spatial extents create dm-level differences in derived geoid models and are the subject of ongoing study at NGS. Similar features are seen in comparison to upward continued terrestrial data held by NGS in the historical database.

A recent terrestrial campaign was completed to test the terrestrial data for the Mobile Bay, Alabama region. The preliminary results of that survey show that the errors lie with the historical, terrestrial data that underlie EGM08.

4 Summary and Conclusions

GPSBM's & aerogravity were compared to EGM08 in an effort evaluate the quality of the model in U.S. states and territories around the world.

EGM2008 performed better than EGM96 in comparison to existing GPSBM data everywhere except in the western Pacific. This improved behavior supports the modifications by NGA to the weighting scheme between GRACE (Tapley et al. 2004) and surface gravity in creating EGM2008. Additionally, the effect of NGS' National Readjustment of 2007 on GPSBM data further improved the comparisons with EGM2008. Differences with recent, internally consistent, high-altitude aerogravity were not conclusive. This possibly due to along-track filtering of the aerogravity but may also be due to potential systematic errors in the historical terrestrial data used to develop the higher degree harmonics in EGM2008.

Previous studies and recent terrestrial surveys support the possibility that these systematic differences exist. Incorporation of these data into the EGM2008 model means that the errors are now embedded into the model. Comparison with the same data will not reveal this weakness, because they reinforce each other in comparisons. The merging techniques used to mitigate the longer wavelength differences cannot be applied at signals to which GRACE is not sensitive.

Instead of further weighting schemes, a more appropriate approach would be to eliminate any remaining systematic errors through refinement and cleansing of the surface gravity data.

If the surface gravity data do not agree with GRACE data as it appears, then this may be resolved by implementing an internally consistent airborne gravity campaign (e.g., GRAV-D) tied to GRACE and GOCE and designed to bridge surface gravity to satellite gravity. This would eventually eliminate or reduce the need for further weighting schemes or modified kernels.

EGM2008 still represents a significant step forward and will remain useful as a unifying reference model. It is consistent with existing data quality and is likely adequate for most applications. Particularly when used in a remove-compute-restore approach.

Using R-C-R and a partially modified kernel would allow adoption of the lower degree harmonics and modification of the higher degree harmonics. It is a mark of how much this model represents an improvement to note that many recent regional models now adopt the "low" degree value to be 360. In deed, recent efforts for the U.S. model have focused on using EGM2008 completely through that level incorporating a modification to the Stokes kernel to affect this. The full signal of EGM2008 (through degree 2160) is removed and the differences between 360 and 2160 and passed through the kernel to restore to the regional model. This approach builds on the EGM2008 model to address those shorter wavelength signals where some discrepancy remains.

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EGM2008 and PGM2007A evaluation for South America

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Abstract. The Earth Gravitational Model (EGM) development team released the Preliminary Gravitational Model (PGM2007A) and the final EGM2008. These models are completed to degree and order 2160 and contain additional spherical harmonic coefficients extending to degree 2190 and order 2160. A total of 1,190 GPS points available on Bench Marks (GPS/BM) in South America and 85,018 mean free air gravity anomalies in a grid of 5' are used to evaluate the following gravity field models: EGM96, EIGEN-GL04S1, EIGEN-GL04C, GGM02S, GGM02C, PGM2007A and EGM2008. The results are presented in terms of statistics and histograms of the discrepancies between GPS geoid heights as well as gravity disturbances and the seven important Global Geopotential Models. The modern models represent a substantial improvement on the gravitational field representation in South America and EGM2008 shows the best result compared to previous models.

Keywords. Geopotential model, geoid modeling, GPS

1 Introduction

There are essentially three classes of Global Geopotential Models (GGMs) according to Rapp (1997), Balmino *et al.* (1997), Featherstone (2002) and Rummel *et al.* (2002):
1-Satellite-only GGMs: derived solely from the analysis of orbits of artificial Earth satellites.
2-Combined GGMs: derived from a combination of satellite, altimetry, land, shiptrack and airborne gravity observation data. The additional information allows an increase of the maximum spherical harmonic degree of the GGMs.
3-Tailored GGMs: an existing satellite-only or combined GGM adjusted with new data, not necessarily used before.

The limitations of satellite information are power-decay of the gravitational field with altitude; the inability to track complete satellite orbits using ground-based stations; imprecise modeling of atmospheric drag, non-gravitational and third-body perturbations; finally incomplete sampling of the global gravity field due to the limited number of satellite orbital inclinations available. Nowadays, with dedicated satellite gravity missions, many old limitations are redressed (Featherstone, 2002). The other limitations are spatial coverage and quality of the additional data used.

The new gravity missions as Challenging Minisatellite Payload (CHAMP) and Gravity Recovery and Climate Experiment (GRACE) are allowing the best knowledge of the long wavelength component of the Earth gravitational field. These missions are the beginning of what is often called the "geopotential international decade" and the scientific community expects a great advance with the Gravity Field and Steady-State Ocean Circulation (GOCE) mission.

This paper mainly focuses on two important combined GGMs: PGM2007A and EGM2008. Geoidal heights derived from GPS/BM and terrestrial gravity data are used to evaluate these models for South America and for Brazil. Other GGMs are validated too: EGM96 (Combined model complete to degree and order 360) (Lemoine *et al.*, 1998a; Lemoine *et al.*, 1998b); EIGEN-GL04S1 (satellite-only GRACE model complete to degree and order 150); EIGEN-GL04C (combined GRACE model complete to degree and order 360) (Förste *et al.*, 2006); GGM02S (satellite-only GRACE model complete to degree and order 160); GGM02C (combined GRACE model complete to degree and order 200).

The height anomaly and gravity disturbances are computed using the very high degree harmonic synthesis program Harmonic_synth_v2 developed by Holmes and Pavlis (2008). It is important to mention that in both PGM2007A and EGM2008 the second-degree zonal harmonic coefficient $\{\overline{C}_{20}\}$ is expressed in the "Zero Tide**"** system, as far as the permanent tide is concerned (Holmes and Pavlis, 2008).

2 GPS data on benchmark

GPS observations carried out on benchmarks of the spirit levelling network in South America, which have been delivered under the SIRGAS (Geocentric Reference System for Americas) project (SIRGAS, 2002), are used for testing the gravimetric determination of the geoid as well as the selected GGMs. At the moment there are GPS/BM data available from the following countries: Brazil, Argentina, Ecuador, Venezuela and Chile (Blitzkow, 1999). A total of 1,190 GPS points are available in South America with 696 points in Brazil (Figure 1).

Table 1 shows the results in terms of mean value, RMS, extreme values of the differences among height anomalies of several GGMs for different degree and order (60, 120, 360 and 2160) and GPS/BM geoidal heights for South America. Table 2 shows the same statistic analysis for Brazil.

In Figures 2 to 6 one can see the histograms of the discrepancies between GGM and GPS/BMs for specific maximum degrees and orders (60, 120, 360). PGM2007A and EGM2008 are more consistent with GPS/BM than the other GGMs, even for low degrees and orders. For degree and order 2160 both models of this order are the best with the final model slightly better than the preliminary. Figure 1 shows the GPS/BM distribution with a colour schedule for differences between EGM2008 height anomalies and GPS/BM geoidal heights. This information is still sparse and not distributed homogeneously, so that this result is geographically limited, but most of the greater differences are in the Andes.

3 Official geoid model for Brazil

The official geoid model in Brazil is MAPGEO2004 (IBGE, 2004; Lobianco *et al.*, 2005). It is computed using EGM96 up to degree and order 180 as the reference field (Figure 7). The reduced Helmert mean gravity anomalies are estimated in blocks of 10' x 10'. For the ocean the KMS-99 satellite altimetry model is used (Andersen and Knudsen, 1998). A DTM is also derived with a resolution of 1' x 1'. It was obtained from digitization of topographic maps, combined with the GLOBE model (Hasting and Dunbar, 1999) where topographic maps were unavailable. The processing of the modified Stokes integral proposed by Featherstone et al. (1998) is carried out using FFT. This modification applies a Meissl (1971) modification to the Vaníček and Kleusberg (1987) kernel.

Table 3 shows statistics of the differences between geoidal heights of MAPGEO2004 and GPS/BM in Brazil (696 points). Looking to the results of PGM2007A, EGM2008 (n=m=360 or 2160) and EIGEN-GL04C (n=m=360) (Table 2), the conclusion is that these models are slightly better than the official geoid model. This probably depends on that the new GGMs have been computed using slightly more gravity information than MAPGEO2004.

Table 1. Statistics of the differences between height anomalies computed by the GGMs and GPS/BM geoidal heights for South America.

n=m	Statistics	EGM2008	PGM2007A	EGM96	EIGEN-GL04C	EIGEN-GL04S1	GGM02C	GGM02S
		(m)	(m)	(m)	(m)	(m)	(m)	(m)
60	**Mean**	-0.15	-0.14	-0.17	-0.20	-0.20	-0.18	-0.18
	RMS dif.	1.75	1.75	1.84	1.77	1.77	1.77	1.77
	Max.	6.2	6.2	5.8	6.3	6.3	6.3	6.3
	Min.	-8.8	-8.8	-8.8	-9.1	-9.1	-9.1	-9.1
120	**Mean**	0.12	0.13	0.10	0.06	0.07	0.09	0.09
	RMS dif.	1.09	1.10	1.16	1.10	1.10	1.08	1.10
	Max.	3.8	3.9	4.3	3.9	3.8	3.8	4.0
	Min.	-4.1	-4.1	-4.4	-4.2	-4.1	-4.2	-4.1
360	**Mean**	0.28	0.30	0.24	0.22			
	RMS dif.	0.72	0.73	0.80	0.70			
	Max.	2.8	2.9	3.7	3.1			
	Min.	-3.3	-3.2	-3.3	-2.9			
2160	**Mean**	0.22	0.24					
	RMS dif.	0.68	0.69					
	Max.	3.4	3.4					
	Min.	-3.3	-3.2					

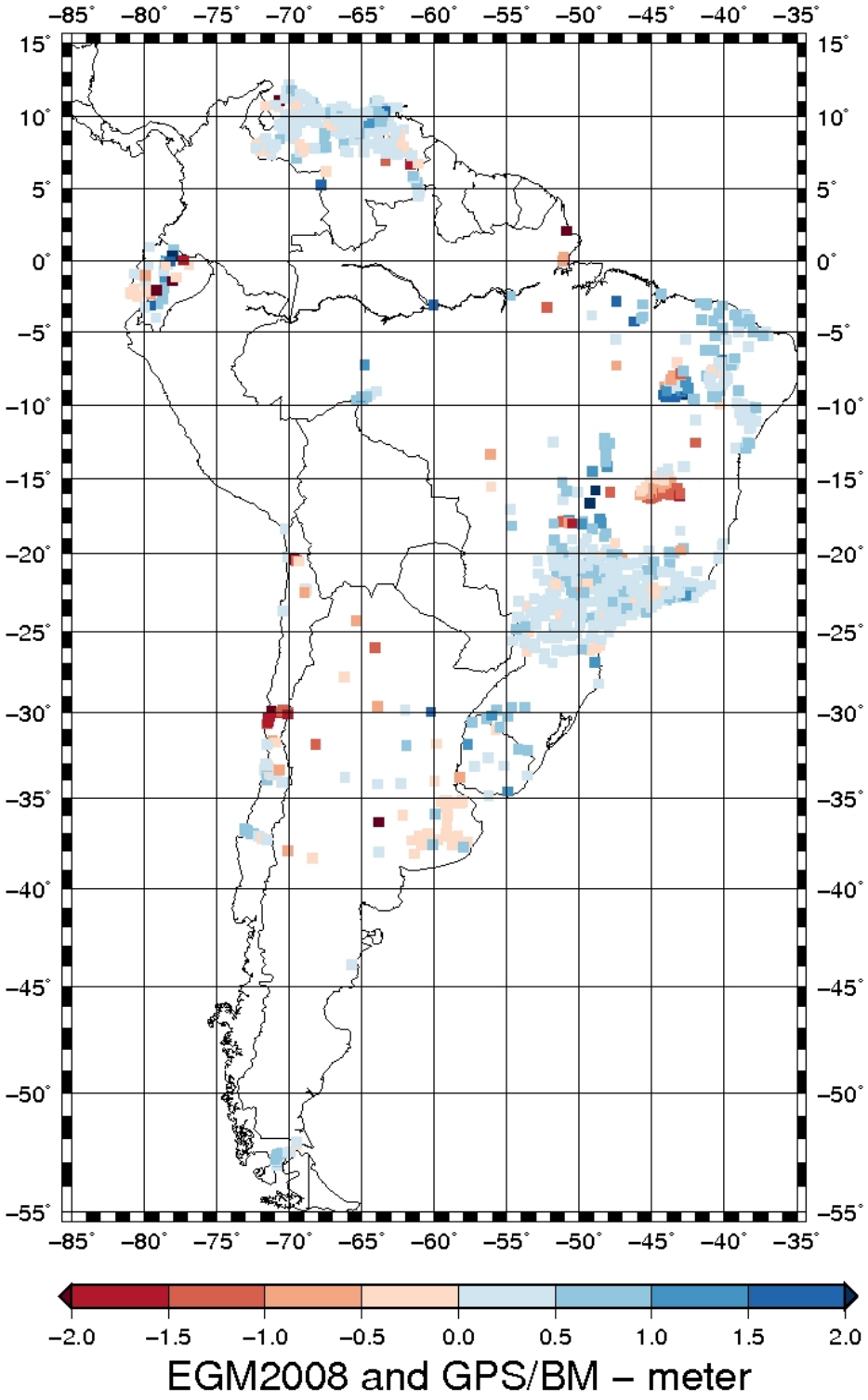


Fig. 1 Distribution of the GPS/BMs and illustration of the differences between EGM2008 height anomalies and GPS/BM geoidal heights.

Table 2. Statistics of the differences between height anomalies of the GGMs and GPS/BM geoidal heights for Brazil

n=m	Dataset	EGM2008 (m)	PGM2007A (m)	EGM96 (m)	EIGEN-GL04C (m)	EIGEN-GL04S1 (m)	GGM02C (m)	GGM02S (m)
60	**Mean**	0.26	0.26	0.34	0.25	0.25	0.27	0.27
	RMS dif.	1.06	1.05	1.15	1.05	1.05	1.05	1.05
	Max.	3.85	3.71	3.95	3.76	3.76	3.78	3.79
	Min.	-4.30	-4.22	-3.71	-4.29	-4.29	-4.26	-4.26
120	**Mean**	0.34	0.34	0.43	0.34	0.34	0.35	0.40
	RMS dif.	0.79	0.81	0.88	0.79	0.80	0.77	0.80
	Max.	2.95	3.00	3.27	2.82	2.97	2.81	3.01
	Min.	-3.38	-3.36	-3.25	-3.42	-3.41	-3.32	-3.32
360	**Mean**	0.31	0.32	0.40	0.32			
	RMS dif.	0.58	0.60	0.75	0.62			
	Max.	2.81	2.90	3.73	3.12			
	Min.	-3.05	-3.03	-3.03	-2.85			
2160	**Mean**	0.29	0.29					
	RMS dif.	0.56	0.58					
	Max.	2.78	2.87					
	Min.	-3.05	-3.03					

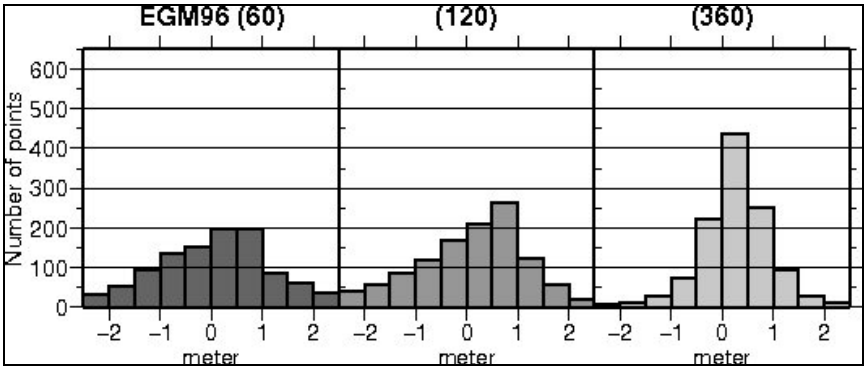


Fig. 2 Histograms of discrepancies between EGM96 height anomalies and GPS/BM geoidal heights.

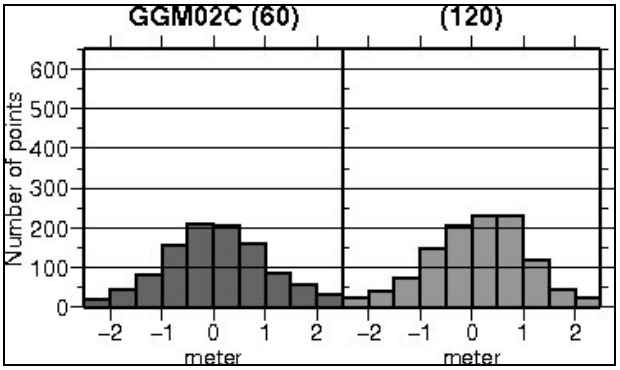


Fig. 3 Histograms of discrepancies between GGM02C height anomalies and GPS/BM geoidal heights.

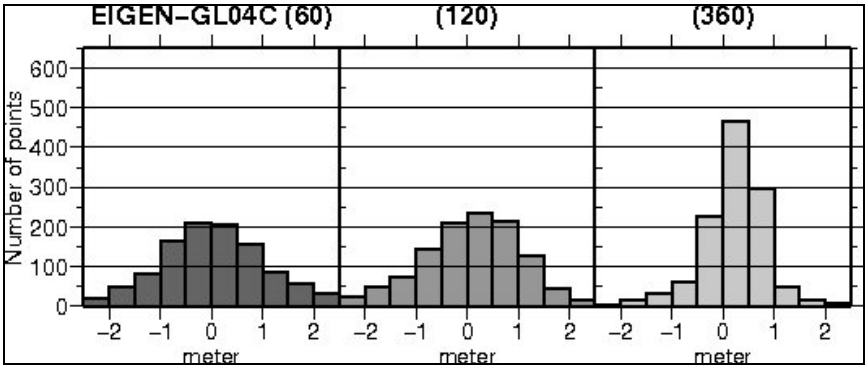


Fig. 4 Histograms of discrepancies between EIGEN-GL04C height anomalies and GPS/BM geoidal heights.

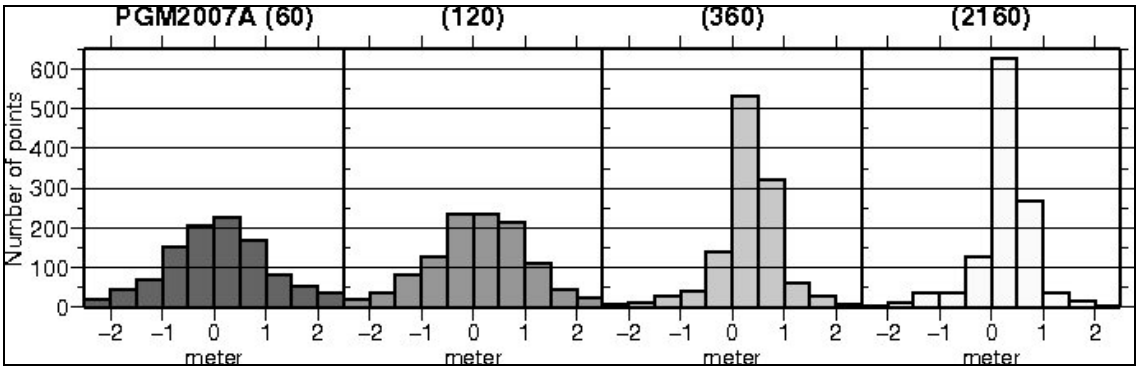


Fig. 5 Histograms of discrepancies between PGM2007A height anomalies and GPS/BM geoidal heights.

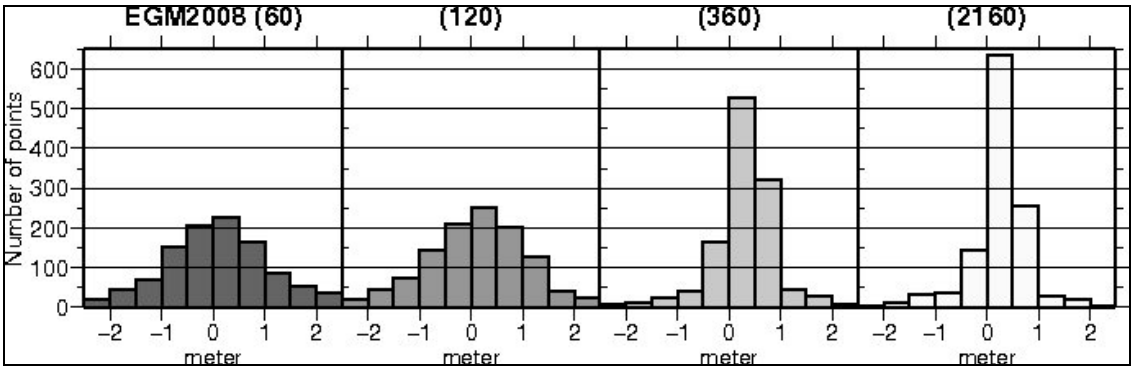


Fig. 6 Histograms of discrepancies between EGM2008 height anomalies and GPS/BM geoidal heights.

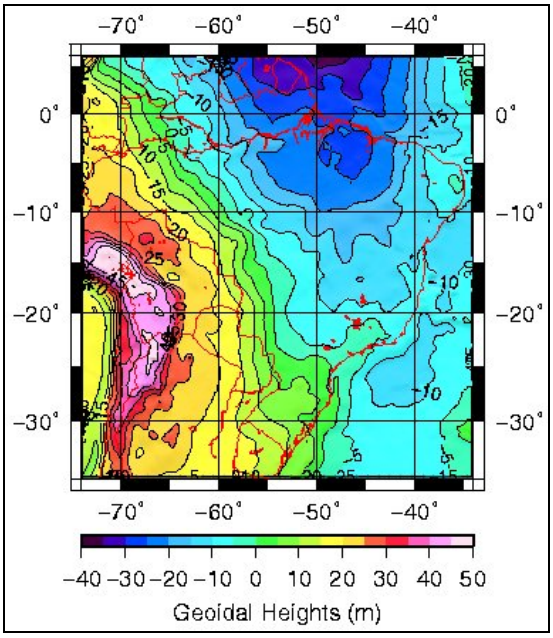


Fig. 7 MAPGEO2004, the official geoid model in Brazil.

Table 3. Statistics of the differences between geoidal heights of MAPGEO2004 and GPS/BM for Brazil (696 points)

	MAPGEO2004 (m)
Mean	-0.57
RMS dif.	0.68
Max.	2.48
Min.	-3.97

4 Terrestrial gravity data

South American Gravity Project (Green and Fairhead, 1991) was the first great effort in collecting and validating gravity data over the continent. This initiative is important to indicate the terrestrial and marine gravity distribution and to identify the major gaps. In 1991, the Anglo-Brazilian Gravity Project (ABGP) started some new efforts to fill in the gaps in Brazil. This project was a cooperation program between LTG/EPUSP (Laboratory of Topography and Geodesy - Polytechnic School, University of São Paulo), Brazilian Institute of Geography and Statistics (IBGE) and Geophysical Exploration Technology (GETECH), supported by U.S. National Geospatial-Intelligence Agency (NGA). After seven years of activities this project was responsible for an outstanding improvement on the gravity point distribution, mainly in the Amazon region, including rivers and airstrips along small villages.

The activities of ABGP were extended to other countries in the continent in 2000 as South America Gravity Studies (SAGS). Presently a total of 849,363 terrestrial gravity points are available in South America.

The gravity anomalies derived from terrestrial gravity data are compared with gravity disturbances derived from GGMs. Mean gravity anomalies in a grid of 5' x 5' (from 25° N to 60° S to 100° W to 25° W) are obtained from the complete Bouguer anomaly using point gravity data, except for Colombia where only mean free air gravity anomalies are available (Rodríguez, 2003). For these computations the SHGEO software is used, developed at the University of New Brunswick, available to EPUSP and IBGE through the Project PIGN (*Projeto de Infraestrutura Geodésica Nacional*). The total grid number is 85,018. The digital terrain model used for different purposes is SAM_3sv2 (Matos and Blitzkow, 2008).

Table 4 shows the results in terms of mean value, RMS and extreme values of the differences between gravity anomalies derived from terrestrial gravity data and gravity disturbances derived from GGMs; the same degrees and orders as before are used. Figures 8 to 12 show the histograms of the discrepancies. One can see that PGM2007A and EGM2008 with n=m=360 are better adjusted to the terrestrial gravity data than the other GGMs.

Figures 13 to 16 show the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM96, EIGEN-GL04C, EGM2008 (n=m=360) and EGM2008 (n=m=2160), respectively. There is a visible improvement between EGM96 and EIGEN-GL04C, although not considerable. To the same degree and order (360) the improvement of EGM2008 is visible in the north and middle of Argentina as well as southeast and south of Brazil. Finally, looking to the full degree and order of EGM2008 (n=m=2160) the improvement is remarkable in the whole South America, mainly around mountainous regions. Nevertheless, the main discrepancies are correlated with high and rough topography, especially over the Andes.

Table 4. Statistics for the discrepancies between terrestrial gravity anomalies and gravity disturbances derived by GGMs (85,018 points).

n=m	Dataset	EGM2008 (mGal)	PGM2007A (mGal)	EGM96 (mGal)	EIGEN-GL04C (mGal)	EIGEN-GL04S1 (mGal)	GGM02C (mGal)	GGM02S (mGal)
60	**Mean**	0.84	0.83	-0.97	-0.78	-0.78	-0.79	-0.79
	RMS dif.	47.54	47.52	48.21	48.16	48.16	48.17	48.17
	Max.	692.48	692.29	696.83	695.62	695.62	695.61	695.59
	Min.	-209.25	-209.51	-215.53	-222.34	-222.34	-222.36	-222.33
120	**Mean**	-0.94	-1.05	-3.04	-2.60	-2.61	-2.62	-2.60
	RMS dif.	42.85	42.84	43.77	43.45	43.42	43.45	43.53
	Max.	581.12	580.22	599.64	585.50	582.38	585.60	582.05
	Min.	-232.66	-232.95	-244.81	-235.10	-233.65	-235.95	-234.54
360	**Mean**	-2.28	-2.42	-4.73	-4.52			
	RMS dif.	28.94	28.97	31.97	31.89			
	Max.	401.83	389.19	377.61	386.99			
	Min.	-338.21	-321.75	-292.86	-303.91			
2160	**Mean**	-0.19	-0.33					
	RMS dif.	20.43	20.41					
	Max.	372.12	356.75					
	Min.	-492.40	-437.62					

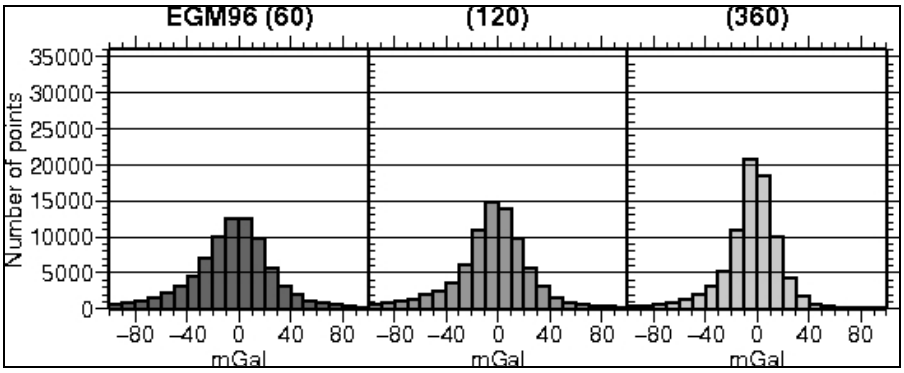


Fig. 8 Histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM96.

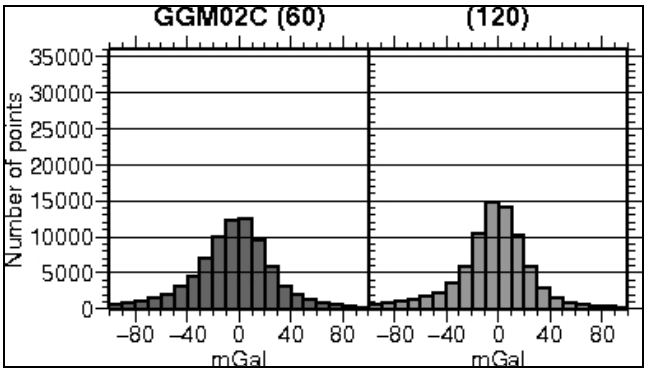


Fig. 9 Histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from GGM02C.

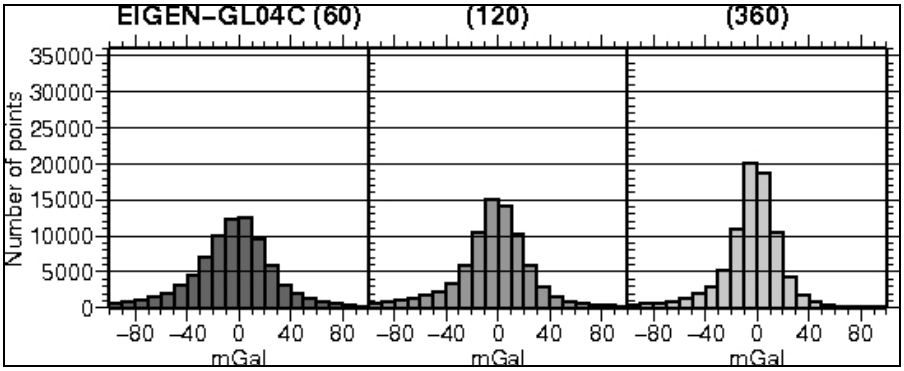


Fig. 10 Histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGEN-GL04C.

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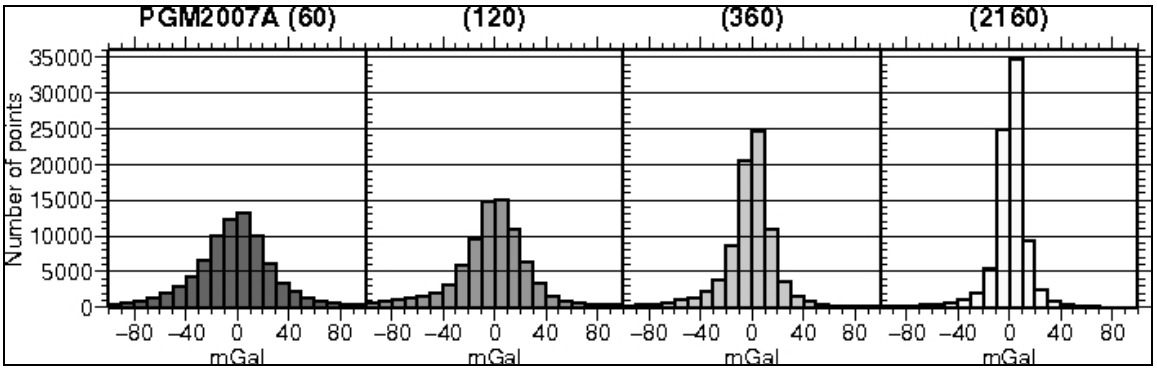


Fig. 11 Histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from PGM2007A.

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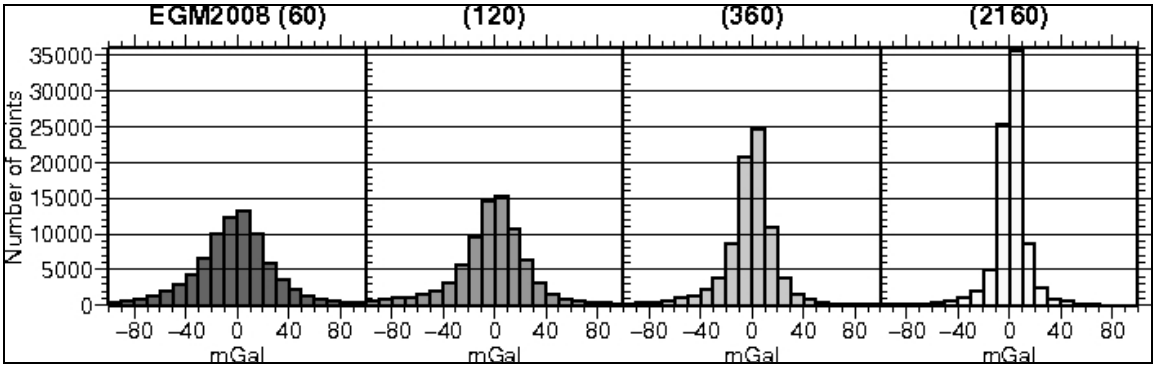


Fig. 12 Histograms of the discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM2008.

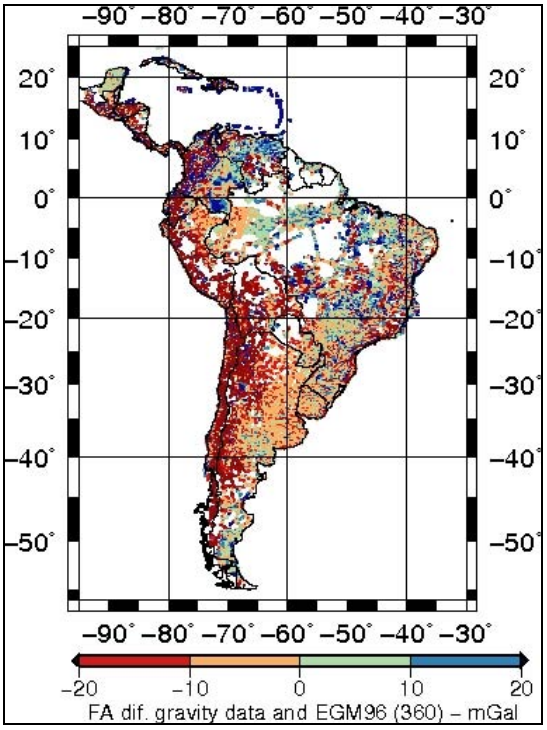


Fig. 13 Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM96.

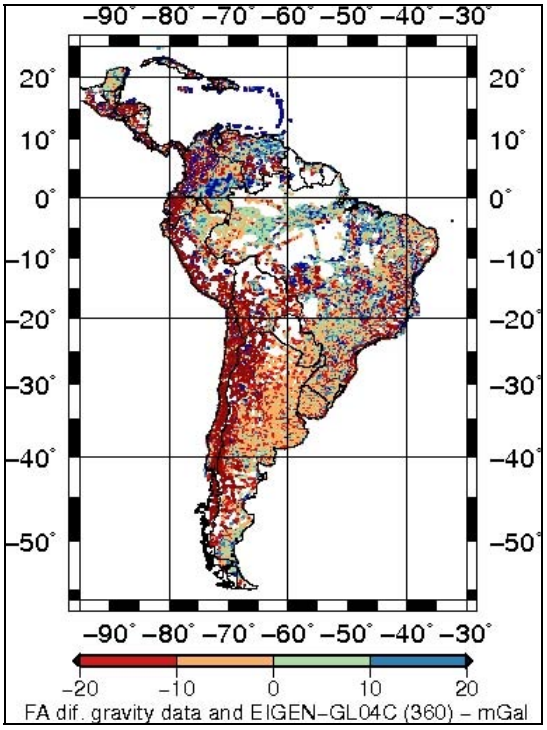


Fig. 14 Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EIGEN-GL04C.

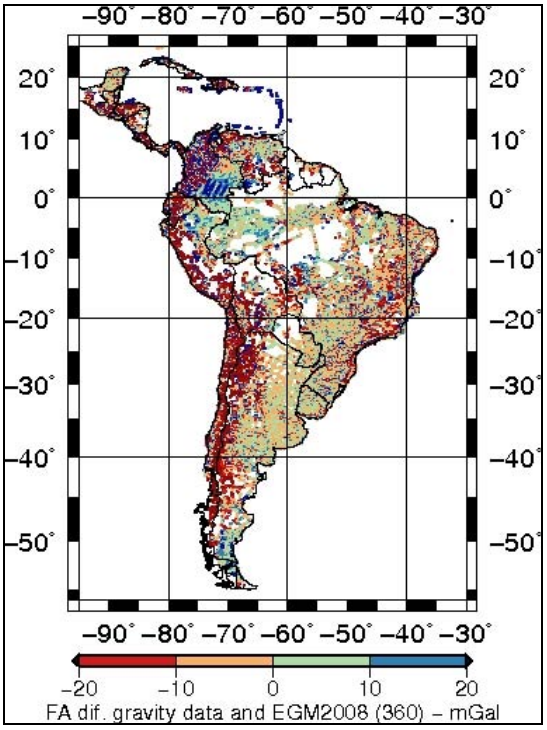


Fig. 15 Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM2008.

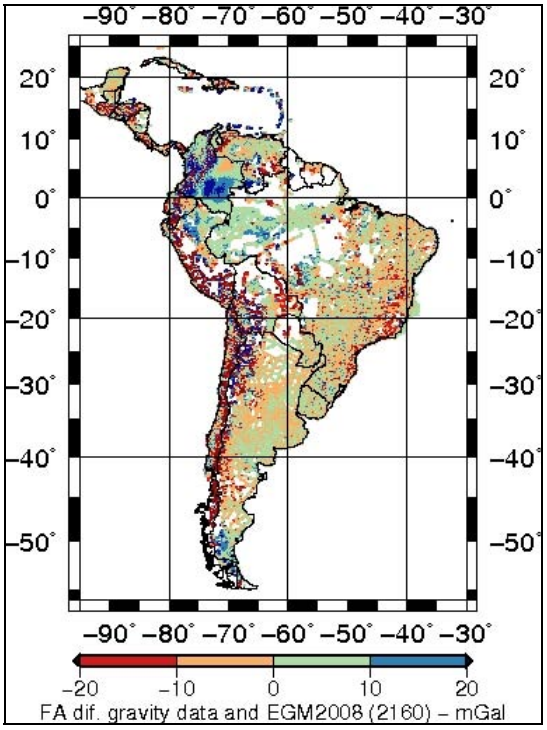


Fig. 16 Discrepancies between terrestrial gravity anomalies and gravity disturbances derived from EGM2008.

5 Conclusions

The validation of the geopotential models PGM2007A and EGM2008 are carried out over South America in terms of:
1 - GPS on Bench Marks;
2 – The geoid model MAPGEO2004;
3 – Terrestrial gravity anomalies.

The global gravity models EGM96, EIGEN-GL04S1, EIGEN-GL04C, GGM02S, GGM02C are also evaluated for different degrees and orders.

The statistics of the differences between the tested geopotential models and GPS/BM show that the best agreement is obtained with EGM2008 (n=m=2160) in South America. In Brazil, this geopotential model shows results slightly better than MAPGEO2004, the official geoid model for Brazil.

The gravity disturbances derived from EGM2008 show the best agreement when compared with terrestrial gravity anomalies. Most of the still existing inconsistencies of this GGM is in mountainous regions, mainly in the Andes.

The general conclusion is that the recent geopotential models, in particular EGM08, represent an important improvement on the knowledge of the gravitational potential in South America.

6 Acknowledgements

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VALIDATION OF EGM2008 OVER ARGENTINA

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Abstract. The EGM96 geopotential model (Lemoine et al., 1998) was for long time the most accurate reference model for many applications in Earth's sciences.
Taking advantage of recent terrestrial gravity, elevation and altimetry data as well as the results from satellite gravity missions, a new global gravity model, EGM2008, has been calculated by the National Geospatial-Intelligence Agency (NGA) of the USA.
The official Earth Gravitational Model EGM2008 has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This model is complete to degree and order 2159, and contains additional spherical harmonic coefficients extending to degree 2190 and order 2159.
This contribution is a report of the validation results of the preliminary pre-released combined geopotential model PGM07A and the final model EGM2008 (Pavlis et. al., 2008) over Argentina. Comparisons with other well-known Earth Gravity Models like EGM96 (Lemoine et. al., 1998) and EIGEN-GL04C (Förste et al., 2006) are also presented.

Keywords. EGM2008, Global Gravity Models (GGMs), Argentina

1. Introduction

One method for assessing the accuracy of the global geopotential models is through comparisons with gravity anomalies and geoid undulations as derived by e.g. GPS and spirit leveling. The main objective of this paper is to validate EGM2008 over all Argentina.
Also the Argentinean regional gravimetric geoid model: ARG06_egm96 has been used for checking the EGM2008 accuracy over Argentina where strong gravity field variations are present.

2. Data description

The validation has been carried out using four data sets: Point land gravity, point shipborne gravity data, GPS/levelling data over Argentina and the Argentinean regional gravimetric geoid model: ARG06_egm96.

2.1. Land Gravity Data

The Argentina Gravity Data Base contains more than 180,000 data points (Figure 1).
Nevertheless, since they come from different sources and were acquired using different procedures, for consistency reasons only the data from IGM (*Instituto Geográfico Militar*) is taken for this evaluation.
In 2002, IGM finished the measurements of the national levelling network, which coincids with the first order national gravity network. It consists of 370 levelling lines containing 16,320 benchmarks, and 225 network nodes, see Figure 2.
The distances between adjacent benchmarks range from 3 km to 9 km. Almost every benchmark in the network has geocentric coordinates. Their accuracy varies from a few centimetres for the most recent ones determined by GPS, up to a few thousand meters in case the coordinates were digitized of those coming from topographic maps, a usual procedure in older days (Pacino et al., 2005).
Most of the gravity values in the network originally referred to the old Potsdam datum, but today they have been converted to IGSN71 through the application of a shift of –14.93 mGal to the measured values. This conversion formula has been tested on more than 800 points that have been measurements

in both systems which results in a mean difference of 0.2 mGal. Apart from the methodology and instrumentation, the overall accuracy for the gravity measurements is better than 0.5 mGal.
Free Air gravity anomalies were calculated in the classical sense as "gravity on the geoid minus normal gravity on the ellipsoid".

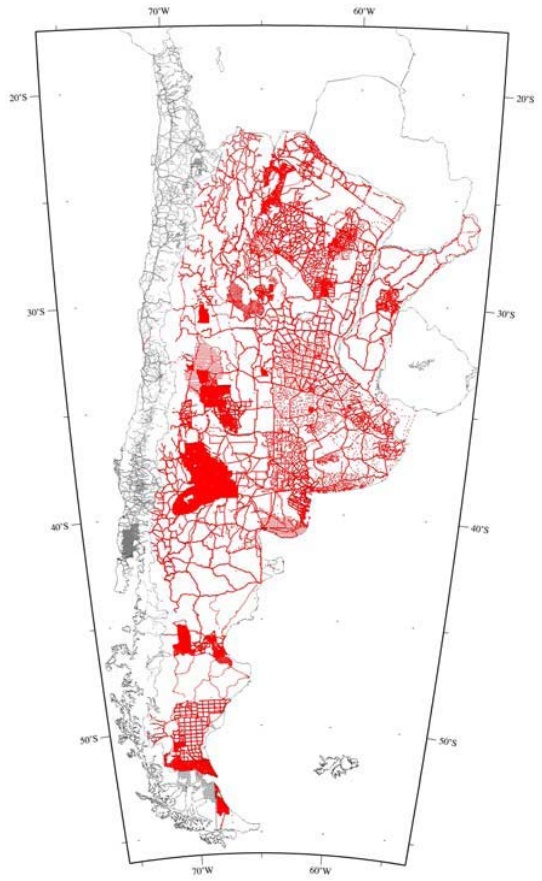


Figure 1: Argentina land gravity data

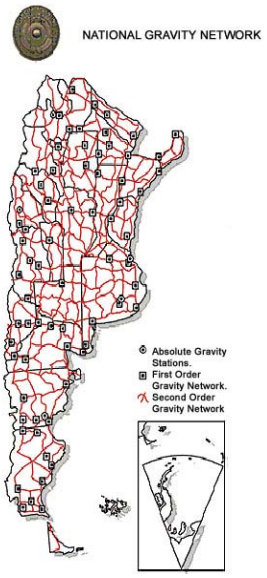


Figure 2: National Gravity Network

The WGS84 gravity formula is used to define normal gravity for each station, according to:

$$\gamma_{84} = 978032.67714 \frac{\left(1 + 0.00193185138639 \sin^2 \varphi\right)}{\sqrt{\left(1 - 0.00669437999013 \sin^2 \varphi\right)}} \text{mGal} \qquad (1)$$

where φ is the latitude

The normal gravity field is generated by a rotating ellipsoid of revolution that includes the atmospheric masses.
The empirically derived atmospheric correction δg_A is:

$$\delta g_A = 0.87 e^{-0.116 h^{1.047}} mGal \qquad (2)$$

where the height h is in km.

The detailed variation of the free air gradient is not known well enough and hence an equation derived from the normal gravity field is used, which depends on height and latitude:

$$FAC = (0.3083293\,357 + 0.00043977\,32\,\sin^2\varphi)\,h + 7.2125\,x\,10^{-8}\,h^2 \qquad (3)$$

where h is the height in meters.

2.2. Shipborne gravity data

The marine gravity data available are 12823 free-air gravity anomalies provided by the Bureau Gravimétrique International (BGI). In Figure 3, the distribution of the shipborne gravity data is depicted for visualization purposes.

2.3. GPS/Levelling Data

Many institutions in Argentina have developed geodetic networks, mainly for cadastral purposes. Some of these points are coincident with levelling benchmarks (Figure 4). These data were acquired in different times using different equipment, techniques and procedures, and are related to different reference frames. The heights of the benchmarks are simple levelling heights without gravity related corrections. Thus, there are neither orthometric nor normal heights.

In order to make a coherent validation, all data was converted into POSGAR94 (**POS**iciones **G**eodesicas **AR**gentinas), the official national reference frame, that realizes the WGS84 system. It was also necessary to clean the data, identifying outliers and blunders.

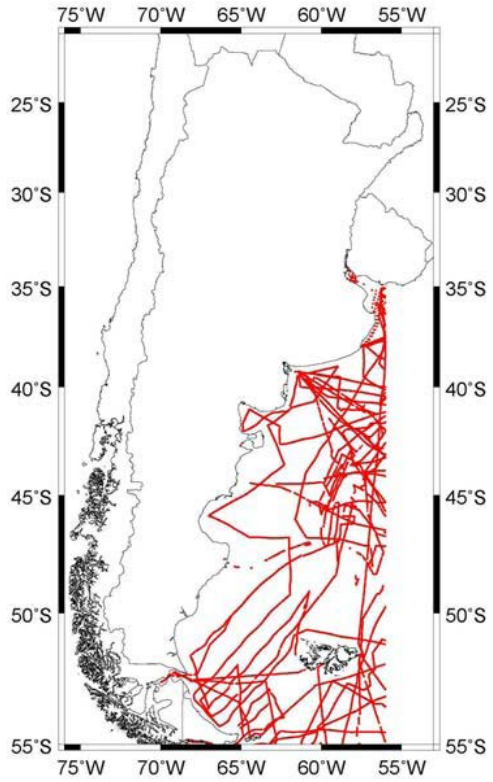


Figure 3: Distribution of shipborne gravity data

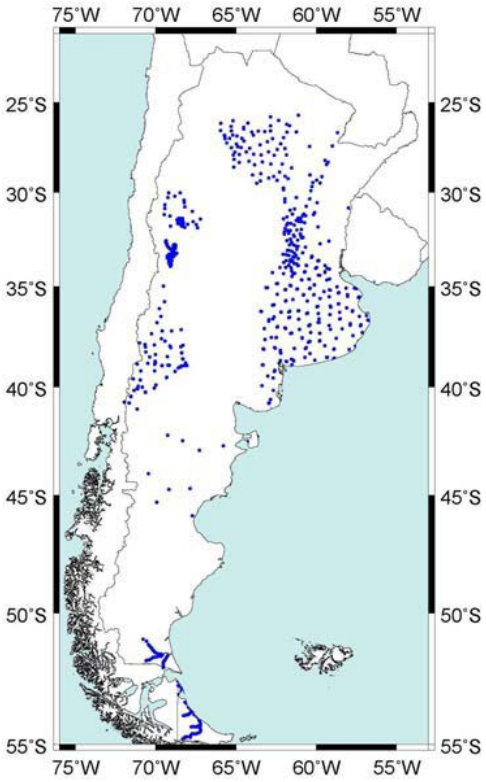


Figure 4: Location of GPS/Levelling points in Argentina

2.4. Argentinean gravimetric geoid model: ARG06_egm96

The gravimetric geoid for Argentina ARG06_egm96 (5´ by 5¨grid) (Tocho et. al., 2005) is based on terrestrial and satellite altimetry - derived gravity anomalies from the KMS02 model (Andersen et al., 2005), which are used to fill in the sparse shipborne data in the Atlantic and Pacific Oceans, offshore Argentina.

The gravimetric geoid was computed using the remove-compute-restore technique, employing Stokes's formula for the prediction of residual geoid heights. Before the prediction of the residual geoid, the free-air gravity anomalies are reduced by the geopotential model EGM96 (Lemoine et. al., 1998) up to degree 360. Furthermore, the effect of the topography is taken into account by Helmert's second method of condensation. The contribution of the local data to the geoid is computed using FFT.

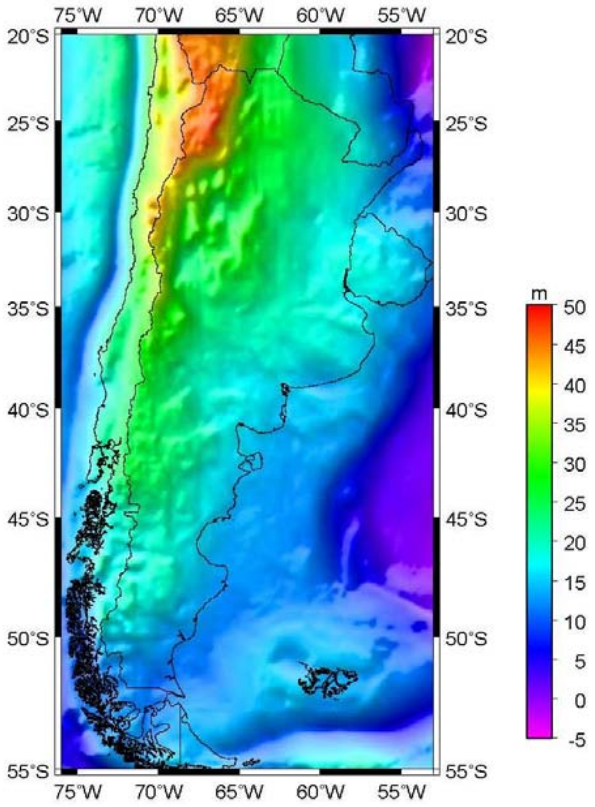


Figure 5: Gravimetric geoid solution ARG06_egm96

3. Results

All the computations are done using the program HARMONIC_SYNTH provided by the EGM2008 development team with the coefficients of EGM2008, EGM96 and EIGEN-GL04C up to different maximum degrees. The computed quantities are spherically approximated gravity anomalies and height anomalies at zero height. No orthometric heights at the computation points are available.

Besides, geoid undulations with respect to WGS 84 are computed using the EGM2008 Tide Free Spherical Harmonic Coefficients and its associated height anomaly to geoid undulation correction model plus a zero-degree term for the height anomaly equal to 41 cm. Geoid undulations are also computed in the same way using EGM96.

3.1. Comparisons with Argentinean GPS/levelling data

Table 1 shows the results in terms of Mean Value, Standard Deviation (σ), Maximum and Minimum of the absolute differences between the 715 GPS/Levelling height anomalies and the Argentinean Geoid Model ARG06_egm96 as well as EGM96, EIGEN-GL04C and EGM2008 up to degree 60, 120, 360, and 2159.

The statistics of the differences between the GPS/levelling derived - geoid and EGM96 and EGM2008 before and after 4-parameter transformation are given in Table 2. The values in parentheses are the results after 4 parameter transformation.

Figure 6 shows the histograms for some comparisons. From Figure 7, we can see that the differences are higher in areas close to mountains.

Table 1: Statistics of the absolute differences between GPS/Levelling and height anomalies computed from Global Gravity Models and the Argentinean geoid model ARG06_egm96. Unit: [m]

n/m	*Model*		*Σ*	*max*	Min
60/60	EGM96	-1.60	±1.87	2.46	-7.65
	EIGEN-GL04C	-1.66	±1.97	2.64	-8.02
	PGM07A	-1.66	±1.97	2.64	-8.03
	EGM2008	-1.66	±1.97	2.64	-8.02
120/120	EGM96	-0.29	±1.10	4.54	-3.28
	EIGEN-GL04C	-0.60	±1.09	3.46	-4.30
	PGM07A	-0.51	±1.02	3.51	-4.11
	EGM2008	-0.59	±1.09	3.47	-4.16
360/360	EGM96	0.13	±0.78	2.29	-1.88
	EIGEN-GL04C	-0.26	±0.38	1.58	-1.94
	PGM07A	0.04	±0.61	2.17	-2.01
	EGM2008	-0.08	±0.44	1.74	-2.10
2159/2159	EGM2008	0.05	±0.48	1.85	-1.41
2159/2190	PGM07A	0.17	±0.67	2.05	-1.58
	EGM2008	0.05	±0.48	1.85	-1.41
	ARG06_egm96	-1.38	±0.53	0.32	-4.41

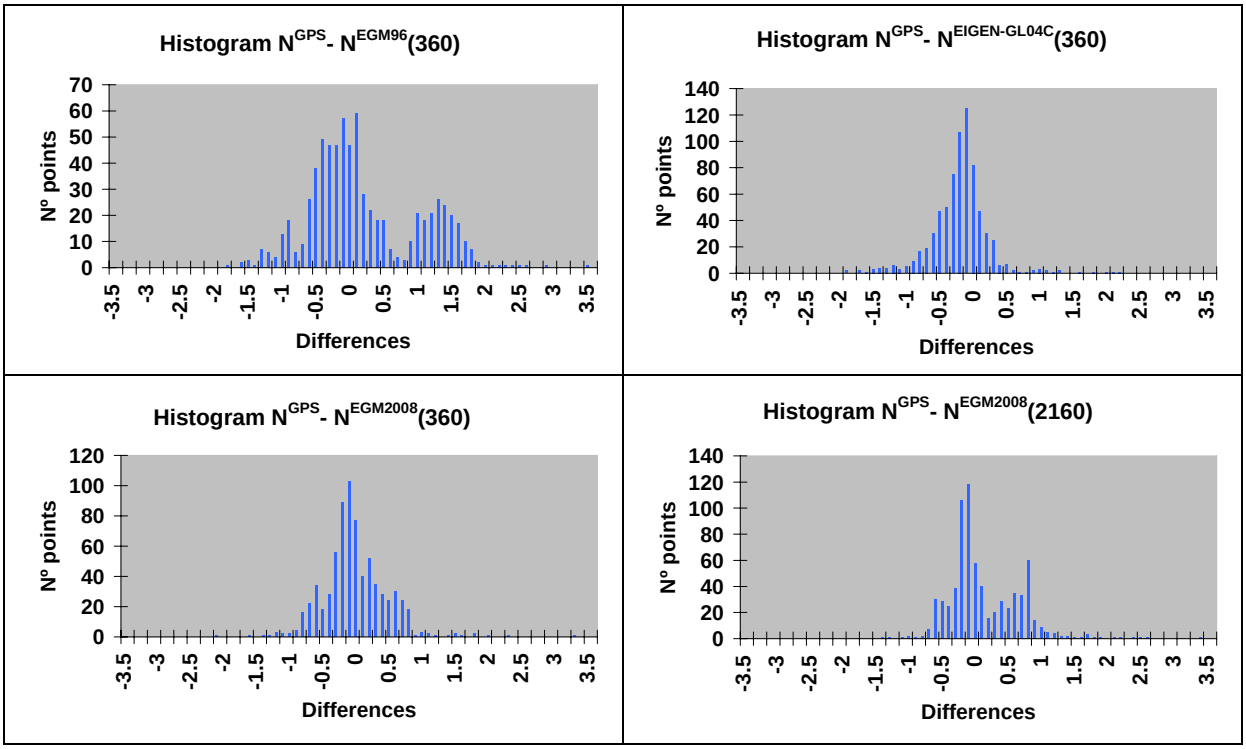


Figure 6: Histograms of the differences between Geoid Undulations from GPS/Levelling Data and from Argentine Geoid Model and Global Gravity Models

Table 2 shows that the global gravity field EGM2008 the long-wavelength structure of the gravity field in Argentina better than EGM96. After transformation EGM2008 fits the GPS/levelling derived geoid with a standard deviation (σ) of near 37 cm while EGM96 fits with a standard deviation of 60 cm. Before the fit, EGM2008 alone reduces the standard deviation of the differences with 40% compared to EGM96 alone.

Table 2: Statistics of the absolute differences between GPS/Levelling and the geoid computed using EGM2008 and EGM96 before and after a 4-parameter transformation. The values in parenthesis are after transformation. Unit: [m]

n/m	*Model*	*mean*	*Σ*	*max*	Min
	N^{GPS}-EGM2008	0.503 (0.000)	±0.510 (±0.374)	2.484 (1.420)	-0.979 (-1.800)
	N^{GPS}-EGM96	0.726 (0.000)	±0.811 ±(0.604)	3.103 (2.21)	-1.313 (-2.19)

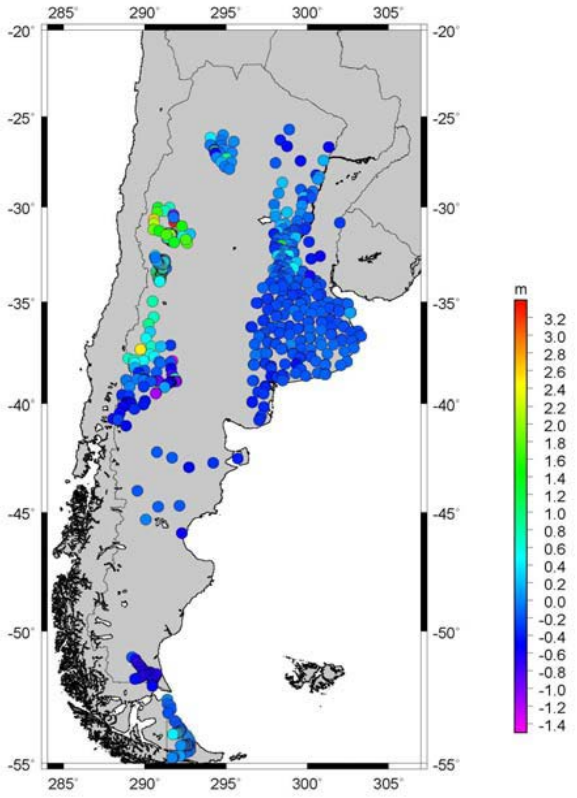


Figure 7: Differences between height anomalies from 715 GPS/levelling and EGM2008

3.2. Comparisons with Argentinean gravity data

Table 3 shows the results in terms of Mean Value, Standard Deviation, Maximum and Minimum of the differences from the comparison of gravity anomalies from shipborne data and gravity anomalies computed using the Global Gravity Models EGM96, PGM2007A and EGM2008 up to different maximum degrees. In Tables 3 and 4, n is the maximum degree and m is the maximum order.

Table 3: Statistics of the differences between shipborne free-air gravity anomalies and EGM96, PGM07A and EGM2008 spherical gravity anomalies. Unit:[mGal]

n/m	*Model*	*mean*	*Σ*	*max*	Min
360 /360	EGM96	0.61	±12.55	62.03	-81.12
	PGM07A	0.93	±12.89	64.71	-67.98
	EGM2008	0.91	±12.82	65.45	-69.07
2159 /2159	EGM2008	1.40	±10.42	68.68	-57.05
	PGM07A	1.43	*±9.68*	66.84	-49.23
2159/2190	EGM2008	1.40	*±9.59*	64.72	-50.25

Table 4 shows the statistics from the differences from the comparison between gravity anomalies from land data and gravity anomalies computed using the Global Gravity Models: EGM96, EIGEN-GL04C, PGM2007A and EGM2008 up to degree 60, 120, 360 and 2159. Figure 8 displays the histograms for some of these comparisons.

Table 4: Statistics of the comparisons between gravity anomalies from land data and gravity anomalies computed using Global Gravity Models with different maximum degrees and orders. Unit: [mGal]

n/m	*Model*	*mean*	*Σ*	*max*	*Min*
60/60	EGM96	-6.68	±35.67	411.69	-298.35
	EIGEN-GL04C	-7.15	±35.75	414.51	-297.76
	PGM07A	5.92	±34.43	498.16	-267.33
	EGM2008	-7.15	±35.75	414.51	-297.76
120/120	EGM96	-5.64	±33.11	389.14	-278.69
	EIGEN-GL04C	-6.33	±33.26	397.02	-276.94
	PGM07A	-6.39	±33.21	396.20	-277.86
	EGM2008	-6.32	±33.22	396.56	-277.85
360/360	EGM96	-5.08	±28.08	390.13	-278.55
	EIGEN-GL04C	-6.02	±28.67	399.64	-271.12
	PGM07A	-5.12	±26.89	388.23	-280.87
	EGM2008	-5.11	±27.09	400.90	-281.15
2159/2159	EGM2008	0.10	±21.59	424.45	-285.09
2159/2190	PGM07A	0.06	±21.21	395.87	-259.23
	EGM2008	0.07	±21.53	426.85	-285.58

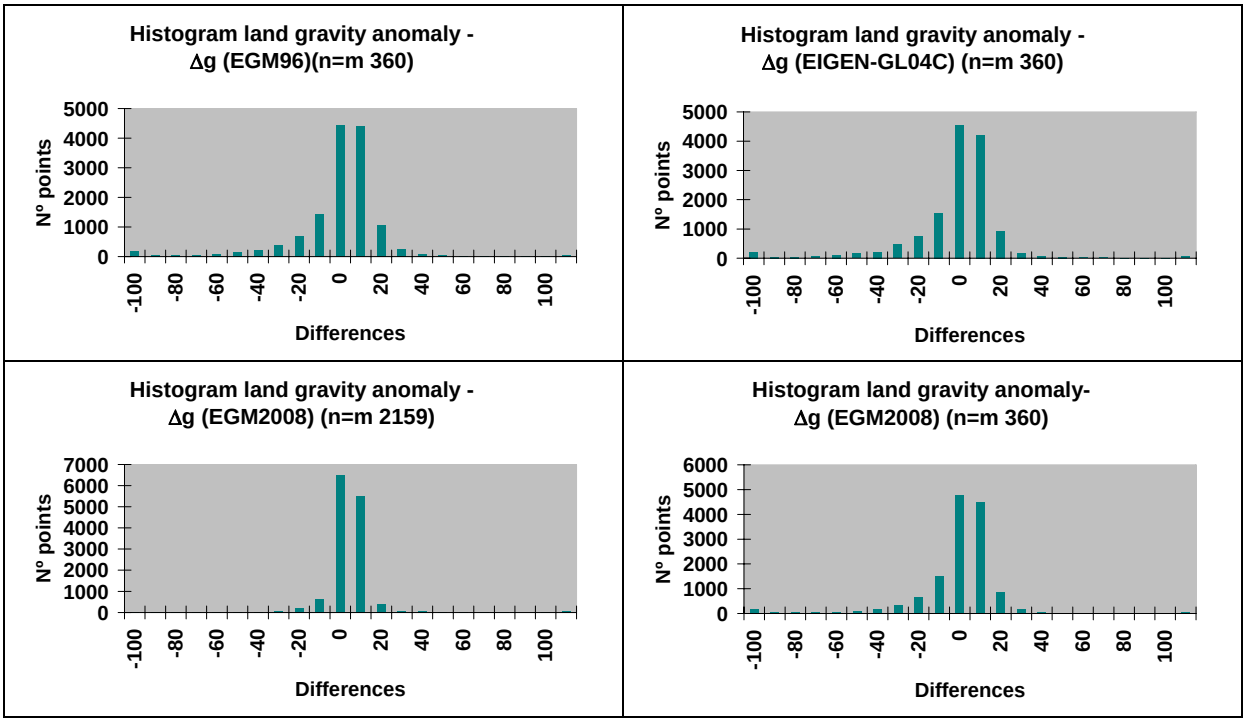


Figure 8: Histograms of the differences between Gravity anomalies from land data and gravity anomalies computed from Global gravity Models

3.3 Comparisons with ARG06_egm96

Height anomalies are first computed using EGM2008 up to degree 2160 on a 5' x 5' grid, which is then compared directly with the gravimetric-only ARG06_egm96 solution (Tocho et. al., 2007). The result

can be seen in Figure 9 and Table 5. This Table also shows the statistics of the differences between height anomalies computed from EGM2008 and EGM96 (used in ARG06_egm96) and the differences between EGM96 and GGM02C.

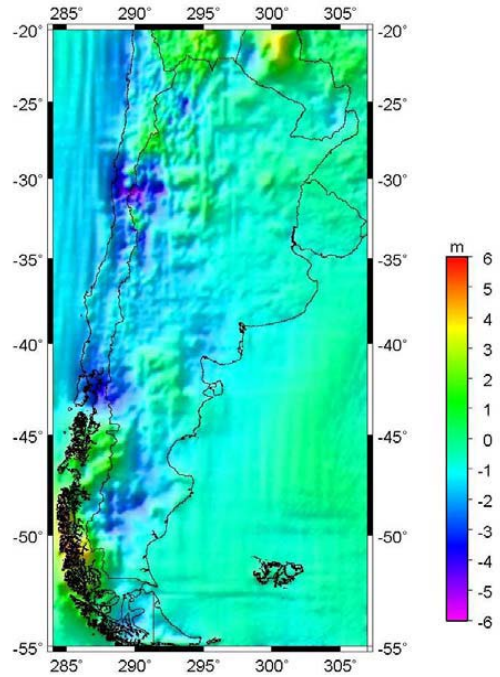


Figure 9: Differences between height anomalies computed from EGM2008 and ARG06_egm96

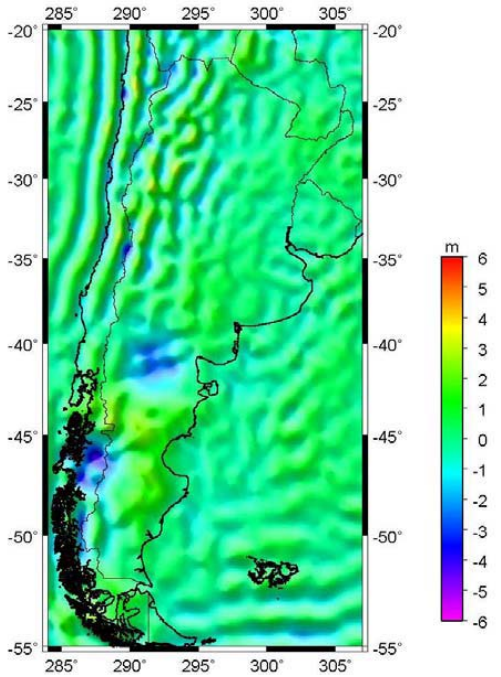


Figure 10: Differences between EGM96 and GGM02C height anomalies over the ARG06_egm96 area

From Figure 9, we conclude that the differences are very high, mainly over the Andes and in a special mainland area located between latitudes 40º S to 42º S and longitudes 70º W to 65 ºW. The gravimetric geoid ARG06_egm96 was computed with EGM96. Then a comparison between the differences between EGM96 and GGM02C (Tapley et al., 2005) height anomalies was performed as we can see in Figure 10. This result shows that the main differences are in the same mentioned areas,we can conclude that probably they are due to the GRACE data-

Table 5: Statistics of the differences between height anomalies computed from EGM2008 and ARG06_egm96 on grid, EGM2008 and EGM96 and EGM96 with GM02C.Unit:[m]

	mean	***Σ***	***max***	Min
EGM20008 minus ARG06_egm96	-0.78	±0.93	5.407 (75.08ºW, 50.33ºS)	-5.642 (70ºW, 30.92ºS)
EGM20008 minus EGM96	0.01	±0.67	5.259 (70.42ºW, 30.42ºS)	-8.044 (72.33ºW, 46.17ºS)
EGM96-GGM02C	0.00	±0.72	3.033 (69.67ºW, 23.83 ºS)	-4.454 (72.42ºW. 46.25 ºS)

4. Summary

A validation of the tide-free version of the geopotential model EGM2008 is carried out over Argentina in terms of:

- Comparison with surface gravity anomalies.
- Comparison with shipborne gravity anomalies.

Comparison with the regional gravimetric geoid model ARG06_egm96

- Comparison with GPS/Levelling points.

In addition, the global gravity models EIGEN-GL04C and EGM96 are also evaluated.
All comparisons are performed up to different degrees and orders.
From the statistics of the differences between the tested geopotential models and GPS/Levelling height anomalies on benchmarks, the best agreement is obtained with EGM2008 (n=m=360) in terms of the mean value and with the EIGEN-GL04C in terms of the standard deviation of the differences.
Geoid undulation values with respect to WGS 84 are calculated using EGM2008 and EGM96. These values are compared with the GPS/levelling derived geoid heights. The global gravity field EGM2008 describes the long-wavelength structure of the gravity field in Argentina better than EGM96. After a 4-parameter transformation, EGM2008 and EGM96 fit the GPS/levelling derived geoid with a standard deviation (σ) of near 37 cm and 81 cm, respectively.
From the comparisons performed using land free-air gravity anomalies, the best results are obtained from the anomalies predicted using EGM2008 up to its maximum degree and order. The main discrepancies are correlated with the high and rough topography, especially over the Andes. Gravity comparisons between free-air shipborne gravity anomalies and gravity anomalies predicted using EGM96 and PGM07A show that the best approximation is obtained with EGM96 (n=m=360) in terms of mean values and with PGM07A in terms of standard deviations.
As future work, new calculations have to be done using the option when the orthometric height is not supplied, and instead it has to be computed for each point by harmonic synthesis of a spherical harmonic model of the elevation, which represents the topographic elevations above mean sea level.

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Evaluation of EGM2008 and PGM2007A over Sweden

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Abstract

An important part of the work performed by the IAG/IGFS Joint Working Group "Evaluation of Global Earth Gravity Models" is to test the new Earth Gravitational Model EGM2008 and its preliminary versions. The purpose of this paper is to present the evaluation of the preliminary PGM2007A and final model EGM2008 over Sweden. The evaluation is done by comparing the model to 195 high quality GPS/leveling observations, to the best regional quasigeoid model presently available and finally to observed gravity anomalies. The regional (gravimetric) quasigeoid model in question has previously been computed in cooperation between Lantmäteriet and the Royal Institute of Technology in Stockholm (KTH) using least squares (stochastic) kernel modification with additive corrections.

The most important result is that EGM2008 agree with the Swedish GPS/leveling data with a RMS of 2.7 cm after a 1-parameter transformation. This is comparable to the corresponding RMS value of 2.2 cm obtained for the regional quasigeoid model. Thus, EGM2008 agrees well with the GPS/levelling data and the regional quasigeoid model in Sweden.

1. Introduction

The purpose of this paper is to present the Swedish evaluation of EGM2008 (Pavlis et al. 2008) and the preliminary model PGM2007A against GPS/leveling observations, the best regional quasigeoid model available over Sweden in 2008 and the gravity anomalies utilised to compute the regional model. This work is done under the umbrella of the IAG/IGFS Joint Working Group "Evaluation of Global Earth Gravity Models"; see the Working Group home page at "http://users.auth.gr/~kotsaki/IAG_JWG/ IAG_JWG.html"

The regional quasigeoid model to which the Earth Gravity Models (EGMs) are compared has been computed in cooperation between Lantmäteriet (Swedish mapping, cadastre and registry authority) and the Royal Institute of Technology in Stockholm (KTH). This work is documented in Ågren et al. (2008). The model is computed using one version of the so-called KTH method, developed by Prof. Lars E Sjöberg and his group at the Royal Institute of Technology in Stockholm. The technique includes least squares (stochastic) kernel modification with additive corrections for the topography, downward continuation, atmosphere and ellipsoidal shape of the Earth. The method is well documented in a long row

of publications; see for instance Sjöberg (1991, 2000, 2003a, 2003b, 2004 and 2007), Sjöberg and Nahavandchi (2000), Ågren (2004a, 2004b) and Kiamehr (2006).

The paper is organised in the following way. The GPS/leveling observations are described in Section 2, which also contains the corresponding evaluations of EGM2008, PGM2007A and EGM96. Section 3 contains a short description of the regional quasigeoid model referred above and the corresponding comparisons to EGM2008. After that, the point gravity anomalies that were used to compute the regional model are compared to the predicted gravity anomalies from EGM2008 and PGM2007A. Finally, the paper ends with our conclusions.

2. Comparison with GPS/leveling observations

The quasigeoids computed by the above EGMs were evaluated using 195 high quality GPS/ leveling height anomalies in the Swedish reference systems SWEREF 99 and RH 2000. More information about SWEREF 99 can be found in Jivall and Lidberg (2000) while RH 2000 is documented in Ågren et al. (2006) and Ågren and Svensson (2007). The normal heights in RH 2000 have either been determined in the RH 2000 adjustment (Ågren and Svensson 2007) or by utilising high quality leveling connections relative to the RH 2000 benchmarks. The stations are divided into two groups depending on the method employed to determine the GPS ellipsoidal height. The two groups are summarised in Table 1, in which approximate standard errors are also given. The distribution of the stations is illustrated in Figure 1.

Table 1: *The GPS/Leveling observations and their approximate standard errors. The permanent GPS stations (SWEPOS) and the stations (SWEREF) determined relative to SWEPOS are shown in red and blue on Figure 1, respectively.*

Data set	#	Short description	Appr. standard errors (mm)		
			GPS height	Normal height	Height anomaly
SWEPOS (red)	24	Permanent GPS stations whose coordinates define SWEREF 99.	5-10	5-10	7-14
SWEREF (blue)	171	Determined relative to SWEPOS using 48 hours of observations, DM T antennas and the Bernese software	10-20	5-10	11-22

It is clear from Table 1 that the observations are of high quality. Figure 1 also shows that the observations are uniformly distributed over the country. The lack of stations in the north-west corresponds to an area with high mountains where no levelling lines are available.

All computations were determined in the zero permanent tide system. Consequently, the GPS/leveling height anomalies were consequently corrected so that they refer to the same system. Since RH 2000 is already defined in a zero permanent tide system, the correction amounts to converting the height above the ellipsoid in SWEREF 99 from non-tidal to zero permanent tide system. This was done using (Ekman 1989):

$$h_{zero} = h_{non-tidal} + h\left(0.099 - 0.296 \cdot \sin^2 \phi\right) \text{ meter} \qquad (1)$$

where the Love number h was chosen to 0.62.

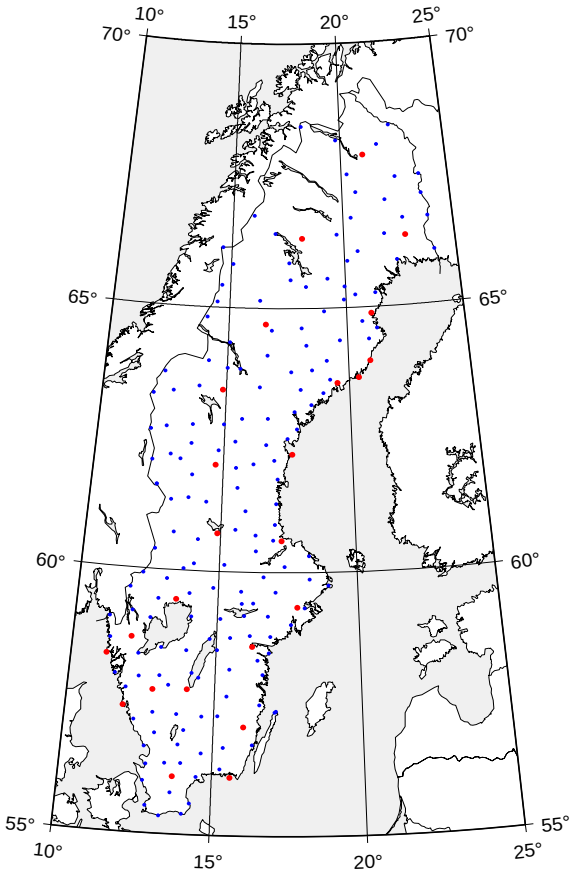


Figure 1: *Locations of the GPS/leveling observations. SWEPOS = Red and SWEREF = Blue.*

The program *Harmonic_synth_v2* is used to compute height anomalies for various spherical harmonic maximum degrees (*M* = 360, 720, 1440 and 2190). The scattered point computation mode (isw=00) is used with the normal height of each station given as input. The statistics of the residuals after a 1-parameter transformation are given in Table 2 for the 4 selected maximum degrees. EGM96 (Lemoine et al. 1998) is here included for comparison. The residuals for both PGM2007A and EGM2008 are illustrated in Figure 2 for the maximum degree 2190.

Table 2: *Statistics for the GPS/leveling residuals of EGM2008, PGM2007A and EGM 96 after a 1-parameter transformation. Unit: m.*

EGM	*M*	# gpslev	Min	Max	Mean	StdDev
EGM2008	2190	195	-0.074	0.095	0.000	0.027
	1440	195	-0.116	0.089	0.000	0.037
	720	195	-0.172	0.124	0.000	0.045
	360	195	-0.266	0.257	0.000	0.099
PGM2007A	2190	195	-0.248	0.085	0.000	0.039
	1440	195	-0.289	0.130	0.000	0.045
	720	195	-0.295	0.118	0.000	0.051
	360	195	-0.428	0.252	0.000	0.103
EGM 96	360	195	-0.376	0.509	0.000	0.172

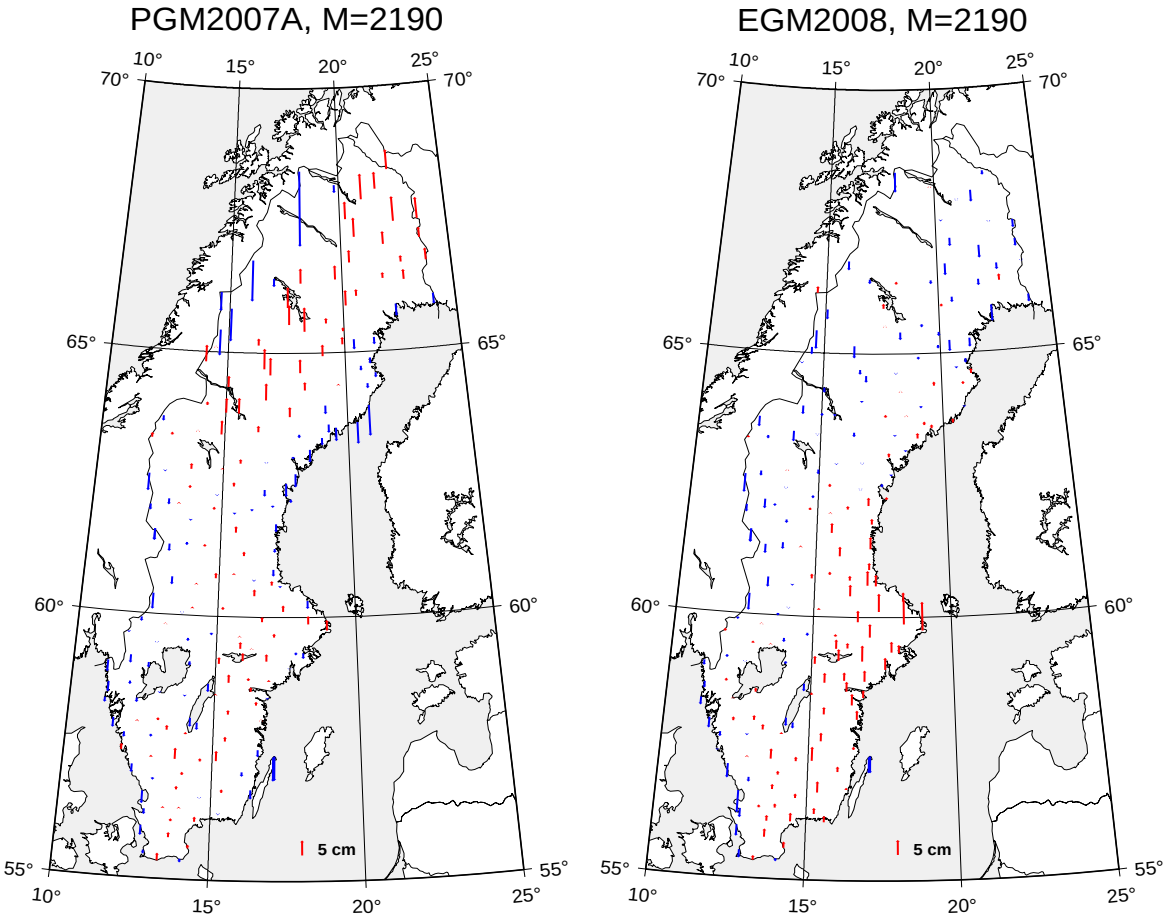


Figure 2: *GPS/leveling residuals of PGM2007A and EGM 2008 for M = 2190 after a 1-parameter transformation. The scale is given by the arrow to the South East.*

The first thing that can be seen in Table 2 and Figure 2 is that EGM2008 fits considerably better with the GPS/leveling data compared to the preliminary PGM2007A. For the latter, the fit is poor in the northern half of the country. For southern Sweden, however, the results are comparable. The fact that EGM2008 is so much better indicates that some improvement must have been made by the processing team based on the preliminary evaluation of PGM2007A. It can further be seen in Table 2 that EGM2008 is a considerable step forward compared to EGM96, also when only the maximum degree M=360 is considered.

It should finally be emphasised that the achieved fit for EGM2008 with M=2190 is impressive. The question now is how this result compares with the best Swedish regional quasigeoid model available in 2008. This is the topic of the next section.

3. Comparison to the best regional quasigeoid model available for Sweden in 2008

As mentioned in the introduction, the best (*gravimetric) regional quasigeoid model* available for Sweden in 2008 has been computed in close cooperation between Lantmäteriet (Swedish mapping, cadastre and registry authority) and the Royal Institute of Technology in Stockholm (KTH); see Ågren et al. (2008). The model is derived by the least squares modification of Stokes' formula using additive corrections (the KTH method). The method can also be described as stochastic modification of Stokes' formula using analytical continuation to point level (Moritz 1980) together with improved atmospheric and ellipsoidal corrections (Ågren et al. 2008). Below a short summary is given of the method used to predict height anomalies. The reason for including this description here is that the comparison between the regional model and EGM2008 not only serves as an evaluation of the models themselves, but also of the corresponding processing strategies.

In the least squares modification of Stokes' formula (e.g. Sjöberg 1991), Stokes' kernel is modified in such a way that the expected global mean square error is minimised. This technique can be applied with the standard remove-compute-restore estimator (e.g. Ågren 2004b), but according to KTH practice the so-called *combined estimator* is preferred (Sjöberg 2003b). This means that Stokes' formula (truncated to a cap) is applied to the uncorrected surface gravity anomaly, Δg. After that, the height anomaly ζ is computed by adding a number of so-called *additive corrections*, which are derived in such a way that the same result is ideally obtained as when the remove-compute-restore technique is utilised (except for numerical effects). We thus have

$$\zeta = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\,\Delta g\,d\sigma + \frac{R}{2\gamma}\sum_{n=2}^{M}\left(s_n + Q_n^M\right)\Delta g_n^{GGM} + \delta\zeta_{TOPO} + \delta\zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL} \qquad (2)$$

where

$S^M(\psi)$ is the modified Stokes' function chosen according to Sjöberg (1991).

$\delta\zeta_{TOPO} = 0$ is the combined topographic correction. Vanishes in the height anomaly case.

$\delta\zeta_{DWC}$ includes analytical continuation to point-level of both the gravity anomalies (Moritz 1980) and the spherical harmonic expansion; cf. Sjöberg (2003a) and Ågren (2004a).

$\delta\zeta_{ATM}$ is the atmospheric correction (Sjöberg and Nahavandchi 2000).

$\delta\zeta_{ELL}$ is the ellipsoidal correction (Sjöberg 2004).

The following data is used to compute the regional quasigeoid model:

- Gravity anomalies from the database of the Nordic Geodetic Commission (NKG).
- The combined model GGM02C (to M=200) extended with EGM 96 up to M=360,
- The Swedish photogrammetric Digital Elevation Model (DEM) thinned out to the resolution of 100 m x 100m.

The principles of the weighting of the terrestrial gravity data in relation to the EGM is described in Ågren et al. (2008).

One problem with using the combined quasigeoid estimator in Eq. (2) is that Stokes' quadrature is made on the rough surface gravity anomaly, which results in large *discretisation errors*. However, by taking advantage of the remove-compute-restore philosophy for the gridding of a comparatively dense surface gravity anomaly grid using a smoothing topographic correction, such errors can be counteracted; see Ågren (2004). This makes it possible to take advantage of the high-frequency information available in the DEM. This strategy was adapted in the present case by utilising the Residual Terrain Model (RTM) correction (Forsberg 1997) as implemented in the TC program (Forsberg 2003). The surface gravity anomaly grid was chosen with a resolution of 0.01°x0.02°, which should be sufficiently dense.

The statistics for the fit of the regional quasigeoid model to the GPS/leveling height anomalies are given in Table 3. The residuals are illustrated in Figure 3 (left hand side), which also includes the depiction of EGM2008 residuals for comparison.

Table 3: *Statistics for the GPS/leveling residuals of the regional quasigeoid model (KTH_080326) after a 1-parameter transformation. Unit: m.*

Model	# gpslev	Min	Max	Mean	StdDev
KTH_080326	195	-0.064	0.061	0.000	0.022

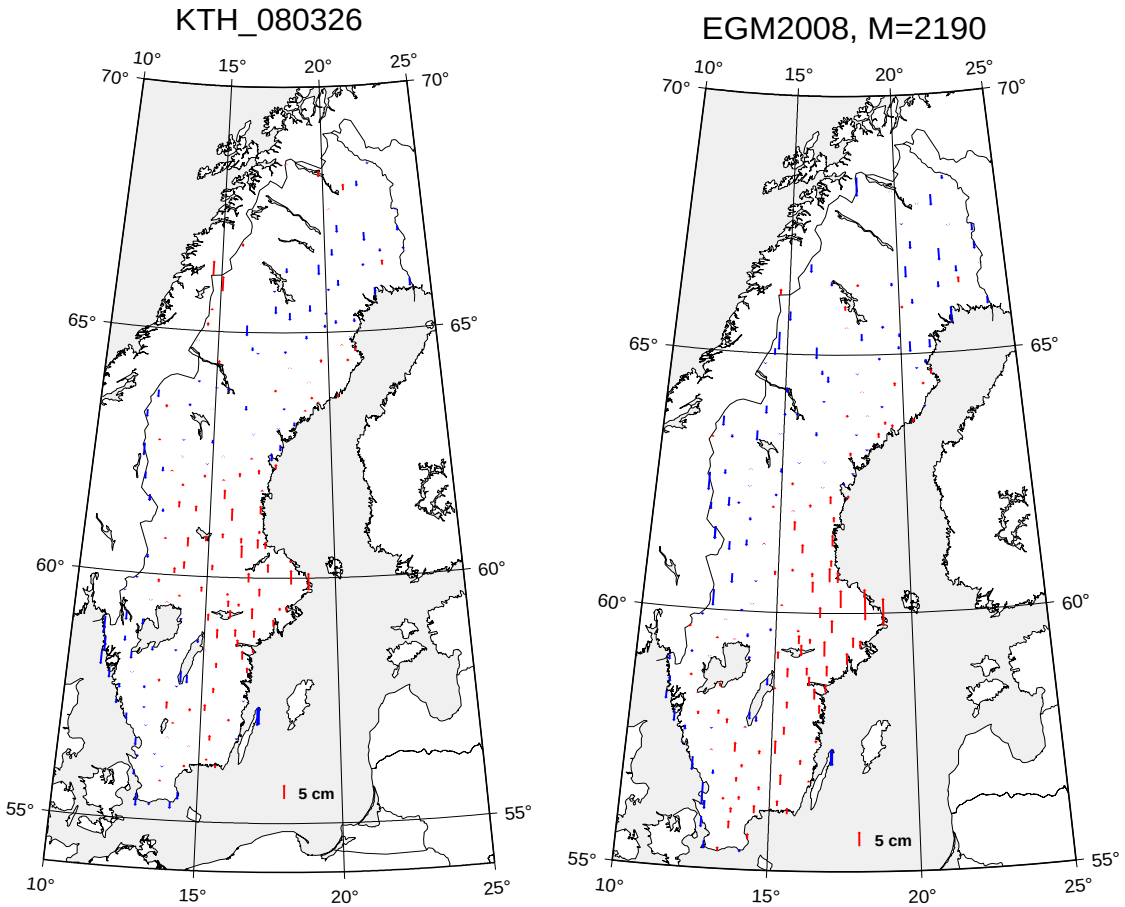


Figure 3: *GPS/leveling residuals of the regional quasigeoid model (KTH_080326) and EGM 2008 after a 1-parameter transformation. The scale is given by the arrow to the South East.*

By comparing the results in Tables 2 and 3, it can be observed that the regional quasigeoid model fits slightly better than EGM2008 to the Swedish GPS/leveling data. As is clear from Figure 3, the agreement between the regional model and EGM2008 is surprisingly good. It is difficult to say, though, how much of the errors in Figure 3 are gravimetric and how much are GPS/leveling. Let us now study the difference in question a little more carefully. For this purpose, the regional height anomaly grid was compared to the same grid computed by EGM2008 using *Harmonic_synth_v2* in the scattered point mode with M = 2190. The comparison was made without transforming the models in any way and without special zero degree corrections. The differences are illustrated in Figure 4 and statistics for the whole grid and for the mainland of Sweden are presented in Table 4.

Table 4: *Statistics for the height anomaly difference between the regional quasigeoid model (KTH_080326) and EGM2008. Unit: m.*

Area	# grid points	Min	Max	Mean	StdDev	RMS
Whole grid	1202351	-0.221	0.242	0.006	0.038	0.038
Mainland of Sweden	390816	-0.187	0.093	-0.002	0.023	0.023

It can be seen that the agreement between the models is within ± 2 cm for most parts of the country. In some areas the discrepancies are larger, but in most cases just a little. The differences are considerably larger in the rough mountains to the North-West of the country, which mainly depends on that the regional model contains more high-frequency information than EGM2008. In these areas the frequencies above *M=2190* are definitely significant. It can also be seen that the models agree reasonably well in the Baltic Sea east of Sweden. We do not comment here on the large deviations outside the coast of Norway. This is not a Swedish matter

Overall, Table 4 and Figure 4 support the conclusion that the two models agree well over Sweden. This is encouraging and shows that the processing strategies for the regional model and for EGM2008 are compatible to a high degree and that very similar gravity data must have been utilised in the processing. Since the regional model has been computed using updated gravity from the NKG database (see above), this shows that good, updated data must have been used for EGM2008 too.

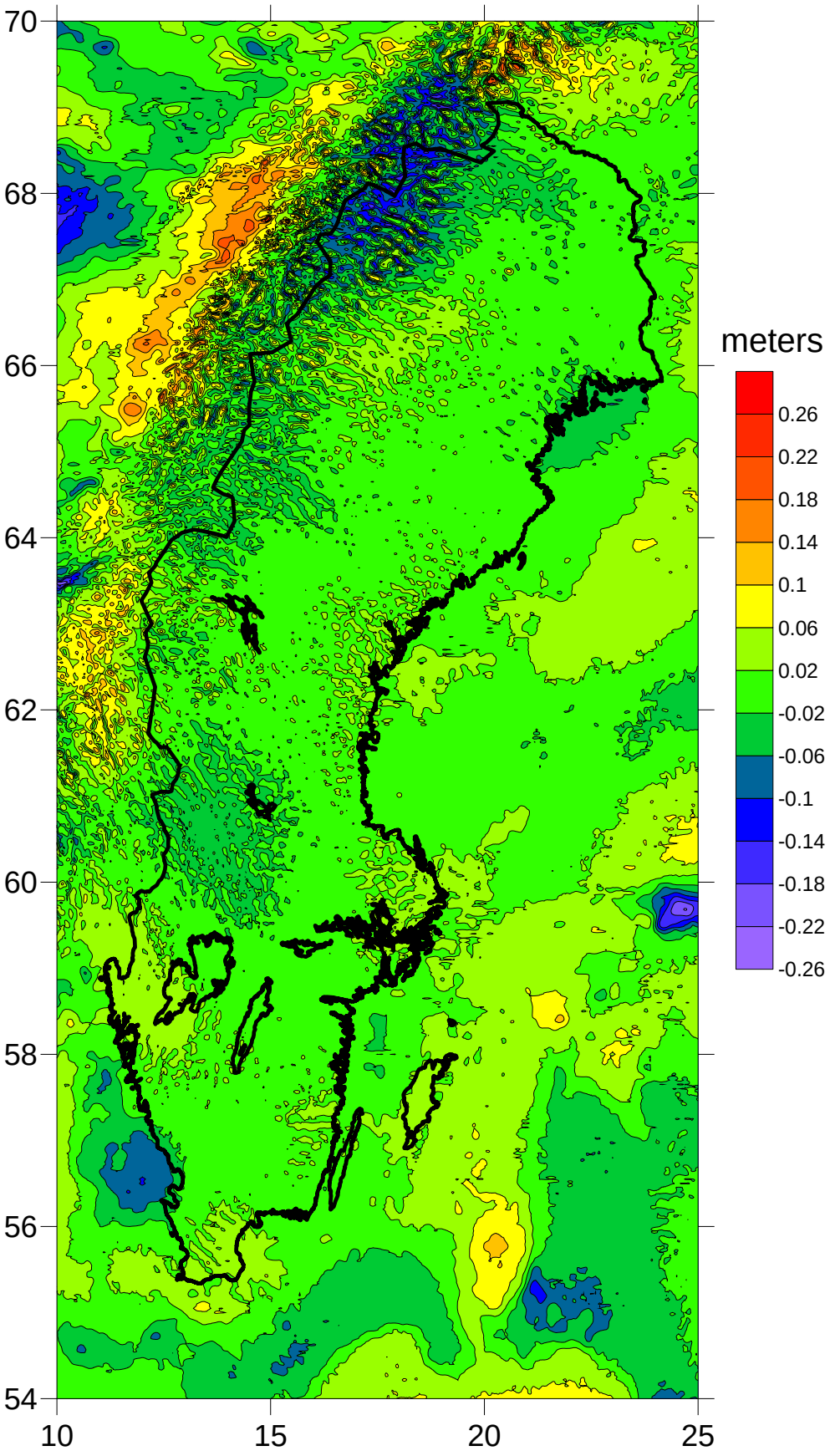


Figure 4: *Difference between the height anomalies from the regional quasigeoid model (KTH_080326) and EGM 2008. Unit: m.*

4. Comparison with gravity anomalies

As mentioned in Section 3, the gravity anomalies used to compute the regional quasigeoid model (KTH_080326) were taken from the NKG (Nordic Geodetic Commission) database, managed by René Forsberg and Gabriel Strykowsky at DTU Space in Denmark. Before estimating the height anomaly using Eq. (2), however, the gravity anomalies were cleaned by

- computing the weighted mean of observations at the same location,
- making a cross validation using the RTM and EGM reduced gravity anomalies and
- finally choosing only the observation with lowest apriori standard error in each compartment of a 0.02°x0.04° grid.

Statistics for the difference between EGM2008 and PGM2007A (using *Harmonic_synth_v2* in the scattered point computation mode with $M = 2190$) and the cleaned gravity anomalies are given in Table 5.

Table 5: Statistics for the difference between the gravity anomalies selected to compute the regional quasigeoid model (KTH_080326) and PGM2007A/EGM2008. Unit: mGal.

EGM	Area	# gravity anomalies	Min	Max	Mean	StdDev	RMS
PGM2007A	Whole grid	270204	-107.2	197.3	1.3	10.2	10.3
	Mainland of Sweden	24570	-60.2	86.9	2.4	7.6	7.9
EGM2008	Whole grid	270204	-99.1	197.8	1.3	10.0	10.1
	Mainland of Sweden	24570	-58.6	80.3	2.6	7.3	7.8

It should be pointed out that no kind of filtering is used in the computation of Table 5. Consequently the differences in question also contain the omission error above the maximum degree 2160, which is definitely not negligible. For instance, the global omission RMS error from the Tscherning and Rapp (1974) degree variance model is 11.2 mGal. With this in mind, it is clear that the observed point gravity data and EGM2008 fit reasonably well over Sweden. No comparisons have been made using filtered gravity anomaly data.

5 Summary

The main purpose of this paper is to present the Swedish evaluation of EGM2008 and its preliminary version PGM2007A. The main conclusions are the following:

- The height anomalies from EGM2008 are almost as accurate as those of the best gravimetric quasigeoid model available for Sweden in 2008. The RMS for the fit to GPS/leveling are 2.7 cm and 2.2 cm, respectively. Considering the standard errors of the GPS/leveling height anomalies, which lies somewhere around 10 – 20 mm, it is clear that both the regional model and EGM2008 are very good.
- The regional quasigeoid model and EGM2008 agree well with each other inside Sweden. This indicates that the respective computation strategies are compatible. Since this is an important result for both techniques, a rather detailed summary has been given of the regional geoid determination method. The good agreement also shows that similar gravity anomaly data is utilized for the two models. Since the regional quasigeoid model has been derived using good, updated data (NKG database), this need to be the case for EGM2008 too.

- EGM2008 is a considerable improvement with respect to PGM2007A as far as Sweden is concerned. The standard deviation improves from 3.9 cm to 2.7 cm when compared to the GPS/leveling-derived height anomalies. This is after correcting for a 1-parameter transformation. The improvements occur in northern half of Sweden, above the 64 degree parallel. Whatever the EGM2008 processing team did to improve PGM2007A, they did the right thing.
- EGM2008 agrees well with the Swedish point gravity anomalies. For the mainland of Sweden, the RMS of the discrepancies is 7.8 mGal. If one considers the magnitude of the omission error for EGM2008, it is clear that this agreement is as good as can be expected.

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Evaluation results of the Earth Gravitational Model EGM08 over the Baltic countries

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Abstract. Earth's geopotential model (EGM) in conjunction with regional terrestrial gravity data are often used in regional geoid determination. Thus, significant enhancements are expected due to release of the new high resolution Earth Gravitational Model EGM08. Accordingly, this study evaluates the performance of the EGM08 model over the Baltic Sea region with emphasis to Estonia, Latvia and Lithuania. Several different sets of the "ground truth" data are used in the comparisons. First, the EGM08-derived height anomalies are compared with an existing regional geoid model. The detected discrepancies range within ± 0.3 m with a mean of -0.02 m, whereas the standard deviation (STD) of the discrepancies amounts to 0.08 m. The largest discrepancies occur in the areas where only a few data points were available either for the regional geoid modeling or at the EGM08 compilation, or both. Second, the free-air gravity anomalies at the terrestrial data-points are compared with the EGM08-derived anomalies. The STD of the anomaly discrepancies is 2.6 mGal. Finally, the EGM08 model is validated with respect to GPS-levelling data. The STD of detected discrepancies is 0.06 m, with a mean of 0.49 m. Thus, the EGM08 based quantities agree reasonably well with the tested datasets. Evidently, most of the available gravity data in the Baltic Sea region appear to be utilised at the EGM08 construction.

Keywords: geopotential model, geoid, GPS-levelling.

1 Introduction

A new combined Earth gravitational model EGM08 (Pavlis et al, 2008) was released to the public in 2008. EGM08 takes advantages of recent satellite, terrestrial gravity, elevation and altimetry data. This activity is conducted by the National Geospatial-Intelligence Agency (NGA) of the USA. The resolution of the EGM08 is 5′ (corresponding to 9 km, i.e. to the spectral degree of ca 2160), also the global accuracy of the EGM08 is expected to be superior over earlier EGM-s.

Regional improvements of global geoid models can be obtained by modifying Stokes's integral formula (Stokes, 1849). When solving the Stokes problem, strictly speaking, gravity anomalies over the entire Earth are required. In practice, however, the data availability is limited to some spatial domain ($\Omega_{\psi 0}$) around the computation point. Modified Stokes's formula (first proposed by Molodenskii et al., 1960) combines local terrestrial gravity anomalies and the EGM-derived long-wavelength component of the geoid. For instance, a generalized Stokes scheme (cf. Vaníček and Sjöberg, 1991) can be used

$$N(\Omega)=\frac{R}{4\pi g_0}\iint_{\Omega_{\psi_0}} S^L(y)\left[\Delta g(R,\Omega)-\sum_{n=2}^{L}\Delta g_n(R,\Omega)\right]d\Omega' + \\ +\frac{R}{2g_0}\sum_{n=2}^{L}\frac{2}{n-1}\Delta g_n(R,\Omega) \quad (1)$$

where R is the mean radius of the Earth; y is the geocentric angle, the modified Stokes function $S^L(y)$

can be computed according to some algorithm (e.g. Wong and Gore (1969); Vaníček and Kleusberg (1987); Sjöberg (1991), among others); g_0 is the normal gravity at the reference ellipsoid, $\Delta g(R,\Omega)$ is terrestrial gravity anomaly on the geoid, Ω denotes a pair of geocentric coordinates (the spherical co-latitude q and longitude λ), $d\Omega'$ is an infinitesimal surface element, y_0 is the radius of the integration cap, L is the modification degree, Δg_n are the harmonics of the EGM-derived gravity anomaly.

Due to availability, quality, and type of data, the characteristics of an EGM vary regionally. Hence, the performance of any EGM needs to be validated in a regional scale by comparisons with other external data sets that depend on the same gravity field. Traditionally, the accuracy of the regional geoid modelling has been assessed by using GPS-levelling points. Apparently, the computations of new regional geoid models will also be based upon the global EGM08 model, the testing of which is necessary to assess its suitability for this task. In this contribution three different sets of the "ground truth" data are used over the three Baltic countries - Estonia, Latvia and Lithuania. First, the EGM08-derived height anomaly is compared with an existing regional geoid model. Second, the free-air gravity anomalies at the terrestrial data-points are compared with the EGM08-derived anomalies. Thereafter the EGM08 model is validated with respect to the GPS-levelling data. The differences between a preliminary PGM07A and the final EGM08 models over the Baltic countries are discussed. Further comparisons reveal that there is still some space for further improvements of the contemporary EGM-s. Actions needed for assembling more consistant combined geopotential models are suggestedas well. A brief summary concludes the paper.

Note that the study results have been reported partly in the international conference Gravity, Geoid and Earth Observation (GGEO), held in June 2008 in Chania, Greece. Since this contribution contains some more details, it can be considered as an extended report of Ellmann (accepted). In addition, this work includes further evaluation results of the EGM08 over the Baltic countries.

2 Target area

The EGM08 performance is examined within the following geographical boundaries: $53.83° < j < 60.06°$; $19.97° < l < 28.52°$, see Fig. 1. Thus, in addition to Estonia, Latvia and Lithuania the target area includes partly also Russia, Belarus, Poland and Finland, together with a large portion of the Baltic Sea. The elevation extremes are 0 m at a shoreline and 318 m in southeast Estonia, whereas most of the target area comprises of sea and topography below 100 m. Due to such low topography no significant numerical differences (3 mm at most) between the geoid and height anomaly occur over the chosen target area.

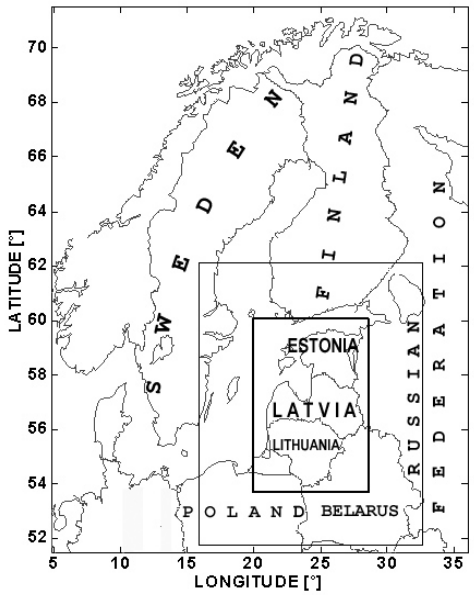


Fig. 1 Location of the target area (enclosed by the bold rectangle). External rectangle denotes the data area borders in the BALTgeoid-04 (Ellmann, 2005) computations.

3 Comparisons with a regional high-resolution geoid model BALTgeoid-04

3.1 Regional BALTgeoid-04 model

A recent Baltic geoid model was computed by Ellmann (2004 and 2005). In his study the geoidal heights were estimated by the least squares modified Stokes's formula (cf. Sjöberg, 1991).

The definition of the main computation criterias (such as the modification limit $L = 67$, the radius of the integration cap $y_0 = 2°$, etc) is explained in detail by (ibid.). An early GRACE-derived („satellite-only") GGM01s model (Tapley et al., 2004) was used as the reference model for computing Δg_n in Eq. (1). The resulting 1.5´x3´ geoid model is depicted in Fig. 2. The geoidal heights in the target area vary between 15 m and 30 m, with the regional downslope trend from southwest toward northeast. The geoid model is mainly smooth (with a STD of the mean ~3 m), but it includes some local irregula-

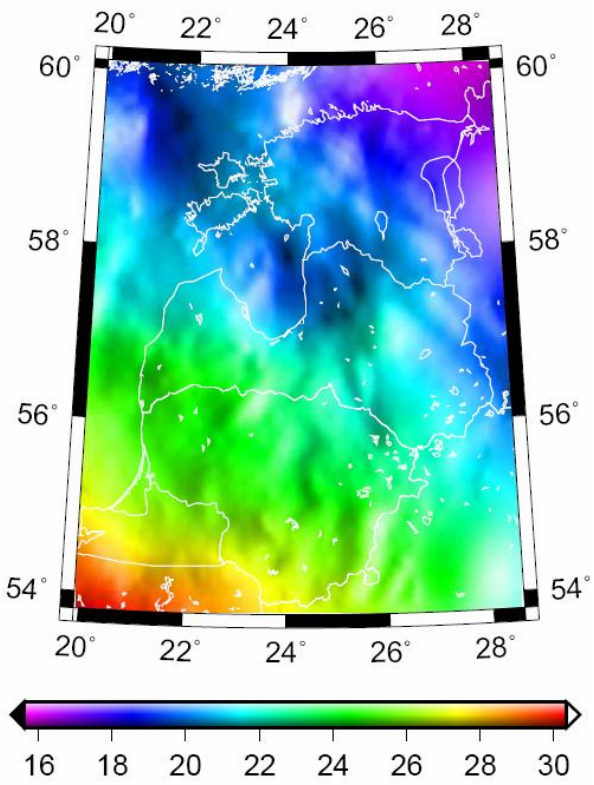


Fig. 2 The Baltic gravimetric geoid model BALTgeoid-04 (Ellmann, 2005). Geoidal heights are given with respect to the GRS-80 reference ellipsoid. Unit is metre. The total area of the image corresponds to 300 000 km^2.

rities in the NW part of the target area. Their location is correlated with the local anomalies of the gravity field (cf. Ellmann, 2004, Fig. 2.3, the anomaly range at the terrestial gravity points are also shown in Fig. 4 of the current paper). The quality of the BALTgeoid-04 model was assessed from the comparisons with the GPS and levelling datasets. The same sets of the control points will also be used for the evaluation of the EGM08-derived height anomalies, therefore some more information on these data is spared for Section 5.

The GPS-levelling points form a surface, which is called here the "geometric geoid model" ($N_{geom} = h - H$). The following STD value of the discrepancies between the BALTgeoid-04 and N_{geom} were achieved: over the whole of the Baltics 5.8 cm, in Estonia 4.0 cm, in Latvia 6.0 cm and in Lithuania 5.7 cm, respectively. It is also concluded that the accuracy of the BALTgeoid-04 model is at least of the same level as is the accuracy of the used control points (Ellmann, 2005).

3.2 Accounting for the differences between the EGM08 and GRS-80 parameters

The BALTgeoid-04 geoidal heights are defined with respect to the GRS-80 (Geodetic Reference System; Moritz, 1992) ellipsoid. Also the physical constants of the GRS-80 are used for computing the normal gravity field in the Baltic countries. Furthermore, the GPS-derived geodetic heights are reckoned from the ETRS89 (European Terrestrial Reference System) oriented GRS-80 ellipsoid.

As is customary in geodesy, the mass of the reference ellipsoid is chosen to be equal to the mass of the Earth, and the origin of the reference ellipsoid is placed at Earth's mass centre. However, in reality the EGM parameters may differ from the corresponding parameters of the adopted geodetic reference ellipsoid. Thus, the differences between the defining constants (i.e. gravity-mass constant *GM*, and the major semi-axis *a* of the ellipsoid versus reference radius for the spherical EGM) of the used GGM and adopted geodetic reference ellipsoid should be considered. The scaling can be introduced via zonal harmonics of the reference ellipsoid by an approach described in Vaníček and Kleusberg (1987, Sect. 5), see also Kirby and Featherstone (1997) and Smith (1998). In the discussion below the EGM-related values will be denoted by the subscript "*EGM*", whereas the subscript "*GRS*" denotes the geodetic reference ellipsoid related quantities

It should be noted that the EGM08 geopotential model utilises $GM_{EGM} = 398600.4415 \cdot 10^9$ m$^3 \cdot$s^{-2}, whereas $GM_{GRS} = 398600.5 \cdot 10^9$ m$^3 \cdot$s^{-2}. The Earth's gravitational potential and its derivatives (such as the disturbing potential, gravity anomaly and geoidal height) can be expressed in terms of an infinite series of spherical harmonics outside the attracting masses of the Earth. Since the EGM08 coefficients are referred to the bounding sphere with some radius *a* (the value $a_{EGM} = 6378136.3$ m is adopted at the compilation of the EGM08, whereas $a_{GRS} = 6378137$ m), then the EGM derived quantities, strictly speaking, ought be computed on the surface of the bounding sphere (or above it). However, the gravity field related quantities can be more or less safely computed inside of this sphere, as long as the evaluation point remains outside the topographic masses. Due this, the EGM-s are better suited for computing the ground related gravity quantities, such as the height anomaly (cf., Molodenskii et al., 1960), rather than the geoid. Note that over the continents the latter would require computations inside the topographic masses.

3.3 The EGM08-derived height anomalies

The „tide-free" version of the EGM08 model (the file *EGM2008_to2190_TideFree.gz*, retrieved from URL:http://users.auth.gr/~kotsaki/IAG_JWG/EGM08_intro.html , retrieved April, 2008) contains fully-normalized, unitless spherical harmonic coefficients, complete to degree and order 2159, plus additional coefficients extending to degree 2190 and order 2159. The EGM08-derived height anomalies z (at the topographic surface, with the geocentric radius of $r_t = r_g + H$) were computed by the following formula:

$$z_{EGM}(\Omega) = \frac{GM_{EGM} - GM_{GRS}}{r_t g_T} + \frac{GM_{EGM}}{r_t g_T} \sum_{n=2}^{2190} \left(\frac{a_{EGM}}{r_t} \right)^n \sum_{m=0}^{n} \{ \Delta\bar{C}_{nm} \cos ml + \bar{S}_{nm} \sin ml \} \bar{P}_{nm}(\cos q) \quad (2)$$

where the normal gravity γ_T is referred to the surface of the telluroid (with the geocentric radius of $r_T = r_{GRS} + H$); $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are fully normalised spherical harmonic coefficients, of degree n and order m; $\bar{P}_{nm}(\cos q)$ are fully normalised associated *Legendre* functions. The first term on the left hand side of Eq. (2) represents the zero degree geoid scaling term, which is due to the difference between the *GM*-values of the EGM08 and that of the reference elliposoid (GRS-80). Using R = 6371 km and g = 981 Gal the zero degree geoid scaling term becomes -0.936 m. This value will be added to the EGM08-derived height anomalies. The (residual) zonal coeffcients $\Delta\bar{C}_{nm}$ account also for the differences between the reference radius of the EGM08 and semi-major axis of the GRS-80.

The above principles have also been realised in the *harmonic_synth_v02.f* code (by Holmes and Pavlis, version 05/01/2006, retrieved from the NGA webpage http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/new_egm.html), which is used in the validation of the EGM08 geopotential model in the present study. However, the program does not account for the influence of the zero-degree term. Therefore, the EGM08-derived quantities are corrected for the missing zero-degree term.

Strictly speaking, Eq.(2) should also account for the difference between the gravity potential on the surface of the geoid (W_0) and the normal gravity potential on the surface of the normal ellipsoid (U_0), i.e. the term $-\frac{W_0 - U_0}{g}$. Recall that in an ideal case $W_0 = U_0$. Several estimates of W_0 have been proposed in the geodetic literature over the past decades. For a recent review of the gradual improvements see Bursa et al (2007) and references therein. Note that many studies of the W_0 rely upon the satellite altimetry (such as TOPEX / Poseidon) results. In this case, however, the data coverage is not truly global, since no data from sub-polar latitudes have been included in such solutions. Some others combine the GPS, levelling and tide-gauge data into a common solution, see e.g. Ardalan et al (2002). Such an approach may provide the best match with the local vertical datum over the given study area. Both approaches can be considered being complimentary to each other to a certain extent. However, at the present the estimates of the W_0 value can still be improved further. Therefore we exclude the $-\frac{W_0 - U_0}{g}$ term from the present comparisons. It is also reasonable to assume, that there may be a certain consistency between the W_0 value and the adopted set of *GM* and *a* values at the compilation of the EGM08 model. All in all, after the proper determination of the W_0 value its contribution can be added to the results of the present study. Note that it will manifest only as a simple one-dimensional bias of the EGM08 derived gravity field quantities.

The EGM08 height anomalies were computed at the grid nodes of the BALTgeoid-04 model. The conceptual differences between the geoid and height anomalies are well known, see e.g., Heiskanen and Moritz (1967, Chap. 8-3). Recall, however, that over the selected target area these differences are numerically insignificant. These differences are neglected in the following comparisons without affecting the objectives of the present study.

3.4 The results

The discrepancies between the BALTgeoid-04 model and EGM08 height anomalies (cf. Eq. (2)) at the BALTgeoid-04 grid nodes (altogether 250 x 172 points) are depicted in Fig. 3. Here we focus only on the general features of the discrepancies. The range of the detected discrepancies varies within ± 3 dm over the whole target area. The largest discrepancies are located outside the borders of Estonia, Latvia and Lithuania. Within the borders of the three countries the absolute range of the discrepancies remains smaller than 15 cm. Full statistics of the comparison can be found in Table 1 (see the last section of this paper).

The nature of the discrepancies between the two models appears to be quite complicated. Note that the discrepancies in the centre of the target area seem to possess a spectral content below degree 100.

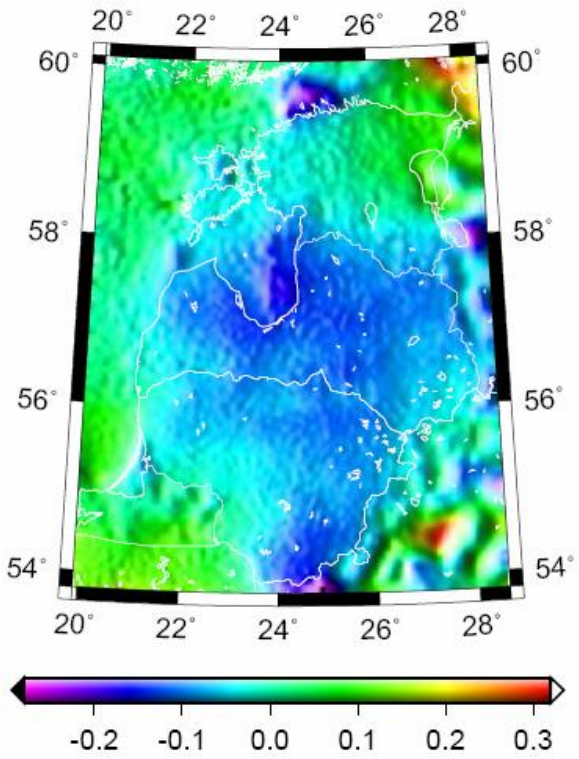


Fig. 3 Discrepancies between the BALTgeoid-04 model and EGM08 height anomalies (n_{max} = 2190) at the BALTgeoid-04 grid nodes (altogether 250 x 172 points). The discrepancies range from -0.289 m to +0.338 m with a mean of -0.025 m. Generally, the EGM08 height anomalies appear to be slightly higher than the BALTgeoid-04 model. Standard deviation of the detected discrepancies amounts to 0.077 m.

It should be noted that the long wavelength component of the GGM01s (which was used as the BALTgeoid-04 reference model, with the degree n_{max} = 67) and EGM08 model is very similar. Their long wavelength differences (both developed up to n_{max} = 67, cf. Eq. (2)) do not exceed ± 4 cm over the target area. This may indicate the presence of the systematic biases among the terrestrial datasets used for the computations of the BALTgeoid-04 and EGM08 model.

Alternatively, the discrepancies could either be due to: (*i*) inadequate reproduction of the spectral content of the disturbing potential from the truncated Stokesian integration (cf. the first term on the right hand side of Eq. (1)); (*ii*) deficiencies of the harmonic analysis when determining the EGM08 spherical harmonic coefficients; or (*iii*) both.

All in all, within the land masses of the three countries (Estonia, Latvia and Lithuania) the agreement between the BALTgeoid-04 and EGM08-derived height anomalies is reasonable, see Fig. 3. It should be noted that the terrestrial data coverage (used for the BALTgeoid-04 model, see Fig 4) is satisfactory there.

Note that the discrepancies between the BALTgeoid-04 and EGM08-derived height anomaly possess shorter wavelength features over the eastern part (especially in SE) of the target area, where only a few data were available for the BALTgeoid-04 computations. Hence, a more complete dataset was most likely available for the compilation of the EGM08 over the eastern part of the target area. Also at some offshore spots, well covered with the terrestrial data, the range of detected discrepancies appears to be unreasonably large. The search for an explanation of the detected discrepancies over the Baltic Sea prompts us to have a closer look at the quality and coverage of the regional terrestrial data.

4 Comparisons with the historical terrestrial gravity survey data

The gravity survey data (altogether 42559 points) used in the current comparisons were obtained (in 2001) from the Danish National Survey and Cadastre, the authorized holder of the Nordic–Baltic gravity database. This international database is created and maintained within the frame of the activities of the Nordic Geodetic Commission. The national contact persons deliver the data to the database, whereas their responsibility is to ensure the quality and internal consistency of the national datasets.

The coverage of the terrestrial data points within the target area is more or less satisfactory, except the eastern part, where only a small number of gravity points is available, see Fig 4.

Data, which are collected during several decades with different methods and equipment and by different nations and specifications, requires careful analysis before further processing.

The Estonian gravity survey was performed by the Institute of Geology at the Estonian Academy of Sciences in 1949-58. The total number of Estonian gravity survey points exceeds 4000, yielding a density of 1 survey point per 10 km^2. The accuracy of these data is (very optimistically) claimed to be < 1 mGal. A register of Latvian and Lithuanian gravity points is mainly reconstructed from the 1: 200 000 paper maps in conjunction with an obsolete global topographic model. The accuracy of such data remains unknown (Kaminskis and Forsberg, 1997). It should be emphasised, however, that the gravity surveys of the three Baltic countries have historically been related to the same vertical system and gravity datum.

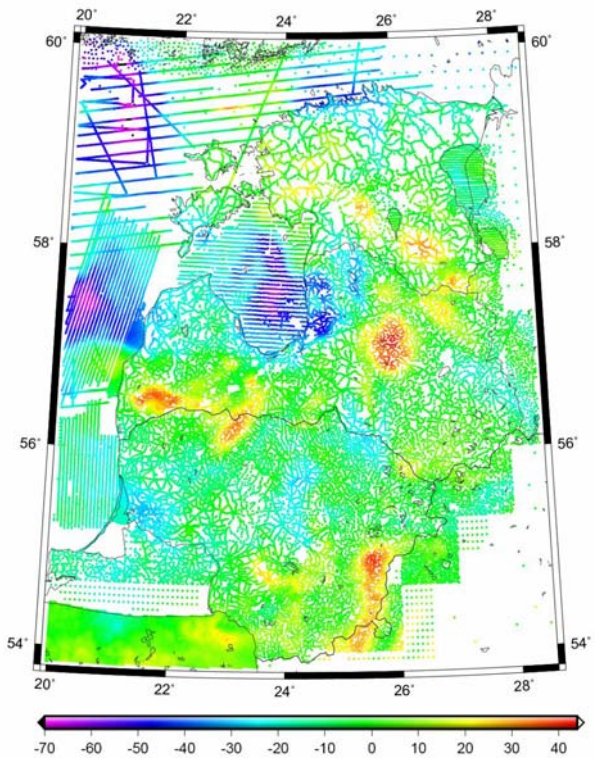


Fig. 4 Free-air anomaly $\Delta g(r_t,\Omega)$ at the terrestrial data points The anomaly values range from -73 to +44 mGal, with a mean of -7 mGal. The STD of the anomaly values is 17 mGal. The colors of the dots are proportional to the range of the anomaly, cf. the colorbar.

Note that the NKG 1997 marine and Baltic Sea 1999 airborne gravity surveys have significantly improved the data coverage over the Baltic Sea. The accuracy of these datasets is estimated to be ~ 2 mGal (Forsberg, 2001). Aligned (mainly in E-W direction) data-points over the Baltic Sea indicate the location of the aero-gravity survey tracks (see Fig. 4). In addition, the marine gravity data within the Riga Gulf and nearby Latvian coastline (aligned in NE-SW direction) have been made available as well.

An extensive analysis of Nordic-Baltic gravity data is summarized in Ellmann (2001). The study revealed the presence of some (presuambly very small) systematic discrepancies between the used datasets. The systematic errors in Nordic-Baltic gravity datasets have also been noticed by other authors, see e.g., Omang and Forsberg (2002), Jürgenson (2003).

The elimination of these possible offsets is outside of the scope of the present study, since it requires a multilateral international involvement. Hence, any possible inherent systematic bias between the national datasets is simply ignored in earlier studies and in this comparison.

The EGM08 derived free-air anomalies Δg (at the topographic surface r_t) are computed at the locations of the Nordic-Baltic terrestrial gravity points by the following formula (Heiskanen and Moritz, 1967, Eq. 2-151c.):

$$\Delta g_{EGM}(r_t,\Omega) = -\frac{GM_{EGM}-GM_{GRS}}{r_t^2}+\frac{GM_{EGM}}{r_t^2}\sum_{n=2}^{2190}(n-1)\cdot$$

$$\cdot\left(\frac{a_{EGM}}{r_t}\right)^n\sum_{m=0}^{n}\{\Delta\bar{C}_{nm}\cos ml+\bar{S}_{nm}\sin ml\}\bar{P}_{nm}(\cos q) \quad (3)$$

Note that the first term on the right hand side is the zero degree scaling term of the gravity anomaly, which amounts to +0.144 mGal (to be added to the EGM08-derived anomalies).

Presuambly, many datasets of the Baltic Sea region most likely do not contain the atmospheric correction on the gravity measurements, which is recommended by the IAG (see Moritz 1992, Sec. 5).

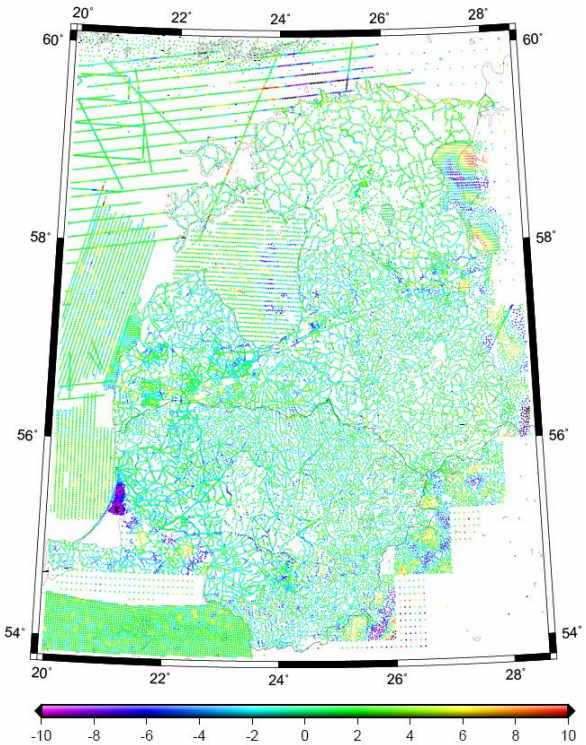


Fig. 5 The discrepancies between the terrestrial and EGM08-derived free-air gravity anomalies. The discrepancies [$\Delta g(r_t,\Omega)$ - $\Delta g_{EGM}(r_t,\Omega)$] range from -18 to +18 mGal, with a mean of 0.05 mGal. The STD of the discrepancies is 2.6 mGal. The colors of the dots are proportional to the range of the detected discrepancies, cf. the colorbar. The black dots denote the locations, where the absolute range of differences exceeds 10 mGal.

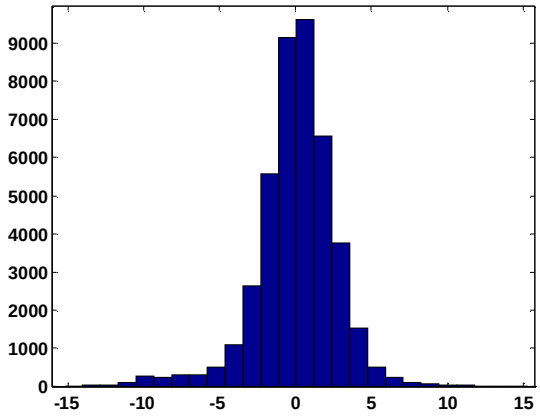


Fig. 6 Histogram of discrepancies between the terrestrial data and EGM08-derived free-air gravity anomalies. Unit is mGal. The total number of the points is 42559.

Recall that at the sea level this correction amounts to +0.87 mGal. In contrast, the attraction of the "beneath" atmospheric masses is naturally embedded in the spaceborne gravity results.

Therefore, for the sake of consistancy of comparisons all the terrestrial data were corrected for the attraction of the atmospheric masses (dg_{atm}) by the following formula (Wenzel, 1985):

$$dg_{atm} \approx 0.874 - 9.9 \cdot 10^{-5} H + 3.5625 \cdot 10^{-9} H^2 \qquad (4)$$

where H is the height above the sea level, the results are in mGal.

The discrepancies between the measured and EGM08-derived gravity anomalies (terrestrial minus EGM08) vary from -18 to +18 mGal, with a mean of +0.05 mGal, see Fig. 5. The histogram of discrepancies is shown in Fig. 6. In general, the EGM08-derived gravity anomalies agree reasonably well with the ground truth. However, there are some (offshore) areas, where the discrepancies are much larger than the regional average, see Fig. 5. This may indicate that different (from those used in this comparison) datasets were used (or no latest data were available) at the compilation of the EGM08. In particular, a relatively powerful negative anomaly over the Gulf of Finland (at 59.5°N & 24.5°E, see Fig. 4) remains "unnoticed" by the EGM08 data. Other such an example is the Kuroshio lagoon at 55.5°N & 21.5°E, where the discrepancies possess a systematic nature. Note that both areas are densely covered with the terrestrial data. Such discrepancies should be studied and ultimately resolved in future gravity field and geoid modelling works.

5 Comparisons with GPS-levelling data

As is well known, inter-comparison of a geoid model, GPS-derived geodetic heights, and spirit-levelled (normal or orthometric) heights at discrete points gives a reasonable indication of the geoid model's accuracy. Thus the further validation of the EGM08 model relays on nationwide sets (one for each country) of high-precision geodetic points, for their locations see Fig. 7.

First, the same constellation of the control points as used at the evaluation of the BALTgeoid-04 model will also be employed here. For all points the geodetic heights from GPS-measurements as well as levelling heights are available. The geodetic coordinates of the control points are related to the respective national realization of the new European Terrestrial Reference System ETRS-89. The spirit-levelled normal heights of all points refer to the Baltic Height System 1977 (Kronstadt tide-gauge).

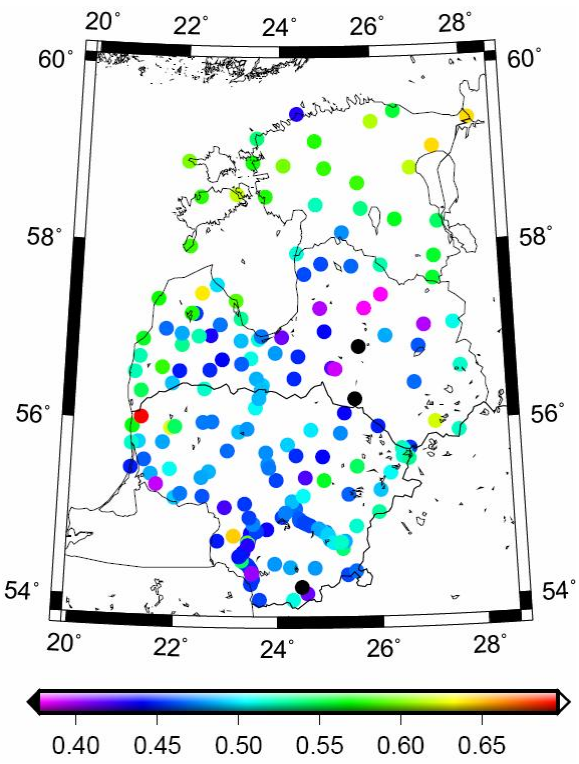


Fig. 7 Distribution of the Baltic GPS-levelling data (altogether 189 points) and their differences from the EGM08 height anomalies (developed up to degree 2190). The discrepancies [N_{geom} - z_{EGM}] range from +0.346 to +0.697 m, with a mean of +0.493 m. The STD of the discrepancies amounts to 0.060 m. The colors of the data-points are proportional to the range of the detected discrepancies (cf. the colorbar). Unit is metre. The levellings are referred to the Kronstadt tide-gauge observations.

The average distance among 26 evenly distributed Estonian control points is 50 km. The combined error of GPS-derived and spirit-levelled heights does not exceed 2-3 cm, most likely. Note that the geodetic heights are computed from the same GPS campaign and most of these points are directly connected to the high-precision levelling network. The Latvian and Lithuanian datasets (53 and 110 points, respectively) are denser. However, the accuracy of the used GPS-levelling points seems to be rather heterogeneous.

The common Baltic geometric geoid is represented by the sum of the three national datasets (189 points). The numerical statistics of the detected differences are presented in Table 1. In particular, the mean of the differences reveals a positive offset (+ 0.49 m) of the Kronstadt vertical datum from the EGM08-derived global geoid. Note however, that this offset depends also on the W_0 value to be adopted in future computations. For instance, a recent estimate W_0 = 62 636 855.75 m^2s^{-2} is published by Ardalan et al (2002). Considering also the GRS-80 related U_0 = 62 636 860.85 then the term $-\frac{W_0 - U_0}{g}$ becomes +0.52 m, which should be added to the EGM08 derived geoid model. In other words, after implementation of the aforementioned assumption the Kronstadt vertical datum will practically coincide with the mean level of the world oceans. On the other hand the Ardalan et al. (2002) approach constrains the regional GPS, levelling and tide-gauge data (surrounding the Baltic Sea only) into a common solutions, which may not necessarily be representable for the whole of the globe.

The resulting STD of differences 6.0 cm indicates almost the same level of accuracy, as it was achieved from the BALTgeoid-04 modelling (Ellmann, 2005). Such comparisons are also produced on a country-by-country basis. The corresponding statistics can be found in Table 1. In particular, the STD of the discrepancies (after removing the mean) as of 0.048, 0.063 and 0.048 m were achieved for the Estonian, Latvian and Lithuanian GPS-levelling points, respectively. Very similar estimates were obtained also from the comparisons with the BALTgeoid-04 model (Ellmann, 2004, Table 2.3).

Note that the Estonian GPS-levelling geoid appears to be somewhat "higher" than the Latvian and Lithuanian geometrical geoid models, see the mean values (+0.56 m versus +0.48 m) in Table 1. It should be noted that no temporal changes in the levelled heights were considered in this study. However, the Estonian points are affected by the Fennoscandian post-glacial rebound. Conversely, the Latvian and Lithuanian points are mostly located outside the land-uplift zone. Since the levellings have been performed over relatively long timespan then the Estonian solution may be contaminated with the land-uplift effect. Thus, the used GPS-levelling points cannot be considered as an entirely errorless dataset.

6 Differences between the EGM08 and a preliminary PGM07A model

As a matter of fact a Working Group (WG) was established by the International Association of Geodesy (IAG) for an independent and coordinated evaluation of the EGM08 quality already in 2006. A preliminary PGM07A model (Pavlis et al., 2007) was released to the WG members for validation in July 2007. The testing results were submitted to the NGA/EGM08 development team for the ultimate „fine-tuning" of the model in October 2007. As a result the final EGM08 model differs somewhat from its preliminary version. The detected discrepancies between the EGM08 and PGM07A derived gravity quantities in the Baltic countries are shown in Figs. 8 and 9. The differences are quite significant, exceeding a dm level in terms of the

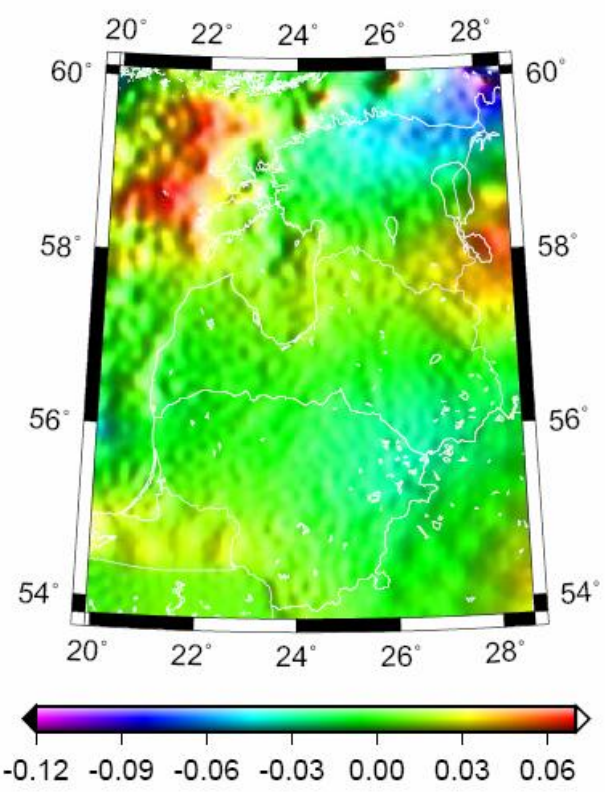


Fig. 8 Discrepancies between the EGM08 and PGM07-derived height anomalies (n_{max} = 2190). Unit is metre.

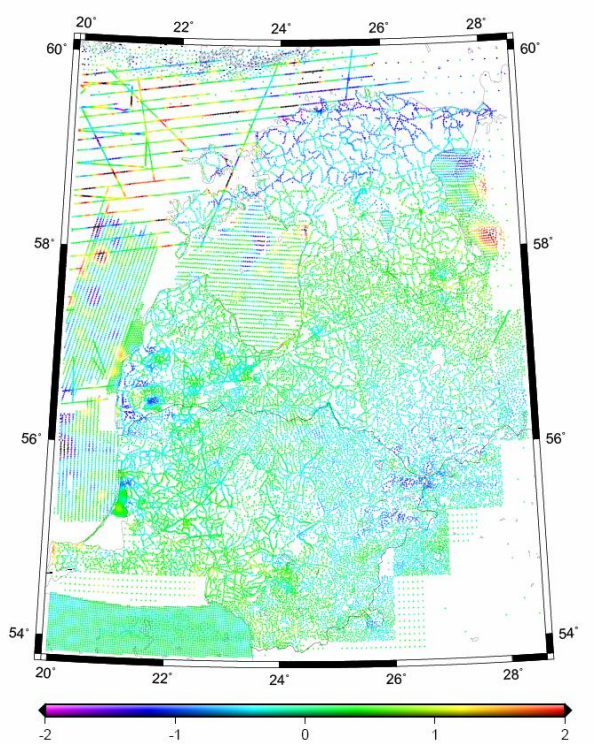


Fig. 9 Discrepancies between the EGM08 and PGM07 derived free-air gravity anomalies (n_{max} = 2190) The colours of the dots are proportional to the range of the detected discrepancies, cf. the colourbar. Unit is mGal. The black dots denote the locations, where the absolute range of differences exceeds 2 mGal

geoidal heights (a few mGal in terms of gravity anomaly). This could be due to the subsequent downweighting of the terrestrial data, at the same time assigning more weight to the satellite info (priv. comm. N. Pavlis, June 2008). In the study area the most significant changes have taken place outside the land masses of Estonia, Latvia and Lithuania.

As a result the accuracy of the EGM08-derived Baltic geoid model at the GPS-levelling points is slightly worse than that of PGM07A. However, the deterioration is just marginal, just some 2-3 mm in terms of STD for each case. For instance, the STD of the PGM07A derived Baltic geometric geoid reached 0.058 m (cf. to that of EGM08: 0.060 m, cf. Table 1).

Apparently, such a downweighting of the terrestrial data has distorted the accuracy of the final EGM08 in the Baltic Sea region. Note that the terrestrial data coverage is rather dense in the study area. Thus, the strategy of assigning more weight to the satellite info at the EGM08 compilation may not be the most optimum in such areas. Conversely, this approach may provide better results in the areas with sparse and less reliable terrestrial data.

7 Toward future combined geopotential models

Intuitively, high-resolution and accurate global geoid models, such as newly released EGM08, create tools for unification of national height systems all over the globe. Can there be any further improvements? The answer is affirmative, since the geodetic community is expecting even more promising results from the dedicated gravimetric satellite missions. In this respect of particular interest is the first satellite gradiometry mission GOCE (Gravity field and steady-state Ocean Circulation Explorer) to be launched by the European Space Agency in 2009. This mission will allow reaching unprecedented accuracy for geopotential coefficients in the global scale and up to degree and order 270 (corresponding to the spatial resolution of 65 km). The GOCE will improve the intermediate wavelength information of the gravity field. However, the usage of the terrestrial data is still unavoidable for the proper recovery of the high-degree spectrum of the gravity field.

Intuitively, for the development of global high-resolution gravity models all the terrestrial data need to be referred to the modern gravity system, which is based on the absolute gravity measurements. Only such a global model will avoid drawbacks, which originate from the usage of different gravity datums. In other words, usage of such a global gravity system at the compilation of the geopotential models is a necessary precondition for the meaningful definition of the offsets among different vertical datums.

One of the main conclusions of the present study was that the EGM08 derived gravity quantities agree reasonably well with the terrestrial survey data in the Baltic Sea region. Apparently most of the historical terrestrial data have been utilised at the compilation of the EGM08.

Note however, that most of the data within the land masses of the Baltic countries have been collected before 1990-ies. Generally, the modern gravity networks were established decades after the historic gravity surveys. In mid 1990-ies a set of absolute gravity points was established in the Baltic countries. After publication of the absolute gravity measurement results (Mäkinen et al., 1996) the national gravity networks were re-adjusted, see e.g. Sas-Uhrynowski et al. (2002) and Oja (2008). Even though attempts were made to convert the historic survey results into the current gravity datum, the connections between the datasets remain still rather loose. More specifically, at areas the discrepancies

between different gravity data are not random at all. The following exercise is a clear example of this.

Most likely the gravity network points and the results of new surveys were not accessible at the compilation of the EGM08. For detecting the discrepancies between the absolute gravity datum, and the EGM08 derived gravity field the free-air anomalies were computed at the locations of the gravity points. Altogether 1957 new gravity points were available for the comparisons: 424 from Estonia, 1485 from Latvia and 48 from Lithuania.

The detected discrepancies between the newly measured and EGM08-derived gravity anomalies (terrestrial minus EGM08) vary from -10 to +10 mGal, with a mean of +0.14 mGal, see Fig. 10. The histogram of discrepancies is shown in Fig. 11. Such comparisons are also produced on a country-by-country basis. The corresponding statistics can be found in Table 1. In particular, the mean of the discrepancies as of +0.37, +0.08 and -0.31 mGal were achieved for the Estonian, Latvian and Lithuanian data-points, respectively. This shows also, that there are some systematic biases between the new terrestrial data and that of the EGM08. In Fig. 10 one may detect a few regions where the discrepancies are having the systematic nature. Clearly, for the further improvement of the global model accuracy the historic gravity surveys need to be revised by the national contact persons. The results should be made available for the future EGM developers. This is a quite burdensome task, requesting international and well coordinated actions. However, this is needed for the sake of the consistency of the global gravity data.

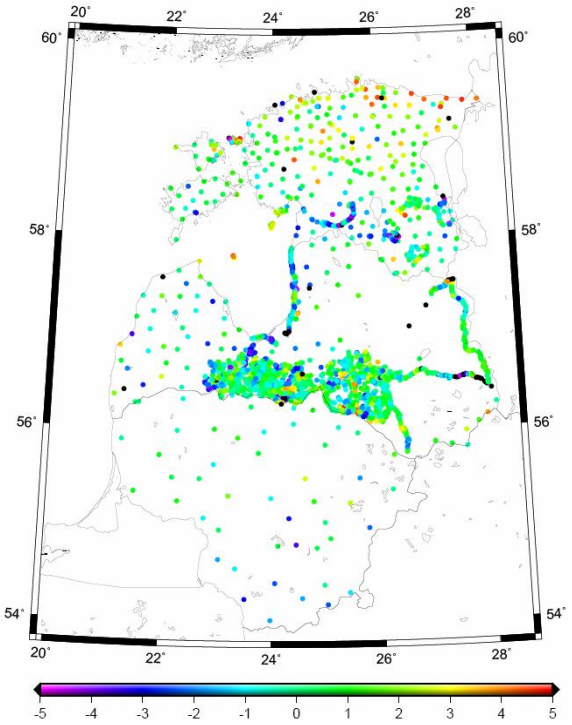


Fig. 10 The discrepancies between the gravity network and new survey points and EGM08-derived free-air gravity anomalies. The discrepancies [$\Delta g(r_t,\Omega)$ - $\Delta g_{EGM}(r_t,\Omega)$] range from -10 to +10 mGal, with a mean of 0.14 mGal. The STD of the discrepancies is 2.0 mGal. The colours of the dots are proportional to the range of the detected discrepancies, cf. the colourbar. The black dots denote the locations, where the absolute range of differences exceeds 5 mGal.

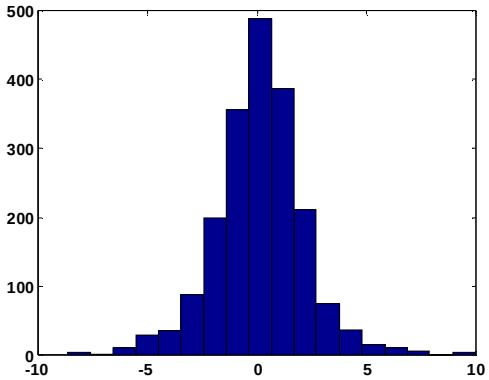


Fig. 11 Histogram of discrepancies between the gravity network and EGM08-derived free-air gravity anomalies. Unit is mGal. The total number of the points is 1957.

8 Summary and conclusions

The performance of the EGM08 model was validated over the three Baltic countries - Estonia, Latvia and Lithuania. Three different sets of the "ground truth" were employed for this task.

First, the EGM08-derived height anomalies were compared with the high-resolution BALTgeoid-04 model. A reasonable agreement between the two models was detected. In particular, within the borders of Estonia, Latvia and Lithuania the absolute ranges of the discrepancies do not exceed 15 cm. Larger discrepancies (but not exceeding $\pm$ 3 dm) are related to the areas where only a few data points were available for the BALTgeoid-04 modelling.

Second, the free-air gravity anomalies at the terrestrial data-points were compared with the EGM08-derived anomalies. This test yielded the STD of the discrepancies ~ 2.6 mGal, which is quite comparable with an average accuracy of the (historical) gravity surveys in the region of interest.

Finally, the quality of the EGM08 model was assessed with several sets of the GPS-levelling data. It is concluded, that the overall accuracy of the EGM08-derived height anomalies in the Baltic countries is almost of the same level as is the accura-

Table 1. The EGM08 evaluation results.

Type of the comparison	Unit	Statistics				
		# of points	Min	Max	Mean	STD*
BALTgeoid-04 minus z_{EGM}	[m]	250 x 172	-0.289	+ 0.338	**-0.025**	0.077
$\Delta g(r_t,\Omega)$- $\Delta g_{EGM}(r_t,\Omega)$						
Historical survey data	[mGal]	42559	-18.713	18.819	**-0.057**	2.599
Modern Baltic data	[mGal]	1957	-8.639	+9.953	**+0.136**	2.046
Modern Estonian data	[mGal]	424	-5.985	+6.446	**+0.375**	2.272
Modern Latvian data	[mGal]	1485	-8.639	+9.953	**+0.082**	1.989
Modern Lithuanian data	[mGal]	48	-3.566	+2.627	**-0.314**	1.388
GPS-levelling (Kronstadt vertical datum)						
Baltic N_{geom} -z_{EGM}	[m]	189	+0.346	+0.697	**+0.493**	0.060
Estonian N_{geom} -z_{EGM}	[m]	26	+0.432	+0.642	**+0.566**	0.048
Latvian N_{geom} - z_{EGM}	[m]	53	+0.346	+0.638	**+0.481**	0.063
Lithuanian N_{geom} -z_{EGM}	[m]	110	+0.375	+0.697	**+0.481**	0.048

*Note that the STD computations are based on the "raw" discrepancies, i.e. without applying any trend removal.

cy of the used control points, i.e. a few cm. Certainly, this conclusion cannot be extended automatically to other regions. Recall, that the Baltic Sea region is well studied gravimetrically, whereas most of the collected gravity data appear to be utilised at the EGM08 construction.

Clearly, the reliability and availability of the terrestrial gravity data (and their subsequent inclusion to the global gravity model) over the region of interest are necessary preconditions for using the EGM08 global geoid (and its follow-ups) for defining offsets among different vertical datums worldwide. Apparently, further EGM improvements could be possible by revising the existing terrestrial data worldwide and also by adding the medium wavelength GOCE information. As a result, the future global geopotential models could lead to unification of different height systems not only within a continent but also overseas.

An appropriate geopotential model is also essential to determine the regional gravimetric geoid model accurately. The present study results indicate that the EGM08 based regional models may possess a great potential in providing further refinement of the regional geoid models.

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Testing EGM2008 on Leveling Data from Scandinavia, adjacent Baltic areas, and Greenland

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Abstract. We tested EGM2008 on GPS/leveling data from Scandinavia and adjacent areas. EGM2008 performs at the same level as the best regional geoid model, NKG2004. However, the direct evaluation of EGM2008 is difficult in Greenland because no leveling data are available. Nevertheless, we show on 78 GPS-MSS data that EGM2008 also performs at the same level as the best regional geoid model GOCINA04.

Keywords. Geopotential models, EGM2008, leveling

1 Introduction

Prior to the official release of the new global geopotential model EGM2008 (Pavlis et al., 2008), complete to the degree and order 2160, the authors contributed to the "EGM2007 Evaluation Project" by testing the preliminary model PGM2007A on leveling data from northern Europe (Scandinavia, the Baltic countries and the adjacent areas around the Baltic Sea) as well as from Greenland. The final report "PGM2007A evaluation on GPS-leveling data in Greenland and Scandinavia and adjacent areas" was submitted to the Joint IGFS/IAG Comission-2 Working Group and included as a "feed back"-contribution to improve the global model. The present work is a repetition of this exercise for the newly released EGM2008.

2 Scandinavia and adjacent areas

Scandinavia and adjacent Baltic areas is a sector covering Norway, Sweden, Finland, Denmark, Estonia, Latvia, Lithuania and a section of Poland. The sector is bounded by the following parallels and meridians: 53°N - 73°N and 1°E - 33°E.

The GPS/leveling data from Scandinavia and adjacent Baltic areas used for the evaluation of the PGM2007A model are the same as those used in the past for the evaluation of the regional Nordic geoid models. In fact, these formerly used GPS/leveling data sets were enhanced by the inclusion of additional leveling data from Norway, Sweden and Finland. Basically, the new data are more consistent (and recent) with respect to the epoch of the GPS campaigns and leveling. Because of the substantial land uplift in the area, this consistency is important.

The leveling data from the different countries consist usually of heights in their national height system, which can differ from country to country (orthometric heights, normal heights), but also in the way they are linked to the Mean Sea Level via a tide gauge (a vertical datum). In assessing the quality of the Nordic geoid model these national data sets are usually used separately (i.e. country wise). The main parameter for the "goodness-of-fit" in a given country is the standard deviation of the misfit between the gravimetric geoid/quasi-geoid model and the "geoid" derived from GPS and leveling measurements: N= h - H.

For the purpose of the regional comparison the GPS/leveling data from individual countries were corrected for relative vertical offset with respect to each other. The biases used were the assessed offset of each countries height datum with respect to the common European Vertical Datum.

We compared the GPS/leveling data to the quasi-geoid models in Table 1.:

Table 1. The quasi-geoid models used in the comparison.

Model	comments
EGM96	(Lemoine et al., 1996)
EGM96+GRACE2S	
NKG96	1996 regional geoid model for the Baltic and Nordic area
NKG2004	2004 regional geoid model for the Baltic and Nordic area
PGM2007A Zero Tide	Preliminary model released by NGA in 2007
EGM2008 Zero Tide	(Pavlis et al., 2008)

The statistics of the comparison for the whole area are shown in Table 2.

Table 2. Scandinavia and adjacent Baltic areas. Statistics of the comparison of GPS/leveling to different quasi-geoid models. (Unit: m).

model	Mean	std. dev.	rms	min	max
EGM96	0.38	0.26	0.46	-0.51	1.80
EGM96 + GRACE2S	0.35	0.20	0.41	-0.66	1.18
NKG96	0.01	0.14	0.14	-0.61	0.51
NKG2004	0.03	0.11	0.11	-0.48	0.39
PGM2007A ZeroTide	-0.59	0.11	0.60	-0.96	-0.18
EGM2008 ZeroTide	-0.55	0.11	0.56	-0.91	-0.13

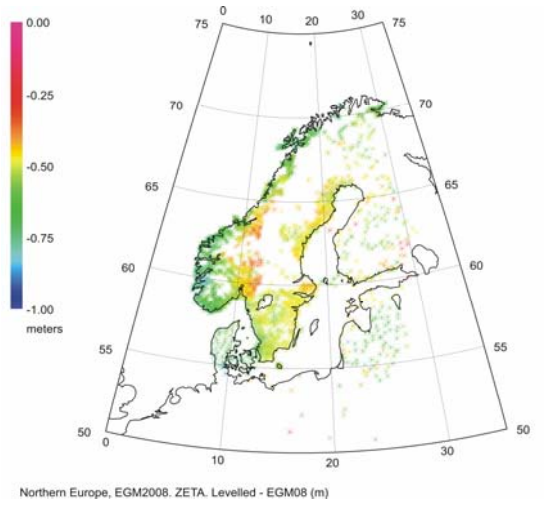


Fig. 1 $\varepsilon=(h-H^{lev})-N_{EGM2008}$ Comparison of the geoid heights from the joint GPS/leveling data set (3144 points) for Scandinavia and adjacent Baltic areas to the EGM2008 model.

From Table 2 and Fig. 1 we conclude, that EGM2008 model is excellent. It is at the same standard as the latest regional gravity model NKG2004. For the joint GPS/leveling data set (see above), this comparison is somehow misleading. On Figure 1, the large misfit in Norway is most certainly caused by problems related to leveling. A problem is the inconsistency between the fixed epoch of the leveling and the fact that, in practice, the leveling was done over many years in the presence of a substantial land uplift caused by the post glacial rebound (Dr. Ove Omang, Norwegian Mapping Authority, personal communication). This leveling and height system problem for Norway is discussed in some details by Lysaker et al. (2007). One can notice that the large misfits to EGM2008 in Norway on Fig. 1 co-locate with the pattern on Fig. 4 in (Lysaker et al., 2007) showing the vertical land uplift velocities. This indicates, as stated, that the misfit in Norway is not a problem with the gravimetric model, but (most probably) with the GPS/leveling. The GPS/leveling data for Finland and Sweden were corrected for the land uplift to a common epoch.

A similar comparison for the individual countries (standard deviation) and for four of the models is shown in Table 3.

Table 3. Scandinavia and its adjacent areas. Standard deviation of the misfit between the best available national GPS/leveling data sets and different quasi-geoid models. (Unit: m).

	N	EGM 96	NKG 2004	PGM 07A Zero Tide	EGM 2008 Zero Tide
Denmark	85	0.35	0.06	0.07	0.07
Estonia	31	0.25	0.07	0.06	0.07
Finland	154	0.14	0.04	0.07	0.07
Latvia	36	0.16	0.10	0.09	0.09
Lithuania	32	0.17	0.08	0.08	0.09
Norway	1693	0.31	0.12	0.12	0.11
Poland	6	0.23	0.05	0.11	0.11
Sweden	910	0.15	0.04	0.03	0.03

The conclusion from this comparison is that EGM2008 is an excellent model for Scandinavia and its adjacent areas. Its accuracy is similar to the current best regional geoid model (NKG2004).

From other reports, the new geopotential model EGM2008, performs also very well in other parts of the world when compared to GPS/leveling data. It renewed the discussion about the need for defining a global vertical datum. For this reason, it is of some interest to list the mean values of the deviation of the national geoid heights of the GPS/leveling data and the EGM2008 model. These values are listed in Table 4.

Table 4 Mean deviation of the best available national GPS/leveling data from EGM2008 quasi-geoid.

	Mean($N_{lev} - N_{EGM2008}$) (m)
Denmark	-0.63
Estonia	-0.66
Finland	-0.51
Latvia	-0.60
Lithuania	-0.60
Norway	-0.57
Poland	-0.53
Sweden	-0.49

It should be emphasized, that the expression "the best available" GPS/leveling data does not mean "the best existing". The national mapping authorities in the involved countries have access to much better and denser GPS/leveling data, which also include a more thorough treatment of, especially, the regional land uplift, as well as a more consistent treatment of the difference in epochs between the GPS- and leveling campaigns. The GPS/leveling data used here are those used in the past under the auspices of the Nordic Geodetic Commission to model and to evaluate the regional geoid models (e.g. NKG96, NKG2004). In other words, there is much more to say about the quality of EGM2008 compared to the "best existing" national GPS/leveling data; especially in the neighboring countries like Poland, where the available data set is most probably not representative at all. In this context, the vertical offsets listed in Table 4 are only rough numbers and do not in any way claim to be an attempt to model the accurate vertical offsets between the national height systems and the global vertical datum.

In this report we are only trying to assess: How does EGM2008 perform compared to the best regional geoid models? The answer to this question is that, for Scandinavia and adjacent Baltic areas, the fit of the new global model is at the same level as the best regional gravimetric geoid model (NKG2004).

2 Greenland

No leveling data exist between settlements in Greenland. A GPS-mean sea-level height data set is selected from recent GPS campaigns. GPS points are all tied to ITRF, mainly through the REFGR Greenland fundamental GPS network. The height above mean sea level (MSL) of the GPS points are mostly based on older MSL determinations, usually from the 1960's, or more recent, often shorter-duration MSL and relative tide gauge campaigns. The GPS ellipsoidal heights and MSL-heights can be used for geoid validation through the following relation:

$$N^{GPS} = h^{GPS} - H_{MSL} - MDT\,, \qquad (1)$$

where MDT is the mean dynamic topography.

We obtained the MDT from the OCCAM oceanographic model. This model does not include the local effects of fjords etc., so "GPS/leveling" data of this type might have significant errors due to local MDT effects, land uplift, and other errors. Fig. 2 shows the locations of the GPS-MSS data.

For the evaluation of PGM2007A model the tide-free model was used. However, since the physical Earth is permanently deformed by tidal forces we are now using the zero tide model for the evaluation of EGM2008. Table 5 shows the effect of the MDT corrections for PGM2007A model. It is seen, that the mean value of the differences is reduced, whereas the standard deviation is only reduced slightly. This indicates that the used MDT model is not adequate for this local application.

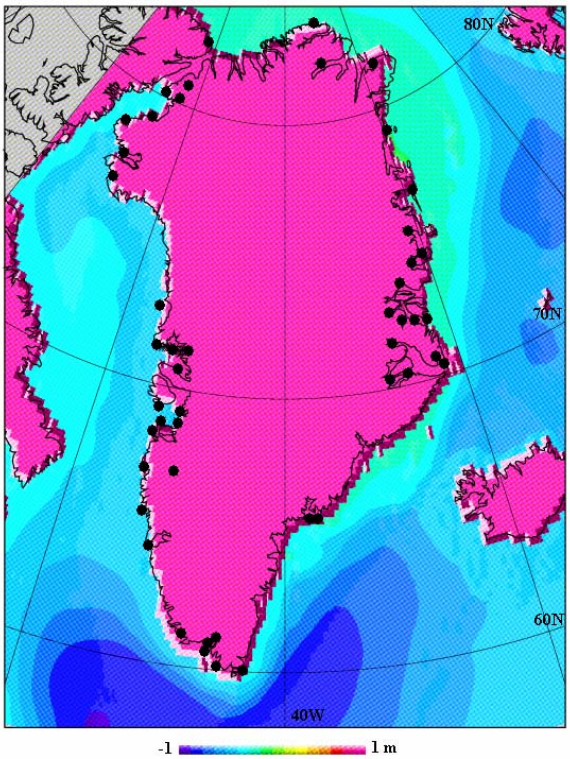


Figure 2. Location of Greenland GPS stations (black dots) and MDT from OCCAM model (color contour)

Table 5 also shows the comparison of the GPS-MSS data (reduced for MDT) to other geoid models: EGM96, GEOID96A and GOCINA04. The latter two are local gravimetric geoid models, derived by somewhat different methods. GEOID96A uses EGM96 as reference, and utilizes least squares collocation to merge high-elevation airborne gravity data (Brozena et al, 1993) and surface data in the ice-free coastal regions; the GOCINA04 model uses a JPL GRACE model as reference, and uses only surface gravity data in the coastal region. Both models use ice thickness information and terrain reductions, and are based on spherical FFT methods.

Table 5. Comparison of EGM2008 and PGM2007A and other models to GPS-MSS data in Greenland (78 points).
MDT – corrected for mean dynamic topography;
no MDT – not corrected for mean dynamic topography.

Unit: m	Mean	Std.dev.	Min	Max
N =GPS-MSS-MDT	35.04	12.66	8.94	57.01
EGM2008, MDT	-0.19	0.40	-0.43	1.60
PGM2007A, no MDT	-0.63	0.45	-1.66	1.08
PGM2007A, MDT	-0.26	0.43	-1.20	1.45
EGM96, MDT	0.71	0.52	-0.52	2.62
GEOID96A, MDT	-1.09	0.83	-2.50	1.76
GOCINA04, MDT	-0.16	0.37	-0.98	1.44

Results in Table 5 indicate that PGM2007A and, especially, EGM2008 perform nearly at the same level as the "best" local geoid model – GOCINA04. The difference might be caused by the different weighting used for the high-altitude airborne gravity data, which might have long-wavelength errors not consistent with GRACE. However, other avenues for systematic errors are the use of terrain and ice reductions since large parts of the ice sheet do not have sufficiently accurate radar echo sounding depth data, especially near the margins of the ice sheet. It is also seen that the earlier geoid models have much larger errors, likely due to the absence of GRACE data. This is as expected.

Conclusions

For Scandinavia and adjacent Baltic areas the new global model EGM2008 is a net improvement over the latest global reference model EGM96 and almost as good as (and sometimes better than) the latest regional quasi-geoid model NKG2004. For Greenland, EGM2008 performs with an accuracy which is comparable to the best local geoid model GOCINA04.

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Testing EGM08 using Czech GPS/leveling data

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Abstract. The new Earth Gravitational Model 2008 (EGM08) was tested using a set of 1024 high-accuracy GPS/leveling stations in the Czech Republic. Values of height anomalies synthesized from the new model were compared with those derived by the combination of leveled Molodensky-type normal heights and GPS-based geodetic (ellipsoidal) heights. The standard deviation computed from the obtained differences was equal to ±3.3 cm. For comparison purposes, the previous model EGM96 has been tested with the same data set. Compared to this model, the new gravitational model fits five times better to the GPS/leveling height anomalies.

Key words : Earth Gravitational Model · EGM08 · leveling · GPS · Czech Republic

1 Introduction

In this report, we describe a test procedure performed in order to verify the quality of the latest Earth Gravitational Model 2008 (EGM08) released in April 2008 (Pavlis et al. 2008) using independent GPS and leveling data on territory of the Czech Republic. The test was conducted in the frame of a joint working group of the International Association of Geodesy (IAG) and the International Gravity Field Service (IGFS) that was established in 2007 in order to test the new model by using various independent data available to members of the working group.

As a test area, the territory of the Czech Republic, a landlocked country in Central Europe, was used. As test data, 1024 discrete values of the height anomaly (ellipsoidal surface normal deviations of the quasi-geoid form the international reference ellipsoid) were utilized. The report describes the origin, distribution and quality of the reference values (Section 2) and provides results in terms of interpolated 2D surfaces based on residual differences as well as their basic statistical values (Section 3). To prove the tremendous improvement the new model represents over the test territory, results based on the previous EGM solution, EGM96 (Lemoine et a. 1998), are also shown.

2 GPS/leveling data

1024 GPS/leveling stations were used to validate EGM08 over the territory of the Czech Republic. These GPS/leveling stations provided a nice sample of independent reference data, although covering only a relatively small region in the Central Europe.

The stations have a known value of the Molodensky-normal height estimated through very precise leveling from closest points of the national leveling network. A length of leveled traverses did not exceed the distance of 5 km. Since the error of precise leveling is ± 1.7 mm/km, the total error can be computed through the well known formula $\pm 1.7 \times \sqrt{5} \doteq \pm 4$ mm (Hrabě and Beneš 1997). Thus, the estimated rms error of the leveled heights of the stations is estimated to be at the order of ± 5 mm. If digital leveling instruments would be used, errors might remain below ± 1 mm over the 5 km distance.

GPS heights of the stations were estimated through static 8-hour GPS observation campaigns with an average length of baselines between the stations at the level of 10 km. Their accuracy estimated from the adjustment is at the order of ± 1 cm. Combining the errors of the leveled and GPS heights, the accuracy of height anomalies estimated at the GPS/leveling stations can approximately be characterized by the rms error of ± 2 cm. Although small in the geographical coverage, the stations have the advantage of being homogeneous with respect to one height system used in the Czech Republic.

The leveling network of the Czech Republic is based on networks of several orders measured in due time. Stability and possible vertical movements of the leveling benchmarks were investigated through repeated leveling campaigns. Namely heights of the first-order vertical network were re-estimated in the recent years. Results of repeated leveling campaigns showed a small tilt in the NW-SE direction, see Figure 1 (Zeman 2001), where detected vertical changes over the period of the last 33 years are shown. The range of the tilt is from $+5$ to -3 cm at the boundaries of the country. The estimated annual vertical motion of the leveling network is then shown in Figure 2 (Zeman et al. 2007). The rate of change is within -1 to

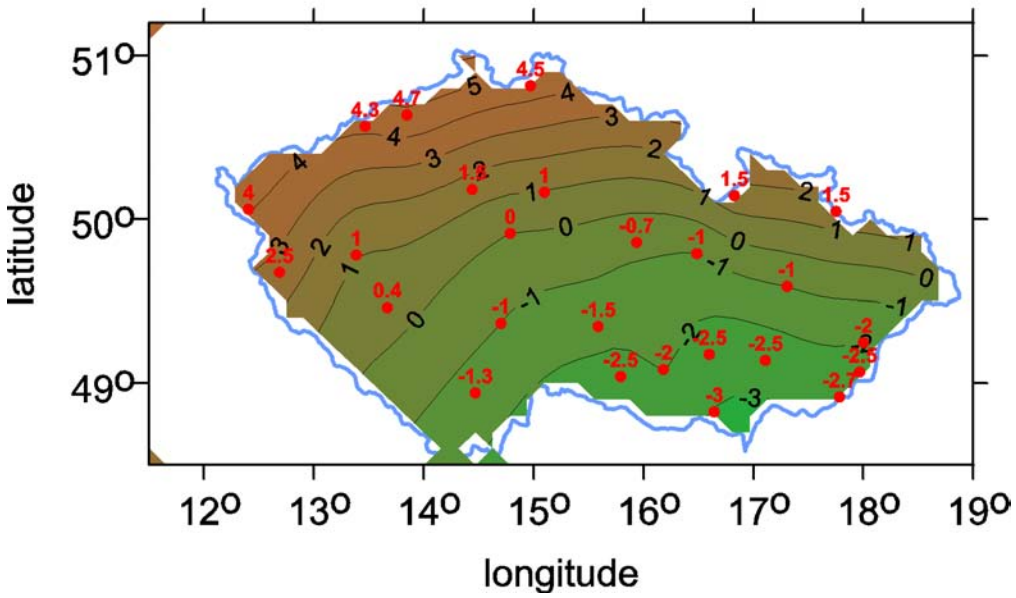


Figure 1: Vertical changes in the Czech leveling network over last three decades (cm)

+5 mm per year depending on the location. The largest changes are along the western boundary of the country. These figures are very important for the analysis of possible trends in the estimated differences between the tested and reference values in our test: trends, that have a different shape or magnitude, cannot be related to the tilt of the leveling network. In contrary, they can be attributed to observation errors in leveling and/or eventual errors in the EGMs.

Besides the accuracy of GPS/leveling values, it is also the size of the data area that is relevant to the testing procedure. In order to detect eventual deficiencies in the entire spectrum of the EGM, the test data should continuously cover the entire Earth's surface. EGM08 is limited to maximum degree and order of the spherical harmonic expansion at the level of 2160 that correspond to the angular resolution of 5 arcmin × 5 arcmin (9 km × 9 km $\cos\phi$ at the Earth's surface, ϕ is latitude). Our test area covers the territory of the Czech Republic, i.e., the area of some 90,000 km^2 that indicates limitations of the test data. Long-wavelength or low-frequency errors in the model cannot be detected through our test data. However, it is their high quality and bias-free characteristics that may provide some insight into the performance of EGM08 in the geographically limited test area. The quality of EGM08 is judged by goodness-of-fit between the EGM08 quasi-geoid and its geometric counterpart estimated through combination of GPS-based geodetic (ellipsoidal) and leveled Molodensky-normal heights.

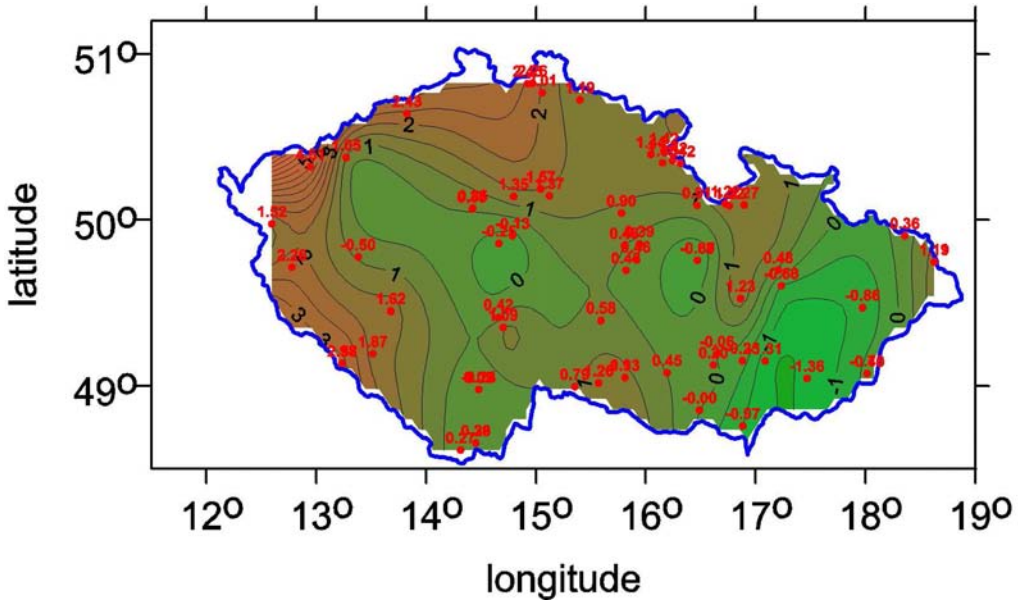


Figure 2: Estimated annual vertical motion over the territory of the Czech Republic (mm/y)

3 Results

Point values of the height anomaly obtained through combination of GPS/leveling heights at the 1024 GPS/leveling stations were compared against their respective values synthesized from EGM08. The estimated differences, reduced for their mean value due to different values of W_0 used in definition of the EGM and local vertical datum (so-called "Baltic System" used in Eastern Europe since 1960's), are tabulated along with their respective standard deviations and other statistical parameters. To show the improvement the new EGM represents, differences for EGM96 are also presented in Table 1.

EGM08 has much higher spatial resolution than EGM96: 5 arcmin for EGM08 vs. 30 arcmin for EGM96. Looking at the statistical values in Table 1, namely at the values of the standard deviation, it is obvious that EGM08 outperforms the older EGM96 also in terms of accuracy. The standard deviation for EGM08 is at the level of ±3.3 cm compared to ±16.8 cm for EGM96, i.e., one could conclude EGM08 is approximately five times more accurate than EGM96. However, this conclusion is valid only for the specific region and the used data. We may recall a test performed by the authors of EGM08 (Pavlis et al. 2008) using approximately 12 thousand GPS/leveling points distributed globally. Values of the standard deviation computed from the obtained differences were ±13 cm for EGM08 and ±30 cm for EGM96, i.e., the improvement globally seemed to be less significant. The much better result of this test is at least partially due to having the test data related only to one vertical datum.

model	minimum	maximum	standard deviation
EGM08	− 0.089	+ 0.094	± 0.033
EGM96	− 0.517	+ 0.505	± 0.168

Table 1: Statistics of estimated differences at 1024 GPS/leveling stations (m)

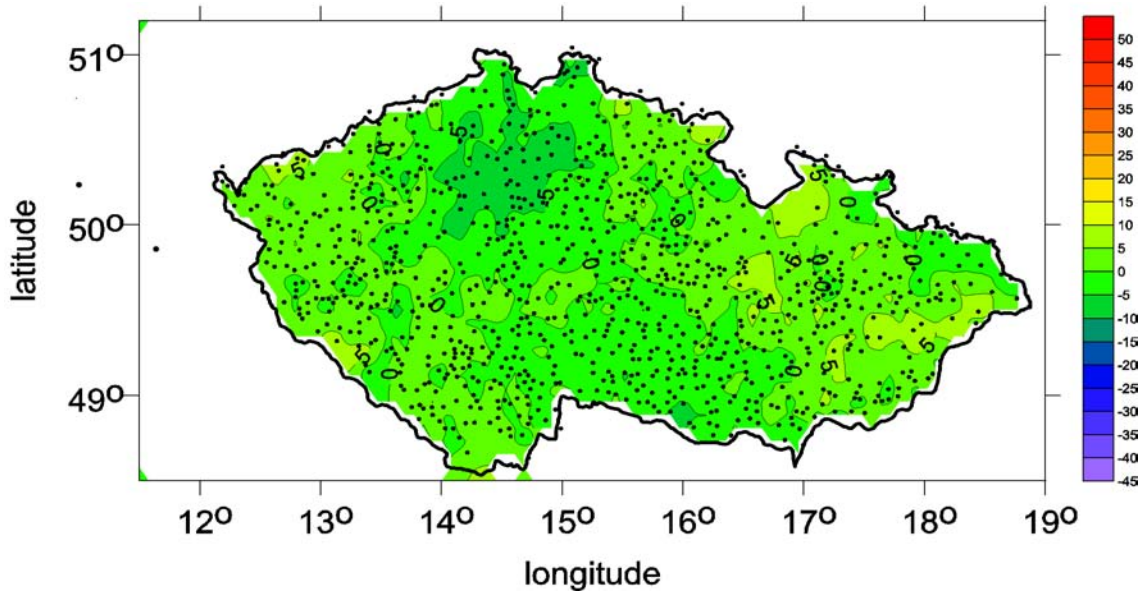


Figure 3: EGM08 vs. GPS/leveling height anomalies (cm)

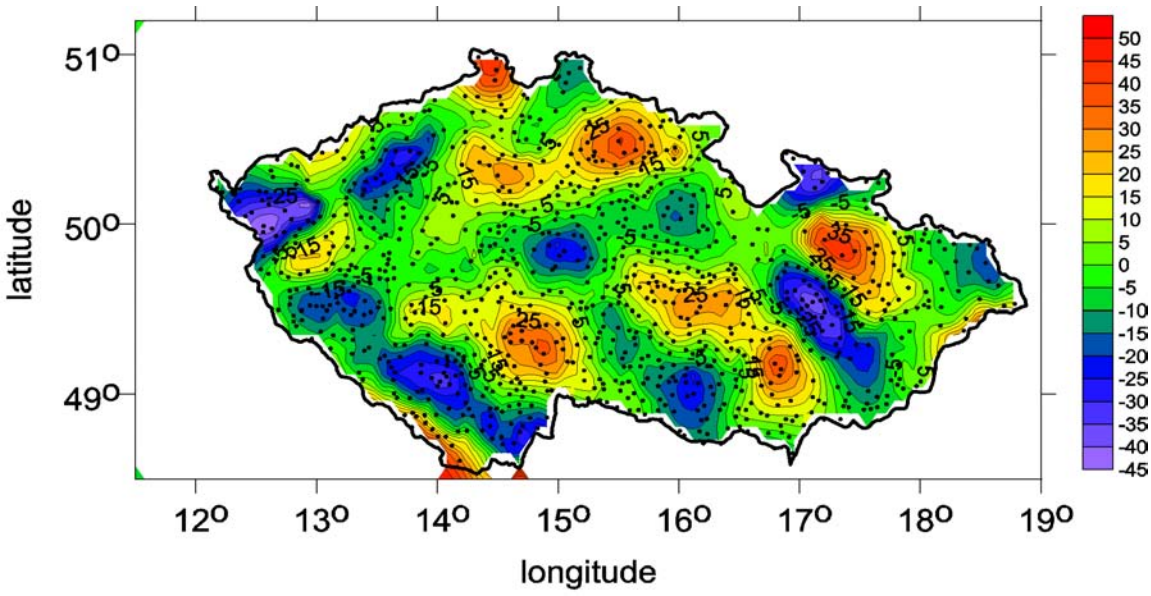


Figure 4: EGM96 vs. GPS/leveling height anomalies (cm)

The fit of the new EGM at the Czech GPS/leveling stations is as good as for a recent high-resolution local quasi-geoid model of the Czech Republic (Novák et al. 2003). Taking into the account the accuracy of leveled and GPS heights discussed in Section 2, one could argue the other way around: the new EGM can be used to test the quality of the national leveling network. Besides the next step in creating a truly global elevation system, this can be yet another important role of the new model: detection of possible long-wavelength (systematic) errors in existing leveling networks (removing vertical movements).

Figure 3 then shows values of the estimated differences for EGM08 and Figure 4 depicts the respective differences for EGM96. Dots represent locations of the GPS/leveling points. Both figures show using the same range of extreme values (those of EGM96) distinct patterns in the computed differences that differ in both geographical distribution and magnitude. Comparing these surfaces with that in Figure 2, there is not much of resemblance (the NW-SE tilt is not visible in the computed differences). The magnitude of the computed differences in Figures 3 and 4 exceeds the size of the estimated vertical changes of the national leveling network in Figure 2, namely those of EGM96. Some of the features in Figures 3 and 4 can at least partially be attributed to systematic observation errors in the national leveling network.

4 Conclusions

Despite the relatively small test area, EGM08 proved to be a significant progress in both accuracy and resolution relatively to EGM96. The new model outperformed EGM96 five times in terms of both characteristics. Moreover, EGM08 reached the accuracy level that was previously achievable only through the combination of global gravitational models and local ground gravity observations. The average fit of the EGM08 quasi-geoid and that computed from the combination of leveled and GPS-based geodetic heights at 1024 points reached the level of ± 3.3 cm. Since this misfit can at least partially be attributed to the observation errors in leveling and GPS positioning, we seem to get closer again – at least regionally – to a long-term target of all geodesists: a one-centimetre geoid model.

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Testing EGM2008 in the Central Mediterranean area

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Abstract

The last geopotential model EGM2008 has been compared with gravity and GPS/leveling data over the Central Mediterranean area. In this area, the gravity field has sharp variations due to strong topography/bathymetry signals and to relevant geophysical features. More than 300.000 gravity values covering the whole area have been considered in the comparisons. These data were used in the last Italian geoid computation and have been validated for datum consistency and for outliers. Further comparisons have been carried out using GPS/leveling data which are homogeneously distributed over the Italian Peninsula. About 1000 available values have been used in the comparison. The results show that, in this area, EGM2008 fits gravity and GPS/leveling data better than other existing global models. The geopotential model performances have been also compared with those of the last Italian geoid estimate, the ITALGEO05 geoid which has been computed in 2005. The statistics of the residuals with GPS/leveling data show that EGM2008 and ITALGEO05 have comparable accuracies, even though EGM2008 is slightly better.

1. Introduction

One feasible method for assessing the accuracy of the global geopotential models is through comparisons with point-wise measured values of functionals of the anomalous potential T(P). Usually, this is done using gravity anomalies at ground level and geoid undulations as derived by e.g. GPS and spirit leveling. Such comparisons are performed over areas with different features of the Earth gravity field in order to test for the global geopotential model performances under different conditions. There are regions where the geopotential field is strongly varying due to topography/bathymetry roughness and/or geophysical features. Tests carried out in these regions are particularly valuable since they prove how such strong variations can be recovered by the geopotential models.

One of these areas is the Central Mediterranean which, in our comparisons, will be considered as the region inside the boundaries: $35° \leq \varphi \leq 48°$, $5° \leq \lambda \leq 20°$. The Alps are on the northern edge of this region and the Apennines crosses it along one of its diagonals. There are shallow water areas in the Adriatic Sea and deep water seas such as the Ionian Sea. Moreover, hilly regions are close to deep waters as it is, e.g., in the southern part of Italy. This gives rise to a highly variable gravity field which is further perturbed by strong geophysical signals. Two examples of such signals are the Calabrian Arc structure along the eastern part of Calabria and the Ivrea body in the western Alps. Thus, over such a relatively small portion of the Earth, relevant changes in the gravity field can be found.

Furthermore, in this area, which has been widely surveyed, quite large ground gravity and GPS/leveling data sets are available. Also, a local geoid estimate has been recently computed in this region, the ITALGEO05 geoid.

These data sets and the ITALGEO05 geoid estimate have been used for checking the EGM2008 accuracy over this area where, as mentioned before, strong gravity field variations are present.

2. Gravity, GPS/leveling and DTM data over the Central Mediterranean area

The gravity data base used in this test has been compiled using different data sources. Most of the data have been supplied by *Servizio Geologico Nazionale* of Italy. Marine gravity comes from surveys which were performed by OGS (Morelli et al., 1975) while data outside the Italian borders are from BGI. In the whole, 310.660 point gravity values were collected in the window $35° \leq \varphi \leq 48°$, $5° \leq \lambda \leq 20°$. In Figure 1, this gravity data base is shown.

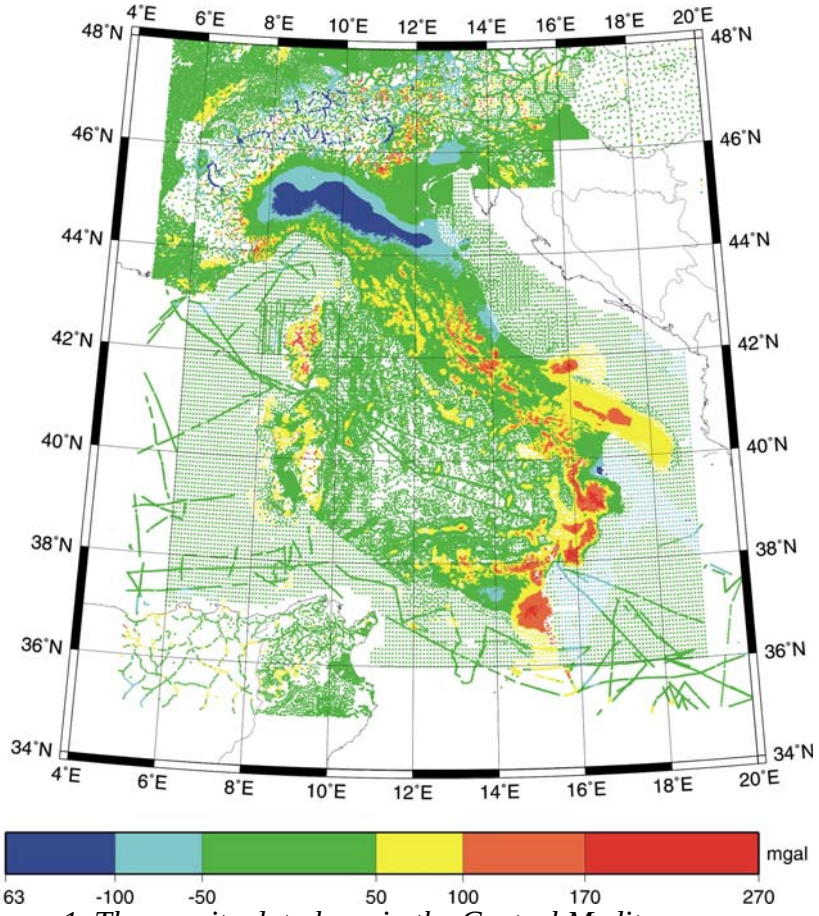


Figure 1. The gravity data base in the Central Mediterranean area.

The range of free-air anomalies is quite large, reflecting the complex structure of the gravity field of the area. As one can see, data are unevenly distributed. The Po plain is densely surveyed while the Alpine region is poorly covered. This reflects the fact that these data are basically ground gravity data which are easier to collect in flat topography areas. Marine gravity is known mostly in grid form even though some ship tack data have been included. Furthermore, it must be underlined that there are areas with no data available over the former Yugoslavia.

GPS/leveling data have been supplied by *Istituto Geografico Militare* (IGM) of Italy. The data base consists of 977 values distributed over the continental part of Italy. These are points belonging to the IGM95 GPS network which was measured in 2003-2004 by IGM (Surace, 1997). The ellipsoidal heights are framed to ETRF89. The leveling lines giving, on the same points, the orthometric heights have been surveyed by IGM over a large time

span. Thus, relevant discrepancies between GPS/leveling undulations and other geoid estimates can occur due to that.
In the context of Italian geoid estimate, both gravity and GPS/leveling data base have been carefully checked.
Gravity data co-ordinate have been referred to WGS84 and observed gravity has been reduced to IGSN71. These data were also checked for outliers following an approach based on a comparison between observed and collocation predicted values.
Check for outliers in GPS/leveling undulations has been performed via comparison with the last estimate of the Italian geoid.
The GPS/leveling data base used in the following comparisons is shown in Figure 2.

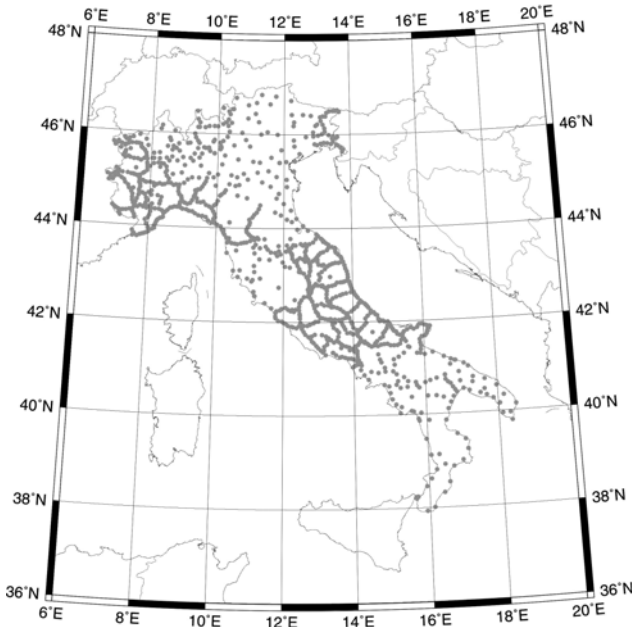


Figure 2. The GPS/leveling data base over the continental part of Italy

DTM has been also used in the computations. This DTM has been prepared for the Italian geoid computation merging different data bases:

- the SRTM3 DEM as a reference elevation model;
- the Italian DTM to fill the SRTM3 gaps over the Italian land region and for a strip of bathymetry near the coasts, where good resolution digitalised bathymetry is available;
- the new 1’ x 1’ NOAA bathymetry in deep seas (https://128.160.23.42/dbdbv/dbvquery.html);
- the GTOPO30 DTM where no other data were available (http://edc.usgs.gov/products/elevation/gtopo30/gtopo30.html).

The area on which this DTM has been assembled is $33° \leq \varphi \leq 50°$, $3° \leq \lambda \leq 22°$. The resulting grid has a regular geographical mesh of 3’’× 3’’ (Borghi et al., 2007).

3. Comparing EGM2008 and other existing geopotential models with gravity data

The EGM2008 geopotential model has been used to reduce gravity values belonging to the described data base. It has been firstly compared to GPM98CR (Wenzel, 1998) which is the model that better fits gravity over this test area (Barzaghi et al., 2008). As it is well known, this is a global geopotential model complete to degree and order 720. The statistics of the data and those of the residuals obtained using the two models are listed in Table 1.

	Δg_{fa} [mGal]	$\Delta g_{fa} - \Delta g_{M_EGM2008}$ [mGal]	$\Delta g_{fa} - \Delta g_{M_GPM98CR}$ [mGal]
#	310660	310660	310660
E	11.52	-5.22	-6.58
σ	63.93	18.38	23.99
Min	-162.55	-243.34	-228.65
Max	269.71	119.49	168.01

Table 1 – Gravity and residuals statistics (after model reduction)

The EGM2008 is remarkably better than GMP98CR model. There are still high residuals that are probably un-removed outliers. However, most of the residuals are contained in the [-50 mgal; 50 mgal] interval, as it can be seen in Figure 2.

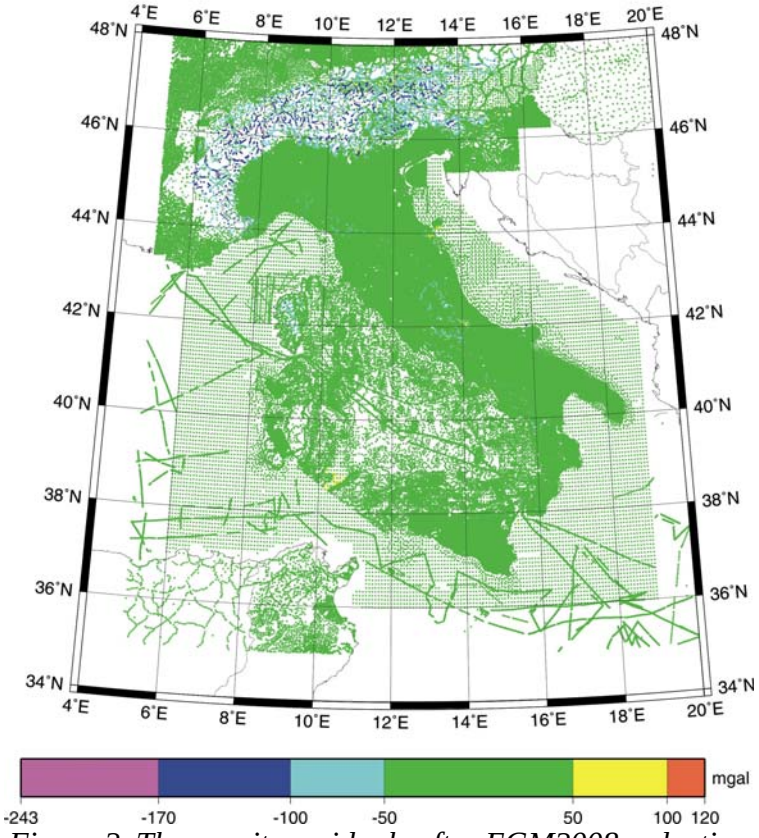


Figure 2. The gravity residuals after EGM2008 reduction

A further comparison has been carried out on the residuals after RTC reduction. RTC effect has been computed using the TC software of the GRAVSOFT package (Tscherning et al., 1994). The DTM described in the previous paragraph has been used as detailed terrain model. The mean DTM has been obtained from this detailed DTM by applying different moving averages. The cap size of the moving average used to get the mean DTM is 10' for the GPM98CR. The one used for the EGM2008 is 3'. They have been selected testing different cap amplitudes and looking for optimal residuals statistics (i.e. minimum mean and standard deviation). As expected, the EGM2008 is associated to a moving average operator that smooth less than the one related to GPM98CR.
The outcomes of this computation are presented in Table 2.

	Δg_{fa} [mGal]	$\Delta g_{fa} - \Delta g_{M_EGM2008} - \Delta g_{rtc}$ [mGal]	$\Delta g_{fa} - \Delta g_{M_GPM98CR} - \Delta g_{rtc}$ [mGal]
#	310660	310660	310660
E	11.52	-0.938	-1.14
σ	63.93	7.884	10.69
Min	-162.55	-287.745	-274.55
Max	269.71	117.257	106.64

Table 2 – Gravity and residuals statistics (after model and RTC reduction)

Apart from a small amount of anomalous values, the final residuals have a negligible mean and a standard deviation which is sharply reduced as compared to the initial value.
Also in this case, the EGM2008 improved the results obtained with GPM98CR. In Figure 3, the plot of the residuals after EGM2008 and RTC reduction is shown. Obviously, the RTC reduction has the strongest impact over the Alps (compare Figure 2 and Figure 3).

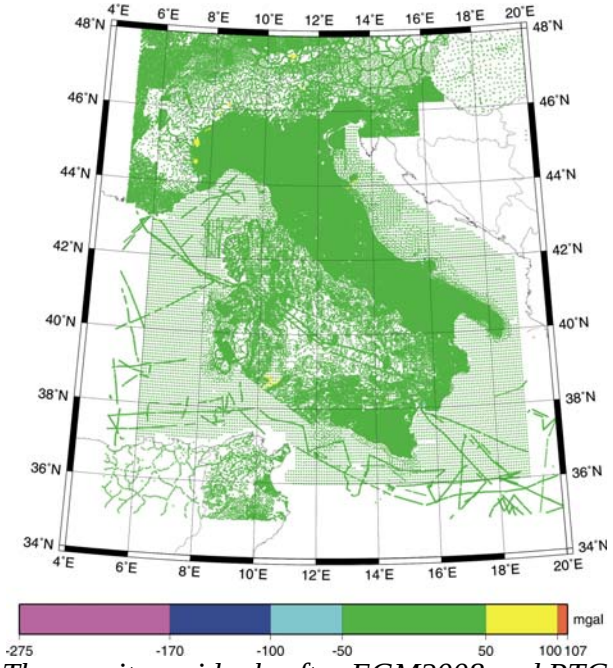


Figure 3. The gravity residuals after EGM2008 and RTC reduction

It is also interesting to compute the empirical covariance functions of the residuals which give an idea of their spatial correlation.
The empirical covariances of gravity after model reduction are plotted in Figure 4 while in Figure 5 those of the residuals after model and RTC reduction are plotted.

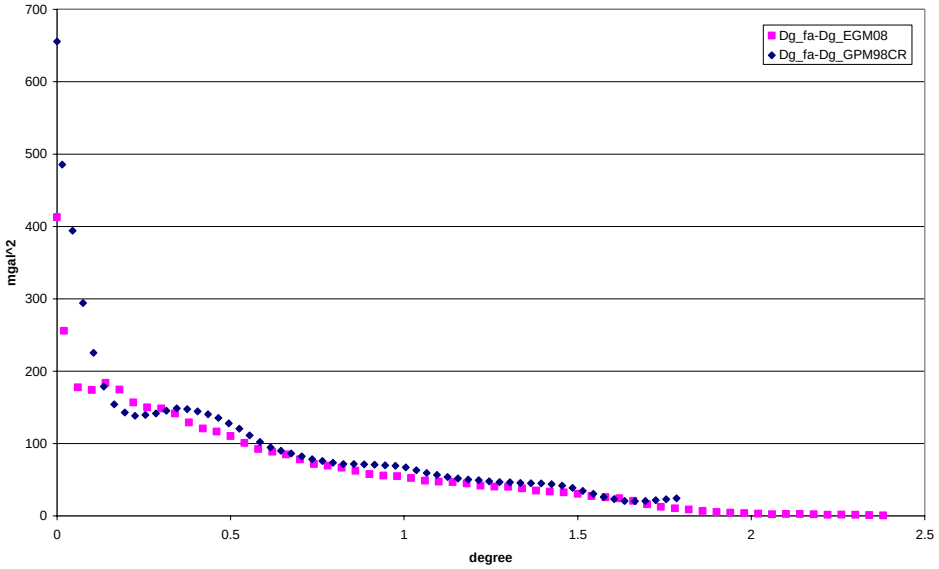


Figure 4. The empirical covariance function of gravity residuals after model reduction

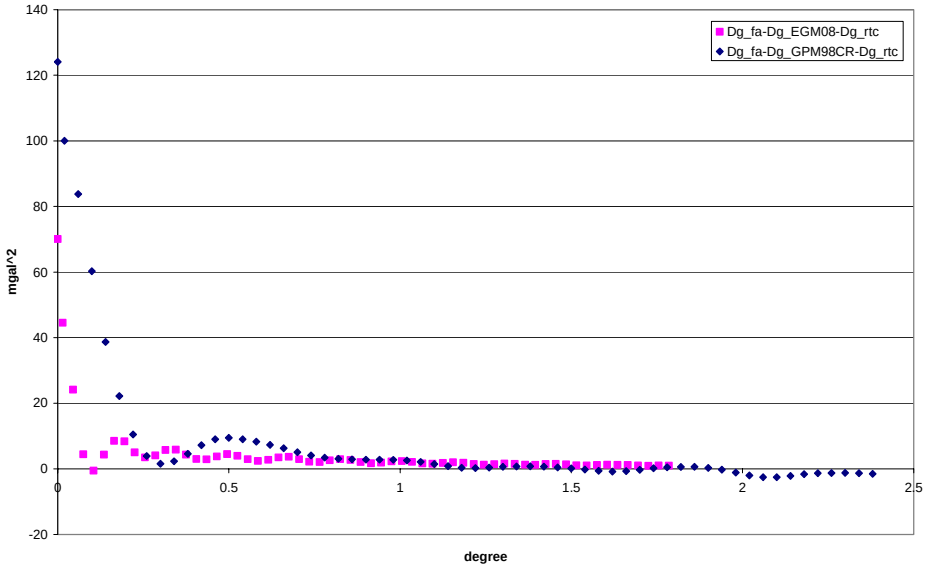


Figure 5. The empirical covariance function of gravity residuals after model and RTC reduction

Covariances after model reduction only are quite similar both for EGM2008 and GPM98CR, but for their values in the origin.
Covariances after model and RTC reduction seems to be more different in their behavior close to the origin. Particularly, the empirical covariance of the residuals obtained using the EGM2008 model displays a very short correlation length which can be hardly fitted using the standard model covariance functions.

Finally, we compared EGM2008 with other two global geopotential models, EIGEN-GL04C and EGM96 which are complete to degree and order 360. This test has been performed on a reduced gravity data set obtained by selecting the observed gravity points closer to the centers of a 1'×1' grid. Again, one can see that the EGM2008 sharply improves the results obtained with previous global geopotential models (see Table 3).

	$\Delta g_{fa} - \Delta g_{M_EGM2008_2190}$ [mGal]	$\Delta g_{fa} - \Delta g_{M_GL04C_360}$ [mGal]	$\Delta g_{fa} - \Delta g_{M_EGM96_360}$ [mGal]
#	142196	142196	142196
E	-5.411	-7.334	-6.415
σ	20.321	32.244	31.135
Min	-241.556	-255.889	-253.978
Max	119.492	194.808	188.235

Table 3 – Residuals statistics using different global geopotential models

4. Fitting GPS/leveling data with EGM2008 and ITALGEO05

The ITALGEO05 is the last Italian geoid estimate (Barzaghi et al., 2008). It is based on a gravity data base which is closely related to the one used for checking the EGM2008 model.

The estimation procedure is the classical "remove-restore" technique (Tscherning, 1994) and the residual geoid component has been estimated using the Fast Collocation approach (Bottoni and Barzaghi, 1993).

The reference global geopotential model adopted in the computation is the GPM98CR, complete up to degree and order 720, while the RTC effect has been estimated using the DTM described in paragraph 2.

The final estimate is given on a regular grid in the area $35° \leq \varphi \leq 48°$, $5° \leq \lambda \leq 20°$ with grid spacing 2' × 2'. This geoid estimate is shown in Figure 6.

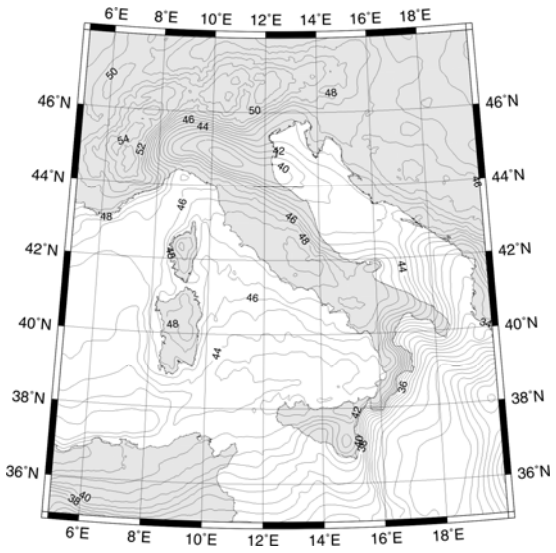


Figure 6 – The Italian geoid ITALGEO05 (equidistance =1m)

EGM2008 has been compared with this geoid estimate and with GPM98CR over the GPS/leveling data set previously described.
The results of this comparisons are summarized in Table 4.

	$N_{Italgeo05} - N_{GPS/lev}$	$N_{GPM98CR} - N_{GPS/lev}$	$N_{EGM2008} - N_{GPS/lev}$
#	977	977	977
E(m)	0.00	0.00	0.00
σ(m)	0.12	0.35	0.10
Min(m)	-0.50	-1.30	-0.33
Max(m)	0.32	0.64	0.34

Table 4 – Residuals statistics on GPS/leveling data using different geoid estimates

Statistics refer to discrepancies after datum shift. The equation used to account for datum shift is the one described in Heiskanen and Moritz (1990):

$$N_{grav} = N_{GPS/lev} + \Delta N(\theta,\lambda) =$$
$$= N_{GPS/lev} + dx\sin\theta\cos\lambda + dy\sin\theta\sin\lambda + dz\cos\theta$$

$$\theta = 90° - \varphi \qquad (dx, dy, dz) = \textit{datum shift parameters}$$

EGM2008 fits GPS/leveling data much better than GPM98CR, as it is reasonable due to the higher degree information contained in EGM2008. Furthermore, it also gives results comparable with those of ITALGEO05 which is the most refined geoid estimate over the test area.
The discrepancies between GPS/leveling data and EGM2008 area shown in Figure 7, while the discrepancies between GPS/leveling data and ITALGEO05 are plotted in Figure 8.

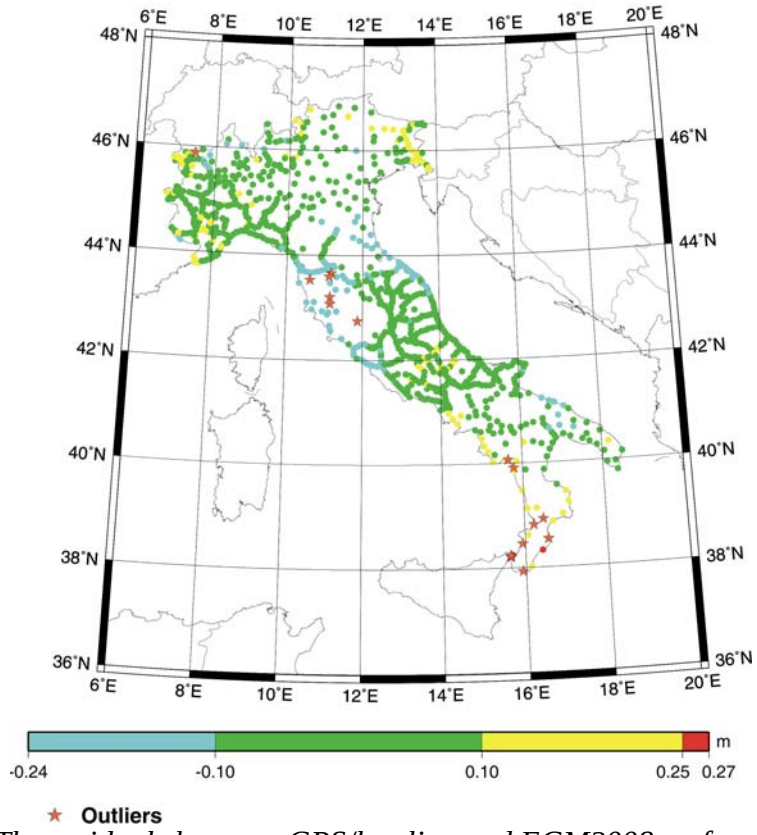


Figure 7. The residuals between GPS/leveling and EGM2008m after datum shift

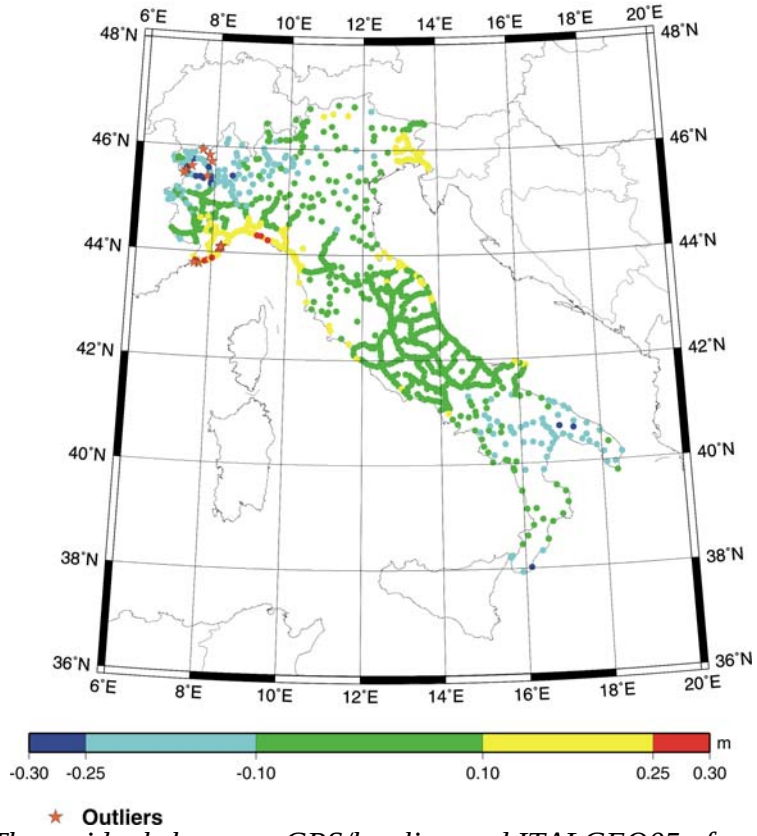


Figure 8. The residuals between GPS/leveling and ITALGEO05 after datum shift

By inspecting the two figures, one can see that these two geoid estimates have, in the whole, the same degree of accuracy. However, there are regions where ITALGEO05 fits GPS/leveling data better than EGM2008 and vice-versa. Also, possible outliers, that are marked during datum shift estimate, are in different regions for EGM2008 and ITALGEO05.

5. Conclusions

The EGM2008 global geopotential model proved to be very effective in fitting gravity and GPS/leveling in the Central Mediterranean area. This model is remarkably better than GPM98CR, previously the best model over the same area. Thus, the EGM2008 coefficients contain, even at high order, valuable information. Furthermore, its accuracy in fitting GPS/leveling data is equivalent (even slightly better) to the geoid estimate ITALGEO05. This regional geoid improves remarkably GPM98CR, which is used to model the low frequency component in the framework of the "remove-restore" technique. However, there are regions in which the EGM2008 fits better GPS/leveling than ITALGEO05, e.g. in the Northern part of the Alps and along the coast of Liguria. This is quite surprising since there the geopotential field is rough and a local geoid estimate should give better results. In fact, the "remove-restore" technique, which allows a detailed modeling of the terrain component, and the local data information should give a refined geoid estimate. This is not the case in the above mentioned regions; thus local geoid estimation procedure should be carefully checked to understand this behavior. Furthermore, an open question is the one related to the reference model to be used for reproducing the "low" frequency geopotential signal in local geoid estimation procedures. One could think to replace GPM98CR (in this test area) with EGM2008 and to apply the "remove-restore" method to get an improved geoid estimate. However, this is not so straightforward since the residuals obtained after EGM2008 and RTC reduction have a covariance structure that cannot be easily fitted with the standard covariance models. Hence, collocation cannot be efficiently applied. So, if collocation is to be used, new models must be studied and implemented. Thus, the results obtained in this test have shown the efficiency of the new geopotential model and have opened new interesting perspectives in geoid computation.

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Evaluation of EGM08 based on GPS and orthometric heights over the Hellenic mainland (*)

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ABSTRACT

This report presents the evaluation results for the new Earth Gravitational Model (EGM08) that was recently released by the US National Geospatial-Intelligence Agency, using GPS and leveled orthometric heights in the area of Greece. Detailed comparisons of geoid undulations obtained from the EGM08 model and other combined global geopotential models (GGMs) with GPS/leveling data have been performed in both absolute and relative sense. The test network covers the entire part of the Hellenic mainland and it consists of more than 1500 benchmarks which belong to the Hellenic national triangulation network, with direct leveling ties to the Hellenic vertical reference frame. The spatial positions of these benchmarks have been recently determined at cm-level accuracy (with respect to ITRF2000) during a nation-wide GPS campaign that was organized in the frame of the HEPOS project. Our results reveal that EGM08 offers a major improvement (more than 60%) for the agreement among geoidal, ellipsoidal and orthometric heights over the mainland part of Greece, compared to the performance of other combined GGMs for the same area.

(*) Final report submitted to the IAG/IGFS Joint Working Group "*Evaluation of Global Earth Gravity Models*" (November 2008)

1. INTRODUCTION

The development of the Earth Gravitational Model EGM08 by the US National Geospatial-Intelligence Agency (Pavlis et al. 2008) unveiled a major achievement in global gravity field mapping. For the first time in modern geodetic history, a spherical harmonic model complete to degree and order 2159, with additional spherical harmonic coefficients (SHCs) extending up to degree 2190 and order 2159, is available for the representation of the Earth's external gravitational potential. This new model offers an unprecedented level of spatial sampling resolution (~ 9 km) for the recovery of gravity field functionals over the entire globe, and it contributes in a most successful way to the continuing efforts of the geodetic community for a high-resolution and high-accuracy reference model of Earth's mean gravity field.

Following the official release of EGM08 to the Earth science community, there is a strong interest among geodesists to quantify its actual accuracy with different validation techniques and 'external' data sets, independently of the estimation and error calibration procedures that were used for its development. In response to the above interest and as part of the related activities that have been coordinated by the IAG/IGFS Joint Working Group on the *Evaluation of Global Earth Gravity Models*, the objective of this report is to present the EGM08 evaluation results that have been obtained for the area of Greece using GPS and leveled orthometric heights. A brief summary of these results has already been given in Kotsakis et al. (2008), lacking though a number of additional tests that are presented for the first time herein (see Sect. 4).

The test network consists of 1542 control points that belong to the Hellenic national triangulation frame, with direct ties to the Hellenic national vertical reference frame through spirit (and in some cases trigonometric) leveling surveys. These control points were recently re-surveyed through a national GPS campaign in the frame of the HEPOS project (more details to be given in Sect. 2) and their spatial positions have been estimated anew at cm-level accuracy with respect to ITRF2000.

Some key features of our study are the extensive national coverage and high spatial density of the test network, corresponding approximately to an average distance of 7 km between adjacent points throughout Greece (Figure 1). These characteristics have been most helpful in identifying the significant improvement that EGM08 yields, over other existing geopotential models, for the representation of gravity field features in certain Hellenic mountainous areas (see Sect. 3). This is actually the first time that a detailed quality analysis for the performance of global geopotential models (GGMs) is carried out over the entire Hellenic mainland with the aid of precise GPS positioning. Consequently, our study also provides a preliminary, yet reliable, assessment about the feasibility of EGM08 for determining orthometric height differences via GPS/geoid-based leveling techniques in Greece (see Sect. 5).

2. DATA SETS

All the evaluation tests and their corresponding results that are presented in the following sections refer to a network of 1542 GPS/leveling benchmarks which covers the entire mainland region of Greece with a relatively uniform spatial distribution (see Figure 1).

Note that some control points which were originally existing in this network, but they were later identified as 'problematic' (mainly due to suspected blunders in their orthometric heights that are provided by the Hellenic Military Geographic Service), have been removed from the following analysis and they are not included in the test network shown in Figure 1.

Although a large number of additional GPS/leveling benchmarks were also available in the Greek islands, they have been deliberately excluded from our current analysis to avoid misleading systematic effects in the evaluation results due to unknown vertical datum differences that exist between the various islands and the mainland region.

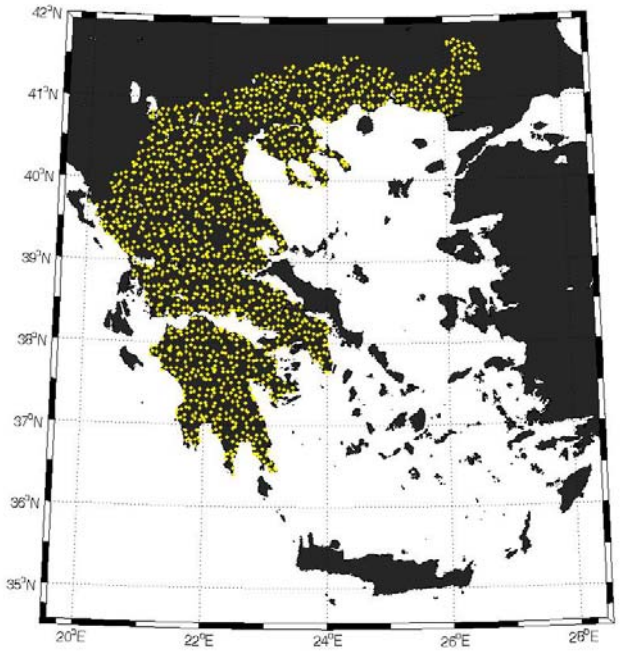


Figure 1. Geographical distribution of the 1542 GPS/leveling benchmarks over the Hellenic mainland.

2.1 Ellipsoidal heights

Within the frame of currently ongoing efforts for the enhancement of the spatial data infrastructure in Greece, a national GPS campaign took place in 2007 in order to acquire a sufficient number of control points with accurately known 3D spatial positions in an ITRF-type coordinate system. These activities have been initiated by the Ministry for the Environment, Planning and Public Works and the financial support of the EU and the Hellenic State, and they are part of the HEPOS (Hellenic Positioning System) project that will lead to the launch of a modern satellite-based positioning service for cadastral, mapping, surveying and other geodetic applications in Greece (Gianniou 2008). The entire project is coordinated by Ktimatologio S.A, a state-owned private sector firm that is responsible for the operation of the Hellenic Cadastral system.

The aforementioned GPS campaign involved more than 2450 geodetic benchmarks within the existing national triangulation network, part of which are the 1542 points shown in Figure 1. The main scope of the campaign was to provide an ample number of control stations

for determining a precise datum transformation model between the official Hellenic Geodetic Reference Frame of 1987 and other ITRF/ETRF-type frames. The actual fieldwork was performed within a 6-month period (March to September 2007) using twelve dual-frequency Trimble 5700/5800 GPS receivers with Zephyr or R8 internal antennas. Thirty three points were used as 'base' reference stations with 24-hour continuous GPS observations, while the rest of the control points were treated as 'rover' stations with observation periods ranging between 1-3 hours. In all cases, a 15-sec sampling rate and an 15° elevation cut-off angle were used for the data collection. Note that the maximum GPS-baseline length that was obtained from the above procedure did not exceed 35 km.

After the processing of the GPS observations using EUREF/EPN ties and IGS precise orbits, the geocentric Cartesian coordinates of all stations (including the 1542 points shown in Figure 1) were determined in ITRF2000 (epoch: 2007.236) and their geometric heights were subsequently derived with respect to the GRS80 ellipsoid. The accuracy of the ellipsoidal heights ranges between 2-5 cm, while the horizontal positioning accuracy with respect to ITRF2000/GRS80 is marginally better by 1-2 cm (1σ level).

2.2 Orthometric heights

Helmert-type orthometric heights at the 1542 test points have been determined through leveling ties to surrounding benchmarks of the national vertical reference frame. These local survey ties were performed in previous years by the Hellenic Military Geographic Service (HMGS) using spirit and/or trigonometric leveling techniques. It should be mentioned that a large number of the test points is located in highly mountainous areas (i.e. 24% of them have orthometric heights $H > 800$ m).

The quality of the known orthometric heights in our test network is mainly affected by two factors: the internal accuracy and consistency of the Hellenic vertical datum (HVD), and the observation accuracy of the local leveling ties to the surrounding HVD benchmarks. Due to the absence of sufficient public documentation from the part of HMGS, the absolute accuracy of these orthometric heights is largely unknown. Their values refer, in principle, to the equipotential surface of Earth's gravity field that coincides with the mean sea level at the HVD's fundamental tide-gauge reference station located in Piraeus port (unknown W_o value, period of tide gauge measurements: 1933-1978); for more details, see Antonopoulos et al. (2001), Takos (1989).

2.3 GPS-based geoid undulations

Based on the known ellipsoidal and orthometric heights, geoid undulations have been computed at the 1542 test points according to the equation

$$N^{GPS} = h - H \tag{1}$$

The above values provide the 'external' dataset upon which the following EGM08 validation tests will be performed.

Note that low-pass filtering or other smoothing techniques have not been applied to the GPS/H geoid heights (N^{GPS}). As a result, the effect of the *omission error* associated with all tested GGMs will be directly reflected in our evaluation results.

2.4 GGM-based geoid undulations

Geoid undulations have also been computed at the 1542 GPS/leveling benchmarks using several different GGMs. For the evaluation results presented herein, we consider the most recent 'mixed' GGMs that have been produced from the combined analysis of various types of satellite data (CHAMP, GRACE, SLR), terrestrial gravity data, and altimetry data; see Table 1.

Table 1. GGMs used for the tests at the 1542 Hellenic GPS/leveling benchmarks.

Models	n_{max}	*Reference*
EGM08	2190	Pavlis et al. (2008)
EIGEN-GL04C	360	Förste et al. (2006)
EIGEN-CG03C	360	Förste et al. (2005)
EIGEN-CG01C	360	Reigber et al. (2006)
GGM02C	200	Tapley et al. (2005)
EGM96	360	Lemoine et al. (1998)

The determination of GGM geoid undulations was carried out through the general formula (Rapp 1997)

$$N = \zeta + \frac{\Delta g^{FA} - 0.1119H}{\bar{\gamma}} H + N_o \tag{2}$$

where ζ and Δg^{FA} denote the height anomaly and free-air gravity anomaly signals, which are computed from spherical harmonic series expansions (up to n_{max}) based on the SHCs of each model and the GRS80 normal gravity field parameters. Only the gravitational potential coefficients with degrees $n \geq 2$ were considered for these harmonic synthesis computations, excluding the contribution of the zero/first-degree harmonics from the GGM-based signals. Note that EIGEN-CG01C and EIGEN-CG03C are the only models among the tested GGMs which are accompanied by non-zero first-degree SHCs. Nevertheless, their omission in the computation of the ζ values has a negligible effect (mm-level) in our evaluation tests.

The term N_o represents the contribution of the zero-degree harmonic to the GGM geoid undulations with respect to the GRS80 reference ellipsoid. It is computed according to the well known formula (e.g. Heiskanen and Moritz 1967)

$$N_o = \frac{GM - GM_o}{R\gamma} - \frac{W_o - U_o}{\gamma} \tag{3}$$

where the parameters GM_o and U_o correspond to the Somigliana-Pizzeti normal gravity field generated by the GRS80 ellipsoid (Moritz 1992)

$GM_o = 398600.5000 \times 10^9 \text{ m}^3 \text{ s}^{-2}$

$U_o = 62636860.85 \text{ m}^2 \text{ s}^{-2}$

The Earth's geocentric gravitational constant (GM) and the constant gravity potential of the geoid (W_o) have been set to the following values

$GM = 398600.4415 \times 10^9$ m^3 s^{-2}

$W_o = 62636856.00$ m^2 s^{-2} *(IERS Conventions 2003)*

while the mean Earth radius R and the mean normal gravity γ on the reference ellipsoid are taken equal to 6371008.771 m and 9.798 m s^{-2}, respectively (GRS80 values). Based on the above conventional choices, the zero-degree term from Eq. (3) yields the value N_o = -0.442 m, which has been added to the geoid undulations obtained from the corresponding SHC series expansions of all GGMs.

Remark. *The numerical computations for the spherical harmonic synthesis of the N values from the various GGMs have been performed with the 'harmonic_synth_v02' software program that is freely provided by the EGM08 development team (Holmes and Pavlis 2006). Note also that the final GGM geoid undulations obtained from Eq. (2) refer to the zero-tide system, with respect to a geometrically fixed reference ellipsoid (GRS80).*

2.5 Height data statistics

The statistics of the individual height datasets that will be used in our evaluation tests are given in Table 2. Note that the statistics for the GGM geoid undulations refer to the values computed from Eq. (2) at the 1542 GPS/leveling benchmarks using the *full* spectral resolution of each model.

From the following table (see, in particular, the *mean* values in the fourth column) it is evident the existence of a large discrepancy (> 25 cm) between the reference surface of the Hellenic vertical datum (which is associated with an unknown W_o value) and the equipotential surface of Earth's gravity field that is specified by the IERS conventional value W_o = 62636856.00 m^2 s^{-2} and realized by the various GGMs over the Hellenic mainland region.

Table 2. Statistics of various height datasets over the test network of 1542 Hellenic GPS/leveling benchmarks (units in m).

	Max	*Min*	*Mean*	σ
h	2562.753	24.950	545.676	442.418
H	2518.889	0.088	510.084	442.077
$N^{GPS} = h\text{-}H$	43.864	19.481	35.592	5.758
N (EGM08)	44.374	19.663	35.968	5.800
N (EIGEN-GL04C)	44.104	19.303	35.874	5.878
N (EIGEN-CG03C)	44.049	19.257	35.861	5.867
N (EIGEN-CG01C)	44.108	19.663	35.823	5.873
N (GGM02C)	44.034	19.771	35.905	5.780
N (EGM96)	44.007	19.687	36.037	5.753

It is also interesting to observe the considerable mean offset of the full-resolution EGM08 geoid (n_{max} = 2190) with respect to the geoid realizations obtained from other GGMs at the GPS/leveling benchmarks. This offset varies from 6 to 15 cm and it should be attributed to long/medium-wavelength systematic differences between EGM08 and the other GGMs over the Hellenic area.

3. POINTWISE EVALUATION TESTS AFTER A SIMPLE BIAS FIT

A series of GGM evaluation tests were performed based on the point values for the ellipsoidal and orthometric heights in the control network. The statistics of the differences between the GPS-based and the GGM-based geoid heights are given in Table 3. In all cases, the values shown in this table refer to the statistics after a least-squares *constant bias fit* was applied to the original misclosures *h-H-N* at the 1542 Hellenic GPS/leveling benchmarks.

The differences in the estimated bias obtained from each model (see last column in Table 3) indicate the existence of systematic regional offsets among the GGM geoids that are likely caused by long/medium-wavelength commission errors in their SHCs and additional omission errors due to their limited spectral resolution. Furthermore, the actual magnitude of the bias between N^{GPS} and N suggests the presence of a sizeable offset between **(a)** the equipotential surface associated with the IERS conventional value W_o = 62636856.00 m^2s^{-2} and realized by the various GGMs over the Hellenic region, and **(b)** the HVD reference surface that is realized through the GPS/H geoid heights N^{GPS} at the test points. For example, based on the results from the full-resolution version of the new model, the HVD reference surface appears to be located 38 cm below the EGM08/W_o/GRS80 geoid realization.

Table 3. Statistics of the residuals N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks (units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.542	-0.437	0.142	-0.377
EGM08 *(n_{max}=360)*	1.476	-1.287	0.370	-0.334
EIGEN-GL04C *(n_{max}=360)*	1.773	-1.174	0.453	-0.283
EIGEN-CG03C *(n_{max}=360)*	1.484	-1.173	0.453	-0.270
EIGEN-CG01C *(n_{max}=360)*	1.571	-1.135	0.492	-0.231
GGM02C *(n_{max}=200)*	2.112	-1.472	0.551	-0.313
EGM96 *(n_{max}=360)*	1.577	-1.063	0.423	-0.446

From the results given in the above table, it is evident that EGM08 offers a remarkable improvement for the agreement among ellipsoidal, orthometric and geoidal heights in Greece. Compared to other GGMs, the standard deviation of the EGM08 residuals N^{GPS}-N over the test network decreases by a factor of 3 (or more). The improvement obtained from the new model is visible even in its 30′ limited-resolution version (n_{max}=360), which matches the GPS/H geoid within ±37 cm (in an average pointwise sense), while all previous GGMs of similar resolution do not perform better than ±42 cm. The major contribution, however, comes from the ultra-high frequency band of EGM08 ($360 < n < 2190$) which enhances the consistency between GGM and GPS/H geoid heights at ±14 cm (1σ level).

In Table 4, we can see the percentage of the GPS/leveling benchmarks whose adjusted residuals *h-H-N* (after a constant bias fit) fall within a specified range of geoid uncertainty. The agreement between EGM08 and GPS/H geoid heights is better than 10 cm for more than half of the total 1542 test points, whereas for the other GGMs the same consistency level is only reached at 18% (or less) of the test points. Furthermore, almost 85% of the test points give an agreement between the full-resolution EGM08 geoid and the GPS/leveling

data that is better than 20 cm, compared to 36% (or less) in the case of all other global models that were tested.

Table 4. Percentage of the 1542 test points whose absolute values of their adjusted residuals N^{GPS}–N *(after a least-squares constant bias fit)* are smaller than some typical geoid accuracy levels.

	< 2 cm	***< 5 cm***	***< 10 cm***	***< 15 cm***	***< 20 cm***
EGM08 *(n_{max}=2190)*	13.3 %	29.8 %	53.5 %	73.0 %	84.6 %
EGM08 *(n_{max}=360)*	4.5 %	11.6 %	22.8 %	32.7 %	43.7 %
EIGEN-GL04C *(n_{max}=360)*	3.6 %	9.3 %	17.7 %	27.5 %	36.0 %
EIGEN-CG03C *(n_{max}=360)*	3.3 %	8.3 %	17.5 %	26.5 %	34.7 %
EIGEN-CG01C *(n_{max}=360)*	2.9 %	7.8 %	15.1 %	23.0 %	29.4 %
GGM02C *(n_{max}=200)*	2.9 %	7.4 %	15.0 %	22.6 %	30.2 %
EGM96 *(n_{max}=360)*	4.3 %	9.8 %	17.5 %	27.7 %	35.5 %

The horizontal spatial variations of the (full-resolution) EGM08 residuals N^{GPS}-N did not reveal any particular systematic pattern within the test network. Both their latitude-dependent and longitude-dependent scatter plots, as shown in Figures 2 and 3, are free of any sizeable north/south or east/west tilts over the Hellenic mainland. In other GGMs, however, some strong localized tilts and systematic oscillations can be identified in the N^{GPS}-N residuals, mainly due to larger commission errors associated with their SHCs and significant omission errors involved in the recovery of the geoid signal (see Figures 2 and 3).

Our evaluation results have also confirmed that EGM08 performs exceedingly better than the other models over the mountainous parts of the Hellenic test network. A strong indication can be seen in the scatter plots of the pointwise residuals N^{GPS}-N (after the constant bias fit) with respect to the orthometric heights of the corresponding GPS/leveling benchmarks (Figure 4). These plots reveal a *height-dependent bias* between the GGM and GPS/H geoid heights, which is considerably reduced in the case of EGM08. Apparently, the higher frequency content of the new model gives a better approximation for the terrain-dependent gravity field features over Greece, a fact that is visible from the comparative analysis of the scatter plots in Figure 4. The remaining height-dependent linear trend in the full-resolution EGM08 residuals N^{GPS}-N (see Figure 4) is caused not only by commission/omission model errors, but it reflects also existing systematic problems in the orthometric heights of the tests points.

Further manifestation for the correlation of GGM and GPS/H geoid differences with the topographic height of the test points can be found in the color plots given in Figure 5. With the visual aid of the ETOPO2 digital elevation model, it is seen that larger values for the residuals N^{GPS}-N occur mostly over geographical areas with strong topographic features. Note that the spatial distribution of the geoid height residuals for the full-resolution model EGM08 is depicted in two separate plots, each with a *different* color-scaling scheme. From the first of these plots, we can verify the overall improvement in the geoid representation over the Hellenic mountains that is achieved with EGM08, compared to the performance of previous GGMs over the same areas. The second scatter plot of the EGM08 geoid residuals N^{GPS}-N (see lower left corner in Figure 5) reveals the remaining inconsistencies with the GPS/leveling data, which are caused by the commission/omission errors of the new model and other unknown systematic distortions in the orthometric heights at the test points.

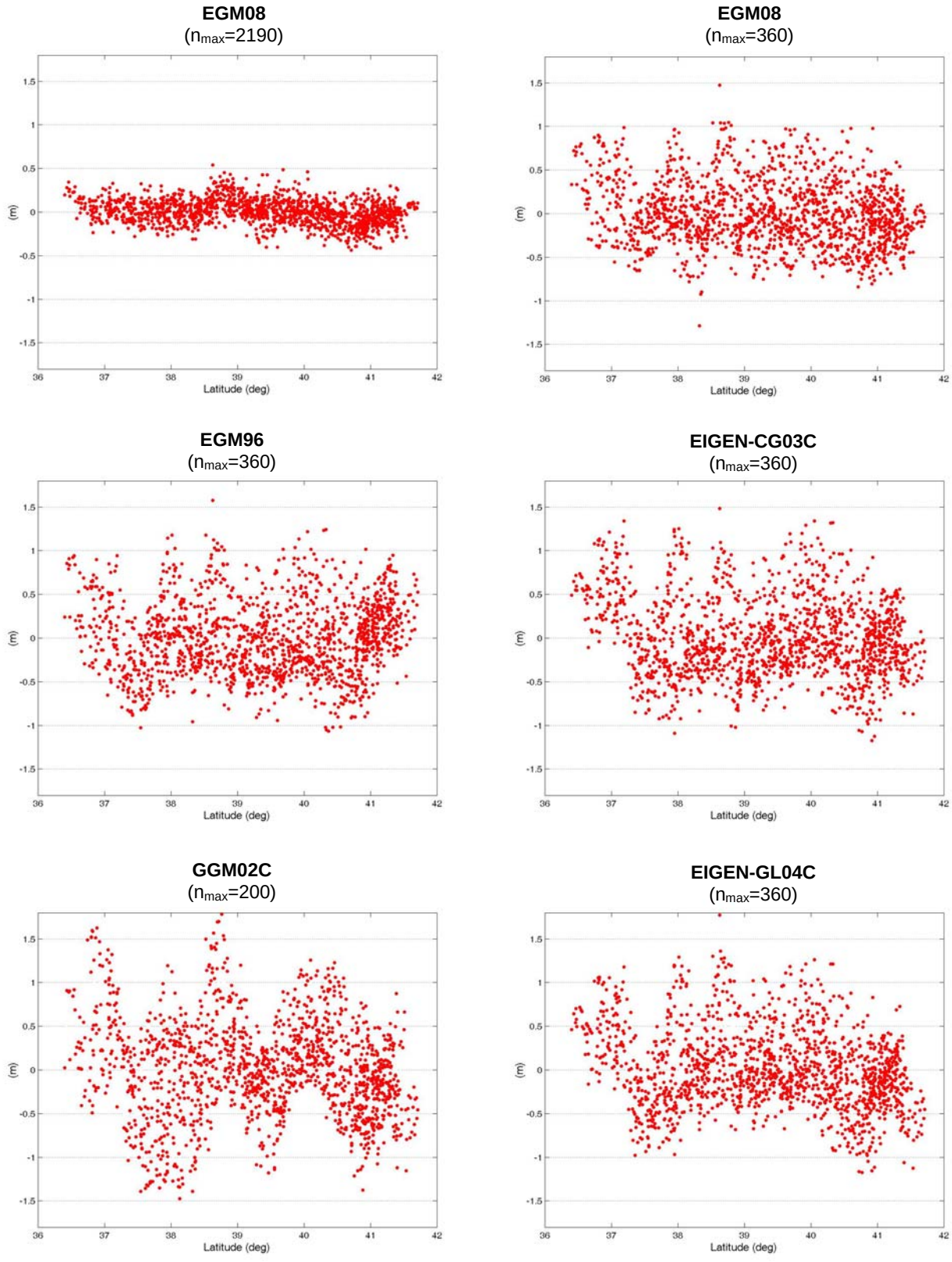


Figure 2. Latitude-dependent variations of the residuals $N^{GPS}–N$ *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

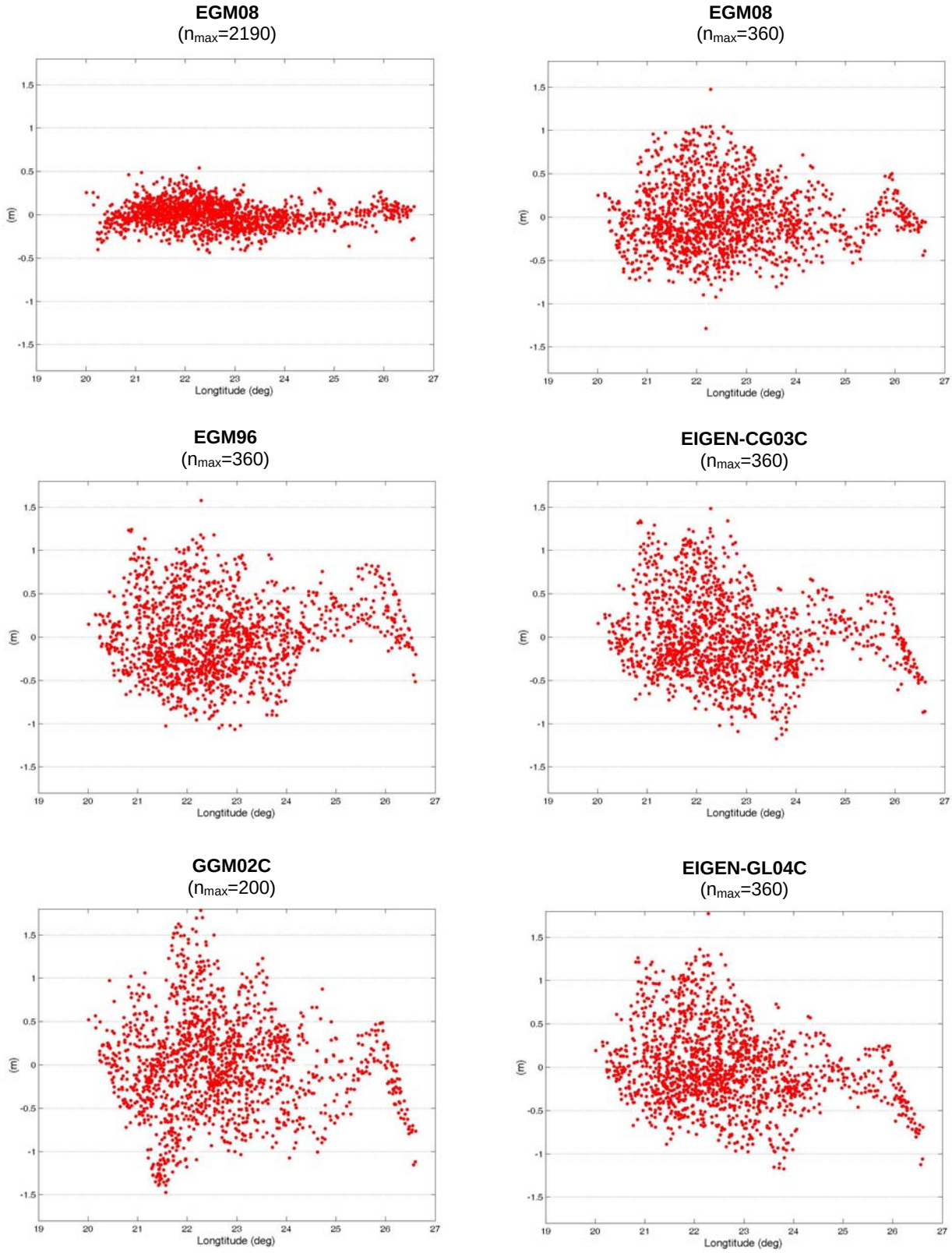


Figure 3. Longitude-dependent variations of the residuals N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

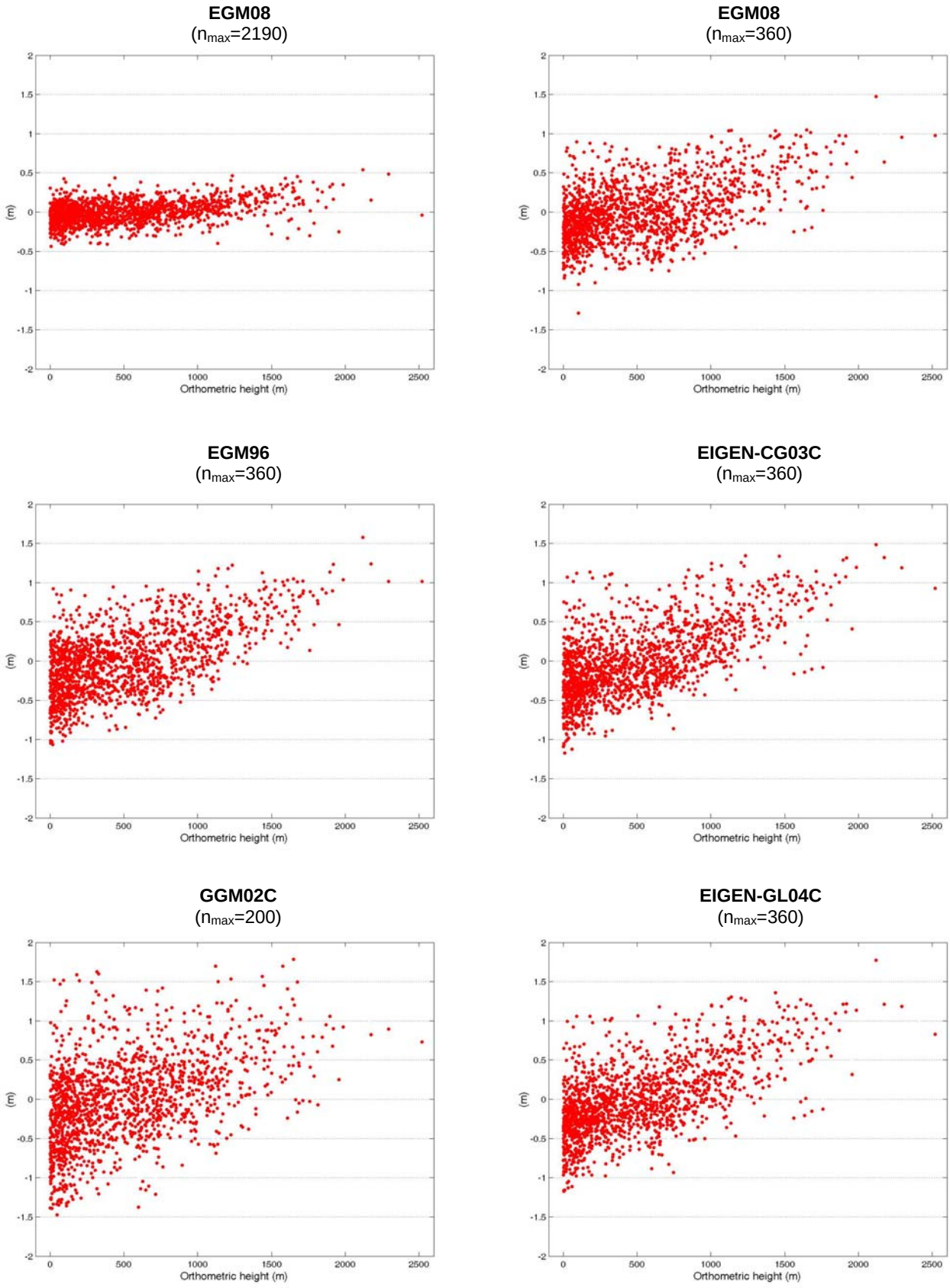


Figure 4. Height-dependent variations of the residuals $N^{GPS}-N$ *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

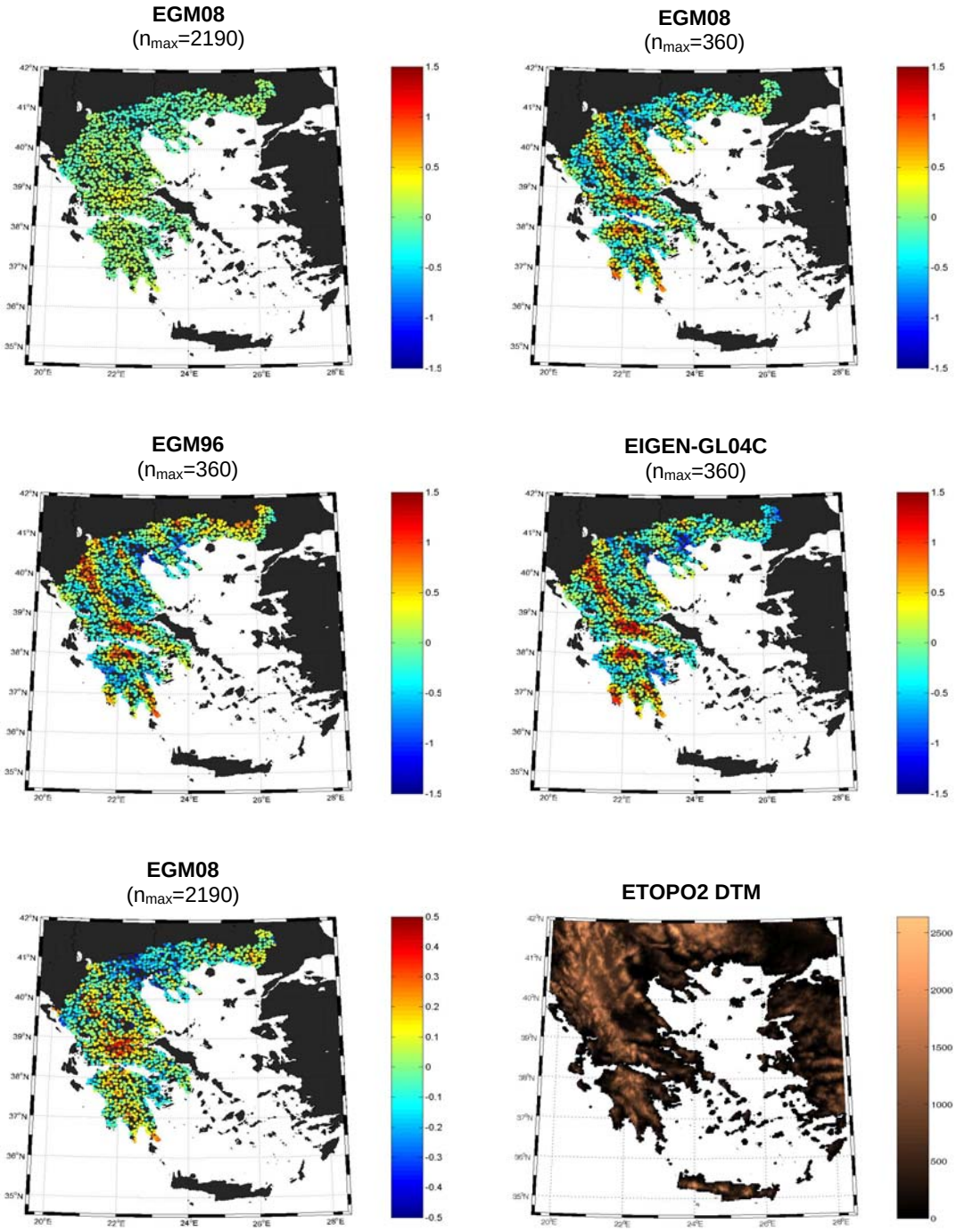


Figure 5. Colored scatter plots showing the geographical distribution of the differences N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

4. POINTWISE EVALUATION TESTS WITH DIFFERENT PARAMETRIC MODELS

In addition to the evaluation results that were presented in the previous section, another set of numerical experiments has been carried out using a number of different parametric models for the least-squares adjustment of the differences $N^{GPS} - N$. The motivation for these additional tests was to investigate the fitting performance of some known linear models that are frequently used in geoid evaluation studies with heterogeneous height data, and to assess their feasibility in modeling the systematic discrepancies between the GGM and GPS/H geoid surfaces over the Hellenic mainland. Although these tests were implemented with all six GGMs that were initially selected for our study, only the results obtained with EGM08 and EGM96 will be presented herein due to space limitations.

The various parametric models that have been fitted to the original misclosures *h-H-N* are given in Eqs. (5)-(10). *Model 1* uses a single constant-bias parametric term and it is actually the same model that was employed for all tests of the previous section. *Model 2* incorporates two additional parametric terms which correspond to an average north-south and east-west tilt between the GGM and GPS/H geoids. *Model 3* is the usual '4-parameter model' which geometrically corresponds to a 3D spatial shift and an approximate uniform scale change of the GGM's reference frame with respect to the underlying reference frame of the GPS heights (or vice versa). Finally, *models 4, 5 and 6* represent height-dependent linear corrector surfaces that constrain the relation among ellipsoidal, orthometric and geoidal heights in terms of the generalized equation

$$h - (1+\delta s_H)H - (1+\delta s_N)N = \mu \tag{4}$$

The above equation takes into consideration the fact that the spatial scale of the GPS heights does not necessarily conform with the spatial scale induced by the GGM geoid undulations and/or the inherent scale of the orthometric heights obtained from terrestrial leveling techniques. Moreover, the GGM geoid undulations and/or the local orthometric heights are often affected by errors that are correlated, to a certain degree, with the Earth's topography (see the results in Figures 4 and 5), a fact that can additionally justify the use of *model 4 or 6* for the optimal fitting between N^{GPS} and N.

Model 1

$$h_i - H_i - N_i = \mu + v_i \tag{5}$$

Model 2

$$h_i - H_i - N_i = \mu + a(\varphi_i - \varphi_o) + b(\lambda_i - \lambda_o)\cos\varphi_i + v_i \tag{6}$$

Model 3

$$h_i - H_i - N_i = \mu + a\cos\varphi_i \cos\lambda_i + b\cos\varphi_i \sin\lambda_i + c\sin\varphi_i + v_i \tag{7}$$

Model 4

$$h_i - H_i - N_i = \mu + \delta s_H H_i + v_i \tag{8}$$

Model 5

$$h_i - H_i - N_i = \mu + \delta s_N N_i + v_i \tag{9}$$

Model 6

$$h_i - H_i - N_i = \mu + \delta s_H H_i + \delta s_N N_i + v_i \quad (10)$$

Remark. *A combination of the above models (e.g. the '4-parameter' or the 'bias and tilt' model merged with a height-dependent scaling term) may also be useful in practice, depending on the behavior of the actual data.*

The statistics of the adjusted residuals $\{v_i\}$ in the test network of 1542 Hellenic GPS/leveling benchmarks, after the least-squares fitting of the previous parametric models, are given in Tables 5 and 6 for the case of EGM96 and EGM08, respectively.

Table 5. Statistics of the differences N^{GPS}–N for the ***EGM96*** geoid heights, after the least-squares fitting of various parametric models at the 1542 GPS/leveling benchmarks (units in m).

	Max	*Min*	*σ*	*Bias (μ)*
Model 1	1.577	-1.063	0.423	-0.446
Model 2	1.587	-1.073	0.422	-0.445
Model 3	1.681	-1.097	0.411	303.983
Model 4	1.198	-0.847	0.341	-0.735
Model 5	1.572	-1.053	0.423	-0.381
Model 6	1.176	-0.861	0.341	-0.656

Table 6. Statistics of the differences N^{GPS}–N for the ***EGM08*** geoid heights, after the least-squares fitting of various parametric models at the 1542 GPS/leveling benchmarks (units in m).

	Max	*Min*	*σ*	*Bias (μ)*
Model 1	0.542	-0.437	0.142	-0.377
Model 2	0.521	-0.398	0.137	-0.377
Model 3	0.522	-0.398	0.137	3.479
Model 4	0.480	-0.476	0.131	-0.440
Model 5	0.528	-0.442	0.135	-0.109
Model 6	0.474	-0.421	0.123	-0.160

From the above results, it can be concluded that the low-order parametric models which are commonly used in the combined adjustment of GPS, geoid and leveled height data (*models 2* and *3*) do not offer any significant improvement for the overall fitting between the EGM08 geoid (or the EGM96 geoid) and the GPS/leveling heights over the Hellenic mainland. On the other hand, a purely height-dependent parametric model (*model 6*) enhances the statistical fit between the EGM08 and the EGM96 geoid with the GPS/leveling heights by 2 cm and 8 cm, respectively (i.e. compared to the performance of the bias-only *model 1*). The improvement in the sigma values obtained from *models 4 and 6* should be attributed to the elimination of the linear correlation trend that was previously identified (see Figure 4) between the misclosures *h-H-N* and the orthometric heights of the test points.

Note that all alternative models which are tested in this section include a common parametric term in the form of a *single constant bias*. However, the various estimates of the common bias parameter μ, as obtained from the least-squares adjustment of each model, exhibit significant variations among each other (see last column in Tables 5 and 6). Specifically, the estimated bias between N^{GPS} and N which is computed from the usual '4-parameter' model appears to be highly inconsistent with respect to the corresponding estimates from the other parametric models. This is not surprising since the intrinsic role of the bias μ in *model 3* is not to represent the average *spatial offset* between the GGM and the GPS/H geoids, as it happens for example in the case of *model 1*. In fact, the three additional parametric terms in *model 3* are the ones that absorb the systematic part of the differences $N^{GPS}-N$ in the form of a three-dimensional spatial shift ($a \rightarrow t_x$, $b \rightarrow t_y$, $c \rightarrow t_z$), leaving to the fourth bias parameter μ the role of a 'scale-change' effect.

At this point, it is perhaps instructive to recall the linearized transformation formula for geoid heights between two parallel geodetic reference frames (see, e.g., Kotsakis 2008)

$$N_i' - N_i = (\mathrm{a}w_i + N_i)\delta s + t_x \cos\varphi_i \cos\lambda_i + t_y \cos\varphi_i \sin\lambda_i + t_z \sin\varphi_i \qquad (11)$$

where a denotes the semi-major axis of the common reference ellipsoid, δs is the differential scale change between the underlying frames, and w_i corresponds to the auxiliary unitless term $(1-e^2\sin^2\varphi_i)^{1/2}$ that is approximately equal to 1 (i.e. the squared eccentricity of the reference ellipsoid is $e^2 \approx 0.0067$). The above formula conveys, in the language of geodetic datum transformation, the basic geometric principles of the '4-parameter' model that is frequently employed for the optimal fitting of GPS, geoid and leveled height data. Given the analytic expression in Eq. (11), the constant bias μ that appears in the formulation of *model 3* emulates the effect of a mean spatial re-scaling rather than a mean spatial offset between two different geoid realizations.

Although less inconsistent with each other, the estimates of the bias parameter μ from the other parametric models show dm-level fluctuations in their values. It should be noted though that the inclusion of additional spatial tilts for the fitting between N^{GPS} and N does not distort the initial estimate of μ that was obtained from *model 1* over the Hellenic mainland. On the other hand, the use of height-dependent scaling terms (*models 4*, *5* and *6*) affects considerably the final estimates of the bias parameter μ, as it can be easily verified from the results in Tables 5 and 6.

All in all, the problem of obtaining a realistic estimate for the average spatial offset between a local vertical datum (e.g. HVD in our case) and a GGM geoid seems to have a strong dependence on the parametric model that is used for the adjustment of heterogeneous height data over a test network of GPS/leveling benchmarks. Since there exist strong theoretical and practical arguments that can be stated in favor of the generalized constraint in Eq. (4), the use of the simple *model 1* is not necessarily the safest choice for estimating the average spatial offset between GGM and GPS/H geoids over a regional network. In view of the frequent absence (or even ignorance) of a complete and reliable stochastic error model for the properly weighted adjustment of the differences $N^{GPS}-N$, a clear geometrical interpretation of the estimated bias μ is not always a straightforward task in GGM evaluation studies.

5. BASELINE EVALUATION TESTS

An additional set of evaluation tests was also performed through the comparison of GGM and GPS/H *geoid slopes* over the Hellenic network of 1542 GPS/leveling benchmarks. For all baselines formed within this network, the following differences of relative geoid undulations were determined

$$\Delta N_{ij}^{GPS} - \Delta N_{ij} = (h_j - H_j - h_i + H_i) - (N_j - N_i) \tag{12}$$

Note that the computation of the above differences took place *after* the implementation of a least-squares bias/tilt fit between the pointwise values of the GGM and GPS/H geoid heights.

Depending on the actual baseline length, the residual values from Eq. (12) were grouped into various spherical-distance classes and their statistics were then evaluated within each class. Given the actual coverage and spatial density of the GPS/leveling benchmarks in our test network, baselines with length from 2 km up to 600 km were considered for this evaluation scheme. The statistics of the differences between the GGM and GPS/H relative geoid heights, for five selected baseline classes, are given in the following tables.

Table 7. Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length **< *3 km*** (number of baselines: 47, units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.142	-0.156	0.058	-0.009
EGM08 *(n_{max}=360)*	0.140	-0.206	0.080	-0.018
EIGEN-GL04C *(n_{max}=360)*	0.156	-0.200	0.087	-0.015
EIGEN-CG03C *(n_{max}=360)*	0.148	-0.205	0.087	-0.016
EIGEN-CG01C *(n_{max}=360)*	0.152	-0.207	0.087	-0.016
GGM02C *(n_{max}=200)*	0.137	-0.230	0.081	-0.021
EGM96 *(n_{max}=360)*	0.136	-0.199	0.081	-0.014

Table 8. Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length **< *5 km*** (number of baselines: 289, units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.643	-0.474	0.111	0.006
EGM08 *(n_{max}=360)*	0.648	-0.534	0.154	0.003
EIGEN-GL04C *(n_{max}=360)*	0.649	-0.542	0.155	0.005
EIGEN-CG03C *(n_{max}=360)*	0.643	-0.540	0.155	0.005
EIGEN-CG01C *(n_{max}=360)*	0.640	-0.536	0.156	0.005
GGM02C *(n_{max}=200)*	0.685	-0.571	0.162	0.003
EGM96 *(n_{max}=360)*	0.643	-0.553	0.154	0.005

Table 9. Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***5-10 km*** (number of baselines: 2119, units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.465	-0.629	0.125	0.001
EGM08 *(n_{max}=360)*	1.022	-1.044	0.248	-0.004
EIGEN-GL04C *(n_{max}=360)*	0.983	-0.988	0.251	-0.000
EIGEN-CG03C *(n_{max}=360)*	0.971	-1.026	0.251	-0.001
EIGEN-CG01C *(n_{max}=360)*	0.976	-1.039	0.252	-0.002
GGM02C *(n_{max}=200)*	0.967	-0.991	0.264	0.002
EGM96 *(n_{max}=360)*	0.963	-1.002	0.251	0.003

Table 10. Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***10-50 km*** (number of baselines: 56575, units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.859	-0.781	0.164	-0.001
EGM08 *(n_{max}=360)*	2.778	-2.417	0.514	-0.012
EIGEN-GL04C *(n_{max}=360)*	2.480	-2.430	0.552	-0.019
EIGEN-CG03C *(n_{max}=360)*	2.335	-2.488	0.550	-0.021
EIGEN-CG01C *(n_{max}=360)*	2.335	-2.445	0.555	-0.021
GGM02C *(n_{max}=200)*	3.221	-2.760	0.627	-0.012
EGM96 *(n_{max}=360)*	2.532	-2.393	0.542	-0.013

Table 11. Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***50-100 km*** (number of baselines: 135970, units in m).

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.891	-0.881	0.189	-0.003
EGM08 *(n_{max}=360)*	2.332	-2.282	0.552	-0.013
EIGEN-GL04C *(n_{max}=360)*	2.410	-2.773	0.658	-0.041
EIGEN-CG03C *(n_{max}=360)*	2.172	-2.568	0.651	-0.043
EIGEN-CG01C *(n_{max}=360)*	2.356	-2.600	0.668	-0.037
GGM02C *(n_{max}=200)*	3.043	-3.611	0.834	-0.067
EGM96 *(n_{max}=360)*	2.226	-2.480	0.623	-0.028

As seen from the results in Tables 7 through 11, the full-resolution EGM08 model performs consistently better than all other GGMs over all baseline classes. The improvement becomes more pronounced as the baseline length increases, indicating the significant contribution of the EGM08 high-degree harmonics ($n > 360$) for the slope representation of the Hellenic geoid over baselines 5-100 km. For example, the resultant σ values of the differences $\Delta N^{GPS} - \Delta N$ are reduced by a factor of 1.4 for baselines <5 km, by a factor of 2 for baselines 5-10 km, and by a factor of about 3.5 for baselines 10-100 km (compared to the performance of EGM96 and other EIGEN-type models).

It is also interesting to observe the considerable bias in the geoid slope residuals $\Delta N^{GPS} - \Delta N$ obtained from all tested GGMs (except from the full-resolution EGM08 model) for base-

lines 10-100 km. This result should be attributed to existing systematic errors in the medium-wavelength SHCs of the tested GGMs and additional omission errors in the pre-EGM08 models, which produce an apparent scale difference between GGM and GPS/H relative geoid undulations for the aforementioned baseline range.

The overall behaviour of the sigma values for the differences between GGM and GPS/H geoid slopes is shown in Figure 6, over all baseline classes that were considered in our tests. The remarkable improvement in the relative geoid accuracy from the EGM08 model is clearly visible, indicating an ΔN-consistency level with the external GPS/leveling data that varies from ±6 cm to ±20 cm (1σ level).

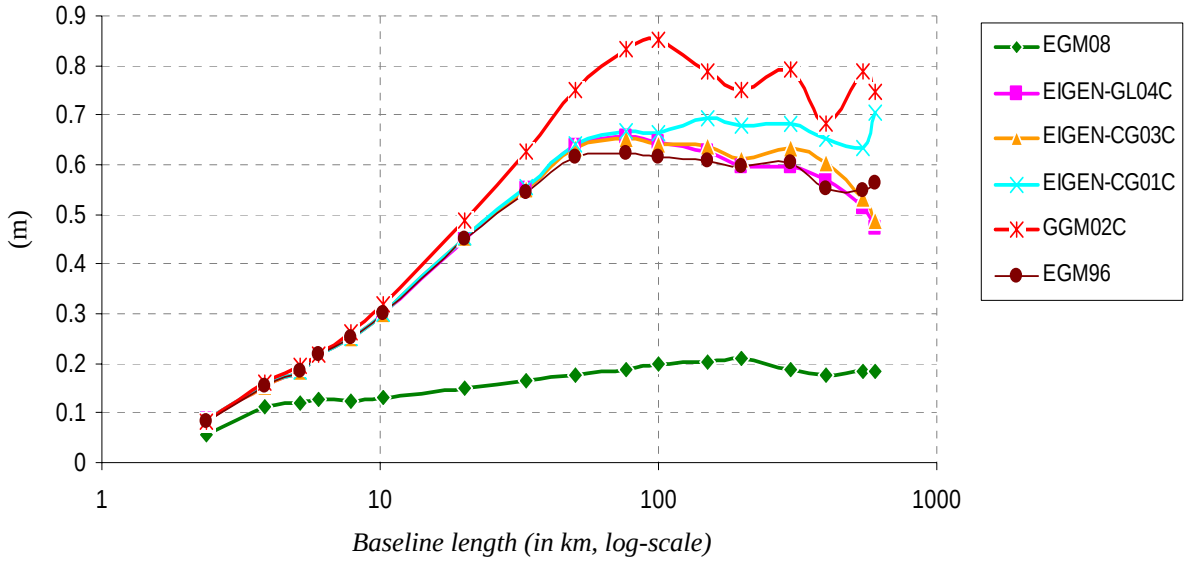


Figure 6. Std of the differences $\Delta N_{ij}^{GPS} - \Delta N_{ij}$ in the test network of 1542 GPS/leveling benchmarks, as a function of the baseline length.

Focusing on the geoid-slope evaluation results for short baselines (up to 30 km) can give us an indication for the expected accuracy in GPS/leveling projects when using an EGM08 reference geoid model over Greece. Our preliminary analysis in the test network showed that the agreement between the height differences ΔH_{ij} computed: **(a)** directly from the known orthometric heights at the GPS/levelling benchmarks and **(b)** indirectly from the GPS/EGM08 ellipsoidal and geoid heights, could be approximated by the statistical error model $\sigma_{\Delta H} = \sigma_o L^{1/2}$ with the a-priori sigma factor σ_o ranging between 3-5 cm/km (for baseline length L<30 km). Although such a performance cannot satisfy mm-level accuracy requirements for vertical positioning (which are 'easily' achievable through spirit leveling techniques), it nevertheless provides a major step forward that can successfully accommodate a variety of engineering and surveying applications. Note that the corresponding performance of EGM96 in our test network is described by a relative accuracy factor of $\sigma_o \approx 9$ cm/km for baselines <30 km.

6. Conclusions

The results of our evaluation tests have revealed the superiority of EGM08 over all existing mixed GGMs for the area of Greece. The new model outperforms the other tested GGMs at the 1542 Hellenic GPS/leveling benchmarks and it improves the statistical fit with the Hellenic GPS/H geoid by approximately 30 cm (or more)! The pointwise agreement among ellipsoidal, orthometric and EGM08-based geoid heights is at ±14 cm (1σ level), reflecting mainly the regional effects of the commission errors in the model's SHCs, as well as other local distortions in the HVD orthometric heights at the control points.

In terms of relative geoid accuracy, EGM08 shows a rather stable performance for the standard deviation of the slope residuals $\Delta N^{GPS} - \Delta N$ over all baseline lengths that were considered in our study. Compared to other tested GGMs whose relative geoid accuracy decreases continuously over baselines 5-100 km (estimated values for $\sigma_{\Delta N}$ reach up to 60 cm), the full-resolution EGM08 model gives a more balanced behavior with the corresponding values of $\sigma_{\Delta N}$ not exceeding 20 cm, even for baselines up to 600 km.

In conclusion, the results presented herein provide a promising testament for the future use of EGM08 in geodetic applications over the Hellenic mainland. However, in view of its possible forthcoming implementation in GPS-based leveling projects throughout Greece (in conjunction with the HEPOS system), a more detailed analysis with additional interpolation methods and spatial 'corrector surfaces' for modeling the differences $N^{GPS} - N$ or $\Delta N^{GPS} - \Delta N$ is required to achieve cm-level consistency for the transformation between GPS/EGM08 and HVD orthometric heights.

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EVALUATION OF THE EARTH GRAVITATIONAL MODEL 2008 IN TURKEY

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Abstract

A new Earth Gravitational Model (EGM08) to degree 2160 has been released to IAG's EGM Evaluation Group. In this study, we evaluate EGM08 Tide Free Model by using regional gravity, quasi-geoid height and GPS/leveling data. The EGM08-derived quantities are compared with (1)the GPS/leveling quasi-geoid heights, (2)an existing GPS/leveling fitted regional quasi-geoid model (TG03), and (3)the surface gravity anomalies in Turkey. The differences between observed/computed and EGM08-derived quantities are investigated. The mean value and standard deviation of the differences between EGM08 derived and observed quantities are found to be -88.8 cm and 24.2 cm for GPS/leveling height anomalies, 27.1 cm and 75.3 cm for TG03 quasi-geoid heights, and 2.8 mGal and 17.1 mGal for surface gravity anomalies. As Turkish proprietary data were not used in EGM08 computations this work is believed to be an external check for EGM08.

1. Introduction

A new Earth Gravitational Model (EGM2008) to degree 2160 has been released to IAG's EGM Evaluation Group. EGM2008 incorporates 5x5 minutes gravity anomalies and has benefited from the latest GRACE solutions. Improved altimetry-derived gravity anomalies and its implied Dynamic Ocean Topography model were also used in computations (Pavlis et al., 2008).

The evaluation and quality assessment of the EGM08 is important for being used in various geodetic and other scientific applications at global and regional scales. The evaluation of the EGM08 is based on the comparisons with other external data. For this aim external data sets, that mainly include GPS/leveling observations, airborne and surface gravity data, sea surface topography, sea surface heights from altimetry band tide gauge, and deflections of the vertical, are to be used in appropriate evaluation procedures.

The objective of this study is to control, validate and perform quality assessment of EGM08 derived data. As Turkish proprietary gravity and GPS/leveling data were not used in the EGM08 computations this work provides an external evaluation for EGM08 derived surface gravity and height anomalies on the physical surface of the earth.

In this study, we compared EGM2008 Tide Free Model with regional gravity, quasi-geoid height and GPS/leveling data. The EGM08-derived quantities are compared with (1)the GPS/leveling quasi-geoid heights, (2)existing GPS/leveling fitted regional quasi-geoid model, and (3)the surface free-air gravity anomalies in Turkey within boundaries 26^{o}E-45^{o}E and 36^{o}N-42^{o}N.

Computations were achieved by using HARMONIC_SYNTHESIS program that is provided with the EGM08 coefficients (Holmes and Pavlis, 2006).

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2. Comparison with GPS/Leveling quasi-geoid heights

Turkish Vertical Control Network (TUDKA99) was re-adjusted due to two Marmara earthquakes occurred in 1999, with of 243 lines of 25680 points with a total length of 29316 km. Vertical datum for TUDKA99 was defined with arithmetic mean of instantaneous sea level measurements recorded at Antalya tide gauge between 1936 and 1971. The geopotential value for the datum point was determined, by making use of gravity value in Potsdam Datum. In the adjustment, geopotential numbers were used as observations; then, geopotential numbers, Helmert orthometric heights and Molodensky normal heights of all network stations were estimated. Gravity values in Potsdam Datum were used for the calculation of geopotential number differences between network stations. The adjustment resulted in point heights of precisions varying from 0.3 cm to 9.0 cm depending on the distance from the datum point. Helmert orthometric height system was selected to be used in Turkey for all geodetic and practical applications, although normal heights of network points were computed as well (Ayhan and Demir, 1992).

Turkish National Fundamental GPS Network (TUTGA) was established in 2001 and a number of the stations have been re-surveyed due to the earthquakes happened in 1999-2003. The total number of network stations is about 600. For each station 3-D coordinates and their associated velocities were computed in ITRF96. Positional accuracy of the stations is about 1-3 cm while the relative accuracy is better than about 0.01 ppm. Besides, the GPS network has been connected to the Turkish Horizontal and Vertical Control Networks through specified points and time-dependent coordinates of all stations were being computed in the context of the maintenance of the network with periodic GPS observations (Ayhan et al., 2002). Total of 197 GPS network stations have been connected to Turkish vertical network by precise leveling (Ayhan et al., 2002; Kılıçoğlu, 2002; Kılıçoğlu and Fırat, 2003).

The quasi-geoid heights ($\zeta_{GPS/lev}$) of the 197 collocated stations were computed by subtracting the Molodensky Normal Heights ($H^*_{TUDKA99}$) from ellipsoidal heights ($h_{TUTGA99A}$) as follows.

$$\zeta_{GPS/lev} = h_{TUTGA99A} - H^*_{TUDKA99} \tag{1}$$

The statistics of the differences are given in Table-1, and a histogram graph of the differences is given in Figure-1. The collocated stations and quasi-geoid differences ($\zeta_{GPS/lev} - \zeta_{EGM08}$) between EGM08 and GPS/leveling are shown in Figure-2.

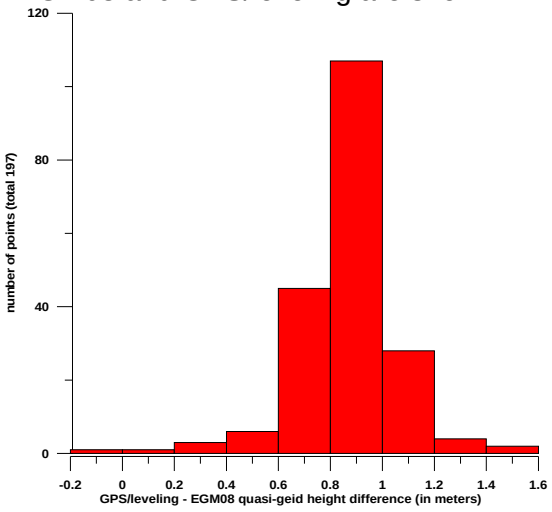


Figure-1: Histogram of the EGM08 - GPS/leveling quasi-geoid height differences

Table-1: Quasi-geoid height differences EGM08 - GPS/leveling.

Number of values	197
Minimum	-0.184 m
Maximum	1.505 m
Mean	0.860 m
Standard deviation	0.189 m

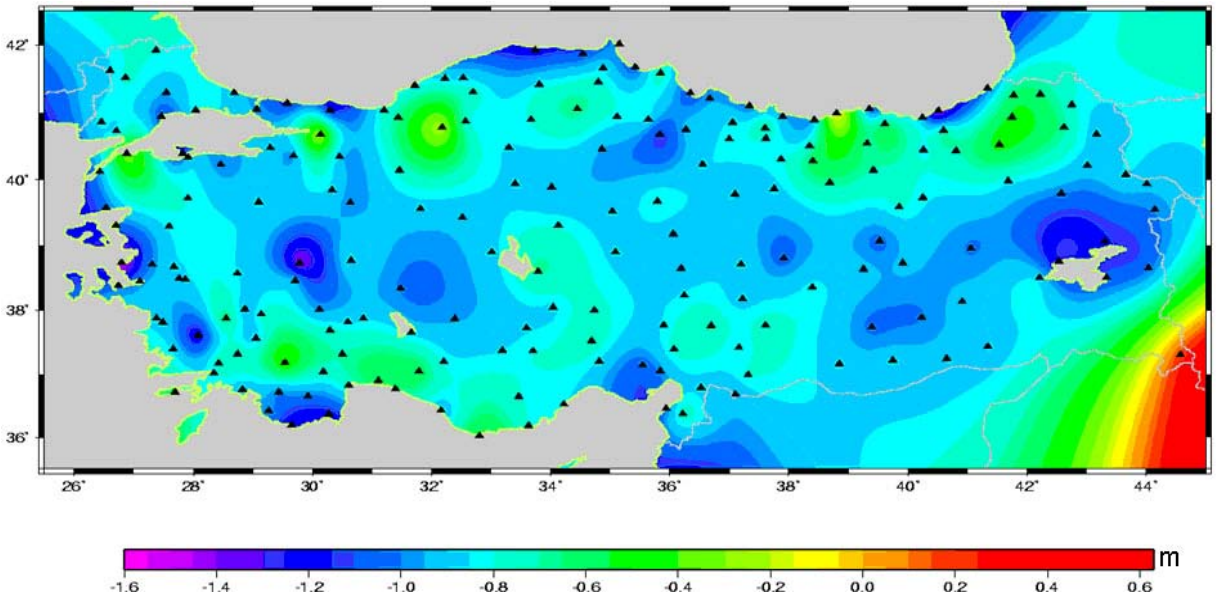


Figure-2: The difference between EGM08 and GPS/leveling quasi-geoid heights in Turkey. Black triangles (197) shows the collocated GPS/leveling stations.

The bias (+86 cm) between two sets is due to definition of local vertical datum (mean sea level at Antalya). GPS observations in Turkey were achieved within the last two decades, whereas precise leveling measurements were achieved during the period of 1936-1994. Since Anatolian plate lies in a very active tectonic region, that is within the boundary zones of Eurasian, African and Arabian tectonic plates, tectonic framework of Mediterranean, in which Anatolia is located, is dominated by collision of Arabian and African Plates with Eurasia. Thus, tectonic regime of Anatolia involves various kinds of tectonic phenomena such as continental collision, strike-slip and thrust faulting, subduction, contraction and extension (Aktug and Kilicoglu, 2006; Aktug et al., 2008). Interseismic vertical rates of GPS stations have been computed by repeated GPS observations within the establishment of TFGN99-A (Ayhan et al, 2002; Aktug et al., 2008). Vertical velocity field of Turkey is given in Figure-3.

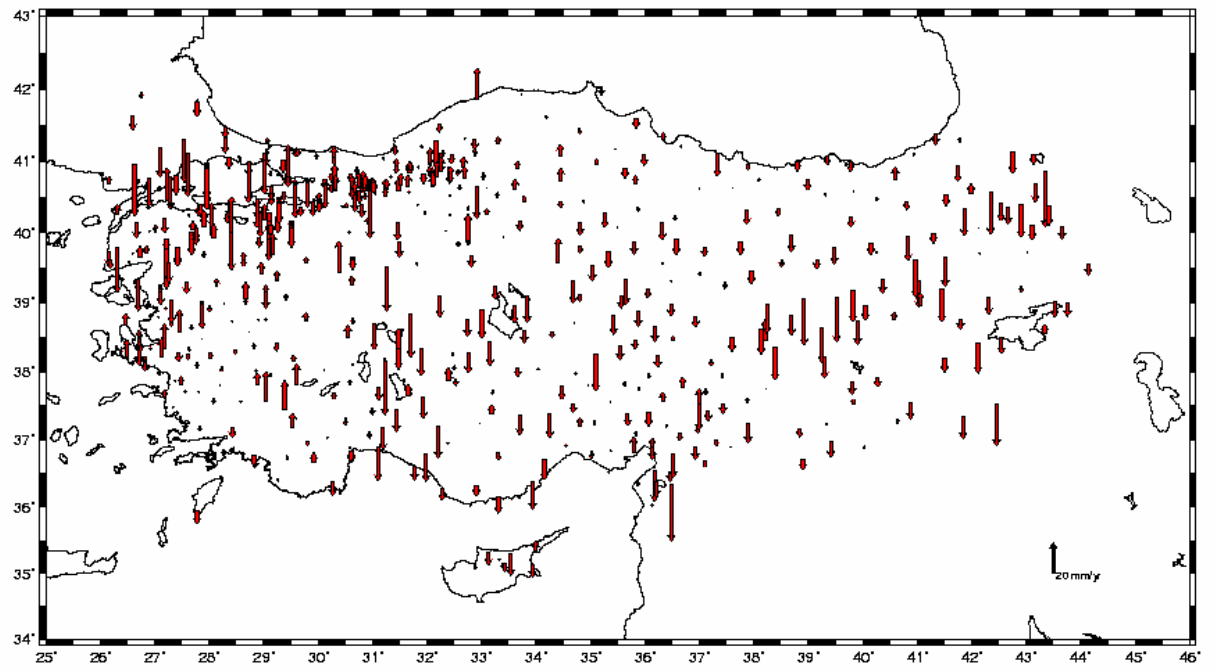


Figure-3 : Vertical velocity field of Turkey as deduced from repeated GPS measurements in the last two decades.

Figure-2 includes interseismic and coseismic (and relatively small effects of postseismic deformation) vertical deformation as well as other network deformations caused by observations and adjustment, whereas Figure-3 shows only interseismic deformation. The differences were also investigated in terms of active fault traces and seismicity. Figure-4 shows that large differences are observed within compresion zones. However, differences in northwestern part of Anatolia (Marmara) which underwent large earthquakes are lower than expected. In this respect, neither active fault traces nor seismicity provide a direct correlation, to which all the differences between EGM08 and GPS/leveling quasi-geoid heights could be attributed. Turkish leveling network has been very much affected by tectonic phenomena and other network distortions. Thus, GPS/leveling quasi-geoid heights may consist of large vertical deformations. Considering the vertical deformation in Turkey in the light of the two figures, the discrepancy between GPS/leveling and EGM08 seems reasonable.

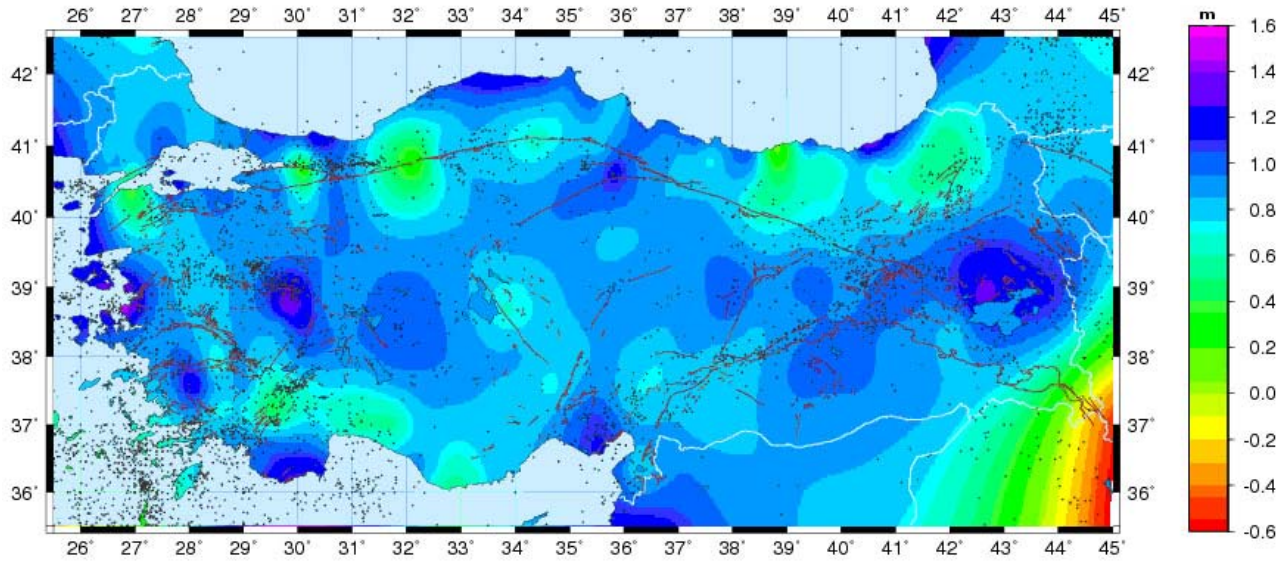


Figure 4. Differences between EGM08 and GPS/Leveling quasi-geoid heights. Earthquakes with magnitudes larger than Mw obtained from KOERI (Kandilli Observatory and Earthquake Research Institue, http://www.koeri.boun.edu.tr). Active fault traces were adapted from (Saroglu et al., 1992).

3. Comparison with a regional gravimetric quasi-geoid height model.

As a precise geoid model referring to a global geocentric datum is essential for determination of the orthometric heights by GPS/leveling, the new Turkish Geoid (TG03) was computed in 2003 as new and more data were available (Kilicoglu, 2005). Heterogeneous data (gravity, topography and geoid heights) were used by Least Squares Collocation (LSC) in a remove-restore procedure (Tscherning et al.,1994). EGM96 (Lemoine et al., 1996) was used as the reference model of the Earth's geopotential model. The data used consist of surface gravity anomalies (on ~ 65000 stations), gravity anomalies derived from ERS1, ERS2 and TOPEX/POSEIDON altimetry data (on ~ 20000 stations), gravity anomalies derived from ship observations (on ~ 10000 stations), GPS/leveling geoid heights (on 197 stations) and topographic heights. Surface gravity values are in Potsdam Datum, and the free air anomalies were computed in GRS80. No surface gravity data were used outside the Turkish border where topographic heights were obtained from GTOPO30 global topography. The residual terrain model (RTM) effect of the topography was computed using a detailed (20"x15") Digital Terrain Model (DTM), coarse (3'x3') DTM and reference (10'x10') DTM. The DTM used consists of high-resolution topographic heights within the borders, and dense bathymetry near the shoreline. Evenly distributed GPS/leveling geoid heights were introduced so as to compute the final geoid in agreement with GPS ellipsoidal heights. As described above, ellipsoidal heights of the GPS/leveling points refer to well-established Turkish National GPS Network (aligned to ITRF96), while orthometric heights refer to Turkish National Vertical Datum.

The geoid heights at 3'x3' grid points within Turkey (25E-46E, 35N-43N) were computed to be further interpolated in practical use in accordance with Turkish National Vertical Datum. The final geoid was tested at GPS/leveling test stations which were not used in the computations, and the external accuracy was found to be within 10 cm as varying with respect to the data distribution and density. Actually, the quasi-geoid heights were computed, then combined with GPS/leveling, and finally transformed to geoid heights by making use of the formula below.

$$N = \varsigma + \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H \cong \varsigma + \frac{\Delta g_{BO}}{\bar{\gamma}} H \qquad (2)$$

Where, $\bar{g}$ is the mean gravity along the plumb line between geoid and the physical earth, $\bar{\gamma}$ is the mean normal gravity along the normal plumb line between ellipsoid and telluroid, Δg_{BO} is Bouguer gravity anomaly and H is orthometric height. (Hoffmann-Wellenhof and Moritz, 2005, p.326).

The quasi-geoid heights from EGM08 and TG03 (unfitted to GPS/leveling) were compared at 3'x3' grid nodes (Figure-5). Figure-6 and Table-2 show the statistics of the differences.

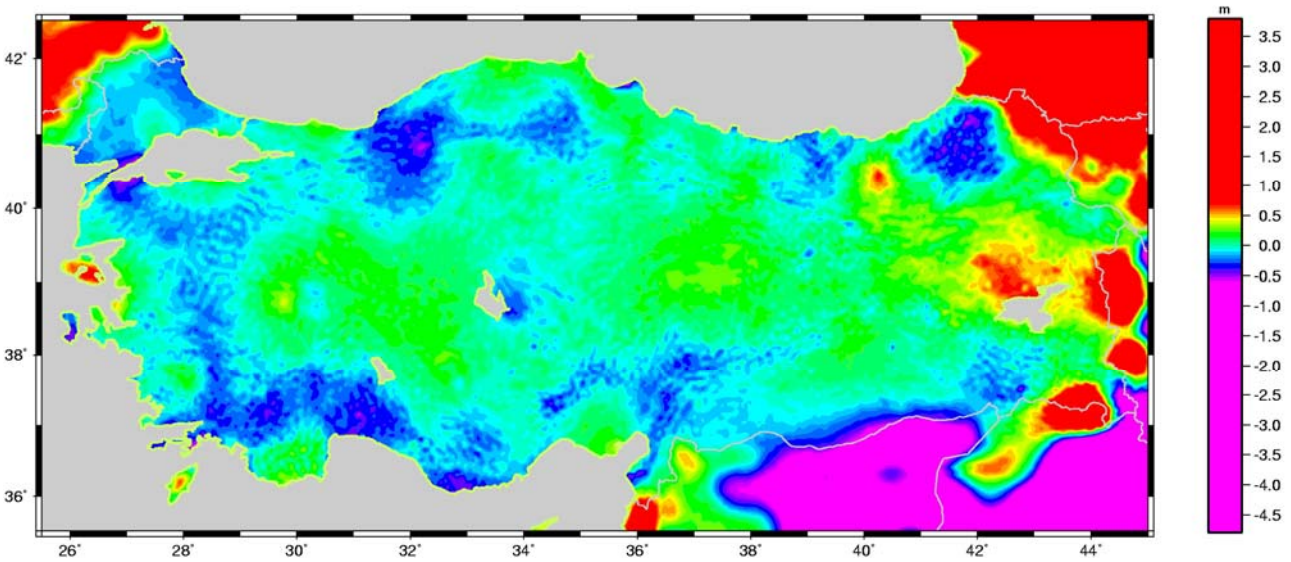


Figure-5: The difference between EGM08 and TG-03 quasi-geoid heights in Turkey. The data within boundaries are taken into account.

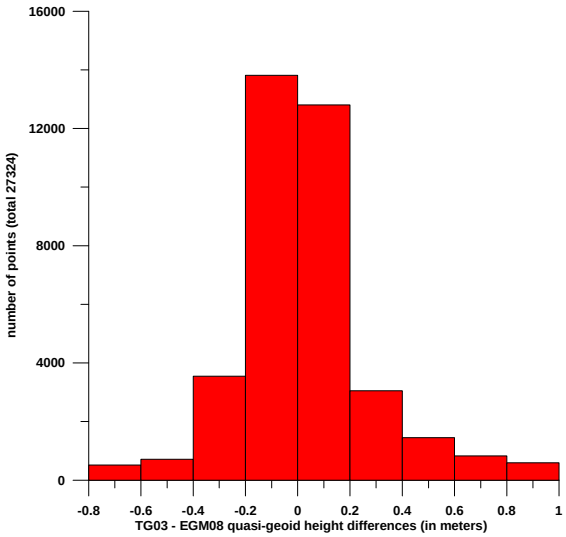


Figure-6: Histogram of the GPS/lev - EGM08 quasi-geoid height differences. Total number of points compared is 37324.

Table-2: Quasi-geoid height differences TG03 – EGM08

Number of values	37324
Minimum	-0.75 m
Maximum	0.99 m
Mean	0.02 m
Standard deviation	0.25 m

4. Comparison with surface free-air gravity anomalies

Surface free air gravity anomalies at 64992 stations in Turkey are used for EGM08 evaluation. Surface gravity anomalies are computed in Potsdam datum and referred to GRS80 normal gravity field. Gravity observations were carried out by using LaCoste&Romberg Gravimeters with reference to Turkish National Gravity Network-1956, of which point gravity accuracies vary between 0.07 to 0.19 mGal (Demir et al., 2006).

EGM08 free air gravity anomalies at observation points are computed as follows (Holmes and Pavlis, 2006; Pavlis et al., 2008).

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r} \tag{3}$$

The distribution of the surface gravity data used in comparison is given in Figure-7 and the differences between EGM08 and surface free air gravity anomalies in Turkey are shown in Figure-8. Figure-9 and Table-3 show the statistics of the differences.

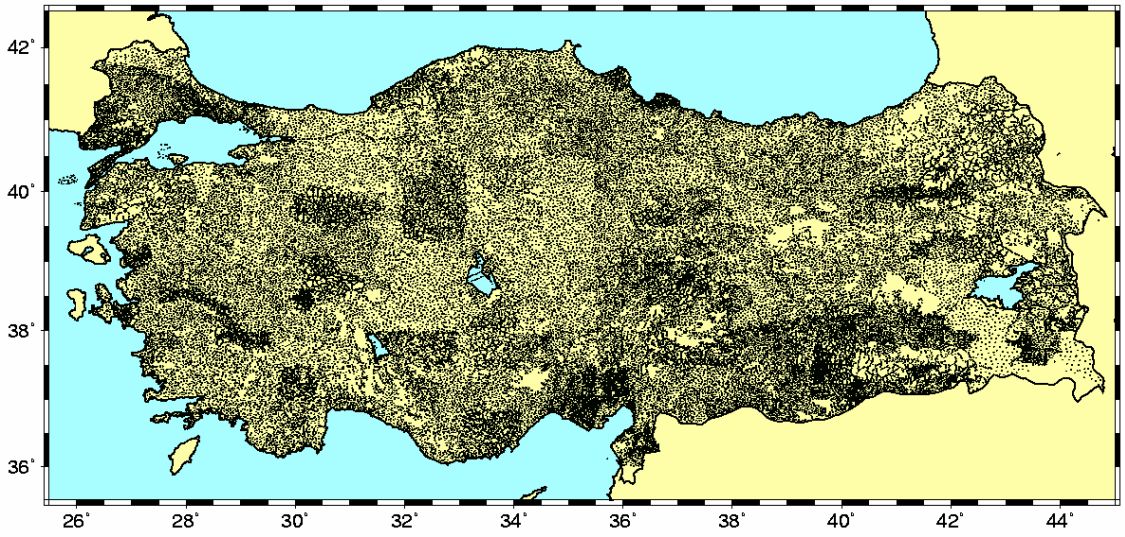


Figure-7: The distribution of the surface gravity data used in comparison in Turkey.

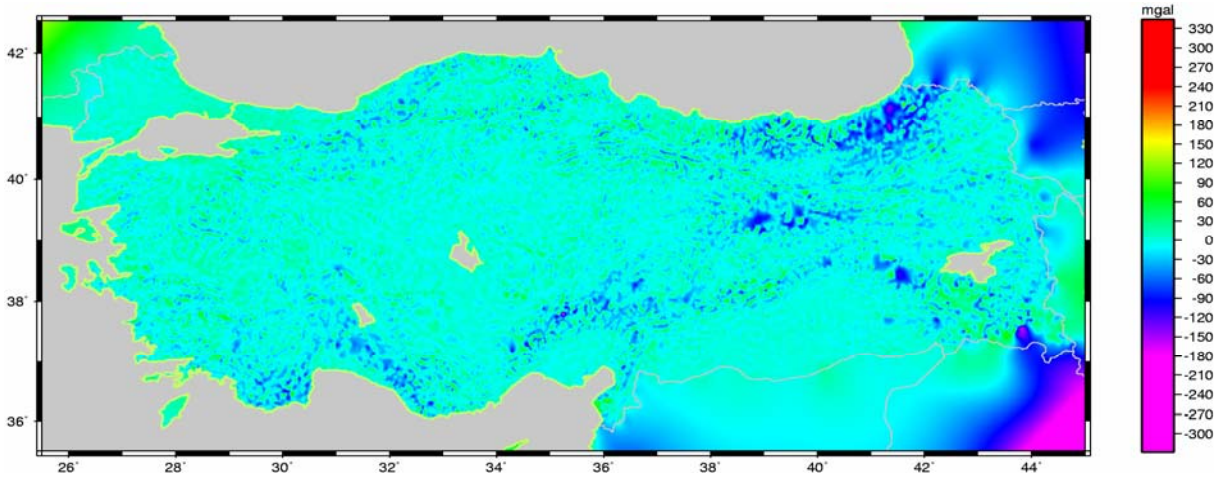


Figure-8: The difference between EGM08 and surface free air gravity anomalies in Turkey. The data within boundaries are taken into account.

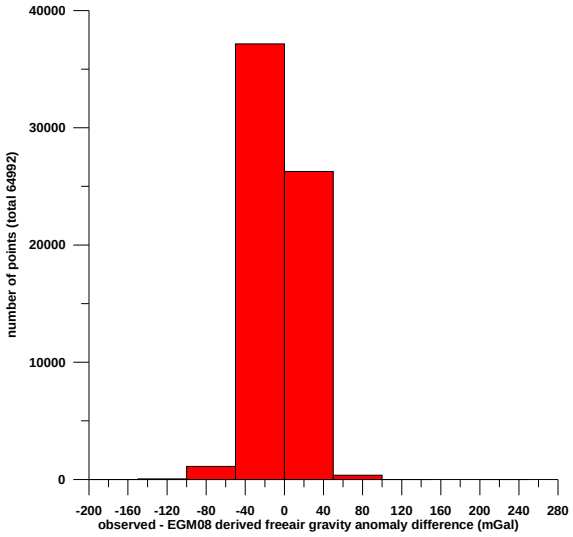


Figure-9: Histogram of the differences between EGM08 and surface free gravity anomalies.

Table-3: Statistics of the differences between EGM08 and surface free gravity anomalies.

Number of values	64992
Minimum	-177.29 mGal
Maximum	238.57 mGal
Mean	-3.28 mGal
Standard deviation	18.36 mGal

5. Conclusions

A new Earth Gravitational Model (EGM2008) to degree 2160 has been released to IAG's EGM Evaluation Group. The objective of this study was to evaluate EGM08 derived quantities with Turkish proprietary surface data.

The evaluation of the EGM08 was based on the comparisons with other external data. In this study, we compared EGM2008 Tide Free Model with regional gravity, quasi-geoid height and GPS/leveling as external data sets. The EGM08 derived quantities were compared with the GPS/leveling quasi-geoid heights, existing GPS/leveling fitted regional quasi-geoid model, and the surface free-air gravity anomalies in Turkey within boundaries 26^{o}E-45^{o}E and 36^{o}N-42^{o}N.

EGM08 derived height anomalies were compared with observed GPS/leveling observed quasi geoid heights at 197 stations spread over the Turkish territory. The statistics of the differences two sets of height anomalies ($\zeta_{GPS/lev} - \zeta_{EGM08}$) are given in Table-1 (mean:+86 cm, st.dev.; 18.9 cm). The bias and the discrepancies between two data sets should be further investigated considering the tectonic phenomena and other network deformations in and around Turkey.

Regional gravimetric quasi-geoid height model for Turkey (TG03) was compared to EGM08. TG03 was computed based on EGM96 with respect to GRS80 ellipsoid. The statistics of the differences (TG03 – EGM08) are given in Table-2 (mean: +2cm, st.dev.: 25 cm). TG03 and EGM08 have no bias factor with respect to each other. The differences show highs and lows in regions with rough topography in Turkey. The large differences might be caused by different resolutions of TG03 and EGM08, and by the lack of data outside Turkish border in TG03 model.

Surface free air gravity anomalies, in Potsdam datum and referred to GRS80 at 64992 stations, in Turkey were used for the evaluation of EGM08. The statistics of the differences (EGM08-observed) of surface gravity anomalies were given in Table-3 (mean: -3.28 mGal, st.dev.: 18.36 mGal). Some highs are seen in the regions with rough topography. This might be caused by the different resolution of DTMs used in EGM08 and reduction of observed gravity.

As Turkish proprietary gravity and GPS/leveling data were not used in the EGM08 computations this work provides an external control for EGM08 derived surface gravity and height anomalies on the physical surface of the earth.

The evaluation of EGM08 over Turkey shows a good agreement in overall. We think that EGM08 will cotribute much to the computation of a new geoid model for Turkey, as well as various geosicence applications.

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Evaluation of the Earth Gravity Model EGM2008 in Algeria

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Abstract

The present work focuses on the comparison between the (EGM2008) that was recently released by the NGA (*National Geospatial-Intelligence Agency, U.S*)/EGM-development team and supplied to a new Joint Working Group established between IGFS and the IAG commission 2 for validation, with land gravity anomalies supplied by the B.G.I. and a pre-processed 5'x 5' grid of the free air anomalies covering the area bounded by the limits $16° \leq \varphi \leq 40°$ and $-10° \leq \lambda \leq 14°$ provided us by GETECH, some of the precise GPS data collected from the international TYRGEONET (*TYRhenian GEOdynamical NETwork*), ALGEONET (*ALGerian GEOdynamical NETwork*) projects with baseline length ranging from about 1 to 1000 km and Algerian gravimeric geoid model based on OSU91A geopotential model in order to assess its quality in Algeria region. Additional comparisons of the terrestrial point data (such as the gravity data and the GPS-based geoid heights used in this study) with the corresponding values obtained from other geopotential models were made. Six global geopotential models were used in this comparison: The new GRACE satellite-only and combined models EIGEN-GRACE02S and GGM02C, the combined CHAMP and GRACE model EIGEN-CG01C, combined CHAMP and LAGEOS model EIGEN-GL04C, OSU91A and EGM96. The comparisons were made at all gravity and GPS levelled data by computation of the residual data (i. e. observed data minus model). The geopotential model that provides the closest statistical fit to these data can be assumed to be the most suitable model to adopt for the determination of the new Algerian gravimetric geoid.

The study shows that all tested models are an improvement over OSU91A geopotential model used in all previous Algerian geoid computations and that new released combined model (EGM2008) is relatively superior to other models in the Algerian region. According to our numerical results, the EGM2008 model fits best the observed values used in this investigation. Its standard deviation fit with GPS/levelling data is 21.2cm and 19.6cm before and after the bias and tilts fitting using four parametric transformation model.

Key words: Geopotential model, TYRGEONET and ALGEONET projects, GPS/Levelling.

1. Introduction

The choice of the best geopotential model to reduce geodetic data is one of the critical steps in computing the geoid. Several studies have shown that the geopotential models tailored to regional or local gravity data are best suited for many applications in solid Earth sciences, as e.g. to study the structure of the Earth, to compute the orbit of a satellite, and for high precision geoid computations.

Over the last 40 years, continuous improvements and refinements to the basic theory have been paralleled by the availability of more accurate and complete data and by improvements in the computational resources available for numerical modelling studies. These advances have led to

the development of a sequence of global geopotential models of increasing spherical harmonic degree and order, and hence resolution. The most recent models are released from satellite gravity missions CHAMP and GRACE and will be mapping the Earth's gravity field with significantly increasing accuracy and spatial resolution. The data obtained from these missions are being and will be used to develop a series of new static satellite-only gravity models down to 150 – 200 km wavelength, as well as combined Earth Gravity Models (EGM's) down to about 20 km wavelength. In 2008, the official Earth Gravitational Model EGM2008 has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This model is complete to degree and order 2159, and contains additional spherical harmonic coefficients (SHCs) extending to degree 2190 and order 2159. It represents a spatial resolution of 5 minutes of arc (about 9 km).The purpose of this work is to give a brief summary of the evaluation results for the Earth gravity model (EGM2008) in Algeria region. In addition, the Geopotential Models (GGMs) derived from the new satellite missions LAGEOS, CHAMP and GRACE (EIGEN-GRACE02S, GGM02C EIGEN-CG01C and EIGEN-GL04C), OSU91A and the most accurate high degree geopotential model EGM96 are also compared with land gravity anomalies and GPS/Levelling geoid heights in Algeria in order to find the GGM that best fits the local gravity field feautures over this region. The first one (EIGEN-GRACE02S) is developed to degree and order 150 while the GGM02C solution was created to degree and order 200. All remaining geopotential models are completed to degree and order 360. These comparisons are performed with and without filtering. Several techniques are possible to do that filtering. A sample for the filtering using a very simple approach is shown in the next paragraph. This is done in order to tune the validation data within the same spectral bandwidth provided by the EGM.

In the next sections the data used for the comparison will be described and the results of the comparison will be shown. The statistical parameters considered in this work are the mean and the standard deviation of the differences between the geopotential models and tested data. The most informative of these statistics is the standard deviation because the mean of the differences is distorted by the exclusion of the zero-order term. Therefore, the best fitting GGM will have the lowest standard deviation between itself and the tested gravity.

2. Data used

2.1. Gravity data

Two data sets of free gravity anomalies were used for the comparison as control data:

- A set of 12472 terrestrial gravity anomalies covering the territory of Algeria, was supplied by the B.G.I. No information concerning the accuracy of the data is available. The data set is referred to the IGSN71 gravity datum and processed using the GRS80 gravity formula. All data have been checked and duplicate points removed in a consistent manner. Figure 1 shows the geographical distribution of the BGI gravity data.

- A pre-processed 5'x 5' grid of the free air anomalies covering the area and bounded by the limits $16° \leq \varphi \leq 40°$ and $-10° \leq \lambda \leq 14°$. This grid containing 289 x 289 points has been provided us by GETECH through the agreement between the National Centre of Space Techniques/Geodetic Laboratory and University of Leeds/GETECH without any information on the accuracy of the different values. Figure 2 gives a graphical representation of the gravity data coverage in the computation area. From this figure, it becomes clear that the coverage with gravity observations is

not sufficient for some land areas particularly in the south of the Algeria and new measurements are needed to accomplish a homogeneous coverage.

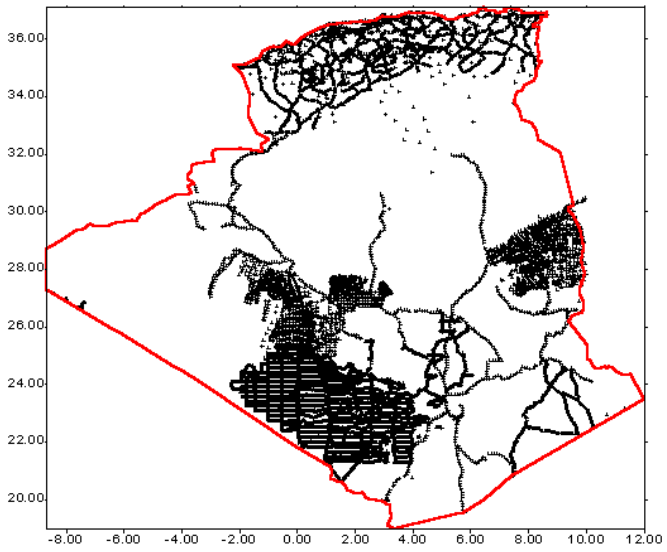


Figure 1. Geographical distribution of BGI gravity measurements

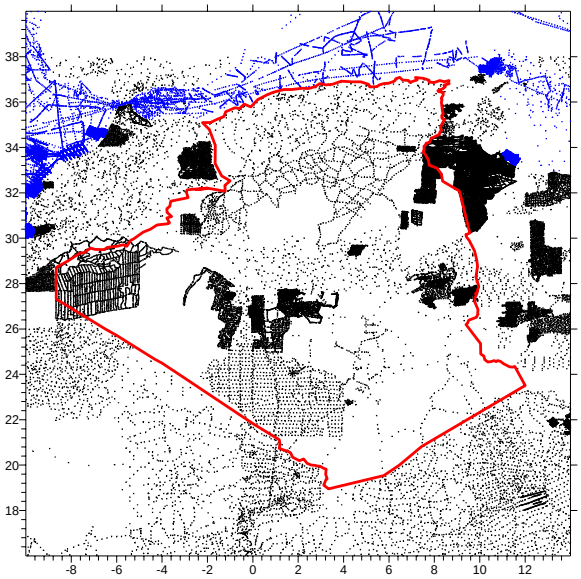


Figure 2. Geographical distribution of GETECH gravity data

2.2. GPS/Levelling data

There are several GPS/levelling points distributed over some regions of Algeria, principally in the northern part of the country. The distribution is fairly good but the total number of the GPS stations is too small in relation to the area of the northern part of Algeria. For this investigation, 71 precise GPS levelled points have been used for the evaluation and validation of the new gravimetric geoid of which 45 are benchmarks of the first order levelling network, and the others belong to the second order levelling network. All of these points are located in the north of Algerian territory between the limits $31° \leq \varphi \leq 37°$ and $-2° \leq \lambda \leq 9°$. The geographical distribution of the available GPS/levelling data is shown in Figure 3. The GPS observations were performed using ASHTECH Z-12 dual frequency receivers with an observation period between 3 and 12 hours and were processed with the Bernese GPS software version 4.2 developed at the Astronomical Institute of the University of Bern (Beutler et al., 2001) using the precise ephemerides supplied by IGS. The computed ellipsoidal heights were referred to WGS84 system and their standard deviations do not exceed 3 cm. All GPS stations have been connected by traditional levelling to the national levelling network, which gives orthometric heights. The accuracy of the levelling heights may be estimated at about 6 cm depending on the type of connection measurements because some GPS points used in this work as benchmarks are located in mountainous regions in which the spirit levelling would be impractical. Unfortunately, the non-availability of GPS levelling data in the whole of the country with a homogeneous distribution and sufficient density does not allow a more reliable assessment, at the national scale, of the quality of the global geopotential model EGM2008.

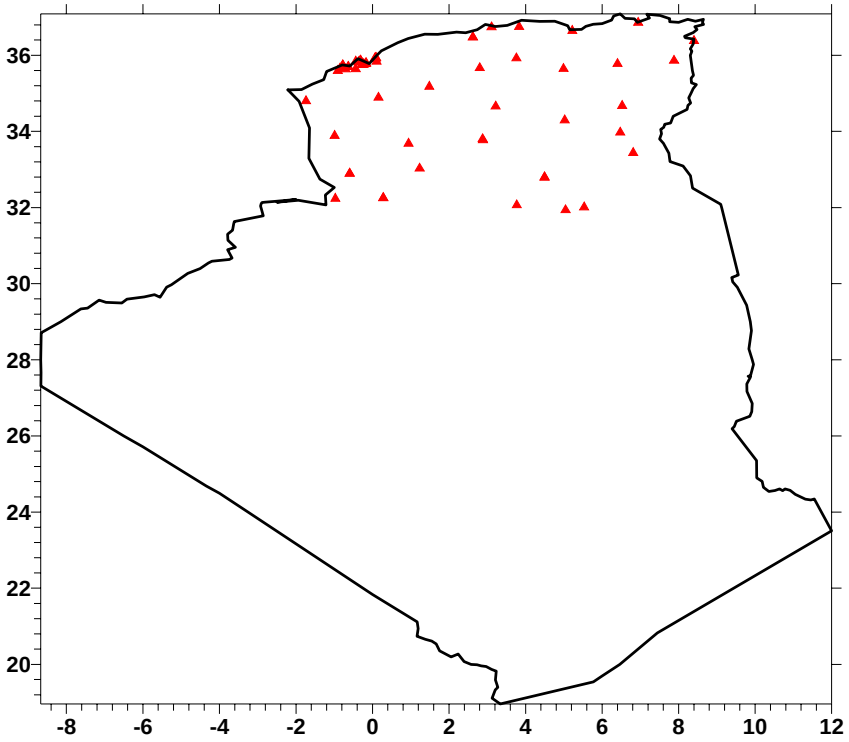


Figure 3. Geographical distribution of GPS/Levelling stations in the northern of Algeria

2.3. Algerian gravimetric geoid model

In view of the use of the GPS for the orthometric height computation, the National Centre of Space Techniques through the national projects of research, has recently focused a part of the current research on the precise geoid determination using different methods. The most recent solution of a preliminary geoid over the Algerian territory was done in 2002 using the spectral combination technique in connection with the remove-restore procedure (Forsberg and Sideris, 1993). For this computation, the pre-processed 5'x 5' grid of the free air anomalies covering the area bounded by the limits $16° \leq \varphi \leq 40°$ and $-10° \leq \lambda \leq 14°$, derived by merging terrestrial gravity data and satellite altimetry data, have been used. This grid contains 289 x 289 points and has been provided us by GETECH. The computation of the effects of the topography according to the RTM reduction modelling method (Forsberg, 1985) is based on the global topographic model GLOBE of 30" x 30". However, for the long wavelength gravity field information the spherical harmonic model OSU91A completed to degree and order 360 (Rapp et al., 1991) was employed. The final quasi-geoid was obtained by adding the geopotential model contribution and the residual terrain effect on the 5' x 5' residual quasi-geoid grid. The major contributions to the final quasi-geoid are coming from the OSU91A geopotential model. The standard deviations of the contributions from gravity data and DEM are ± 1.4 m and ± 0.007 m respectively. So, we will note that in the Algerian territory, the indirect quasi-geoid effect is significant only in mountainous areas where it reaches values from one to a few decimeters. On the remaining territory it is on the level of a few millimeters. However, the Algerian height system is based on orthometric heights, so the gravimetrically determined quasi-geoid has been transformed to a geoid model and then compared to geoid undulations provided by GPS and levelling (Benahmed Daho and Fairhead, 2004).

3. Evaluation results

3.1. Comparison with free gravity anomalies

The free air gravity anomalies from BGI and GETECH are compared with corresponding values computed from the tested geopotential models, and the smallest residuals imply the best Earth Geopotential Model. These comparisons are performed in both cases with and without filtering. In first case (with filtering), before comparing them and in order to make a fair comparison and taking into account the EGM's omission error, all tested geopotential models were truncated to degree and order 150 that represents the current limit for the new GRACE satellite-only EIGEN-GRACE02S. BGI and GETECH free air gravity anomalies are low-pass filtered using the high degree geopotential model EGM2008 from degree 151 to degree 2190, i.e. free air gravity information in this spectral range is subtracted from the BGI and GETECH free-gravity anomalies data sets before they are compared to the corresponding quantities obtained from the tested EGM (Gruber, 2004). The results for these comparisons in both cases are summarised for BGI free air gravity anomalies in Table 3, and for GETECH gravity data in Table 4 (the statistics given in bold within the parentheses refer to the values obtained with filtering). All the original data are referred to GRS80. The computations were carried out using the FORTRAN program harmonic_synth_v02 developed by NGA and supplied to a new Joint Working Group established between IGFS and the IAG Commission 2 for validation and quality assessment of GRACE, CHAMP and GOCE-based satellite-only and combined solutions for the Earth's static gravity field. Figure 4 shows the histograms of the differences between BGI free air anomalies and those obtained from all tested geopotential models without filtering.

Geopotential models	Minimum	Maximum	Mean	σ
OSU91A	-97.035 **(-106.680)**	125.561 **(68.215)**	0.300 **(1.308)**	13.164 **(6.534)**
EGM96	-100.959 **(-101.977)**	112.026 **(56.941)**	-2.170 **(-0.942)**	13.542 **(6.683)**
EIGEN-CG01C	-98.959 **(-103.739)**	110.187 **(58.515)**	-2.103 **(-0.704)**	14.056 **(6.649)**
EIGEN-GL04C	-99.362 **(-104.522)**	112.031 **(58.750)**	-1.924 **(-0.531)**	14.029 **(6.659)**
GGM02C	-93.885 **(-104.938)**	123.040 **(57.294)**	-1.123 **(-0.566)**	14.910 **(6.345)**
EIGEN-GRACE02S	-90.684 **(-106.680)**	144.810 **(68.215)**	-1.136 **(1.308)**	17.196 **(8.413)**
EGM2008	-104.094 **(-104.094)**	55.925 **(55.925)**	-0.610 **(-0.610)**	6.119 **(6.119)**

Table 3. Statistics of the reduced data between the BGI gravity data and the geopotential models (mgal).

From the Table 3 and in both cases (with and without filtering), we can see that the free air gravity anomalies computed from EGM2008 model have been significantly improved as compared to other tested geopotential models.

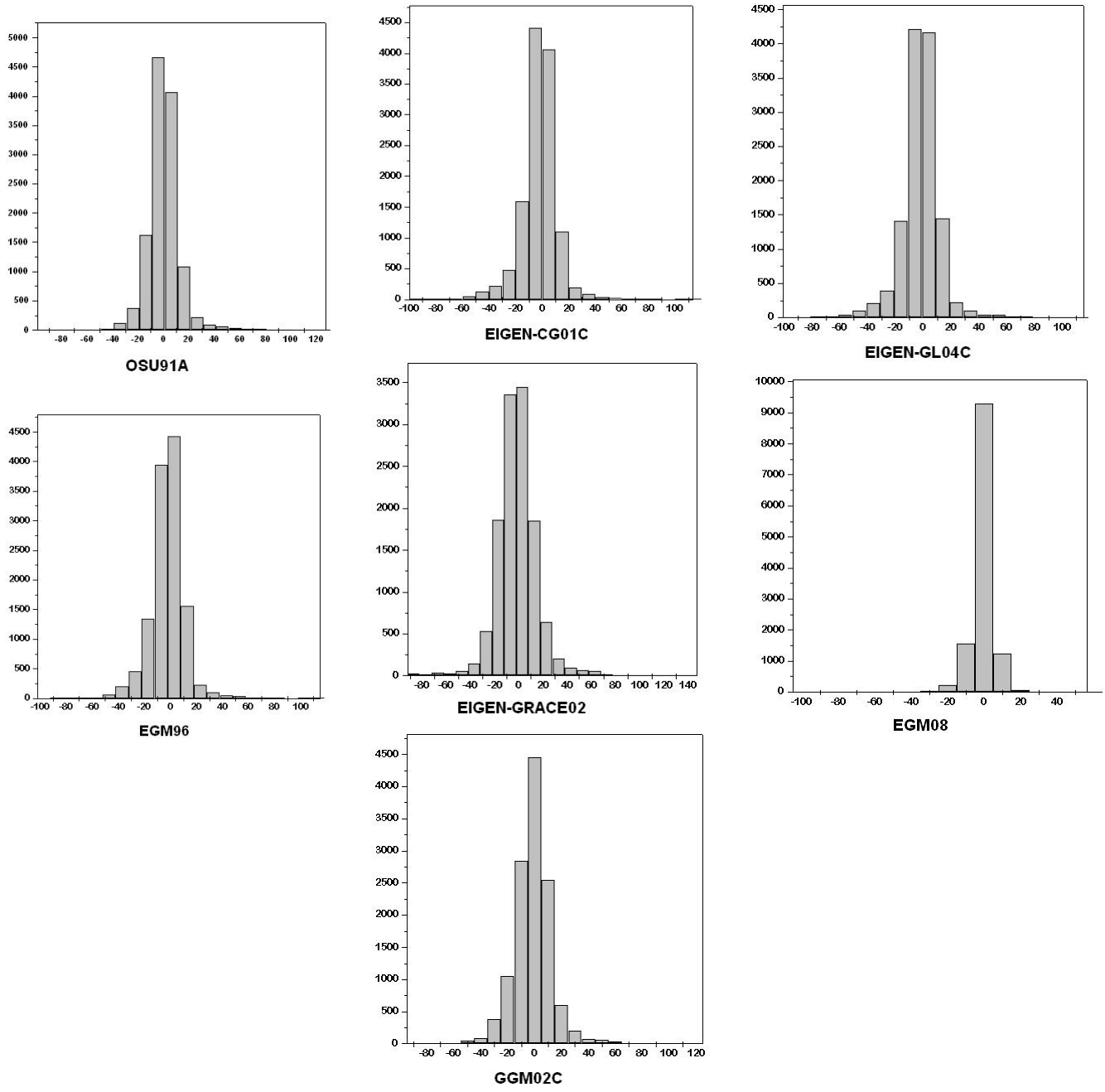


Figure 4. Histograms of difference between BGI free air anomalies and those computed from tested geopotential models. The full spectral range for all tested geopotential models has been used.

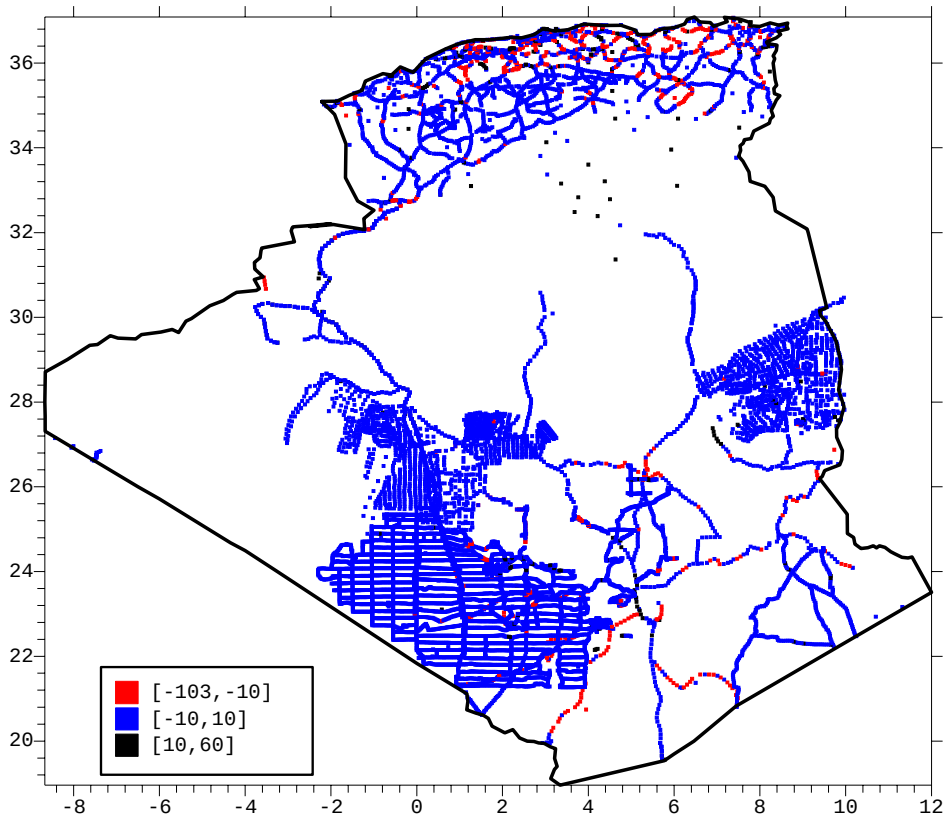


Figure 5. Free air gravity anomalies differences between the BGI gravity data and EGM2008 model (mgal).

The figure 5 represents the classed post map of the differences between the BGI free air anomalies and corresponding ones from EGM2008 model. According to the used BGI land gravity data, the EGM2008 geopotential model is able to recover gravity anomalies over 92.4% of the Algerian territory to within 10 mgal (see figure 5). The majority of the free-air gravity anomalies computed from EGM2008 over land show a good correspondence with the land free-air gravity anomalies, even in areas where there are large gravity anomaly gradients such as central Algeria (see figure 5). The largest differences are in the mountainous regions and along Algerian coastline.

Geopotential models	**Minimum**	**Maximum**	**Mean**	σ
OSU91A	-122.11 **(-261.27)**	190.57 **(108.86)**	-2.25 **(-2.24)**	14.82 **(15.27)**
EGM96	-143.06 **(-269.45)**	167.09 **(113.80)**	-2.57 **(-2.56)**	15.51 **(15.50)**
EIGEN-CG01C	-133.90 **(-270.82)**	170.06 **(111.68)**	-2.62 **(-2.60)**	15.66 **(15.50)**
EIGEN-GL04C	-133.85 **(-269.63)**	170.78 **(114.37)**	-2.60 **(-2.58)**	15.66 **(15.50)**
GGM02C	-139.58 **(-271.64)**	183.38 **(113.27)**	-2.57 **(-2.58)**	17.65 **(15.42)**
EIGEN-GRACE02S	-142.36 **(-283.99)**	215.04 **(121.03)**	-2.60 **(-2.62)**	19.69 **(16.75)**
EGM2008	-271.94 **(-271.94)**	110.91 **(110.91)**	-2.60 **(-2.58)**	15.35 **(15.35)**

Table 4. Statistical comparisons between geopotential models and GETECH free-air gravity anomalies (mgal).

From the statistics of Table 4, it is unclear which model consistently gives the best agreement with GETECH free-air anomalies. We can see that all geopotential models excepting GRACE satellite-only gravity model give almost the same results in terms of the standard deviation. This is because no new gravity data have been used in this region for the determination of the tested GGM models compared to OSU91A model.

4.2. Comparison with GPS/Levelling data

The global geopotential models discussed above were also compared with a number of GPS/Levelling data set available only in the northern of Algeria. The comparisons were performed with and without filtering using the same procedure described above. At these 71 points, both h (ellipsoidal height) and H (orthometric height) are known. The GPS/levelling height N^{GPS} is the result of the difference between the ellipsoidal height obtained by GPS and the orthometric one obtained by spirit levelling and gravity information. However, the geoid undulation derived from GPS/Levelling refers to the GRS80 ellipsoid and their corresponding values computed from GGM refer to a mean Earth ellipsoid that does not have the same dimensions as the WGS84 ellipsoid and hereby, it is necessary to take into account the effect of the different equatorial radius in the computation of the geoid undulation using the spherical harmonic expansion for each model. The statistics of the differences in benchmarks before and after fitting the systematic biases and tilts using a four-parameter model for both cases (without and with filtering) are summarised in Table 5 and Table 6 respectively. In all cases, the statistics shown refer to the values without fitting, whereas the statistics given within the parentheses refer to the values of the residuals obtained after the fitting of the following four-parameter transformation model (Heiskanen and Moritz , 1967):

$$\mathbf{N_i^{EGM} - N_i^{GPS} = \cos\Phi_i \cos\lambda_i . x_1 + \cos\Phi_i \sin\lambda_i . x_2 + \sin\Phi_i . x_3 + x_4 + v_i} \quad \textbf{(1)}$$

where ($\mathbf{N^{EGM}}$) is interpolated geoid undulation at a network of GPS benchmarks from geopotential models, ($\mathbf{N^{GPS}}$) is the corresponding GPS/levelling-derived geoid height, $\mathbf{x_4}$ is the shift parameter between the vertical datum implied by the GPS/levelling data and the gravimetric datum, $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ are the shift parameters between two 'parallel' datums and v_i denotes a residual random noise term. The vector of unknown parameters is solved by minimizing the quantity $\mathbf{v^T v}$. The adjusted values for the residuals $\mathbf{v_i}$ give a realistic picture of the level of absolute agreement between the tested geopotential models based geoid and the GPS/levelling data.

Geopotential models	Minimum	Maximum	Mean	σ
OSU91A	-3.006 **(-1.500)**	1.747 **(1.400)**	0.202 **(0.000)**	1.194 **(0.491)**
EGM96	-0.896 **(-0.750)**	0.779 **(0.520)**	-0.028 **(0.000)**	0.340 **(0.294)**
EIGEN-CG01C	-0.740 **(-0.687)**	0.681 **(0.532)**	-0.009 **(0.000)**	0.357 **(0.302)**
EIGEN-GL04C	-0.633 **(-0.530)**	0.637 **(0.676)**	-0.016 **(0.000)**	0.333 **(0.295)**
GGM02C	-0.987 **(-1.195)**	0.924 **(0.805)**	0.258 **(0.000)**	0.480 **(0.381)**
EIGEN-GRACE02S	-1.817 **(-1.908)**	1.437 **(1.140)**	0.468 **(0.000)**	0.834 **(0.582)**
EGM2008	-0.666 **(-0.558)**	0.610 **(0.517)**	-0.077 **(0.000)**	0.212 **(0.196)**

Table 5. Comparison of all geoid undulations from geopotential models with GPS/Levelling heights before and after the bias and tilt fitting (m). The full spectral range for all tested geopotential models has been used.

When filtering was not applied (see Table 5), we can see that the Earth Gravity Model EGM2008 fits better than the other models the GPS/Levelling heights. The geoid undulations computed from this model have been significantly improved as compared to other tested models.

Geopotential models	Minimum	Maximum	Mean	σ
OSU91A	-2.804	1.635	-0.152	0.997
	(-1.271)	**(1.293)**	**(0.000)**	**(0.565)**
EGM96	-1.008	0.693	-0.252	0.317
	(-0.566)	**(0.377)**	**(0.000)**	**(0.191)**
EIGEN-CG01C	-0.787	0.529	-0.207	0.222
	(-0.574)	**(0.543)**	**(0.000)**	**(0.214)**
EIGEN-GL04C	-0.777	0.301	-0.220	0.207
	(-0.516)	**(0.469)**	**(0.000)**	**(0.197)**
GGM02C	-0.679	0.527	-0.071	0.212
	(-0.571)	**(0.495)**	**(0.000)**	**(0.199)**
EIGEN-GRACE02S	-0.990	1.725	-0.078	0.447
	(-1.213)	**(0.600)**	**(0.000)**	**(0.396)**
EGM2008	-0.666	0.610	-0.077	0.212
	(-0.558)	**(0.517)**	**(0.000)**	**(0.196)**

Table 6. Comparison of all geoid undulations from geopotential models with GPS/Levelling heights before and after the bias and tilt fitting (m). The filtering has been applied to the GPS based geoid heights.

From the statistics of Table 6 (with filtering), the standard deviations values of the differences show significant improvements with respect to OSU91A. All other models excepting GRACE satellite-only gravity model present very similar results. The best agreement (EIGEN-GL04C) is at the ± 20.7cm level in terms of the standard deviation of the differences, before the bias and tilt fit. For the OSU91A, EGM96, EIGEN-CG01C, GGM02C, EIGEN-GRACE02S and EGM2008 models, it is at the ± 99.7cm, ± 31.7cm, ± 22.2cm, ± 21.2cm, ± 44.7cm and ± 21.2cm respectively. After the bias and tilt fit, the improvement is almost at the 43.2cm level for OSU91A, the 12.6cm for EGM96, and the 8mm for EIGEN-CG01C, the 1cm for EIGEN-GL04C, the 1.3cm for GGM02C model, the 5.1cm for GRACE02S model, and 1.6cm for the new released model EGM2008. In addition, we can see that the results from EGM96 after fitting remain very close to those given by the EGM2008 model in terms of the standard deviation. The two curves present the same behaviour (see Figure 6). No significant differences between these curves are shown. Based on the data sets used in the present work, the new released Earth Gravity Model EGM2008 is consistently superior to other tested geopotential models.

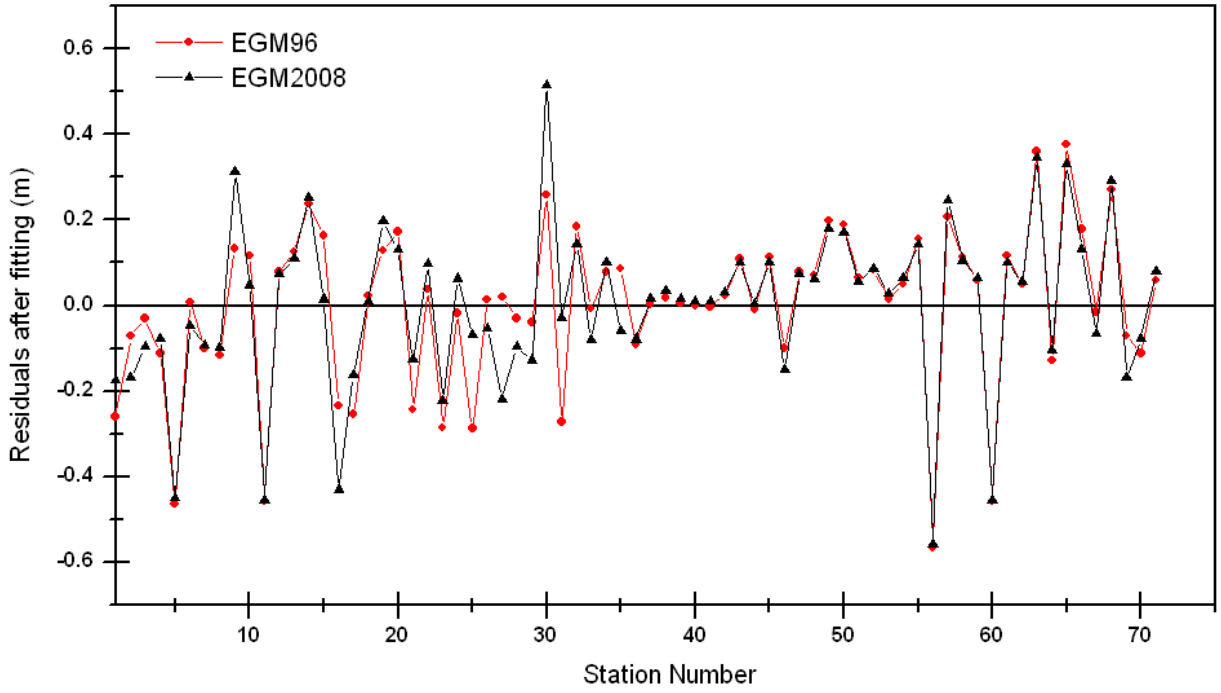


Figure 6. Residuals in benchmarks, after fitting and with filtering, from EGM96 and their corresponding from EGM2008 (m).

3.3. Comparison of the EGM2008 with gravimetric geoid model

The difference between gravimetric geoid model for Algeria and geoid undulations computed using EGM2008 model complete to degree and order 2190 range between -6.63m and 6.11m with an average of 0.54m and a standard deviation of about ±1.29m. The large discrepancies in Algeria, with the maximum (+6m) occurring in the South and South-Western regions. No terrestrial data were available for these areas. The maximum negative difference occurs in Mediterranean Sea, outside the area of interest (see figure 7). These large discrepancies are attributed, principally, to GETECH gridded gravity data quality used in the computation of the gravimetric geoid solution for Algeria (Benahmed Daho and Fairhead, 2004).

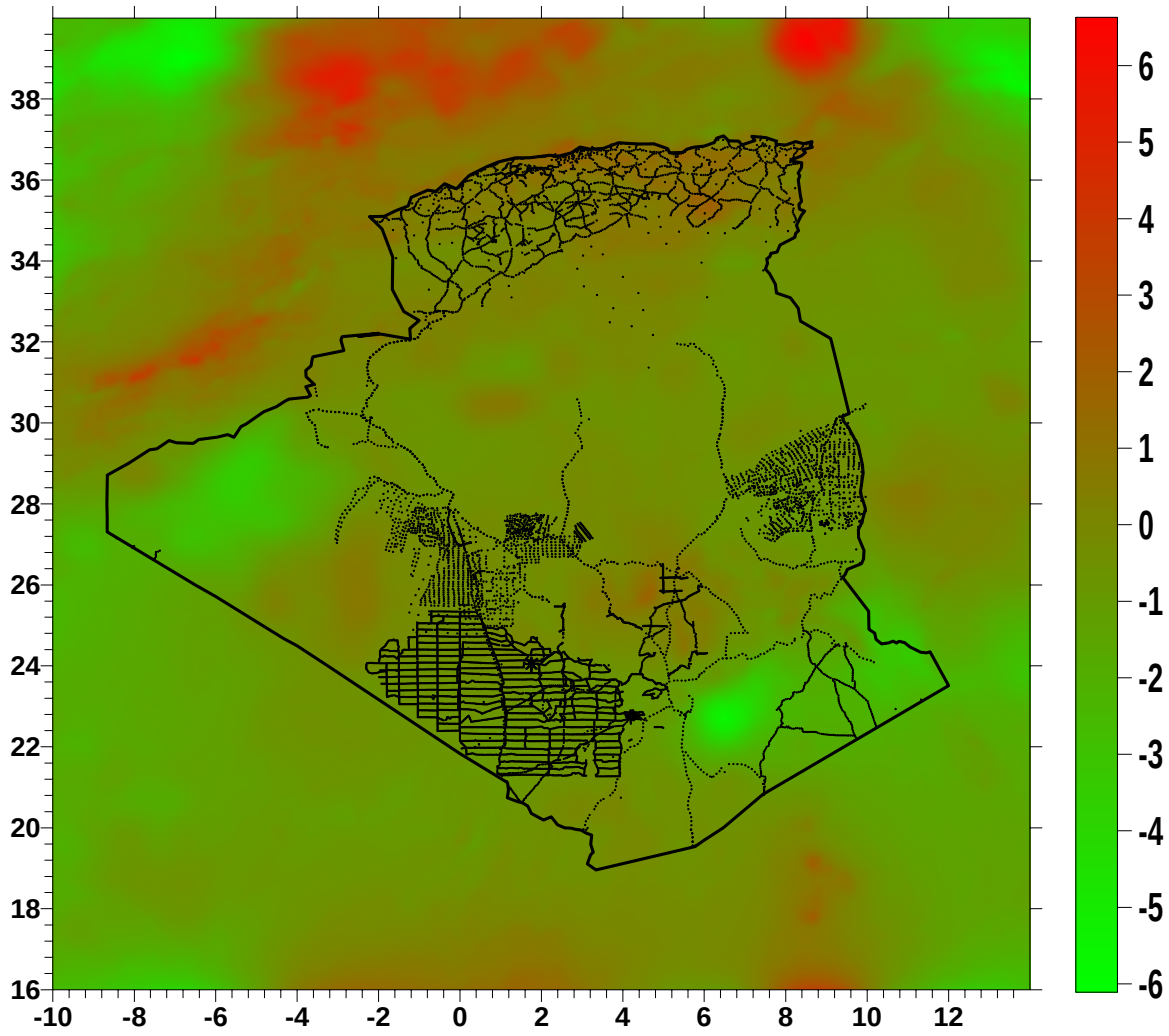


Figure 7. Map of differences between gravimetric geoid model for Algeria and geoid undulations computed by using EGM2008 model complete to degree and order 2190 (mgal).

4. Conclusion

This paper has described the comparisons of the GPS/Levelling geoid undulations and gravity anomalies in Algeria with those computed from the new released Earth global model EGM2008. Additional comparisons of the terrestrial point data with the corresponding values obtained from the geopotential models OSU91A, EGM96, the new Global Gravity Models from the recent satellite gravimetric missions CHAMP and GRACE were made. Based on the data sets used in the present work to evaluate the performances the new realised geopotential model EGM2008 within Algeria one has to note that in general the new model is an improvement over OSU91A geopotential model used in all previous Algerian geoid computations. According to our numerical results, the EGM2008 model fits better the observed values. The geoid undulations computed from this model have been significantly improved as compared to other tested models. The overall best agreement (±21.2cm) in the experimental area before fitting is achieved when the global geopotential model EGM2008 was used.

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Evaluation of the EGM2008 Geopotential Model for Egypt

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1. Introduction

EGM2008 (Pavlis et al., 2008) is the most recent earth geopotential model complete to degree and order 2159, and contains some additional spherical harmonic coefficients up to degree 2190. It represents a spatial resolution of 5 minutes of arc (about 9 km). The scaling parameters GM, a associated with this model have the numerical values:

$$\begin{aligned} GM &= 3986004.415 \times 10^8 \text{ m}^3 \text{ s}^{-2}\,, \\ a &= 6378136.3 \text{ m}\,. \end{aligned}$$

The EGM2008 zero tide model is used within this investigation for the validation process. The coefficient C_{20} of the EGM2008 zero tide model has the numerical value:

$$C_{20} = -0.484169317366974 \times 10^{-03}\,.$$

In this paper, an evaluation of EGM2008 for Egypt is given. The validation data include GPS/levelling, the latest Egyptian geoid model EGGG2008, point gravity anomalies and topographic heights derived by DTM2006.0 model for Egypt. A summary of the results and a conclusion are provided at the end of the paper.

2. Basic Equations

The gravitational potential V can be expressed in spherical harmonic expansion as (Torge, 1989, p. 28; Dragomir et al., 1982, p. 53)

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\right]\,, \qquad (1)$$

where GM is the geocentric gravitational constant, r is the geocentric radius, θ is the polar distance, λ is the geodetic longitude, a stands for the equatorial radius of the mean earth's ellipsoid, $\bar{P}_{nm}$ denotes the associated fully normalized Legendre functions and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized potential coefficients. The polar distance θ can simply be expressed in terms of the geocentric latitude ψ as:

$$\theta = 90^{\circ} - \psi\,, \qquad (2)$$

where ψ is related to the geodetic latitude ϕ through the following expression (Torge, 1980, p. 50):

$$\tan\psi = (1-f)^2\tan\phi\,, \qquad (3)$$

where f is the flattening of the earth's ellipsoid. The geocentric radius r can easily be expressed by

$$r = \sqrt{x^2 + y^2 + z^2}\,, \tag{4}$$

where x, y, z are the geodetic cartesian coordinates given by (Moritz, 1980, p. 8)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\rho + h)\cos\phi\cos\lambda \\ (\rho + h)\cos\phi\sin\lambda \\ [\rho(1 - e^2) + h]\sin\phi \end{bmatrix}, \tag{5}$$

where ρ is the radius of curvature in the prime vertical plane, given by

$$\rho = \frac{a}{(1 - e^2\sin^2\phi)^{\frac{1}{2}}}\,. \tag{6}$$

Here h stands for the ellipsoidal height and e is the first eccentricity of the ellipsoid.

The disturbing potential T is defined by

$$T(r, \theta, \lambda) = V(r, \theta, \lambda) - U(r, \theta) \tag{7}$$

where U is the normal gravitational potential of the mean earth's ellipsoid, given by (Torge, 1989, p. 37)

$$U(r, \theta) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \bar{C}^u_{n0}\bar{P}_{n0}(\cos\theta)\right]. \tag{8}$$

Here $\bar{C}^u_{n0}$ denotes the fully normalized harmonic coefficients implied by the reference equipotential ellipsoid. Because of the rotational symmetry of the mean earth's ellipsoid, there will be only zonal terms. And because of the symmetry with respect to the equatorial plane, there will be only even zonal harmonics $\bar{C}^u_{2n,0}$ (Heiskanen and Moritz, 1967, p. 72). The even degree zonal harmonic coefficients $\bar{C}^u_{2n,0}$ converge quickly toward zero, so that (8) may safely be truncated after $n = 6$. The degree zonal harmonics of the equipotential earth's ellipsoid J_n can be given by (Rapp, 1982, p. 7)

$$J_2 = \frac{2}{3}\left[f(1 - \frac{f}{2}) - \frac{m}{2}(1 - \frac{2f}{7} + \frac{11f^2}{49})\right], \tag{9}$$

$$J_4 = -\frac{4f}{35}(1 - \frac{f}{2})\left[7f(1 - \frac{f}{2}) - 5m(1 - \frac{2f}{7})\right], \tag{10}$$

$$J_6 = \frac{4f^2}{21}(6f - 5m)\,, \tag{11}$$

where m is given by (Torge, 1980, p. 58)

$$m = \frac{\omega^2 a^3(1 - f)}{GM}\,, \tag{12}$$

where ω is the angular velocity. The degree zonal harmonic coefficients J_n are related to the fully normalized coefficients of the reference ellipsoid $\bar{C}^u_n$ through the following relationship (Rapp, 1982, P. 7)

$$\bar{C}^u_n = -\frac{J_n}{\sqrt{2n + 1}}\,. \tag{13}$$

Thus inserting (1) and (8) into (7), the disturbing potential T can be expressed as (Torge, 1989, p. 43)

$$T(r,\theta,\lambda) = \frac{GM}{r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\,, \tag{14}$$

where $\bar{C}^*_{nm}$ is the difference between the actual coefficients $\bar{C}_{nm}$ and those implied by the reference equipotential ellipsoid $\bar{C}^u_{nm}$. In view of the above discussion, one may write the following relation for $\bar{C}^*_{nm}$:

$$\begin{array}{rcll} \bar{C}^*_{n0} & = & \bar{C}_{n0} - \bar{C}^u_{n0} & \text{if } m = 0\,, \\ \bar{C}^*_{nm} & = & \bar{C}_{nm} & \text{if } m \neq 0\,. \end{array} \tag{15}$$

The gravity anomaly Δg can be expressed by (Moritz, 1980, p. 14)

$$\Delta g(r,\theta,\lambda) = -\frac{\partial T}{\partial r} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial r}\,T(r,\theta,\lambda)\,, \tag{16}$$

where γ is the normal gravity. The normal gravity γ may be expressed in terms of the normal gravity on the surface of the ellipsoid $\gamma_\circ$, with sufficient accuracy for our purpose, as

$$\gamma = \gamma_\circ - 0.3086\,h\,, \tag{17}$$

where h stands for the height above the reference ellipsoid. Here γ and $\gamma_\circ$ are in mgal and h is in meter. The normal gravity on the surface of the ellipsoid $\gamma_\circ$ can be expressed as (Heiskanen and Moritz, 1967, p. 76):

$$\gamma_\circ = \gamma_e \frac{1 + k\sin^2\phi}{\sqrt{1 - e^2\sin^2\phi}}\,, \tag{18}$$

where k is given by

$$k = \frac{(1-f)\,\gamma_p - \gamma_e}{\gamma_e}\,. \tag{19}$$

Here γ_e and γ_p stand for the normal gravity at the equator and the pole, respectively.

Using the spherical approximation, we may write (ibid, p. 87)

$$\frac{1}{\gamma}\frac{\partial \gamma}{\partial r} = -\frac{2}{r}\,. \tag{20}$$

Then (16) becomes

$$\Delta g(r,\theta,\lambda) = -\frac{\partial T}{\partial r} - \frac{2}{r}\,T(r,\theta,\lambda)\,. \tag{21}$$

Inserting (14) into (21), one may write the following expression for the gravity anomaly Δg

$$\Delta g(r,\theta,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{\infty}(n-1)\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\,. \tag{22}$$

The height anomaly ζ can be given by the generalized Bruns formula as (Moritz, 1980, p. 353)

$$\zeta(r,\theta,\lambda) = \frac{T(r,\theta,\lambda)}{\gamma}\,. \tag{23}$$

Inserting (14) into (23) gives

$$\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta)\,. \tag{24}$$

It should be noted that (23) can also be used for the calculation of the geoid undulation N but with the evaluation of T on the surface of the geoid by an appropriate choice of r.

3. Gravity Anomalies Comparison

The available point gravity anomalies data set consists of 13566 points. This data set has been used for the EGGG2002 geoid for Egypt (Abd-Elmotaal, 2003). Figure 1 shows the distribution of the free-air gravity anomalies for Egypt used for the current investigation. The distribution of the free-air gravity anomaly stations on-land is very poor, concentrated mainly along the Nile valley. Many areas are empty.

The point free-air gravity anomalies range between -190.51 mgal and 294.74 mgal with an average of -3.28 mgal and a standard deviation of about 60.36 mgal. Highest values are in sea area. These values are gridded at $5'\times5'$ grid using krigging interpolation technique. Figure 2 shows the gridded $5'\times5'$ free-air gravity anomalies for Egypt used for the current investigation.

The program GRVABD (Abd-Elmotaal, 1998) has been used to create $5'\times5'$ free-air gravity anomalies based on EGM2008 model till degree and order 2159. These anomalies are shown in Figure 3. These free-air anomalies range between -186.60 mgal and 389.59 mgal with an average of 0.86 mgal and a standard deviation of about 38.49 mgal. Figure 3 shows more rough structure for the free-air gravity anomalies than Fig. 2.

It should be noted that the steep structure of the free-air anomalies produced by EGM2008 model at Qena ($\phi = 25.3°$, $\lambda = 32°$) refer to wrong gravity data at that region (as it has been proved in previous investigations (cf. Abd-Elmotaal, 2003)), which is believed that they have been included in producing EGM2008.

The steep structure of the free-air anomalies produced by EGM2008 model in Sinai might come from the lack of data included in producing EGM2008, beside the steep structure of topography there (cf. Fig. 5).

Figure 4 shows the difference between the gridded point free-anomalies and those computed using EGM2008 model till degree and order 2159. These differences range between -185.11 mgal and 135.88 mgal with an average of -2.73 mgal and a standard deviation of about 20.37 mgal. White areas mean differences less than 20 mgal in magnitude. Figure 4 shows very small differences in sea areas, but larger differences in land areas, especially in Sinai, where no data point are available, and also at Qena, where wrong gravity data were included in producing the EGM2008 model.

More interesting is the evaluation of the EGM2008 model at the point gravity stations. Program GRVHRM (Abd-Elmotaal, 1998) has been used to compute the free-air

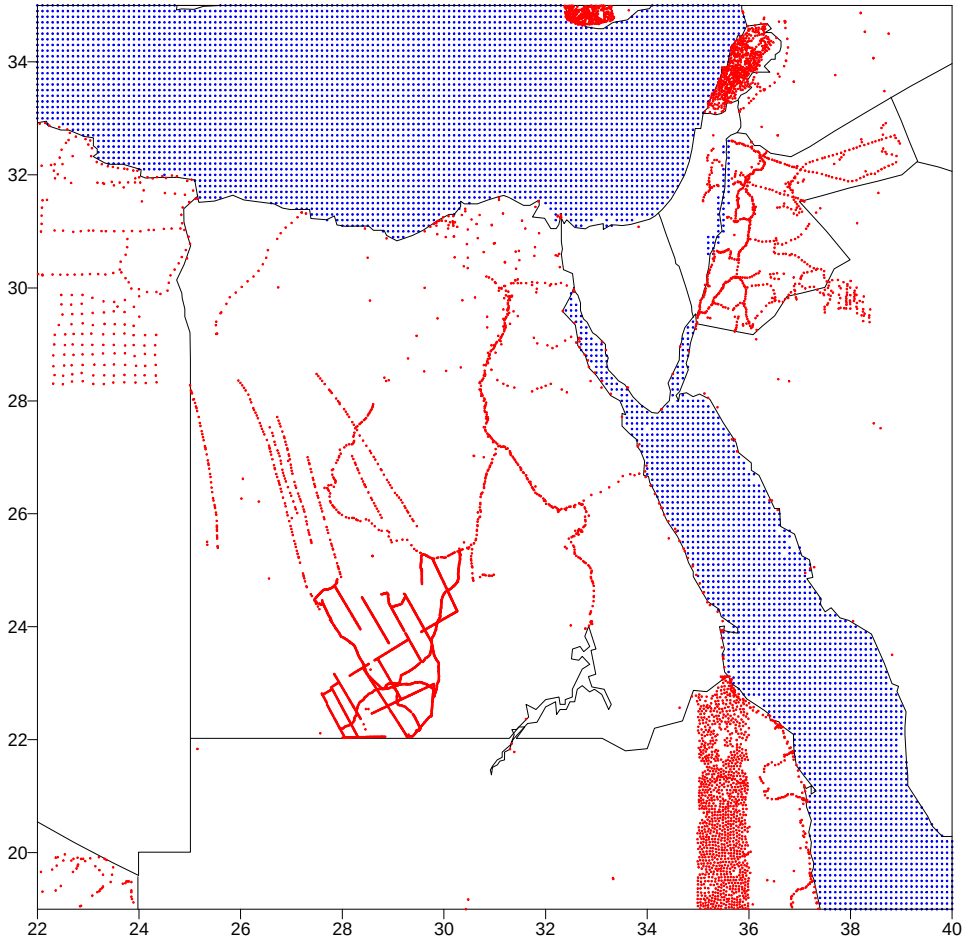


Figure 1: Distribution of the free-air gravity anomalies for Egypt.

gravity anomalies at the gravity data points using the EGM2008 model complete to degree and order 2159. For the sake of comparison, EGM96 and OSU91A models complete to degree and order 360 have also been used to compte free-air gravity anomalies at the gravity data points. No topographic-isostatic reduction has been made at this stage. Table 1 shows the statistics of the point free-air gravity anomalies, free-air anomalies produced using EGM2008, EGM96 and OSU91A models, and their differences. Table 1 shows that EGM2008 model fits best to the point free-air data in view of the smallest standard deviation of the differences between the point free-air and produced model anomalies, however, it produces a larger range difference than that of EGM96. As far as the mean difference is concerned, all three geopotential models give nearly the same good centered differences.

Table 2 shows the differences between the point free-air anomalies and those produced using the EGM2008, EGM96 and OSU91A models after removing the effect of the topographic-isostatic masses employed by the Airy-Heiskanen isostatic model, using the following parameters:

$$T_\circ = 30 \text{ km}, \tag{25}$$

$$\Delta\rho = 0.4 \text{ g/cm}^3. \tag{26}$$

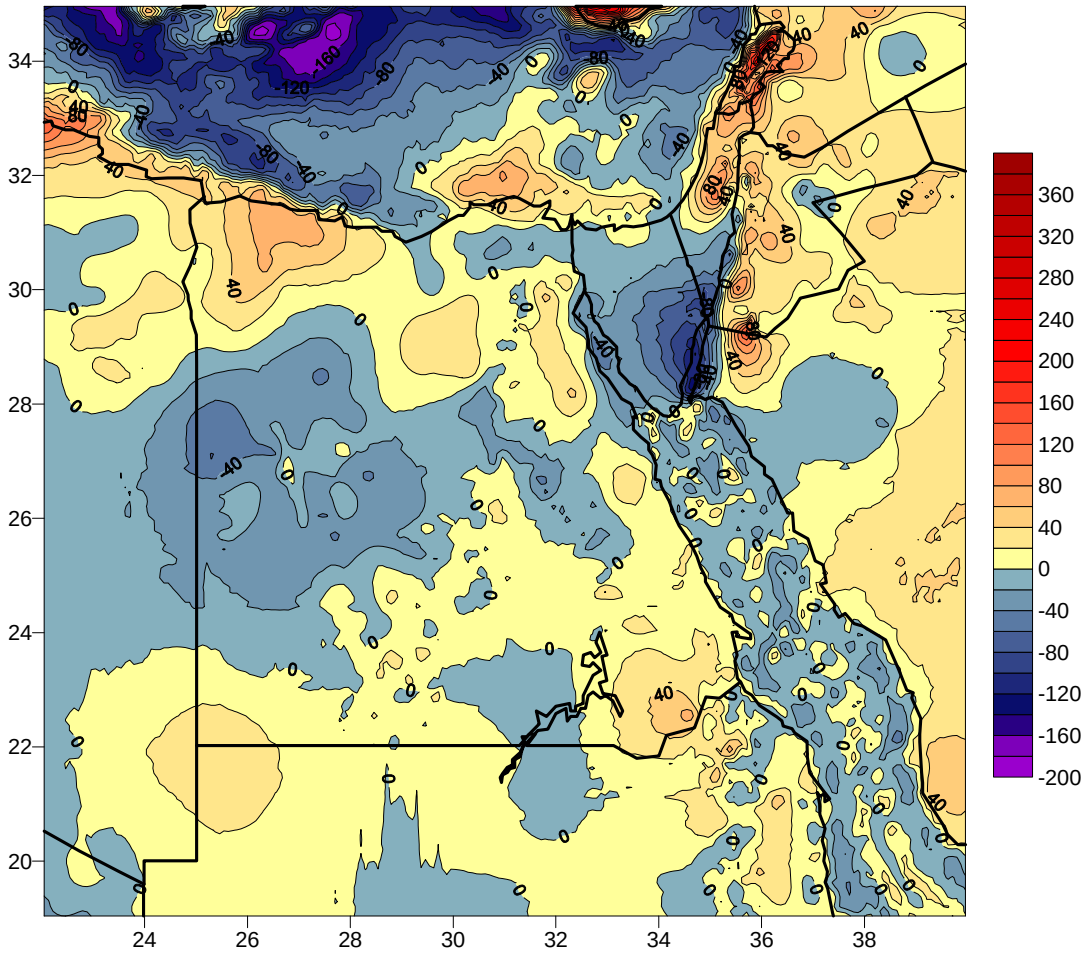


Figure 2: Gridded $5' \times 5'$ free-air gravity anomalies for Egypt using the points gravity anomalies. Contour interval: 20 mgal.

Table 1: Statistics of the gravity anomalies (13566 gravity stations) without topographic reductions

gravity anomalies	statistical parameters			
	min.	max.	mean	st. dev.
	mgal	mgal	mgal	mgal
Point free-air	−190.51	294.74	−3.28	60.36
OSU91A	−165.64	169.37	−3.59	50.36
EGM96	−183.31	211.61	−2.72	56.62
EGM2008	−211.13	290.92	−3.51	61.82
Point free-air − OSU91A	−108.16	137.13	0.30	23.39
Point free-air − EGM96	−96.37	100.20	−0.56	19.12
Point free-air − EGM2008	−83.44	140.39	0.22	11.39

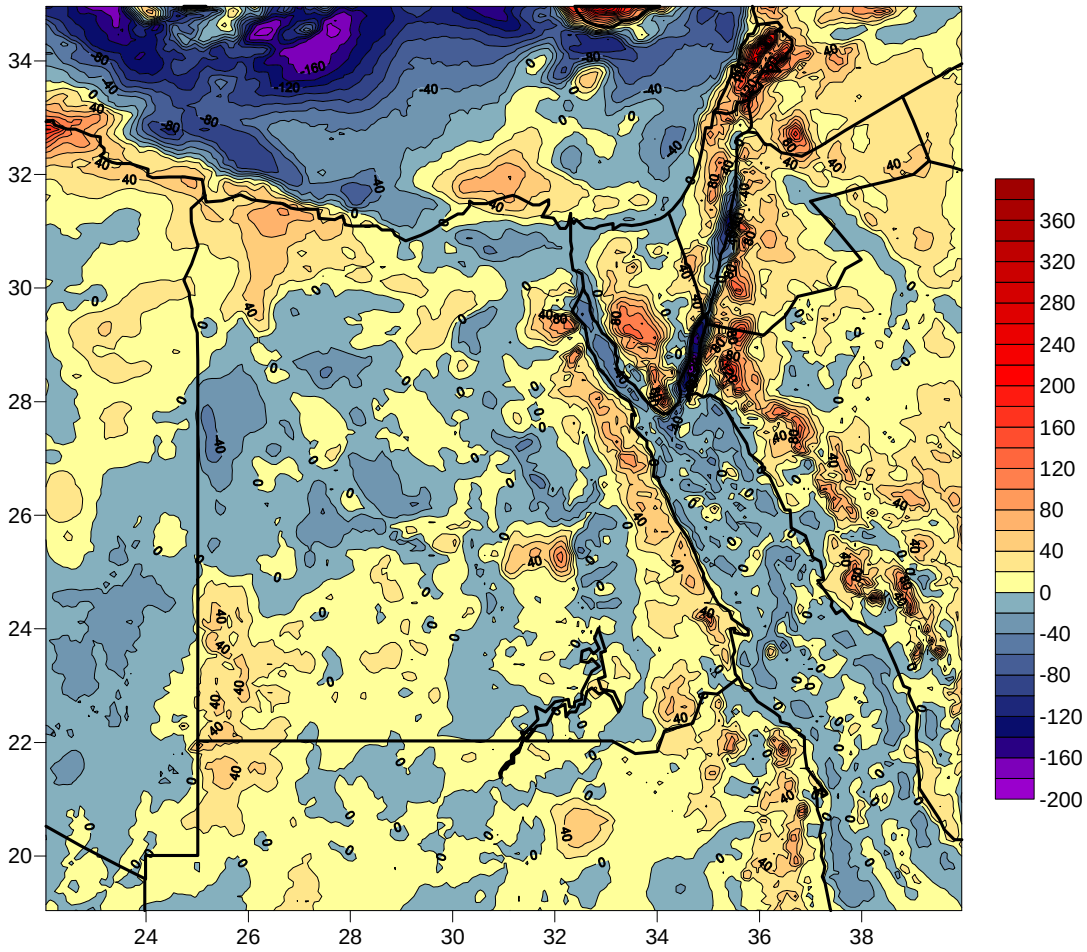


Figure 3: $5' \times 5'$ free-air gravity anomalies for Egypt using EGM2008 model till degree and order 2159. Contour interval: 20 mgal.

Comparing Tables 1 and 2 shows that after removing the effect of the topographic-isostatic masses, the differences became worse. This derives us to study the topographic height, which will be discussed in the following section.

Table 2: Statistics of the gravity anomaly differences (13566 gravity stations) after topographic-isostatic reduction

gravity anomalies	statistical parameters			
	min.	max.	mean	st. dev.
	mgal	mgal	mgal	mgal
Point free-air − OSU91A	−83.47	152.12	5.04	26.65
Point free-air − EGM96	−115.49	122.64	4.17	29.54
Point free-air − EGM2008	−213.58	199.36	4.96	32.47

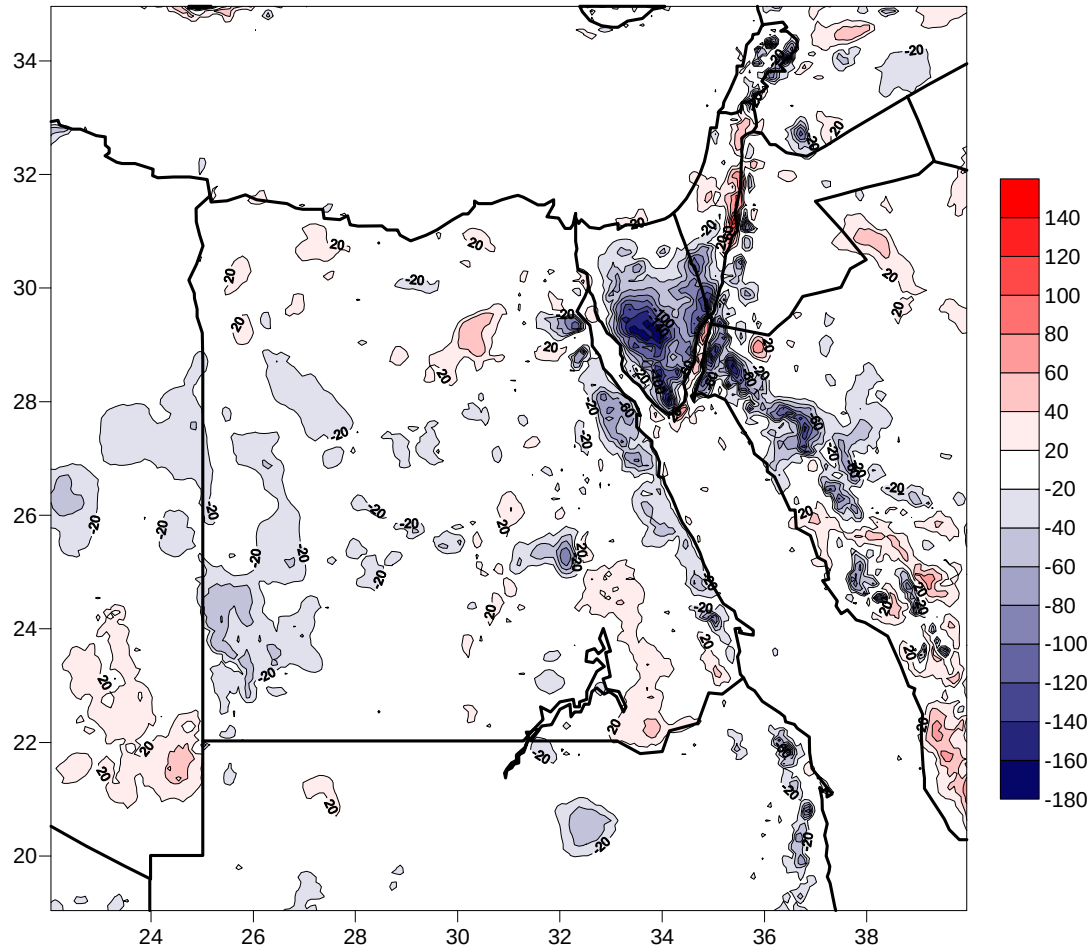


Figure 4: Difference between gridded point free-air anomalies and those computed using EGM2008 model till degree and order 2159. Contour interval: 20 mgal.

4. Digital Height Models Comparison

Abd-Elmotaal (2004 a) has produced Digital Heights Models for Africa with different resolutions, 30″, 3′ and 5′. This represents the most recent DHM for Africa. A window for Egypt out of the African DHM AFH04M5 ($5' \times 5'$) has been created and illustrated in Fig. 5. The heights within this window range between -4192 m and 2668 m with an average of -19.5 m and a standard deviation of about 993 m.

The DTM2006.0 model (Pavlis et al., 2007) contains fully-normalized spherical harmonic coefficients of the elevation $(\bar{C}_{nm}, \bar{S}_{nm})$ in units of meters, complete to degree and order 2190. Heights H above Mean Sea Level (MSL) can be computed by:

$$H(\theta,\lambda) = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right) \bar{P}_{nm}(\cos\theta)\,. \tag{27}$$

The DTM2006.0 model has been used to compute the heights for the Egyptian window by using FIELDCRE program (Abd-Elmotaal, 2004 b). The used upper maximum degree N_{max} has been set to 2160.

Figure 6 illustrates a $5' \times 5'$ DHM for Egypt derived by using DTM2006.0 model

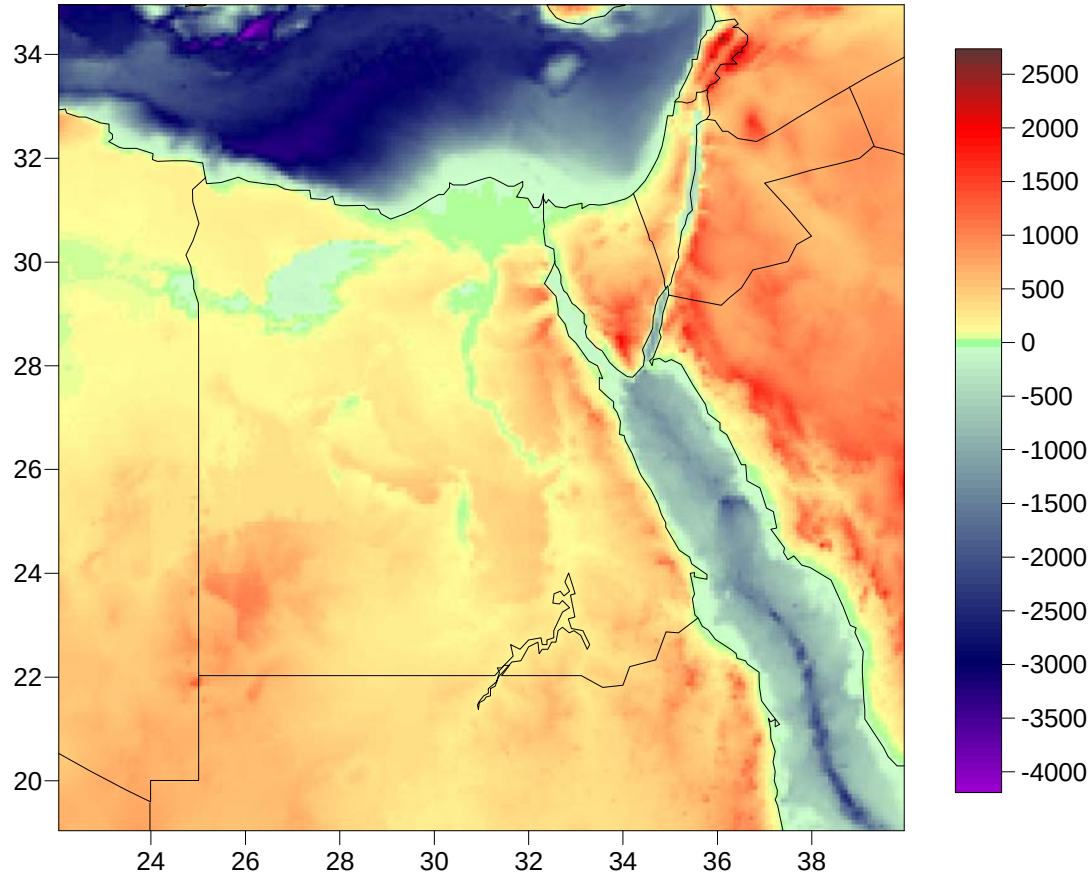


Figure 5: $5' \times 5'$ DHM for Egypt derived from the African DHM AFH04M5 (after Abd-Elmotaal, 2004 a).

complete to degree and order 2160. The heights within the Egyptian window range between −4136 m and 2734 m with an average of −29 m and a standard deviation of about 998 m.

Comparing Figs. 5 and 6 shows a general agreement of the topography. Figure 7 illustrates the difference between AFH04M0 DHM and heights computed by using DTM2006.0 model complete to degree and order 2160. These difference range between −2904 m and 2284 m with an average of 9.5 m and a standard deviation of about 214 m. White areas mean differences less than 100 m in magnitude. Figure 7 shows very small differences in land areas, but larger differences in sea areas, especially in the Mediterranean sea. Some few hundred meters differences in topographic heights in Sinai are remarkable. This might be responsible for the large free-air anomaly differences in Sinai (cf. Fig 4).

5. Geoid Comparison

Figure 8 shows the most recent geoid solution for Egypt EGGG2008 (Abd-Elmotaal, 2008). It is a gravimetric geoid computed using a tailored global geopotential model for Egypt (created basically using the EGM96 global geopotential model) within the window remove-restore technique described by Abd-Elmotaal and Kühtreiber (2003). The geoid

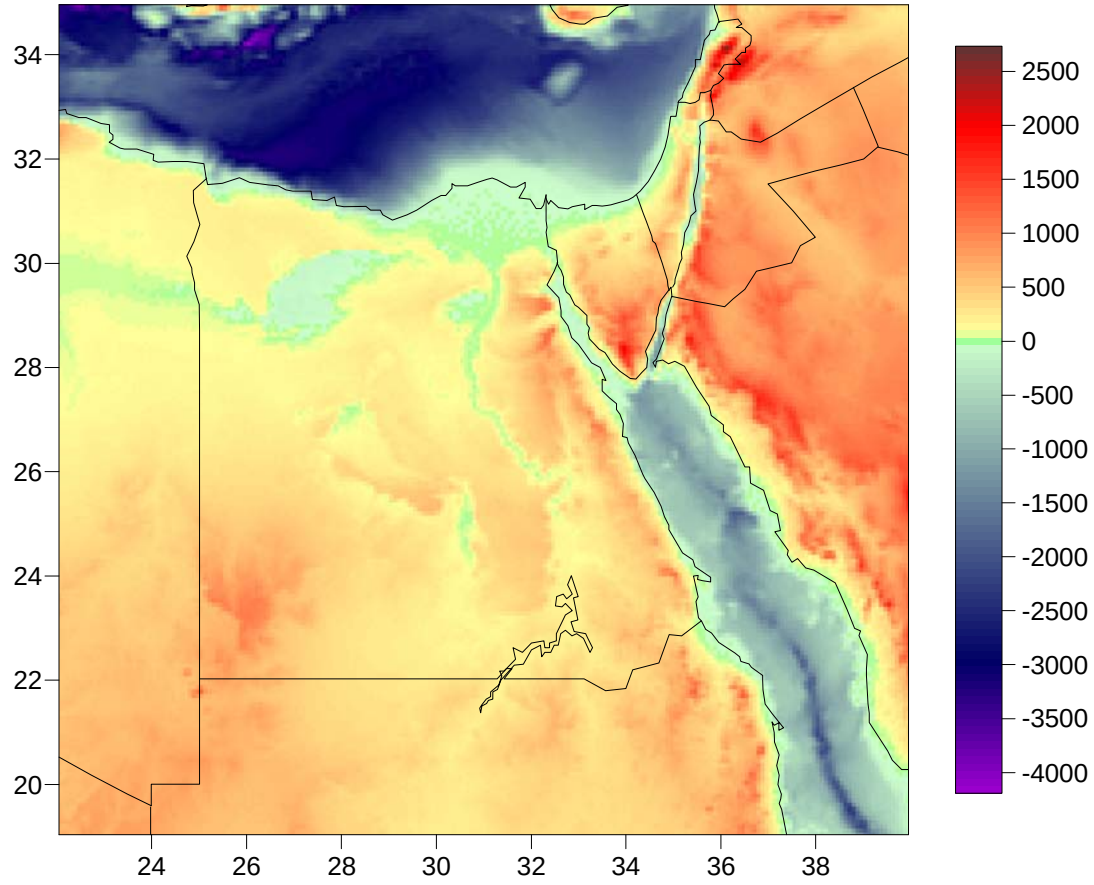


Figure 6: $5' \times 5'$ DHM for Egypt derived by using DTM2006.0 model complete to degree and order 2160.

heights range between 5.95 m and 23.89 m with an average of 14.52 m and a standard deviation of about 3.65 m.

The EGM2008 model has been used to create a geoid model for Egypt using GRVABD program (Abd-Elmotaal, 1998). Figure 9 shows the geoid undulations for Egypt computed by using EGM2008 model complete to degree and order 2159. The geoid heights range between 6.37 m and 22.89 m with an average of 14.80 m and a standard deviation of about 3.16 m.

Figure 10 shows the difference between EGGG2008 geoid model and geoid undulations computed by using EGM2008 model complete to degree and order 2159. These geoid undulation differences range between -7.83 m and 3.40 m with an average of -0.28 m and a standard deviation of about 1.92 m. White areas mean differences less than 0.5 m in magnitude. Figure 10 shows very large differences especially in Sinai and North-Western desert.

6. GPS Comparison

Figure 11 shows the difference of geoid undulations at GPS stations between GPS/levelling undulations and those computed using EGM2008 model complete to degree and order

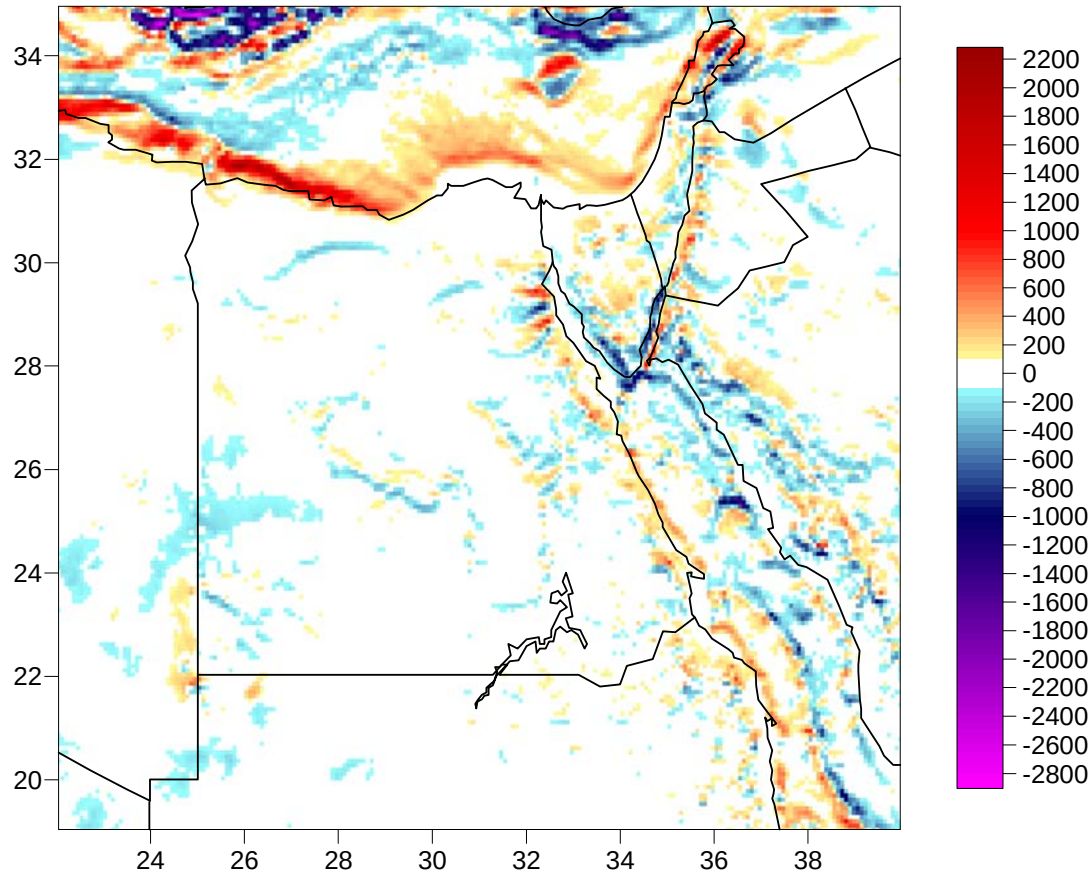


Figure 7: Difference between AFH04M5 DHM and heights computed by using DTM2006.0 model complete to degree and order 2160.

2159. These differences range between −5.01 m and 3.32 m with an average of −0.04 m and a standard deviation of about 2.11 m. Figure 11 also shows the differences of the undulations at the GPS stations (30 stations). Figure 11 confirms the conclusion drawn in the previous section that there are large difference at both Sinai and North-Western desert.

Table 3 illustrates the statistics of the geoid undulations at GPS stations of both GPS/levelling and those computed using EGM2008 model complete to degree and order 2159, as well as their differences. It also shows, for comparison purposes, same statistics for the case of the EGM96 global geopotential model as well as the EGGG2008 local geoid model. Table 3 confirms the already stated conclusion that EGM2008 doesn't fit the geoid undulations of Egypt.

7. Conclusion

A validation scheme of the recently generated EGM2008 geopotential model over Egypt has been carried out in this investigation. The validation includes the followings:

- Comparison of gridded $5' \times 5'$ gravity anomalies for Egypt
- Comparison of point gravity anomalies for Egypt

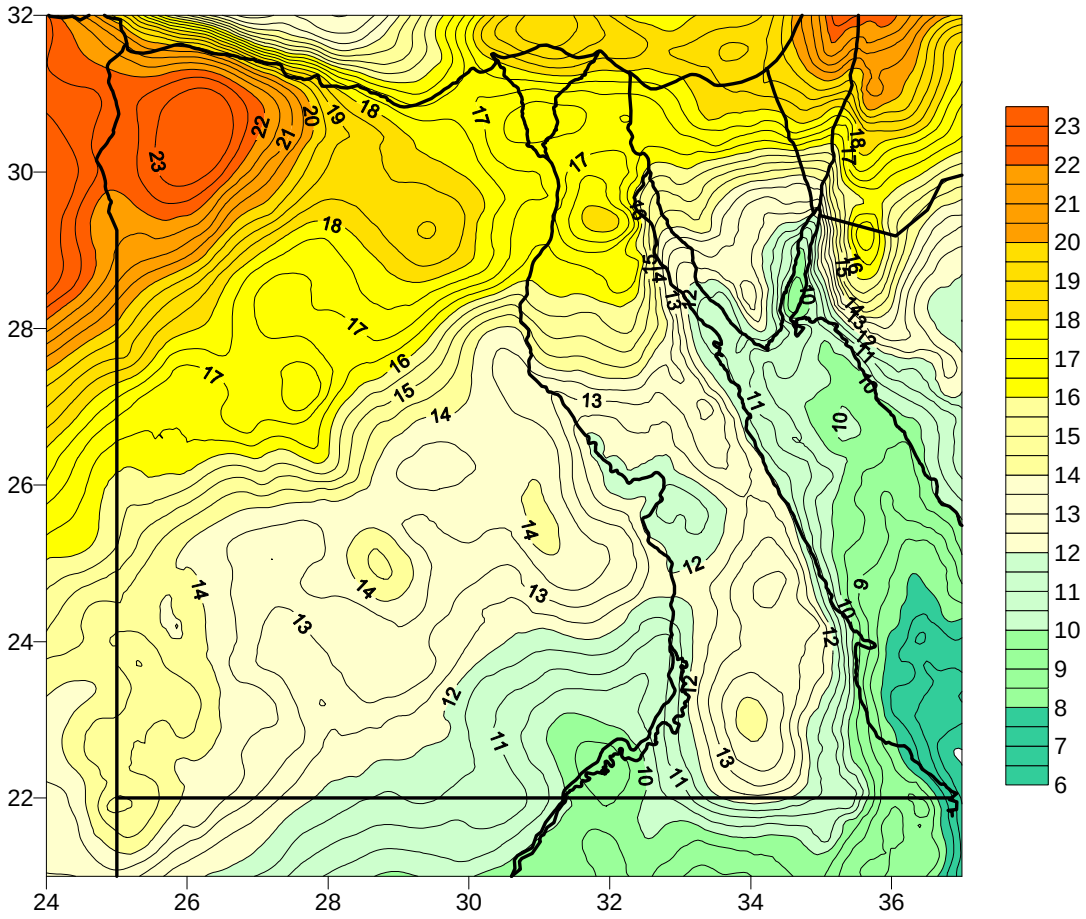


Figure 8: EGGG2008 geoid model for Egypt (after Abd-Elmotaal, 2008). Contour interval: 0.5 m.

Table 3: Statistics of the geoid undulations at GPS stations (30 stations)

geoid undulation	statistical parameters			
	min.	max.	mean	st. dev.
	m	m	m	m
GPS/levelling	7.29	21.51	14.85	3.73
EGM2008	7.39	20.96	14.89	3.06
EGM96	7.53	21.25	14.74	3.06
EGGG2008	7.28	21.56	14.78	3.75
GPS/levelling − EGM2008	−5.01	3.32	−0.04	2.11
GPS/levelling − EGM96	−2.42	2.84	0.10	1.37
GPS/levelling − EGM96	−0.05	1.75	0.07	0.32

- Comparison of geoid model for Egypt
- Comparison of geoid undulations and GPS stations for Egypt

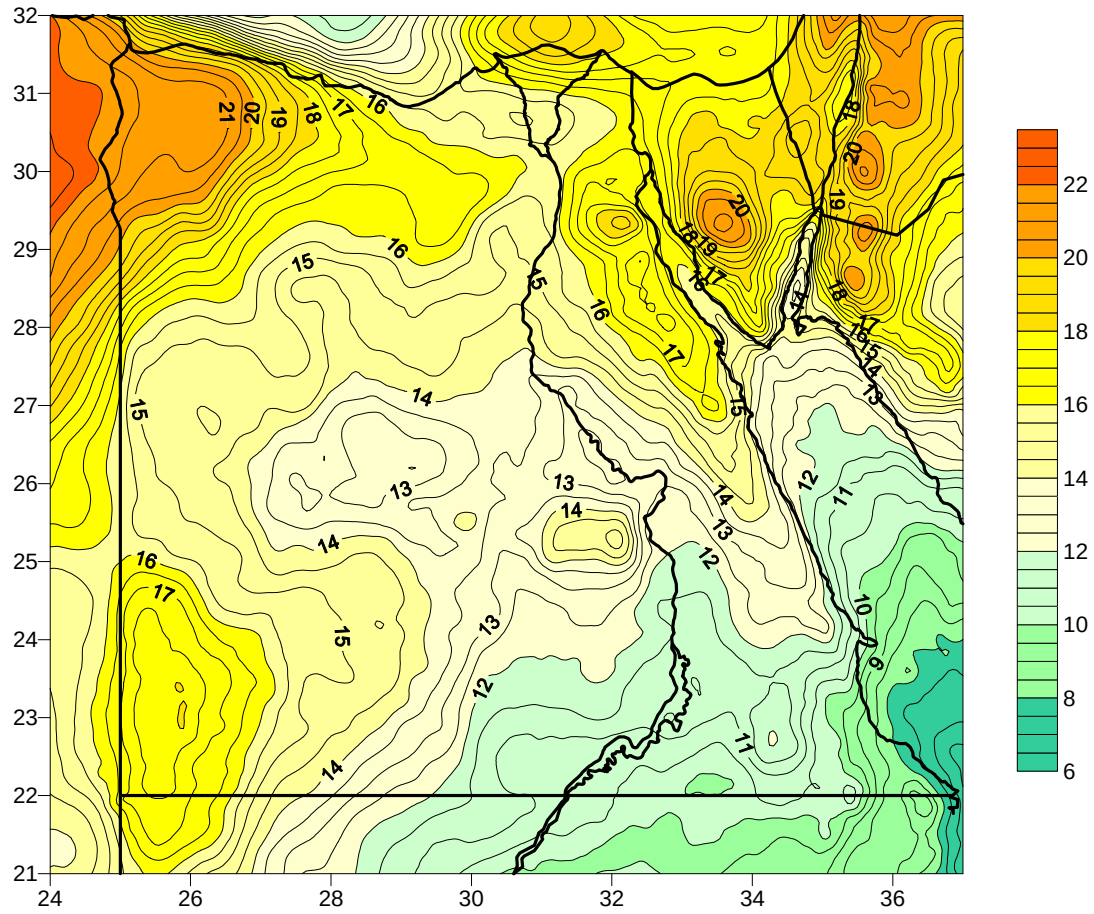


Figure 9: Geoid model for Egypt computed by using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

The validation also includes the comparison of the most updated DHM for Egypt and the DTM2006.0 DHM.

The results proved that the EGM2008 model gives generally good agreement with gravity anomalies in Egypt without applying topographic-isostatic reduction. After performing topographic-isostatic reduction, worse results were obtained.

As far as geoid undulations are concerned, EGM2008 proved to produce incompatible geoid undulations for Egypt. Large difference were obtained, especially in Sinai and North-Western desert.

It should also be pointed out that it is believed that some wrong gravity data at Qena ($\phi = 25.3°$, $\lambda = 32°$) have been included in producing EGM2008. It is, therefore, recommended to remove them in producing any upcoming geopotential model.

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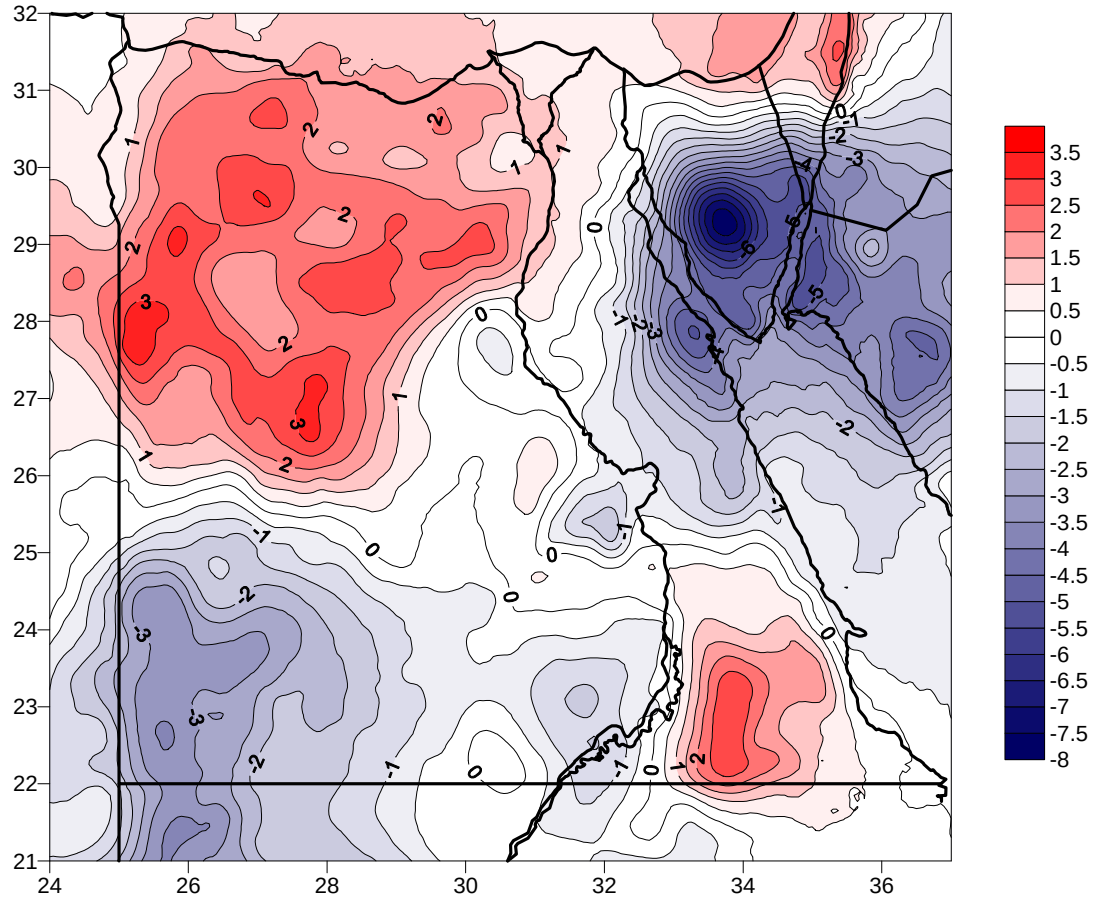


Figure 10: Geoid difference for Egypt between EGGG2008 model and geoid undulations computed by using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

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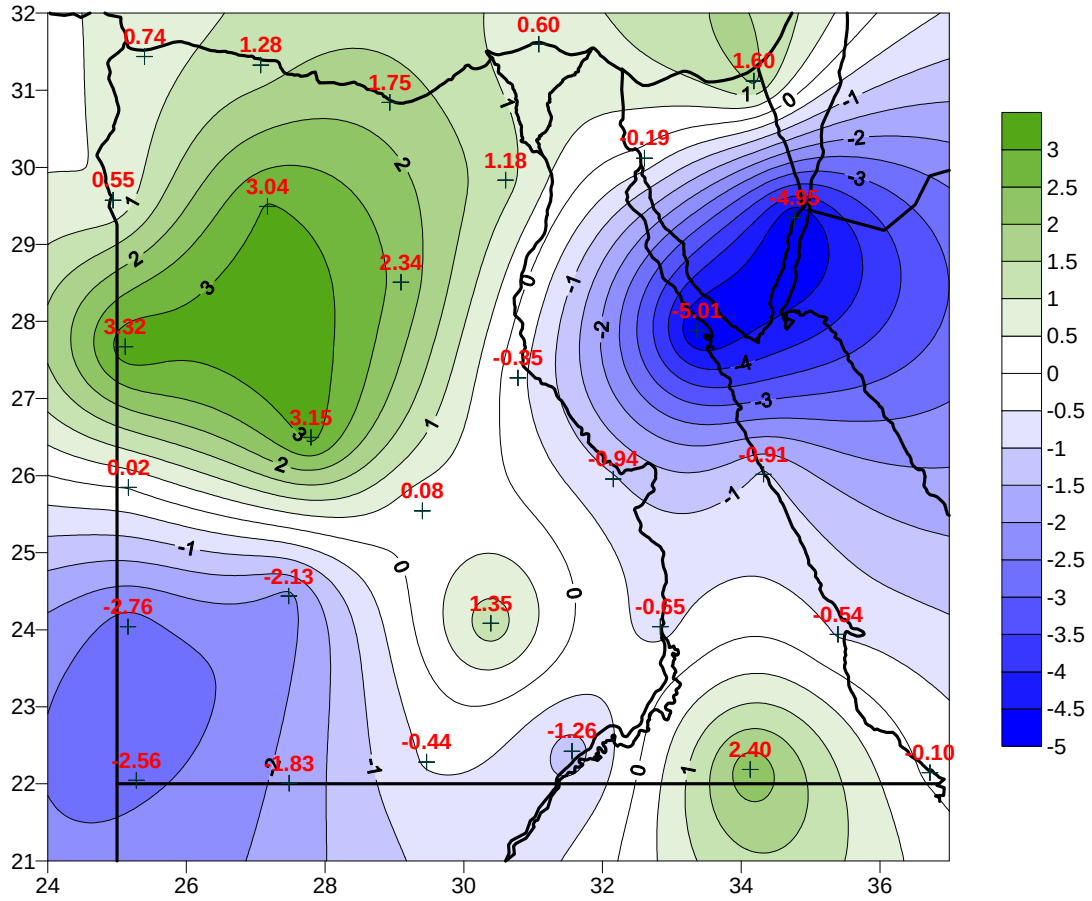


Figure 11: Difference of geoid undulations at GPS stations between GPS/levelling undulations and those computed using EGM2008 model complete to degree and order 2159. Contour interval: 0.5 m.

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EGM2008 EVALUATION FOR AFRICA

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1 Introduction

The Earth Geopotential Model 2008 (EGM2008) is the latest version of a series of geopotential models developed under the leadership of the National Geospatial-Intelligence Agency (Pavlis et al., 2008). It incorporates harmonic coefficients derived from the GRACE satellite mission, marine gravity anomalies derived from satellite altimetry, and a comprehensive set of terrestrial gravity anomalies. It is likely to become the standard geopotential model used for many applications including orbit modelling and geoid modelling. As such, it is important that it be assessed by means of comparisons with independent or quasi-independent data sets. This paper focusses the assessment on the continent of Africa, probably the continent for which the details of the Earth's geopotential are least well known. This evaluation has been carried out using three data sets:

- African Geoid Project 2007 geoid model (5' by 5' grid)
- Point gravity anomalies for southern Africa
- GPS/levelling data for 79 points in South Africa

The first two data sets are not truly independent of the EGM2008. Most of the point gravity anomalies in the second data set also formed part of the terrestrial data used in EGM2008. The AGP2007 geoid model (Merry, 2007) used gravity anomalies which were also used in EGM2008, and it also used harmonic coefficients derived from GRACE tracking data. Nevertheless, there are sufficient differences in the way the data were used for the two data sets to provide a meaningful comparison.

2 Geoidal Heights

The AGP2007 geoidal height model uses the following data in a remove-restore model:

- Eigen GL04C geopotential model, truncated at degree 120, tide-free system, referenced to the WGS84 ellipsoid, a=6378137m.
- A gridded set of 5' free-air gravity anomalies, derived from three major data sets – the holdings of the University of Cape Town (UCT) for Africa south of 8° S; the holdings of the University of Leeds for the rest of Africa (Fairhead et al. 1988); and the KMS02 marine gravity anomaly data set from the Danish National Survey and Cadastre (Andersen et al., 2005). Where

insufficient measured values existed, the Eigen GL04C field was used to generate grid values.

- The Molodensky G_1 correction term and the correction N-ζ to convert from height anomalies to geoidal heights were computed using the gridded anomalies and the SRTM 30" DEM (Farr and Kobrick, 2000).

EGM2008 geoidal heights for Africa, referred to the WGS84 ellipsoid in the tide-free system, were extracted from a data set available on the NGA EGM2008 web page (Pavlis et al., 2008). These data are on a 2.5' grid - this set was decimated to a 5' grid to enable a direct comparison with the AGP2007 model. The results are summarised in Table 1 and Figures 1 and 2.

Difference	# of points	Minimum	Maximum	Mean	Std. Dev.
EGM2008 minus AGP2007	593832	-4.58m	+6.31m	+0.02m	0.73m

Table 1: Difference between EGM2008 and AGP2007 Geoid Models

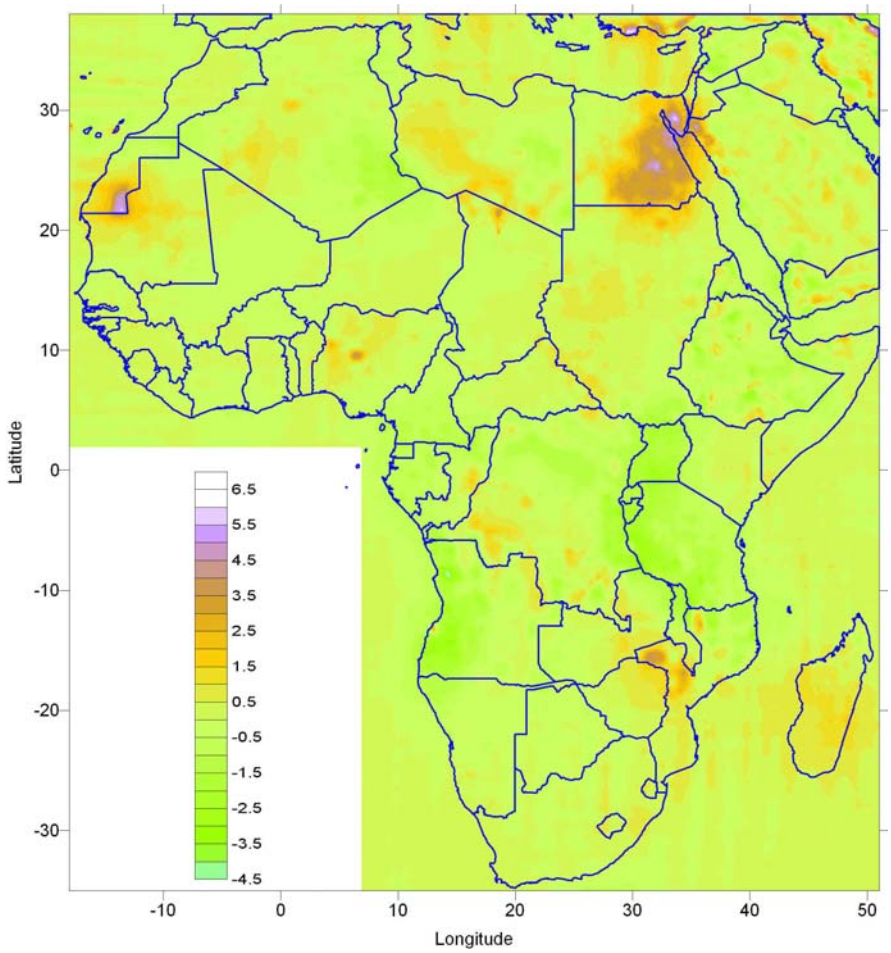


Figure 1: EGM2008 Geoid minus AGP2007 Geoid (metres)

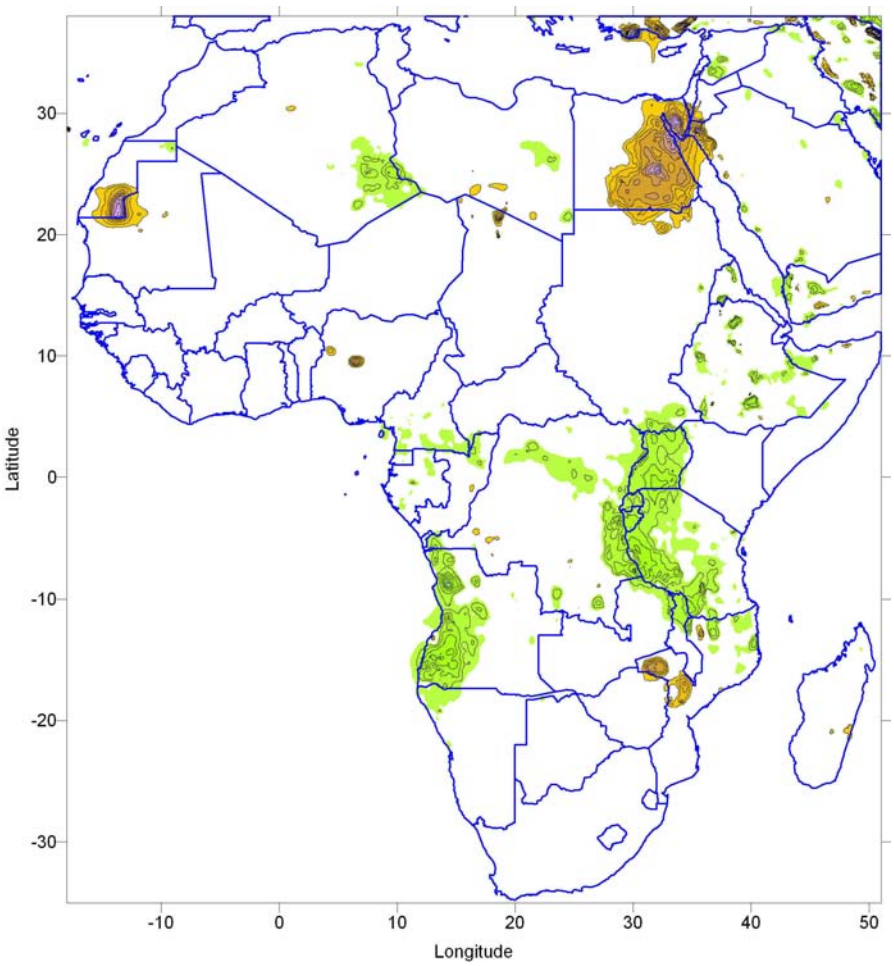


Figure 2: EGM2008 Geoid minus AGP2007 Geoid
(contour interval: 0.5m)
(showing differences where the magnitude exceeds one metre)

The striping that is a feature of many of the GRACE models, including Eigen GL04C used for AGP2007, is apparent in Figure 1. As mentioned in Pavlis et al. (2008) the GRACE data have been re-processed to remove this effect from EGM2008. Figure 2 highlights the larger discrepancies (magnitude larger than one metre). Negative values are blue/green, positive values are in yellow/brown/purple. The minimum discrepancy listed in Table 1 occurs in northwest Angola, in a region where no measured gravity values were available for AGP2007. The maximum discrepancy listed in Table 1 occurs in southern Turkey, outside the area of interest. Looking at some of the discrepancies within Africa:

- There are large positive discrepancies in Egypt, with the maximum (+5.7m) occurring in the Sinai region. No terrestrial data were available for AGP2007 in the Sinai region, and it could be that EGM2008 has made use of new data. Terrestrial data are available for AGP2007 in the remainder of Egypt, and the large positive discrepancies are unexplained.

- The large positive discrepancy in the southeast of Western Sahara is unexplained. Terrestrial gravity data are available in this region.
- The small regions of positive discrepancy in northern Chad and central Nigeria are in regions where there are no terrestrial data available for AGP2007. Presumably such data were available for EGM2008.
- The large (-4.6m) negative discrepancy in northwest Angola is in an area where no data were available to AGP2007. Further south, data are available, but a negative discrepancy persists.
- The positive discrepancies in central and northern Mozambique are in areas where no gravity data were available to AGP2007.

There are large gravity data gaps in Africa (Merry, 2007). For AGP2007 these data gaps were filled using the Eigen GL04C model only. For EGM2008 these gaps were filled using a combination of geopotential coefficients from GRACE and from gravity anomalies deduced from a 30" DEM using the Residual Terrain Model (RTM) approach (Pavlis et al., 2006). As is evident from the discussion above, the two different approaches have yielded substantially different results for geoidal heights in areas lacking observed terrestrial gravity data. Overall, the discrepancies are larger than expected. A more detailed investigation would need more detailed information regarding the terrestrial data used in EGM2008. Unfortunately there are no GPS/levelling data sets available in the areas of the major discrepancies which could be used to resolve these discrepancies.

3 Gravity Anomalies

The computation of AGP2007 used a 5' grid of free-air gravity anomalies. These were interpolated from point values. Only the point values in the University of Cape Town's holding are available for comparison (essentially south of 8° South). The EGM2008 harmonic coefficients were used to generate a 5' grid of gravity anomalies for southern Africa. In turn, these gridded data were used to interpolate gravity anomalies at the data points The comparison between the two sets is summarised in Table 2 and in Figure 3.

Difference	# of points	Minimum	Maximum	Mean	Std. Dev.
Measured Δg minus EGM2008	157495	-132.7mgal	+81.4mgal	-0.8mgal	9.3mgal

Table 2: Difference between Measured and EGM2008 Gravity Anomalies

There are some interesting discrepancies between the measured gravity anomalies and those deduced from the EGM2008 harmonic coefficients:

- With some exceptions the positive discrepancies are correlated with regions of high or rough topography. This is especially so in Lesotho, where elevations routinely exceed 2000m.

- The "peak" on the coast of Namibia correlates with the Brandberg mountain, which EGM2008 has failed to model (no doubt due to the short wavelength nature of the feature).
- The "peak" to the west of Lesotho corresponds to the position of the Trompsburg gravity anomaly – like the Brandberg a short-wavelength feature.
- The linear feature on the border between South Africa and Mozambique corresponds with a known steep gradient in the observed anomalies.
- The extensive area of positive discrepancies in Angola corresponds to a similar discrepancy between the EGM2008 and AGP2007 geoid models. As the discrepancy is of the order of 14mgal, the possibility exists that measured data have not been converted from the Potsdam gravity datum to the IGSN71 gravity datum.
- There is a further positive discrepancy in the Caprivi region of northeast Namibia. This area is virtually flat and the cause of the discrepancy has not been identified.

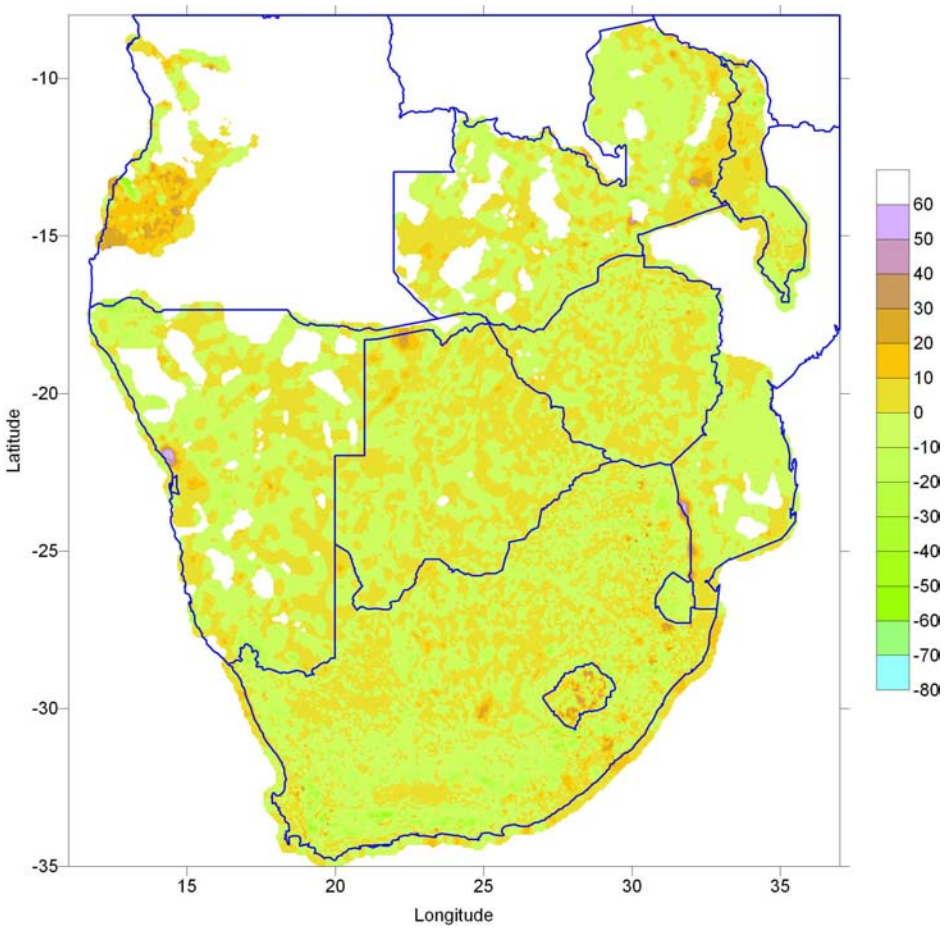


Figure 3: Measured minus EGM2008 Gravity Anomalies (mgals)

Generally, within South Africa, where there is good gravity coverage, the agreement is good. As with the geoidal heights, it is difficult to comment further upon the discrepancies without knowing in detail what observed gravity data were used in forming the EGM2008 model.

4 GPS/Levelling

Precise GPS/levelling geoidal heights are few and far between in Africa. Not all data sets are freely available, and where there are such data, there is often no information on the quality of the data. Recently, a new GPS/levelling data set became available for South Africa (S. Koch, personal communication, 2008). The GPS measurements have been made at 79 benchmarks of the precise levelling network of South Africa by staff of the Chief Directorate: Surveys & Mapping (Figure 4). The GPS co-ordinates are in the ITRF2005 reference frame, and ellipsoidal heights refer to the WGS84 ellipsoid. The South African height system is essentially a modified normal height system, so for the purposes of comparison EGM2008 height anomalies were computed at these 79 points. The results are summarised in Table 3, which includes results for AGP2007 and EGM96 height anomalies.

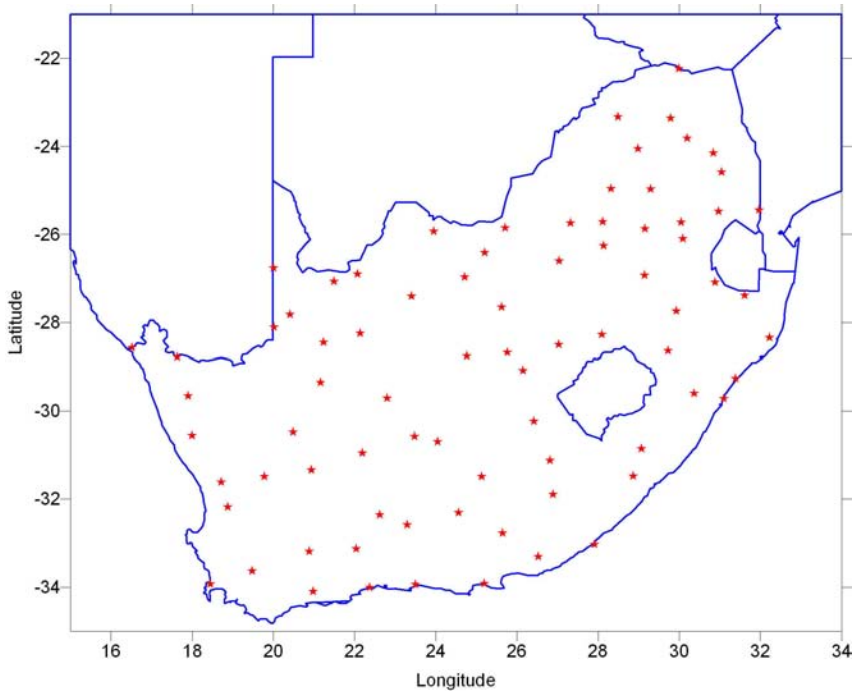


Figure 4: GPS/Levelling Data Points – South Africa

Difference	Minimum	Maximum	Mean	Std. Dev.
GPS/levelling – EGM2008	-0.84m	+0.02m	-0.42m	0.17m
GPS/levelling – AGP2007	-0.92m	+0.36m	-0.44m	0.24m
GPS/levelling – EGM96	-0.95m	+0.68m	-0.24m	0.35m

Table 3: Difference between GPS/Levelling and EGM2008 Height Anomalies

EGM2008 provides the most consistent agreement with the GPS/levelling data and is a substantial (two-fold) improvement over EGM96. The AGP2007 result is disappointing considering that (at least within South Africa) EGM2008 and AGP2007 used essentially the same terrestrial gravity data. It is possible that the "striping" inherent in the underlying GL04C model used for AGP2007 may have contributed to this comparatively poor result.

5 Conclusions

EGM2008 is a significant improvement over EGM96. Within South Africa there is a two-fold improvement in the agreement with GPS/levelling. EGM2008 appears to be free of the striping effect evident in geoidal heights computed from other models based upon the GRACE mission.

There are significant small scale discrepancies with the AGP2007 geoid model, and with the free-air gravity anomaly data in southern Africa. These discrepancies could be due to:

- differences in the available terrestrial gravity data sets.
- differences in the way in which data gaps have been filled.

Further investigation into the sources of these discrepancies is warranted.

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Is Australian data really validating EGM2008, or is EGM2008 just in/validating Australian data?

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Abstract. The tide-free release of the EGM2008 combined global geopotential model and its tide-free pre-release PGM2007A are compared with Australian land, marine and airborne gravity observations, co-located GPS-levelling on the [admittedly problematic] Australian Height Datum, astrogeodetic deflections of the vertical, and the AUSGeoid98 regional gravimetric quasigeoid model.

In all comparisons, EGM2008 performs better than any previous global gravity model. The standard deviation of the differences between free-air gravity anomalies from EGM2008 and free-air gravity anomalies from Australian land gravity observations is ±5.5 mGal, compared to, e.g., ±11.7 mGal for EGM96. Furthermore, the standard deviation of the differences between height anomalies from EGM2008 and a nation-wide set of 254 GPS-levelling points is ±17.3 cm, compared to, e.g., ±33.4 cm

for EGM96. In the comparisons with GPS-levelling, EGM2008 also outperforms AUSGeoid98 (standard deviation of ±19.1 cm in the differences with the nation-wide set of 254 GPS-levelling points), and the same holds for the comparison to astrogeodetic deflections of the vertical.

However, due to the poor quality of some of the Australian data, we cannot legitimately claim to truly validate EGM2008. Instead, EGM2008 confirms the already-known problems with the Australian data, as well as revealing some previously unknown problems. If one wants to claim validation, then EGM2008 is validated implicitly because it can confirm the errors in our regional data. Simply, EGM2008 is a good model over Australia.

1. Introduction

Australia, as a significant landmass in the Southern Hemisphere with reasonable geodetic data coverage, has been used over the years for 'ground truthing' global geopotential models (GGMs). Several studies have addressed this, mainly with a view to the later production of regional gravimetric geoid/quasigeoid models (e.g., Kearsley and Holloway 1989, Zhang and Featherstone 1995, Kirby et al. 1998, Amos and Featherstone 2003). Here, this effort is continued by comparing the tide-free version of the EGM2008 GGM (Pavlis et al. 2008) and its tide-free pre-release PGM2007A (Pavlis et al. 2007), with Australian gravity-field-related data. This is part of the International Association of Geodesy's (IAG's) Inter-Commission Working Group 2 *Evaluation of Global Earth Gravity Models* (http://users.auth.gr/~kotsaki/IAG_JWG/IAG_JWG.html). In an attempt to provide a more complete and useful 'validation', we use some newer data not used before.

We have maintained quite a close working relationship with the EGM2008 development team, providing them with access to a recent release Australian gravity database, the latest Australian digital elevation model (DEM), a nationwide set of 254 GPS-levelling data, and a nationwide set of 1080 historical astrogeodetic vertical deflections. Despite this, we have found quite a few discrepancies in this comparison that indicate problems with the Australian data, some of which were known, but some that were not.

Indeed, our attempted 'validation' has proven to be a two-way process, where EGM2008 has confirmed problems that were already known (e.g., with the Australian quasigeoid model in the coastal zone), but it has identified some problems (e.g., with

the Australian gravity data) that we were previously unaware of. This alone is testament to the quality of EGM2008, i.e., an implicit validation. In this report, we first describe the Australian data and their perceived deficiencies, followed by EGM2008's confirmation of these, showing our primary conclusion that EGM2008 is implicitly validated over Australia. Results of computations from EGM2008's pre-release PGM2007A are also shown for comparison.

2. Description of the Australian Data

2.1 Australian gravity data

The Australian national gravity database (Fraser et al. 1976, Murray 1997) is now freely available via a web-based delivery system (http://www.ga.gov.au/gadds), subject to licence conditions. For this study, the July 2007 and June 2008 releases of the gravity data base are used, Compared to the 1996 data release used for AUSGeoid98, there is now much more metadata and information on the individual records in the database. However, not all individual records are accurate (e.g., marine gravity measurements are specified on the Australian Height Datum (AHD), which is impossible because the AHD is simply not defined offshore). Therefore, some caution is needed. The July 2007 release of the database contains 1,245,026 land and marine gravity observations (Fig. 1) while the June 2008 release contains 1,304,904 land observations and no marine observations (Fig. 2). The marine gravity observations were removed by Geoscience Australia during the review cycle of Featherstone (in press), which demonstrated them to be in gross error (up to 900 mGal!) because no cross-over adjustment had been applied.

The gravity datum for the June 2007 release is ISOGal84 (Wellman et al. 1985), which is tied to the IGSN71 (Morelli et al. 1971). The gravity datum for the July 2008 release is the Australian Absolute Gravity Datum 2007 (AAGD07; Tracey et al. 2007), which is not specifically tied to the IGSN71. Instead, it is based on a nation-wide set of 60 absolute gravity measurements made with a portable A10 gravimeter. AAGD07 is 0.078 mGal less than ISOGal84.

The broad-scale coverage of land gravity observations was collected on an ~11 km grid (~7 km in South Australia), mostly after the 1950s so as to promote the development of resources in Australia (Fraser et al. 1976, Murray 1988). Since most of these data were collected before the establishment of the AHD (Roelse et al. 1971, 1975), most of the heights of the gravity observations were determined by barometers

(Bellamy and Lodwick 1968), though some surveys were conducted along spirit-levelling lines available at the time (datum usually at a nearby tide-gauge). Barlow (1977) estimates the barometric elevation error of these earlier surveys to be between 3 m and 10 m; the quality of the pre-AHD levelling remains unknown.

Since these Australia-wide reconnaissance gravity surveys, additional in-fill gravity data have been added to the database by State/Territory geological and geophysical mapping agencies, the private sector, academic institutions and others. Interrogation of the 2007 release database indicates that around 30,000 of these are on AHD benchmarks giving far more precise heights (but see the later discussion on distortions in the AHD). However, the 2008 release database no longer indicates which observations are on AHD benchmarks. Though this information must be held by Geoscience Australia, it is not provided via the web-based delivery system.

Over the last decade, most of the newer gravity data in Australia has been coordinated using carrier-phase relative GPS techniques. However, this needs a quasi-geoid model to convert them to normal heights. [The AHD uses a truncated variant of the normal orthometric height system (Featherstone and Kuhn 2006; Roelse et al. 1971, 1975)]. Unfortunately, however, the quasi/geoid models used for this GPS height transformation are not stored in the Geoscience Australia database, nor are the original ellipsoidal heights, but the GPS-coordinated gravity surveys were identified in the 2007 database. From Featherstone's [unnamed] contacts with the major GPS-gravimetry contractors in Australia, these GPS surveys have used a variety of models, ranging from OSU91A (Rapp et al. 1991) and EGM96 (Lemoine et al. 1998) to AUSGeoid91 (Kearsley and Govind 1991), AUSGeoid93 (Steed and Holtznagel 1994) and AUSGeoid98 (Featherstone et al. 2001).

As such, the later 'validation' is broken down according to the perceived quality of the land gravity data (all data, GPS-coordinated gravity, and ship-track gravity). Hopefully, the relative accuracy of these datasets will give a more informed evaluation, rather than the 'wholesale' approach taken previously of using all data with equal weight (cf. Kearsley and Holloway 1989, Zhang and Featherstone 1995, Kirby et al. 1998, Amos and Featherstone 2003).

Second-order, atmospherically corrected, free-air gravity anomalies were recomputed from the primary observations (gravity values and 3D coordinates) in the Australian gravity databases. The formulas used are summarised in Featherstone and Dentith (1997) and Hackney and Featherstone (2003). The database claims to provide

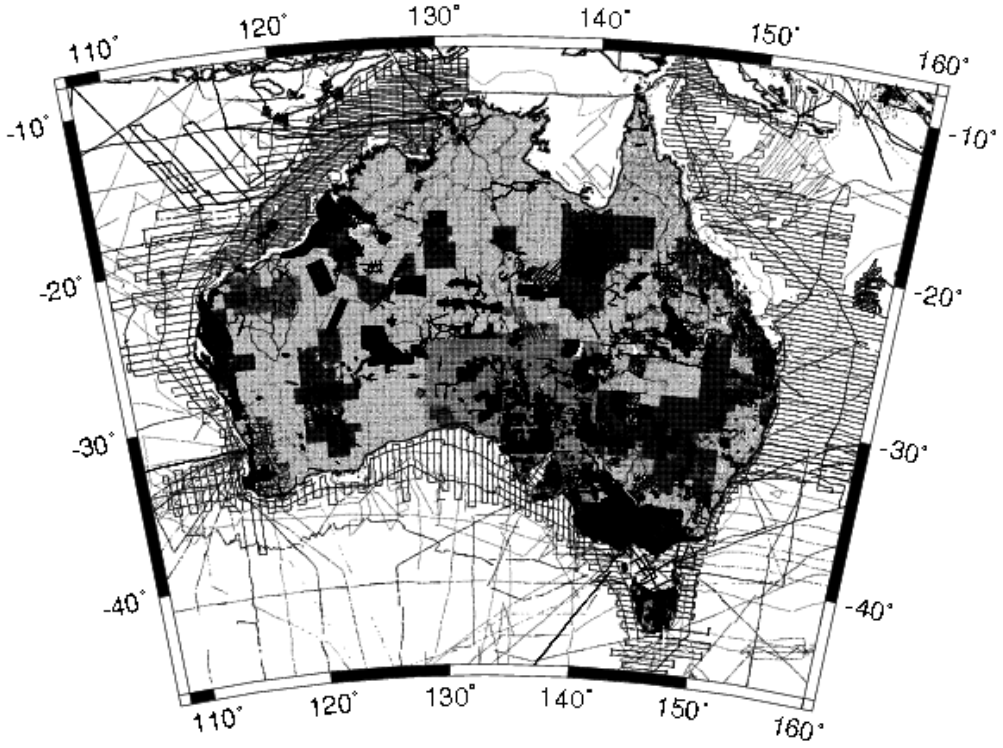


Fig 1. Coverage of the 1,245,026 Australian land and marine gravity observations in the July 2007 data release from Geoscience Australia (Lambert projection)

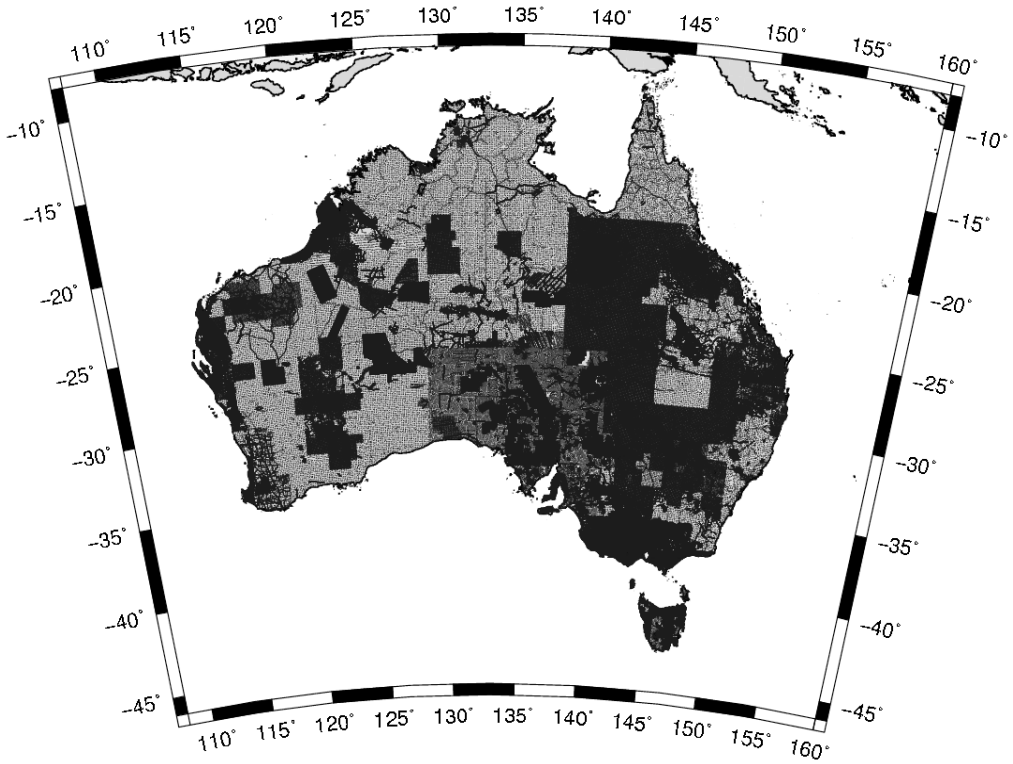


Fig 2. Coverage of the 1,304,904 Australian land gravity observations in the June 2008 data release from Geoscience Australia (Lambert projection)

horizontal coordinates on the Geocentric Datum of Australia 1994 (GDA94), but no information is given about the transformation method used (if at all). For instance, pre-1966 gravity observations were collected before the nation-wide adoption of the Australian Geodetic Datum, so transformation to GDA94 will technically be impossible. Featherstone (1995) shows that the use of a non-geocentric datum to compute gravity anomalies causes small (0.1 mGal), yet systematic, errors in the computed gravity anomalies.

The ship-track gravity data around Australia (Symonds and Willcox 1976, Mather et al. 1976) are far more problematic. In AUSGeoid98, these data were [incorrectly] assumed to have previously been crossover adjusted (Featherstone et al. 2001). However, they were not, as shown through comparison with multi-mission satellite altimetry data (Featherstone, in press) or via point-mass modelling (Claessens et al. 2001). Indeed, the later 'validation' of EGM2008 using AUSGeoid98 clearly shows that the erroneous ship-track data have distorted AUSGeoid98 in offshore regions. Therefore, rather than 'validating' EGM2008 using AUSGeoid98, EGM2008 is 'invalidating' AUSGeoid98 in some coastal areas, but this problem has been known for some time now.

Petkovic et al. (2001) readjusted these ship-track data [note that AUSGeoid98 used the 1996 data release], but the ship-tracks were constrained to Sandwell and Smith's satellite-altimeter-derived gravity anomalies (version unknown). Since satellite-altimeter data are notoriously problematic in the coastal zone (e.g., Andersen and Knudsen 2000, Deng and Featherstone 2006), it is highly likely that the so-adjusted Australian ship-track gravity data have become distorted in this region. For instance, Petkovic (2004, pers comm) commented that they had significant problems in the Bass Straight between the Australian mainland and Tasmania. Therefore, the evaluations using Australian ship-track data should be treated very sceptically. We did attempt to crossover-adjust the Australian ship-track observations ourselves, but the adjustment failed because it is very poorly conditioned in many places because of the large distances involved and the scarcity of ship tracks (cf. Fig. 1).

Later, it will be shown that the ship-track gravity observations in the 2007 Australian gravity database are not the readjusted values from Petkovic et al. (2001). This works on the assumption that the Australian ship-track data have *not* been used in the development of EGM2008, where some tracks show large consistent offsets. Moreover, these are consistent with the differences shown in Featherstone (in press).

As such, the Australian ship-track data simply should not be used to try to 'validate' EGM2008. Instead, EGM2008 invalidates these data. As stated, the ship-track gravity observations have all been removed in the 2008 release of the gravity database, during the review cycle of Featherstone (in press).

Many of the land gravity observations in the July 2008 release of the Australian gravity database have not been used in the computation of EGM2008. Therefore, these observations can provide a more independent validation of EGM2008. The EGM2008 development team (Factor 2008, pers. comm.) provided us with the horizontal locations of all 905,483 land gravity observations that were used in the computation of EGM2008. Matching of these locations (after application of a datum shift to the GDA94) with locations of observations in the 2008 gravity database revealed that 548,787 points in the Australian gravity database do not match any observation used in EGM2008 to within 100m. These form an independent set of observations (Fig. 3).

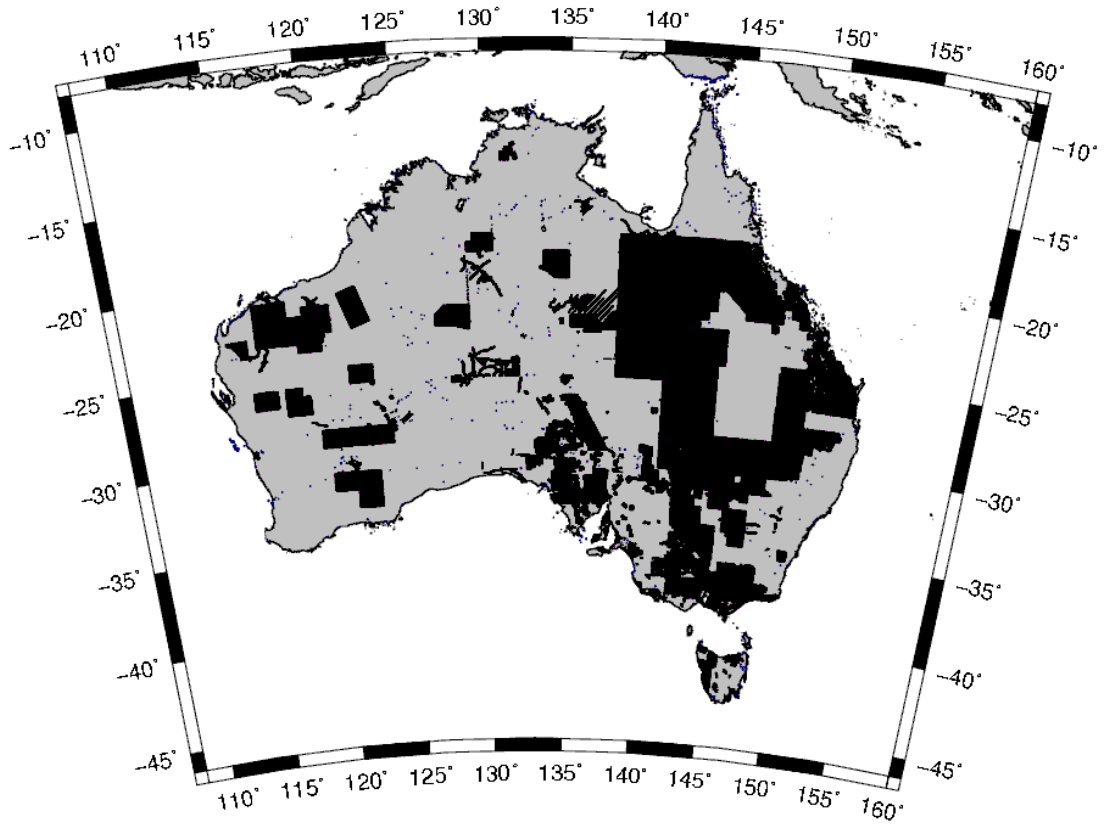


Fig 3. Coverage of the 548,787 Australian land and marine gravity observations in the June 2008 data release from Geoscience Australia that were not used in the computation of EGM2008 (Lambert projection)

It was also found that 156,269 observations used in the computation of EGM2008 do not match any of the points in the Australian gravity database to within 100 m. The reason for this is probably that NGA holds gravity observations not stored in the Australian gravity data base.

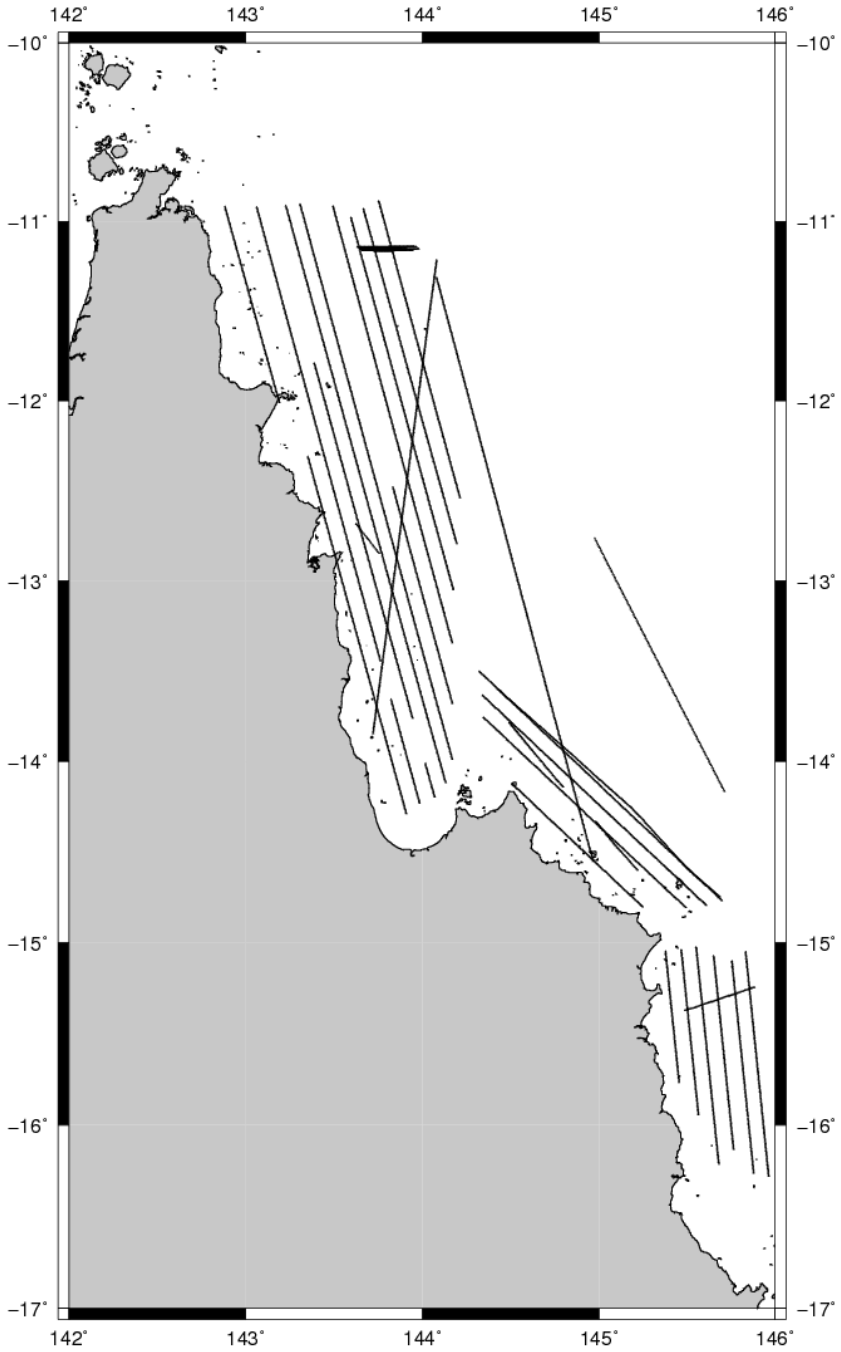


Fig 4. Coverage of the 6,725 observations from the Barrier Reef Airborne Gravity Survey 1999 (BRAGS'99) (Mercator projection)

An additional dataset of gravity observations used in this study consists of airborne gravimetry from the Barrier Reef Airborne Gravity Survey (BRAGS'99) (Sproule et al. 2001), provided by Forsberg (2004, pers. comm.). This survey covers an area over the shallow waters of the Great Barrier Reef to the north-east of Australia (Fig. 4). The airborne gravity data were taken at a flight altitude of ~500 m and low-pass filtering was applied with filter parameters set such that the survey has a spatial resolution of 8 km. Sproule et al. (2001) estimate the noise level of the data is 2.8 mGal, based on a crossover analysis. Molodensky-type free-air gravity anomalies were computed from the raw gravity observations at flight altitude to allow for a comparison with EGM2008 at flight altitude.

2.2 Australian GPS-levelling data

Although Featherstone et al. (2001) and Featherstone and Guo (2001) used a set of 1013 GPS-levelling data across Australia (Fig. 5) to 'validate' AUSGeoid98, it has since been discovered that an unknown number of these ellipsoidal heights were observed *indirectly*. The term indirectly means that a GPS survey was tied to a base

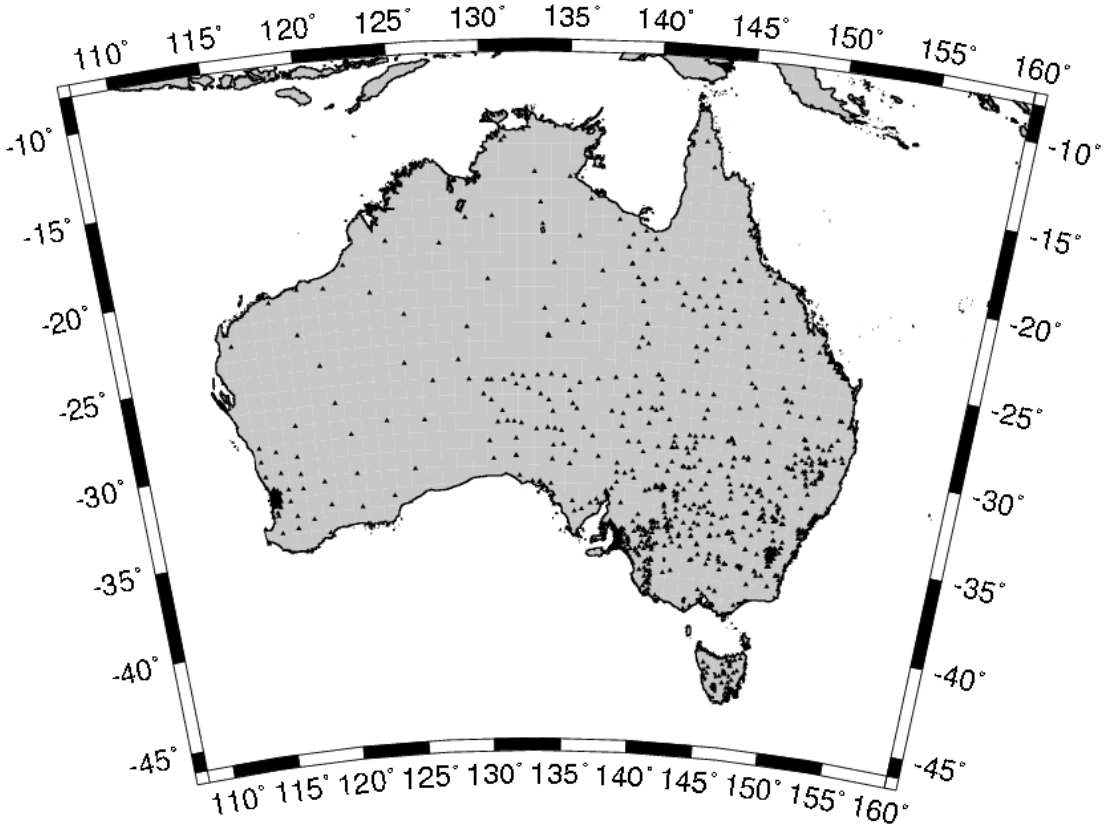


Fig 5. Coverage of the older 1013 GPS-levelling points (Lambert projection)

station whose ellipsoidal height had been calculated from the AHD height and a quasi/geoid model. Although the ellipsoidal height at the other end of the baseline was used to populate this database of 1013 points, they are considered 'impure' because the starting ellipsoidal height will have been contaminated by AHD and quasi/geoid model errors, thus propagating into some of the 1013 heights used.

Since then, a newer 'pure' GPS ellipsoidal height dataset has been observed at 254 junction points of the AHD (cf. Soltanpour et al. 2006, Featherstone and Sproule 2006). These ellipsoidal heights (Fig.6) used typically five or more days of observations and most were post-processed with the AUSPOS on-line GPS processing service (http://www.ga.gov.au/bin/gps.pl). However, there are still some problems with these ellipsoidal heights because they are not all on the same realisation of the International Terrestrial Reference Frame (ITRF). Current metadata prevents this being rectified immediately by transformation (e.g., just ITRF is specified for some States/Territories instead of the exact ITRF realisation and the epoch used for the GPS data processing). The differences are estimated to be a few centimetres.

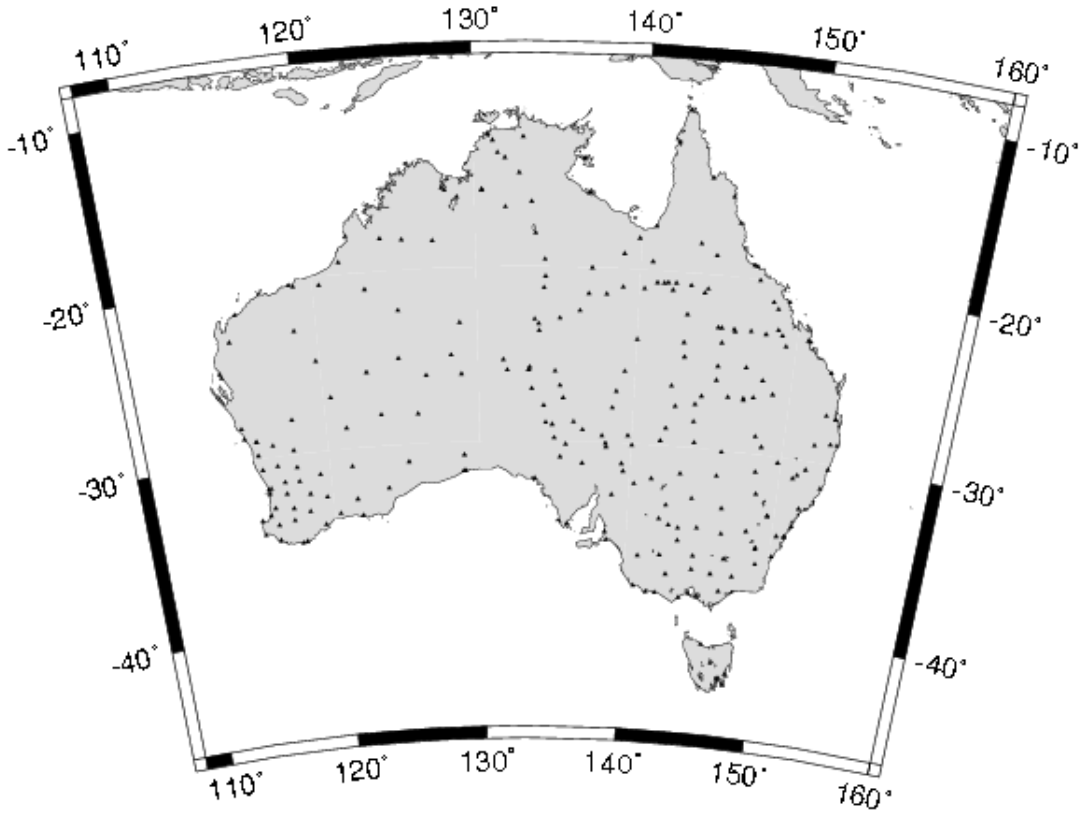


Fig 6. Coverage of the newer 254 GPS-levelling points (Lambert projection).

These and other GPS observations will be reprocessed by Geoscience Australia [the custodian of these data] to bring them to ITRF 2005 (Altamimi et al. 2007), thus homogenising this 254-point dataset, as well as including newer GPS surveys (Johnston 2007, pers comm). However, this reprocessed dataset is not yet available, so we have had to work with the same data used by Soltanpour et al. (2006) and Featherstone and Sproule (2006).

Two more reliable GPS-levelling datasets available in Australia are over the regional areas of the southwest seismic zone (SWSZ) in Western Australia (cf. Featherstone 2004, Featherstone et al. 2004) and the South Australian Seismic Zone (SASZ) near Adelaide. While they do not cover huge areas (Fig. 7), the dual-frequency GPS data were collected for at least seven days per station (some for a month) and processed with Bernese v5.0 and IGS (International GNSS Service) precise 'final orbits' (e.g., Featherstone et al. 2004). The levelled heights were later collected by the relevant State geodetic agencies by two-way closed levelling to the nearest AHD benchmarks.

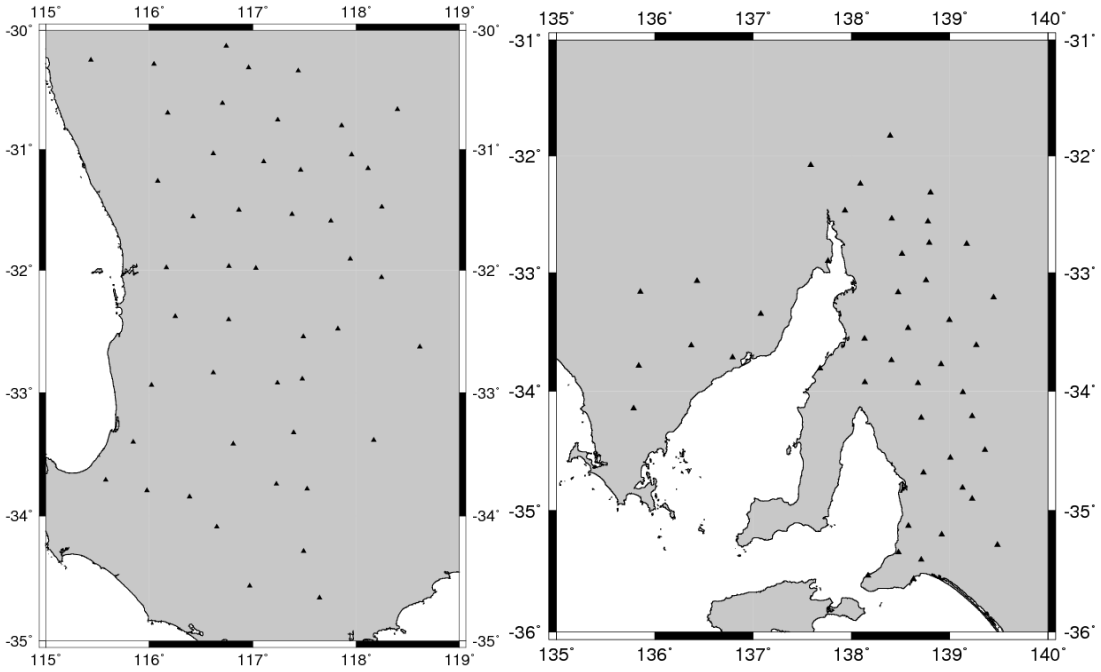


Fig 7. Coverage of the **(a: left)** 48 GPS-levelling points in the SWSZ, and **(b: right)** 45 GPS-levelling points in the SASZ (Mercator projections).

The final GPS-levelling dataset used in this study is a set of 243 points in Western Australia (Fig. 8). The GPS observations for this dataset were taken between 1995 and 2007 over a period of at least six hours using dual-frequency receivers. The

data were processed with Bernese v5.0 and IGS precise ‘final’ orbits in the ITRF2005 reference frame, and corrected for ocean tide loading effects. The mean of the estimated formal standard deviations of the ellipsoidal heights is 2.0 mm, though this is probably overoptimistic by a factor of 5 to 10. .

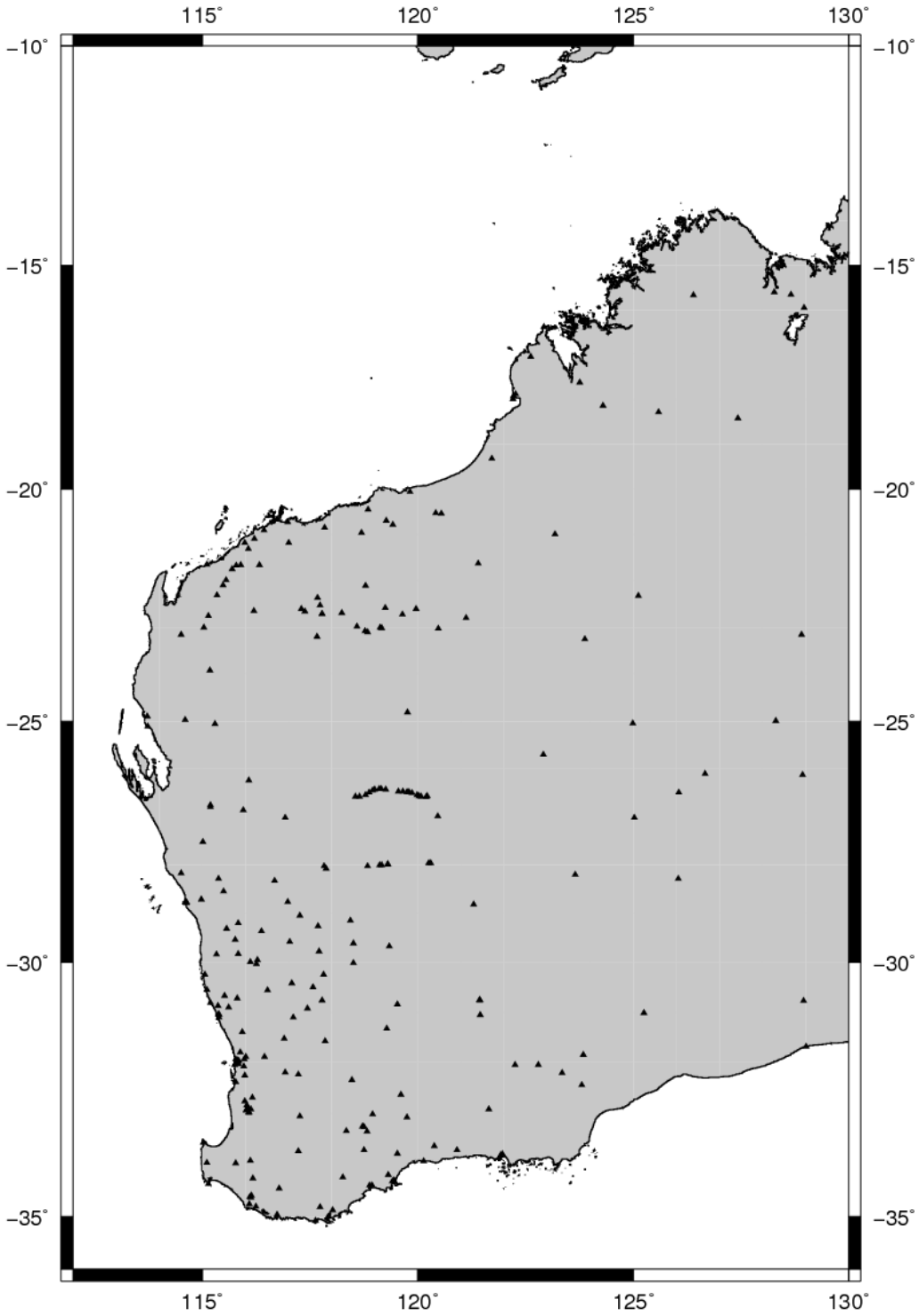


Fig 8. Coverage of the 243 GPS-levelling points in Western Australia (Mercator projection)

Due to the differences in processing strategies and perceived quality of the different GPS-levelling datasets, as with the gravity data, the evaluation of EGM2008 is conducted for the separate datasets.

Of more concern in any GPS-levelling evaluation in Australia is the quality of the levelling data. The AHD is principally a third-order vertical datum (Roelse et al. 1971, 1975; Morgan 1992), where third-order spirit-levelling measurements in Australia allow for a 12 root km millimetre misclose (ICSM 2007), which is considerably worse than in most parts of Europe (Adam et al. 1999) and North America (Zilkoski et al. 1992) for example. Moreover, the AHD was realised by a fixed-network adjustment constrained to mean sea level (MSL) observed over a three-year period at 30 tide gauges around the Australian mainland and two tide gauges in Tasmania (e.g., Featherstone 2001). Finally, the AHD uses a truncated version of the normal orthometric height system (Roelse et al. 1971, 1975; Featherstone and Kuhn 2006).

The largest problem in the spirit-levelled and MSL-fixed-adjusted AHD heights is a predominantly north-south-oriented distortion of around 1-2m (Featherstone 2001, 2004, 2006, 2007), which presents the major limitation to using GPS-levelling in Australia to 'validate' any quasigeoid model. We believe that most of this distortion has been caused by the constraints to MSL, in which mainly north-south-oriented sea-surface topography around Australia causes the adjustment to be north-south-tilted with respect to an equipotential surface. As such, the GPS-levelling 'validation' presented later should be given less weight, but some interpretation of the north-south, AHD-induced, tilt in the differences will be included in an attempt to rate their relative credibility.

To overcome the distortions in the AHD, we readjusted the levelling observations, provided by Geoscience Australia (Johnston 2007, pers. comm.), fixing the height of one tide-gauge only, so that the network is minimally constrained. The normal orthometric heights of the national and Western Australian GPS-levelling datasets were fixed to the tide gauge at Albany on Western Australia's south coast, while the normal orthometric heights of the South Australian Seismic Zone dataset were fixed to the tide gauge at Port Lincoln on the Eyre Peninsula. These minimally constrained readjusted heights do not show the north-south oriented distortion that the AHD contains and are therefore more useful for validation of EGM2008.

2.3 Australian astrogeodetic vertical deflections

During correspondence with the EGM2008 development team, we provided them with 1080 Australian astrogeodetically observed vertical deflections/deviations (Fig. 9). Vertical deflection data, being higher order derivatives of the Earth's disturbing potential, provide a better validation of high-degree GGMs (cf. Jekeli 1999; Müller et al. 2007a; Hirt et al. 2007; Featherstone and Morgan 2008). The provenance and estimated quality of these data are described in Featherstone (2006, 2007), Featherstone and Morgan (2007) and Featherstone and Lichti (2008). The accuracy is crudely estimated to be around one arc-second in each deflection component, but this is difficult to ascertain as the original records no longer seem to exist. [At least, neither we nor Geoscience Australia could locate them.]

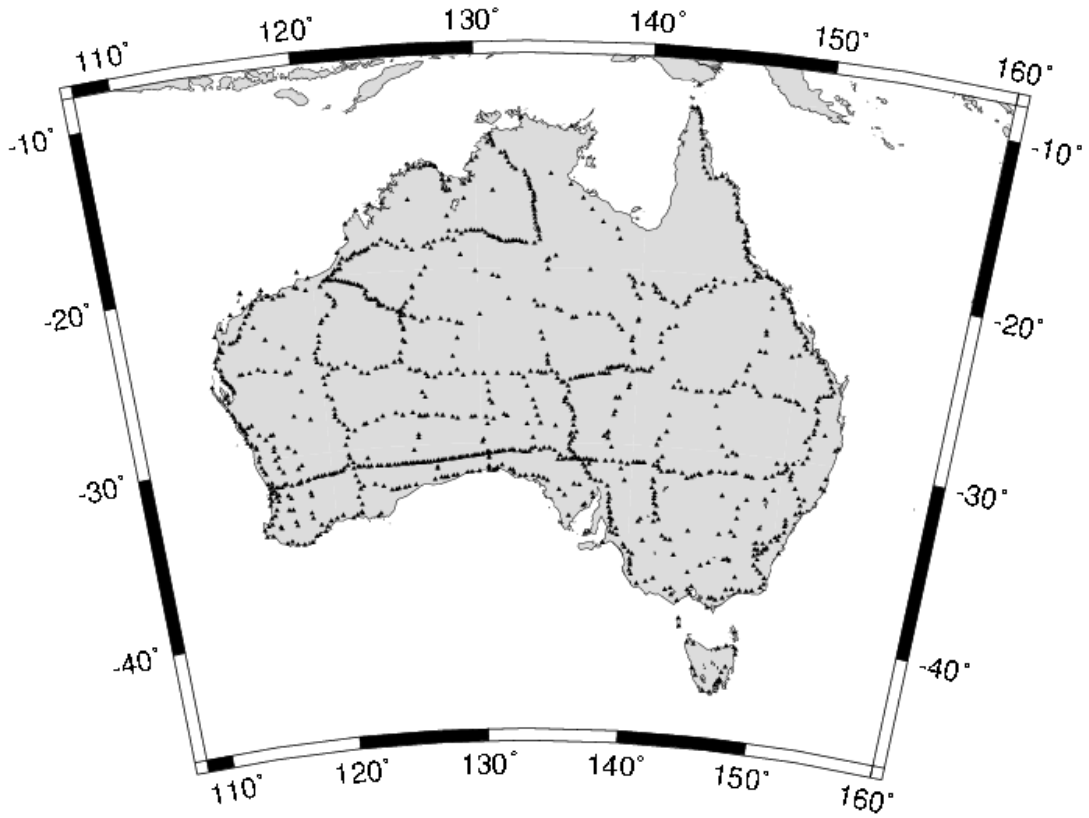


Fig 9. Coverage of the 1080 astrogeodetically observed vertical deflections [Lambert projection]

As such, the main problem with the reliability of the Australian vertical deflection is the vintage of the data (cf. Kearsley 1976). Most, if not all [no dates are available], observations were made before or during the establishment of the AGD66 (i.e., pre-1966; Bomford 1967), so are subject to timing, instrumental and star-almanac er-

rors over 40 years ago (cf. Featherstone and Lichti 2008). While new digital zenith cameras, coupled with GPS, are now producing high precision vertical deflection data (Hirt and Bürki 2002; Hirt and Seeber 2007; Müller et al. 2007b), no such data are available in Australia, yet. As such, the Australian 'validation' of EGM2008 using vertical deflections must account for the poorer quality of the data.

2.4 AUSGeoid98

The AUSGeoid98 regional gravimetric quasigeoid model (Featherstone et al., 2001) remains the nationally recognised standard in Australia for the transformation of GPS-derived ellipsoidal heights to the AHD, despite being computed nearly a decade ago. It refers to the GRS80 ellipsoid. A new model is currently being computed based on EGM2008 (e.g., Featherstone et al. 2007). However, it is informative to compare EGM2008 with AUSGeoid98 to see if there are any spatial differences that warrant further investigation. Indeed, this 'validation' highlights known problems with AUSGeoid98 in marine areas, as well as identifying some previously unknown ones. As such, EGM2008 'invalidates' AUSGeoid98 in some regions.

AUSGeoid98 (Fig. 10) was computed from EGM96 (Lemoine et al. 1998) to degree and order 360, the 1996 release of Geoscience Australia's land and marine gravity data (note the earlier comments on the quality of the Australian ship-track gravity data), marine gravity anomalies from Sandwell and Smith (1997; version 9.2) warped to fit the [incorrect] ship-track data using least-squares collocation (Kirby and Forsberg 1998), and terrain corrections from the version 1 Australian digital elevation model (DEM). The latter had to be generalised to 27 arc-seconds because of errors in the DEM (Kirby and Featherstone 1999, 2001).

The computation method chosen for AUSGeoid98 was a hybrid of the remove-compute-restore and deterministically modified kernel approach with the degree-20 Featherstone et al. (1998) kernel for a 1.5 degree spherical cap. The zero-degree term of ~1m (including any vertical datum offset for the AHD) was estimated by computing a mean difference between the 1013 GPS-levelling data described earlier and AUSGeoid98, but no tilts were estimated nor applied. AUSGeoid98 is shown in Fig. 10.

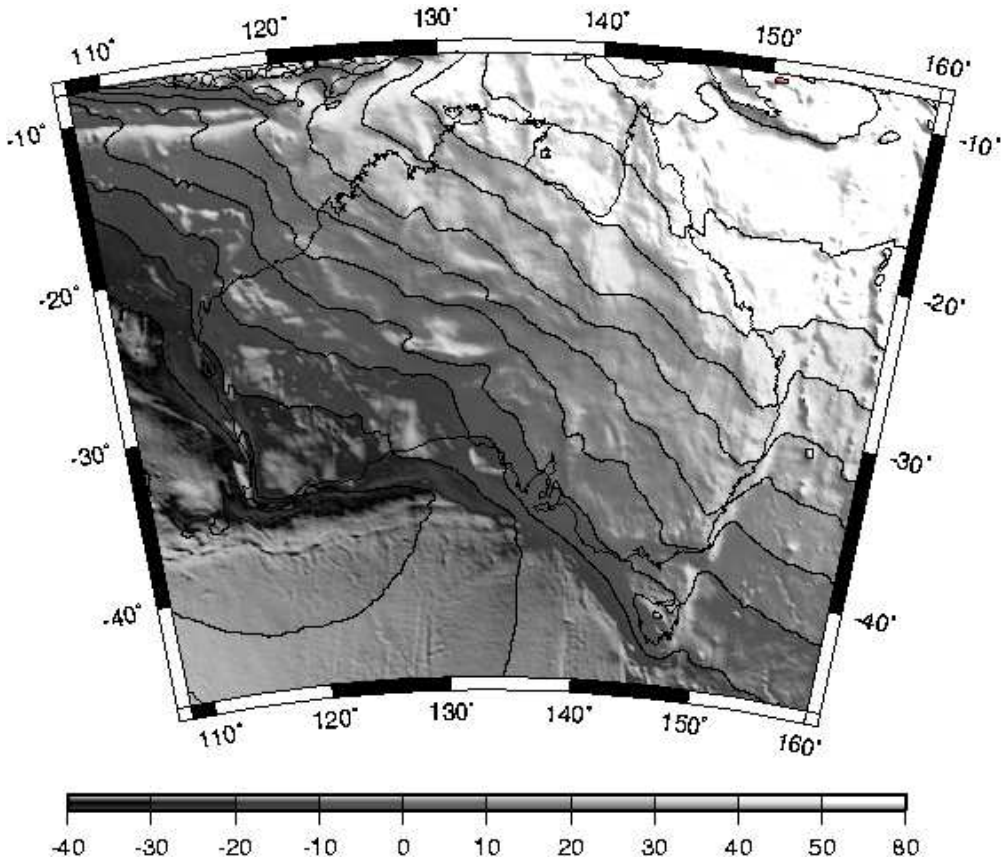


Fig 10. AUSGeoid98 with respect to GRS80 [Lambert projection; units in metres]

3 Results

All gravity-field-related quantities computed from PGM2007A and EGM2008 in this Australian 'validation' used the `HARMONIC_SYNTH.f` FORTRAN software provided by the EGM2008 development team. This software was adapted slightly so as to run on the Western Australian Centre for Geodesy's Sun UNIX workstations. It was tested using the sample datasets, also provided by the EGM2008 development team, and compared with our in-house code, showing that the insignificant differences were only due to computer-dependent algebra.

In order to enforce compatibility with the GRS80 ellipsoid used for all the Australian data, GRS80 parameters were set in the 'parameter input' files for the `HARMONIC_SYNTH.f` so that the zero-degree term and scaling of the even-degree coefficients were computed according to the algorithm in Lemoine et al. (1998). [Note that the previous Australian treatment of the zero degree term, neglecting differences in potential (Kirby and Featherstone 1997) is incorrect.]

3.1 Comparisons with Australian gravity data

First, the computer time required to evaluate a GGM up to degree and order 2160 at a large number of scattered points is very long, even though the accelerated routines of Holmes and Featherstone (2002) are used in `HARMONIC_SYNTH.f`. Due to the large number of gravity observations (~1.3 million), and because gravity observations are generally irregularly spaced, spherical harmonic recursions along parallels cannot be utilised to accelerate the computations. Some of the results presented below for the Australian land gravity anomalies have therefore used pre-evaluation of PGM2007A and EGM2008 on a 2 arc-minute grid, followed by bicubic interpolation to the gravity observations' locations.

The `HARMONIC_SYNTH.f` software needs to 'know' the 3D location of the gravity observation with respect to the geometrical surface of the reference ellipsoid used (GRS80 in the case of this Australian 'validation'). This will not yield gravity disturbances because `HARMONIC_SYNTH.f` is configured to deliver gravity anomalies at the point of observation (i.e., Molodensky-type free-air gravity anomalies). However, since only AHD heights of gravity observations are available in the Australian national gravity database, height anomalies (quasigeoid heights) were first computed at the gravity observation locations from PGM2007A/EGM2008, and these were added to the AHD heights to obtain an ellipsoidal height for each gravity observations. These ellipsoidal heights were used to compute (linearly approximated) free-air gravity anomalies at the gravity observation points via the fundamental equation of physical geodesy (boundary condition). Tables 1 and 2 show results from comparisons of various GGMs with the 2007 and 2008 releases of the Australian gravity database, respectively.

The majority of the free-air gravity anomalies computed from PGM2007A and EGM2008 over land show a good correspondence with the land free-air gravity anomalies (Fig. 11), even in areas where there are large gravity anomaly gradients such as in central Australia. Figure 11 shows that the largest differences are in the mountainous regions (cf. Fig. 12), notably in Tasmania and along the Great Dividing Range along the eastern coastline. This could be caused by erroneous Australian data, but internal validation (Sproule et al. 2006) does not show such a problem.

model	*# points*	*degree*	*max*	*min*	*mean*	*std*
raw data (all data)	1,245,026	n/a	+931.029	–229.847	+4.292	±26.565
PGM2007A (land data)	1,095,065	2160	+68.741	–79.860	–0.296	±4.954
EGM2008 (land data) Fig. 11	1,095,065	2160	+68.728	–78.169	–0.296	±4.924
EGM96 (land data)	1,095,065	360	+111.393	–95.202	–0.307	±11.756
PGM2007A (marine data)	149,961	2160	+972.004	–171.687	–0.810	±12.104
EGM2008 (marine data) Fig. 15	149,961	2160	+970.963	–171.681	–0.748	±12.034
EGM96 (marine data)	149,961	360	+988.674	–124.895	–1.278	±17.641
PGM2007A (all data)	1,245,026	2160	+972.004	–171.687	–0.358	±6.266
EGM2008 (all data)	1,245,026	2160	+970.963	–171.681	–0.350	±6.226
EGM96 (all data)	1,245,026	360	+988.674	–124.895	–0.424	±12.611

Table 1 Fit of the geopotential models to Australian free-air gravity anomalies in the 2007 release database [units in mGal]

model	*# points*	*degree*	*max*	*min*	*mean*	*std*
PGM2007A (all data)	1,304,904	2160	+192.523	–88.485	+0.486	±5.557
EGM2008 (all data)	1,304,904	2160	+192.294	–88.756	+0.498	±5.541
EGM96 (all data)	1,304,904	2160	+189.775	–110.010	+0.296	±11.678
EGM2008 (independent data)	548,787	2160	+191.677	–67.641	+0.566	±6.373

Table 2 Fit of the geopotential models to Australian free-air gravity anomalies in the 2008 release database [units in mGal]

It can be seen from Fig. 13 that differences between EGM2008 and gravity observations at high altitude (> 1000 m) are more dispersed than those at lower altitudes. Figure 13 also shows that the differences have a small negative correlation with terrain height. Curiously, some surveys in mountainous regions that are part of the Australian gravity database appear to show a much larger correlation with terrain height than the total database. This requires further investigation.

The larger differences in mountainous areas are more likely to be a combination of problems modelling the variable gravity field in these mountainous regions (topographical and downward continuation corrections) and the omission error in EGM2008, where gravity field variations with a wavelength shorter than 5 arc-minutes will not be modelled. The omission error can be seen in Fig. 11, where a 'cantaloupe' pattern can be discerned throughout the image. Figure 14 shows a zoom-in on the southern Australian Alps for the GPS-coordinated gravity data from the

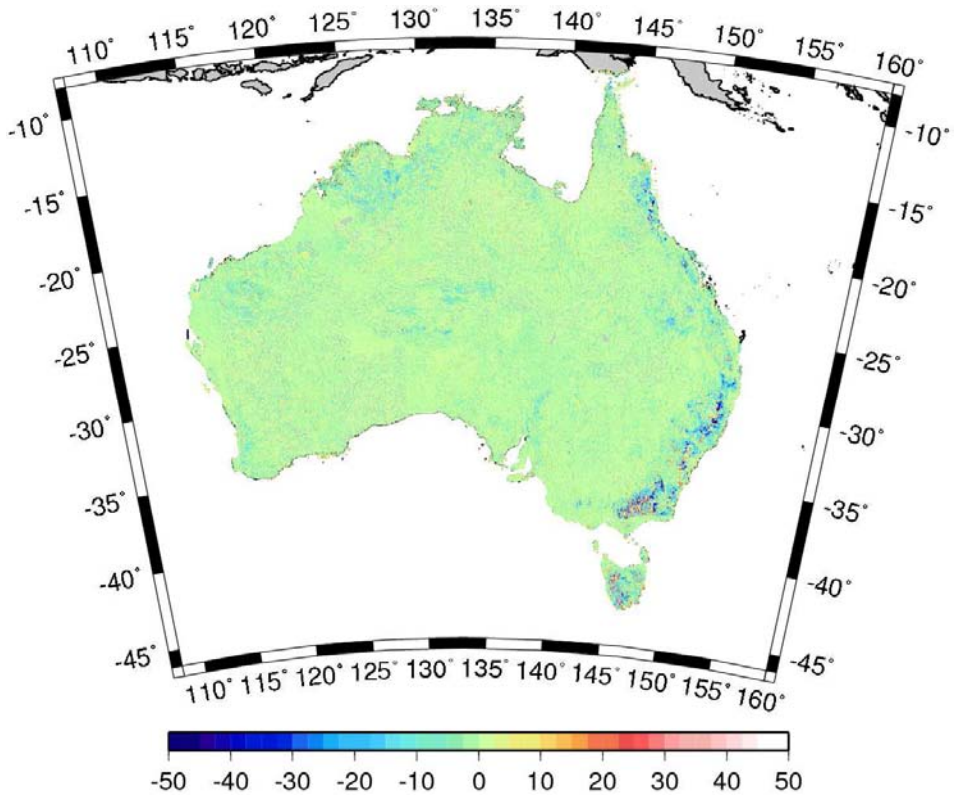


Fig 11. Differences between free-air gravity anomalies from EGM2008 and Australian free-air gravity anomalies on land [Lambert projection; units in mGal].

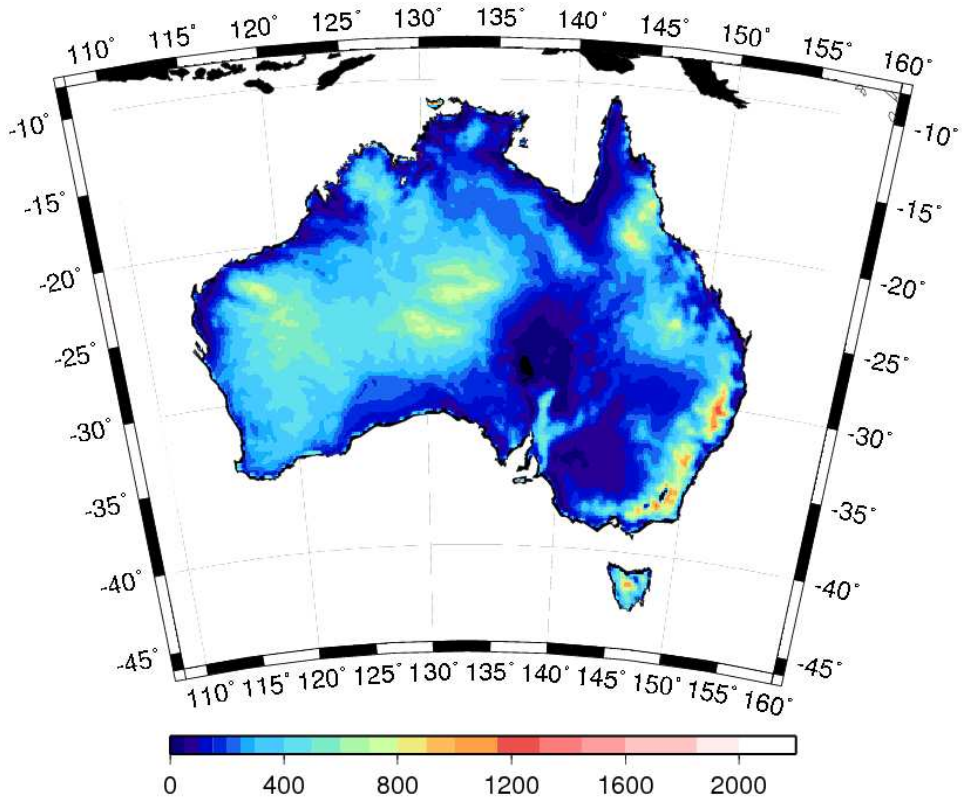


Fig 12. Australian topography from the version 2.1 DEM [Lambert projection; units in metres]

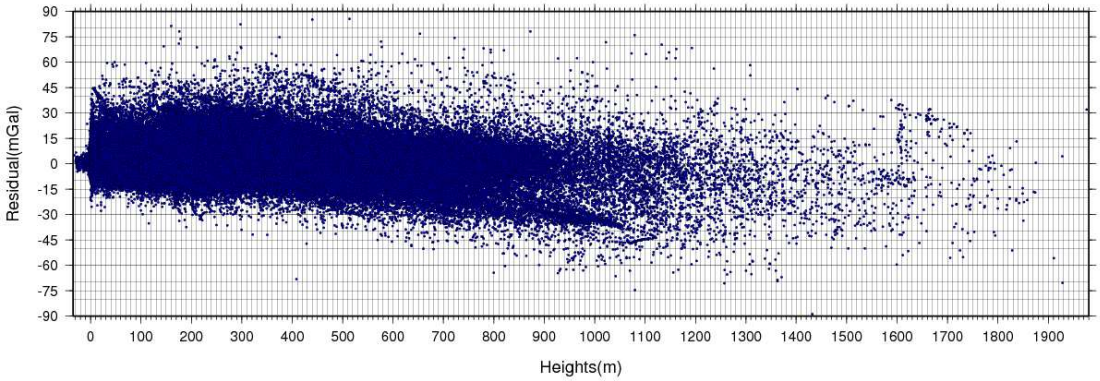


Fig 13. Differences between free-air gravity anomalies from EGM2008 and the 2008 release of the Australian gravity database as a function of terrain height

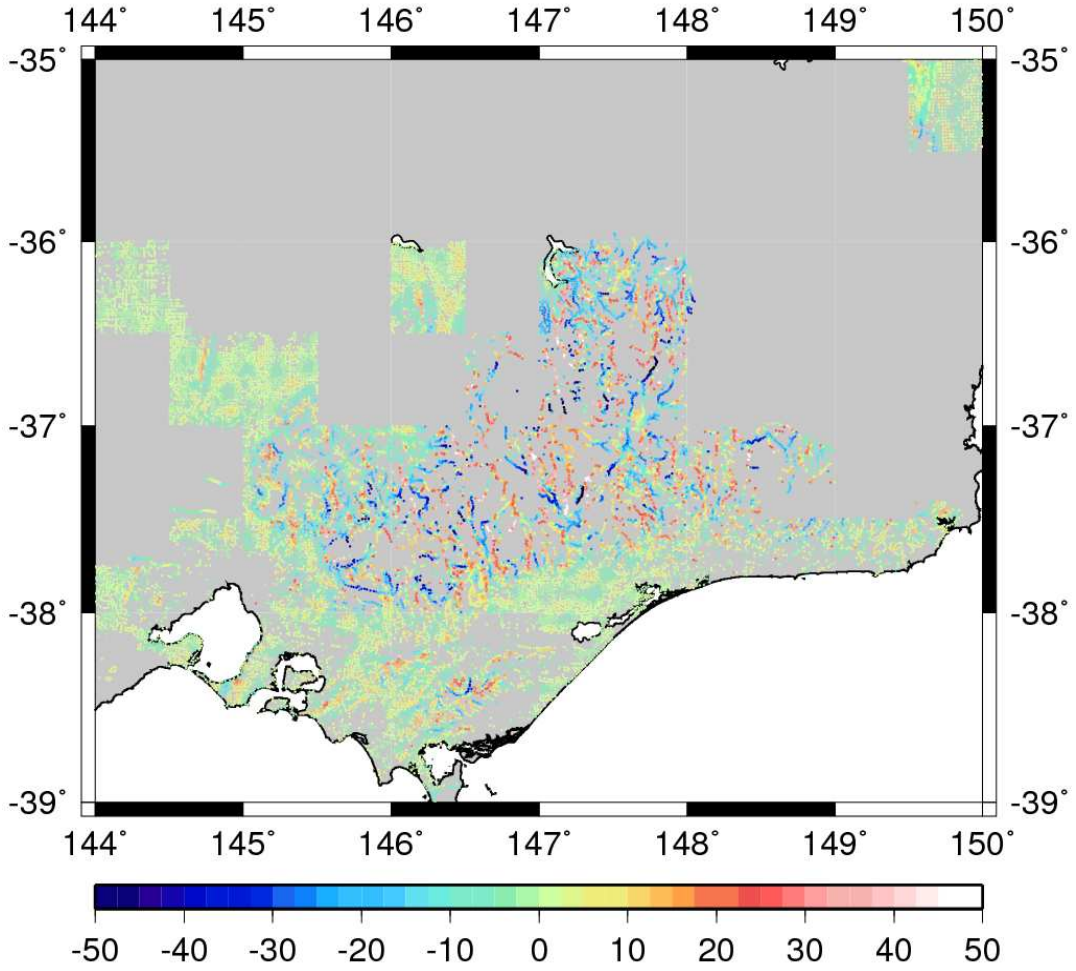


Fig 14. Differences between free-air gravity anomalies from PGM2007A and Australian free-air gravity anomalies over the Australian Alps [Mercator projection; units in mGal]

2007 data release. We suspect that these are not GPS-coordinated surveys, which are more usually conducted on a regular grid, and this is probably an error in the metadata in the 2007 data release.

There are also some larger differences in Fig. 11 close to the coastline (the land gravity database also includes a few hundred sea-bottom gravity observations and gravity observations made on sandbanks at low tide). We will revisit this later, but it is plausible that the satellite altimeter-derived gravity anomalies used in PGM2007A and EGM2008 remain in error in the problematic coastal zone (cf. Deng and Featherstone 2006).

From Fig. 15, the bulk of the free-air gravity anomalies computed from EGM2008 agree with the ship-track gravity anomalies to within ~5 mGal. However, several ship tracks show considerable biases of over 50 mGal (reaching over 900 mGal; Table 1), as was noted by Featherstone et al. (2001) who deleted most but not all of these (see later). This confirms that the Australian ship-track gravity database

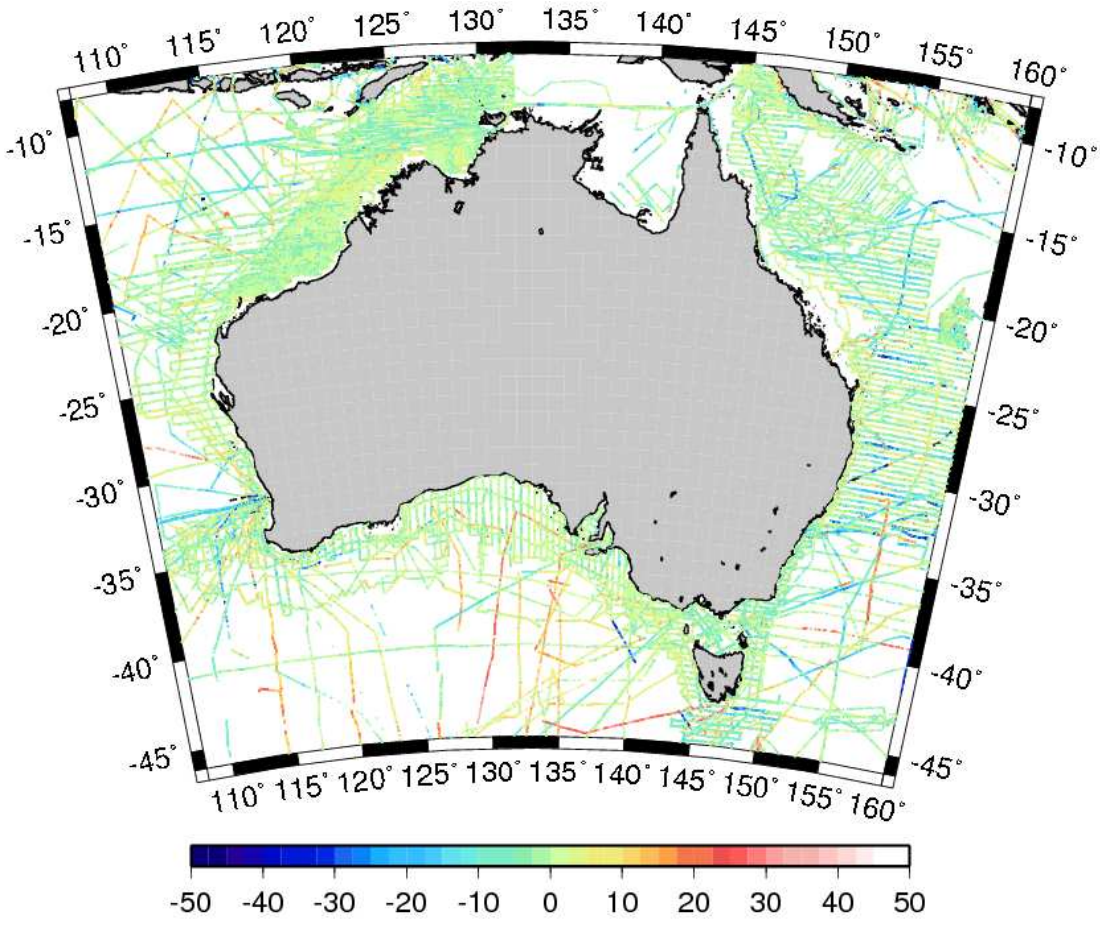


Fig 15. Differences between free-air gravity anomalies from EGM2008 and Australian ship-track gravity anomalies [Lambert projection; units in mGal].

has not been crossover adjusted. Though uncertain, we suspect that no ship-track data were used in EGM2008, so these differences essentially reflect the difference between altimeter-derived gravity anomalies in EGM2008 and the ship-tracks. Unlike the comparison in Featherstone (2003), large differences are not seen near the coast, indicating that the altimeter data have been improved in these regions.

The airborne gravity observations show a good agreement with EGM2008 (Fig. 16), with a standard deviation of the differences of 4.0 mGal (Table 3). This is only slightly larger than the expected noise level of the airborne gravity observations,

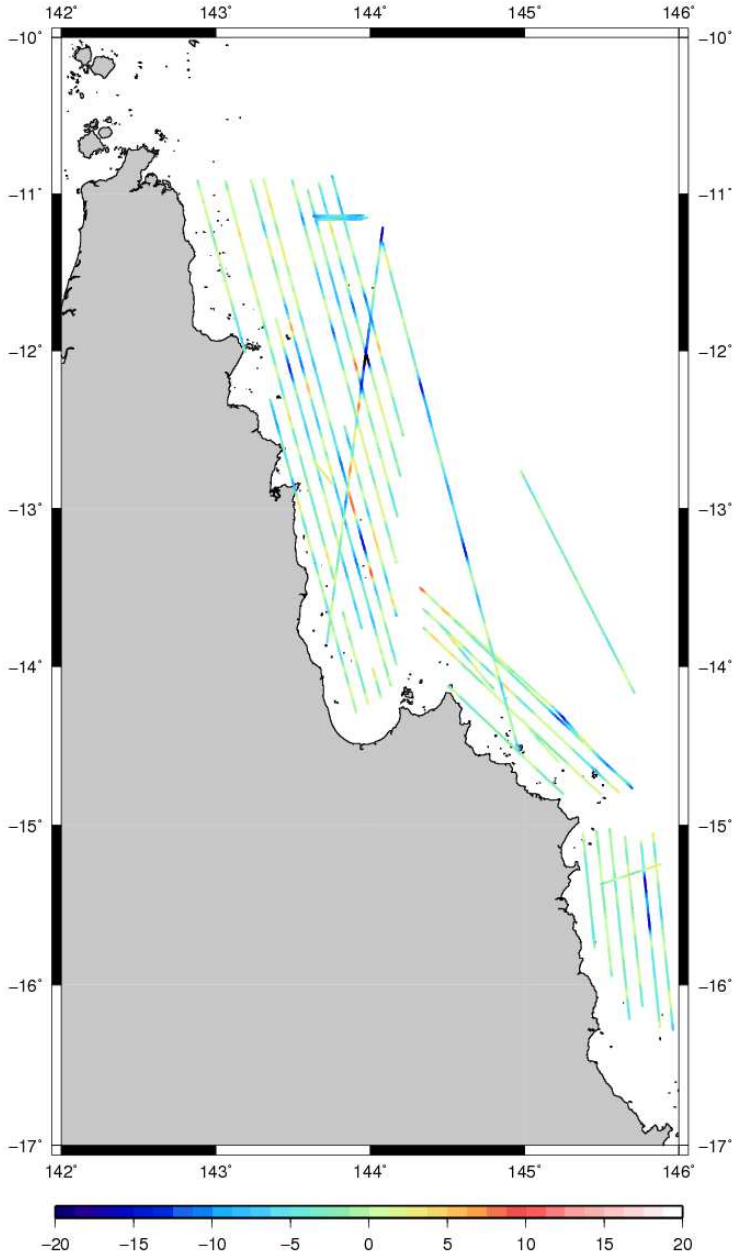


Fig 16. Differences between gravity anomalies from EGM2008 and airborne gravity anomalies at flight height [Lambert projection; units in mGal].

model	*# points*	*degree*	*max*	*min*	*mean*	*std*
raw data	6,725	n/a	212.008	–88.205	+89.038	±65.109
EGM96	6,725	360	41.356	–88.526	–13.107	±22.324
PGM2007A	6,725	2160	12.830	–19.682	–3.842	±3.962
EGM2008	6,725	2160	13.239	–22.434	–2.495	±3.954

Table 3 Descriptive statistics of the airborne gravity anomalies and of the relative differences with gravity anomalies computed from various GGMs [units in mGal]

which is estimated to be 2.8 mGal from crossover analysis (Sproule et al., 2001). Figure 16 shows that the differences are mainly of a very short wavelength nature, reflecting the low-pass filtering that is applied to airborne gravimetry. It can be seen in Table 3 that the comparisons with PGM2007A and EGM2008 give similar statistics and are a significant improvement on EGM96.

3.2 Comparisons with AUSGeoid98

Height anomalies (quasigeoid heights) were computed from PGM2007A and EGM2008 up to degree and order 2160 on a 2' x 2' grid and compared directly with the gravimetric-only AUSGeoid98 solution (Featherstone et al. 2001). This provides some of the most interesting (to us) results (Fig. 17 and Table 4).

	PGM2007A minus AUSGeoid98	*EGM2008 minus AUSGeoid98*
Number of points	1,781,101	1,781,101
% of area	3.842	3.842
Min	-2.472 m (120.917˚E, 10.633˚S)	-2.476 m (159.633˚E, 9.900˚S)
Max	13.062 m (125.217˚E, 8.567˚S)	12.983 m (147.367˚E, 8.400˚S)
Arithmetic mean	0.057 m	0.064 m
Area mean	0.072 m	0.081 m
Arithmetic RMS	0.458 m	0.462 m
Area RMS	0.504 m	0.509 m
Arithmetic STD	0.454 m	0.458 m
Area STD	0.499 m	0.504 m

Table 4 Descriptive statistics of the relative differences between quasigeoid heights computed from PGM2007A/EGM2008 and AUSGeoid98 on a 2’x2’ grid.

The differences are mainly of a medium-wavelength nature over the Australian mainland (Fig. 17). From a comparison with the differences between EGM96 (used in AUSGeoid98) and GGM02C (Tapley et al. 2005) (Fig. 18), these differences seem to come mostly from the GRACE data. The largest medium-wavelength difference in Fig. 17 appears in the Gulf of Carpentaria (centred on 140˚E, 12˚S), where only a very limited number of ship-track gravity observations is available (cf. Figs. 1 and 15). It could be that the altimeter-derived gravity anomalies are in error in this shallow sea. However, Tregoning et al. (2008) show that a weather-driven annual sea surface height variation of ~40 cm amplitude affects GRACE gravity field solutions. Therefore, the differences in this region are more likely due to aliasing in the GGMs, but errors in the altimeter data cannot be ruled out. Clearly, this needs further attention. The differences in Fig. 17 to the north of Australia are because no gravity data were available in this region for the computation of AUSGeoid98.

Figure 19 shows the differences between height anomalies from PGM2007A and EGM2008 over the AUSGeoid98 area. The differences over land are in the range of ±20 cm and mainly of medium wavelength nature. This is due to the use of a different GRACE-derived satellite only GGMs in PGM2007A and EGM2008. Over the oceans, a short wavelength noise is also visible. This is due to the use of different satellite altimeter gravity anomalies in PGM2007A and EGM2008. The difference over the Gulf of Carpentaria is now much less, suggesting that the aliasing was larger in EGM96. However, care still needs to be exercised in this region.

The next most noticeable features in Fig. 17 are the stripes offshore (e.g., to the east of Queensland and northern New South Wales). These stripes are due to the unadjusted ship-track data used in AUSGeoid98 (discussed earlier). We are unsure whether ship-track data were used in the computation of EGM2008, but from these analyses it appears not, or if they were, they have been crossover adjusted properly.

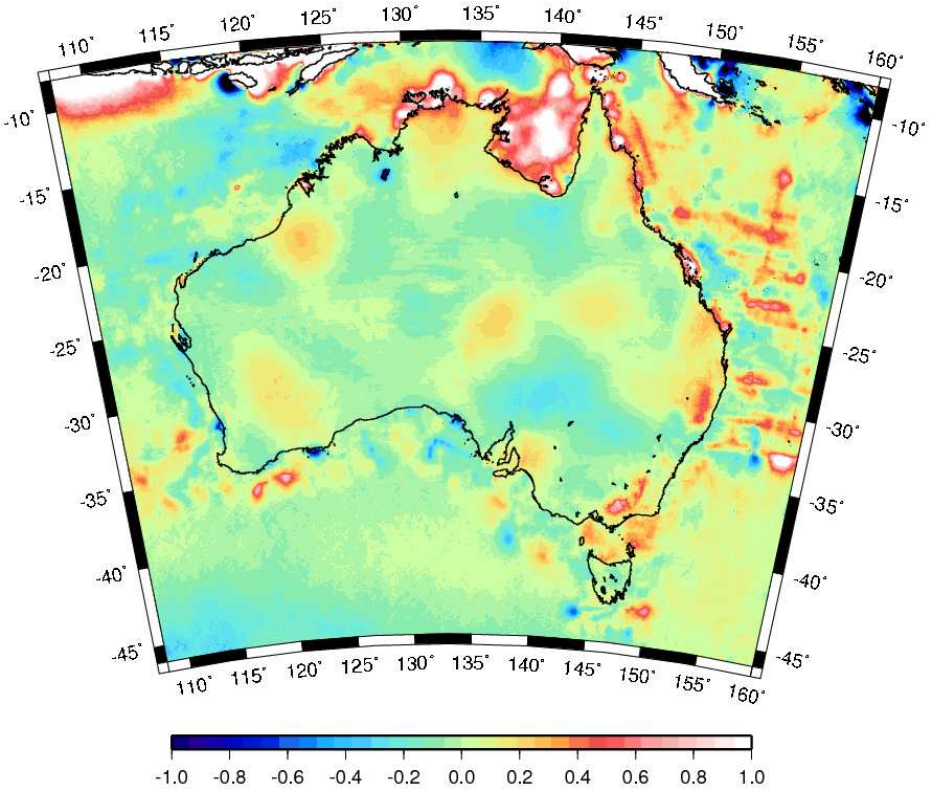


Fig 17. Differences between height anomalies computed from EGM2008 and AUSGeoid98 [Lambert projection; units in metres]

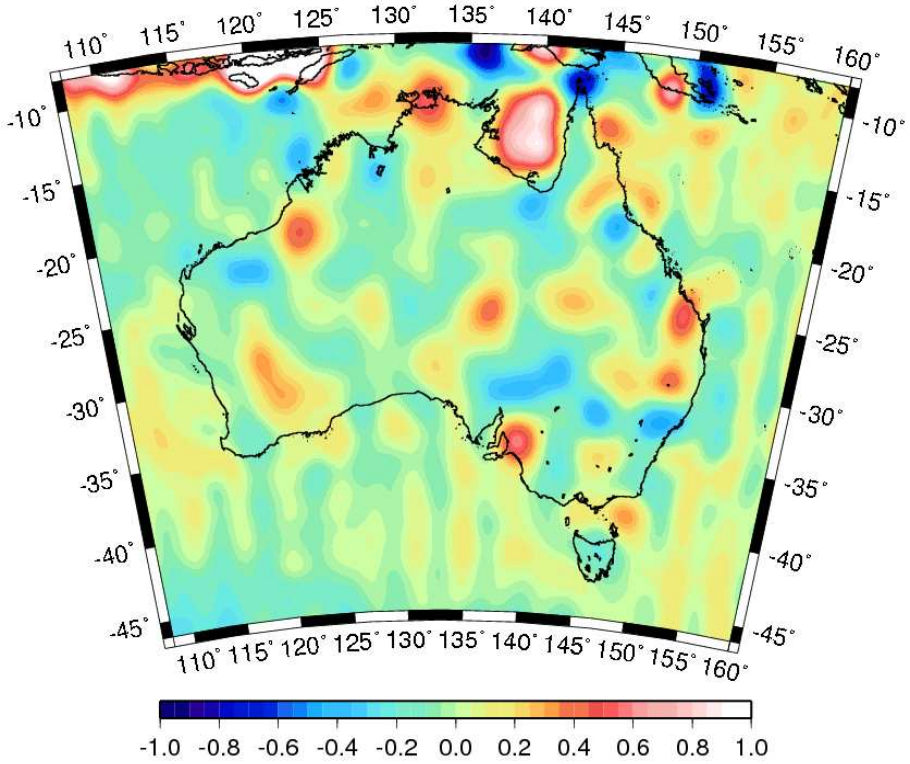


Fig 18. Differences between EGM96 and GGM02C quasigeoid heights to degree 200 (~100 km resolution) over the AUSGeoid98 area [Lambert projection; units in metres].

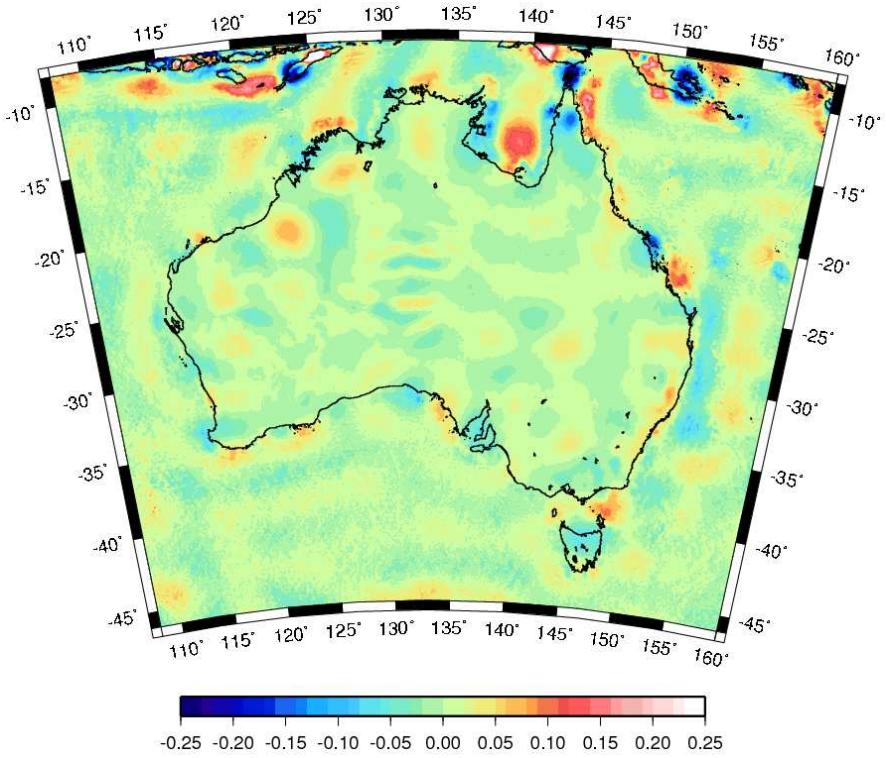


Fig 19. Differences between PGM2007A and EGM2008 quasigeoid heights to degree 2160 over the AUSGeoid98 area [Lambert projection; units in metres].

Short-wavelength differences in Fig. 17 occur in some of the mountainous regions (e.g., the Australian Alps; ~147˚E, ~37˚S). However, this only occurs for part of the Great Dividing Range, unlike the differences with the gravity data (Fig. 11). The large difference in Fig. 17 over the Australian Alps (~147˚E, ~37˚S) correlates almost exactly with the differences between gravity anomalies in Fig. 11. This indicates that the Australian data may be in error here, which will be investigated further. The same applies for the difference centred on (~151˚E, ~30˚S).

There are also large short-wavelength differences in Fig. 17 in many coastal regions (e.g., off the coast of Perth, Western Australia; ~116˚E, ~32˚S). These are in some cases due to differences in altimeter data used in EGM2008 and AUSGeoid98, and in other cases due to the use of unadjusted ship-track gravity data in AUSGeoid98, which will be elaborated upon next.

Claessens et al. (2001) and Kirby (2003) have shown that the inclusion of ship-track gravity data in the computation of AUSGeoid98 have probably caused an erroneous rise in AUSGeoid98 quasigeoid heights in marine areas offshore Perth. The negative differences between height anomalies from EGM2008 and AUSGeoid98 in these areas (Fig. 20) are therefore expected. However, the differences in Fig. 20 do

not show a strong spatial correlation with the poor-quality ship-track data. This is because the least-squares collocation draping of the altimeter-derived gravity anomalies onto the land and ship-track data has smeared out the effect. It is then smeared out further when the residual gravity anomalies were Stokes-integrated.

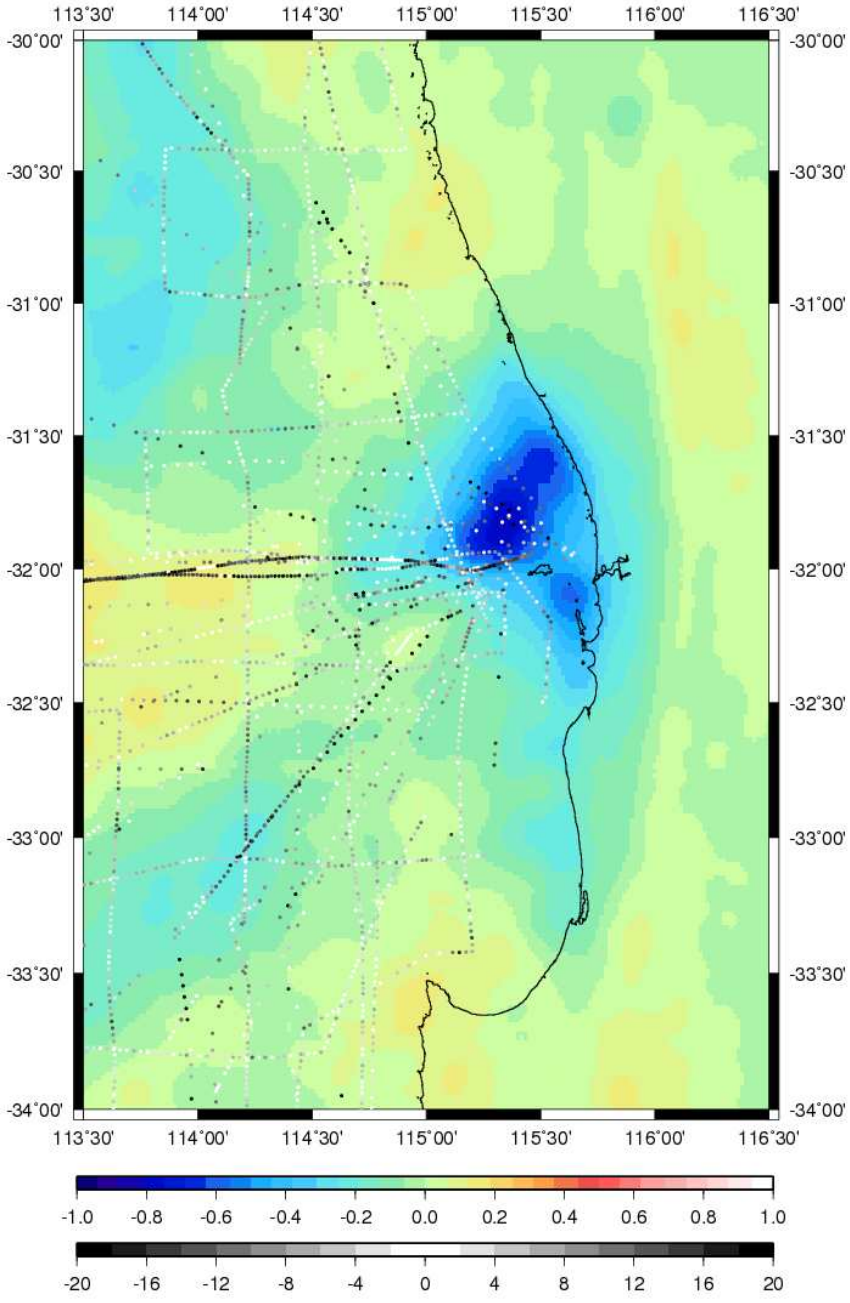


Fig 20. Differences between height anomalies computed from EGM2008 and AUSGeoid98 in the Perth region (colour scale) and differences between free-air gravity anomalies from EGM2008 and from ship-track observations (greyscale)
[Mercator projection; units in metres and mGal]

Figure 21 shows differences in height anomalies from EGM2008 and AUSGeoid98 over the eastern part of the Great Australian Bight (around and to the west of Adelaide). The central western part of Fig. 21 contains a particularly clear example of the distortion in AUSGeoid98 due to the inclusion of faulty ship-track data. The influence of faulty ship-track data can also be seen in Fig. 22, which shows differences in height anomalies from EGM2008 and AUSGeoid98 off the Queensland coast.

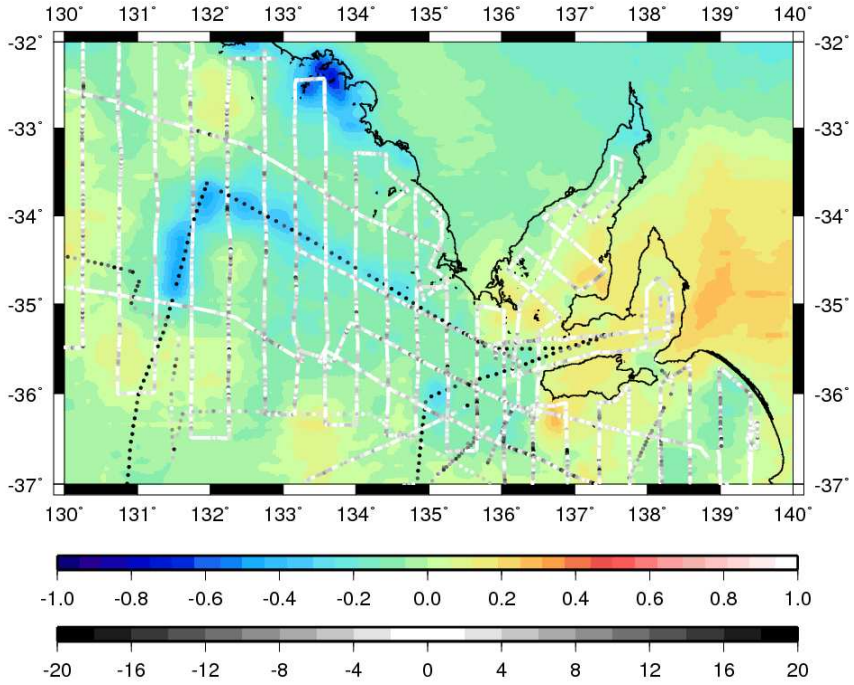


Fig 21. Differences between height anomalies computed from EGM2008 and AUSGeoid98 in the eastern part of the Great Australian Bight (colour scale) and differences between free-air gravity anomalies from EGM2008 and from ship-track observations (greyscale) [Mercator projection; units in metres and mGal]

However, larger differences closer to the coast, e.g., near Ceduna (~133.5E, ~32.5S; Fig. 21), near Mackay (~149E, ~21S; Fig. 22) and near Bundaberg (~152E, ~25S; Fig. 22), cannot be explained by inaccurate ship-track data, and are more likely explained by differences in the altimeter data used in the computation of EGM2008 and AUSGeoid98. We cannot isolate which altimetry dataset is in error in these areas. The differences offshore Queensland (Fig. 22) are exacerbated by the presence of the Great Barrier Reef, which prevents dense ship-track surveys and complicates tidal modelling in this region. The relatively large differences in height anomalies over land near Adelaide (~139E, ~35S; Fig. 21) will be discussed in the next section.

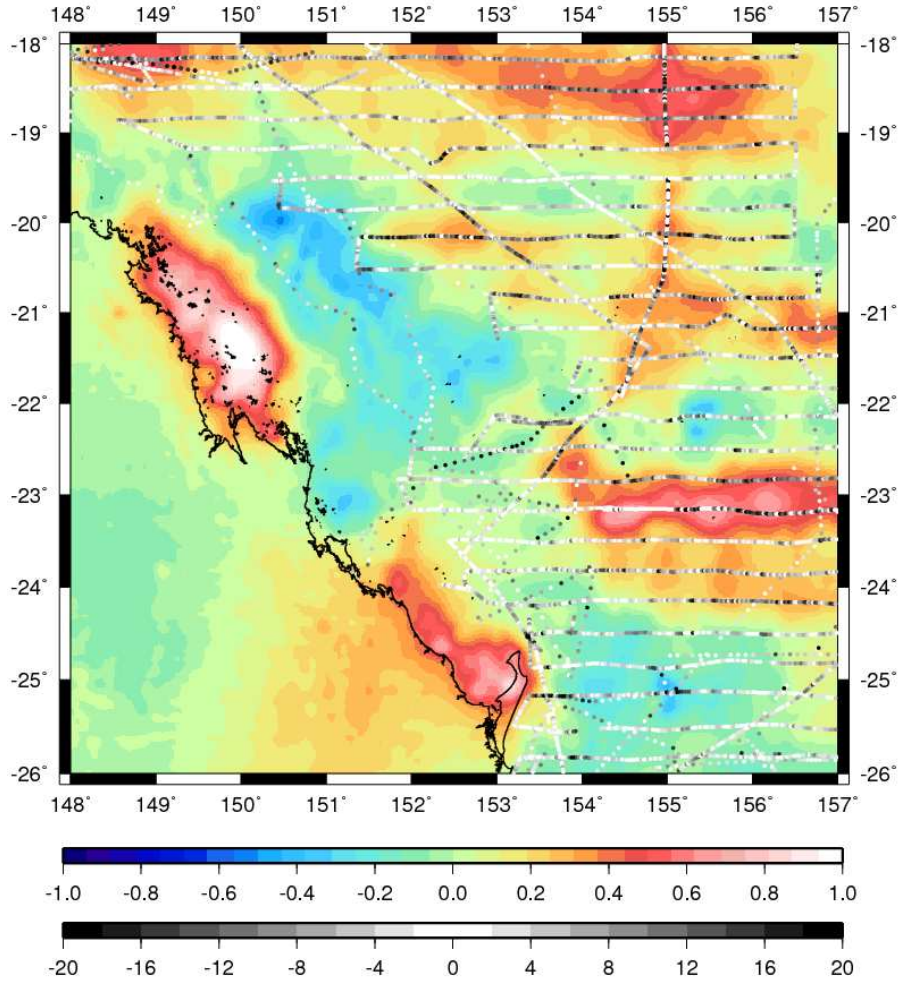


Fig 22. Differences between height anomalies computed from EGM2008 and AUSGeoid98 off Queensland (colour scale) and differences between free-air gravity anomalies from EGM2008 and from ship-track observations (greyscale)
[Mercator projection; units in metres and mGal]

3.3 Comparisons with Australian GPS-levelling data

Table 5 indicates that PGM2007A and EGM2008 improve on many earlier GGMs in terms of standard deviation (STD) of the differences with respect to the 254 GPS-levelling points across Australia. It should, however, be recalled that the levelling data suffers from a north-south-oriented trend in the AHD (see earlier), which is clearly visible in Fig. 23. The STD of the differences between AUSGeoid98 and PGM2007A over the 254 GPS-levelling points is ±0.171m, and the STD of the differences between AUSGeoid98 and EGM2008 is ±0.164m (both not shown in Table 5). These numbers are considerably smaller than any of the standard deviations reported in Table 5. Comparisons with a larger set of 1013 GPS-levelling points of more dubious quality (see Section 2.2) are shown in Table 6. The GPS-levelling dataset of 243

points in Western Australia shows better agreement with all tested quasigeoid models (see Table 7).

Quasigeoid	*Degree*	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
EGM96	360	No	+0.894	–0.961	+0.009	±0.334
GGM02C	200	No	+0.950	–1.318	+0.007	±0.415
EIGEN-GL04C	360	No	+0.789	–0.653	+0.059	±0.293
AUSGeoid98	~5400	No	+0.865	–0.721	+0.077	±0.284
PGM2007A	2160	No	+0.663	–0.536	+0.068	±0.249
EGM2008	2160	No	+0.648	–0.535	+0.063	±0.242
AUSGeoid98	~5400	Yes	+0.518	–0.756	+0.000	±0.191
PGM2007A	2160	Yes	+0.551	–0.769	+0.000	±0.179
EGM2008	2160	Yes	+0.571	–0.701	+0.000	±0.173

Table 5 Descriptive statistics of the absolute differences between quasigeoid models and 254 co-located GPS-AHD points [units in metres]

Quasigeoid	*Degree*	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
AUSGeoid98	~5400	No	+3.558	–2.572	–0.003	±0.317
PGM2007A	2160	No	+3.153	–2.695	–0.021	±0.278
EGM2008	2160	No	+3.180	–2.711	–0.025	±0.273
AUSGeoid98	~5400	Yes	+3.346	–2.460	+0.000	±0.267
PGM2007A	2160	Yes	+3.055	–2.581	+0.000	±0.230
EGM2008	2160	Yes	+3.087	–2.596	+0.000	±0.228

Table 6 Descriptive statistics of the absolute differences between quasigeoid models and 1013 co-located GPS-AHD points [units in metres]

Quasigeoid	*Degree*	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
AUSGeoid98	~5400	No	+0.416	–0.740	–0.027	±0.204
PGM2007A	2160	No	+0.430	–0.583	–0.059	±0.175
EGM2008	2160	No	+0.378	–0.578	–0.060	±0.172
AUSGeoid98	~5400	Yes	+0.392	–0.743	0.000	±0.178
PGM2007A	2160	Yes	+0.358	–0.567	0.000	±0.132
EGM2008	2160	Yes	+0.364	–0.562	0.000	±0.126

Table 7 Descriptive statistics of the absolute differences between quasigeoid models and 243 co-located GPS-AHD points in Western Australia [units in metres]

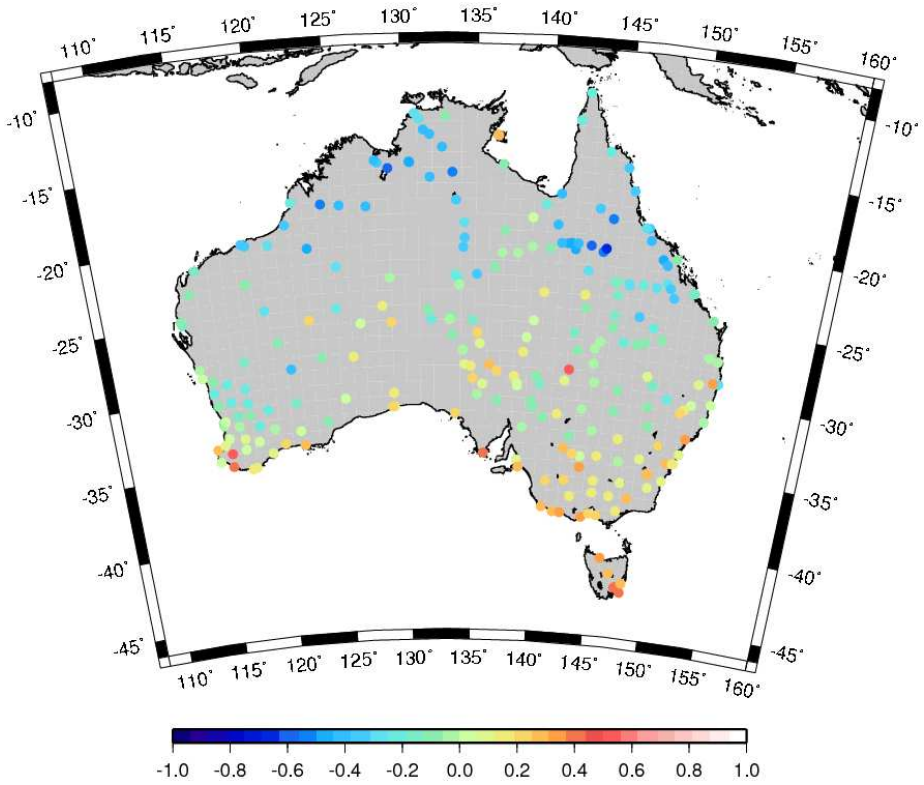


Fig 23. Differences between height anomalies from 254 GPS-levelling observations and EGM2008 [Lambert projection; units in metres]

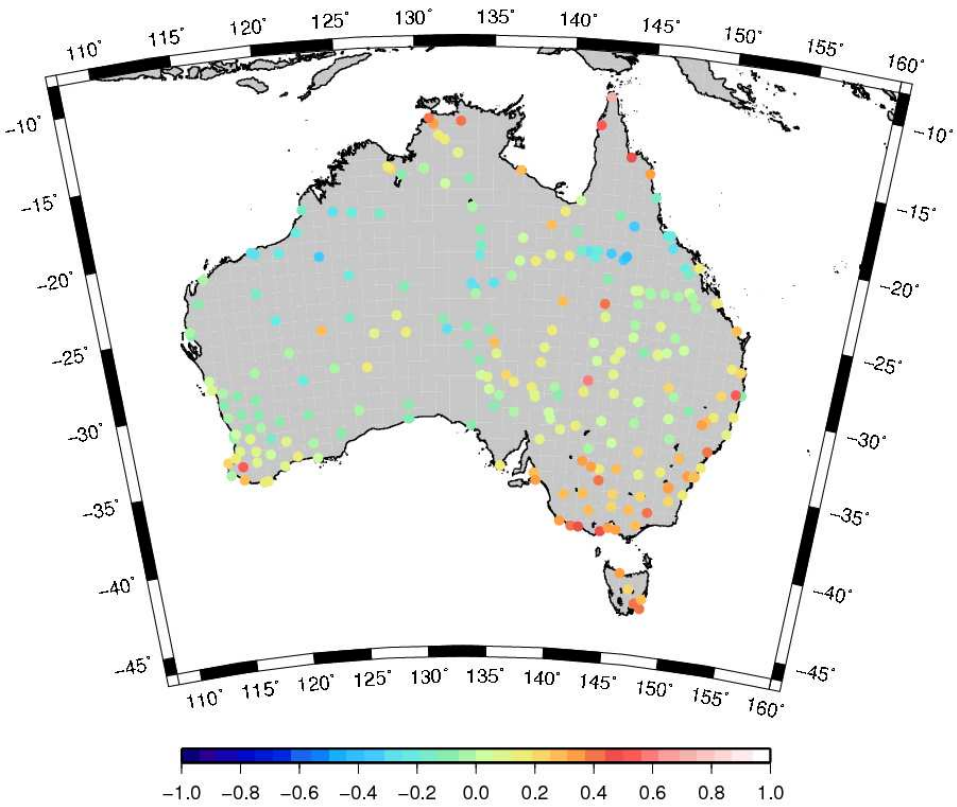


Fig 24. Differences between height anomalies from 254 minimally constrained GPS-levelling observations and EGM2008 [Lambert projection; units in metres]

Comparisons were also made to the regional GPS-levelling data in the SASZ and the SWSZ (Tables 8 and 9 and Fig. 25). The SWSZ data (published in an Appendix to Featherstone (2004)) were inadvertently not supplied to the EGM2008 development team. For the SASZ dataset, the STDs of PGM2007A and EGM2008 are larger than that of AUSGeoid98, but this is reversed when a bias and tilt are removed (see Table 8). In the SWSZ, EGM2008 has the smallest STD, but after removal of a bias and tilt the STD of AUSGeoid98 is the same (see Table 9).

Quasigeoid	*Degree*	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
EGM96	360	No	+1.637	–0.401	+0.246	±0.466
AUSGeoid98	~5400	No	+0.313	–0.211	+0.010	±0.117
PGM2007A	2160	No	+0.396	–0.322	–0.044	±0.133
EGM2008	2160	No	+0.402	–0.286	–0.036	±0.127
EGM96	360	Yes	+1.154	–0.732	+0.000	±0.396
AUSGeoid98	~5400	Yes	+0.373	–0.210	+0.000	±0.105
PGM2007A	2160	Yes	+0.394	–0.196	+0.000	±0.102
EGM2008	2160	Yes	+0.374	–0.183	+0.000	±0.100

Table 8 Descriptive statistics of the absolute differences between quasigeoid models and 45 co-located GPS-AHD points in the South Australian Seismic Zone [units in metres]

Quasigeoid	*Degree*	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
EGM96	360	No	+1.174	–0.211	+0.512	±0.280
AUSGeoid98	~5400	No	+0.196	–0.277	–0.010	±0.128
PGM2007A	2160	No	+0.160	–0.335	–0.002	±0.120
EGM2008	2160	No	+0.144	–0.305	–0.006	±0.106
EGM96	360	Yes	+0.543	–0.606	+0.000	±0.244
AUSGeoid98	~5400	Yes	+0.097	–0.133	+0.000	±0.046
PGM2007A	2160	Yes	+0.092	–0.138	+0.000	±0.050
EGM2008	2160	Yes	+0.092	–0.130	+0.000	±0.046

Table 9 Descriptive statistics of the absolute differences between quasigeoid models and 48 co-located GPS-AHD points in the South West Seismic Zone [units in metres]

The STDs for EGM2008 are consistently smaller than the STDs for PGM2007A in Tables 5, 6, 7, 8 and 9, both with and without removal of a bias and tilt. Thus, although there is little difference between PGM2007A and EGM2008 in

the comparisons with gravity anomalies, EGM2008 is an improvement on PGM2007A in the comparison with GPS-levelling data.

It is interesting to note that the STDs of AUSGeoid98, PGM2007A and EGM2008 are very similar for the regional GPS-levelling data sets in South Australia and Western Australia (see Tables 8 and 9), whereas PGM2007A and EGM2008 agree significantly better with the nation-wide GPS-levelling data sets than AUSGeoid98 (see Tables 5 and 6). This is probably caused by the improved accuracy of the low degrees in PGM2007A and EGM2008 compared to EGM96, which was used in AUSGeoid98, due to the inclusion of GRACE data.

Table 10 shows the biases, tilts and directions of the least-squares fitted planes used in the generation of the statistics in Tables 5 to 9. The tilts in the differences with PGM2007A and EGM2008 are slightly smaller than the tilt in the differences with AUSGeoid98 for the nation-wide data sets. The tilts reported here for AUSGeoid98 are slightly larger than those reported by Featherstone and Guo (2001) (~0.26 mm/km for the nation-wide data set of 1013 GPS-levelling observations) and Featherstone (2004) (~0.81 mm/km for the SWSZ data set in Western Australia). This is because the tilts computed in this 'validation' were not constrained to be in a north-south direction, as was the case for Featherstone and Guo (2001) and Featherstone (2004).

Nevertheless, it can be seen from the azimuths in Table 10 that most of the fitted planes are generally close to a north-south direction. This is consistent with the known north-south distortion in the AHD (discussed earlier). As such, PGM2007A and EGM2008 are again implicitly validated because they confirm the known north-south tilt in the AHD. The only exception to this is the South Australian data set, which is discussed next.

Figure 25 (right) shows that the differences between the GPS-AHD heights and height anomalies from EGM2008 in the SASZ follow a north-south trend, similar to the well-known trend in the AHD (cf. Table 7), but more than three times as steep as the trend over the whole of Australia (~0.77/~0.71 mm/km versus ~0.23 mm/km). However, the differences between the GPS-AHD heights and AUSGeoid98 show a very different pattern (Fig. 25 left) with a 'bulge' in the south-east part of the map of up to ~0.3 m. This has caused the azimuth of the least-squares fitted plane to be skewed from the expected north-south direction, which indicates a problem with AUSGeoid98 in this region.

Quasigeoid	*Degree*	*Dataset*	*Bias (m)*	*Tilt (mm/km)*	*Azimuth (deg)*
AUSGeoid98	~5400	Australia-wide (254)	+0.076	+0.281	+3.590
PGM2007A	2160	Australia-wide (254)	+0.068	+0.226	–1.738
EGM2008	2160	Australia-wide (254)	+0.063	+0.221	–1.999
AUSGeoid98	~5400	Australia-wide (1013)	–0.003	+0.267	+2.700
PGM2007A	2160	Australia-wide (1013)	–0.021	+0.232	–9.015
EGM2008	2160	Australia-wide (1013)	–0.025	+0.223	–8.510
AUSGeoid98	~5400	Western Australia (243)	–0.027	+0.178	+14.088
PGM2007A	2160	Western Australia (243)	–0.058	+0.212	–2.758
EGM2008	2160	Western Australia (243)	–0.060	+0.215	+3.190
AUSGeoid98	~5400	South Australia (45)	+0.010	+0.478	+142.366
PGM2007A	2160	South Australia (45)	–0.044	+0.767	–14.166
EGM2008	2160	South Australia (45)	–0.036	+0.708	–21.566
AUSGeoid98	~5400	SW Western Australia (48)	–0.010	+0.922	+22.143
PGM2007A	2160	SW Western Australia (48)	–0.003	+0.902	+31.033
EGM2008	2160	SW Western Australia (48)	–0.006	+0.783	+30.360

Table 10 Bias, tilt and azimuth of maximum positive gradient for planes fitted in a least-squares sense to the differences between GPS-levelling observations and several quasigeoid models. The positive tilt values, coupled with the azimuths show that the differences generally increase northwards.

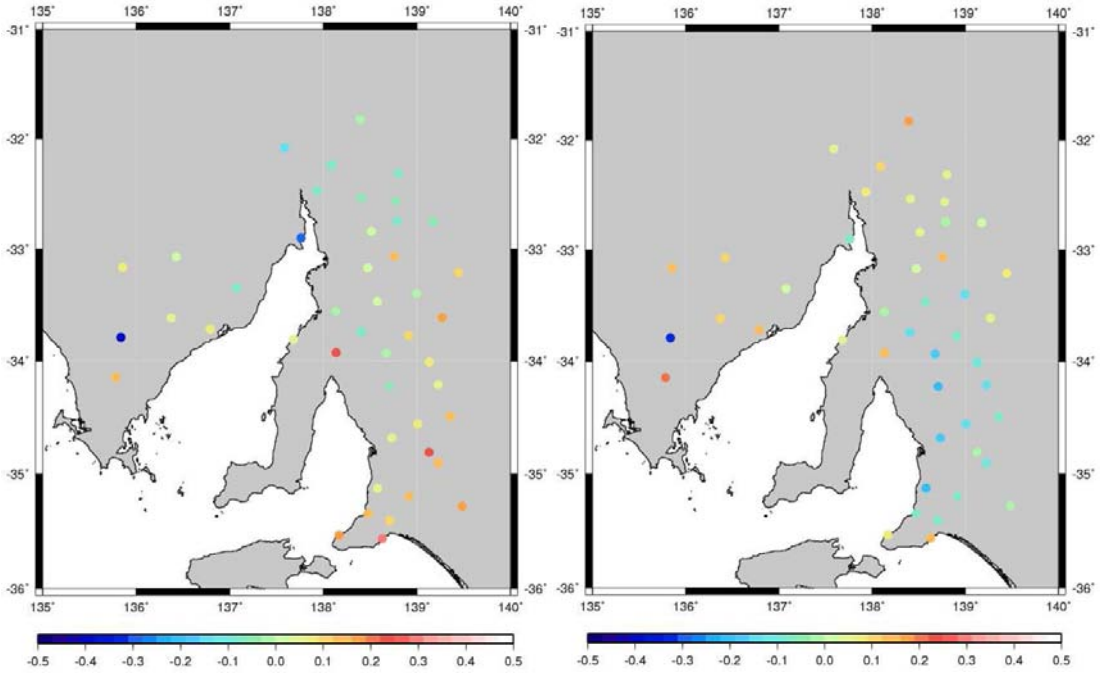


Fig 25. Differences between height anomalies from GPS-levelling observations and AUSGeoid98 (left) and EGM2008 (right) over the SASZ [Mercator projection; units in metres]

This ‘bulge’ is also apparent in the differences between the GPS-AHD heights and height anomalies computed from EGM96, and the differences in this case are much larger (see Fig. 26). This indicates that EGM96 contains an error in this region, which has propagated into AUSGeoid98, albeit mitigated by local gravimetric data. The residual quasigeoid computed for this region in AUSGeoid98 was around one metre, being one of the largest ‘corrections’ to EGM96 in that computation. Thus, despite the fact that AUSGeoid98, PGM2007A and EGM2008 give similar standard deviations in the comparison with GPS-levelling data over the SASZ, spatial analysis of the differences appear to indicate that PGM2007A and EGM2008 are the more accurate models in this region.

Historically, the Adelaide region has always been a problematic area for GGMs (see, e.g., Kearsley and Holloway 1989, Zhang and Featherstone 1995, Kirby et al. 1998, Amos and Featherstone 2003). PGM2007A and EGM2008 appear now to be free from this error, which is a positive validation for these models.

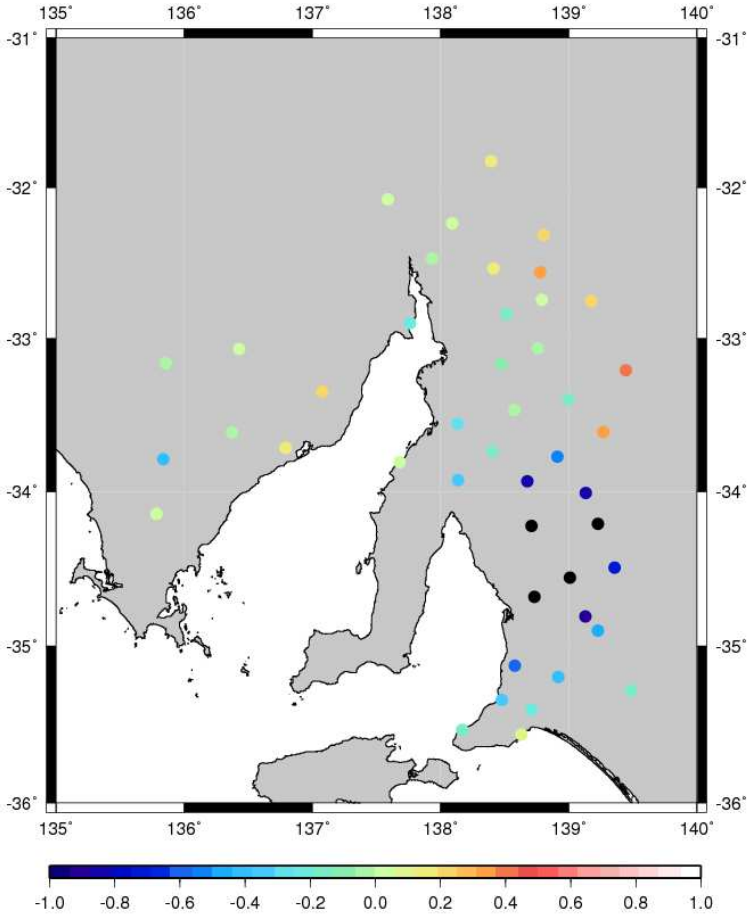


Fig 26. Differences between height anomalies from GPS-levelling observations and EGM96 [Mercator projection; units in metres]

It is obvious from all GPS-levelling comparisons that the slope in the AHD is complicating the evaluation. Removal of a bias and tilt cannot completely account for the deficiencies in the AHD. The main reason for the deficiencies is the fact that the AHD is constrained to 32 tide gauges around the coast. This was overcome for this 'validation' by performing a minimally constrained adjustment on the levelling, fixing the height of one tide-gauge only. All datasets are tied to the Albany tide-gauge on the south coast of Western Australia, except for the SASZ dataset, which is tied to the tide-gauge at Port Lincoln on the Eyre Peninsula in South Australia.

Table 11 shows the results of comparisons of EGM2008 to the GPS-levelling datasets, where the levelling observations were adjusted using a minimally constrained adjustment. Minimally constraining the levelling observations decreases the STD of the differences with EGM2008 for all datasets, except for the SASZ. In South Australia, the differences between GPS-levelling and EGM2008 show an east-west trend of ~1 mm/km. The reason for this trend is probably erroneous spirit-levelling data in the file used for the adjustment (cf. Steed 2006).

Dataset	*Bias/tilt removed?*	*Max*	*Min*	*Mean*	*STD*
Australia-wide (248)	No	+0.727	–0.437	+0.062	±0.203
Western Australia (243)	No	+0.300	–0.402	–0.007	±0.125
South Australia (45)	No	+0.632	–0.397	+0.391	±0.180
SW Western Australia (48)	No	+0.225	–0.048	+0.063	±0.059
Australia-wide (248)	Yes	+0.710	–0.569	+0.000	±0.182
Western Australia (243)	Yes	+0.400	–0.420	+0.000	±0.102
South Australia (45)	Yes	+0.250	–0.491	+0.000	±0.113
SW Western Australia (48)	Yes	+0.084	–0.097	+0.000	±0.039

Table 11 Descriptive statistics of the absolute differences between EGM2008 and various minimally constrained GPS-levelling datasets [units in metres]

The absolute differences between height anomalies from GPS-levelling and from EGM2008 at two points can be subtracted from one another to compute a relative baseline difference (cf. Featherstone 2001) to evaluate the quasigeoid gradients. This was done for all possible baselines between the 254 GPS-levelling points in the nationwide dataset and the relative differences are plotted against baseline length (Fig. 27). The majority of relative differences fall within the 12 root km misclosure toler-

ance (ICSM 2007), especially for long baselines, but many fall outside this level. This is probably due to errors in the AHD, most notably the north-south slope.

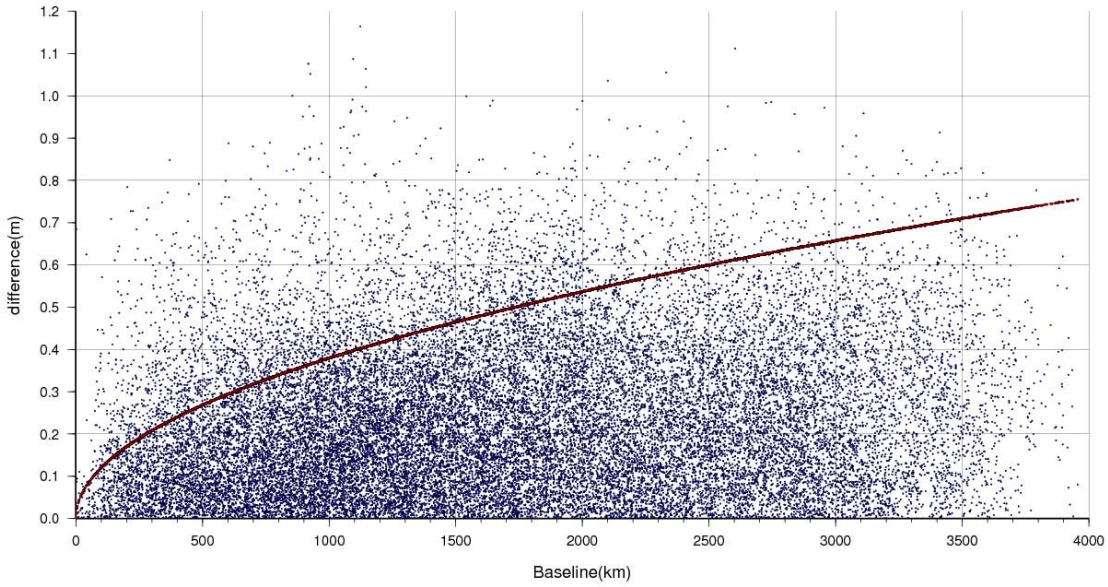


Fig 27. Relative baseline differences between height anomalies from 254 GPS-levelling observations in Australia and height anomalies from EGM2008, where the black line indicates the 12 root km allowable misclose for third-order Australian spirit-levelling

The effect of the north-south slope in the AHD on the relative baseline differences can be seen most clearly in Figs. 28 and 29. Figure 28 shows the relative baseline differences for the 243 GPS-AHD points in Western Australia. The longest baselines in this dataset (>1800 km) are all north-south oriented. Almost all of these show large relative differences. However, after minimally constraining the levelling observations in a readjustment, the relative baseline differences become much smaller (see Fig. 29). This is especially visible in the longest baselines, but holds for all baseline lengths.

Statistics for all GPS-levelling datasets are shown in Tables 12 and 13. Table 12 shows the baseline statistics when 'official' AHD heights are used, and Table 13 shows the baseline statistics when the levelling observations are minimally constrained in a readjustment. Minimally constraining the levelling observations decreases the relative baseline differences for all datasets except the SASZ. As mentioned earlier, the levelling data in this area requires further investigation.

The majority of all relative baseline differences is below the formal precision threshold of third-order levelling (12 root km in Australia; ICSM 2007). The differ-

ences are likely to be affected more by errors in the levelling than by errors in EGM2008. Therefore, true validation of EGM2008 from comparisons to GPS-levelling data cannot be claimed.

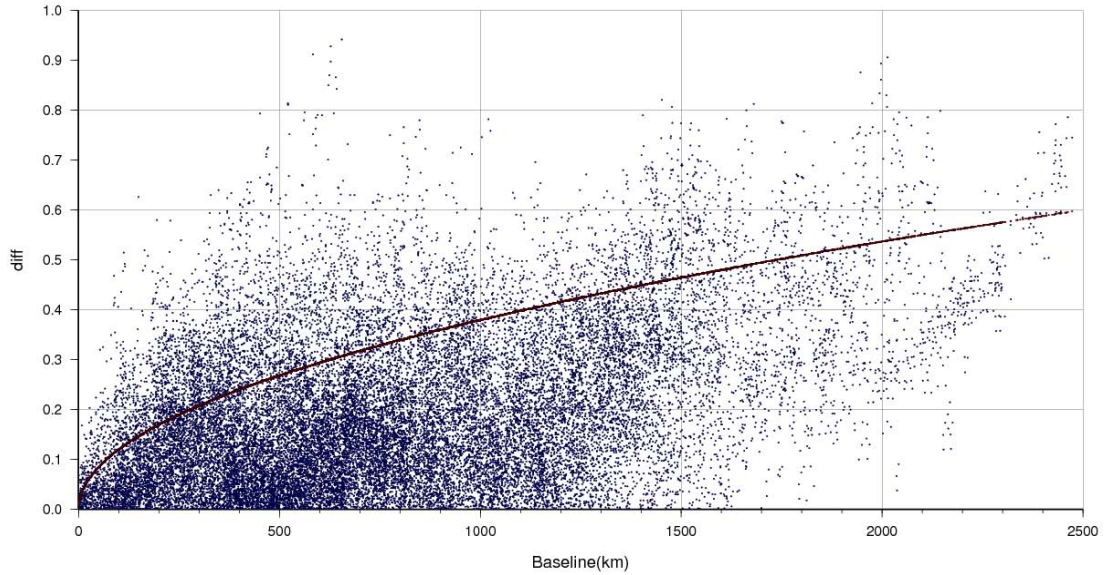


Fig 28. Relative baseline differences between height anomalies from 243 GPS-levelling observations in Western Australia and height anomalies from EGM2008, where the black line indicates the 12 root km allowable misclose for third-order Australian spirit-levelling

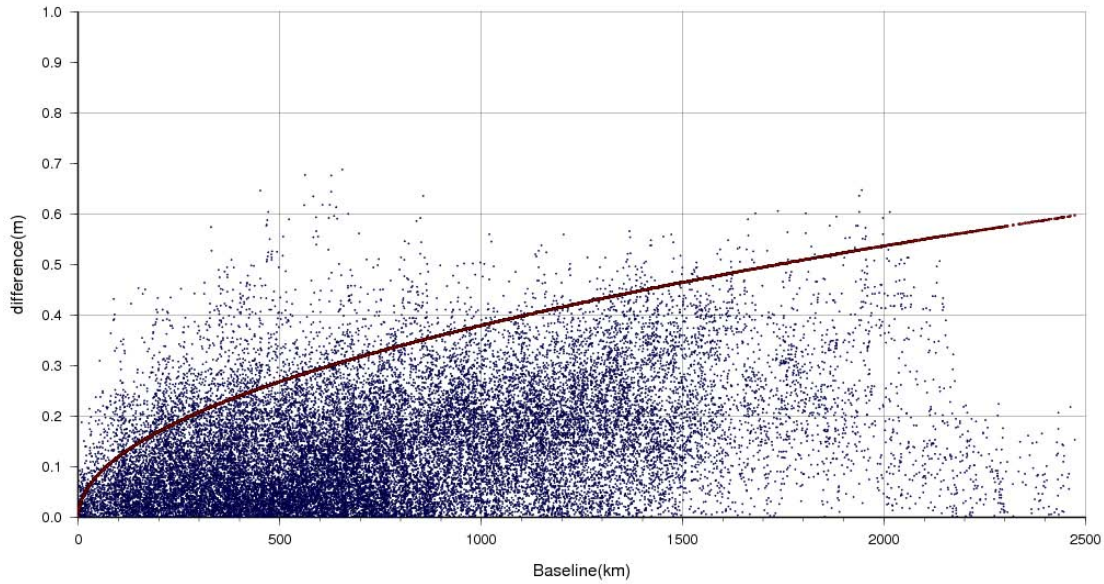


Fig 29. Relative baseline differences between height anomalies from 243 minimally constrained GPS-levelling observations in Western Australia and height anomalies from EGM2008, where the black line indicates the 12 root km allowable misclose for third-order Australian spirit-levelling

Dataset	*Mean baseline length*	*Max*	*Min*	*Mean*	*STD*	*Percentage baselines below 12 root km*
Australia-wide (254)	1,700,060	+1.116	–1.182	+0.042	±0.341	81.86%
Western Australia (243)	783,286	+0.893	–0.942	–0.047	±0.241	81.43%
South Australia (45)	415,100	+0.546	–0.689	–0.090	±0.158	65.76%
SW Western Australia (48)	530,677	+0.448	–0.394	+0.094	±0.118	76.77%

Table 12 Descriptive statistics of the relative baseline differences between EGM2008 and various GPS-AHD datasets [units in metres]

Dataset	*Mean baseline length*	*Max*	*Min*	*Mean*	*STD*	*Percentage baselines below 12 root km*
Australia-wide (254)	1,700,060	+1.164	–1.052	+0.081	±0.276	89.03%
Western Australia (243)	783,286	+0.592	–0.688	–0.019	±0.178	93.04%
South Australia (45)	415,100	+0.954	–1.029	–0.090	±0.240	52.63%
SW Western Australia (48)	530,677	+0.273	–0.203	+0.043	±0.073	97.25%

Table 13 Descriptive statistics of the relative baseline differences between EGM2008 and various minimally constrained GPS-levelling datasets [units in metres]

3.4 Comparisons with Australian vertical deflections

The results of comparisons of vertical deflections computed from PGM2007A and EGM2008 to a set of 1080 historic astrogeodetically observed vertical deflections over Australia are shown in Table 14 and Figs. 30 and 31. The results for AUSGeoid98 and PGM2007A agree exactly with the statistics given by Pavlis et al. (2007). This validation is probably the strongest from the Australian data, even though the vintage of the Australian astrogeodetic observations are not ideal because they were mostly observed over 40 years ago. Nevertheless, because vertical deflections are higher order derivatives, they sense high-frequency variations in the gravity field and are thus better for validating GGMs in the high degrees (Jekeli 1999).

PGM2007A and EGM2008 seemingly slightly outperform AUSGeoid98 in Table 14. This may partly be because the `HARMONIC_SYNTH.f` software uses the height of the astrogeodetic observation to evaluate a Helmert deflection directly at the surface of the Earth, so is more compatible with the astrogeodetic observations that yield Helmert deflections. On the other hand, AUSGeoid98 deflections are Pizzetti vertical

deflections at the geoid because they were computed from the horizontal gradients of AUSGeoid98. As such, the curvature and torsion of the plumbline through the topography is neglected, which will account for part of the worse comparison for AUSGeoid98 in Table 14.

Deflection	*Model*	*Degree*	*Max*	*Min*	*Mean*	*STD*
north-south (ξ)	AUSGeoid98	~5400	+17.83	–7.76	–0.25	±1.28
north-south (ξ)	PGM2007A	2160	+17.79	–6.95	–0.17	±1.24
north-south (ξ)	EGM2008	2160	+17.69	–6.99	–0.62	±1.17
east-west (η)	AUSGeoid98	~5400	+9.11	–12.65	–0.17	±1.36
east-west (η)	PGM2007A	2160	+8.77	–11.35	–0.10	±1.18
east-west (η)	EGM2008	2160	+8.70	–11.34	+0.10	±1.28

Table 14 Descriptive statistics of the differences between 1080 astrogeodetic observations of vertical deflections and AUSGeoid98, PGM2007A and EGM2008 [units in arc seconds]

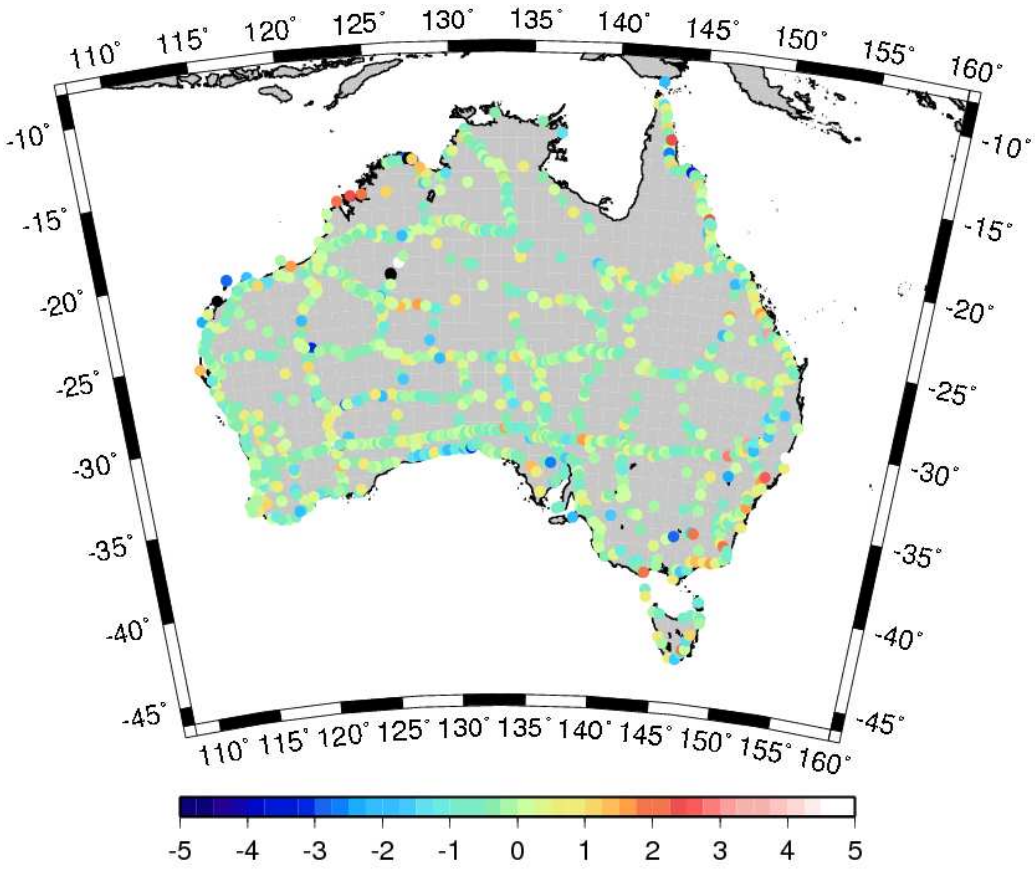


Fig 30. Differences between north-south vertical deflections from 1080 astrogeodetic measurements and EGM2008 [Lambert projection, units in arc seconds]

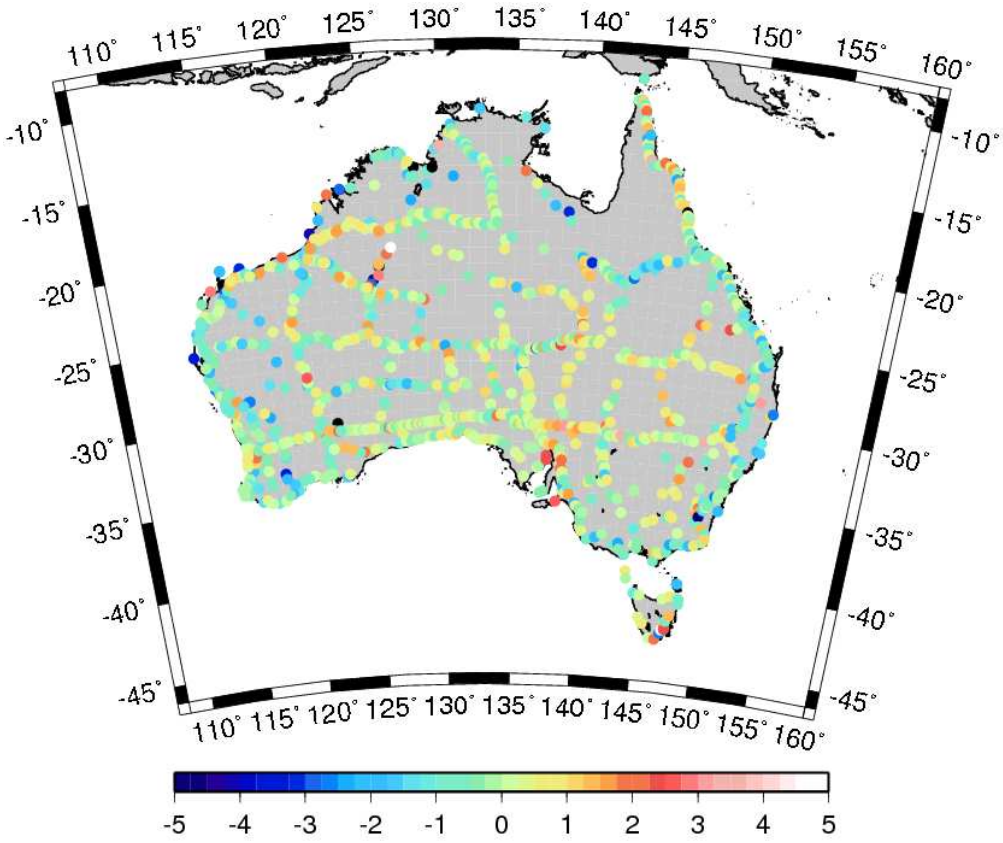


Fig 31. Differences between east-west vertical deflections from 1080 astrogeodetic measurements and EGM2008 [Lambert projection, units in arc seconds]

4. Conclusion

The tide-free combined global geopotential model EGM2008 and its preliminary version PGM2007A were compared with Australian land, marine and airborne gravity observations, co-located GPS-levelling, the AUSGeoid98 regional gravimetric quasi-geoid model, and astrogeodetic deflections of the vertical. The results show that we cannot legitimately claim to truly validate EGM2008. Instead, these global models confirm the already-known problems with the Australian data, as well as revealing some previously unknown problems. If one wants to claim validation, then EGM2008 is validated because it can confirm the errors in our regional data. Simply, EGM2008 is a good model over Australia.

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Evaluation of the Earth Gravitational Model 2008 using GPS-Leveling and Gravity data in China

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Abstract

The evaluation of both PGM07A and EGM08 has been carried out in China using the observed data sets and high resolution regional geoid models including the observations of 652 GPS/Leveling points within Chinese national A/B-order GPS networks and 1160 GPS/Leveling points distributed over seven provincial regions, as well as the data sets of 183201 2′×2′ gridded mean gravity anomalies collected from five provincial regions. In the evaluation of PGM07A and EGM08, the other recent released Earth Geopotential Models (EGMs) including EIG01C, EIG03C, EIG04C, GGM01C, GGM02C, GGM01C* and GGM02C* are also used for comparisons.

The statistic results of the comparisons show that the RMS and Std.D of the differences between the quasi-geoid heights derived from PGM07A/EGM08 and their corresponding ones determined from the GPS/Leveling data of 652 A/B-order testing points are ±0.358m, ±0.338m for PGM07A, and ±0.284m, ±0.257m for EGM08 respectively, which are remarkably less than the RMS and Std.D corresponding to the other EGMs used in the comparisons. The RMS and Std.D of the differences between the quasi-geoid heights derived from PGM07A/EGM08 and their corresponding ones determined from the GPS/Leveling data of total 1160 points in seven provincial networks are in the range of ±0.09m ~±0.31m with a mean of ±0.19m for PGM07A, and ±0.07m ~±0.24m with a mean of ±0.16m for EGM08. For the other seven EGMs, however, the corresponding range of the statistics is ±0.26m ~±0.80m with a mean of ±0.44m. The comparisons of the quasi-geoid derived from PGM07A and EGM08 as well as the other EGMs with those determined from the GPS/Leveling data of all 1812 points within both A/B-order GPS/Leveling networks and seven provincial ones are also made. The RMS and Std.D of the differences obtained from the comparisons are ±0.269m, ±0.243m for

PGM07A, ±0.222m, ±0.186m for EGM08, respectively, and for the other seven EGMs, the corresponding statistics are in the range of ±0.375m ~±0.680m. The results of comparisons above show that the quasi-geoid derived from PGM07A and EGM08 can provide a better approximation to the quasi-geoids determined from Chinese GPS/Leveling data including both the data of national A/B-order networks and the data of provincial GPS/leveling networks than an approximation to the same quasi-geoids provided by the quasi-geoids derived from the other seven EGMs. According to all statistical results of comparisons between the EGMs-derived quasi-geoids and GPS/Leveling based ones, however, EGM08 is better than PGM07A as applied in Chinese mainland. The quasi-geoid heights derived from EGM08 are more accurate than those derived from PGM07A by a range of ±0.01m ~±0.07m with an average of ±0.03m in the RMS and Std.D of comparisons.

In addition, the comparisons of gravity anomalies derived from PGM07A and EGM08 with those measured in five provincial regions are made, and the statistical results of the comparisons show that the RMS and Std.D of the differences between PGM07A/EGM08-derived gravity anomalies and those measured in the regions are within the range of ±13 ~29 mGal, with a mean of ±18 mGal. It also indicates that the gridded mean anomalies derived from EGM08 are more accurate than those derived from PGA07 by a range of ±1~4 mGal with a mean of ±2.7 mGal in the RMS and Std.D comparison.

In the general, the quasi-geoid and gravity anomalies computed from PGM07A and EGM08 have significantly improved the representation of the Chinese local gravity field as compared with the other seven models used in the evaluation. However, our testing results show that for both EGM08 and PGM07, there are some large (or small) biases of the quasi-geoids which deviate from the general level of the biases in some regions.

1. Introduction

Since 2004, the National Geospatial-Intelligence Agency (NGA) of the USA has embarked upon the development of a new Earth Gravitational Model (EGM) to support future realizations of NGA's World Geodetic System (Pavlis et al, 2004), and NGA along with its partner SGT, Inc. has been leading an effort to develop a Preliminary Gravitational Model (PGM) series in yearly succession including PGM04A through PGM07A complete to degree and order 2160 or higher degree. The PGM07A complete to

degree 2190 and order 2160, which is to be used as reference model in a last iteration leading to the final EGM, has been provided to geodetic community and related earth scientists only for evaluation of the PGM. Now the final EGM, i.e. EGM08, has been completed in the early 2008, so PGM07A along with EGM08 need to be evaluated according to the information from a meeting held in Chania, Greece in June of 2008.

The IAG/IGFS Joint Working Group (JWG) is commissioned for the organization and guidance of the evaluation affairs. This report will present an evaluation of the models PGM07A and EGM08 by use of the GPS/Leveling and the terrestrial gravity data within the Chinese territory. We adopt the evaluation methods and some necessary ancillary models/information for the quality assessment of the PGM and the EGM including the computational models, geodetic parameters, as well as software, etc., provided by JWG. In this report, the testing for the evaluation of PGM07 and EGM08 is focused on two kinds of data comparisons, i.e. GPS/Leveling data and terrestrial gravity anomaly data in China. However, besides PGM07A and EGM08, some recent released satellite combined gravitational models including two series of EIGEN01-04C and GGM01-02C are also included in the comparisons.

2. Data used for the evaluation

2.1 GPS/Leveling data

(1) National data

652 GPS/Leveling points within Chinese national A/B-order GPS networks are used for the evaluation of PGM07A and EGM08, among the points, 28 A-order points and 624 B-order points are included, respectively (see Fig. 1). The A-order GPS campaign was carried out during two separately operational periods, i.e. in 1992 and in 1996. The GPS results are referred to ITRF93 at epoch 1996.365 and WGS84. The mean accuracy of GPS ellipsoidal heights at B-order GPS points is about 0.10m. The leveling at the A/B order GPS points, referred to the China Yellow Mean Sea Level 1985 Datum, was performed before the period of GPS campaigns, and the mean accuracy of the normal heights derived from the leveling is better than the level of 0.10m. Finally, the mean accuracy of the height anomalies (quasi-geoid heights) derived from the GPS/Leveling is the level of 0.14m.



Fig. 1 The distribution of 652 national A/B-order GPS/Leveling points

(2) Provincial data

Seven regional GPS/Leveling data sets collected from the provincial regions including Gansu, Guangdong, Guangxi, Hebei, Jiangsu, Qinghai and Shanxi, are used for the tests and evaluation (see Fig. 2). The total number of the regional GPS/Leveling points is 1160, and the mean accuracy of the GPS/Leveling-derived quasi-geoid heights from the data sets is the level of 0.05m. The GPS ellipsoidal heights are also referred to WGS84. The information about the regional GPS/Leveling points in detail is shown in Table 1.



Fig. 2 The distribution of the provincial GPS/Leveling points

Table 1. The number of GPS/leveling points for each provincial region

province	Gansu	Guangdong	Guangxi	Hebei	Jiangsu	Qinghai	Shanxi	total
number	131	88	94	114	177	31	525	1160

2.2 Digital Terrain Model (DTM)

The digital terrain models with 3″×3″ resolution generated from SRTM (Shuttle Radar Topography Mission) are used for provincial gravity reductions.

2.3 Terrestrial gravity data

Five regional terrestrial point gravity data sets are used for the evaluation, which are distributed over five provincial regions including Gansu, Guangdong, Guangxi, Qinghai and Shanxi. The total number of the gravity point values used is 746,135, which were tied to IGSN71 gravity datum, but their accuracies in detail are not clear enough. The method of topographic-isostatic gravity reduction is adopted for generating high-resolution gridded mean free air gravity anomaly set on the Earth's surface based on remove-restore principle and an approach of interpolation/extrapolation (*Heiskanen W.A. & H. Moritz, 1967*). In the gravity reduction computations, a spherical integral formula taking account of the Earth's curvature is used for both terrain corrections and isostatic compensation

corrections with the SRTM 3″×3″ DTM. The related information for the provincial data sets is shown in Table 2.

Table 2. The number of the terrestrial gravity point values for each provincial region

Province	Gansu	Guangdong	Guangxi	Qinghai	Shanxi	total
Number	55,467	417,336	28,372	213,862	31,098	746,135

3. Quasi-geoid comparisons

Hereafter, the comparisons are to compute the differences of observed values minus corresponding model ones.

3.1 National GPS/Leveling-derived quasi-geoid vs. quasi-geoids derived from PGM07A, EGM08 and other EGMs

The observed quasi-geoid heights are computed at all testing points in national A/B-order GPS/Leveling networks, and the corresponding model ones are derived from the Earth gravitational models, including PGM07A, EGM08 and some recent released models derived from the data of satellite and terrestrial gravity (EIG01C, EIG03C, EIG04C, GGM01C, GGM02C, GGM01C*, and GGM02C*). The results of the comparisons are tabulated in Table 3. From Table 3, we can see that the RMS and Std.D of the differences between the quasi-geoid heights derived from the EGMs (PGM07A and EGM08) and their corresponding observed ones determined from the data of GPS/Leveling at 652 A/B order testing points are ±0.358m and ±0.338m for PGM07A, ±0.284m and ±0.257m for EGM08, respectively, which are remarkably better than the RMS and Std.D corresponding to the other seven models used in the tests. For the latter models, the order of magnitude is in the range ±0.499m to ±0.764m with a mean of ±0.584m (RMS) and ±0.435m to ±0.699m with a mean of ±0.521m (Std.D), respectively. The result of the above comparisons shows that the accuracy of PGM07A-derived quasi-geoid as applied in China in the sense of its approximation to the observed quasi-geoid by GPS/Leveling, is significantly improved by about 0.2 m versus the recent released combined satellite EGMs, and the accuracy of EGM08-derived quasi-geoid is higher than that of PGM07A-derived one by about 0.08m.

Table 3. The statistics of the comparisons of the quasi-geoid heights derived from PGM07A/EGM08 and the other EGMs with those determined from the GPS/Leveling data of 652 A/B-order points (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	1.670	-2.314	-0.250	±0.527	±0.464
EIG03C	1.670	-2.020	-0.250	±0.502	±0.435
EIG04C	1.805	-2.262	-0.247	±0.504	±0.439
GGM01C	1.408	-3.221	-0.249	±0.596	±0.541
GGM02C	1.788	-2.539	-0.236	±0.499	±0.440
GGM01C*	2.903	-4.730	-0.308	±0.764	±0.699
GGM02C*	3.295	-4.048	-0.296	±0.695	±0.629
PGM2007A	1.989	-2.441	-0.117	±0.358	±0.338
EGM2008	1.641	-1.886	-0.121	±0.284	±0.257

3.2 Provincial GPS/Leveling-based quasi-geoids vs. quasi-geoids derived from PGM07A, EGM08 and the other EGMs

Comparisons of the quasi-geoid heights derived from PGM07A and EGM08 as well as the other seven EGMs used in the evaluation with those determined based on the GPS/Leveling data of the regional networks are performed. The statistic results of the comparisons are shown in Table 4 ~ Table10.

These tables show that the RMS and Std.D of the differences between PGM07-derived quasi-geoid heights and the corresponding GPS/Leveling-based ones are in the range ±0.107m to ±0.309m with a mean of ±0.214m (RMS) and ±0.090m to ±0.308m with a mean of ±0.172m (Std.D), respectively; for EGM08, the corresponding RMS and Std.D are in the range ±0.099m to ±0.238m with a mean of ±0.188m (RMS) and ±0.073m to ±0.237m with a mean of ±0.138m (Std.D), respectively; for the other seven EGMs used in the evaluation, the corresponding RMS and Std.D are in the range ±0.263m to ±0.805m with a mean of ±0.439m (RMS) and ±0.190m to ±0.740m with a mean ±0.373m (Std.D).

According to these tables and the above simple statistical analyses, Firstly, we can see that the quasi-geoids derived from both PGM07A and EGM08 can provide a better approximation to those determined by the GPS/Leveling data of provincial networks than the approximation provided by the quasi-geoids derived from the other seven EGMs used in the comparisons, with respect to the order of magnitude of the improved approximation, for PGM07A, it is the level of 0.21m in both RMS and Std.D on an average; for EGM08, is 0.24m in both RMS and Std.D on an average. We can also see that EGM08 is better than PGM07A. The statistical results of the differences between EGM08-derived

quasi-geoid heights and the corresponding provincial GPS/Leveling-based ones are smaller than the corresponding statistical results for PGM07A by the level of 0.03m in both RMS and Std.D on average, however, in the provincial regions such as Gansu, Jiangsu and Qinghai, the results for EGM08 are much less than the corresponding results to PGM07A by the level of 0.06m in both RMS and Std.D on an average.

Secondly, the results show that the statistics (RMS, Std.D) of the differences between the PGM07A or EGM08-derived quasi-geoid and the provincial GPS/Leveling-based quasi-geoid are smaller than those between PGM07A or EGM08-derived quasi-geoid and the national A/B-order GPS/Leveling-based quasi-geoid by about 0.1~0.2m on average(comparing Table 3 with Table 4~10), for this case, one reason is the fact that the density of provincial GPS/Leveling points is higher than that of the used A/B-order GPS/Leveling points, and the mean accuracy of the former is also higher than the latter, and other causes may be included; Thirdly, the comparisons also indirectly show that the accuracies of the quasi-geoids determined from the data of GPS/Leveling in the networks of three provinces, i.e. Guangdong, Guangxi and Shanxi, are higher than the accuracies of those in the networks of the other provinces by about the level of 0.1~0.2m; Finally, the biases between all EGM-derived quasi-geoids and GPS/Leveling-based ones in some regions are stably within about -0.1~-0.2m.

Table 4. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and other EGMs with those from GPS/Leveling (131 points) in Gansu (Uint: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.717	-1.283	-0.268	±0.495	±0.417
EIG03C	0.620	-1.330	-0.252	±0.479	±0.407
EIG04C	0.662	-1.270	-0.228	±0.456	±0.395
GGM01C	0.988	-1.678	-0.211	±0.535	±0.491
GGM02C	0.636	-1.077	-0.205	±0.421	±0.368
GGM01C*	1.509	-2.399	-0.234	±0.776	±0.740
GGM02C*	1.282	-2.121	-0.227	±0.672	±0.632
PGM2007A	0.734	-0.491	-0.069	±0.229	±0.219
EGM2008	0.663	-0.502	-0.077	±0.184	±0.167

Table 5. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those from GPS/Leveling (88points) in Guangdong (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.583	-0.780	-0.157	±0.301	±0.257
EIG03C	0.576	-0.749	-0.177	±0.321	±0.268
EIG04C	0.458	-0.825	-0.194	±0.322	±0.257
GGM01C	0.760	-0.817	-0.148	±0.391	±0.362
GGM02C	0.376	-0.739	-0.186	±0.316	±0.256
GGM01C*	0.699	-0.859	-0.148	±0.358	±0.326
GGM02C*	0.546	-0.763	-0.186	±0.336	±0.279
PGM2007A	0.142	-0.369	-0.154	±0.179	±0.090
EGM2008	0.131	-0.416	-0.163	±0.186	±0.091

Table 6. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those from GPS/Leveling (94points) in Guangxi (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.478	-0.718	-0.130	±0.263	±0.228
EIG03C	0.292	-0.635	-0.119	±0.264	±0.236
EIG04C	0.500	-0.615	-0.131	±0.274	±0.240
GGM01C	1.121	-0.917	-0.147	±0.396	±0.367
GGM02C	0.520	-0.686	-0.129	±0.265	±0.231
GGM01C*	0.948	-1.087	-0.148	±0.403	±0.375
GGM02C*	0.536	-0.857	-0.130	±0.299	±0.270
PGM2007A	0.140	-0.485	-0.051	±0.107	±0.094
EGM2008	0.107	-0.275	-0.066	±0.099	±0.073

Table 7. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those form GPS/Leveling (114points) in Hebei (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.529	-0.832	-0.158	±0.355	±0.318
EIG03C	0.598	-1.004	-0.168	±0.385	±0.347
EIG04C	0.616	-0.930	-0.160	±0.366	±0.329
GGM01C	0.854	-0.898	-0.129	±0.464	±0.446

GGM02C	0.568	-0.903	-0.178	±0.373	±0.328
GGM01C*	0.895	-0.917	-0.174	±0.429	±0.393
GGM02C*	0.581	-0.921	-0.223	±0.388	±0.317
PGM2007A	0.131	-0.721	-0.142	±0.224	±0.174
EGM2008	0.136	-0.690	-0.160	±0.231	±0.167

Table 8. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those from GPS/Leveling (117points) in Jiangsu (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.360	-0.820	-0.191	±0.344	±0.285
EIG03C	0.228	-0.719	-0.195	±0.272	±0.190
EIG04C	0.341	-0.681	-0.181	±0.282	±0.216
GGM01C	0.313	-0.909	-0.223	±0.351	±0.271
GGM02C	0.291	-0.741	-0.202	±0.284	±0.199
GGM01C*	0.447	-0.919	-0.219	±0.360	±0.285
GGM02C*	0.449	-0.754	-0.198	±0.307	±0.235
PGM2007A	0.366	-0.714	-0.213	±0.288	±0.194
EGM2008	0.268	-0.634	-0.190	±0.237	±0.140

Table 9. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those from GPS/Leveling (31points) in Qinghai (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.510	-1.493	-0.323	±0.524	±0.412
EIG03C	0.458	-1.327	-0.257	±0.466	±0.389
EIG04C	0.538	-1.377	-0.305	±0.512	±0.411
GGM01C	1.011	-1.631	-0.337	±0.665	±0.573
GGM02C	0.798	-1.454	-0.305	±0.538	±0.443
GGM01C*	0.655	-1.801	-0.457	±0.805	±0.663
GGM02C*	0.442	-1.475	-0.425	±0.669	±0.516
GGM01S	2.085	-1.828	-0.429	±0.956	±0.854
GGM02S	2.876	-3.331	-0.421	±1.595	±1.539
PGM2007A	0.674	-0.747	0.021	±0.309	±0.308
EGM2008	0.585	-0.511	-0.015	±0.238	±0.237

Table 10. The statistics of the comparisons of the quasi-geoid heights from PGM07A/EGM08 and the other EGMs with those from GPS/Leveling (525points) in Shanxi (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	0.822	-1.342	-0.301	±0.520	±0.424
EIG03C	0.635	-1.327	-0.304	±0.496	±0.391
EIG04C	0.618	-1.308	-0.307	±0.504	±0.400
GGM01C	0.785	-1.495	-0.350	±0.548	±0.421
GGM02C	0.703	-1.271	-0.285	±0.467	±0.370
GGM01C*	1.370	-1.966	-0.378	±0.760	±0.660
GGM02C*	1.196	-2.115	-0.313	±0.726	±0.655
PGM2007A	0.375	-0.375	-0.098	±0.160	±0.127
EGM2008	0.343	-0.368	-0.107	±0.140	±0.090

3.3 Overall comparisons

The overall comparisons of the quasi-geoids derived from PGM07A and EGM08 as well as the other seven EGMs with those determined from the data of GPS/Leveling of all used 1812 points distributed in the whole Chinese mainland are made. The distribution of the differences of the comparisons is plotted in Fig. 3a and Fig. 3b. The statistic results of the comparisons are shown in Table 11. From this table, we can see that the corresponding statistical results (RMS, Std.D) of the comparisons are better than those listed in Table 3, but somewhat worse than those listed in Table 4 ~ Table 10.

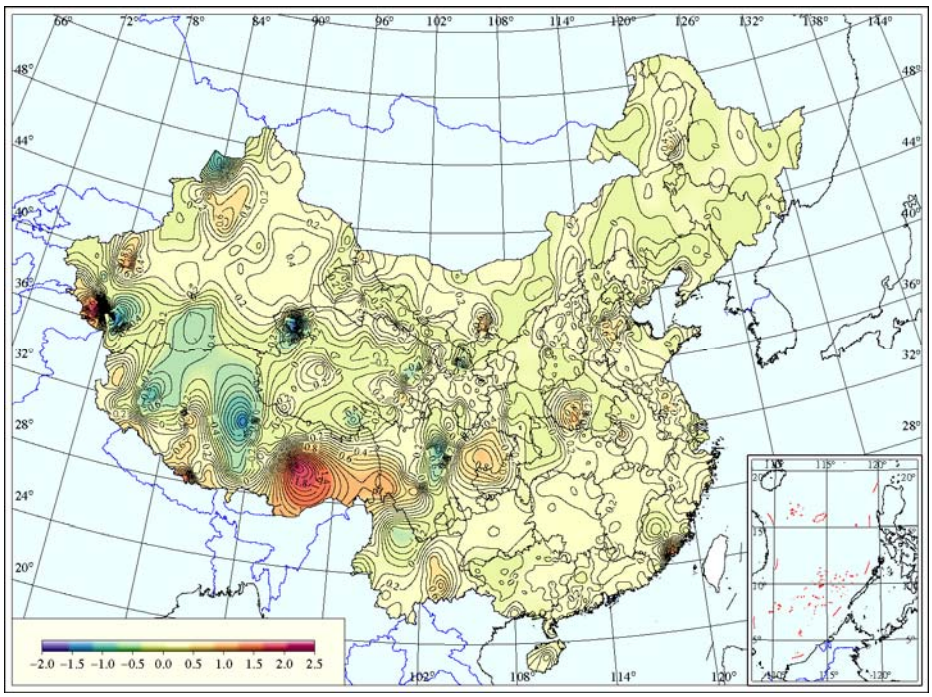


Fig. 3a The differences of the comparisons with PGM07A (Unit: m)

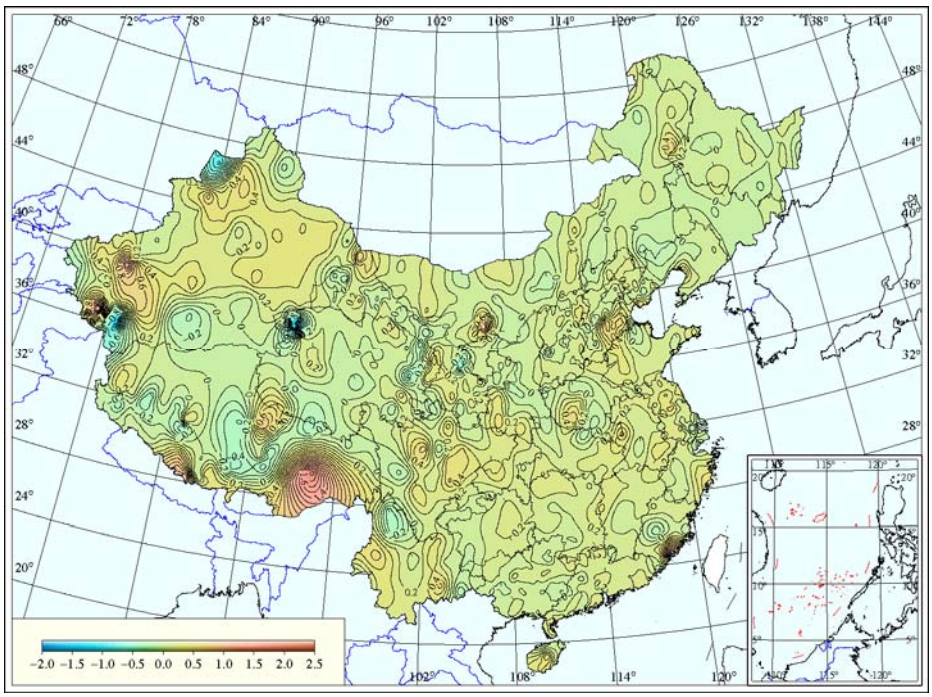


Fig. 3b The differences of the comparisons with EGM08 (Unit: m)

Table 11. The statistics of the overall comparisons of the quasi-geoids from PGM07A/EGM08 and the other EGMs with those from all 1812 used GPS/Leveling in Chinese mainland (Unit: m)

Model	Max	Min	Bias	RMS	Std. D
EIG01C	1.670	-2.314	-0.245	±0.478	±0.410
EIG03C	1.670	-2.020	-0.245	±0.455	±0.384
EIG04C	1.805	-2.262	-0.243	±0.458	±0.388
GGM01C	1.408	-3.221	-0.257	±0.533	±0.467
GGM02C	1.788	-2.539	-0.234	±0.443	±0.375
GGM01C*	2.903	-4.730	-0.292	±0.686	±0.621
GGM02C*	3.295	-4.048	-0.270	±0.629	±0.568
PGM2007A	1.989	-2.441	-0.115	±0.269	±0.243
EGM2008	1.641	-1.886	-0.120	±0.222	±0.186

4. Gravity anomaly comparisons

The comparisons of PGM07A/EGM08-derived free air gravity anomalies with those in-situ measurements in the five provincial regions are made using 2'×2' gridded mean value data sets. The distribution of the differences between PGM07A/EGM08-derived gridded mean free air gravity anomalies and those corresponding gridded observation values in the regions is shown in Fig. 4 ~ Fig. 8 respectively. The statistics of the comparisons with total 183201 gridded values are listed in Table 12.

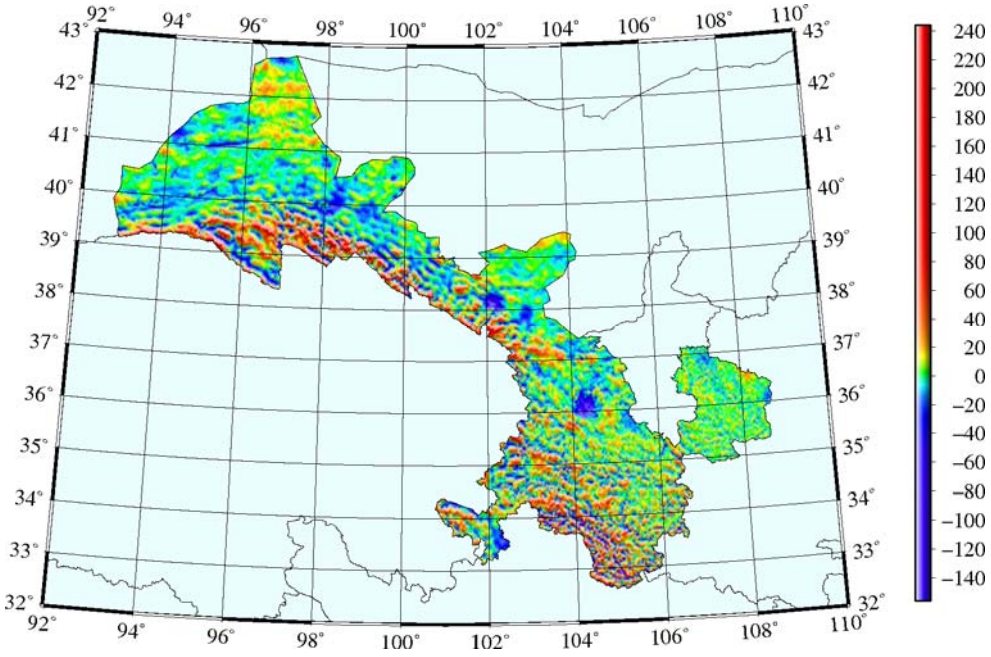


Fig. 4a The differences between 2'×2' gridded free-air gravity anomalies from PGM07A and the corresponding observation values in Gansu province region (Unit: mGal)

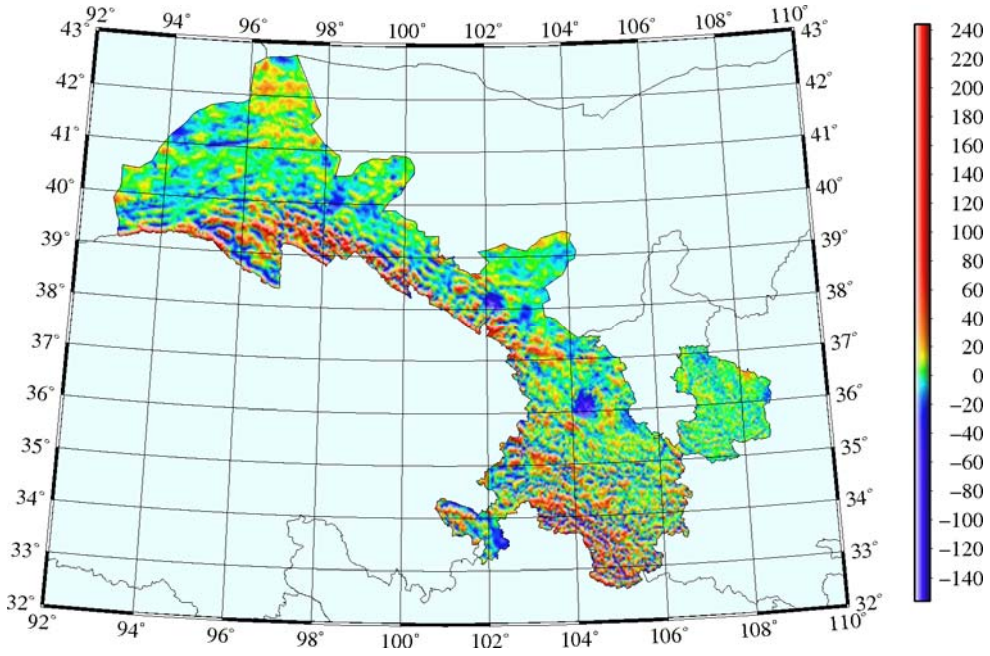


Fig. 4b The differences between 2'×2' gridded free-air gravity anomalies from EGM08 and the corresponding observation values in Gansu province region (Unit: mGal)

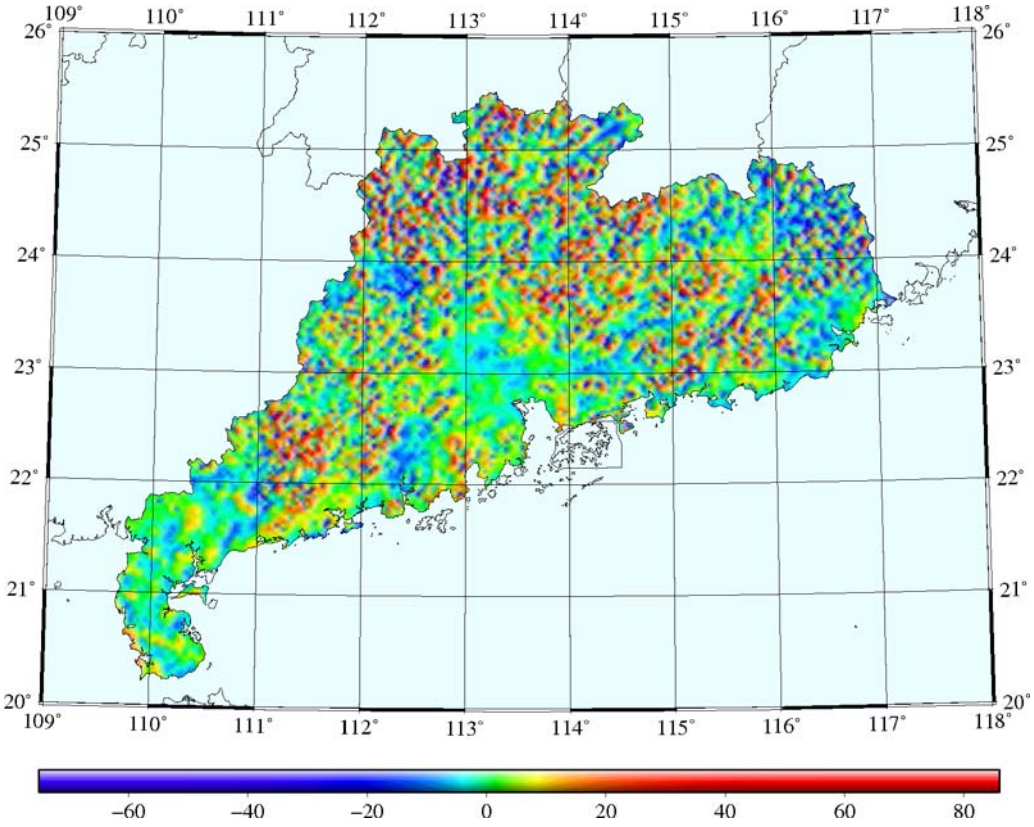


Fig. 5a The differences between 2'×2' gridded free-air gravity anomalies from PGM07A and the corresponding observation values in Guangdong province region (Unit:

mGal)

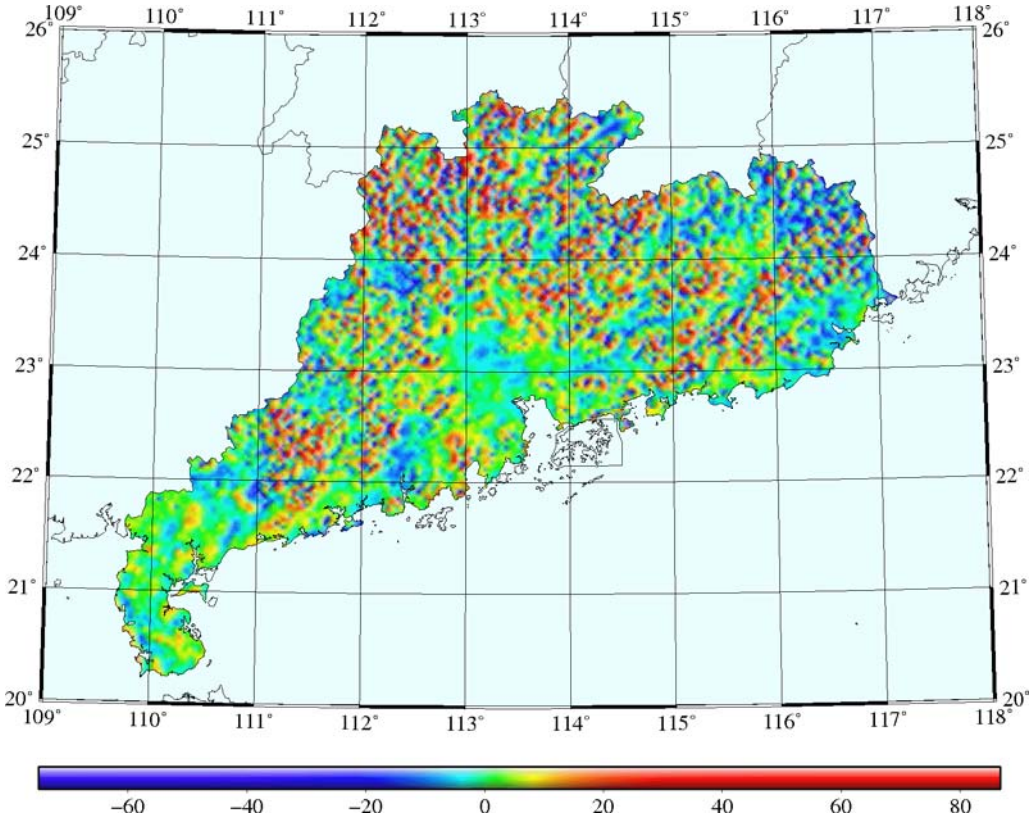


Fig. 5b The differences between 2'×2' gridded free-air gravity anomalies from EGM08 and the corresponding observation values in Guangdong province region (Unit: mGal)

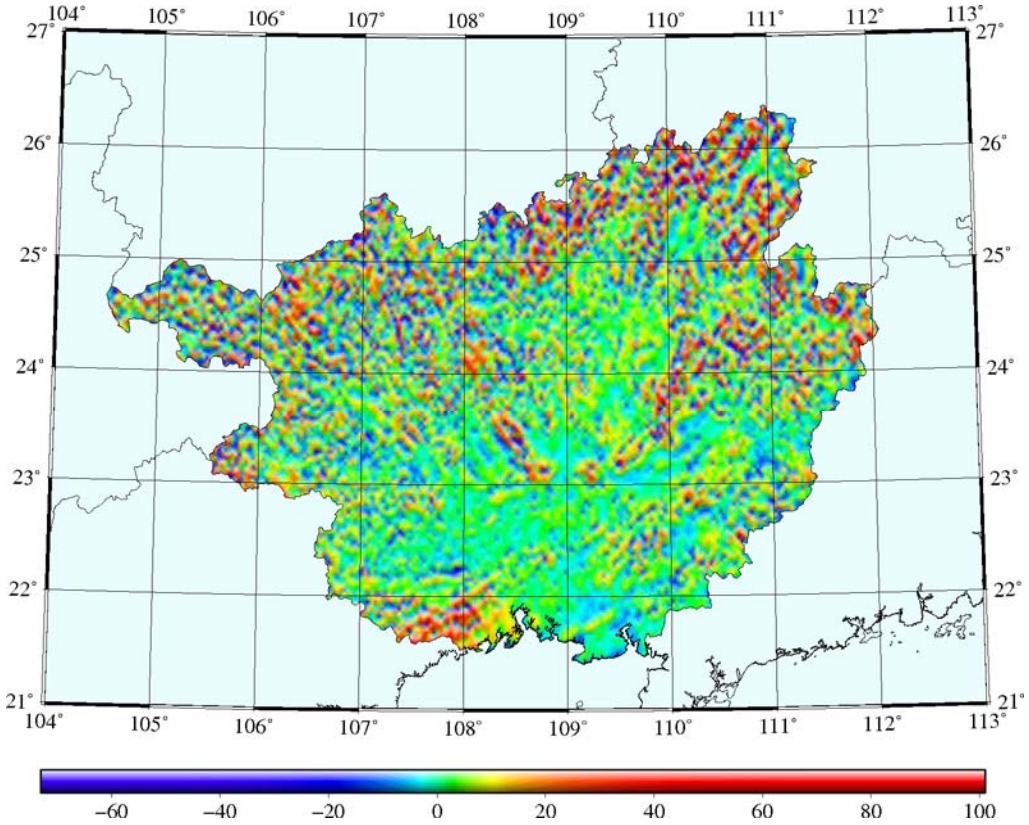


Fig. 6a The differences between 2'×2' gridded free-air gravity anomalies from PGM07A and the corresponding observation values in Guangxi province region (Unit: mGal)

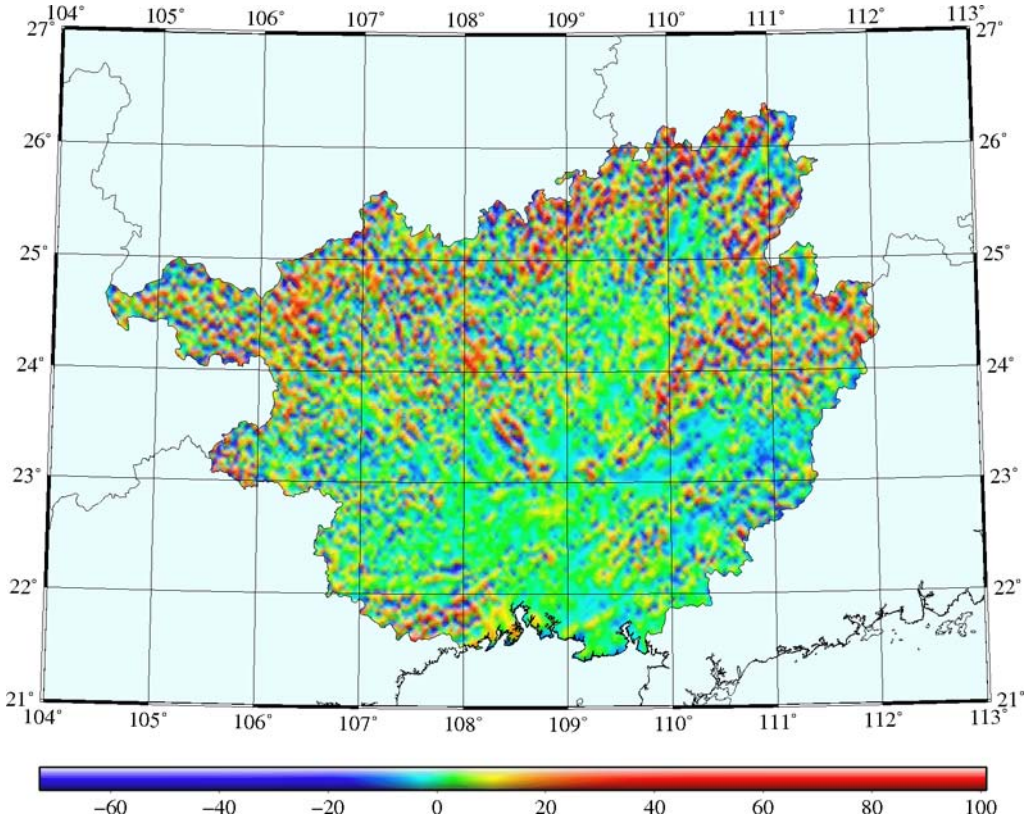


Fig. 6b The differences between 2'×2' gridded free-air gravity anomalies from EGM08 and the corresponding observation values in Guangxi province region (Unit: mGal)

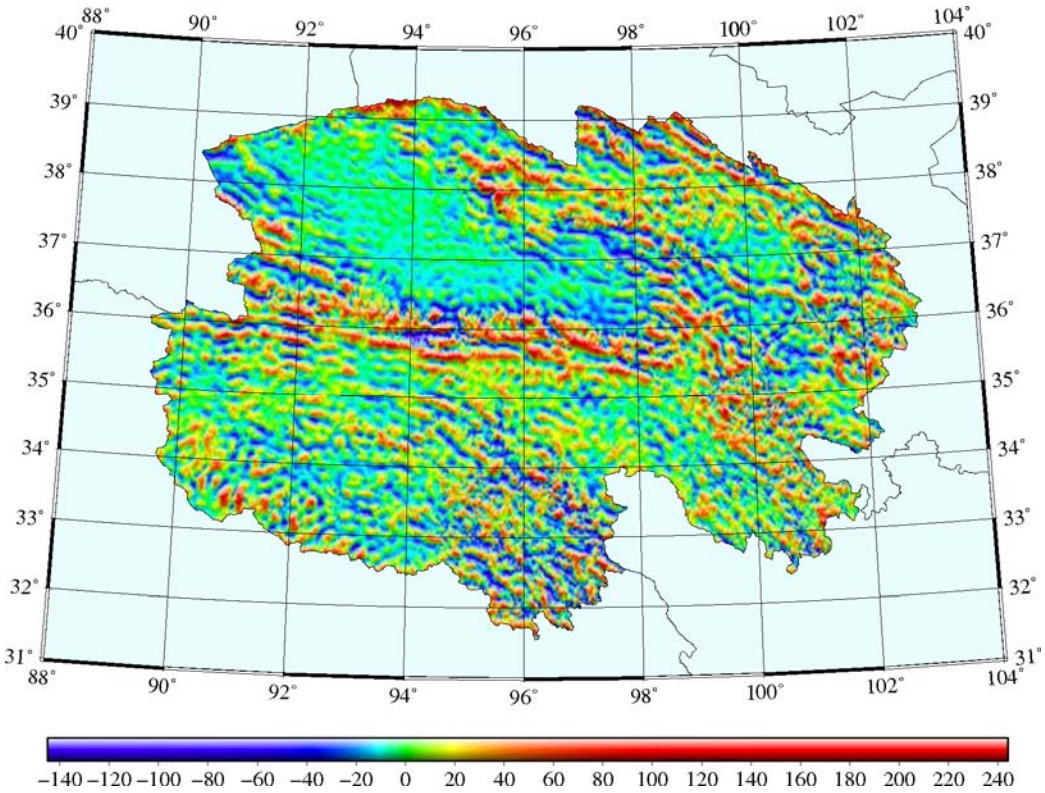


Fig. 7a The differences between 2'×2' gridded free-air gravity anomalies from PGM07A and the corresponding observation values in Qinghai province region (Unit: mGal)

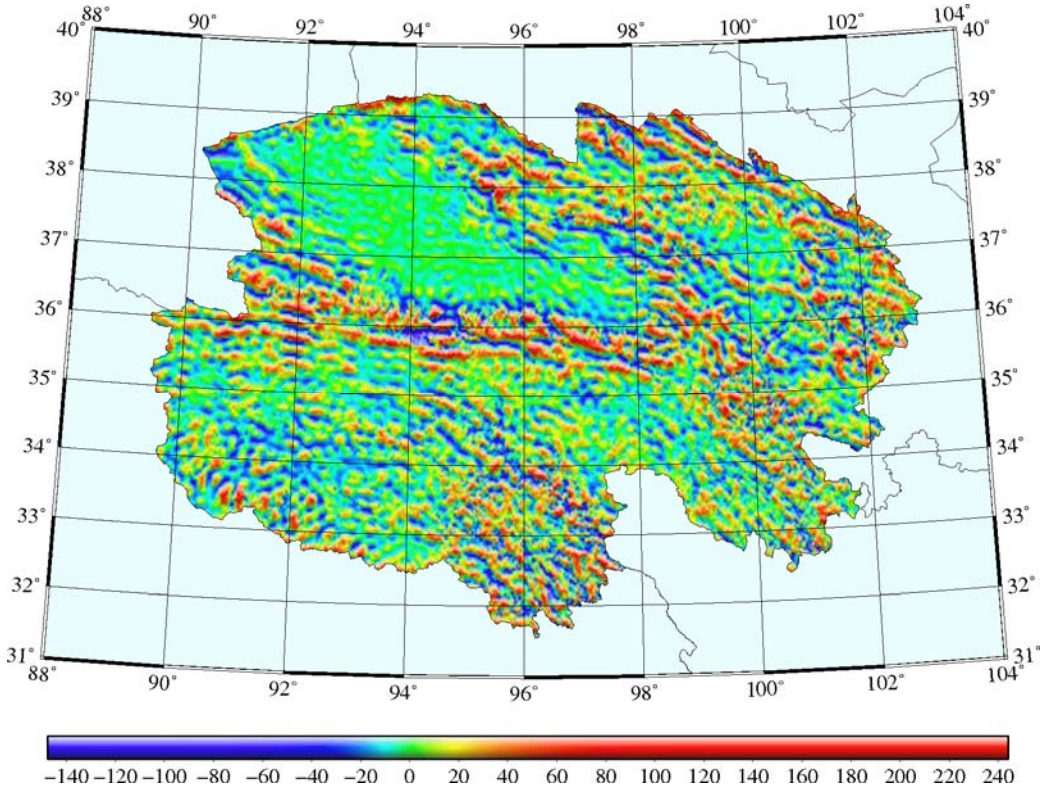


Fig. 7b The differences between 2'×2' gridded free-air gravity anomalies from EGM08 and the corresponding observation values in Qinghai province region (Unit: mGal)

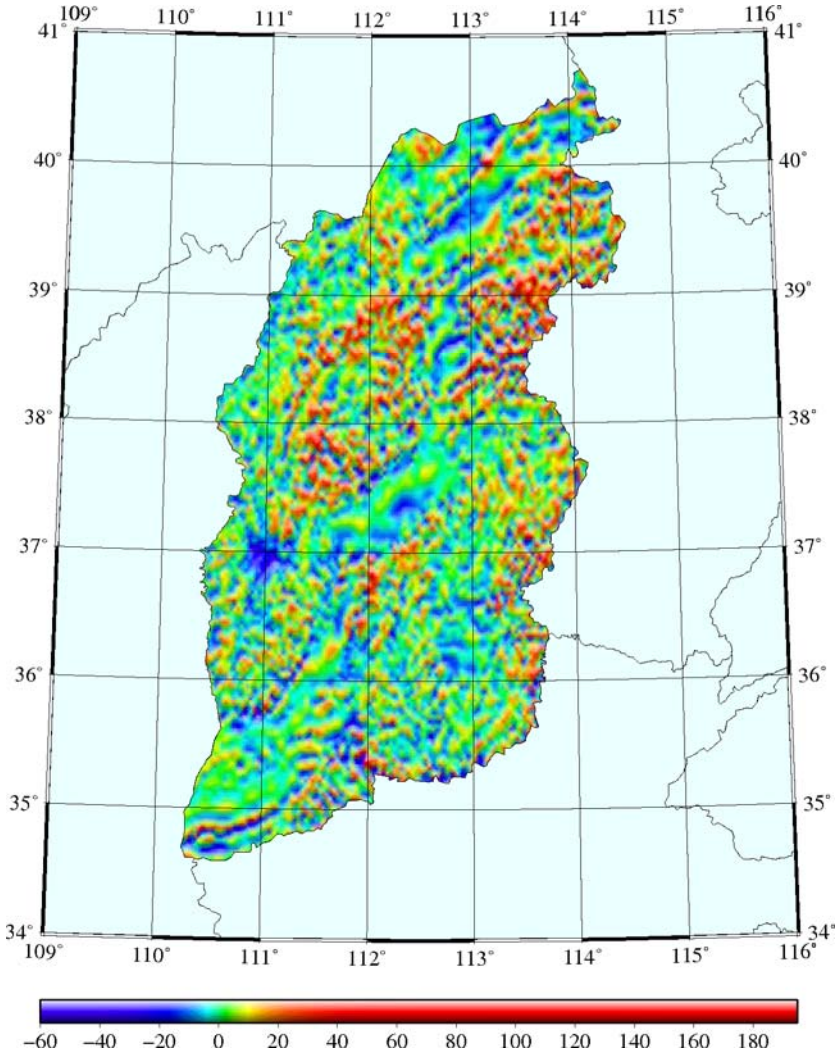


Fig. 8a The differences between 2'×2' gridded free-air gravity anomalies from PGM07A and the corresponding observation values in Shanxi province region (Unit: mGal)

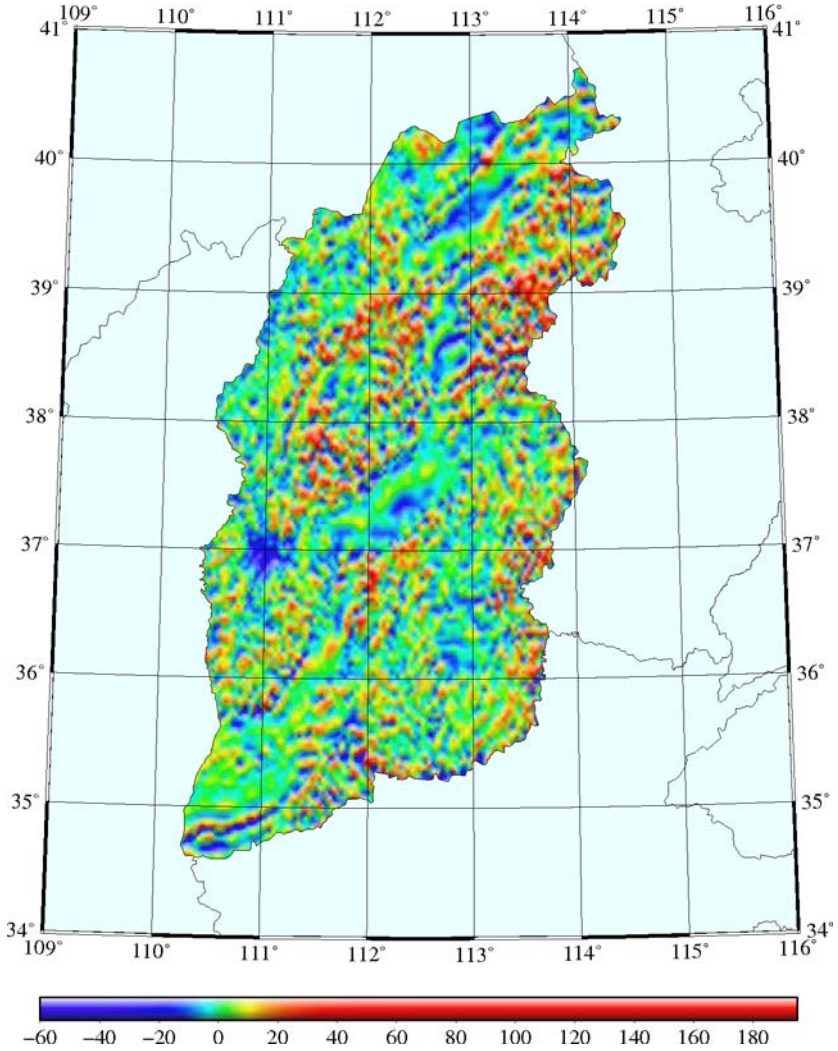


Fig. 8b The differences between 2'×2' gridded free-air gravity anomalies from EGM08 and the corresponding observation values in Shanxi province region (Unit: mGal)

Table 12a. The statistics of the differences in the comparisons of the total 183201 measured gravity anomalies with the corresponding ones from PGM07(Unit: mGal)

Area	Block	Max	Min	Bias	RMS	Std. D
Gansu	48929	243.996	-155.599	4.491	±24.751	±25.155
Guangdong	18860	85.897	-74.683	1.364	±12.664	±12.590
Guangxi	23673	100.108	-72.876	3.232	±14.078	±13.702
Qinghai	73432	243.996	-144.039	4.206	±29.161	±28.856
Shanxi	18307	194.780	-59.150	2.817	±13.464	±13.167

Table 12b. The statistics of the differences in the comparisons of the total 183201 measured gravity anomalies with the corresponding ones from EGM08 (Unit: mGal)

Area	Block	Max	Min	Bias	RMS	Std. D
Gansu	48929	218.938	-143.032	4.264	±24.167	±23.788
Guangdong	18860	86.801	-74.908	1.346	±12.547	±12.475
Guangxi	23673	95.884	-71.460	3.357	±14.015	±13.608
Qinghai	73432	218.938	-147.677	4.125	±27.683	±27.373
Shanxi	18307	194.375	-56.505	3.056	±13.111	±13.462

Table 12c. The statistics of the differences between the differences of EGM08-derived anomalies and PGM07A-derived ones from the corresponding total 183201 measured anomoalies(Unit: mGal)

Area	Block	Max	Min	Bias	RMS	Std. D
Gansu	48929	15.380	-25.058	-0.227	±3.247	±3.239
Guangdong	18860	6.561	-14.187	-0.018	±1.721	±1.721
Guangxi	23673	11.201	-29.411	0.126	±2.350	±2.347
Qinghai	73432	15.030	-25.058	-0.080	±4.718	±4.717
Shanxi	18307	4.426	-2.798	0.239	±1.368	±1.347

5. Conclusions

This report mainly focuses on the comparisons of the quasi-geoids derived from PGM07A and EGM08 as well as the other recent released Earth Gravitational Models (EGMs) with the quasi-geoids determined by the national A/B-order GPS/Leveling data and some provincial GPS/Leveling data. The statistical results of the comparisons show that the quasi-geoids derived from both PGM07A and EGM08 can provide a better approximation to the quasi-geoids determined from the provincial GPS/Leveling data than an approximation to the same quasi-geoids provided by the quasi-geoids derived from the other seven EGMs used in comparisons. The statistics (RMS and Std.D) of the differences of the above mentioned approximations is in the range of ±0.1~ ±0.5m. The statistical results of overall comparisons using all 1812 nationwide GPS/Leveling point data, also show that the quasi-geoids derived from PGM07A and EGM08 are better than those derived from the other seven EGMs in terms of the approximation level to the GPS/Leveling-based quasi-geoids by the range of ±0.1~ ±0.4m in RMS and Std.D.

According to the approximation levels to the GPS/Leveling-based quasi-geoids, the comparisons for the evaluations of EGM08 also indicate that the quasi-geoid derived from EGM08 is better than that derived from PGM07A by the range of ±0.01~ ±0.07m with an average of ±0.03m in RMS and Std.D.

In addition, the comparisons of gravity anomalies derived from PGM07A and EGM08 with corresponding ones measured in the five provincial regions are made, and the statistical results of the comparisons show that the RMS and Std.D of the differences between PGM07A/EGM08-derived gravity anomalies and those corresponding ones measured in the regions are within the range of ±13~±29 mGal with an average of ±18 mGal. According to the approximation levels to the in-situ measured gravity anomalies, the statistical results of the comparisons also indicate that the gridded mean anomalies derived from EGM08 are more accurate than those derived from PGM07A by the range of ±1~4 mGal with an average of ±2.7 mGal in RMS and Std.D of differences of comparisons.

In the general, the quasi-geoid and gravity anomalies computed from PGM07A /EGM08 have significantly improved the representation of the Chinese local gravity field as compared with the other seven EGMs used in the comparisons. However, there are some anomalous (large or small) biases of quasi-geoid which deviate from the general level of the biases in some regions. Therefore, the biases are locally inconsistent with each other, for example, the difference of biases between quasi-geoids of Jiangsu and Qinghai regions for both PGM07A and EGM08 is up to 0.2m, and the differences between even two adjacent provinces also have relatively large values, such as Guangdong and Guangxi, as well as Qinghai and Gansu. On the contrary, the biases of the quasi-geoids derived from both EIGEN and GGM series models, which deviate from the corresponding ones determined by GPS/Leveling, are basically consistent with each other in different regions of China. We know that there also exist some large bias differences of geoids in some states of USA, e.g., the maximum of the difference between Washington and Florida reaches 1.14m (*Roman et al, 2007*). The large biases variation of PGM07A/EGM08-derived geoid occurring in some regions may be a problem, and it will lead to inconsistency and incompatibility in regional quasi-geoid or geoid determination by the models.

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Gravity and geoid estimate in South India and their comparison with EGM2008

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Abstract

The geopotential model EGM2008 has been tested in the Southern part of India by fitting gravity data. These data have been collected by the National Geophysical Research Institute of India and have been also used to estimate the geoid in this region. For the comparisons a total of 16013 gravity values have been considered. Previous global geopotential models have been also computed on these gravity points to analyze their performance as related to EGM2008. Furthermore, this new model has been compared with the local geoid estimate that is available in this region. The results prove that, in terms of gravity anomaly, in the Southern India region, EGM2008 is better than any other existing model. As for the geoid undulation, the differences between the local South India geoid and EGM2008 are less than 2 m (in absolute value) and are partially correlated with the topography.

1. Introduction

The EGM2008 is the last geopotential model estimated at the agreement NGA/NASA. This estimate, with maximum degree 2160, proved to be very effective in modeling different functionals of the anomalous potential. We present a comparison over the Southern India region using a quite large data base collected by National Geophysical Research Institute of India. The gravity field of this area is quite regular and its structure is mainly connected to the topography**;** in fact the major variations are in the area $10° \leq \varphi \leq 12°$, $76° \leq \lambda \leq 78°$ where the principal relieves are concentrated. The topography varies from sea level to high mountains (about 2500 meters) and the area is surrounded by deep ocean. Land gravity is known inside the area $8° \leq \varphi \leq 15°$, $74° \leq \lambda \leq 81°$ where 16013 gravity values were measured.
Over this area, a local geoid has been recently computed by the National Geopysical Research Institute of India in co-operation with IGeS (International Geoid Service). This is an updated and detailed geoid which was computed using the "remove-restore" technique and collocation.
Both gravity and this geoid estimate were used to check for the accuracy of EGM2008 in this area

2. Fitting different geopotential models with gravity data in South India.

The distribution of the gravity point values used in the comparison is shown in Figure 1. Data density is not uniform; however, the global coverage is satisfactory since there are only limited data gaps (as compared to the whole area). The data base contains 16013 gravity values.

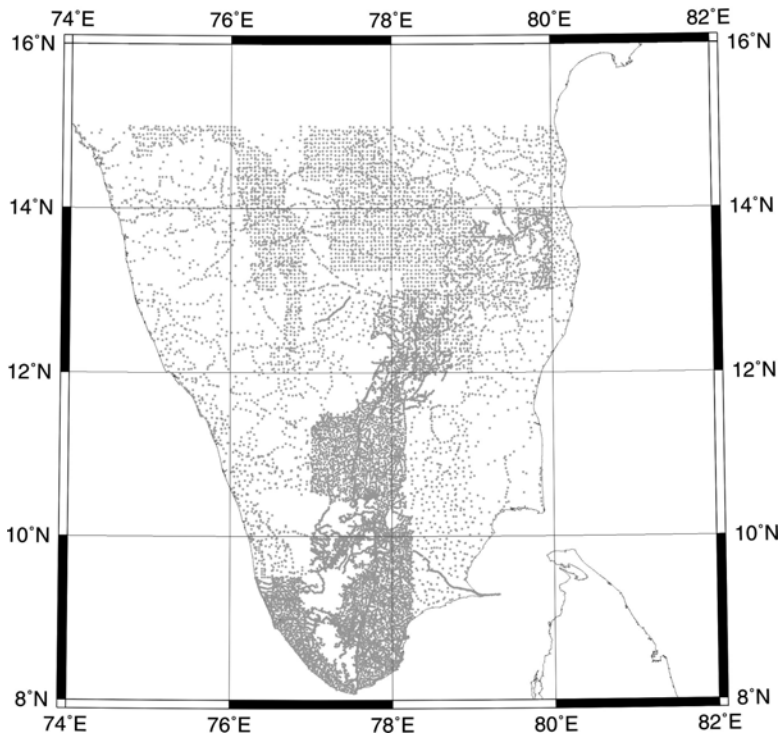


Figure 1. The gravity data base in South India.

Point positions refer to WGS84; gravity is in IGSN71 and the adopted normal field is GRS80.
Different available geopotential models have been used to reduce these gravity data in order to be compared to EGM2008.
The considered models are:

- EGM96 (Lemoine et al., 1998)
- GPM98CR (Wenzel, 1998)
- EIGEN_GL04C (Förste et al., 2007)
- EGM2008 (Holmes and Pavlis, 2008).

For each model, the gravity effect has been computed using the full coefficient expansion.
The statistics of the data and those of the residuals obtained using the tested models are listed in Table 1.

Gravity	E (mgal)	σ(mgal)	Min (mgal)	Max(mgal)
Δg_{fa}	-33.93	29.13	-120.93	192.77
Δg_{fa}-Δg_{EGM96}	-6.45	21.85	-94.46	182.32
Δg_{fa}-$\Delta g_{GPM98CR}$	-7.56	22.42	-83.08	200.56
Δg_{fa}-Δg_{GL04C}	-6.08	21.65	-92.55	183.23
Δg_{fa}-$\Delta g_{EGM2008}$	-0.08	10.88	-112.54	79.40

Table 1 – Gravity and residuals statistics (after model reduction)

The statistics show that EGM2008 is able to describe the gravity field of this area better than the other considered models. The standard deviation of EGM2008 residuals is, roughly speaking, one third the value of the unreduced data. When using the other models, the standard deviations of the residuals decrease only to about two third of the original value. Particularly, the GPM98CR model, which is complete to degree and order 720, gives, as compared to EGM96 and EIGEN_GL04C, the worst result. Furthermore, the EGM2008 residuals have also a very small mean, less than one tenth of mgal.
The plot of the residual values after EGM2008 reduction is shown in Figure 2.

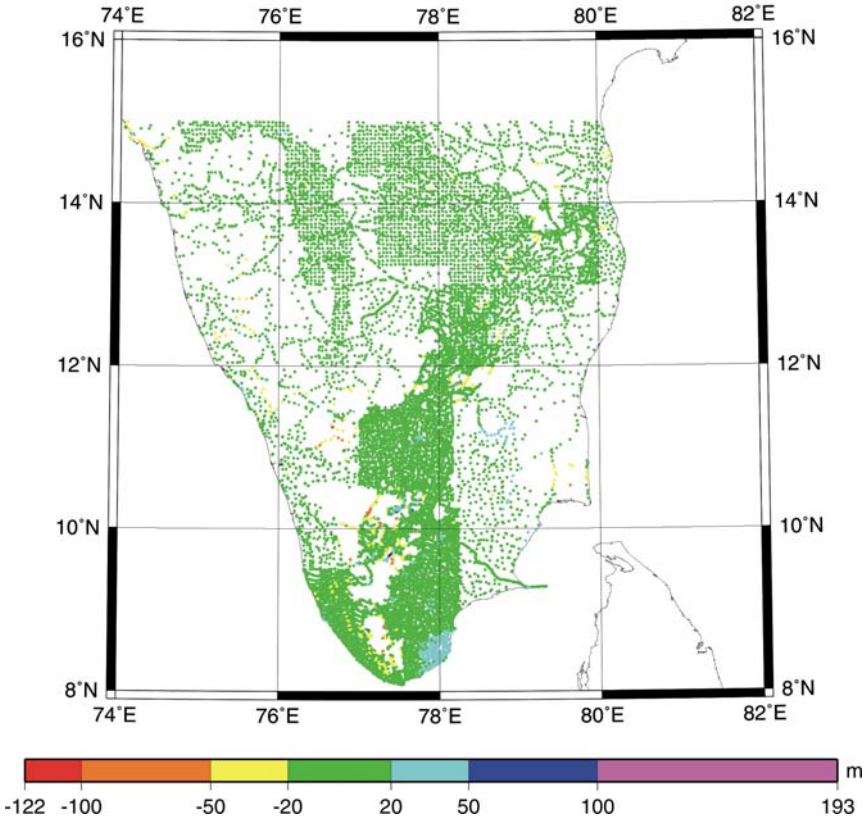


Figure 2. The gravity residuals after EGM2008 reduction

Most of the residuals are in the range [-20 mgal, 20 mgal] and only some gravity lines have larger residuals (probably related to unreliable data). This plot can be compared with the one displaying e.g. the EIGEN_GL04C residuals (see Figure 3).

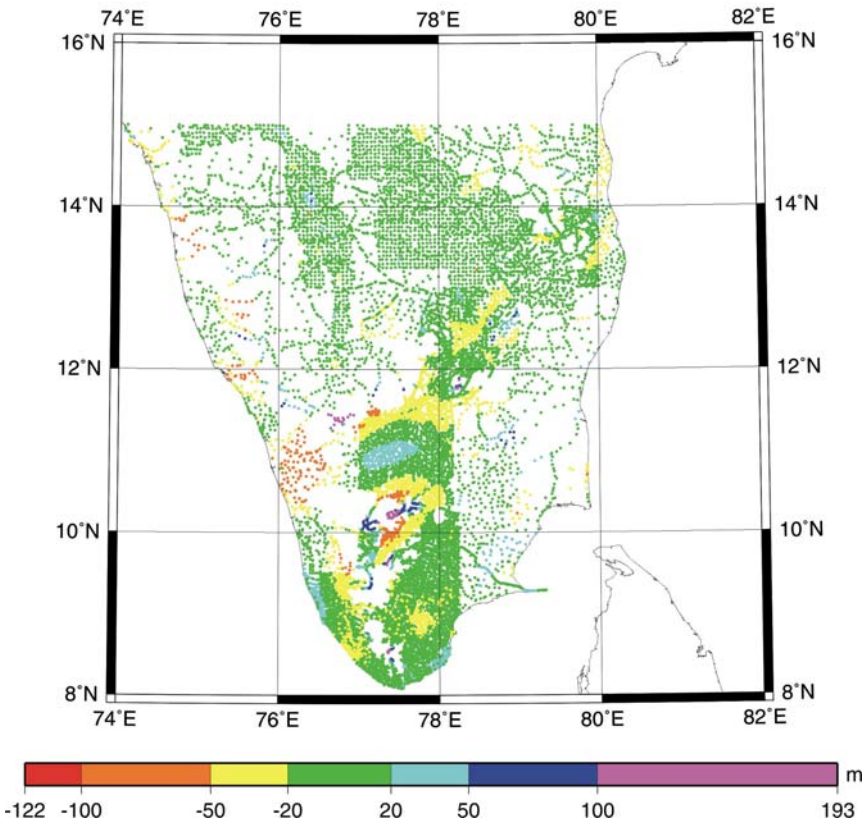


Figure 3. The gravity residuals after EIGEN_GL04C reduction

As one can see in Figure 3, the residuals show a higher degree of variability. Equivalent results have been obtained using all the other tested models.
EGM2008 model includes a higher frequency signal as compared to the other models, so a smother residuals gravity field is obtained. This also reflects in the statistics of Table 1.
Assuming that this high frequency signal is basically due to the terrain effect, one should obtain results which are similar to EGM2008 by removing also the RTC effect while using the "lower" frequency geopotential models.
A further computation has been then carried out to get the residuals after RTC reduction. This has been done for the EIGEN_GL04C model only, the one giving the better results (if EGM2008 is not considered).
RTC effect has been computed using the TC software of the GRAVSOFT package (Tscherning et al., 1994). The DTM used in the computation has been obtained by merging SRTM3 and the 1'×1' NOAA bathymetry (see next paragraph).
The residual statistics are listed in Table 2.

Gravity	E (mgal)	σ(mgal)	Min (mgal)	Max(mgal)
$\Delta g_{fa}-\Delta g_{gl04C}-\Delta g_{RTC}$	-1.10	15.69	-85.26	83.01

Table 2 – Residuals statistics after EIGEN_GLC04 model and RTC reduction.

Even though there is an improvement in the residual statistics, we have better results using EGM2008 only.

Thus, the full remove step based on EIGEN_GL04C gives a result which is poorer than the one obtained using the EGM2008 model only.

3. The comparison between EGM2008 and the South India geoid estimate.

The height anomaly implied by the EGM2008 model has been compared with a recent geoid estimate available over the Southern region of India.
The South India Geoid estimate is an updated computation in this area. It is based on a gravity data base which is assembled using the 16013 land gravity values described in the previous paragraph and gravity at sea derived from altimetry (Andersen et al, 2005). The global gravity data base consists of 63968 values.
The estimation procedure is the "remove-restore" technique (Tscherning, 1994) and the residual geoid component has been computed using the Fast Collocation approach (Bottoni and Barzaghi, 1993).
The reference global geopotential model adopted in the computation is the EIGEN_GL04C model, complete up to degree and order 360.
The RTC effect has been estimated using a DTM which has been obtained by merging SRTM3 (Farr et al., 2007) and a 1'×1' NOAA bathymetry (https://128.160.23.42/dbdbv/dbvquery.html). In such a way, over the area $4° < \varphi < 16°$, $72° < \lambda < 83°$ a 100m× 100m DTM was derived, plotted in Figure 4.

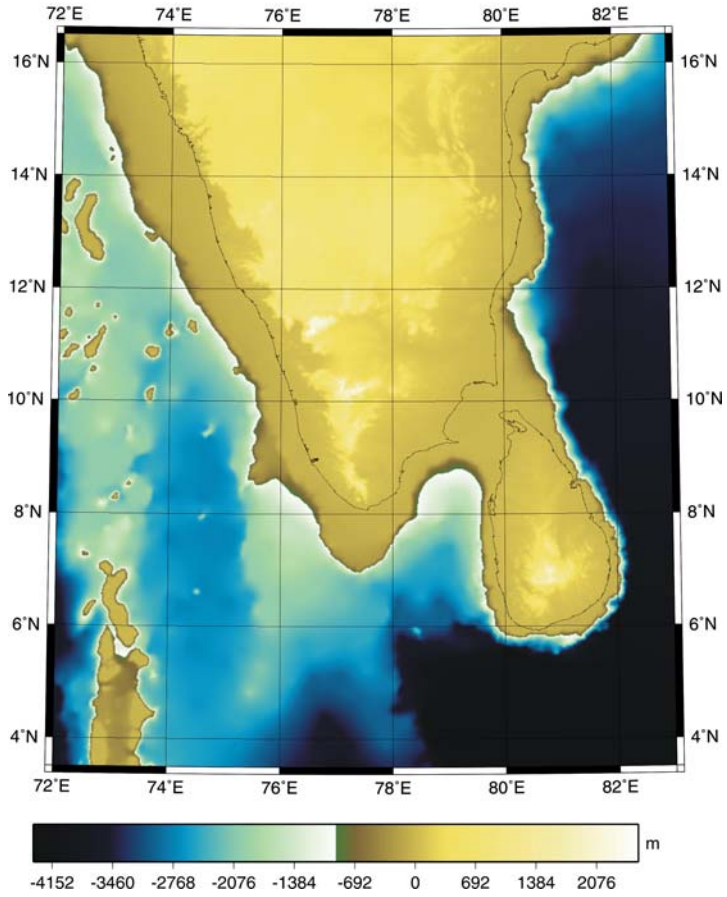


Figure 4 – The South India DTM

RTC effect has been computed using the TC software of the GRAVSOFT package (Tscherning et al., 1994).
Residual gravity data were then gridded over a regular 2' × 2' geographical grid covering the area $6° < \varphi < 14°$, $74.5° < \lambda < 80.5°$.
Statistics of the remove step are presented in Table 3.

	Δg_0 [mGal]	$\Delta g_0 - \Delta g_M$ [mGal]	Δg_r [mGal]	Δg_r^G [mGal]
#	63968	63968	63968	43621
E	-39.40	-0.89	0.42	-0.55
σ	31.92	17.59	14.96	14.80
Min	-173.79	-92.66	-132.96	-84.14
Max	192.77	183.14	83.01	72.75

Table 3- Statistics of the remove step
($\Delta g_r = \Delta g_0 - \Delta g_M - \Delta g_{RTC}$; Δg_r^G=gridded residual gravity)

The empirical covariance of the Δg_r^G values is plotted in Figure 5 together with the best fit model function which has been adopted for computing the residual geoid undulation by fast collocation.

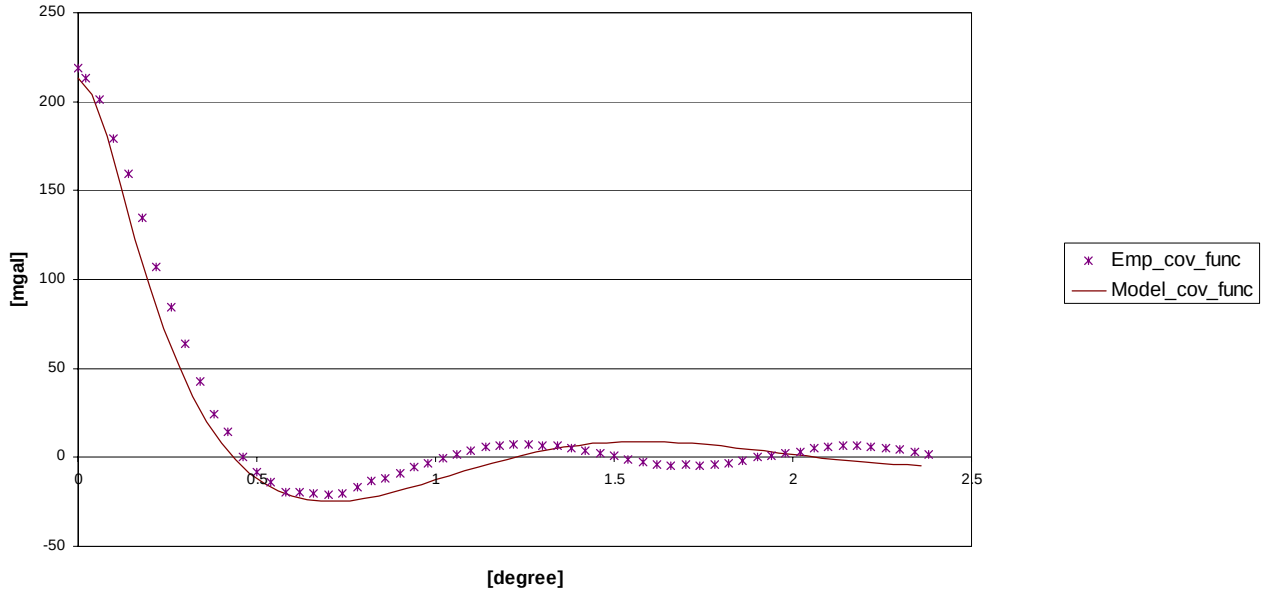


Figure 5 – The empirical covariance of Δg_r^G and the best fit model

The final geoid estimate, given on the same regular grid used for gravity, is shown in Figure 6.

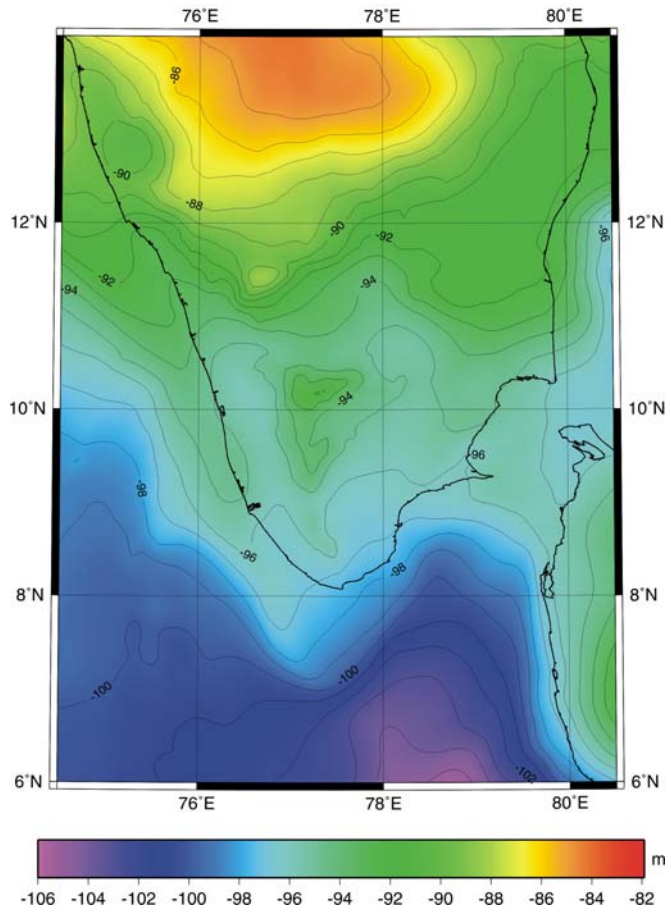


Figure 6 – The South India Geoid (equidistance =1m)

The South India Geoid undulation deepens from north to south**;** this behaviour should be related to the geophysical structures of the area. It is still possible to single out on the Geoid the configuration of the main relieves in the area 10°≤ φ ≤12°, 76°≤ λ ≤78° and of the deepest waters in its southern part.

This geoid estimate has been compared to EGM2008 height anomaly which has been computed on the same grid.

The statistics of the differences, after datum shift, are described in Table 4. The equation used to account for datum shift is the one described in Heiskanen and Moritz (1990):

$$N_{tot} = N_{EGM\,2008} + \Delta N(\theta,\lambda) = \\ = N_{EGM\,2008} + dx\sin\theta\cos\lambda + dy\sin\theta\sin\lambda + dz\cos\theta$$

$$\theta = 90° - \varphi \qquad (dx, dy, dz) = \textit{datum shift parameters}$$

	E (m)	σ(m)	Min (m)	Max(m)
N_{tot}-$N_{EGM2008}$-ΔN	0.00	0.22	-1.51	1.08

Table 2 – Statistics of the residuals in geoid undulation after datum shift.

The plot of these residuals are given in Figure 7. The area with the higher discrepancies is the one included in $10° < \varphi < 12°$, $76° < \lambda < 78°$. This is a region with high mountains (see Figure 4) and it is reasonable to assume that EGM2008 is not able to model completely the terrain signal, as it is the case for the estimated local geoid.

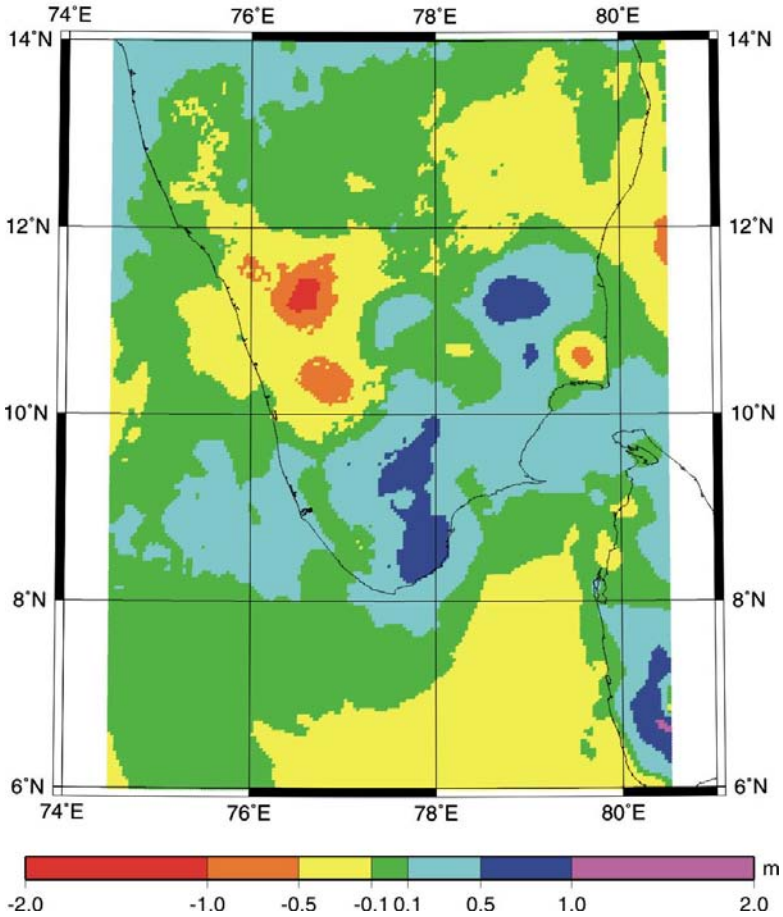


Figure 7 – Undulation differences between geoid estimate and EGM2008 after datum shift.

Another area where one can see high frequency differences is in $10° < \varphi < 12°$, $78° < \lambda < 80°$. In this area, the correlation of discrepancies with terrain signal is less clear and only the maximum in the North-Western part could be related to that. Also, medium frequency differences are present in the whole area. However, this is only a relative comparison between the two geoid estimates. Only a check using GPS/levelling can prove which is the best estimate. Unfortunately, at the moment, no data of this kind are available for this area.

4. Conclusions

In the Southern part of India, EGM2008 and other three global geopotential models, EGM96, GPM98CR and EIGEN_GL04C, have been used to reduce a set of 16013 gravity data. The statistics of the gravity residuals prove the high accuracy of EGM2008 which is the best fit model. The other tested models, are practically equivalent. Thus, it seems that the high frequencies in GMP98CR (complete to degree and order 720) don't give any substantial contribution over this area. Also, EGM2008 gives better results even if RTC is applied in combination with EIGEN_GL04C model. A further comparison has been carried out using a geoid estimate available in this area. Most of the high frequency differences

can be explained in terms of terrain effect. Furthermore, a medium frequency residual field, having a variation around 1 m, is clearly visible. Comparison with GPS/leveling data, not available at the moment, could give a deeper insight on these discrepancies.

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Assessment of EGM2008 over Sri Lanka, an area where 'fill-in' data were used in EGM2008

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Abstract. The tide-free EGM2008 combined global geopotential model is compared with land and marine gravity observations and co-located GPS-levelling on and around Sri Lanka (formerly Ceylon). Not all these data are in the public domain, so offer an informative test of how the 'fill-in' methodology used in EGM2008 performs versus observed data. Sri Lanka is also in an area where the geoid exhibits its lowest elevation with respect to a geocentric reference ellipsoid. A -1.75 m bias between the GPS-levelling and EGM2008 led to an investigation into the Sri Lankan geodetic datums, showing a bias in the ellipsoidal heights. After rejection of 15 outliers, the standard deviation of the difference between 207 Sri Lankan GPS-levelling points and EGM2008 is ±0.184 m. The difference between the gravity anomalies and EGM2008 showed that the Sri Lankan gravity data is based on the old Potsdam datum. The Sri Lankan land gravity data, after rejection of outliers, yielded standard deviations of ±6.743 mGal for 20 GPS-coordinated gravity points on fundamental benchmarks, ±14.704 mGal for 42 gravity points on fundamental benchmarks but with coarse locations, and ±6.367 mGal for 1032 digitised and reconstructed free-air anomalies from a Bouguer anomaly map. The ship-track gravity data have not been crossover adjusted, and yield a standard deviation of ±43.683 mGal. Importantly, the ability of EGM2008 to identify datum deficiencies is an implicit validation and leads to its application in other areas to search for datum deficiencies.

1. INTRODUCTION

From Pavlis et al. (2008), Sri Lanka (called Ceylon before 1972) is one of the regions where 'fill-in' 5-arc-minute mean terrestrial gravity anomalies were used to generate EGM2008. To our understanding, the fill-in procedure takes Bouguer gravity anomalies from commercially sensitive or confidential data sources, then 'reconstructs' free-air anomalies using the elevations from the DEM2006.0 digital elevation model (Pavlis et al. 2006), which is derived principally from the Shuttle Radar Topography Mission (SRTM; Werner 2001). This reconstruction procedure is described in Lemoine et al. (1998); also see Featherstone and Kirby (2000).

In this assessment of EGM2008, we use previously unavailable GPS-levelling data across Sri Lanka, gravity observations at fundamental benchmarks (FBMs) of the Sri Lankan geodetic levelling network (Udayakantha and Tennakone 1993), ship-track gravity anomalies offshore (NGDC 1999), and gravity anomalies digitised from a Bouguer gravity anomaly map of Sri Lanka (Hatherton and Ranasinghe 1972). The GPS-levelling comparisons show a -1.75 m bias (standard deviation of ±0.18 m), which is attributed mostly to a problem with the origin of the ellipsoidal heights. The gravity comparisons show that the Sri Lankan data are referred to the old Potsdam gravity datum, but the gravity anomalies used in EGM2008 appear to use the International Gravity Standardisation Network 1971 (IGSN71).

2. DESCRIPTION OF THE SRI LANKAN DATA

2.1 Land and ship-track gravity data

2.1.1 International gravity links to Sri Lanka

The first recorded gravity observations in Sri Lanka were made by Glennie (1935), of the Survey of India, which involved a set of 21 pendulum gravity observations. These observations resulted in two map compilations showing Hayford anomalies and 'crustal warp' anomalies (Hatherton and Ranasinghe 1972). A definition of crustal warp anomalies could not be found, but it is likely they are isostatic gravity anomalies. However, the original data are not now available for further analysis.

There were several later surveys that made gravity observations to connect Sri Lanka to international gravity networks. Relative observations were made at two airports: Ratmalana (Colombo) and Katunayake. Woollard and Rose (1963) made observations at Ratmalana airport, giving a value of 978132.3 mGal. Gravity observations at both airports were made again in 1969 when the British Institute of Geologi-

cal Sciences used a LaCoste & Romberg (L&R) gravimeter (serial number G97) to give gravity differences among New Delhi, Yangon (Rangoon), Singapore and Colombo.

The gravity difference between Singapore University (Geography Department) and Katunayake airport was 40.78 mGal and between Singapore and Ratmalana airport was 50.14 mGal. The absolute values were assigned 978122.24 mGal at Katunayake and 978131.6 mGal at Ratmalana based on the Singapore value of 978081.5 mGal (Hatherton at al. 1975), which was on the Potsdam gravity datum (J Mäkinen 2008, pers. comm.). These two Potsdam-related gravity values were referred as Evans's values in subsequent Sri Lankan gravity surveys (e.g., Hatherton and Ranasinghe 1972; Udayakantha and Tennakone 1993); described in Section 2.1.2. As such, Sri Lankan gravity data are offset from IGSN71 (Morelli et al. 1974) by around 14 mGal, which will be shown later in Sections 3.2.3 and 3.2.4.

Another control gravity survey was carried out in 1973 by Evans (Hatherton at al. 1975, Appendix) using the same G97 L&R meter at the base stations occupied by Hatherton and Ranasinghe (1972) (Section 2.1.2) and connected to Evans's 1969 value (978131.6 mGal) at Ratmalana airport. The IGSN71 value at Ratmalana was later calculated by using the IGSN71-derived calibration factor for the L&R G97 difference measured between the same Singapore and Ratmalana points in 1969. The IGSN71 value at Ratmalana is 978116.81 mGal based on the IGSN71 value (978066.68 mGal) at Singapore University (Hatherton et al. 1975).

Ratmalana airport has also been tied to IGSN71 by NAVOCEANO (formerly the U.S. Naval Oceanographic Office), giving the value as 978116.900 mGal. While this point was also located at Ratmalana, it is not at the same ground mark observed by Evans. As such, the datum of the Sri Lankan gravity anomalies described next is based on the old Potsdam gravity datum, even though IGSN71 values are available. This will lead to an expected bias of around 14 mGal between Sri Lankan gravity anomalies and any gravity anomalies referred to IGSN71 (Section 3.2).

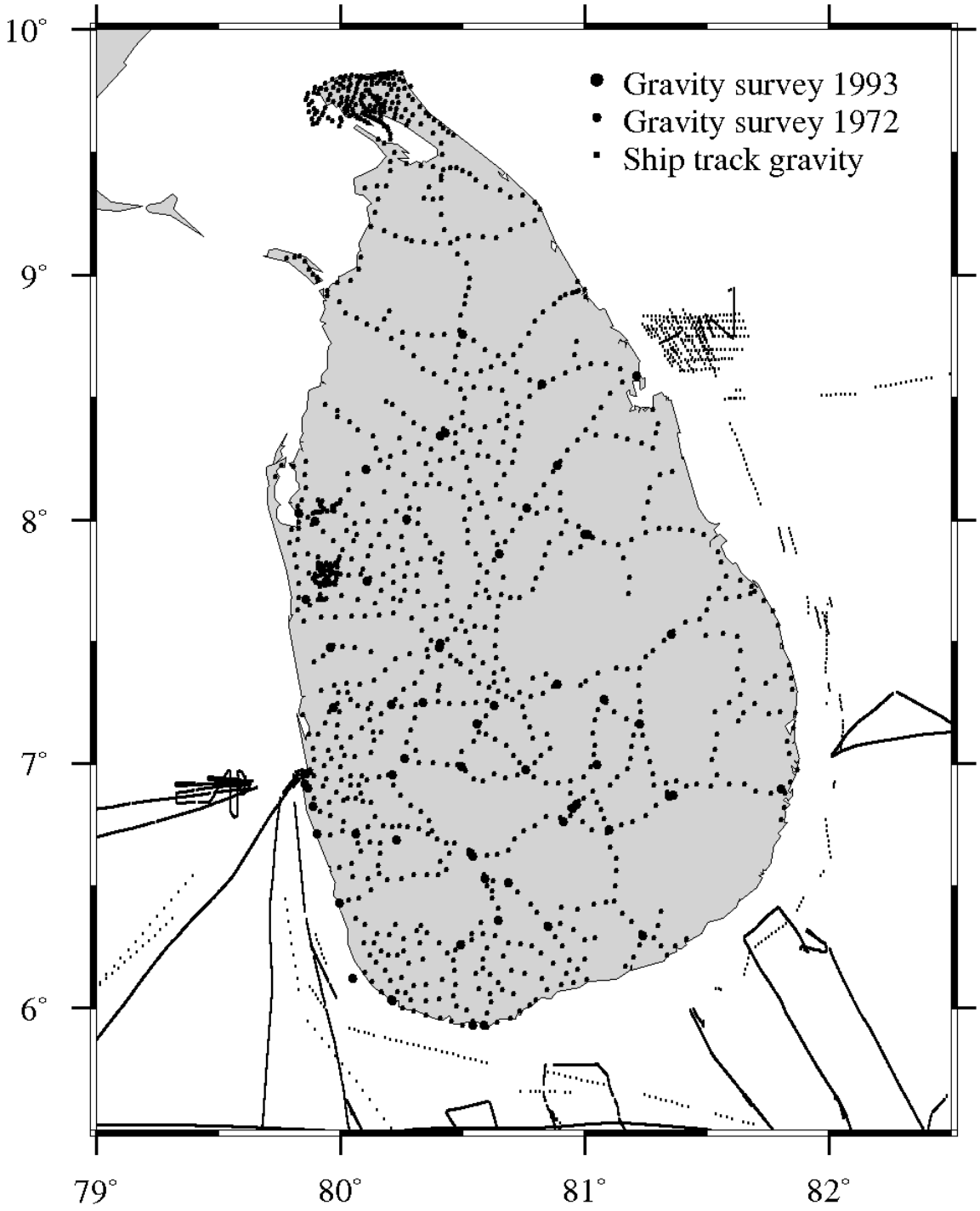


Fig 1 *Locations of the Sri Lankan gravity data (Mercator projection): the digitised 1972 land gravity survey of Hatherton and Ranasinghe (1972) (1,070 points), the 1993 land gravity survey of Udayakantha and Tennakone (1993) (52 points), and ship-track gravity from NGDC (1999) (12,192 points)*

2.1.2 National terrestrial gravity surveys

a) 1971 survey of 1170 points

A significant gravity survey was conducted over the whole of Sri Lanka in 1971, and led to two maps showing Bouguer anomalies (Hatherton and Ranasinghe 1972, Hatherton et al. 1975) and isostatic anomalies (Hatherton and Hutchings 1972). According to Hatherton and Ranasinghe (1972), the survey was to produce a gravity anomaly map of the whole country, together with detailed surveys of several areas to use the gravity method for studying the geology of the country.

The observations were made using a Worden gravimeter (serial number W283), which was a high-drift meter (~0.35 mGal/day). The survey was carried out by establishing 19 base stations relative to Evans's 1969 Potsdam-related gravity value (978131.6 mGal) at Ratmalana airport (Hatherton and Ranasinghe 1972). Unfortunately, the original data are no longer available. However, Hatherton and Ranasinghe (1972) provide a contour map of complete Bouguer anomalies with the locations of the gravity observations, which we have manually digitised (Section 3.2.2).

A total of 1170 gravity observations were performed during this 1971 survey relative to these 19 base stations, with 87% of observations in areas where the altitude is less than 500 ft (~150m). Very few observations were made at higher altitudes in the central southern mountainous region (Fig. 2) due to instrumental limitations (the travel time in mountains versus the need to regularly occupy a base station to correct for the gravimeter's drift, and avoiding calibrating the extended range dial in the gravimeter). Also, this 1971 survey did not establish base stations on permanent monuments, and the method of adjustment of the network was not made clear. The 19 base stations were reoccupied and remeasured in 1973 with the L&R G97 meter, and the differences were found to agree within 0.25 mGal, except for two stations where the difference was in excess of 1 mGal.

The horizontal positions and the heights of these 1971 gravity observations were taken from the Sri Lankan one-inch topographic map series (1:63,360). The observations were made approximately every four miles (~6.5km) along roads (Hatherton and Ranasinghe 1972, Hatherton and Hutchings 1972); see Fig. 1.

The complete Bouguer anomalies in Hatherton and Ranasinghe's (1972) contour map were based on the International Gravity Formula 1930 with a standard rock density of 2670 kg/m^3. Terrain corrections were computed out to Hammer's (1939) zone M, and were found to be small, except in the highlands. The largest terrain cor-

rection was 11.9 mGal at the southern scarp of the highlands (cf. Fig. 2). The complete Bouguer gravity anomalies were in the range of -75 mGal to +45 mGal (Hatherton and Ranasinghe 1972).

Since the accuracy of the locations and heights are rather coarse and the datums implicit to the computation of gravity anomalies are somewhat ambiguous, this survey might not reflect the actual gravity field over Sri Lanka and thus not be such a strong validation of EGM2008. Digitising these data from a contour map also adds further uncertainty to the veracity of this dataset to validate EGM2008. The digitisation process is described in Section 3.2.2.

b) 1993 survey of 52 points

Udayakantha and Tennakone (1993) established a new systematic gravity network by making observations with a newer L&R gravimeter (serial number D186) at 52 permanent monuments, which included 49 points on FBMs of the Sri Lankan geodetic levelling network (Price 1932). [The geodetic datums in Sri Lanka will be described in Section 2.2]. This 1993 gravity survey was also based on Evans's 1969 Potsdam-related gravity value (978131.6 mGal) at Ratmalana airport, and the network was least-squares adjusted (Udayakantha and Tennakone 1993).

Nine identifiable base stations from Hatherton and Ranasinghe (1972) were also remeasured and compared. The surveys agreed to within 0.32 mGal, except one station with a difference of ~1 mGal. However, these nine stations were not adjusted in the 1993 survey because they were not on permanent monuments (Udayakantha and Tennakone 1993). As well as the adjusted 52 points, an additional 11 unadjusted points were available from this gravity survey, but are not used in this assessment of EGM2008 because they are unadjusted.

The horizontal positions of the FBMs are not very accurately known as they were compiled from topographic maps available at that time, so are probably accurate to around 0.5-1.0 km. However, 20 of the FBMs have now been incorporated in the national GPS-based geodetic network (Section 2.2.1) and hence their locations are known to a few centimetres (Geodetic Survey Unit 2000), but there remains some uncertainty around their ellipsoidal heights, which will be discussed in Section 2.2.1.

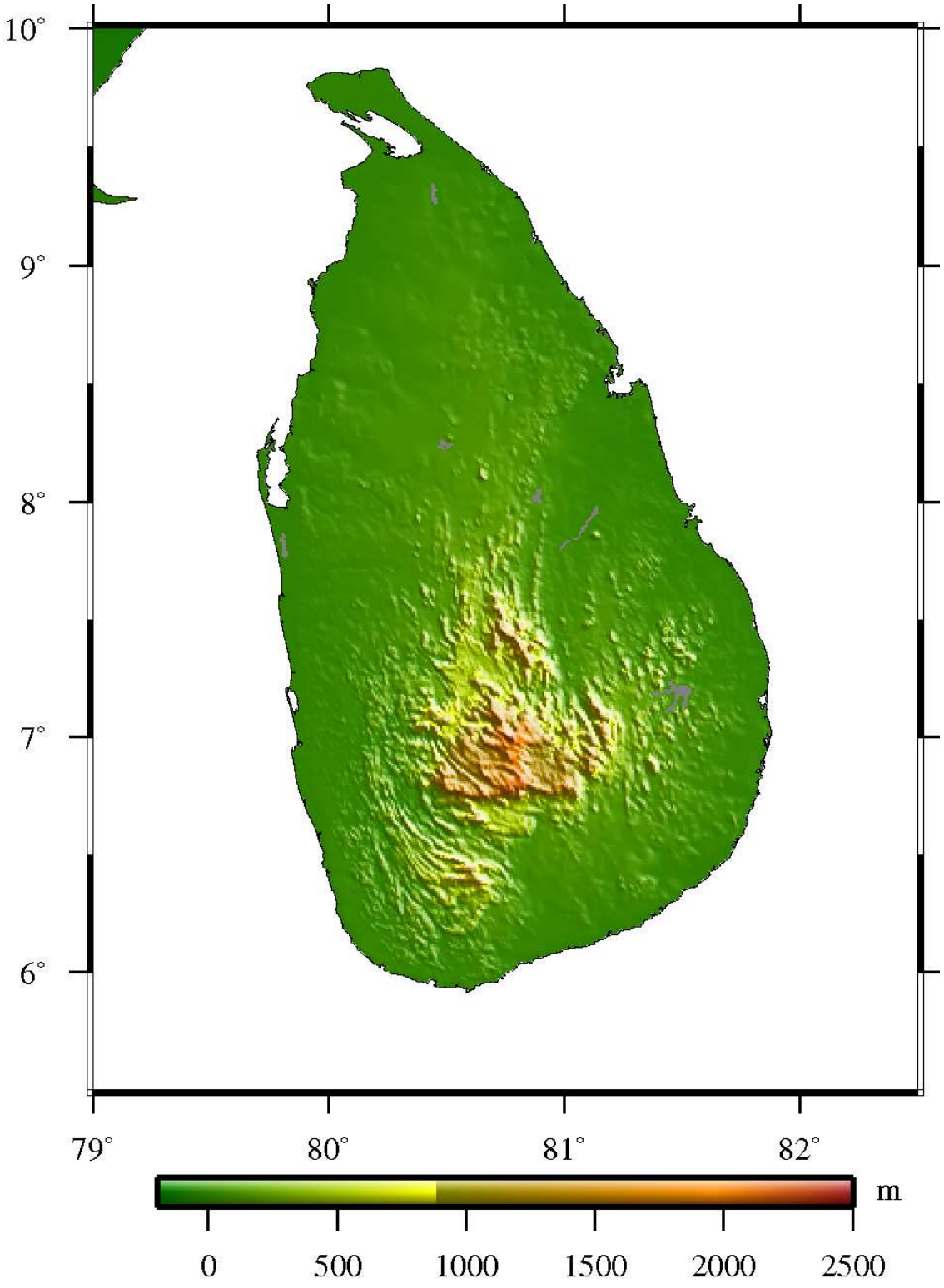


Fig 2 *Sri Lankan topography from a 1 km resolution DEM based on digitised contours from the Survey Department of Sri Lanka (Mercator projection). The maximum elevation is 2,524 m in the central southern mountains.*

2.1.3 Ship-track gravity

As well as the land gravity data, some ship track gravity data are also available around Sri Lanka (NGDC 1999; Fig. 1). The NGDC database contains 12,192 sea surface gravity observations between 79.0°E to 83.0°E and 5.0°N to 10.0°N. According to the metadata, only 643 measurements (one cruise) were crossover adjusted for gravimeter drift, and no evidence could be found for tide corrections for the whole set of data. As such, these data will not be able to provide a useful validation of EGM2008 around Sri Lanka.

2.2 The geodetic datums of Sri Lanka

2.2.1 Horizontal datums

The principal triangulation network of Sri Lanka began in 1857 and was completed by 1885 for the Topographical Survey of Ceylon, which was on the scale of half mile to an inch (Geodetic Survey Unit 2000). The most significant revision to the network is documented in Jackson (1933). It involved re-measuring the distance and azimuth of each baseline at Negombo (Kandawala to Halgastota) and Batticaloa (Tavelamunai to Vaunativu) with invar tapes and a Gautier 5" micrometer theodolote.

The two baselines were about ~5.5 miles (~8.8 km) long and separated by 127 miles (~205km). The triangulated value of the Negombo base computed from the Batticaloa base was found to be 1:116,000 (Jackson 1933). Astronomical coordinates of Kandawala and the astronomical azimuth of the Negombo baseline were kept fixed to form the astronomical datum of Sri Lanka. This Sri Lankan horizontal datum is referred to as Kandawala in most of the literature (e.g., Jackson 1933; Geodetic Survey Unit 2000; NIMA 2000).

The Sri Lankan horizontal geodetic control network has continuously been upgraded by using newer technology and computational methodologies. A major breakthrough took place in 1993 for densification of the network and upgrading its accuracy by using GPS (Geodetic Survey Unit 2000). The new control network was used to form the Sri Lanka Datum 1999 (SLD99), and consists of one base station (ISMD, Institute of Surveying and Mapping Diyatalawa), 10 secondary base stations, 48 existing Kandawala triangulation stations, 20 FBMs (cf. Price 1932) and 194 new control stations. SLD is not a geocentric datum.

The GPS baselines were observed with Leica 300 receivers and processed with SKI v1.2. The network was least-squares adjusted with GPSENV v3.32 in the

Geolab v2.6a software under a minimal constraint by fixing the 3D coordinates of ISMD (see the discussion below). The average ppm precision of the 1,265 baselines in the network was 0.127229, and the computed distance accuracy was 1:7,900,000. The highest (A) and the lowest (2-I) rank of FGCC (United States' Federal Geodetic Control Committee) order (Bossler 1984) of the baselines were found to be 5 each, while 496 and 759 baselines were ranked as B and 1 respectively (Geodetic Survey Unit 2000).

The old triangulation stations were used to determine datum transformation parameters from WGS84 to Kandawala and from WGS84 to SLD99. The transformation parameters were determined based on the seven-parameter Bursa-Wolf model for each datum (Geodetic Survey Unit 2000). SLD99 was made available for use in Sri Lanka since 2000, and the national grid coordinates (SL_GRID_99) are computed using the transverse Mercator projection on the Everest 1830 (1937 Adjustment) ellipsoid, which was also used for the Kandawala datum (e.g., NIMA 2000). Both horizontal geodetic datums are still being used separately in Sri Lanka.

Prompted by a significant bias of -1.75 m in the GPS-levelling comparisons (Section 3.1.1), we scrutinised the GPS ellipsoidal height and spirit-levelling datums. We were satisfied that the MSL-based spirit-levelling datum (Section 2.2.2) could not account for this bias, so focussed on the GPS ellipsoidal height datum. According to Geodetic Survey Unit (2000), the starting point for the GPS surveys was the DORIS (Doppler Orbit Determination and Radiopositioning Integrated on Satellite) station at Colombo (COLA; DOMES ID 23501S001), which was active from 1991 to 2004.

Since the ground mark beneath the ~3-m-high DORIS beacon could not be occupied by GPS, Geodetic Survey Unit (2000) occupied another GPS base station nearby. The coordinates in Geodetic Survey Unit (2000) are not the same as the GPS marker (DOMES ID: 23501M001) listed at the IDS (International DORIS Service) website (http://ids.cls.fr/), though we have been advised that that same GPS marker was occupied. The coordinates of this GPS base station at the Survey General's Office (SGO), Colombo, are latitude: 6°N 53' 30.8699", longitude 79°E 52' 26.3102", ellipsoidal height: -76.238 m.

The site-log for COLA at the IDS website gives the tie vector between the DORIS and their GPS marker, which gives the ITRF2000 coordinate as latitude 6°N 53' 30.8611", longitude 79°E 52' 26.3146", ellipsoidal height -75.692 m. The DOMES 23501M001 GPS mark was local-tied to COLA during the Epoch'92 GPS

campaign by IGN (Institut Géographique National) using conventional geodetic techniques (total station), and no GPS observations were made on the DORIS ground mark (H Fagard 2008, pers. comm.). Apparently, the GPS points are physically the same, but different coordinates seem to have been inaccurately incorporated into the GPS survey. This introduces a bias in the ellipsoidal heights of 0.456 m.

The uncertainty of the origin of the GPS survey is compounded further by the fact that the origin point was chosen to be ISMD, which is approximately 120 km east of COLA. From Geodetic Survey Unit (2000), two GPS baselines (i.e., radiations) were used to establish this site from SGO over an ellipsoidal height difference of ~1.2 km. The differential tropospheric delays in this near-equatorial region are probably poorly modelled by commercial software in single baseline mode, resulting in a height error (e.g., Rothacher 2002). The coordinates of the ISMD origin are: latitude: 6°N 49' 02.68716", longitude: 80°E 57' 40.88000", ellipsoidal height: 1164.366 m.

Geodetic Survey Unit (2000) also occupied ISMD for seven days and determined a sequential code GPS position solution using Leica's SKI v2.1 software (note that this site was occupied before GPS selective availability was discontinued). This sequential code GPS position was 3.555 m higher than that from the ~120 km baseline from SGO, though the latitude and longitude were comparatively closer to each other. This indicates (neglecting the effect of selective availability) that the differential tropospheric bias over this ~1.2 km height difference is an additional problem that will cause the GPS ellipsoidal heights to be biased.

While the realisation of ellipsoidal heights in this way is not of concern to geodetic surveyors in Sri Lanka (because relative GPS surveys will be insensitive to any bias), it will cause a problem for an absolute GPS-levelling evaluation (cf. Featherstone 2001) of EGM2008. At this stage, and before further investigations can be completed, we believe the combination of the ambiguous connection to the COLA DORIS site and the long GPS baseline observed over a substantial height difference to ISMD could account for much of the -1.75 m mean difference in the GPS-levelling comparisons (Section 3.1). In addition, COLA was identified as one of 17 stations of having poor antenna stability at the end of 1999 by the IDS (Fagard 2006) and was not recommended for DORIS core network for ITRF2005 (Willis et al. 2005).

Importantly, and as a validation of EGM2008, this uncertainty in the ellipsoidal height datum would not have been investigated so thoroughly if we had not found a large bias with a small standard deviation between the GPS-levelling and

EGM2008. As such, it could be argued that EGM2008 is already contributing to vertical datum unification.

2.2.2 Vertical datum

The vertical geodetic control network of Sri Lanka (originally termed as the geodetic levelling of Ceylon) was established between 1926 and 1930 using parallel glass plate micrometer attachments mounted on precise levels with staves graduated to fiftieths of a foot on a strip of invar fixed at one end (Price 1932).

For the datum's origin, tidal observations of mean sea level (MSL) were carried out between 1923 and 1933 at two harbours, Colombo and Trincomalee, using self-recording tide gauges. Beforehand, MSL was determined at three harbours by the Great Trigonometrical Survey of India using self-recording tide gauges over the following periods: 1884-1889 at Colombo and Galle; and 1889-1896 at Tricomalee (Jackson 1936).

Initially, the levelling network was supposed to be fixed at the Colombo and Trincomalee tide gauges, with redetermination of the MSL by the newer observations. However, since the tidal observations were underway at Colombo and Trincomalee when the levelling network was adjusted in 1932, the MSL at Colombo as determined by the Great Trigonometric Survey of India during 1884-1889 was used instead. However, after the local tidal observations were completed at Colombo and Tricomalee during 1929-1933, a small rise of MSL of 0.074 ft (~0.023 m) at Colombo and 0.199 ft (~0.061 m) at Tricomalee relative to the 1884-1889 and 1889-1896 values was seen (Price 1932, Jackson 1936).

According to Price (1932), the main part of the Sri Lankan levelling network comprises 27 circuits of ~2,400 miles (~4,300 km) of levelling in total. Originally, there were 53 FBMs, though some have been destroyed or disturbed since then. The probable error of the levelling is ±0.42 mm/√km and the accuracy of determination is ±0.0005 ft (~±0.02 mm). However, these values are for spirit-levelling conducted over 70 years ago, so are probably too overoptimistic.

Normal-orthometric corrections (quoted incorrectly as orthometric corrections in Price 1932) were applied to observed height differences to account for the non-parallelism of level surfaces using the formula $-0.005302(\Delta\phi)H\sin 2\phi_M$ ft (Bomford

1971, chapter 3), where H is the mean height of the section of levelling; ϕ_M is the mean latitude and $\Delta\phi$ is the difference in latitude of the terminal points of the section.

The network was adjusted using least squares, holding the MSL at Colombo fixed to zero. Dynamic heights were also calculated (e.g. Bomford, 1971, chapter 3) for the adjusted benchmarks by considering 7°N as the standard latitude along with normal gravity at the mean latitude of the terminals of the levelling section. The whole network was based on precise geodetic levelling observations, and no evidence was found that observed gravity data were used in the computations.

From Price and Grice (1932), the height system of Sri Lanka is a largely geometrical system, which does not reflect the actual Sri Lankan gravity field, as would be more of the case for an orthometric-type height system. Since the corrections were computed with a normal gravity field, it is more appropriate to say that the Sri Lankan heights are based on normal orthometric height system (cf. Featherstone and Kuhn 2006).

However, Sri Lanka has claimed to use an orthometric height system since establishing the geodetic levelling network of the country til present by almost every user of heights (e.g., Price 1932, Price and Grice (1932), Geodetic Survey Unit 2000) and the Sri Lankan State authorities. This basic misunderstanding surrounding height systems leads to confusion (cf. Bomford, 1971, Chapters 3 & 7; Featherstone and Kuhn 2006; Heiskanen and Moritz 1967, Chapters 4 & 8; Jekeli 2000).

In actuality, the Sri Lankan vertical datum is based on five years of MSL observations at Colombo and an approximation of the normal orthometric height system.

2.2.3 GPS-levelling data

There are 222 GPS-levelling points in Sri Lanka, comprising 20 FBMs and 202 new points, which have been used for this assessment of EGM2008 (Fig. 3). The [normal-orthometric] heights of the FBMs relative to MSL at Colombo were taken from Price and Grice (1932) and the heights of new points were obtained by spirit levelling (Geodetic Survey Unit 2000). No control points have been used for this study whose heights were derived by trigonometric levelling techniques.

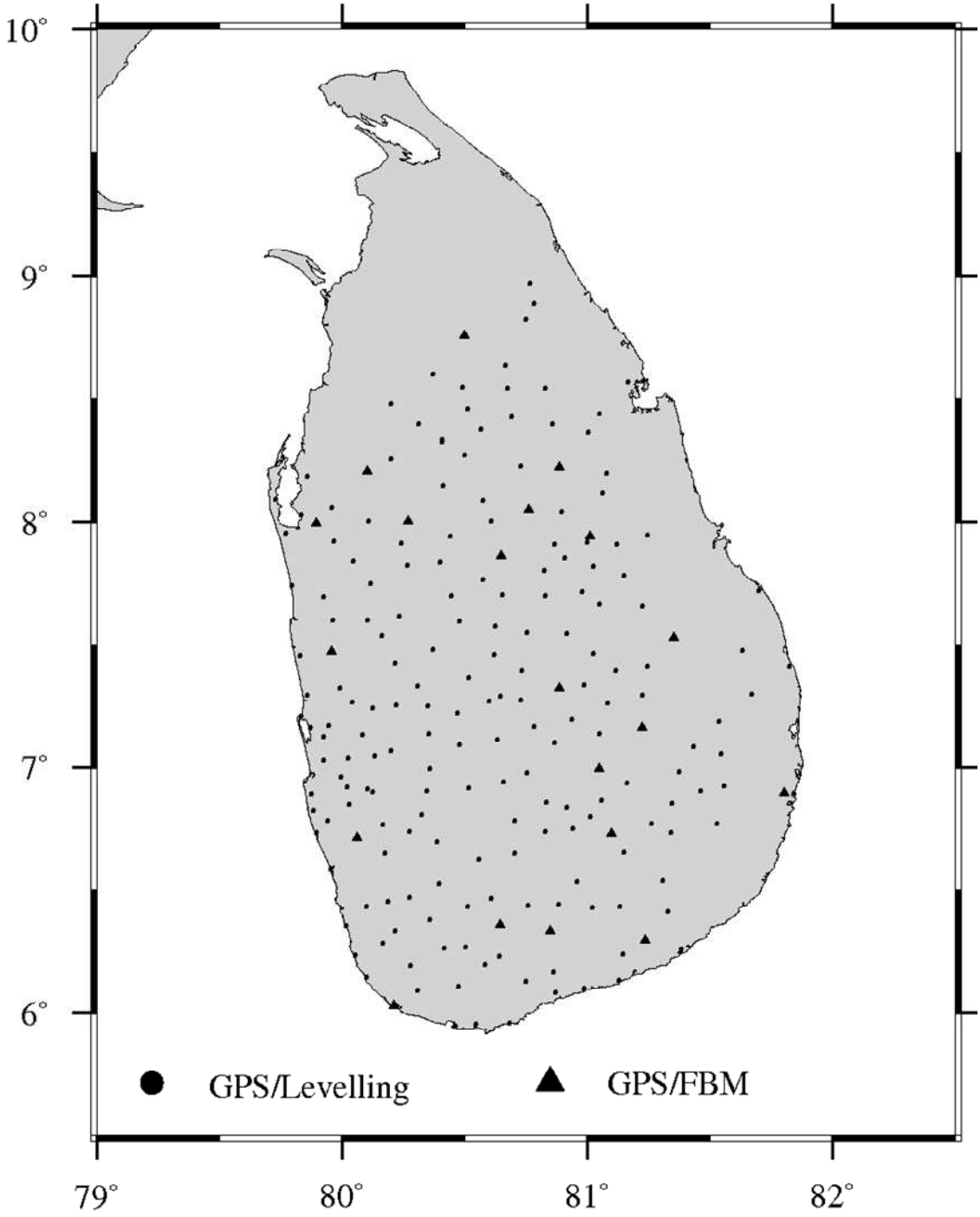


Fig 3 *Coverage of the 207 GPS-levelling points in Sri Lanka (Mercator projection). Spurious heights were removed as blunders after initial investigation (Section 3.1)*

However, the accuracy of the levelled heights at the new control points is uncertain, except for the 20 FBMs (Section 2.1.2). According to the levelling standards of Sri Lanka (Goonewardena 1970), the detailed levelling has the lowest accuracy with a 24 mm/√km allowable misclose, and the misclosure factors for minor levelling and third-order levelling are 14 mm/√km and 8 mm/√km, respectively.

Due to uncertainty of the heights at these new control points, eight were identified and rejected after initial investigation as blunders. The procedure for selecting the control points used in the assessment is described in Section 3.1.1. Figure 3 shows the 207 points used for the assessment after the outliers were removed.

3 RESULTS AND DISCUSSION

All gravity-field-related quantities computed from EGM2008 used the `HARMONIC_SYNTH.f` FORTRAN-77 software provided by the EGM development team. GRS80 parameters were set in the 'parameter input' so that the zero-degree term and scaling of the even-degree coefficients were computed according to the algorithm in Lemoine et al. (1998).

3.1 Comparisons with GPS-levelling data

The Sri Lankan GPS-levelling data have not been compared with previously available geopotential models, which are considerably lower degree and order than EGM2008. We have not traced any publication on testing geopotential models for the study area.

GPS-levelling data can be used to assess geoid and quasigeoid models in absolute or relative modes (Featherstone 2001). The absolute tests can tell something about biases in EGM2008 or the local vertical datum, though these are inseparable (cf. Featherstone 2004). The relative tests can tell something about slopes in EGM2008 or the local vertical datum, but again being inseparable.

3.1.1 Absolute GPS-levelling tests

Height anomalies at the GPS-levelled points were compared with synthesised values from EGM2008 by using `HARMONIC_SYNTH.f` in scattered point computation mode. The differences are summarised statistically in Table 1 after the outliers ($\pm 3\sigma$) were removed. The analysis classified the available GPS-levelling data by the perceived level of accuracy of the normal orthometric heights on the Sri Lankan vertical datum. Since the height accuracy is not exactly known at points other than the FBMs, these

two categories are assessed separately (Table 1). For comparison, the statistics were also computed for EGM96 (Table 2).

Data type	Min (m)	Max (m)	Mean (m)	STD (m)
20 (FBMs)	-1.885	-1.528	-1.727	±0.098
187 (levelled)	-2.465	-0.991	-1.763	±0.190
207 (All)	-2.465	-0.991	-1.760	±0.184

Table 1 *Statistics of the differences of height anomalies between GPS-levelling and EGM2008. There are 20 FBMs where the height is known accurately and 187 points were spirit levelled but with unknown height accuracy.*

Data type	Min (m)	Max (m)	Mean (m)	STD (m)
20 (FBMs)	-2.571	-0.751	-1.902	±0.398
187 (levelled)	-2.929	-0.724	-1.888	±0.423
207 (All)	-2.929	-0.724	-1.890	±0.420

Table 2 *Statistics of the differences of height anomalies between GPS-levelling and EGM96. There are 20 FBMs where the height is known accurately and 187 points were spirit levelled but with unknown height accuracy.*

Eight points were removed as clear (>5 m) blunders from the comparisons due to spurious differences of height anomalies, which were separated from the main cluster in Fig. 4, which shows the differences before the blunders were identified. The positional accuracy of the GPS data was confirmed by the Geodetic Survey Unit (2000), so the blunders are most likely due to inferior levelled heights. This left 214 usable out of 222 GPS-levelled points, which gave a mean difference of -1.737 m and a standard deviation of ±0.249 m before detection of outliers. Using ±3σ, a further seven points were rejected, leaving 207 usable GPS-levelled points (Fig. 3).

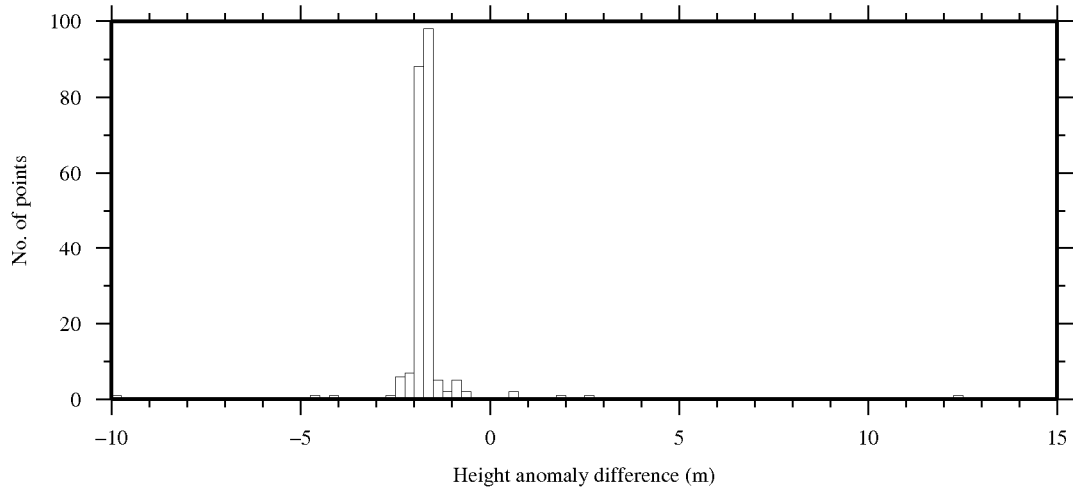


Fig 4 *Histogram of differences of height anomalies for the whole set of GPS-levelled points (222). Most of points are clustered (between -2.75 m and -0.5 m) with a bias of -1.76 m. The eight isolated points are identified as blunders due to inferior levelled heights.*

A large mean difference (-1.760 m for all points) in Table 1 is larger than the difference of MSL values used for the vertical datum of Sri Lanka (Section 2.2.2). The mean dynamic topography (MDT) from the DNSC08 model, which has been derived from the DNSC08 mean sea surface and EGM2008 (Andersen and Knudsen 2008), around the island is ~ +0.7 m above EGM2008. According to the results in Table 1, the geometric quasigeoid associated with the observed MSL at the Colombo tide gauge is 1.760 m below EGM2008.

The relatively small standard deviation of ±0.184 m for the whole data set (±0.098 m for the 20 FBMs) therefore suggests that there is a bias between EGM2008 and the Sri Lankan GPS-levelling data. The ~+0.7 m MDT cannot explain this -1.760 m offset (it is in the opposite direction), so it is more likely to originate from the ellipsoidal heights starting from a point that is not properly connected to the COLA DORIS GPS mark at SGO and the 'unstable' radiated GPS baseline vector from SGO to ISMD (Section 2.2.1). This is an implicit validation of EGM2008, as it led to the investigation of the ellipsoidal height datum in Sri Lanka (Section 2.2.1).

The calculated height anomaly differences are plotted against the normal orthometric height of the point in Fig. 5 to determine if there is any correlation with height. Unfortunately, there is only a limited number of points available in the mountainous areas above 1,000 m, with most GPS-levelling points located below 250 m. The correlation coefficient for the linear regression in Fig. 5 is -0.195 and hence no significant height-dependent trend in the differences is observed.

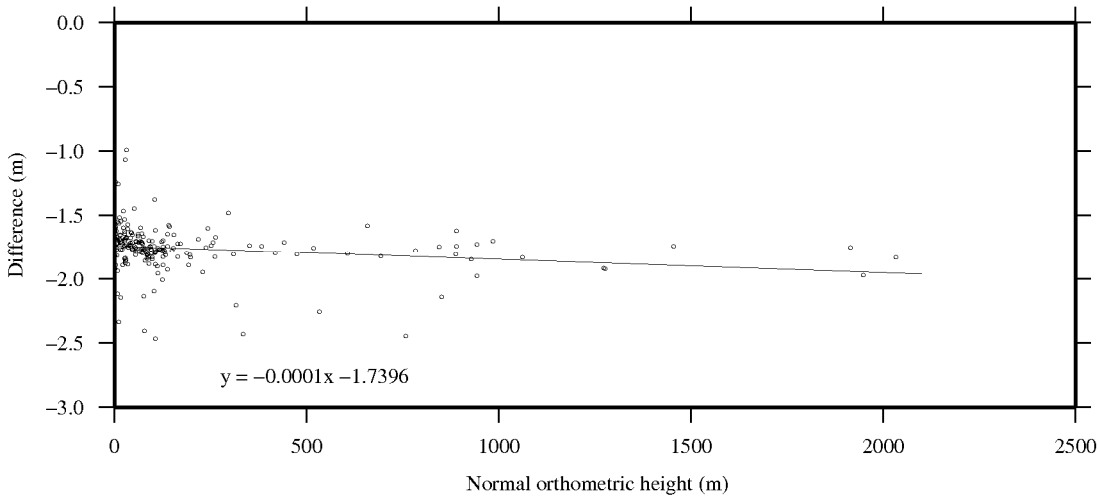


Fig 5 *Difference of height anomalies versus normal orthometric height. The correlation coefficient is -0.195 and no significant trend is seen.*

Figure 6 shows a map of the height anomaly difference derived by subtracting the geometric GPS-levelled height anomalies and EGM2008-synthesised height anomalies (in scattered point mode). The surface was created by gridding with continuous curvature splines in tension (Smith and Wessel 1990) available in Generic Mapping Tools (GMT) open-source software (Wessel and Smith 1998). From Fig. 6, some spurious points appear to remain that were not rejected by the ±3σ threshold. All these are not at FBMs, indicating that levelling errors occur that these points too, but they could not be justifiably rejected by statistical outlier detection. Further investigation (and re-levelling) would be needed to properly isolate these. Also, there is no correlation of the differences with the southern central mountains (cf. Fig. 2), as already seen in Fig. 5.

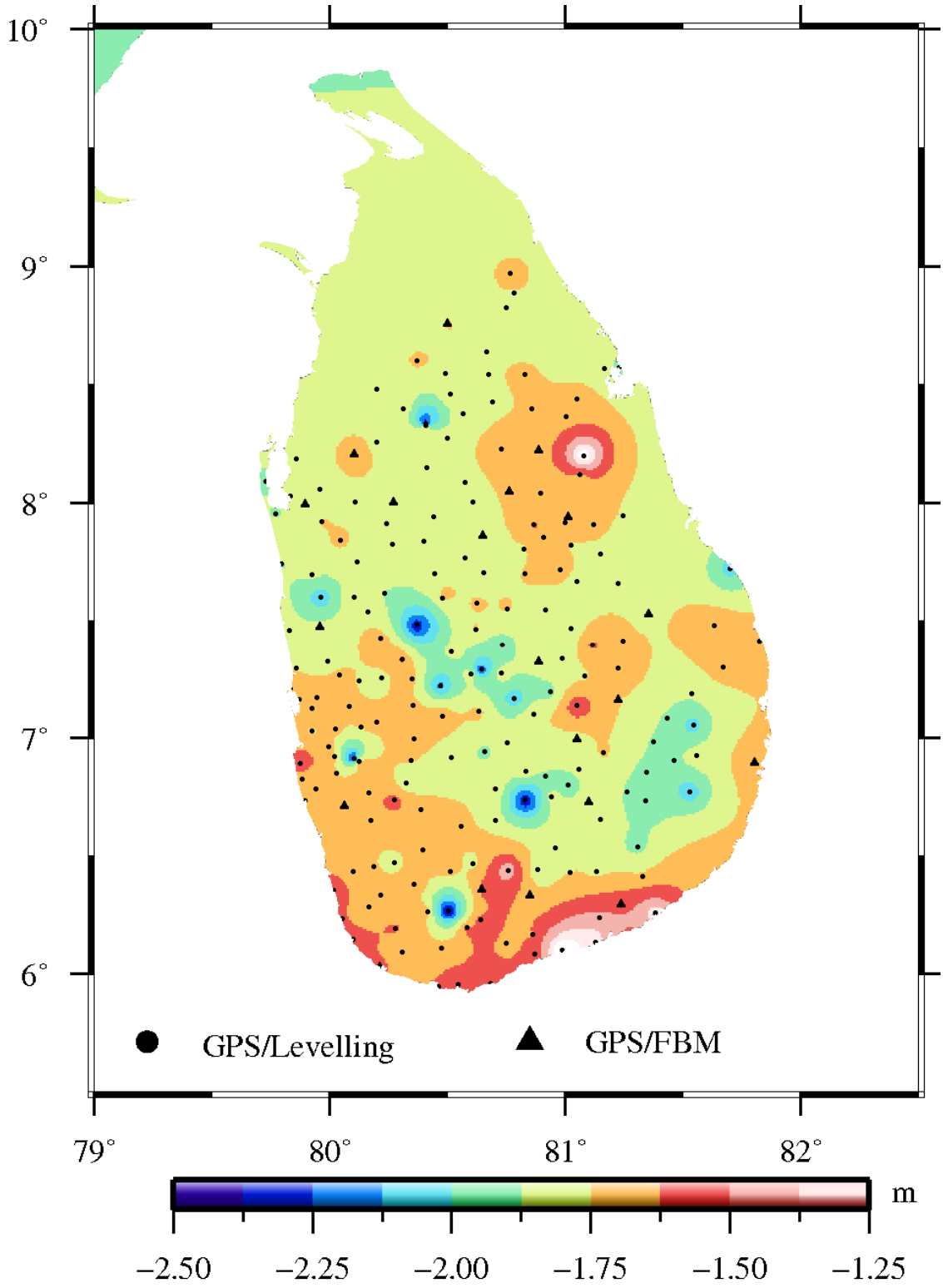


Fig 6 *Difference of the height anomalies derived from geometric quasigeoid and synthesised values from EGM2008 (Mercator projection). The majority of points have differences near to the mean (-1.760 m). A few points show relatively larger differences, indicating that levelling errors remain.*

North-south and east-west trends in the differences of height anomalies were calculated by performing linear regressions in latitude (Fig. 7) and longitude (Fig. 8). The north-south tilt equates to ~0.41 mm/km when converting degrees to kilometres (one degree is ~111 km at the equator). The east-west tilt equates to ~0.04 mm/km. However, the correlation coefficient for the north-south tilt is ~-0.025 and for the east-west tilt is ~-0.013, so these slopes are not significant.

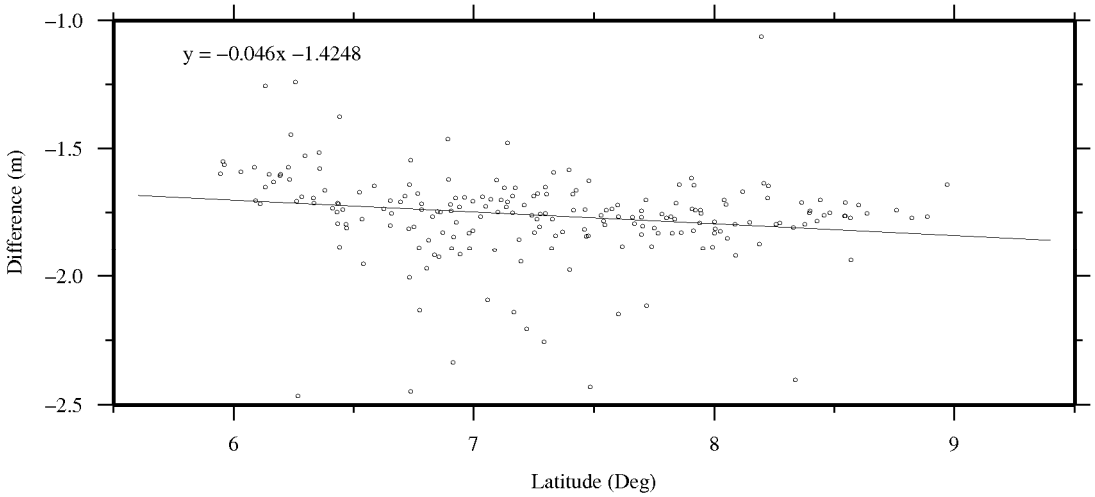


Fig 7 *Linear regression of the height anomaly differences between GPS-levelling and EGM2008 in latitude. The north-south tilt equates to ~0.41 mm/km and the linear correlation coefficient is ~-0.025. No significant slope is observed.*

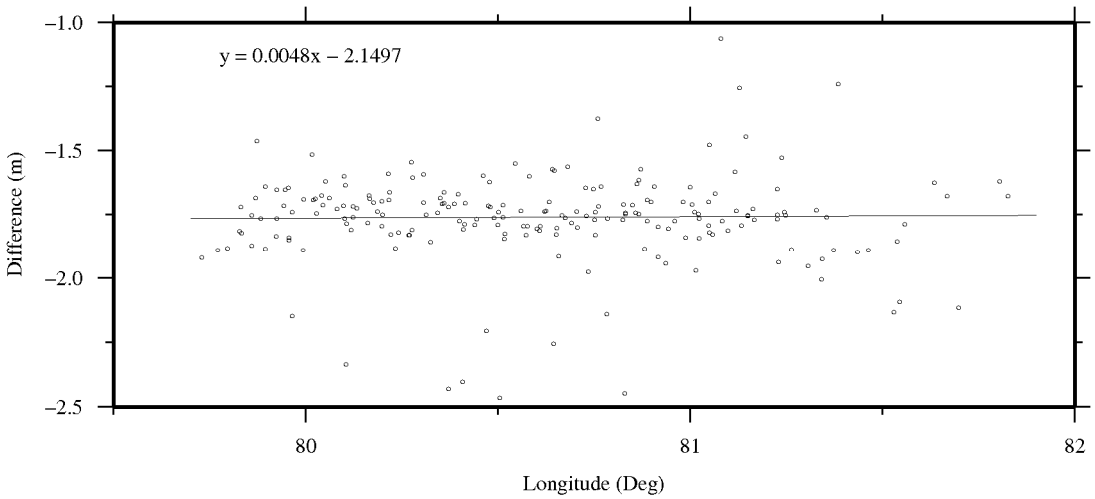


Fig 8 *Linear regression of the height anomaly differences between GPS-levelling and EGM2008 in longitude The east-west tilt equates to ~0.04 mm/km and the linear correlation coefficient is ~-0.013. No significant slope is observed.*

3.1.2 Relative GPS-levelling tests

In addition to the absolute tests, the relative (i.e., quasigeoid slope) differences between EGM2008 and the GPS-levelling have been obtained from all 21,321 possible baselines between the 207 GPS-levelling stations (i.e., after the outlier rejection in Section 3.1.1.) using the `GEOID_REL_TESTER.f` software in Featherstone (2001). Geometric height anomalies were determined from the GPS-levelling data and EGM2008 height anomalies at these points were interpolated from an equiangular 1 arc-minute (synthesised height anomalies) grid using bi-cubic methods.

Table 3 was created from the output of `GEOID_REL_TESTER.f` and shows the statistics of the differences of the relative height anomalies or gradients in height anomaly differences. The statistics of absolute differences are in Table 3 are similar to the values in Table 1. The small differences are due to interpolation error, but they are much smaller than the expected errors in the levelling. As such, the absolute differences in Table 1 should be interpreted as the more accurate values.

	Baseline length (km)	Absolute differences (m)	Relative differences (m)	Misclosure (ppm)
Max	336.157	-0.927	1.557	653.796
Min	0.912	-2.484	-1.504	0.000
Mean	121.964	-1.764	0.026	2.284
STD	±60.625	±0.218	±0.282	±5.937
RMS	±136.201	±1.778	±0.283	±6.361

Table 3 *Statistics of the baseline analysis. Absolute and relative differences were calculated from the geometric quasigeoid and a one arc min equiangular EGM2008 quasigeoid grid. The large maximum misclosure value is due to shorter baselines.*

In relative quasigeoid testing, it is immaterial where the heights are tied to, because any constant bias cancels on differencing (e.g., Featherstone 2001). This results in a relatively small mean difference (0.026 m), where the -1.760 m bias has cancelled (cf. Figs. 9 and 10). However, the standard deviation of ±0.282 m is a small increase on the absolute differences (±0.184 m in Table 1) because uncorrelated errors do not cancel on differencing. The misclosure is calculated in mm per km (ppm) by dividing

the relative difference by the length (geodesic distance) of the baseline. The mean ppm value shows that, on average, EGM2008 can recover normal orthometric height differences with a precision of about ~2.3 ppm (mm/km).

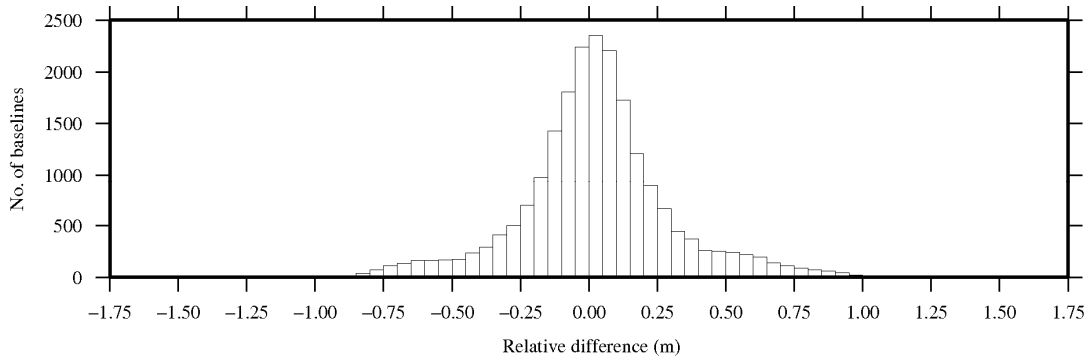


Fig 9 *Histogram showing the unbiased distribution of the relative differences of height anomalies among 21,321 baselines.*

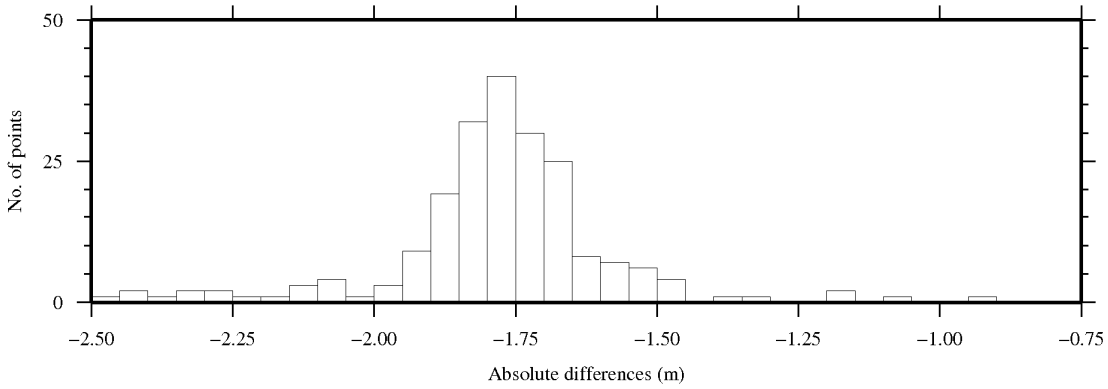


Fig 10 *Histogram showing the absolute differences calculated from* `GEOID_REL_TESTER.f`

The allowable misclose of third order levelling (A; 8 mm/√km), minor levelling (B; 14 mm/√km) and detail levelling (C; 24 mm/√km) of Sri Lankan standards (Goonewardena 1970) are represented by curves in Fig. 11. 34% of the relative differences are under curve A, ~54% are under curve B, and ~72% are under curve C, which is the lowest accuracy (detail) levelling standard generally used for colleting coarse height information for elevation contours.

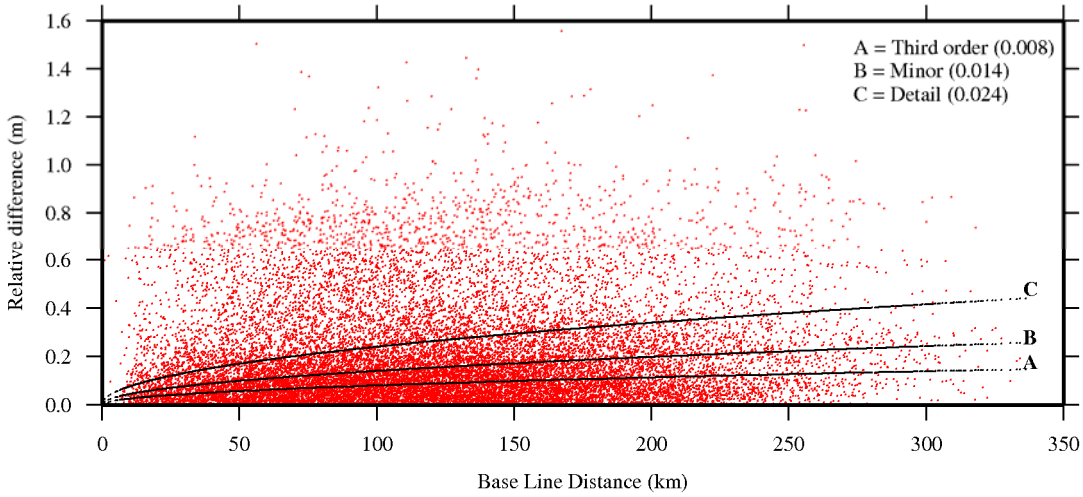


Fig 11 *Relative differences of the height anomalies from GPS-levelling and EGM2008 computed over 21,321 possible baselines. Misclosure standards of third order, minor levelling and detail levelling are shown as A, B and C.*

3.2 Comparison with Sri Lankan gravity data

3.2.1 Computation of gravity anomalies

The Sri Lankan free air gravity anomalies were re-calculated with respect to the Geodetic Reference System 1980 (Moritz 1980). The linear free air correction, *F*, was computed with normal gravity γ_{1980} from the International Gravity Formula 1980 with

$$F = -\frac{\partial \gamma}{\partial H} H = +0.3086\, H \;\; mGal \qquad (1)$$

$$\gamma_{1980} = 978\;032.7\left(1 + 0.005\,302\,4 \sin^2 \phi - 0.000\,005\,8 \sin^2 2\phi\right) \; mGal \qquad (2)$$

No atmospheric correction was applied, but which is only 0.871 mGal at sea level (Moritz 1980). The linear free-air correction was considered sufficient because most of the gravity observations are made below ~250m (Section 2.1.1)

These were compared with synthesised values from `HARMONIC_SYSNTH.f` software, which gives the gravity anomaly (i.e., Molodensky free air gravity anomalies) at the observation point. For the synthesis, the 3D location with respect to the geometrical surface of the reference ellipsoid (GRS80 was used for this Sri Lankan assessment) is needed. Therefore, the geometric quasigeoid was interpolated in order to find the height anomalies for the non GPS-coordinated points in Table 4, except for the marine gravity 9described later). `HARMONIC_SYSNTH.f` was used to synthesise free air anomalies in scattered point mode in this comparison.

3.2.2. Digitised and reconstructed gravity anomalies

Gravity data were digitised from the complete/refined Bouguer anomaly map of Hatherton and Ranasinghe (1972). The locations of gravity observations (marked on the map) were compiled by onscreen digitisation and transformed to a geocentric datum using ESRI's ArcMap v9.2. The Bouguer anomalies were interpolated from the 5-mGal-interval contours to the observation locations on the map. The International Gravity Formula 1924 was used to compute the Bouguer anomalies in Heatherton and Ranasinghe (1972). Therefore, the normal gravity field was converted from International 1924 (γ_{1924}) to GRS80 (γ_{1980}) for latitude ϕ using (e.g., Li and Götze 2001).

$$\gamma_{1980} - \gamma_{1930} = -16.3 + 13.7 \sin^2 \phi \quad \text{mGal} \tag{3}$$

To check the reliability of the digitised values, simple Bouguer anomalies were calculated at the 20 GPS-coordinated FBMs (Section 2.2.1) and compared with values obtained by interpolating the digitised Bouguer anomalies to the FBMs using GMT. The mean difference was -0.476 mGal with a standard deviation of ±1.815 mGal, showing the digitisation to be reasonably accurate. However, terrain corrections had been applied out to Hammer's (1939) zone M on Hatherton and Ranasinghe's (1972) map, whereas they were not applied at the 20 FBMs. Only a small difference can be expected because most of the digitised observations were in low-lying regions.

Next it was necessary to 'reconstruct' free-air anomalies (actually Faye anomalies that have a terrain correction applied to a free-air anomaly) from the digitised complete Bouguer anomalies (cf. Featherstone and Kirby 2000). The normal orthometric heights (H) required to calculate the Bouguer slab correction ($2\pi G\rho$) were bicubically interpolated from a 100 m DEM (Fig. 2) and subsequently added to the Bouguer anomalies (Δg_B) to find the free air anomaly (Δg_F)

$$\Delta g_F = \Delta g_B + 2\pi G\rho H \tag{4}$$

with the standard rock density ρ of 2670kg/m^3. These 1055 digitised and reconstructed gravity anomalies will be compared with EGM2008.

3.2.3 Results for land gravity

The results presented in Table 4 for the Sri Lankan gravity observations, not all of which are available in the public domain, indicates how EGM2008 agrees with the

local gravity field. The terrestrial gravity data are grouped as (*a*), (*b*) and (*c*) in descending order the level of perceived accuracy of location, height and gravity (Section 2.2). The statistics of differences between the Sri Lankan and EGM2008-synthesised free air anomalies are shown in Table 4 after removing outliers (±3σ).

The mean difference for all 63 points under (*a*) and (*b*) is 13.736 mGal. Only one value was an outlier with the standard deviation of ±8.485 mGal used for the outlier detection (cf. Fig. 12). For dataset (*c*), 23 points were found to be outliers for the 1055 digitised points (Fig. 13) with a prior mean of 12.578 mGal and standard deviation of ±7.984 mGal used for the outlier detection.

Gravity data	# points	Min	Max	Mean	Std
a) GPS-coordinated gravity on FBMs	20	-7.589	19.430	10.194	±6.743
b) FBM and permanent positions with coarse locations	42	-26.039	35.799	8.711	±14.704
c) Digitised and reconstructed free-air anomalies from a Bouguer anomaly map	1032	-10.532	33.660	12.841	±6.367
d) Ship track gravity	12,192	-96.768	107.198	9.552	±43.683

Table 4 *Agreement of EGM2008 with Sri Lankan free air anomalies after removing outliers (±3σ). Points under (a) are where gravity was measured with a L&R meter and also belong to the geodetic levelling network of Sri Lanka. Points under (b) have coarse locations (compiled from maps). Points under (c) are reconstructed (digitised) free air anomalies from a Bouguer anomaly map of Ceylon. Points under (d) are marine gravity anomalies. All values in mGal.*

The results in Table 4 are all based on the old Potsdam gravity datum (Ratmalana airport) as it is the datum of previous national gravity surveys in Sri Lanka (Section 2.1.2). The conversion (about -14 mGal) between the Potsdam and IGSN71 gravity datums (e.g., Morelli et al. 1974; Hatherton et al. 1975) has not been applied, and thus remains present in the mean differences for EGM2008 over the region. The positive biases (~10 to 15 mGal in the means) in Table 4 are consistent with this gravity datum offset. This shows that EGM2008 has used ISGN71-based gravity anoma-

lies over Sri Lanka, unlike some previous geopotential models that used Potsdam (e.g., Kim and Rapp 1990).

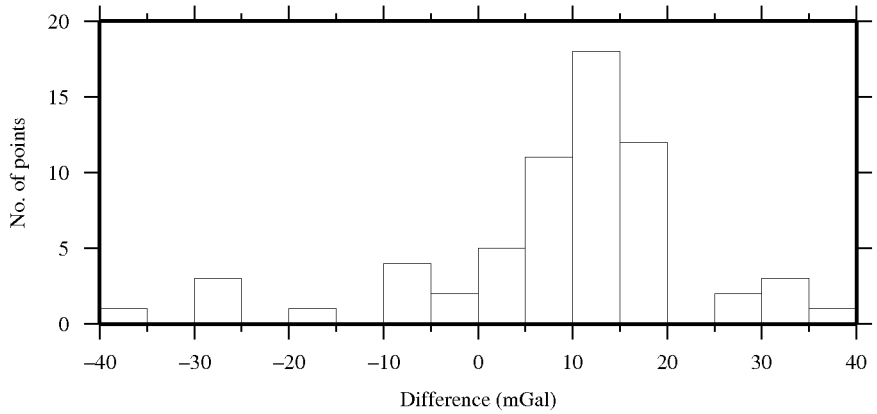


Fig 12 *Distribution of differences between free air anomalies of 63 terrestrial gravity observations from subsets (a) and (b) and EGM2008. The mean is 8.485 mGal and the standard deviation (1σ) is ±13.736 mGal for the whole set of data. One point was outside ±3 σ.*

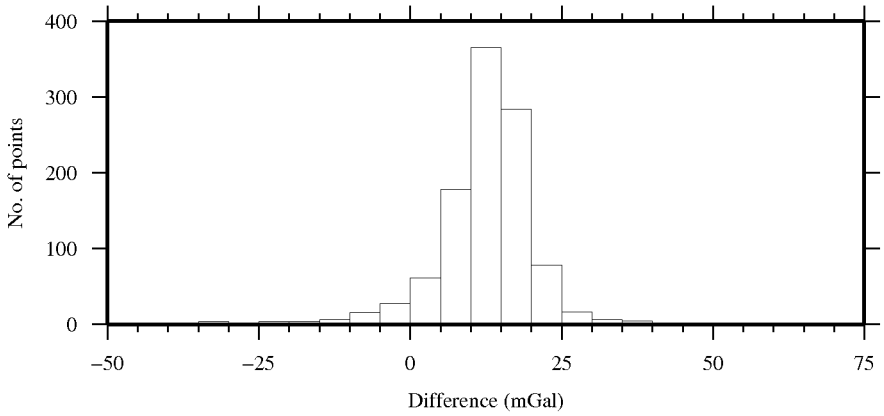


Fig 13 *Distribution of differences between free air anomalies of 1055 terrestrial gravity observations from subset (c) and EGM2008. The mean is 12.578 mGal and the standard deviation (1σ) is ±7.984 mGal for the whole set of data. Twenty three points were outside ±3 σ.*

The frequency distribution of the differences for datasets (*a*) and (*b*) combined spans over ~-40 to ~+40 mGal (Fig. 12). The 20 GPS-coordinated gravity points (*a*) have a smaller standard deviation (±6.743 mGal) than the gravity points (*b*) whose locations were scaled from maps (±15.997 mGal). The reason is the coarse locations of the latter. This inaccurate location would give an uncertainty in height, especially in the hilly areas, and therefore the EGM2008 free air anomaly will not be synthesised at the same 3D location by `HARMONIC_SYNTH.f`. This effect is shown in Fig 14,

where the differences for the non-GPS-coordinated points have a fairly large deviation from the mean for heights greater than 500m.

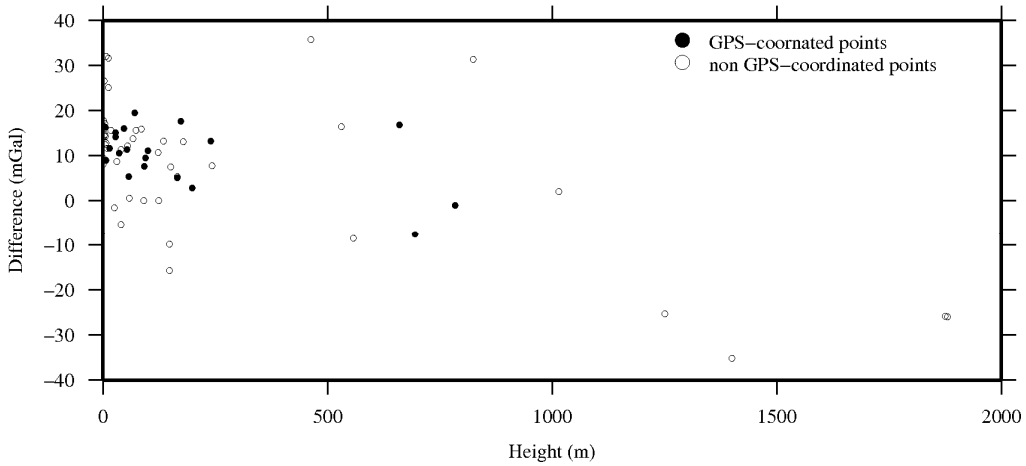


Fig 14 *Differences of free air anomalies for (a) and (b) with height. Non GPS-coordinated points have large deviation than GPS-coordinated points at higher altitudes.*

Figure 15 shows the difference between free air anomalies from Sri Lankan gravity and EGM2008. Though the most of differences are consistent with the ~14 mGal gravity datum bias (Section 2.1.1), the central southern mountain region (cf. Fig. 2) shows notable extreme values. The poorer Sri Lankan gravity positions would provide some explanation for this, but the omission error and downward continuation corrections in EGM2008 may also be responsible. From Pavlis et al. (2008), the standard deviation of the 'fill-in' gravity anomalies on and around Sri Lanka is about 3-5 mGal, with the larger values being in the mountainous central south of the island.

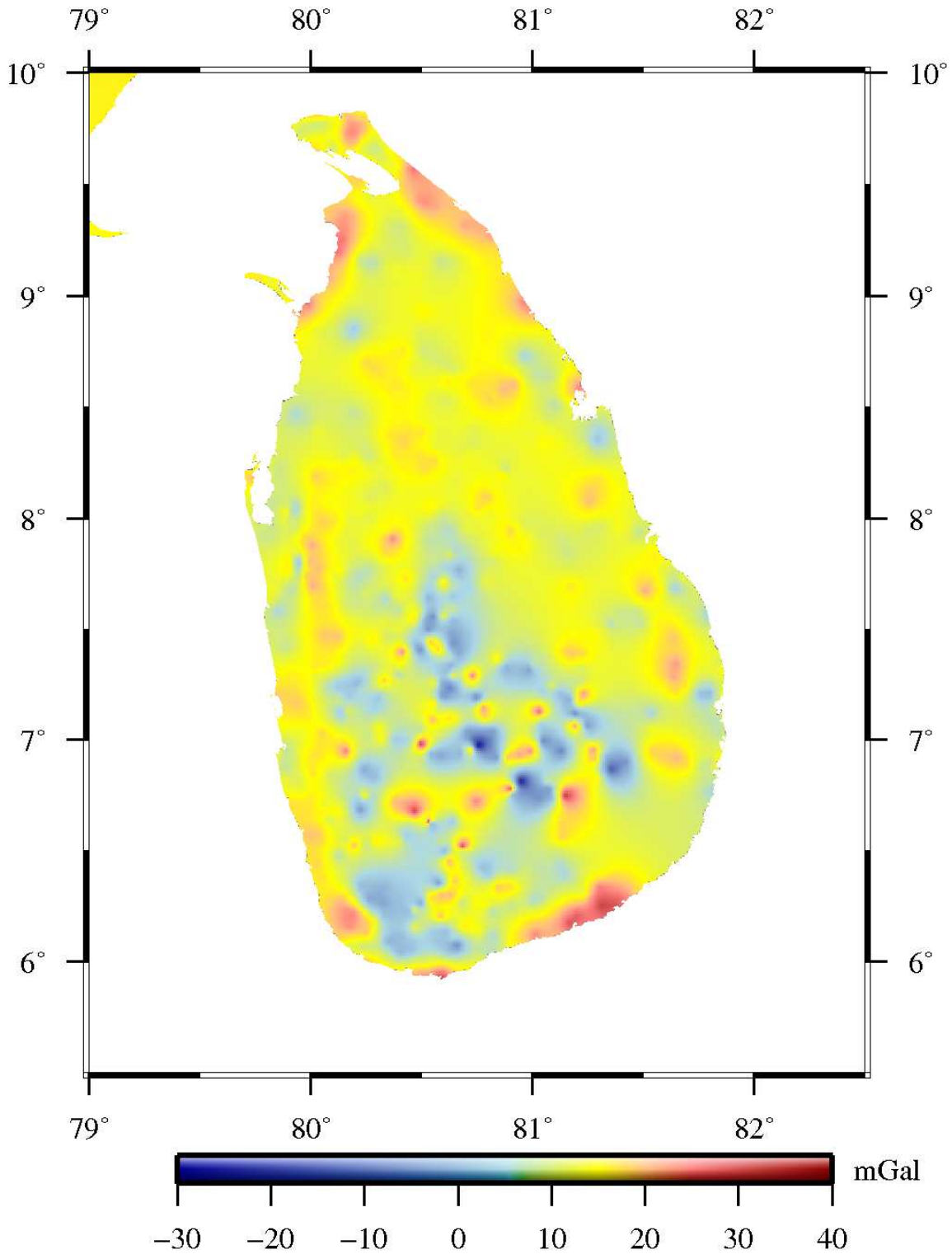


Fig 15 *Difference between free air anomalies of Sri Lankan gravity and EGM2008 (Mercator projection). The larger differences are seen in the central southern mountains.*

3.2.3 Results for ship-track gravity

Figure 16 shows a histogram of the differences between the NGDC (1999) ship-track gravity data and EGM2008, and Fig. 17 shows a chart of the differences along the cruise lines. To enable the `HARMONIC_SYNTH.f` software to compute gravity anomalies on the mean sea surface where the ship-track gravity observations are taken, the ellipsoidal heights of the ship-track gravity points were obtained by applying the MDT model from the Danish National Space Institute (DNSC) (Andersen and Knudsen 2008). The 2 arc-minute grid was interpolated to the locations of the ship-track observations, then `HARMONIC_SYSNTH.f` was run in scattered point mode, where is added the EGM2008 height to the MDT to compute the gravity anomaly on the mean sea surface.

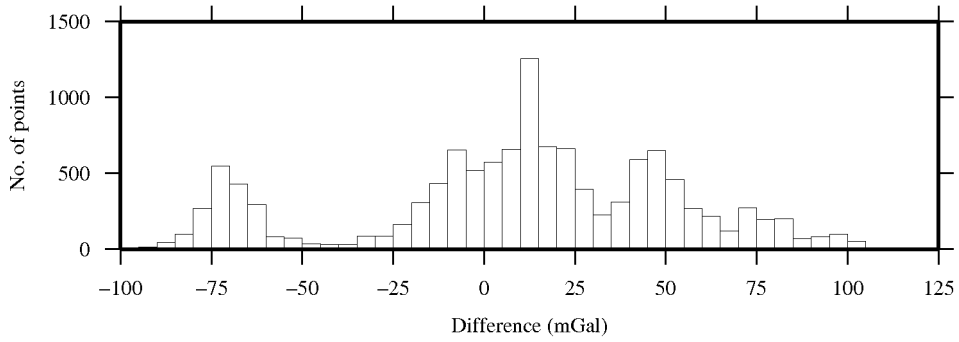


Fig 16 *Frequency distribution of differences in marine free air gravity anomalies and EGM2008. Clusters indicate that the data are not crossover adjusted. The largest central peak is consistent with the datum bias in the terrestrial gravity anomalies.*

The offsets in the differences among the ship tracks appear as widely spread and clustered peaks in the histogram (Fig. 16), confirming further that they have not been crossover adjusted. We did not attempt a crossover adjustment because the tracks are widely spaced with few crossovers, rendering any adjustment ill conditioned. Also, the majority of differences are around 10–15 mGal which correlates with the bias in the terrestrial gravity observations due to the Ratmalana datum being tied to Potsdam.

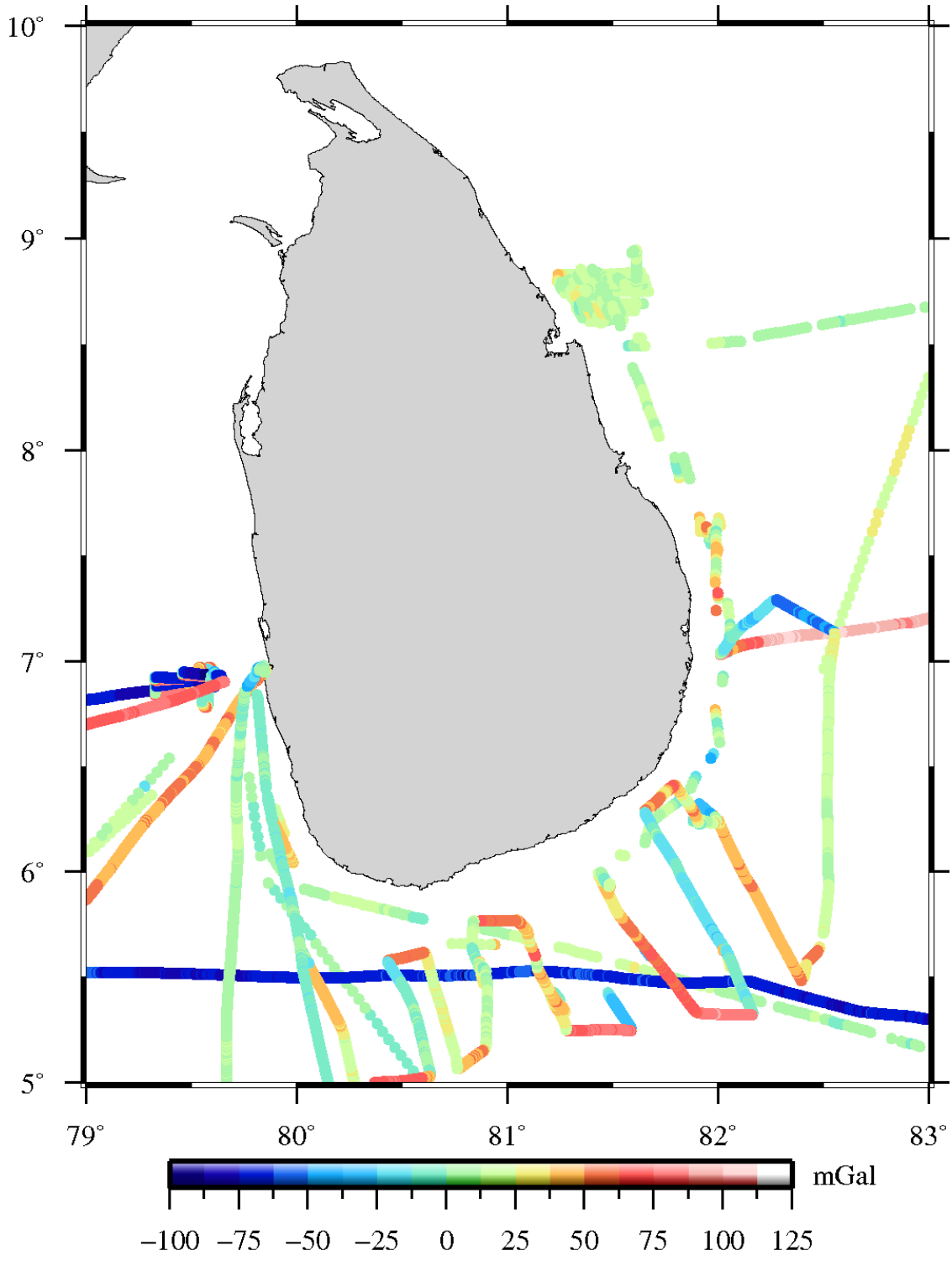


Fig 17 *Differences between marine free air gravity anomalies and EGM2008. The significant offsets among overlapping tracks shows that the cruise lines have not been crossover adjusted. In addition, there is a consistent bias of ~14 mGal in the cruises coming from Colombo (~7N, 80E), indicating that they were tied to the Potsdam datum via Ratmalana airport. (Mercator projection)*

4. CONCLUSIONS

We have used Sri Lankan gravity and GPS-levelling data to assess EGM2008, which is where 'fill-in' gravity anomalies were used in EGM2008. The analysis eventually proved to be more useful for detecting problems in the Sri Lankan data, particularly with respect to the datums. From the initial quick-look comparisons, we detected a ~1.7 m bias between the GPS-levelling and EGM2008, which led to an in-depth investigation into the Sri Lankan vertical geodetic datums. This uncovered a probable bias in the ellipsoidal heights used due to a combination of an incorrect local tie at the COLA DORIS site and a ~120-km-long radiated GPS baseline over a 1.2 km height difference. The difference between the gravity anomalies and EGM2008 showed that the Sri Lankan data, tied to an absolute value at Ratmalana airport, is based on the old Potsdam datum, which is offset from IGSN71 by about -14 mGal. This is similar to the mean differences observed, indicating that EGM2008 uses IGSN71-refernced gravity anomalies over Sri Lanka, unlike some earlier geopotential models (e.g. Kim and Rapp 1990).

After these datum-related biases are neglected, the standard deviations of the differences become more informative. These were calculated for different subsets of the Sri- Lankan GPS-levelling and gravity data according to their perceived reliability. After rejection of 15 outliers, the standard deviation of the difference between 207 Sri Lankan GPS-levelling points and EGM2008 is ±0.184 m, which reduces to ±0.098 m for a subset of 20 stations at fundamental benchmarks. Over baselines, the standard deviation was ±0.282 m, showing the presence of uncorrelated errors. The Sri Lankan land gravity data, after rejection of outliers, yielded standard deviations of ±6.743 mGal for 20 GPS-coordinated gravity points on fundamental benchmarks, ±14.704 mGal for 42 gravity points on fundamental benchmarks but with coarse locations, and ±6.367 mGal for 1032 digitised and reconstructed free-air anomalies from a Bouguer anomaly map. The ship-track gravity data have not been crossover adjusted, and yield a standard deviation of ±43.683 mGal, but showed that several tracks originating from Colombo had been tied to the Potsdam datum at Ratamala airport.

This study has therefore implicitly validated EGM2008 because the standard deviations of the differences are reasonable, and the large biases can be explained by peculiarities in the Sri Lankan geodetic and gravity datums. Indeed, this is an auxiliary application of EGM2008, where datum deficiencies can be detected and then in-

vestigated further. The detection of ellipsoidal height bias shows that EGM2008 is already making a contribution to vertical datum unification.

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Evaluating EGM2008 over East Antarctica

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Abstract

The release of EGM2008 and associated products such as grids of mean dynamic ocean topography offer the possibility of utilising the extensive historical record in Antarctica with today's modern satellite sensing techniques. In this study, we use data acquired at the Mawson, Davis, Casey and Scott and McMurdo stations in East Antarctica to investigate the performance of EGM2008 over this region. EGM2008 over Antarctica is entirely dependent on the EGM2008-adopted global GRACE satellite-derived gravity field. This is in contrast to most other regions of the Earth, where there are also contributions from terrestrial gravity and/or altimeter satellites. We determine, over East Antarctica, and at our four test sites that EGM2008 should be used with caution when precisions better than one metre are required. The precisions at the test sites are better than this, but the evidence is that the four test sites are probably not representative of the large area of East Antarctica they are being forced to represent. Notwithstanding any of the above, EGM2008 represents a significant step forward in East Antarctica and that the use of test stations and regions where there is little or no complementary data is a valid method of investigating the performance of the model.

1. Introduction

The needs and uses for heights relative to the geoid in Antarctica are as great as elsewhere over the Earth's surface; it is just that the applications are different. Of particular importance, at this moment, are studies aimed at re-evaluating and connecting historical surveys with modern surveys for the task of deducing ice mass change over decadal time periods.

In the Australian ANARE (Australian National Antarctic Research Expeditions) context, extensive optical levelling surveys were done on the Amery Ice Shelf and the Wilkes Local Ice Cap (cf. Figure 1).

- On the Amery Ice Shelf, Corry (1986, 1987 and 1996) observed a central flow line of some 400 km in 1968. In 1996, Phillips and Craven (Phillips 1999) recovered eight of the original poles placed by Corry in 1968. King et al. (2007) performed a complete re-adjustment of Corry's horizontal observations, and then made a comparison with GPS and INSAR data. The height data has now been reprocessed and comparative studies made with ICESat and GPS data (King et al. in press).
- On the Wilkes Local Ice Cap, optical levelling was undertaken by McLaren in 1965 (McLaren 1968) and Pfitzner the following year (Pfitzner 1980). A re-occupation program was trialled in the Austral Summer of 2004-2005 with GPS and ICESat observations. This data is not yet fully analysed due, in part, to the datum connection difficulties and, in part, due to difficulties associated with estimating ice flow velocities.

Figure 1 is an AVHRR (Advanced Very High Resolution Radiometer) image of Antarctica. It shows the Trans Antarctic Mountains, which divide the continent into East and West Antarctica. The two regimes are very different. East Antarctica is dominated by the high plateau, in-excess of 3000 m altitude, and steep slopes to the coast, which is generally in close proximity to the Antarctic Circle. West Antarctica is lower, generally about 2200 m in elevation. The West Antarctic coastline is far from uniform with two major seas, Ross and Weddell, extending to 78°S and a third smaller sea, Bellingshausen, extending to 72.5°S.

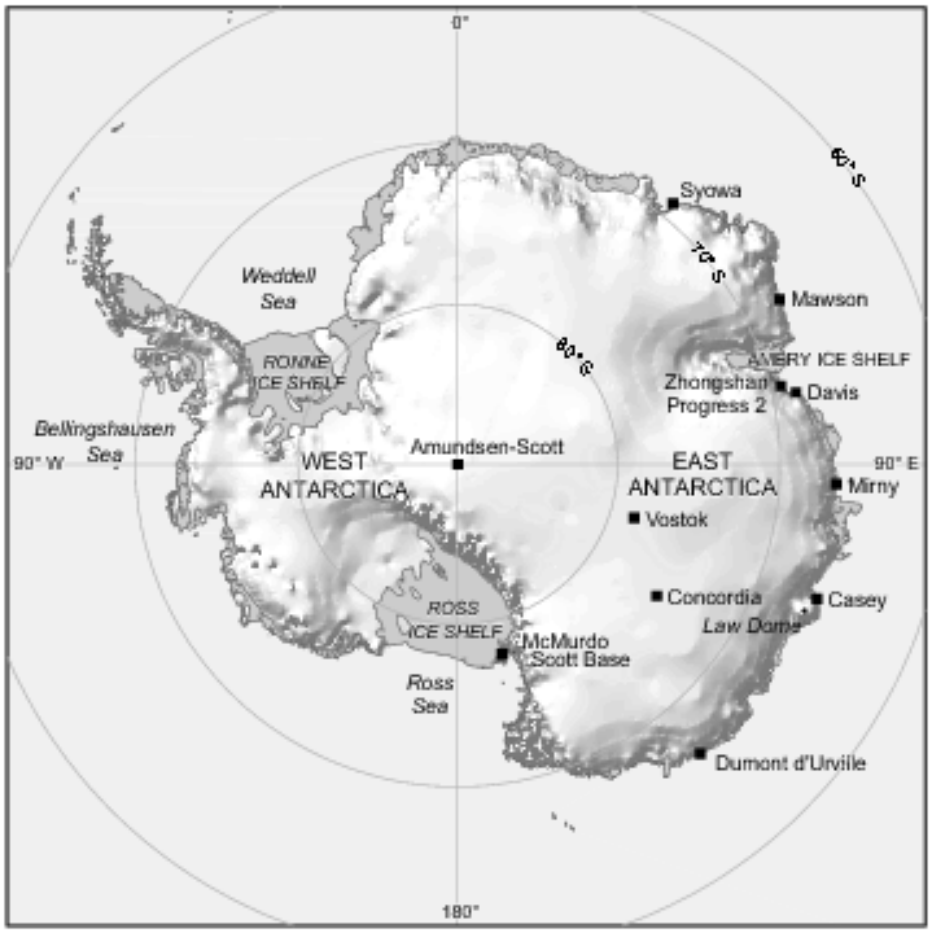


Figure 1: Map of Antarctica derived from AVHRR imagery. Polar stereographic projection with elevation shading and principal ice shelves and the three major regional seas.

EGM2008 (Pavlis et al. 2008) uses three principal data types to derive a new Earth Gravity Model, which seeks to overcome many of the limitations of the earlier EGM96 model (Lemoine et al. 1998).

1. Terrestrial gravity anomalies. In Antarctica, terrestrial gravity observations were an integral part of the major over-snow traverses programmes conducted during the IGY(International Geophysical Year) 1957-1958 and the decade there after (see, e.g., Thiel et al. 1959, Hollin 1961 and Walker 1966). Unfortunately, these positions were poorly constrained until satellite Doppler positioning was introduced in the late 1970s, at which time the height system was changed to the geometric ellipsoidal system. Thus, it is not too surprising that EGM2008 contains no terrestrial gravity data over Antarctica.
2. Altimeter satellite-derived anomalies. There are two sources of gravity anomalies derived from altimeter satellites.

The first is the Sandwell data (cf. Sandwell and Smith 1997; http://topex.ucsd.edu/marine_grav/). Sandwell uses data from GEOSAT and ERS1, which imposes two limitations on the data set. The first is that the inclination of GEOSAT, 108 degrees, limits GEOSAT data to the band 72°S to 72°N. ERS1 has an inclination of 92 degrees and therefore significantly extends coverage in the Polar Regions. The second limitation is the footprint of the imaging system. In radar satellites such as GEOSAT, the effective size of this footprint varies from 2 km to 10 km depending antenna characteristics, the width of the transmitted pulse and surface roughness (e.g., Rees 2001). The impact is that as the footprint size increases, the reliability of heights decrease, especially when there is significant surface roughness or surface slope.

The second data set is that from the Danish National Space Center, DNSC, (http://www.space.dtu.dk/english.aspx). DNSC use data from many more satellites including ICESat, which has a 70 m footprint and a 94 degree inclination. The use of ICESat data for the recovery of gravity anomalies was pioneered by DNSC staff (Forsberg and Skourup 2005). Zwally et al. (2008) have also shown that gravity anomalies and sea-ice fee board data can be recovered from ICESat data using data over the Weddell Sea offshore West Antarctica (cf. Figure 1). The caveats for such processing include the level of bias in the "lowest-level" filtering scheme and the level of *a priori* knowledge assumed.

EGM2008 seeks to use the strengths of both the Sandwell and DNSC data sets. Thus, EGM2008 uses Sandwell data over the open oceans, while the DNSC data is used for the 195-km-wide coastal zone. There is also a transition zone over which this change occurs (Pavlis 2008, pers. comm.).

3. Global satellite gravity fields are regularly determined from the GRACE satellites in several modes (http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html). GRACE-only solutions have been published by: The Center for Space Research at the University of Texas (//http://www.csr.utexas.edu/grace/), GeoForschungsZentrum Potsdam (http://op.gfz-potsdam.de/grace/index_GRACE.html) and Institut für Geodäsie und Geoinformation, University of Bonn (http://www.geod.uni-bonn.de/), among others. EGM2008 uses the ITG-GRACE03s model, extending to degree and order 180 (http://www.geod.uni-bonn.de/itg-grace03.html). The limiting degree of the model is controlled by the crossover between signal recovery and calibrated, formal, errors. The initial presentation of this model at the Joint

International GSTM and DFG SPP Symposium in Potsdam on 15 October 2007 discussed model striations, particularly their likely causes. An alternative static model by Tapley et al. (2005) only extends to degree 120.

The problem faced in East Antarctica is that only data type 3 contributes to EGM2008, whereas most other regions, including the Arctic, have at least one additional data type.

2. The Functional Model

This study uses the well-known relationship between ellipsoidal height, h; orthometric height, H; mean dynamic topography of the ocean, MDT, and the geoid-ellipsoid separation, N:

$$h = (H_msl + MDT) + N \quad (1)$$

where H_msl is the mean sea level (MSL) height of the tide gauge bench mark (TGBM), which needs to be 'corrected' to the geoid with the prevailing MDT.

3. Description of the Data

Some 30 nations operate Antarctic stations or bases (http://en.wikipedia.org/wiki/List_of_research_stations_in_Antarctica). Many of these nations contribute GPS data to the IGS (International GNSS Service) network, (http://igscb.jpl.nasa.gov/network/netindex.html). Seven stations in East Antarctica also contribute tide gauge data to the GLOSS network (http://www.gloss-sealevel.org/). The Antarctic programmes undertaken by most nations, especially in this instance by Australia and New Zealand, include precision levelling between the TGBM and the IGS GPS antenna, and precision GPS observations at the TGBM (http://www.antarcticanz.govt.nz and http://www.ga.gov.au/geodesy/antarc/antgauge.jsp).

This data provides the classical estimates of both the levelled height with respect to MSL, H_msl, and the ellipsoidal height, h, at the station IGS GPS receivers, the local TGBM and associated reference and intermediate marks of interest, e.g., marks used in previous geodetic missions such as PAGEOS (http://en.wikipedia.org/wiki/PAGEOS) or old IGS sites. In addition to this need, there have been special needs for levelled heights to determine the elevations of raised beaches, and aircraft runways. A notable example is the Vestfold Hills Survey, in the vicinity of the Davis Station (Johnston and Digney 2001).

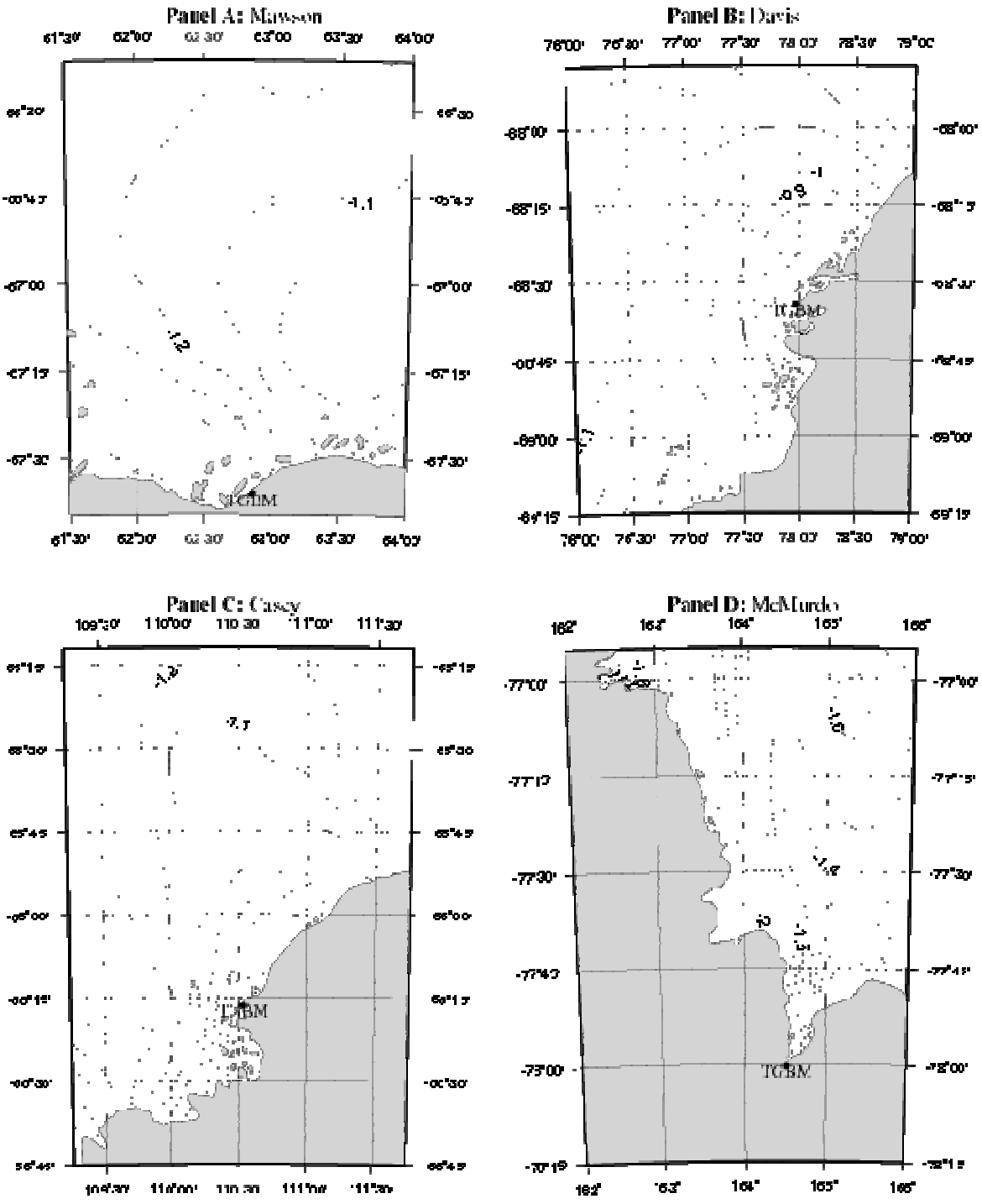


Figure 2: Panels showing contours, 0.1 m interval, of MDT from DNSC08 near the Mawson, Davis, Casey and McMurdo stations.

For the MDT in this study, we used the DNSC08 model (Andersen and Knudsen 2008). The behaviour of this model at the Mawson, Davis, Casey and McMurdo stations is shown in Figure 2, which plots MDT with a 0.1 m contour level. Because this data set uses the same altimeter satellites used in developing EGM2008, it will be beset by similar problems of coastline contamination and data continuity. It is estimated from Figure 2 and a general understanding of coastal effects that values of MDT are likely to be unreliable within 10 km to 20 km of the coast. For this reason, the value at the TGBN is likely to be in error by several contour intervals before an open water steady-state condition is reached. This is especially so at Davis and Casey, where there are many small peninsulas, bays and off-shore islands in the immediate vicinity. The case in McMurdo Sound is complex because data is limited and sea-ice covers the ocean for much of the year. The use of the GLOSS tide gauge at Cape Roberts will alleviate some of these problems. As such, GPS and spirit-levelling connections have been programmed for the forthcoming field season.

4. Data Analysis

Our analysis is based on the remove-restore principle since the summation of individual spectral bands is equal to the full model. This approach readily allows for the contribution of individual bands to be determined by differencing two sequences, usually a sequence that is near full with a sequence that is band limited. This concept is shown in Figure 3.

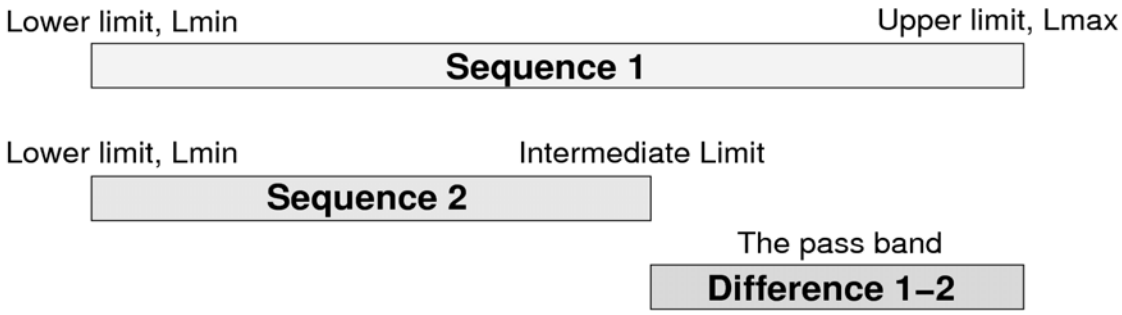


Figure 3: Schematic explanation of generating the contribution of a pass band.

Figure 4, with four panels, shows the effect of increasing the pass band by progressively reducing the maximum degree to which the coefficients of sequence 2 (cf. Figure 3) are evaluated.

Panel A in Figure 4 shows that there is little or no high-frequency contribution over

East Antarctica. Additionally, there is some low-level noise that parallels the Antarctic coast. These two features are almost certainly consistent with the EGM2008 model. The first is consistent with no terrestrial Antarctic gravity data and a complete reliance upon the GRACE-only ITG-GRACE03s global gravity field model. The second is consistent with the use of satellite altimeter data, which abruptly ends as the satellite crosses from an ocean to land environment or begins with a crossing from an ice to ocean environment. Additionally, there is a transition zone, 195 km off the coast of Antarctica, where the principal altimeter data set changes from the open-ocean Sandwell data set to the coastal DNSC data set, which is expected to have better performance characteristics close to the Antarctic coastline (Pavlis 2008, pers comm.).

Panel C in Figure 4, whose residual scale is twice that of Panels A and B, shows further increases in noise. It is also clear that the four test regions of Table 1 are not in regions of particularly bad noise and hence the determined residuals may not be reflective of the magnitudes that exist in other regions; see, for example, the Amery Ice Shelf south of Davis and the Bunger Hills/Mirny region to the west of Casey at about 105°E.

Table 1: A summary of data used in the study. The positions are the mean values for sites in the local region. In the case of Mawson and Casey, all locations were within 10 km of each other and hence have been assigned a point attribute with no residual, (h-H_msl-MDT-N), standard deviation. N has been computed using EGM2008 to degree 2190.

Station Area	**Latitude (degrees)**	**Longitude (degrees)**	**Data Type**	**No. of sites**	**MDT at TGBM (m)**	**Mean Residual (m)**	**STD of residual (m)**
Mawson	-67.60276	62.87097	Point	5	-1.166	-0.796	NA
Davis	-68.55535	78.13456	Area	21	-0.422	-0.267	+/- 0.063
Casey	-66.28012	110.53078	Point	5	-0.551	-0.924	NA
McMurdo	-77.82861	165.72620	Area	7	-1.859	0.236	+/-0.123

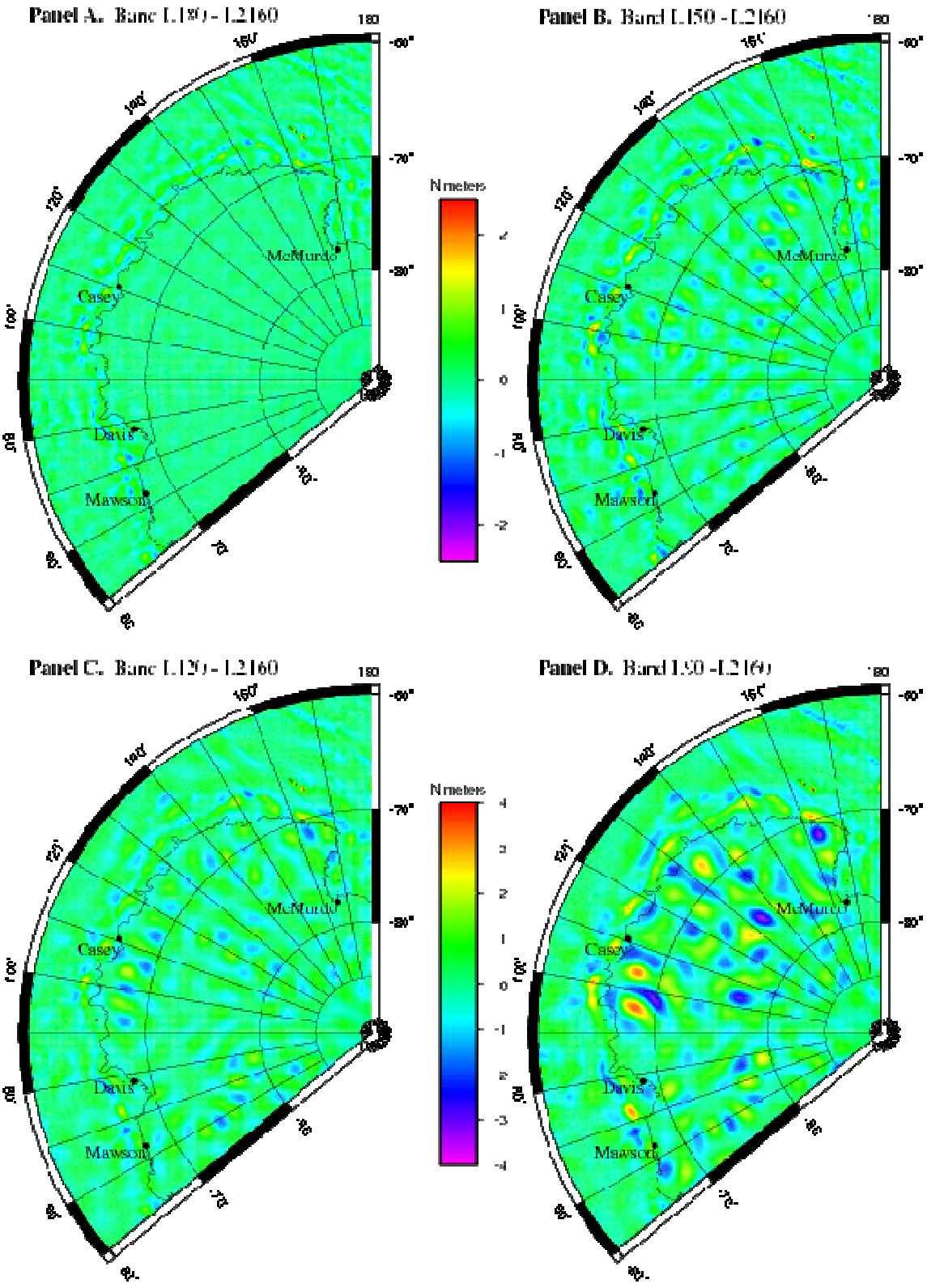


Figure 4: Four panels showing increasing information from a near zero level for pass band L180-L2160, Panel A, to a possible noise-dominated pass band L90-L2160, Panel D. Panel B shows an increase in noise as the pass band is increased at the low frequency end. The noise does not appear to be random because striations start to appear as well as regions where the residuals oscillate.

Panel D in Figure 4, with the same residual scale as Panel C, shows an amplification of noise as the bandwidth is further increased. Since the patterns in Panels C and D are similar and regionally repetitious, there is the strong suggestion that it is structured noise rather than signal that is causing the difference.

Figure 5, showing EGM2008 in the band between L=90 and L=120, Panel A, and the band between L=120 and L=150, Panel B, supports the structured noise hypothesis as the patters are similar, although of different magnitudes in the two panels. The reduced signal level in Panel B is consistent with the expected lower signal levels that can be detected as the degree of the model is increased. The patterns to the west of Casey at 100°E and to the north of McMurdo are two regions where pattern similarity is high.

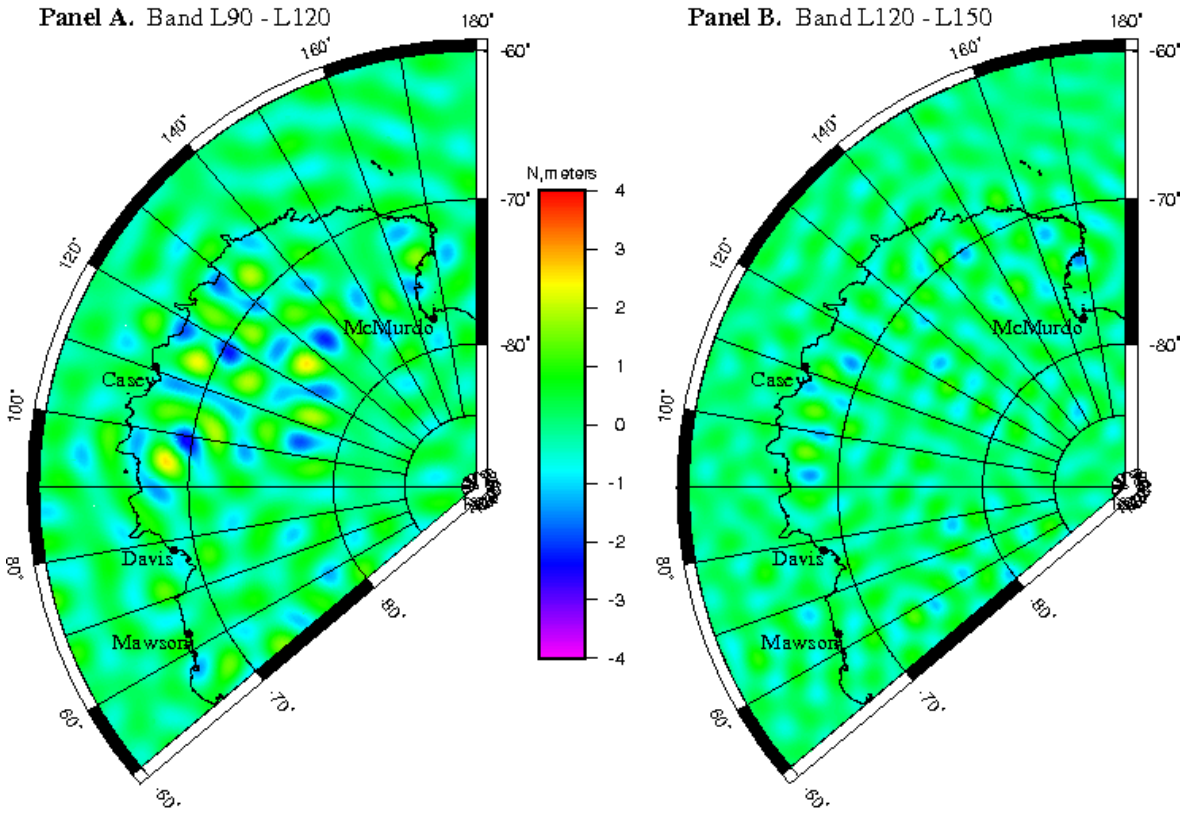


Figure 5: Two pass bands, L90-L120 and L120-L150, which highlight low-level regional signal or noise in the EGM2008 model.

We computed formal correlation coefficients between the two panels in Figure 5 in an attempt to support these impressions. However, we found that the correlation coefficient was only -0.03, a value that is indicative of no correlation. An investigation of this null result showed, using auto-correlation techniques, that the correlation coefficient

rises to 0.7 when the grid is displaced, relative to itself, by 0.5 degrees. A fall to 0.17 occurs when the displacement is 1 degree. The visible displacements of the features of Panels A and B in Figure 5 are of this order. We therefore conclude that the panels of both Figures 4 and 5 show a subtle mixture of signal and noise, which cannot be separated or characterised in this case. This is entirely consistent with the behaviour of spherical harmonics which are oscillatory by their very nature and depend on superposition cancellation and addition to represent local features (e.g., Moritz 1980).

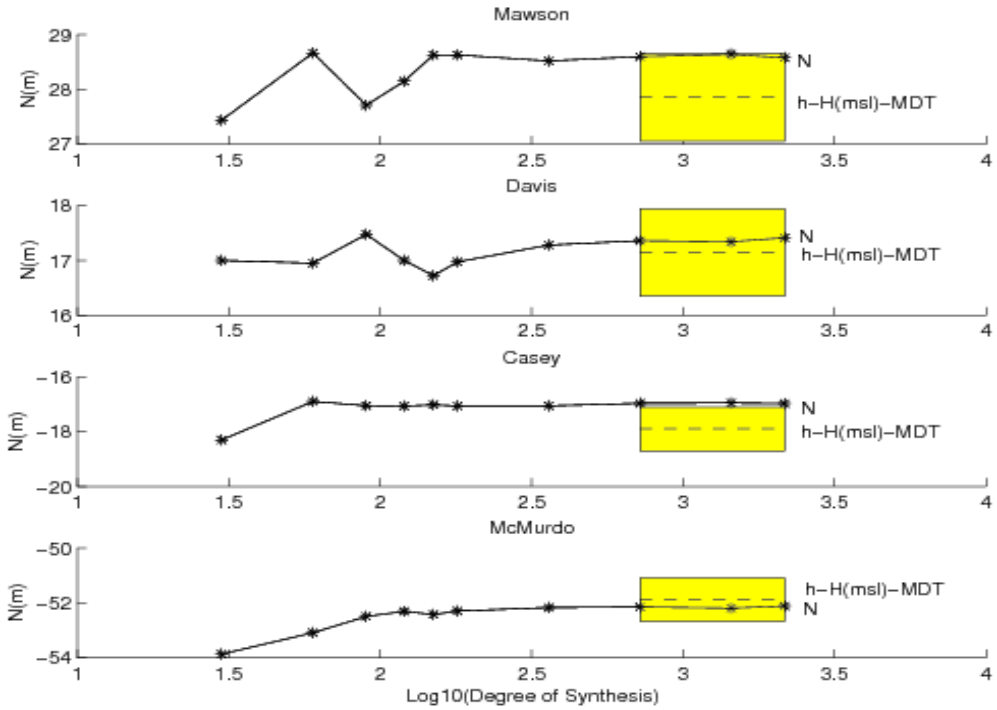


Figure 6: Characterization geoid separation and residuals at the four East Antarctic sites. The solid lines represent a mean position N computed from the EGM2008 model while the dashed lines represent the observed mean value after correcting for MDT. The yellow patch is an estimated 95% confidence interval.

The second part of our data analysis concerned the individual station data of Table 1. Figure 6 shows the evolution of *N* from EGM2008 as a function of the degree of the model and the value computed by differencing the observed ellipsoidal height with MDT-adjusted MSL heights. Figure 6 uses a log10 abscissa scale so that the high degree expansions do not dominate. No error bounds are shown for *N*, even though it has clearly been demonstrated that there are errors due to model inadequacies in East Antarctica. Figure 6 also shows the mean difference between the GPS-determined ellipsoidal heights and the MDT-adjusted MSL height. This estimate of *N* is plotted as a yellow patch, which

has a width equal to a 95% confidence level (two sigma) of 0.8 m. The major contributor to this error is the reliability of the MDT. While the sample is too small to draw reliable conclusions, we are heartened by the fact that the sign of the residual is not constant and that an estimate of sigma based on the range of residuals is consistent with the adopted one-sigma level of ±0.4 m.

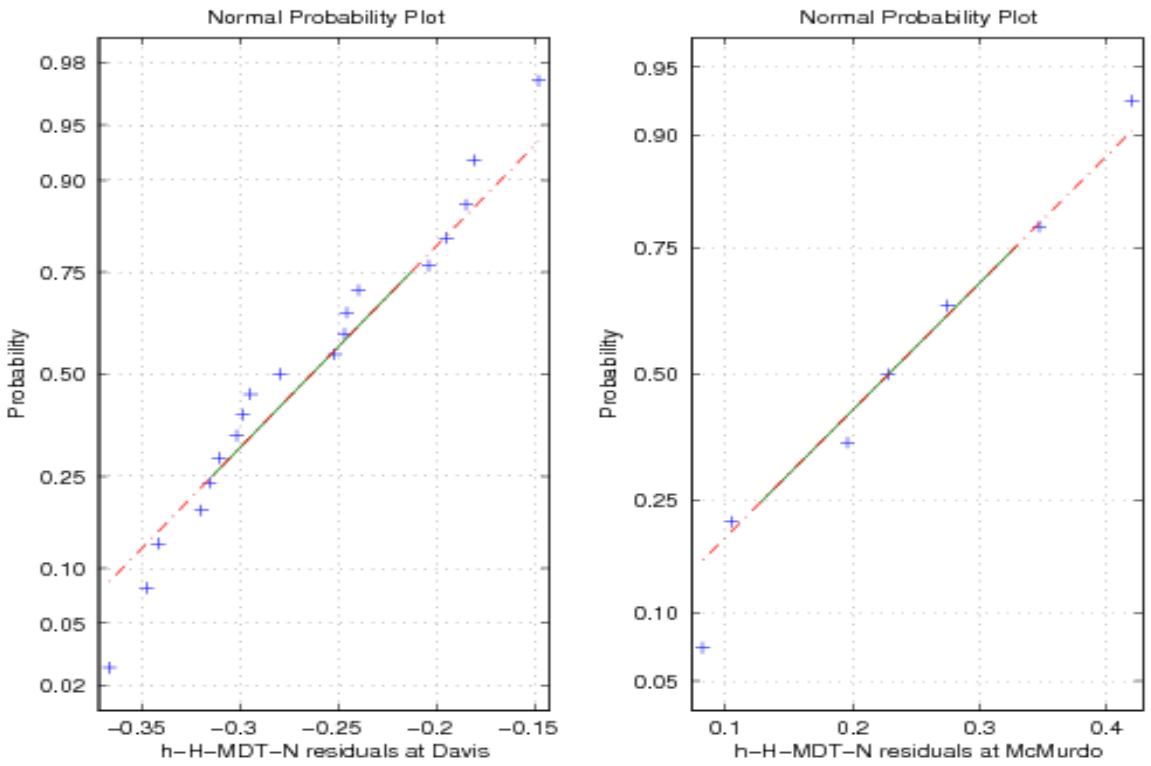


Figure 7: Normal probability plots of the residuals, h-H-N, at Davis and McMurdo. The solid line is the regression line from the inter-quartile range of 25 to 75 percent. The dashed line is the extension of this regression line to the full data set.

Figure 7 shows the behaviour of residuals at Davis and McMurdo where levelling data extended beyond the immediate station perimeter. Figure 7 shows that the data is normally distributed. This local normally distributed feature supports the hypothesis that the bias is principally due to regional issues in EGM2008 and/or DNSC08_MDT and that Antarctic levelling operations, which are frequently performed in adverse conditions, are usually consistent with third-order geodetic standards (Intergovernmental Committee on Surveying and Mapping 2007), which is the normal operational goal.

5. Discussion and Conclusion

It is clear from the pass band and individual station data that there are significant biases in EGM2008 and/or DNSC08_MDT over East Antarctica. From the very small sample, it is clear that these biases will approach and may even exceed one metre even though the formal one-sigma value is ±0.4 m. This study indicates that separating the relative contributions of the Earth Gravity Model and the MDT model will remain a challenge for the immediate future. We think that this is achievable by two methods.

1. By collecting data in regions showing anomalous behaviour. Two candidate areas are the Mirny/Bunger Hills region at about 100°E and Synder Rocks, about 100 km west of Casey. Both regions are currently regularly visited for other activities including the maintenance of Automatic Weather Stations, AWS.
2. By utilising the resources being assembled by the Antarctic Geoid Project (Scheinert 2005) and ICECAP (http://www.ig.utexas.edu/research/projects/icecap/) for local and regional models.

The importance of this study for Antarctic glaciology is clear. It is that EGM2008 and/or DNSC08 in their current forms are not entirely reliable products in East Antarctica for climate studies, where surface changes are needed to a precision level of 0.1 m to 0.2 m, which is consistent with the precision level of the old optical surveys and more modern satellite altimeter data. However, EGM2008 is likely to be of great value for many other applications including airfield construction and preparations.

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