

Evaluation results of the Earth Gravitational Model EGM08 over the Baltic countries

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Abstract. Earth's geopotential model (EGM) in conjunction with regional terrestrial gravity data are often used in regional geoid determination. Thus, significant enhancements are expected due to release of the new high resolution Earth Gravitational Model EGM08. Accordingly, this study evaluates the performance of the EGM08 model over the Baltic Sea region with emphasis to Estonia, Latvia and Lithuania. Several different sets of the "ground truth" data are used in the comparisons. First, the EGM08-derived height anomalies are compared with an existing regional geoid model. The detected discrepancies range within ± 0.3 m with a mean of -0.02 m, whereas the standard deviation (STD) of the discrepancies amounts to 0.08 m. The largest discrepancies occur in the areas where only a few data points were available either for the regional geoid modeling or at the EGM08 compilation, or both. Second, the free-air gravity anomalies at the terrestrial data-points are compared with the EGM08-derived anomalies. The STD of the anomaly discrepancies is 2.6 mGal. Finally, the EGM08 model is validated with respect to GPS-levelling data. The STD of detected discrepancies is 0.06 m, with a mean of 0.49 m. Thus, the EGM08 based quantities agree reasonably well with the tested datasets. Evidently, most of the available gravity data in the Baltic Sea region appear to be utilised at the EGM08 construction.

Keywords: geopotential model, geoid, GPS-levelling.

1 Introduction

A new combined Earth gravitational model EGM08 (Pavlis et al, 2008) was released to the public in 2008. EGM08 takes advantages of recent satellite, terrestrial gravity, elevation and altimetry data. This activity is conducted by the National Geospatial-Intelligence Agency (NGA) of the USA. The resolution of the EGM08 is $5'$ (corresponding to 9 km, i.e. to the spectral degree of ca 2160), also the global accuracy of the EGM08 is expected to be superior over earlier EGM-s.

Regional improvements of global geoid models can be obtained by modifying Stokes's integral formula (Stokes, 1849). When solving the Stokes problem, strictly speaking, gravity anomalies over the entire Earth are required. In practice, however, the data availability is limited to some spatial domain (Ω_{ψ_0}) around the computation point. Modified Stokes's formula (first proposed by Molodenskii et al., 1960) combines local terrestrial gravity anomalies and the EGM-derived long-wavelength component of the geoid. For instance, a generalized Stokes scheme (cf. Vaniček and Sjöberg, 1991) can be used

$$N(\Omega) = \frac{R}{4\pi g_0} \iint_{\Omega_{\psi_0}} S^L(y) \left[\Delta g(R, \Omega) - \sum_{n=2}^L \Delta g_n(R, \Omega) \right] d\Omega' + \frac{R}{2g_0} \sum_{n=2}^L \frac{2}{n-1} \Delta g_n(R, \Omega) \quad (1)$$

where R is the mean radius of the Earth; y is the geocentric angle, the modified Stokes function $S^L(y)$

can be computed according to some algorithm (e.g. Wong and Gore (1969); Vaníček and Kleusberg (1987); Sjöberg (1991), among others); g_0 is the normal gravity at the reference ellipsoid, $\Delta g(R, \Omega)$ is terrestrial gravity anomaly on the geoid, Ω denotes a pair of geocentric coordinates (the spherical co-latitude q and longitude λ), $d\Omega'$ is an infinitesimal surface element, y_0 is the radius of the integration cap, L is the modification degree, Δg_n are the harmonics of the EGM-derived gravity anomaly.

Due to availability, quality, and type of data, the characteristics of an EGM vary regionally. Hence, the performance of any EGM needs to be validated in a regional scale by comparisons with other external data sets that depend on the same gravity field. Traditionally, the accuracy of the regional geoid modelling has been assessed by using GPS-levelling points. Apparently, the computations of new regional geoid models will also be based upon the global EGM08 model, the testing of which is necessary to assess its suitability for this task. In this contribution three different sets of the “ground truth” data are used over the three Baltic countries - Estonia, Latvia and Lithuania. First, the EGM08-derived height anomaly is compared with an existing regional geoid model. Second, the free-air gravity anomalies at the terrestrial data-points are compared with the EGM08-derived anomalies. Thereafter the EGM08 model is validated with respect to the GPS-levelling data. The differences between a preliminary PGM07A and the final EGM08 models over the Baltic countries are discussed. Further comparisons reveal that there is still some space for further improvements of the contemporary EGM-s. Actions needed for assembling more consistent combined geopotential models are suggested as well. A brief summary concludes the paper.

Note that the study results have been reported partly in the international conference Gravity, Geoid and Earth Observation (GGEO), held in June 2008 in Chania, Greece. Since this contribution contains some more details, it can be considered as an extended report of Ellmann (accepted). In addition, this work includes further evaluation results of the EGM08 over the Baltic countries.

2 Target area

The EGM08 performance is examined within the following geographical boundaries: $53.83^\circ < j < 60.06^\circ$; $19.97^\circ < l < 28.52^\circ$, see Fig. 1. Thus, in addition to Estonia, Latvia and Lithuania the target area includes partly also Russia, Belarus, Poland and Finland, together with a large portion of the Baltic

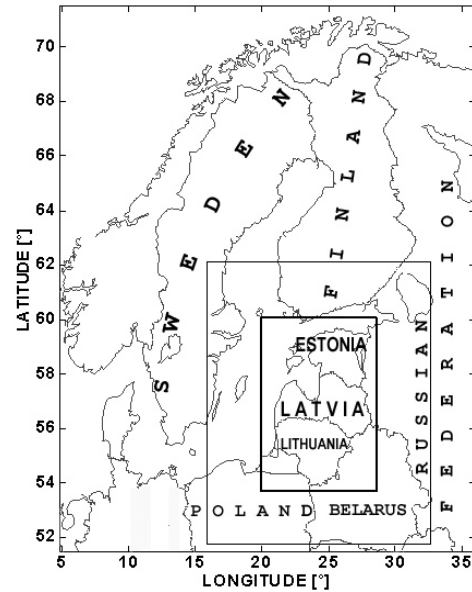


Fig. 1 Location of the target area (enclosed by the bold rectangle). External rectangle denotes the data area borders in the BALTgeoid-04 (Ellmann, 2005) computations.

Sea. The elevation extremes are 0 m at a shoreline and 318 m in southeast Estonia, whereas most of the target area comprises of sea and topography below 100 m. Due to such low topography no significant numerical differences (3 mm at most) between the geoid and height anomaly occur over the chosen target area.

3 Comparisons with a regional high-resolution geoid model BALTgeoid-04

3.1 Regional BALTgeoid-04 model

A recent Baltic geoid model was computed by Ellmann (2004 and 2005). In his study the geoidal heights were estimated by the least squares modified Stokes's formula (cf. Sjöberg, 1991).

The definition of the main computation criterias (such as the modification limit $L = 67$, the radius of the integration cap $y_0 = 2^\circ$, etc) is explained in detail by (ibid.). An early GRACE-derived („satellite-only“) GGM01s model (Tapley et al., 2004) was used as the reference model for computing Δg_n in Eq. (1). The resulting $1.5^\circ \times 3^\circ$ geoid model is depicted in Fig. 2. The geoidal heights in the target area vary between 15 m and 30 m, with the regional downslope trend from southwest toward northeast. The geoid model is mainly smooth (with a STD of the mean ~ 3 m), but it includes some local irregula-

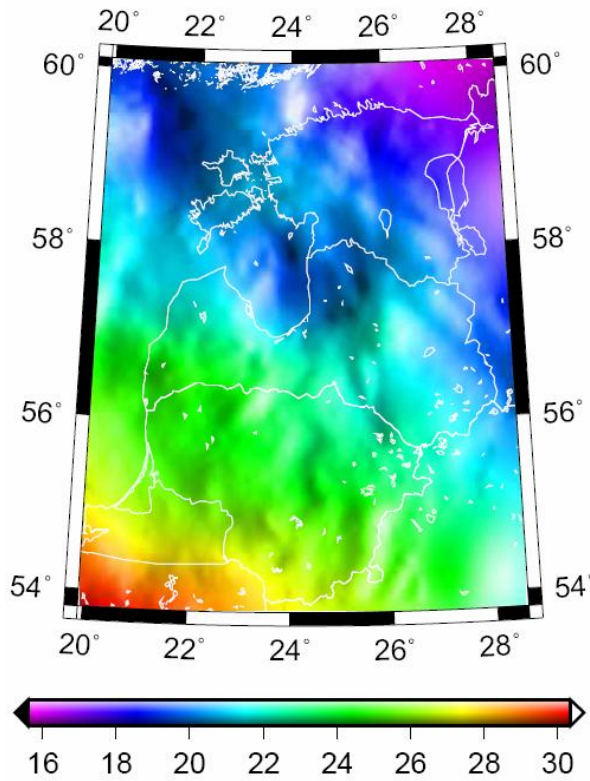


Fig. 2 The Baltic gravimetric geoid model BALTgeoid-04 (Ellmann, 2005). Geoidal heights are given with respect to the GRS-80 reference ellipsoid. Unit is metre. The total area of the image corresponds to 300 000 km².

rities in the NW part of the target area. Their location is correlated with the local anomalies of the gravity field (cf. Ellmann, 2004, Fig. 2.3, the anomaly range at the terrestrial gravity points are also shown in Fig. 4 of the current paper). The quality of the BALTgeoid-04 model was assessed from the comparisons with the GPS and levelling datasets. The same sets of the control points will also be used for the evaluation of the EGM08-derived height anomalies, therefore some more information on these data is spared for Section 5.

The GPS-levelling points form a surface, which is called here the “geometric geoid model” ($N_{geom} = h - H$). The following STD value of the discrepancies between the BALTgeoid-04 and N_{geom} were achieved: over the whole of the Baltics 5.8 cm, in Estonia 4.0 cm, in Latvia 6.0 cm and in Lithuania 5.7 cm, respectively. It is also concluded that the accuracy of the BALTgeoid-04 model is at least of the same level as is the accuracy of the used control points (Ellmann, 2005).

3.2 Accounting for the differences between the EGM08 and GRS-80 parameters

The BALTgeoid-04 geoidal heights are defined with respect to the GRS-80 (Geodetic Reference System; Moritz, 1992) ellipsoid. Also the physical constants of the GRS-80 are used for computing the normal gravity field in the Baltic countries. Furthermore, the GPS-derived geodetic heights are reckoned from the ETRS89 (European Terrestrial Reference System) oriented GRS-80 ellipsoid.

As is customary in geodesy, the mass of the reference ellipsoid is chosen to be equal to the mass of the Earth, and the origin of the reference ellipsoid is placed at Earth’s mass centre. However, in reality the EGM parameters may differ from the corresponding parameters of the adopted geodetic reference ellipsoid. Thus, the differences between the defining constants (i.e. gravity-mass constant GM , and the major semi-axis a of the ellipsoid versus reference radius for the spherical EGM) of the used GGM and adopted geodetic reference ellipsoid should be considered. The scaling can be introduced via zonal harmonics of the reference ellipsoid by an approach described in Vaniček and Kleusberg (1987, Sect. 5), see also Kirby and Featherstone (1997) and Smith (1998). In the discussion below the EGM-related values will be denoted by the subscript “EGM”, whereas the subscript “GRS” denotes the geodetic reference ellipsoid related quantities

It should be noted that the EGM08 geopotential model utilises $GM_{EGM} = 398600.4415 \cdot 10^9 \text{ m}^3 \cdot \text{s}^{-2}$, whereas $GM_{GRS} = 398600.5 \cdot 10^9 \text{ m}^3 \cdot \text{s}^{-2}$. The Earth’s gravitational potential and its derivatives (such as the disturbing potential, gravity anomaly and geoidal height) can be expressed in terms of an infinite series of spherical harmonics outside the attracting masses of the Earth. Since the EGM08 coefficients are referred to the bounding sphere with some radius a (the value $a_{EGM} = 6378136.3 \text{ m}$ is adopted at the compilation of the EGM08, whereas $a_{GRS} = 6378137 \text{ m}$), then the EGM derived quantities, strictly speaking, ought be computed on the surface of the bounding sphere (or above it). However, the gravity field related quantities can be more or less safely computed inside of this sphere, as long as the evaluation point remains outside the topographic masses. Due this, the EGM-s are better suited for computing the ground related gravity quantities, such as the height anomaly (cf., Molodenskii et al., 1960), rather than the geoid. Note that over the continents the latter would require computations inside the topographic masses.

3.3 The EGM08-derived height anomalies

The „tide-free“ version of the EGM08 model (the file *EGM2008_to2190_TideFree.gz*, retrieved from URL: http://users.auth.gr/~kotsaki/IAG_JWG/EGM08_intro.html, retrieved April, 2008) contains fully-normalized, unitless spherical harmonic coefficients, complete to degree and order 2159, plus additional coefficients extending to degree 2190 and order 2159. The EGM08-derived height anomalies z (at the topographic surface, with the geocentric radius of $r_t = r_g + H$) were computed by the following formula:

$$z_{EGM}(\Omega) = \frac{GM_{EGM} - GM_{GRS}}{r_t g_T} + \frac{GM_{EGM}}{r_t g_T} \sum_{n=2}^{2190} \left(\frac{a_{EGM}}{r_t} \right)^n \sum_{m=0}^n \{ \Delta \bar{C}_{nm} \cos ml + \bar{S}_{nm} \sin ml \} \bar{P}_{nm}(\cos q) \quad (2)$$

where the normal gravity γ_T is referred to the surface of the telluroid (with the geocentric radius of $r_T = r_{GRS} + H$); $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are fully normalised spherical harmonic coefficients, of degree n and order m ; $\bar{P}_{nm}(\cos q)$ are fully normalised associated Legendre functions. The first term on the left hand side of Eq. (2) represents the zero degree geoid scaling term, which is due to the difference between the GM -values of the EGM08 and that of the reference ellipsoid (GRS-80). Using $R = 6371$ km and $g = 981$ Gal the zero degree geoid scaling term becomes -0.936 m. This value will be added to the EGM08-derived height anomalies. The (residual) zonal coefficients $\Delta \bar{C}_{nm}$ account also for the differences between the reference radius of the EGM08 and semi-major axis of the GRS-80.

The above principles have also been realised in the *harmonic_synth_v02.f* code (by Holmes and Pavlis, version 05/01/2006, retrieved from the NGA webpage http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/new_egm.html), which is used in the validation of the EGM08 geopotential model in the present study. However, the program does not account for the influence of the zero-degree term. Therefore, the EGM08-derived quantities are corrected for the missing zero-degree term.

Strictly speaking, Eq.(2) should also account for the difference between the gravity potential on the surface of the geoid (W_0) and the normal gravity potential on the surface of the normal ellipsoid (U_0), i.e. the term $-\frac{W_0 - U_0}{g}$. Recall that in an ideal case

$W_0 = U_0$. Several estimates of W_0 have been proposed in the geodetic literature over the past

decades. For a recent review of the gradual improvements see Bursa et al (2007) and references therein. Note that many studies of the W_0 rely upon the satellite altimetry (such as TOPEX / Poseidon) results. In this case, however, the data coverage is not truly global, since no data from sub-polar latitudes have been included in such solutions. Some others combine the GPS, levelling and tide-gauge data into a common solution, see e.g. Ardalan et al (2002). Such an approach may provide the best match with the local vertical datum over the given study area. Both approaches can be considered being complimentary to each other to a certain extent. However, at the present the estimates of the W_0 value can still be improved further. Therefore we exclude the $-\frac{W_0 - U_0}{g}$ term from the present

comparisons. It is also reasonable to assume, that there may be a certain consistency between the W_0 value and the adopted set of GM and a values at the compilation of the EGM08 model. All in all, after the proper determination of the W_0 value its contribution can be added to the results of the present study. Note that it will manifest only as a simple one-dimensional bias of the EGM08 derived gravity field quantities.

The EGM08 height anomalies were computed at the grid nodes of the BALTgeoid-04 model. The conceptual differences between the geoid and height anomalies are well known, see e.g., Heiskanen and Moritz (1967, Chap. 8-3). Recall, however, that over the selected target area these differences are numerically insignificant. These differences are neglected in the following comparisons without affecting the objectives of the present study.

3.4 The results

The discrepancies between the BALTgeoid-04 model and EGM08 height anomalies (cf. Eq. (2)) at the BALTgeoid-04 grid nodes (altogether 250×172 points) are depicted in Fig. 3. Here we focus only on the general features of the discrepancies. The range of the detected discrepancies varies within ± 3 dm over the whole target area. The largest discrepancies are located outside the borders of Estonia, Latvia and Lithuania. Within the borders of the three countries the absolute range of the discrepancies remains smaller than 15 cm. Full statistics of the comparison can be found in Table 1 (see the last section of this paper).

The nature of the discrepancies between the two models appears to be quite complicated. Note that the discrepancies in the centre of the target area seem to possess a spectral content below degree 100.

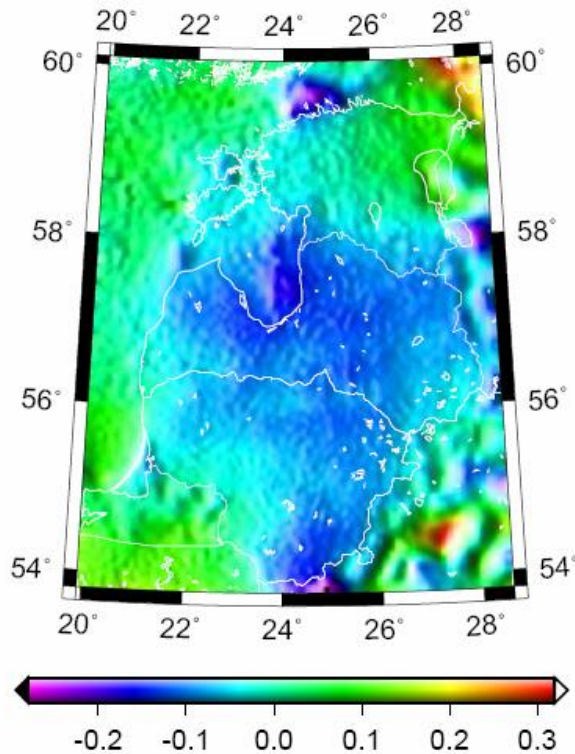


Fig. 3 Discrepancies between the BALTgeoid-04 model and EGM08 height anomalies ($n_{\max} = 2190$) at the BALTgeoid-04 grid nodes (altogether 250×172 points). The discrepancies range from -0.289 m to $+0.338$ m with a mean of -0.025 m. Generally, the EGM08 height anomalies appear to be slightly higher than the BALTgeoid-04 model. Standard deviation of the detected discrepancies amounts to 0.077 m.

It should be noted that the long wavelength component of the GGM01s (which was used as the BALTgeoid-04 reference model, with the degree $n_{\max} = 67$) and EGM08 model is very similar. Their long wavelength differences (both developed up to $n_{\max} = 67$, cf. Eq. (2)) do not exceed ± 4 cm over the target area. This may indicate the presence of the systematic biases among the terrestrial datasets used for the computations of the BALTgeoid-04 and EGM08 model.

Alternatively, the discrepancies could either be due to: (i) inadequate reproduction of the spectral content of the disturbing potential from the truncated Stokesian integration (cf. the first term on the right hand side of Eq. (1)); (ii) deficiencies of the harmonic analysis when determining the EGM08 spherical harmonic coefficients; or (iii) both.

All in all, within the land masses of the three countries (Estonia, Latvia and Lithuania) the agreement between the BALTgeoid-04 and EGM08-derived height anomalies is reasonable, see Fig. 3. It should be noted that the terrestrial data coverage

(used for the BALTgeoid-04 model, see Fig 4) is satisfactory there.

Note that the discrepancies between the BALTgeoid-04 and EGM08-derived height anomaly possess shorter wavelength features over the eastern part (especially in SE) of the target area, where only a few data were available for the BALTgeoid-04 computations. Hence, a more complete dataset was most likely available for the compilation of the EGM08 over the eastern part of the target area. Also at some offshore spots, well covered with the terrestrial data, the range of detected discrepancies appears to be unreasonably large. The search for an explanation of the detected discrepancies over the Baltic Sea prompts us to have a closer look at the quality and coverage of the regional terrestrial data.

4 Comparisons with the historical terrestrial gravity survey data

The gravity survey data (altogether 42559 points) used in the current comparisons were obtained (in 2001) from the Danish National Survey and Cadastre, the authorized holder of the Nordic-Baltic gravity database. This international database is created and maintained within the frame of the activities of the Nordic Geodetic Commission. The national contact persons deliver the data to the database, whereas their responsibility is to ensure the quality and internal consistency of the national datasets.

The coverage of the terrestrial data points within the target area is more or less satisfactory, except the eastern part, where only a small number of gravity points is available, see Fig 4.

Data, which are collected during several decades with different methods and equipment and by different nations and specifications, requires careful analysis before further processing.

The Estonian gravity survey was performed by the Institute of Geology at the Estonian Academy of Sciences in 1949-58. The total number of Estonian gravity survey points exceeds 4000, yielding a density of 1 survey point per 10 km^2 . The accuracy of these data is (very optimistically) claimed to be $< 1 \text{ mGal}$. A register of Latvian and Lithuanian gravity points is mainly reconstructed from the 1: 200 000 paper maps in conjunction with an obsolete global topographic model. The accuracy of such data remains unknown (Kaminskis and Forsberg, 1997). It should be emphasised, however, that the gravity surveys of the three Baltic countries have historically been related to the same vertical system and gravity datum.

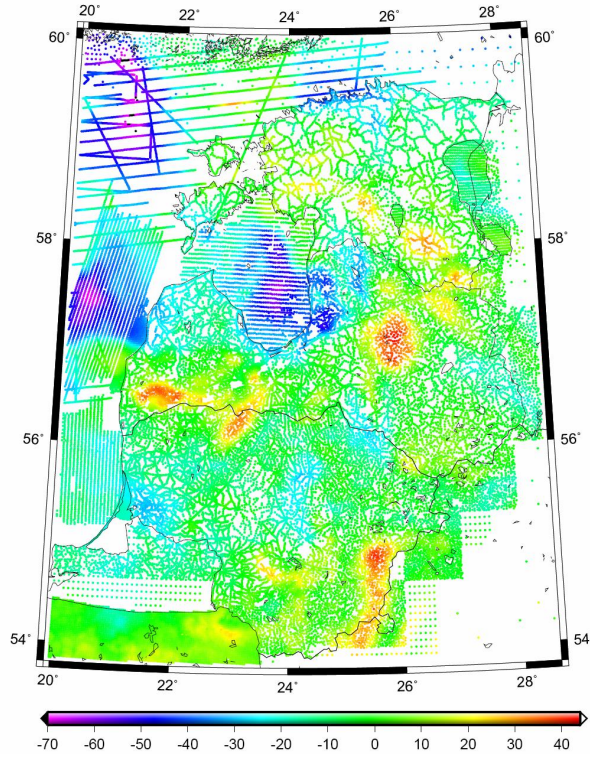


Fig. 4 Free-air anomaly $\Delta g(r_t, \Omega)$ at the terrestrial data points. The anomaly values range from -73 to +44 mGal, with a mean of -7 mGal. The STD of the anomaly values is 17 mGal. The colors of the dots are proportional to the range of the anomaly, cf. the colorbar.

Note that the NKG 1997 marine and Baltic Sea 1999 airborne gravity surveys have significantly improved the data coverage over the Baltic Sea. The accuracy of these datasets is estimated to be ~ 2 mGal (Forsberg, 2001). Aligned (mainly in E-W direction) data-points over the Baltic Sea indicate the location of the aero-gravity survey tracks (see Fig. 4). In addition, the marine gravity data within the Riga Gulf and nearby Latvian coastline (aligned in NE-SW direction) have been made available as well.

An extensive analysis of Nordic-Baltic gravity data is summarized in Ellmann (2001). The study revealed the presence of some (presumably very small) systematic discrepancies between the used datasets. The systematic errors in Nordic-Baltic gravity datasets have also been noticed by other authors, see e.g., Omang and Forsberg (2002), Jürgenson (2003).

The elimination of these possible offsets is outside of the scope of the present study, since it requires a multilateral international involvement. Hence, any possible inherent systematic bias

between the national datasets is simply ignored in earlier studies and in this comparison.

The EGM08 derived free-air anomalies Δg (at the topographic surface r_t) are computed at the locations of the Nordic-Baltic terrestrial gravity points by the following formula (Heiskanen and Moritz, 1967, Eq. 2-151c.):

$$\Delta g_{EGM}(r_t, \Omega) = -\frac{GM_{EGM} - GM_{GRS}}{r_t^2} + \frac{GM_{EGM}}{r_t^2} \sum_{n=2}^{2190} (n-1) \cdot \left(\frac{a_{EGM}}{r_t} \right)^n \sum_{m=0}^n \{ \Delta \bar{C}_{nm} \cos ml + \bar{S}_{nm} \sin ml \} \bar{P}_{nm}(\cos q) \quad (3)$$

Note that the first term on the right hand side is the zero degree scaling term of the gravity anomaly, which amounts to +0.144 mGal (to be added to the EGM08-derived anomalies).

Presumably, many datasets of the Baltic Sea region most likely do not contain the atmospheric correction on the gravity measurements, which is recommended by the IAG (see Moritz 1992, Sec. 5).

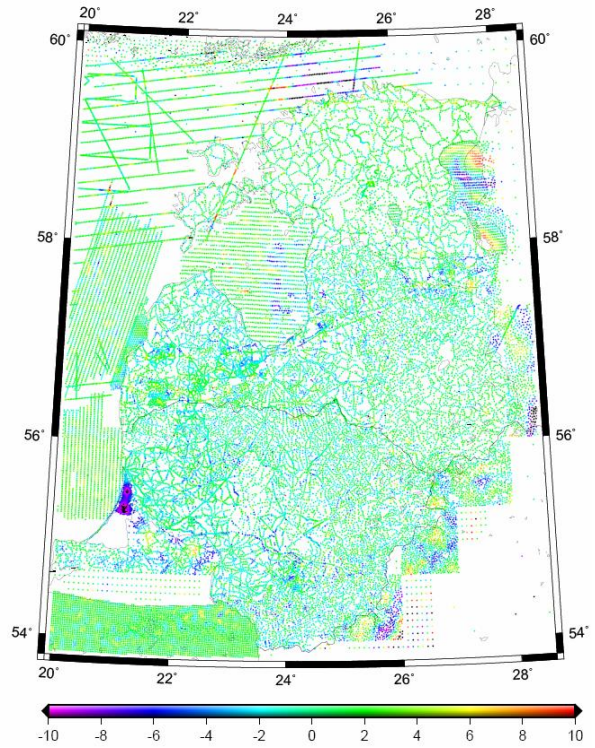


Fig. 5 The discrepancies between the terrestrial and EGM08-derived free-air gravity anomalies. The discrepancies $[\Delta g(r_t, \Omega) - \Delta g_{EGM}(r_t, \Omega)]$ range from -18 to +18 mGal, with a mean of 0.05 mGal. The STD of the discrepancies is 2.6 mGal. The colors of the dots are proportional to the range of the detected discrepancies, cf. the colorbar. The black dots denote the locations, where the absolute range of differences exceeds 10 mGal.

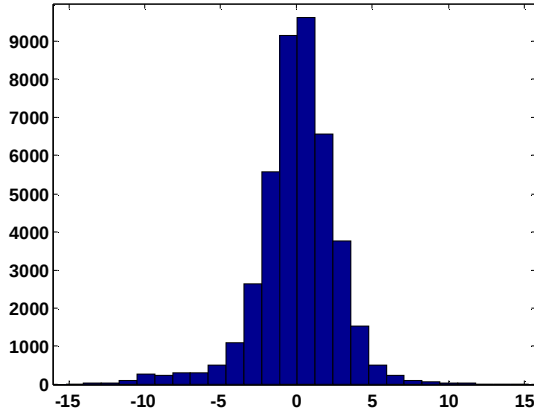


Fig. 6 Histogram of discrepancies between the terrestrial data and EGM08-derived free-air gravity anomalies. Unit is mGal. The total number of the points is 42559.

Recall that at the sea level this correction amounts to +0.87 mGal. In contrast, the attraction of the “beneath” atmospheric masses is naturally embedded in the spaceborne gravity results.

Therefore, for the sake of consistency of comparisons all the terrestrial data were corrected for the attraction of the atmospheric masses (dg_{atm}) by the following formula (Wenzel, 1985):

$$dg_{atm} \approx 0.874 - 9.9 \cdot 10^{-5} H + 3.5625 \cdot 10^{-9} H^2 \quad (4)$$

where H is the height above the sea level, the results are in mGal.

The discrepancies between the measured and EGM08-derived gravity anomalies (terrestrial minus EGM08) vary from -18 to +18 mGal, with a mean of +0.05 mGal, see Fig. 5. The histogram of discrepancies is shown in Fig. 6. In general, the EGM08-derived gravity anomalies agree reasonably well with the ground truth. However, there are some (offshore) areas, where the discrepancies are much larger than the regional average, see Fig. 5. This may indicate that different (from those used in this comparison) datasets were used (or no latest data were available) at the compilation of the EGM08. In particular, a relatively powerful negative anomaly over the Gulf of Finland (at 59.5°N & 24.5°E, see Fig. 4) remains “unnoticed” by the EGM08 data. Other such an example is the Kuroshio lagoon at 55.5°N & 21.5°E, where the discrepancies possess a systematic nature. Note that both areas are densely covered with the terrestrial data. Such discrepancies should be studied and ultimately resolved in future gravity field and geoid modelling works.

5 Comparisons with GPS-levelling data

As is well known, inter-comparison of a geoid model, GPS-derived geodetic heights, and spirit-levelled (normal or orthometric) heights at discrete points gives a reasonable indication of the geoid model’s accuracy. Thus the further validation of the EGM08 model relies on nationwide sets (one for each country) of high-precision geodetic points, for their locations see Fig. 7.

First, the same constellation of the control points as used at the evaluation of the BALTgeoid-04 model will also be employed here. For all points the geodetic heights from GPS-measurements as well as levelling heights are available. The geodetic coordinates of the control points are related to the respective national realization of the new European Terrestrial Reference System ETRS-89. The spirit-levelled normal heights of all points refer to the Baltic Height System 1977 (Kronstadt tide-gauge).

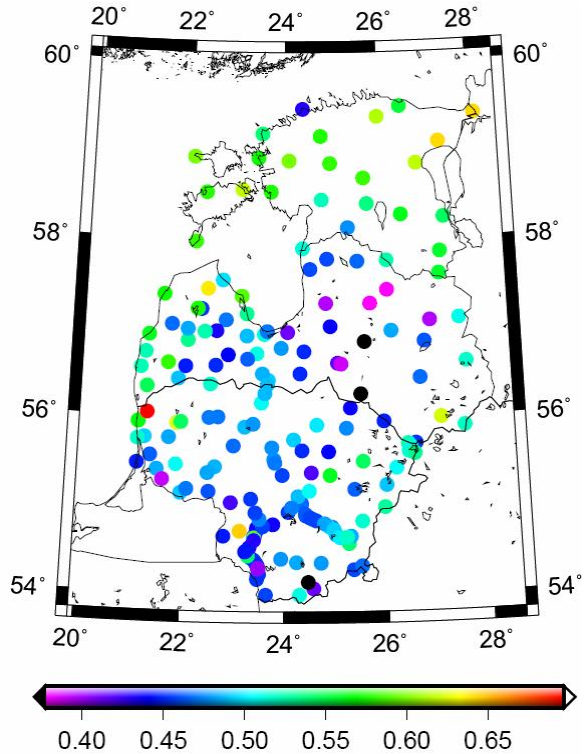


Fig. 7 Distribution of the Baltic GPS-levelling data (altogether 189 points) and their differences from the EGM08 height anomalies (developed up to degree 2190). The discrepancies $[N_{geom} - z_{EGM}]$ range from +0.346 to +0.697 m, with a mean of +0.493 m. The STD of the discrepancies amounts to 0.060 m. The colors of the data-points are proportional to the range of the detected discrepancies (cf. the colorbar). Unit is metre. The levellings are referred to the Kronstadt tide-gauge observations.

The average distance among 26 evenly distributed Estonian control points is 50 km. The combined error of GPS-derived and spirit-levelled heights does not exceed 2-3 cm, most likely. Note that the geodetic heights are computed from the same GPS campaign and most of these points are directly connected to the high-precision levelling network. The Latvian and Lithuanian datasets (53 and 110 points, respectively) are denser. However, the accuracy of the used GPS-levelling points seems to be rather heterogeneous.

The common Baltic geometric geoid is represented by the sum of the three national datasets (189 points). The numerical statistics of the detected differences are presented in Table 1. In particular, the mean of the differences reveals a positive offset (+ 0.49 m) of the Kronstadt vertical datum from the EGM08-derived global geoid. Note however, that this offset depends also on the W_0 value to be adopted in future computations. For instance, a recent estimate $W_0 = 62\,636\,855.75 \text{ m}^2\text{s}^{-2}$ is published by Ardalan et al (2002). Considering also the GRS-80 related $U_0 = 62\,636\,860.85$ then the term $-\frac{W_0 - U_0}{g}$ becomes +0.52 m, which should be

added to the EGM08 derived geoid model. In other words, after implementation of the aforementioned assumption the Kronstadt vertical datum will practically coincide with the mean level of the world oceans. On the other hand the Ardalan et al. (2002) approach constrains the regional GPS, levelling and tide-gauge data (surrounding the Baltic Sea only) into a common solutions, which may not necessarily be representable for the whole of the globe.

The resulting STD of differences 6.0 cm indicates almost the same level of accuracy, as it was achieved from the BALTgeoid-04 modelling (Ellmann, 2005). Such comparisons are also produced on a country-by-country basis. The corresponding statistics can be found in Table 1. In particular, the STD of the discrepancies (after removing the mean) as of 0.048, 0.063 and 0.048 m were achieved for the Estonian, Latvian and Lithuanian GPS-levelling points, respectively. Very similar estimates were obtained also from the comparisons with the BALTgeoid-04 model (Ellmann, 2004, Table 2.3).

Note that the Estonian GPS-levelling geoid appears to be somewhat “higher” than the Latvian and Lithuanian geometrical geoid models, see the mean values (+0.56 m versus +0.48 m) in Table 1. It should be noted that no temporal changes in the levelled heights were considered in this study. However, the Estonian points are affected by the Fennoscandian post-glacial rebound. Conversely, the

Latvian and Lithuanian points are mostly located outside the land-uplift zone. Since the levellings have been performed over relatively long timespan then the Estonian solution may be contaminated with the land-uplift effect. Thus, the used GPS-levelling points cannot be considered as an entirely errorless dataset.

6 Differences between the EGM08 and a preliminary PGM07A model

As a matter of fact a Working Group (WG) was established by the International Association of Geodesy (IAG) for an independent and coordinated evaluation of the EGM08 quality already in 2006. A preliminary PGM07A model (Pavlis et al., 2007) was released to the WG members for validation in July 2007. The testing results were submitted to the NGA/EGM08 development team for the ultimate „fine-tuning“ of the model in October 2007. As a result the final EGM08 model differs somewhat from its preliminary version. The detected discrepancies between the EGM08 and PGM07A derived gravity quantities in the Baltic countries are shown in Figs. 8 and 9. The differences are quite significant, exceeding a dm level in terms of the

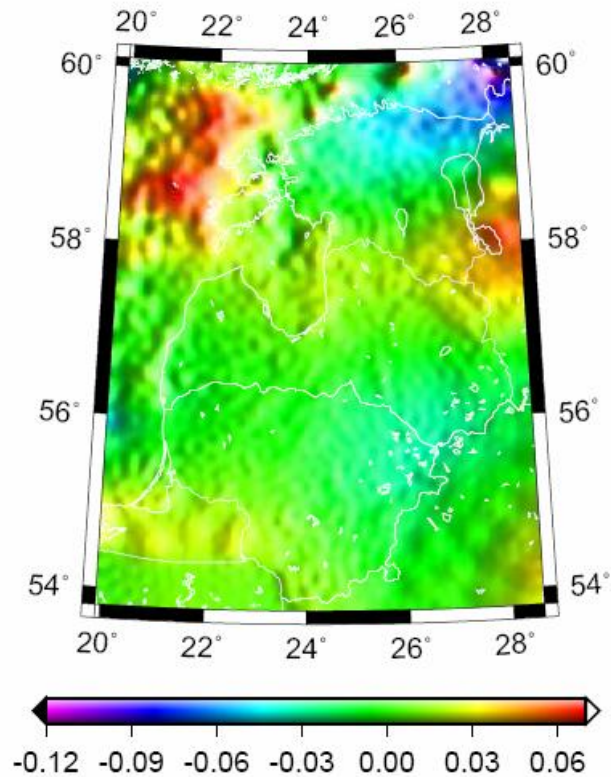


Fig. 8 Discrepancies between the EGM08 and PGM07A-derived height anomalies ($n_{\max} = 2190$). Unit is metre.

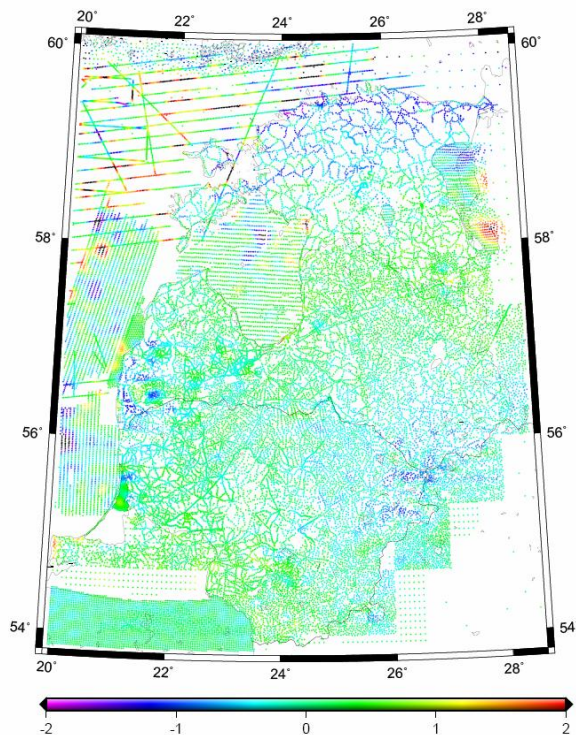


Fig. 9 Discrepancies between the EGM08 and PGM07 derived free-air gravity anomalies ($n_{\max} = 2190$). The colours of the dots are proportional to the range of the detected discrepancies, cf. the colourbar. Unit is mGal. The black dots denote the locations, where the absolute range of differences exceeds 2 mGal

geoidal heights (a few mGal in terms of gravity anomaly). This could be due to the subsequent downweighting of the terrestrial data, at the same time assigning more weight to the satellite info (priv. comm. N. Pavlis, June 2008). In the study area the most significant changes have taken place outside the land masses of Estonia, Latvia and Lithuania.

As a result the accuracy of the EGM08-derived Baltic geoid model at the GPS-levelling points is slightly worse than that of PGM07A. However, the deterioration is just marginal, just some 2-3 mm in terms of STD for each case. For instance, the STD of the PGM07A derived Baltic geometric geoid reached 0.058 m (cf. to that of EGM08: 0.060 m, cf. Table 1).

Apparently, such a downweighting of the terrestrial data has distorted the accuracy of the final EGM08 in the Baltic Sea region. Note that the terrestrial data coverage is rather dense in the study area. Thus, the strategy of assigning more weight to the satellite info at the EGM08 compilation may not be the most optimum in such areas. Conversely, this approach may provide better results in the areas with sparse and less reliable terrestrial data.

7 Toward future combined geopotential models

Intuitively, high-resolution and accurate global geoid models, such as newly released EGM08, create tools for unification of national height systems all over the globe. Can there be any further improvements? The answer is affirmative, since the geodetic community is expecting even more promising results from the dedicated gravimetric satellite missions. In this respect of particular interest is the first satellite gradiometry mission GOCE (Gravity field and steady-state Ocean Circulation Explorer) to be launched by the European Space Agency in 2009. This mission will allow reaching unprecedented accuracy for geopotential coefficients in the global scale and up to degree and order 270 (corresponding to the spatial resolution of 65 km). The GOCE will improve the intermediate wavelength information of the gravity field. However, the usage of the terrestrial data is still unavoidable for the proper recovery of the high-degree spectrum of the gravity field.

Intuitively, for the development of global high-resolution gravity models all the terrestrial data need to be referred to the modern gravity system, which is based on the absolute gravity measurements. Only such a global model will avoid drawbacks, which originate from the usage of different gravity datums. In other words, usage of such a global gravity system at the compilation of the geopotential models is a necessary precondition for the meaningful definition of the offsets among different vertical datums.

One of the main conclusions of the present study was that the EGM08 derived gravity quantities agree reasonably well with the terrestrial survey data in the Baltic Sea region. Apparently most of the historical terrestrial data have been utilised at the compilation of the EGM08.

Note however, that most of the data within the land masses of the Baltic countries have been collected before 1990-ies. Generally, the modern gravity networks were established decades after the historic gravity surveys. In mid 1990-ies a set of absolute gravity points was established in the Baltic countries. After publication of the absolute gravity measurement results (Mäkinen et al., 1996) the national gravity networks were re-adjusted, see e.g. Sas-Uhrynowski et al. (2002) and Oja (2008). Even though attempts were made to convert the historic survey results into the current gravity datum, the connections between the datasets remain still rather loose. More specifically, at areas the discrepancies

between different gravity data are not random at all. The following exercise is a clear example of this.

Most likely the gravity network points and the results of new surveys were not accessible at the compilation of the EGM08. For detecting the discrepancies between the absolute gravity datum, and the EGM08 derived gravity field the free-air anomalies were computed at the locations of the gravity points. Altogether 1957 new gravity points were available for the comparisons: 424 from Estonia, 1485 from Latvia and 48 from Lithuania.

The detected discrepancies between the newly measured and EGM08-derived gravity anomalies (terrestrial minus EGM08) vary from -10 to +10 mGal, with a mean of +0.14 mGal, see Fig. 10. The histogram of discrepancies is shown in Fig. 11. Such comparisons are also produced on a country-by-country basis. The corresponding statistics can be found in Table 1. In particular, the mean of the discrepancies as of +0.37, +0.08 and -0.31 mGal were achieved for the Estonian, Latvian and Lithuanian data-points, respectively. This shows also, that there are some systematic biases between the new terrestrial data and that of the EGM08. In Fig. 10 one may detect a few regions where the dis-

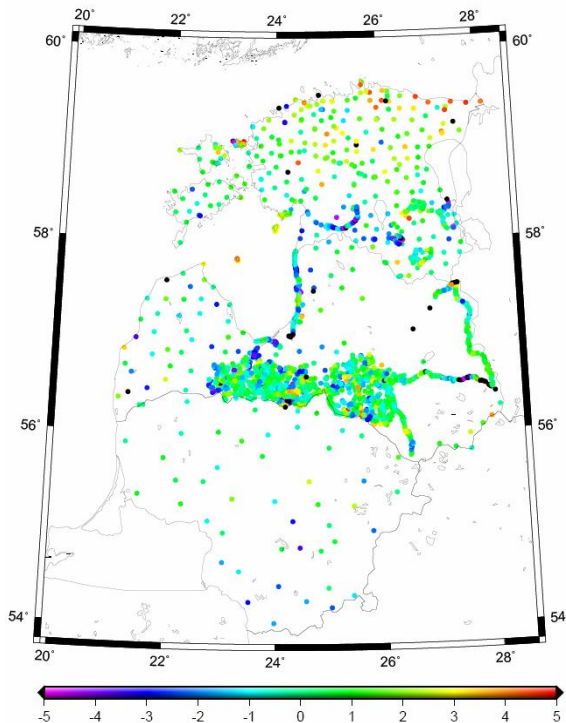


Fig. 10 The discrepancies between the gravity network and new survey points and EGM08-derived free-air gravity anomalies. The discrepancies $[\Delta g(r_t, \Omega) - \Delta g_{EGM}(r_t, \Omega)]$ range from -10 to +10 mGal, with a mean of 0.14 mGal. The STD of the discrepancies is 2.0 mGal. The colours of the dots are proportional to the range of the detected discrepancies, cf. the colourbar. The black dots denote the locations, where the absolute range of differences exceeds 5 mGal.

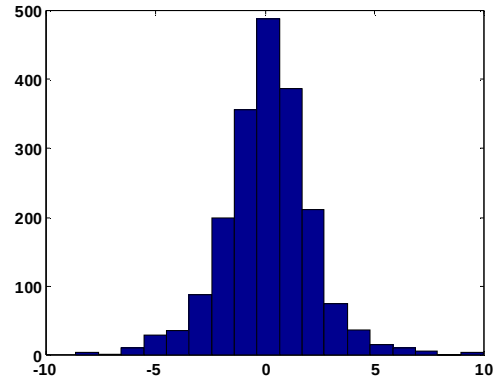


Fig. 11 Histogram of discrepancies between the gravity network and EGM08-derived free-air gravity anomalies. Unit is mGal. The total number of the points is 1957.

crepancies are having the systematic nature. Clearly, for the further improvement of the global model accuracy the historic gravity surveys need to be revised by the national contact persons. The results should be made available for the future EGM developers. This is a quite burdensome task, requesting international and well coordinated actions. However, this is needed for the sake of the consistency of the global gravity data.

8 Summary and conclusions

The performance of the EGM08 model was validated over the three Baltic countries - Estonia, Latvia and Lithuania. Three different sets of the “ground truth” were employed for this task.

First, the EGM08-derived height anomalies were compared with the high-resolution BALTgeoid-04 model. A reasonable agreement between the two models was detected. In particular, within the borders of Estonia, Latvia and Lithuania the absolute ranges of the discrepancies do not exceed 15 cm. Larger discrepancies (but not exceeding ± 3 dm) are related to the areas where only a few data points were available for the BALTgeoid-04 modelling.

Second, the free-air gravity anomalies at the terrestrial data-points were compared with the EGM08-derived anomalies. This test yielded the STD of the discrepancies ~ 2.6 mGal, which is quite comparable with an average accuracy of the (historical) gravity surveys in the region of interest.

Finally, the quality of the EGM08 model was assessed with several sets of the GPS-levelling data. It is concluded, that the overall accuracy of the EGM08-derived height anomalies in the Baltic countries is almost of the same level as is the accura-

Table 1. The EGM08 evaluation results.

Type of the comparison	Unit	Statistics				
		# of points	Min	Max	Mean	STD*
BALTgeoid-04 minus z_{EGM}	[m]	250 x 172	-0.289	+ 0.338	-0.025	0.077
$\Delta g(r_t, \Omega) - \Delta g_{EGM}(r_t, \Omega)$						
Historical survey data	[mGal]	42559	-18.713	18.819	-0.057	2.599
Modern Baltic data	[mGal]	1957	-8.639	+9.953	+0.136	2.046
Modern Estonian data	[mGal]	424	-5.985	+6.446	+0.375	2.272
Modern Latvian data	[mGal]	1485	-8.639	+9.953	+0.082	1.989
Modern Lithuanian data	[mGal]	48	-3.566	+2.627	-0.314	1.388
GPS-levelling (Kronstadt vertical datum)						
Baltic $N_{geom} - z_{EGM}$	[m]	189	+0.346	+0.697	+0.493	0.060
Estonian $N_{geom} - z_{EGM}$	[m]	26	+0.432	+0.642	+0.566	0.048
Latvian $N_{geom} - z_{EGM}$	[m]	53	+0.346	+0.638	+0.481	0.063
Lithuanian $N_{geom} - z_{EGM}$	[m]	110	+0.375	+0.697	+0.481	0.048

*Note that the STD computations are based on the “raw” discrepancies, i.e. without applying any trend removal.

cy of the used control points, i.e. a few cm. Certainly, this conclusion cannot be extended automatically to other regions. Recall, that the Baltic Sea region is well studied gravimetrically, whereas most of the collected gravity data appear to be utilised at the EGM08 construction.

Clearly, the reliability and availability of the terrestrial gravity data (and their subsequent inclusion to the global gravity model) over the region of interest are necessary preconditions for using the EGM08 global geoid (and its follow-ups) for defining offsets among different vertical datums worldwide. Apparently, further EGM improvements could be possible by revising the existing terrestrial data worldwide and also by adding the medium wavelength GOCE information. As a result, the future global geopotential models could lead to unification of different height systems not only within a continent but also overseas.

An appropriate geopotential model is also essential to determine the regional gravimetric geoid model accurately. The present study results indicate that the EGM08 based regional models may possess a great potential in providing further refinement of the regional geoid models.

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