Validation of GOCE Gravity Field Models in Germany

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Abstract

The fifth release of global gravity field models (GGMs) from the GOCE mission published in mid-2014 are based on observation data from the whole mission lifetime. High precision gravity field data sets in Germany are utilized to validate the GOCE GGMs of the fourth and fifth release processed by the time-wise (TIM) and the direct approach (DIR) as well as their internal uncertainty estimates. The comparisons are performed between EGM2008 and the GOCE GGMs up to spherical harmonic degree and order (d/o) 200 in terms of quasigeoid heights, gravity anomalies and vertical deflections. For the comparisons of GPS/levelling data and astrogeodetic vertical deflections with the GOCE GGMs, the different spectral characteristics of the data sets are considered by modelling the high frequency signals above d/o 200 by EGM2008 and detailed topographic information within the residual terrain model (RTM) approach. The RMS differences of all comparisons are introduced as observations within a least squares adjustment in order to estimate the commission errors of the GOCE GGMs as well as the uncertainties from all other terms involved in the comparisons, i.e. EGM2008, RTM effects and GPS/levelling. The commission errors of the fifth release GOCE GGMs up to d/o 200 are estimated to 2.5 cm and 2.4 cm for TIM5 and DIR5, respectively, as well as 0.66 mGal and 0.10″ for both models in terms of quasigeoid heights, gravity anomalies and vertical deflections, respectively. This is in good agreement with the formal error degree variances of TIM5, while for DIR5 a factor of 3 is proposed. Moreover, both fifth release GOCE GGMs show significant improvements of approx. 30% compared to their predecessors.

1 Introduction

The GOCE mission aims at providing the geoid and gravity anomalies with an uncertainty of 1-2 cm and 1 mGal, respectively, both at a resolution of 100 km, corresponding to spherical harmonic degree and order (d/o) 200 (e.g. Pail et al. 2011); this is equivalent to 0.15" for single vertical deflection components (Voigt and Denker 2014). The GOCE High-level Processing Facility (HPF) generated five releases of global gravity field models (GGMs), which are based on 2, 6, 12 and 27 months as well as 4 years of GOCE observations. The processing follows three different approaches, i.e. the direct (DIR), the time-wise (TIM) and the space-wise (SPW) approach. However, the releases of the space-wise (SPW) approach GGMs stopped after release two. The approaches are outlined in Pail et al. (2011). While the TIM GGMs are based on GOCE observations only, the DIR GGMs build combination solutions of GOCE data combined with multi-year GRACE and SLR (see also Table 1). Detailed information on the fifth release GOCE GGMs TIM5 and DIR5 can be found in Brockmann et al. (2014) and Bruinsma et al. (2014), respectively.

In order to ensure high quality GOCE products, various internal and external calibration and validation techniques are of vital importance. Focusing on the external validation of GOCE GGMs by external gravity field data sets, numerous studies have been carried out using various validation methods and global as well as regional data sets since the first release GOCE GGMs were published in 2010. The first release GOCE GGMs were evaluated e.g. by Gruber et al. (2011) and Hirt et al. (2011). Studies on the first to the third release GOCE GGMs can be found e.g. in Voigt and Denker (2014a), while the fourth release GOCE GGMs are analyzed e.g. by Yi and Rummel (2014), Godah et al. (2014) and Rexer et al. (2013). The performance analyses of GOCE GGMs up to the fifth release by HPF were recently presented by Gruber and Rummel (2014).

The objective of this study is to validate the GOCE GGMs of the fifth release by gravity field data sets in Germany. This includes analyses of the improvement relative to the fourth release GOCE GGMs and of the absolute performance as well as of the uncertainty estimates. In this context, the ultra-high degree spherical harmonic model EGM2008 (Pavlis et al. 2012) is employed as well as the current GPS/levelling data set and astrogeodetic vertical deflections in Germany. All GGM and terrestrial quantities were referred to GRS80 and the zero-tide system. Section 2 describes the internal uncertainty estimates of the GOCE GGMs under evaluation. In section 3, the gravity field data sets in Germany used for the evaluation of the GOCE GGMs are introduced and compared in terms of quasigeoid heights. Section 4 explains the applied validation methods and gives the results of the comparisons with the GOCE GGMs, which are used in section 5 within a common least squares adjustment in order to estimate the uncertainty contributions of all terms involved in the comparisons.

2 Internal Uncertainty Estimates of the Global Gravity Field Models

Error degree variances are commonly used for internal uncertainty estimates of GGMs. Figure 1 shows the positive square roots of the sum of the error degree variances in terms of geoid heights, gravity anomalies and single vertical deflection components for the GOCE GGMs under evaluation as well as for EGM2008 up to degree 250, hereinafter referred to as cumulative errors and used as commission errors. For the computation of the error degree variances for single vertical deflection components see Voigt and Denker (2014). Regarding the aspired GOCE spatial resolution of 100 km, corresponding to d/o 200, the cumulative errors are 2.2 cm, 0.59 mGal and 0.09″ for TIM5, while DIR5 provides 0.8 cm, 0.21 mGal and 0.03″ in terms of geoid heights, gravity anomalies and vertical deflections, respectively. These numbers represent an improvement of approx. 40% to the predecessor models. However, the formal estimates of TIM5 and DIR5 differ by a factor of 3 and one of the key questions is how realistic these formal estimates really are and whether they should be calibrated, e.g. with regard to proper weighting procedures in the context of high resolution geoid determination based on GGMs. In Bruinsma et al. (2014), a calibration factor of 2 is stated for DIR5 based on the comparisons with GPS/levelling data in Germany (see also sections 3 and 5).

Some remarks on the error degree variance approach are appropriate. First, the error degree variances do not take into account correlations among the coefficients. Second, the handling

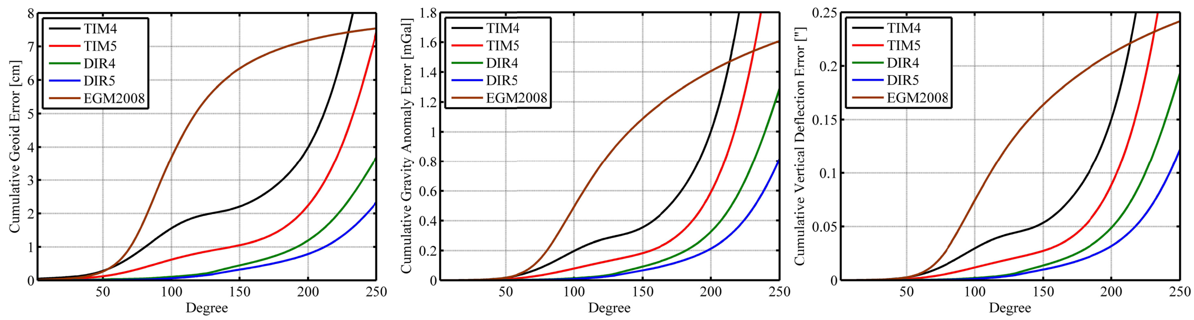

Figure 1 Cumulative errors of current global gravity field models in terms of geoid (left), gravity anomalies (middle) and vertical deflections (right)

Table 1 Cumulative errors for various GGMs up to d/o 200 and 2159, respectively, in terms of geoid (ζ), gravity anomalies (Δg), and vertical deflections (ξ,η)

GGM	Data	d/o	ζ [cm]	Δg [mGal]	ξ,η ["]
TIM4	GOCE (27 months)	2-200	4.0	1.00	0.15
TIM5	GOCE (4 years)	2-200	2.2	0.59	0.09
DIR4	GOCE (27 months) , GRACE, SLR	2-200	1.2	0.32	0.05
DIR5	GOCE (4 years), GRACE, SLR	2-200	0.8	0.21	0.03
EGM2008	GRACE, terrestrial and altimetric data	2-200	7.2	1.40	0.21
EGM2008	GRACE, terrestrial and altimetric data	2-2159	8.2	4.22	0.63

of the polar gap problem in the GOCE processing, affecting the zonal and near-zonal coefficients, has also an impact on the supplied spherical harmonic coefficient standard deviations, particularly on the GOCE only TIM models. And third, the uncertainties of the GOCE GGMs depend on the geographical latitude. Hence, latitude-dependent standard deviations from the propagation of the full variance covariance matrix of the spherical harmonic coefficients should be preferred. Brockmann et al. (2014) specify a global mean geoid accuracy of 2.4 cm for TIM5 up to d/o 200. For latitudes between 47° and 55° covering Germany, the corresponding values are 1.5 cm and 1.3 cm, respectively, from the variance covariance propagation. This is a decrease by a factor of 1.6 compared to the cumulative geoid error of 2.2 cm in Table 1.

3 Comparisons of the External Data Sets in Germany

For the evaluation of the GOCE GGMs, high precision gravity field data sets in Germany are used. Besides the ultra-high degree spherical harmonic model EGM2008 complete to d/o 2159 and additional coefficients up to degree 2190 (Pavlis et al. 2012) based on ITG-Grace03s as well as high precision terrestrial and altimetric data (σ=0.1-1.0 mGal in Germany), a data set of 954 GPS/levelling points (σ=1-2 cm) exist; further details on the terrestrial data sets can be found e.g. in Ihde et al. (2010). In addition, a unique data set of 370 astrogeodetic vertical deflections (σ=0.08″) along two 600 km long profiles are available, traversing through a subset of 46 GPS/levelling points in total. These are 29 additional vertical deflection stations in comparison to previous studies, e.g. Voigt and Denker (2014a) resulting from the connection to the future GPS/levelling data set from the renewal of the German height reference network (2006-2012). The method of astronomical topographic levelling (astro levelling) is used to compute quasigeoid heights with regard to a vertical datum fixed in the starting points of the profiles. For details on this data set and comprehensive studies on astrogeodetic gravity field modelling see Voigt (2013) with the first results published in Voigt et al. (2009).

One of the main advantages of the three data sets, i.e. EGM2008, GPS/levelling and the astrogeodetic vertical deflections, is that they are fully independent and thus provide valuable uncertainty estimates by applying a cross-validation in terms of quasigeoid heights. However, the different spectral characteristics of the terrestrial data and EGM2008, which is limited to the maximum degree and order, should be considered. The high frequency signal beyond degree 2190 and order 2159 are modelled by detailed topographic information from SRTM3 v4.1 (Jarvis et al. 2008) and the residual terrain model (RTM) approach with regard to Earth2012 (Hirt et al. 2012), a spherical harmonic model of the Earth's topography (with water) used as reference topography (see also Gruber et al. 2011).

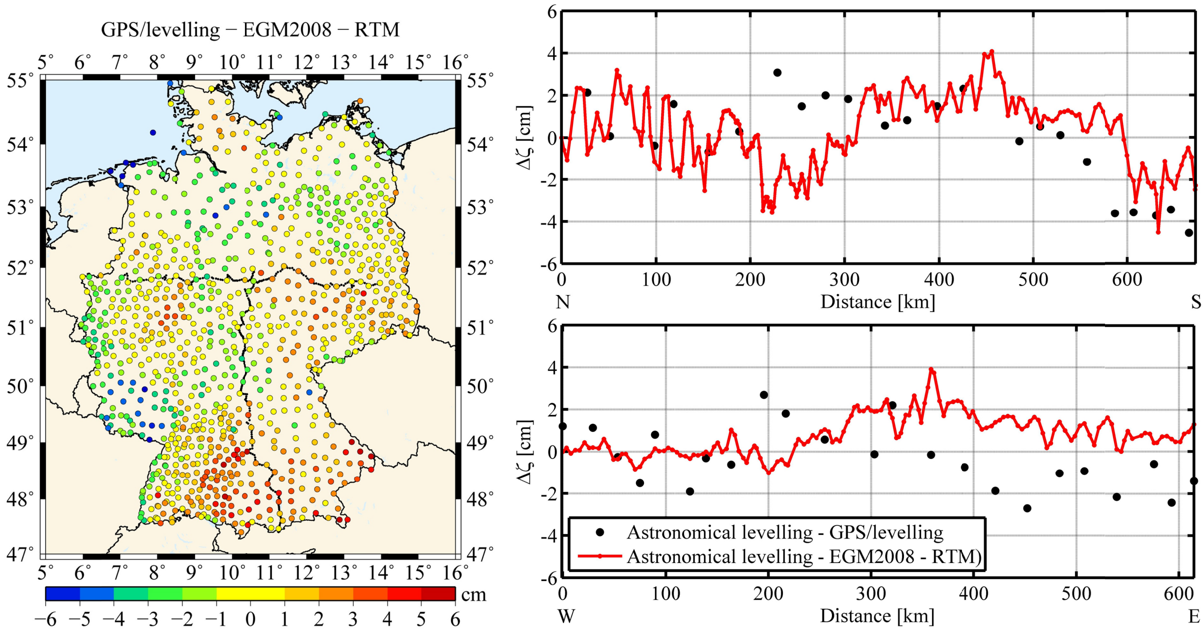

Figure 2 Differences between GPS/levelling data and quasigeoid heights from EGM2008+RTM (left) with the astrogeodetic vertical deflection stations (black dots); quasigeoid differences between astro levelling and GPS/levelling as well as EGM2008+RTM along the North-South (top right) and the West-East profile (bottom right)

Figure 2 shows the quasigeoid differences between the three data sets in Germany without applying any fitting or correction surface. The comparison between GPS/levelling and EGM2008 augmented by RTM effects on 954 stations in Germany yield an RMS difference of 2.1 cm with a range from -7.3 cm to 5.8 cm. This number represents an improvement of approx. 25% to the RMS difference of 2.8 cm specified in previous studies, where the reference topography was built by a moving average filter in conjunction with a spatial low pass filter, e.g. Voigt and Denker (2014a).

In addition, the astrogeodetic profile lines are shown in Figure 2 (left) with RMS differences of 2.2 cm and 1.6 cm between GPS/levelling and EGM2008+RTM along the North-South profile (from North N to South S) and the West-East profile (from West W to East E). This is in good agreement with the slightly larger value of 2.1 cm on the GPS/levelling stations, implying that the subset of 46 points along the profiles is quite representative for the whole data set of 954 points in Germany. The differences of the quasigeoid heights between astro levelling and GPS/levelling as well as EGM2008+RTM (Figure 2, right) are in the range of ± 4 cm with RMS differences of 2.2 cm and 1.7 cm along the North-South profile as well as 1.5 cm and 1.3 cm, respectively, along the West-East profile. The consistently larger differences along the North-South profile can be attributed to the rugged terrain.

The uncertainties of the three data sets along the profiles can be determined from the RMS differences between them. In Table 2, the merged RMS differences between two data sets in each case along both profiles are shown as off-diagonal elements. On the main diagonal, the uncertainties of the individual data sets resulting from the RMS differences are displayed with 1.0, 1.6 and 1.1 cm for the quasigeoid heights from astro levelling, GPS/levelling and EGM2008+RTM, respectively. For example, the merged RMS difference of 1.9 cm between GPS/levelling and EGM2008+RTM along both profiles can be explained by the uncertainties of the data sets with $\sqrt{1.6^2+1.1^2}$. Hence, the three data sets are well suited for the validation of the GOCE GGMs with aspired (and stated) commission errors of 1-2 cm at a resolution of 100 km. Gruber and Rummel (2014) provide an RMS difference of 2.0 cm between the GPS/levelling data and EGM2008+RTM in Germany after applying a planar fit, while esti-

Table 2 RMS differences (off-diagonal) between the data sets and uncertainties (main diagonal) of the data sets in terms of quasigeoid heights along the profiles in Germany [cm]

Data Set	Astro levelling	GPS/levelling	EGM2008+RTM
Astro levelling	1.0	1.8	1.4
GPS/levelling		1.6	1.9
EGM2008+RTM			1.1

mating the uncertainty of the (planarly fitted) GPS/levelling data to 2.1 cm, which seems rather pessimistic as also EGM2008+RTM uncertainties contribute to the differences.

4 Comparisons between the External Data Sets and the GOCE Gravity Field Models in Germany

The three data sets introduced in the last section are used for the evaluation of the GOCE GGMs. In order to consider the spectral characteristics of the GGM and the terrestrial data, three different methods are applied for the comparisons of EGM2008, GPS/levelling data and the astrogeodetic vertical deflections with the GOCE GGMs. It should be noted that the GPS/levelling data and the astrogeodetic vertical deflections are fully independent of all GOCE GGMs, while EGM2008 is also independent of the TIM models. However, the long-wavelength signals of EGM2008 and the DIR models are correlated by the GRACE data included in both solutions.

The first method is the direct comparison of quasigeoid heights, gravity anomalies and vertical deflections from EGM2008 up to a certain maximum degree and order with corresponding functionals of the GOCE GGMs. This method has the advantage that signals of the same spectral band can be compared directly. The spherical harmonic syntheses are carried out on a 1′x1′ grid over Germany containing 316,800 grid points in total. Figure 3 shows the differences between the quasigeoid heights from EGM2008 and the GOCE GGMs up to d/o 200. The RMS differences, also displayed in Table 3, decrease from 3.4 to 2.7 cm (TIM4 vs. TIM5) and from 3.8 to 2.7 cm (DIR4 vs. DIR5) with the fifth release GGMs performing quite similar but DIR5 being slightly superior to TIM5, probably because of the GRACE data included. The typical difference patterns significantly decrease from release 4 to 5, indicating the increasing accuracy of the GOCE GGMs in areas with high quality terrestrial gravity data included in EGM2008. Corresponding differences are computed as well in terms of gravity anomalies and vertical deflection components with RMS values of 0.72 mGal and 0.11″, respectively, for both TIM5 and DIR5 showing again an improvement of approx. 30% against their predecessors. These results already demonstrate the high accuracy level of the fifth release GOCE GGMs with the aim of providing gravity anomalies with an uncertainty of 1 mGal up to d/o 200 well achieved.

The second method applies the GPS/levelling data in Germany. For a thorough validation of the GOCE GGMs, the different spectral characteristics of both data sets are considered, i.e., the high-frequency signals beyond the maximum d/o 200, which are not included in the GOCE GGMs but represent a significant portion of the entire spectrum, are taken into account. For this purpose, the high frequency signals above d/o 200 are modelled by EGM2008 up to degree 2190 and order 2159, while beyond this degree RTM effects are utilized (see section 3). The differences between the GPS/levelling data and quasigeoid heights from GOCE GGMs augmented by EGM2008 signals and RTM effects are depicted in Figure 4. Again, the RMS differences (also shown in Table 3) decrease significantly from 3.8 to 3.1 cm (TIM4 vs. TIM5) and from 4.1 to 3.0 cm (DIR4 vs. DIR5). These differences are strongly correlated with the differences shown in Figure 3 to approx. 85% for the release 4 and 75% for the re-

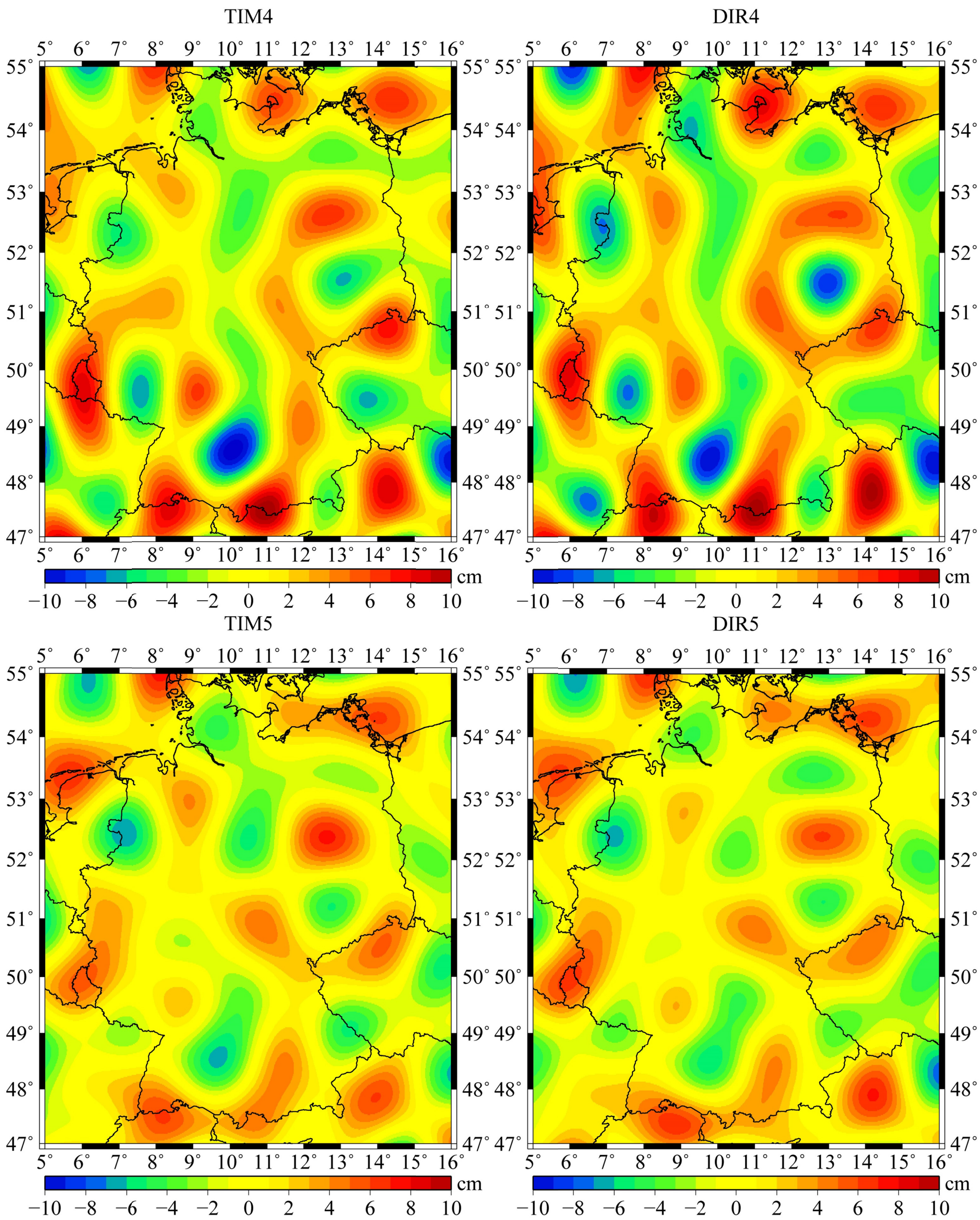

Figure 3 Differences between quasigeoid heights from EGM2008 and GOCE GGMs up to d/o 200

lease 5 GGMs. This implies two aspects. First, the uncertainties of the GOCE GGMs contribute most to the differences and, second, both validation methods are consistent. However, the RMS quasigeoid differences between GPS/levelling and the GOCE GGMs are slightly larger than the corresponding values between EGM2008 and the GOCE GGMs. This shows that EGM2008 up to d/o 200 is somewhat superior to the sum of uncertainties from GPS/levelling, EGM2008 between d/o 201 and 2159 and RTM effects. In addition, uncertainties arising from the spectral break between the GOCE GGMs and EGM2008 at d/o 200 could have an effect on this comparison. This can be reduced by applying a proper weighting function, e.g. by using the error degree variances of the GOCE GGMs and EGM2008 within a certain spectral band around the maximum degree of the GOCE GGMs. However, this is not done here in order to use identical GOCE signals in the comparisons.

Within the third method, the astrogeodetic vertical deflections along the profiles are compared with corresponding values from the GOCE GGMs. On the one hand, vertical deflections are quite suitable for the validation of the higher degree signals, where quasigeoid heights are not that sensitive. But on the other hand, the comparison on the basis of vertical deflections lacks

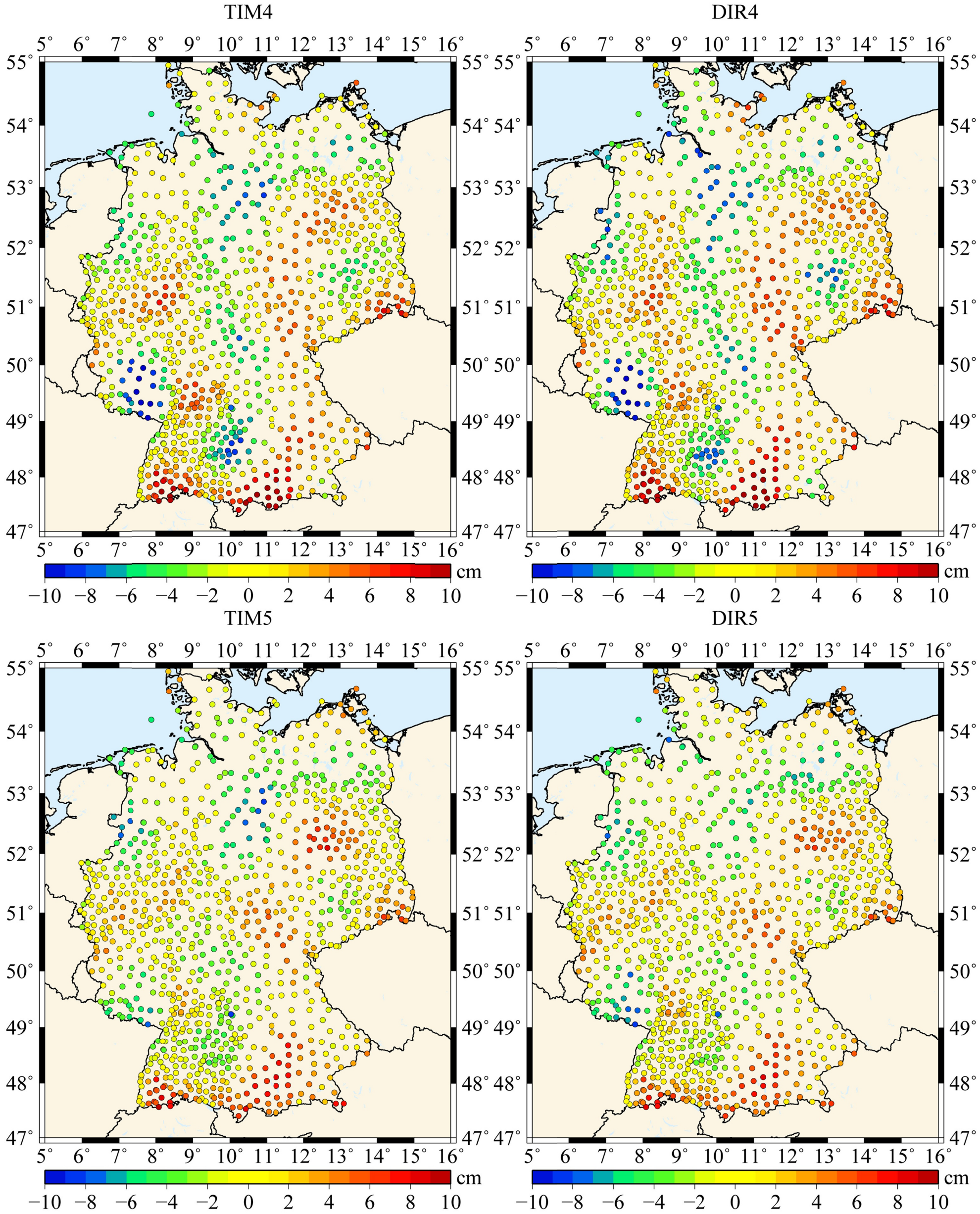

Figure 4 Differences between GPS/levelling data and quasigeoid heights from GOCE GGMs up to d/o 200 augmented by EGM2008 signals up to d/o 2159 and RTM effects

Table 3 RMS differences between external data sets and GOCE GGMs in terms of quasigeoid heights (ζ), gravity anomalies (Δg), and vertical deflections (ξ,η)

	EGM2008			GPS/levelling	Astrogeodetic vertical deflections
GGM	ζ [cm]	Δg [mGal]	ξ,η ["]	ζ [cm]	ξ,η ["]
TIM4	3.4	0.94	0.15	3.8	0.30
TIM5	2.7	0.72	0.11	3.1	0.29
DIR4	3.8	1.06	0.16	4.1	0.31
DIR5	2.7	0.72	0.11	3.0	0.28
EGM2008+RTM	-	-	-	2.1	0.27

of the strong influence of high frequency effects around the observation points, e.g. the vertical deflection signal beyond d/o 2159 is 1.65″ while it is just 2.3 cm for quasigeoid heights (see Voigt and Denker 2014). In principle, the validation is based on the same strategy as the second method but with an additional spatial Gaussian filtering of 9 km width (corresponding to d/o 2159) in order to further reduce unmodelled high-frequency effects; see also Voigt and Denker (2014). The RMS differences of these comparisons are shown in Table 3, where the statistics of the individual components along the two profiles are already merged, only slightly decreasing from 0.30 to 0.29″ (TIM4 vs. TIM5) and from 0.31 to 0.28″ (DIR4 vs. DIR5). This result clearly indicates the impact of unmodelled high frequency effects coming mostly from density variations not captured by the RTM effects.

5 External Uncertainty Estimates of the Global Gravity Field Models

The uncertainties of the GOCE GGMs are estimated externally on the basis of the comparisons carried out in the last section. For this purpose, the RMS differences of Table 3 are introduced as observations within a common least squares adjustment. The general relation between the observation vector **l** and the vector **x** of the unknown parameters is given by **l**+**v**=**Ax** with the residuals **v** and the design matrix **A** aiming at the minimization of $\mathbf{v}^T\mathbf{P}\mathbf{v}$, where **P** is the weighting matrix. The unknown parameters are the scaling factors of the cumulative error degree variances of the GOCE GGMs TIM4 and TIM5 as well as DIR4 and DIR5 up to d/o 200. In addition, a scaling factor of the cumulative error degree variances of EGM2008 and another for an a priori uncertainty of the RTM effects are estimated as well as the uncertainty of the GPS/levelling data, as the RMS differences result from the contributions of all data sets involved in the comparisons. A redundancy of 15 results follows from the 22 RMS differences and the 7 unknown parameters. The uncertainty of the astrogeodetic vertical deflections is known to be σ_{Astro}=0.08″ from the comparisons with gravimetric vertical deflections in flat areas (Voigt 2013) as well as from repeated observations (Hirt and Seeber 2008) and set as a constant.

The relations between the observations and the unknown parameters are shown in order to set up the design matrix **A**. The first method of section 4 gives RMS differences between EGM2008 and the GOCE GGMs in terms of quasigeoid heights, gravity anomalies and vertical deflections up to d/o 200, which can be explained by

$$\mathrm{RMS(EGM2008\text{-}GOCE)}=(m_{GOCE}\cdot\sigma^2_{GOCE}+m_{EGM2008}\cdot\sigma^2_{EGM2008})^{1/2} \qquad (1)$$

with the unknown scaling factors m_{GOCE} and $m_{EGM2008}$ of the cumulative error degree variances σ^2_{GOCE} and $\sigma^2_{EGM2008}$ for the GOCE GGMs and EGM2008 up to d/o 200, respectively. The positive square roots of the cumulative error degree variances, referred to as cumulative errors, are shown in Table 1. From the second method, the RMS differences between GPS/levelling (GPS/lev) and the GOCE GGM quasigeoid heights follow from

$$\mathrm{RMS(GPS/lev\text{-}GOCE)}=(m_{GOCE}\cdot\sigma^2_{GOCE}+m_{EGM2008}\cdot\sigma^2_{EGM2008}+m_{RTM}\cdot\sigma^2_{RTM}+\sigma^2_{GPS/lev})^{1/2}, \qquad (2)$$

where $\sigma^2_{EGM2008}$ herein is the cumulative error degree variance between of EGM2008 between d/o 201 and 2159. Two additional unknown scaling factor are introduced, i.e. the scaling factor m_{RTM} for the RTM effects in terms of quasigeoid heights with respect to an a priori uncertainty σ_{RTM} (see below) and the uncertainty of the GPS/levelling data $\sigma_{GPS/lev}$. The RMS differences between the astrogeodetic vertical deflections (Astro) and corresponding values from the GOCE GGMs within the third method results from

$$\mathrm{RMS(Astro\text{-}GOCE)}=(m_{GOCE}\cdot\sigma^2_{GOCE}+m_{EGM2008}\cdot\sigma^2_{EGM2008}+m_{RTM}\cdot\sigma^2_{RTM}+\sigma^2_{Astro})^{1/2} \qquad (3)$$

Table 4 Structure of the design matrix **A** with the non-zero elements symbolized by *

Method	Functional		m_{GOCE}	$m_{EGM2008}$	m_{RTM}	$\sigma_{GPS/lev}$
EGM2008-GOCE	ζ		*	*	0	0
EGM2008-GOCE	Δg		*	*	0	0
EGM2008-GOCE	ξ,η		*	*	0	0
GPS/lev-GOCE-EGM2008-RTM	ζ	**A**=	*	*	*	*
Astro-GOCE-EGM2008-RTM	ξ,η		*	*	*	0
GPS/lev-EGM2008-RTM	ζ		0	*	*	*
Astro-EGM2008-RTM	ξ,η		0	*	*	0

with the unknown scaling factors of the cumulative error degree variances and the RTM effects of (2), but now in terms of vertical deflections. A constant a priori value of σ_{RTM}=0.20″ is chosen for the uncertainty of the RTM effects in terms of vertical deflections with regard to the RMS differences in Table 3. This corresponds to σ_{RTM}=0.07 cm for the RTM effects in terms of quasigeoid heights according to the degree variance model of Tscherning and Rapp (1974). In Table 4, the principal structure of the design matrix **A** is given showing the individual contributions of the comparisons with regard to the estimated parameters. It should be noted that this matrix is of full rank. For the set-up of the stochastic model, uncorrelated observations are assumed with empirically determined a priori standard deviations for the RMS differences of $\sigma_{RMS(\zeta)}$=0.05 cm, $\sigma_{RMS(\Delta g)}$=0.1 mGal and $\sigma_{RMS(\xi,\eta)}$=0.01″ in terms of quasigeoid heights, gravity anomalies and vertical deflections, respectively. However, the ratios of the a priori standard deviations mostly affect the estimated standard deviations of the parameters, but not the parameters themselves. The relations between the observations and the unknown parameters (1) to (3) are non-linear. Hence, they have to be linearized at approximate values for each iteration step.

The parameters estimated within the least squares estimation are shown in Table 5 together with their standard deviations. Applying the square roots of the estimated scaling factors to the cumulative errors of the GGMs in Table 1 and to the a priori standard deviations of the RTM effects leads to the estimated uncertainties shown in Table 6. The commission error of EGM2008 is 1.0 cm, 0.20 mGal and 0.03″ up to d/o 200 in terms of quasigeoid heights, gravity anomalies and vertical deflections, respectively. The uncertainty of 0.1 cm for the RTM effects in terms of quasigeoid heights is quite negligible, while it amounts to 0.24″ in terms of vertical deflections. In total, this method proposes a cumulative error of 1.2 cm for EGM2008+RTM and the full spectral band in terms of quasigeoid heights, while the uncertainty of the GPS/levelling data is 1.7 cm. This is fully compatible with the numbers of 1.1 cm and 1.6 cm, respectively, determined along the profiles (section 3). The commission errors of the release 5 GOCE GGMs up to d/o 200 are estimated to 2.5 cm for TIM5 and 2.4 cm for DIR5 as well as 0.66 mGal and 0.10″ for both GGMs. The slightly lower value of DIR5 in terms of quasigeoid heights is probably related to the GRACE data included. The estimated

Table 5 Estimated parameters and standard deviations from the least squares adjustment

Parameters	
m_{TIM4}	0.69 ± 0.02
m_{TIM5}	1.26 ± 0.04
m_{DIR4}	9.41 ± 0.19
m_{DIR5}	9.64 ± 0.32
$m_{EGM2008}$	0.02 ± 0.00
m_{RTM}	1.46 ± 0.06
$\sigma_{GPS/lev}$ [cm]	1.7 ± 0.04

Table 6 Estimated uncertainties of all terms involved in the comparisons and standard deviations in terms of quasigeoid heights (ζ), gravity anomalies (Δg), and vertical deflections (ξ,η) from the least squares adjustment

Data Set	d/o	ζ [cm]	Δg [mGal]	ξ,η ["]
TIM4	2-200	3.3 ± 0.04	0.83 ± 0.009	0.13 ± 0.001
TIM5	2-200	2.5 ± 0.04	0.66 ± 0.011	0.10 ± 0.002
DIR4	2-200	3.7 ± 0.04	0.99 ± 0.010	0.15 ± 0.001
DIR5	2-200	2.4 ± 0.04	0.66 ± 0.011	0.10 ± 0.002
EGM2008	2-200	1.0 ± 0.07	0.20 ± 0.013	0.03 ± 0.002
EGM2008	2-2159	1.2 ± 0.08	-	0.09 ± 0.006
RTM	>2159	0.1 ± 0.00	-	0.24 ± 0.005
GPS/levelling	2-∞	1.7 ± 0.04	-	-
Astrogeodetic vertical deflections	2-∞	-	-	0.08 (const)

standard deviations of the commission errors are on the level of <0.1 cm, 0.01 mGal and <0.01″, respectively, for quasigeoid heights, gravity anomalies and vertical deflections confirming a high consistency of the applied data sets and the validation methods.

The estimated parameters shown in Table 5 suggest that the cumulative errors of the DIR models from Table 1 should be factorized by 3.1 for the latitudes of Germany, while the corresponding values for the TIM models seem to be quite reasonable with factors of 0.8 and 1.1 (TIM4 and TIM5, respectively). However, a higher factor of 1.8 for the commission error results from the propagation of the TIM5 full variance covariance matrix of 1.4 cm up to d/o 200. The estimates are somewhat pessimistic for the GOCE GGMs with regard to a calibration factor of 2 for DIR5 (Bruinsma et al. 2014) and an overall performance of 1.8 cm or better for both release 5 GGMs up to d/o 200 (Gruber and Rummel 2014).

This method can be used in all countries with GPS/levelling data sets, e.g. the countries of the EUVN-DA (European Vertical Reference Network - Densification Action), provided that no systematic errors are present in the terrestrial data sets. For the case that only GPS/levelling data and no astrogeodetic vertical deflections exist or should be used, it has to be noted that the uncertainties of GPS/levelling and the RTM effects should be estimated together. While the uncertainties of EGM2008 and GPS/levelling can strongly vary between countries, the accuracy of the GOCE GGMs shall only show a latitude dependency. However, it should be further noted that such comparisons represent just a small spot check with regard to the aspired GOCE resolution of 100 km and that the quality of the results depends on the size of the test area.

6 Summary and Conclusions

For the validation of the fourth and fifth release GOCE GGM and their internal uncertainty estimates, comparisons have been carried out with external data sets in Germany, i.e. EGM2008, GPS/levelling data and astrogeodetic vertical deflections. While the gravity field functionals from EGM2008 and the GOCE GGMs are compared directly up to d/o 200, the comparisons between the GOCE GGMs and the GPS/levelling data as well as the astrogeodetic vertical deflections presuppose the consideration of the different spectral characteristics. This is done by modelling the high frequency signals above d/o 200 by EGM2008 as well as detailed topographic information and the RTM approach.

The RMS differences of the three comparisons reveal a high consistency, and are introduced as observations within a least squares adjustment for the estimation of the uncertainties of all terms involved in the comparisons. While the uncertainties of EGM2008 quasigeoid heights and GPS/levelling in Germany are estimated to 1.2 and 1.7 cm, respectively, the commission errors of the GOCE GGMs up to d/o 200 are 2.4 and 2.5 cm for TIM5 and DIR5, respectively, as well as 0.66 mGal and 0.10″ for both release 5 models in terms of quasigeoid heights, gravity anomalies and vertical deflections. These figures are fully compatible with the formal error degree variances of TIM5, while a factor of 3 should be applied to the formal cumulative errors of DIR5.

The validation results can be used for the calibration of the formal error degree variances of the GGMs and be used for setting up a realistic weighting procedure in the context of high resolution quasigeoid modelling. The release 5 GOCE GGMs based on observation data from the whole mission lifetime do not fully reach the level of EGM2008 up to d/o 200 in Germany with high quality terrestrial gravity data included in the solution. But nevertheless, GOCE could come into play in countries with high quality terrestrial data, as the major uncertainty contributions of high resolution quasigeoid solutions come from the spectral band between d/o 50 and 360 (Denker 2013).

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